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Isospin dependence of proton and neutron radii within relativistic mean field theory ★

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Abstract

The proton and neutron radii of even–even β-stable nuclei with $A \geqslant 40$ and a few chains of isotopes with $Z = 50, 56, 82, 94$ protons and isotones with $N = 50, 82, 126$ neutrons are analyzed. The average isospin dependence of the radii evaluated within the relativistic mean field theory is studied. A simple, phenomenological formula for neutron radii is proposed. © 1998 Published by Elsevier Science B.V.

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1. Introduction

The nuclear radii are ones of the crucial quantities good for testing of every theoretical model of nucleus. The radii are measured with a high accuracy for the charge density distributions [1–3] and less precise for the neutron ones [4]. The data concerning nuclear sizes and shapes are already known for many nuclei, especially for those close to the β stability line, but also more exotic nuclei are explored extensively by experimentalists at present. It would be worthwhile to know what the relativistic mean field theory (RMFT) predicts for such nuclei and to approximate the results in terms of a simple, practical formulae for radii.

The nuclear radii depend chiefly on the number of nucleons (A) [5]

$$R_0 = r_0 A^{1/3}, \qquad (1)$$

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where the radius constant $r_0 \approx 1.2$ fm. It is the result of saturation property of nuclear forces which is manifested in the experimental fact that the volume of nucleus is roughly proportional to the mass number A. However, the formula (1) is not valid any more for nuclei in which numbers of protons (Z) and neutrons (N) differ significantly.

It was found in Ref. [6] that the isospin dependent formula for the nuclear charge radius constant

$$r_0^{\rm ch} = 1.25 \left(1 - 0.2\frac{N-Z}{A}\right) \text{ fm}, \tag{2}$$

describes much better than Eq. (1) the experimentally known charge mean square radii of even-even nuclei with $A \geqslant 60$. The formula (2) was obtained assuming the uniform charge distribution within deformed nucleus. In such an approximation the root mean square radius (RMSR) of deformed nucleus is given by the following formula

$$\langle r^2 \rangle^{1/2} = \sqrt{\tfrac{3}{5}} R_0 g(\varepsilon, \varepsilon_4), \tag{3}$$

where the function g describes the dependence of the mean square radius on deformation. The equilibrium deformations of nuclei were taken from Ref. [7], where the two-dimensional (ε, ε_4) space of deformation parameters was used. The potential energy surfaces were calculated in Ref. [7] by the Strutinsky prescription with the zero-point energy correction terms according to the generator coordinate method [8].

Lateron it was found, after more broad calculations with the equilibrium deformations taken from Ref. [9], that the additional term $\sim 1/A$ in the formula (2) should appear when reproducing the mean square radii of all nuclei beginning from the lightest ones up to the actinides [10]

$$r_0^{\rm ch} = 1.240 \left(1 - 0.191\frac{N-Z}{A} + 1.646\frac{1}{A}\right) \text{ fm}. \tag{4}$$

A further development of the formula for nuclear radius was made in Refs. [11,12]. After extensive Hartree-Bogolubov calculations with various sets of Skyrme effective interactions for proton and neutron density distributions in a few spherical nuclei the authors of Ref. [11] postulate new terms proportional to $1/A^2$ in the formula for $r_0^{\rm ch}$.

The aim of our present work is to find simple formulae which approximate the results obtained for proton and neutron radii within the RMFT [13,14]. We investigate all even-even β stable nuclei as well as the isotopes and isotones corresponding to major magic Z and N numbers. When discussing the nuclides far from stability we change the mass number A up to the zero neutron or proton separation energy.

In Section 2 we describe briefly the theoretical model and its parameters. In Section 3 we analyze the results of the calculations and we write formulae for the radii of the proton and neutron distributions, which approximate the theoretical results of the RMFT. A simple, phenomenological formula for neutron radii is proposed as the result of the above analysis. The conclusions and further investigations perspectives are described in Section 4.

