

# ON SHAPE ISOMERS OF Pt–Pb ISOTOPES IN THE 4D FOURIER PARAMETRISATION\* \*\*

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Using the Fourier shape parametrisation of deformed nuclei developed by us recently, potential-energy landscapes for isotopes of nuclei between platinum ( $Z = 78$ ) and lead ( $Z = 82$ ) are analysed in a 4-dimensional deformation space, searching for local extrema, ridges and valleys. A certain number of yet unknown super- and hyper-deformed shape isomers in even–even Pt, Hg and Pb isotopes are predicted. Quadrupole moments in the relevant minima are evaluated. A nice agreement of the theoretical predictions with the experimental ground-state data for these quantities gives strong confidence on the quality of our results for the corresponding isomeric states.

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## 1. Introduction

Potential energy surfaces (PES) describe the energy of nuclei as a function of some deformation parameters. In order to exploit in an optimal way all the relevant deformation degrees of freedom, while keeping computation time within reasonable limits, necessitates a clever way of parametrising nuclear deformations, a shape parametrization that relies on only very few

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deformation parameters, but that allows, on the other hand, to control its convergence. Such an analytical form should, if possible, be applicable in addition to all nuclear systems throughout the periodic table. With the Fourier shape parametrization recently developed by us [1, 2], we are convinced to have such a tool at hand. We have, indeed, shown [2–4] that with only 4 deformation parameters, corresponding to elongation, left–right (reflection) asymmetry, neck formation, and non-axiality, we are able to describe or predict ground-state properties, as well as shape and fission isomeric states. We have even been able to predict the mass partitioning in the fission process at low excitation energy, and its evolution with proton and neutron number [2].

In what follows, we are going to exploit this powerful tool to scan these 4-dimensional deformation energy landscapes for a total of 66 isotopes of Pt, Hg and Pb nuclei in order to localize ground states, local extrema, ridges and valleys [4].

## 2. Results and discussion

The deformation-energy landscapes discussed below have been obtained within the macroscopic–microscopic model using the Lublin–Strasbourg Drop (LSD) [5] for the macroscopic part together with Strutinsky shell corrections [6] and BCS pairing correlations [7] for the quantum mechanical corrections, where a Yukawa-folded mean-field potential [8] has been used to determine the single-particle energy spectrum for protons and neutrons. For the description of nuclear deformations, the Fourier shape parametrization [1, 2] has been used, as already mentioned above, where it has been shown [3, 4] that with only 4 deformation parameters  $q_1$ – $q_4$  corresponding respectively to non-axiality, elongation, left–right asymmetry and neck degree of freedom, one is able to describe any (non-pathological) nuclear deformation from the oblate side up to the extreme deformations that occur in the fission process close to the scission configuration [4]. A more detailed description of the theoretical background of our calculations can be found *e.g.* in Ref. [9].

We start by showing in Fig. 1 (left column) the energies relative to the one of the (spherical) liquid-drop (LSD) minimum, for the ground state and the two lowest energy shape isomers for the Pt, Hg and Pb isotopic chains. Also shown (right column) are the charge quadrupole moments (absolute values) for the three isotopic chains, which agree very nicely with the available experimental ground-state data [10]. On both quantities (energies and quadrupole moments), the spherical  $N = 126$  shell closure is clearly visible. Notice, however, that for the Pt isotopic chain, no isotope has been synthesized beyond  $^{204}\text{Pt}$ , *i.e.* beyond the  $N = 126$  shell closure. Whereas at shell closure the isotopes in the three chains are evidently spherical, they

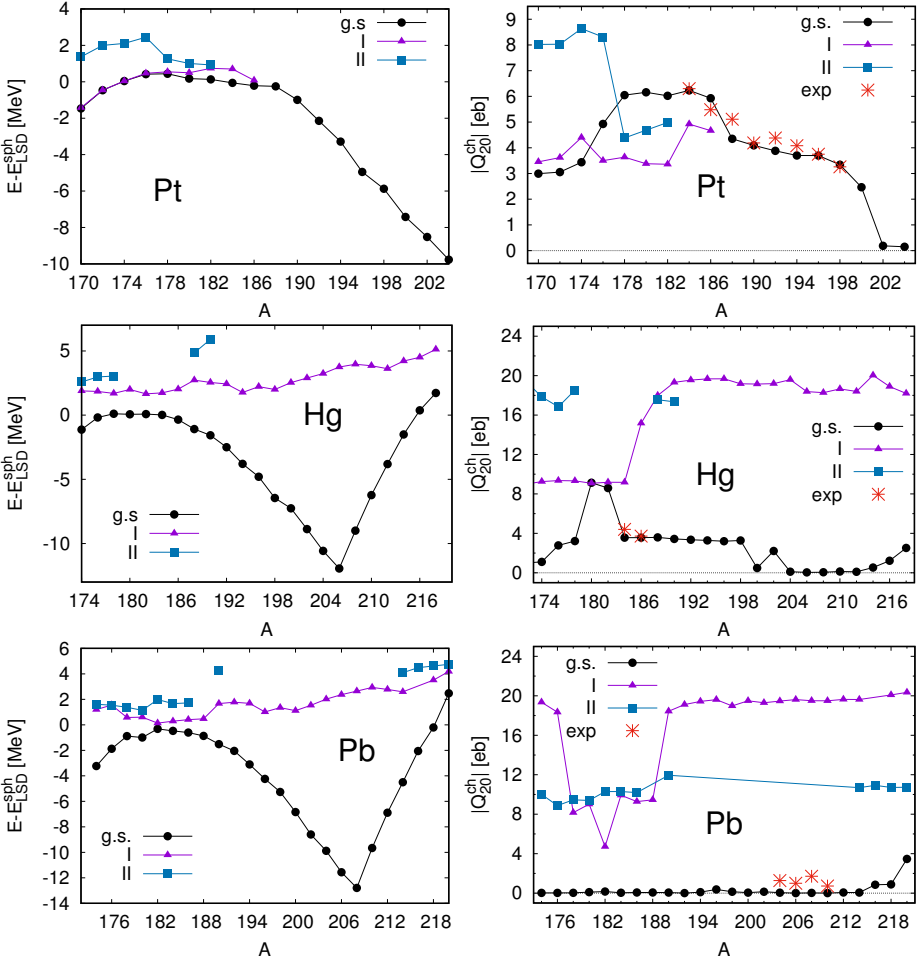


Fig. 1. Energy at the ground state and the first and second shape-isomers of Pt, Hg and Pb isotopes (from top to bottom) relative to the spherical liquid-drop (LSD) minimum (left column) and corresponding charge quadrupole moments  $|Q_{20}^{\text{ch}}|$  (right column) as compared to the experimental data [10].

can be either prolate (corresponding to positive  $q_2$  values) or oblate (negative  $q_2$  values) as one goes away from the  $N = 126$  magic number. This is demonstrated in Figs. 2, 3 and 4 where the deformation of the different isotopes in each of the three isotopic chains is indicated in the  $(q_2, q_4)$  deformation space, where, as mentioned above,  $q_4$  stands for the hexadecapole (neck) degree of freedom. It should be made clear that the energies of the nuclear ground state and the shape isomers in Fig. 1 are always obtained by a minimisation in the above-introduced 4-dimensional deformation space,

and that the ground-state deformations in the  $(q_2, q_4)$  space given in Fig. 2 result, as well, by a minimisation with respect to the two additional deformation parameters  $q_1$  and  $q_3$ . Since the different shape isomers could be