2. Theoretical model

The relativistic mean field theory [13] is a variational model based on a standard Lagrangian density [14]

$$\mathcal{L} = \bar{\psi}_i \left[\gamma^\mu \left(i\partial_\mu - g_\omega \omega_\mu - g_\rho \boldsymbol{\rho}_\mu \boldsymbol{\tau} - e\frac{1-\tau_3}{2} A_\mu \right) - M - g_\sigma \sigma \right] \psi_i$$
$$+ \tfrac{1}{2}(\partial\sigma)^2 - U(\sigma) - \tfrac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \tfrac{1}{2}m_\omega^2 \omega^2$$
$$- \tfrac{1}{4}\boldsymbol{R}_{\mu\nu}\boldsymbol{R}^{\mu\nu} + \tfrac{1}{2}m_\rho^2 \boldsymbol{\rho}^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \quad (5)$$

consisting of nucleon ψ, mesons $\sigma, \omega, \boldsymbol{\rho}$ and electromagnetic A fields. The σ mesons potential has been taken in the non-linear form:

$$U(\sigma) = \tfrac{1}{2}m_\sigma^2 \sigma^2 + \tfrac{1}{3}g_2 \sigma^3 + \tfrac{1}{4}g_3 \sigma^4\,. \quad (6)$$

It was found in Ref. [15] that the NL-3 [16] parameters set of the mean field Lagrangian (5) reproduced well binding energies, proton and neutron separation energies, electric quadrupole moments and radii of all nuclei along the whole β-stability line. The NL-3 parameters are:

– nucleon mass	$M = 939$ MeV,
– meson masses	$m_\sigma = 508.194$ MeV, $m_\omega = 782.501$ MeV,
	$m_\rho = 763$ MeV,
– meson coupling constants	$g_\sigma = 10.217$, $g_\omega = 12.868$, $g_\rho = 4.474$,
– σ meson field constants	$g_2 = -10.431$ fm^{-1}, $g_3 = -28.885$.

The relativistic Hartree equations are solved by iterations: one starts with some estimate of the meson and electromagnetic fields, then solving the Dirac equation one finds the Dirac spinors. They give the densities and currents as sources for the Klein–Gordon equations for the meson fields. After their solution the new set of the meson and electromagnetic fields is found as the starting point for the next iteration. When the self-consistency is achieved, Hartree-Bogolubov wave functions $\Psi^{p(n)}$ of protons and neutrons are used for evaluation of the mean values of operators in interest. We assume here the product of the BCS-type functions

$$|\Psi\rangle = \prod_{\nu>0} (u_\nu + v_\nu a_\nu^+ a_{-\nu}^+)|vac\rangle\,, \quad (7)$$

for protons and neutrons as the ground state wave function of nucleus.

To get the strength of the pairing interaction for nuclei from very different regions, we have taken simply the experimental energy gaps (Δ) and the lowest in energy Z (or N) single-particle levels when solving the BCS equations. Such a procedure is justified in our calculations because the quantities which we evaluate depend rather weakly on the choice of the pairing force. All Δ-s are extracted from the experimental mass differences taken from the Wapstra-Audi tables [17] but for the isotopes for which the experimental data do not exists we have used the estimates: $\Delta_{p(n)} = 12/\sqrt{A}$ MeV.

The monopole moments of protons and neutrons distributions are

$$Q_0^{\mathrm{p(n)}} = \langle\Psi|\sum_\nu r_\nu^2|\Psi\rangle^{\mathrm{p(n)}} . \tag{8}$$

The mean square radii (MSR) are defined

$$\langle r^2\rangle_{\mathrm{p}} = \frac{Q_0^p}{Z} , \qquad \langle r^2\rangle_{\mathrm{n}} = \frac{Q_0^n}{N} . \tag{9}$$

and the root mean square radii (RMSR) are

$$r_{p(n)} = \sqrt{\langle r^2\rangle_{p(n)}} = \sqrt{\tfrac{3}{5}} R_{p(n)} . \tag{10}$$

In Eqs. (9) and (10) we have neglected corrections originating from the center of mass motion. For heavier nuclei which we discuss these corrections are small. The quadrupole moments of proton and neutron distributions are given by

$$Q_2^{\mathrm{p(n)}} = \left\langle \Psi \left| \sum_\nu 2r_\nu^2 P_2(\cos\vartheta_\nu) \right| \Psi \right\rangle^{\mathrm{p(n)}} , \tag{11}$$

where P_2 is the Legendre polynomial of the order 2. The quadrupole deformation parameter of proton or neutron distribution is approximately equal to

$$\beta_2^{\mathrm{p(n)}} \approx \sqrt{\frac{4\pi}{5}} \frac{Q_2^{\mathrm{p(n)}}}{Q_0^{\mathrm{p(n)}}} . \tag{12}$$

We have evaluated also the binding energies of nuclei, the reduced electric quadrupole transition probabilities, the proton and neutron separation energies. The results agree with the experimental data taken from Refs. [1–4,17].