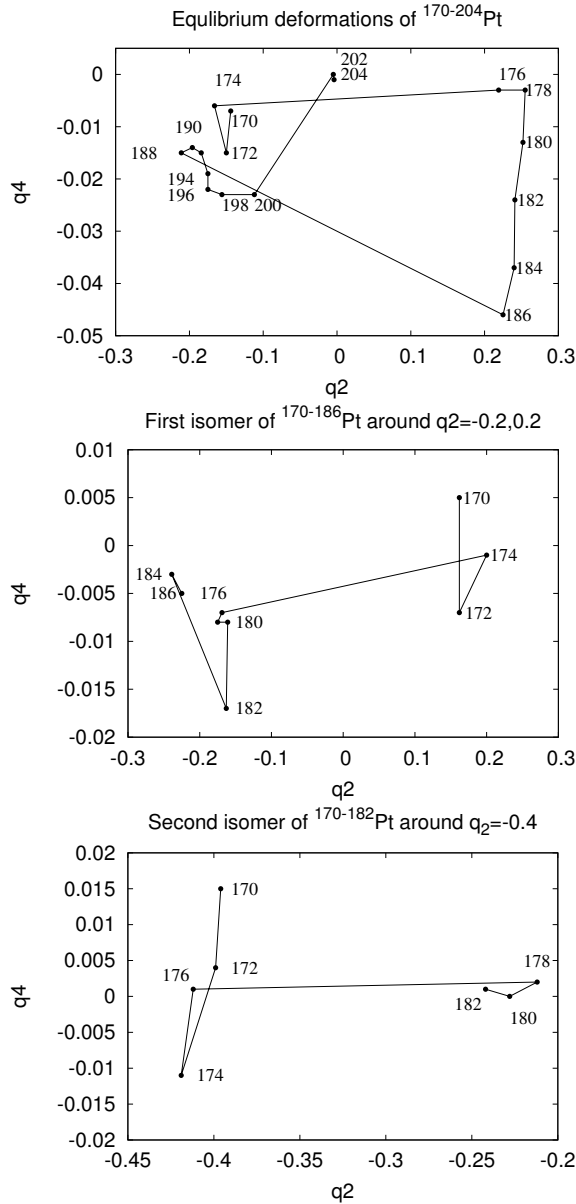


Fig. 2. Ground state, first and second isomer quadrupole  $q_2$ , and hexadecapole  $q_4$  Fourier deformations of Pt isotopes.

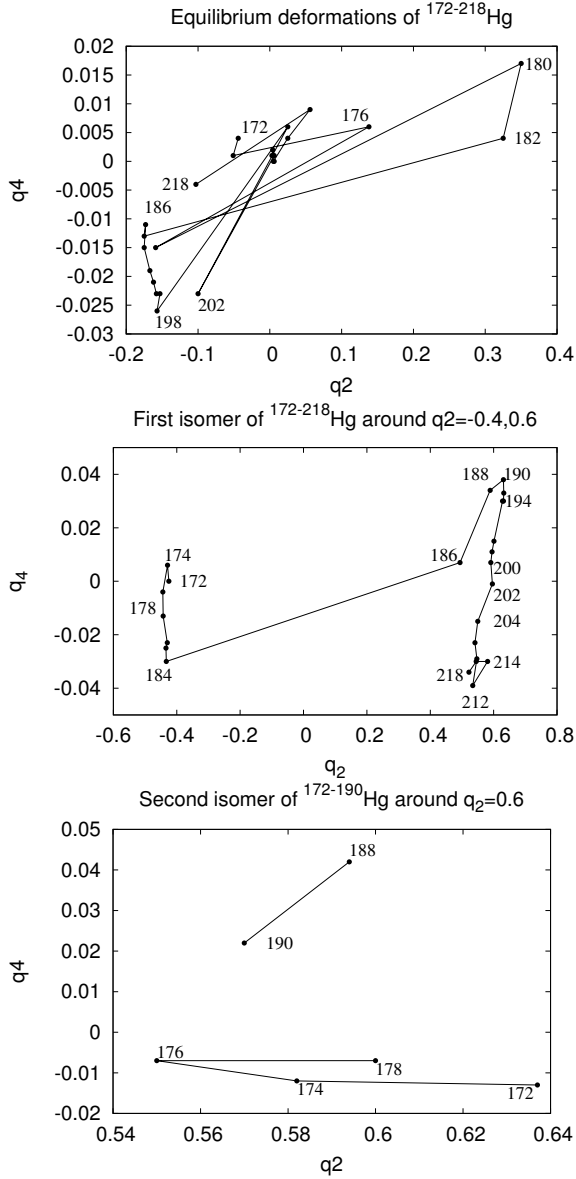


Fig. 3. Ground state, first and second isomer quadrupole  $q_2$ , and hexadecapole  $q_4$  Fourier deformations of Hg isotopes.

quite close in energy, as this is already seen in Fig. 1, it can well happen that shape isomer I (first in energy) and shape isomer II (second in energy) change roles, or that another shape isomer comes down in energy to take the place of either of the two. This would explain the sudden jumps that are ob-

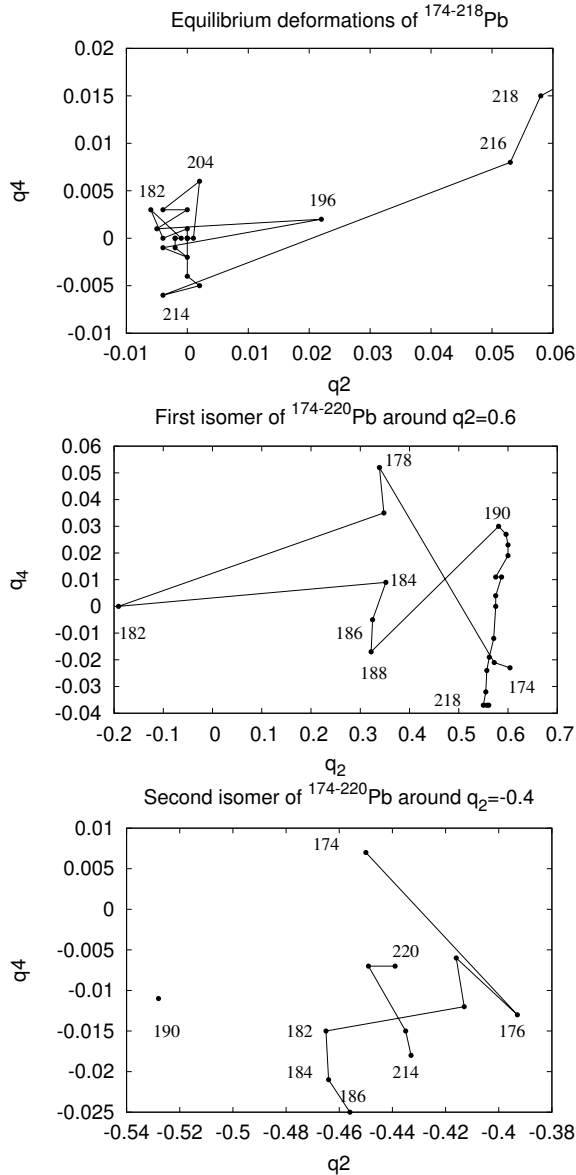


Fig. 4. Ground state, first and second isomer quadrupole  $q_2$ , and hexadecapole  $q_4$  Fourier deformations of Pb isotopes.

served both in the right column of Fig. 1 and in Fig. 2, like when going from  $^{174}\text{Pt}$  to  $^{176}\text{Pt}$  where, obviously ground state (oblate in  $^{174}\text{Pt}$  but prolate in  $^{176}\text{Pt}$ ) and shape isomer I (prolate in  $^{174}\text{Pt}$  but oblate in  $^{176}\text{Pt}$ ) change roles.

Due to the variation of shell and pairing effects, when going from one isotope to the next, it can also easily happen that local minima in the deformation energy landscape appear or disappear. In general, we have observed that the energy landscape is rather soft in the lighter isotopes of the nuclei studied here, while much more pronounced minima are observed in the heavier ones. For the most deformed of these, it is certainly worth to study them in the context of the debated existence of a second very elongated minimum in this region of nuclei [11].

Till now, only the elongation  $q_2$  (quadrupole) and the neck  $q_4$  (hexadecapole) degrees of freedom of our 4-dimensional deformation space have been discussed and one necessarily will ask about the importance of left–right asymmetry and non-axiality. It turns out, however, that all investigated nuclei, without any exception, have been found to be left–right symmetric ( $q_3 = 0$ ). All lead isotopes are, in addition, found to be axially symmetric. When Pt and Hg isotopes are concerned, some of them exhibit non-vanishing  $q_1$  values and, therefore, have some non-axial deformation. A close analysis shows, however, that this non-axiality is, indeed, extremely small, as well in the ground-state as in the shape-isomeric states. Translated into a  $(\beta, \gamma)$  deformation space (see *e.g.* Ref. [9] for a comparison of the  $(q_2, q_1)$  and the  $(\beta, \gamma)$  deformation spaces), the observed here non-axiality deformation parameters  $q_1$  would correspond to a non-axiality parameters  $\gamma$  only a couple of degrees away from either the prolate or the oblate axis, so that one could safely say that also these isotopes are practically axially symmetric.

### 3. Summary

Potential-energy landscapes have been computed within the macroscopic–microscopic approach and the new Fourier shape parametrization for 66 even–even isotopes of the Pt, Hg and Pb nuclei, locating local minima in the 4-dimensional deformation space. Ground state and the two first isomeric states closest in energy to it are identified. We have shown that a clever choice of the Fourier deformation parameters [2] yields, indeed, a very rapidly converging expression for the nuclear deformation. We have thus identified several shape isomeric states and their evolution with  $Z$  and  $N$  that, according to our preliminary investigation, match the experimental data where available [12]. Predictions for moments of inertia and rotational energies in these local minima are currently determined and will be presented in a forthcoming publication. Our present study should be understood as a pilot investigation, aimed at demonstrating the potential power of our approach. More detailed quantitative investigations are in progress.

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