3. Results

Various functions of radii and density moments were investigated in order to extract the isospin dependence of proton and neutron density distributions. The calculations were performed for the nuclei along the β stability line with the mass number $40 \leqslant A \leqslant 256$ and for all potentially existing isotopes with $Z = 50, 56, 82, 94$ and isotones with $N = 50, 82, 126$. For the β-stable nuclei only one isotope with the smallest mass for given A is chosen. For the nuclei out of β-stability line the calculation was made until the proton or neutron drip line was reached.

The difference between the binding energy of a nucleus calculated by the RMFT with the NL-3 parameters set and its experimental value taken from Ref. [17] does not exeed 5 MeV in most cases. Only for the heaviest isotones with $N = 126$ it reaches 10 MeV. The separation energies of neutron and proton evaluated theoretically agree with the experimental data very well. Also the reduced electric quadrupole transition probabilities $B(\mathrm{E2})$ obtained within RMFT are close to the experimental data in spite of

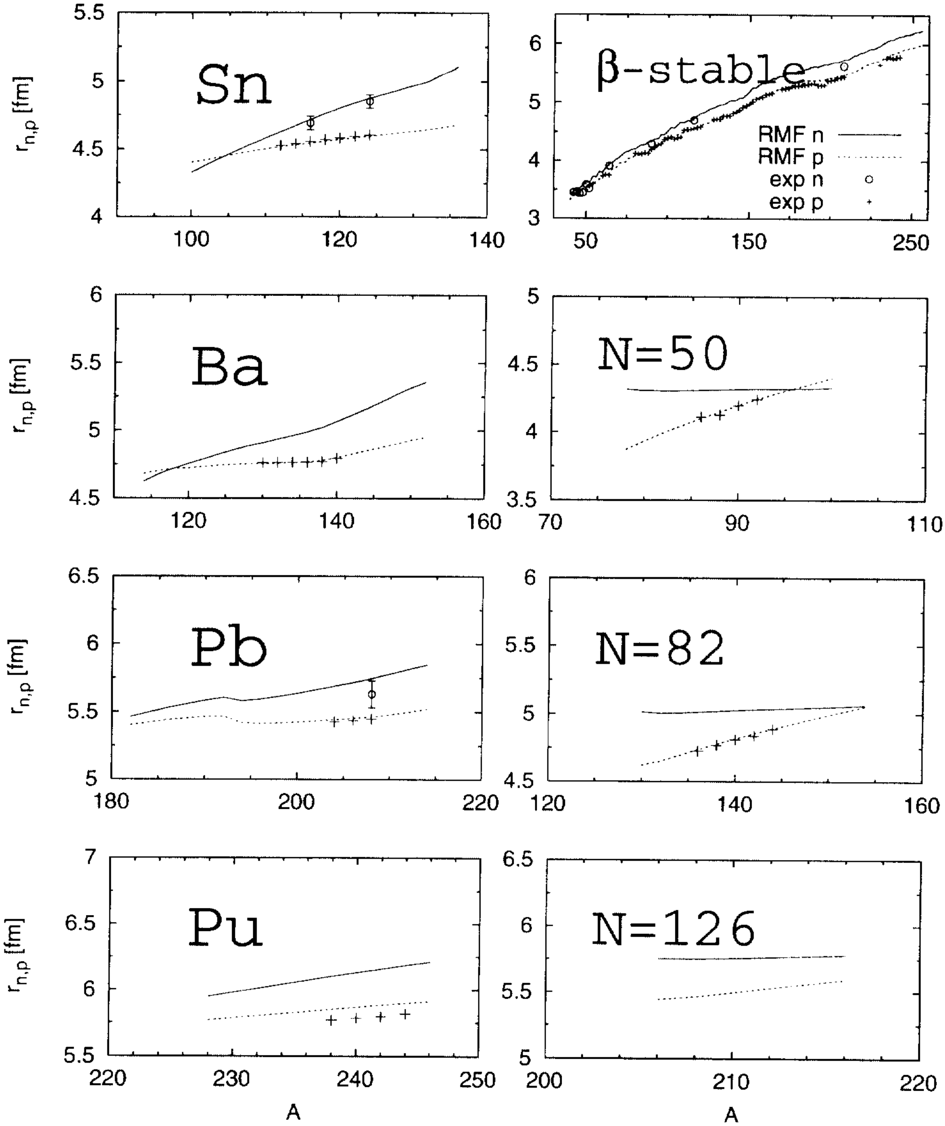

Fig. 1. The neutron (solid lines) and proton (dashed lines) root mean square radii r_n, r_p obtained within the RMFT+BCS model with the NL-3 parameters [16] for the β-stable nuclei and a few sets of isotopes and isotones compared with the experimental data (circles for neutrons, crosses for protons) as functions of mass number A.

too large theoretical predictions for the nuclei about $A \sim 170$ and 250. The deformations of the proton and neutron distribution are close to each other. The absolute difference between the quadrupole deformation β of both distributions is smaller than 0.03.

The neutron r_n and proton r_p root mean square radii calculated by the RMFT are presented in Fig. 1. The RMFT radii of neutron and proton distributions differ from each other up to 0.5 fm for the heaviest isotopes, what is connected with the difference

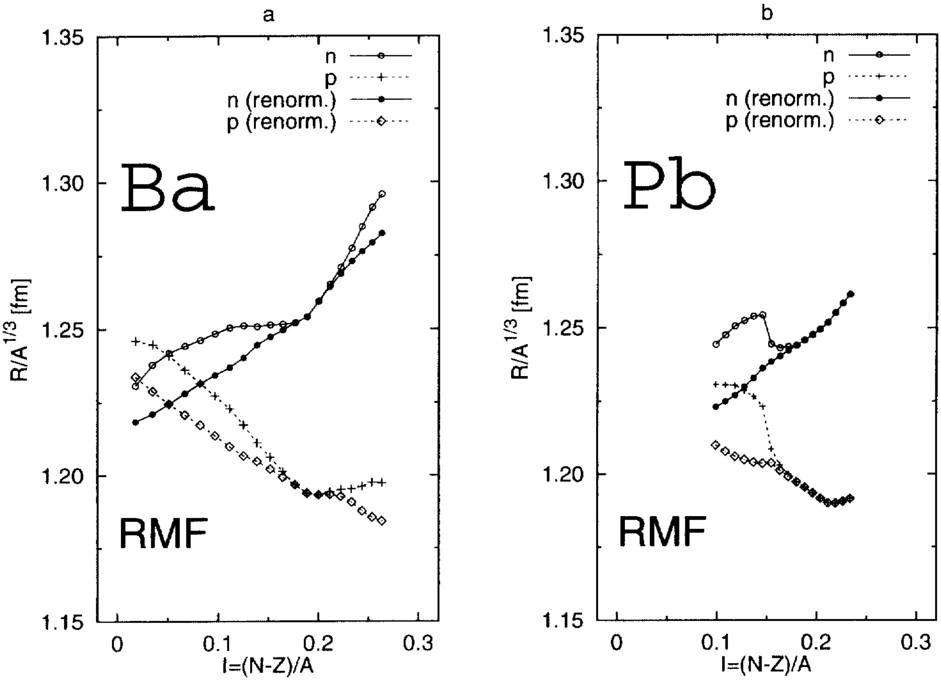

Fig. 2. The proton (crosses) and neutron (open circles) nuclear radii divided by $A^{1/3}$ as functions of the relative neutron excess I for Ba (a) and Pb (b) isotopes. Nuclear radii renormalized to sphere are plotted with diamonds and dots, respectively.

in size and deformation of the proton and neutron density distributions. The results for both radii are well confirmed by the experimental data.

In order to extract the average isospin dependence of the radius constant from the root mean square radii evaluated within the RMFT we have divided the RMSR values by the factor $\sqrt{3/5}A^{1/3}$ (see Eq. (10)). As one can see in Figs. 2a and 2b the radius constants of protons (crosses) and neutrons (circles) are gathered around 1.25 and 1.20 fm, respectively as functions of the relative neutron excess $I = (N - Z)/A$. In order to analyze better the results one has to remove the influence of deformation on nuclear radii. We have renormalized them to the sphere using the volume conservation rule. It was made approximately by dividing the nuclear radii constants by factor g:

$$g(\beta_2) = 1 + \frac{5}{4\pi}\beta_2^2 . \tag{13}$$

This renormalized to sphere quantities are shown on Figs. 2a,b for Ba and Pb isotopes, respectively. Now one can see that the renormalized proton radii (diamonds) decrease almost linear with I while the neutron ones (dots) increase.

The above investigation has convinced us that the microscopic radius constant $r_0^{\mathrm{p,n}}$ must depend also on the isospin I. The additional dependence on $1/A$ was noticed when comparing the results for different isotope chains. In Figs. 3 and 4 the results are shifted

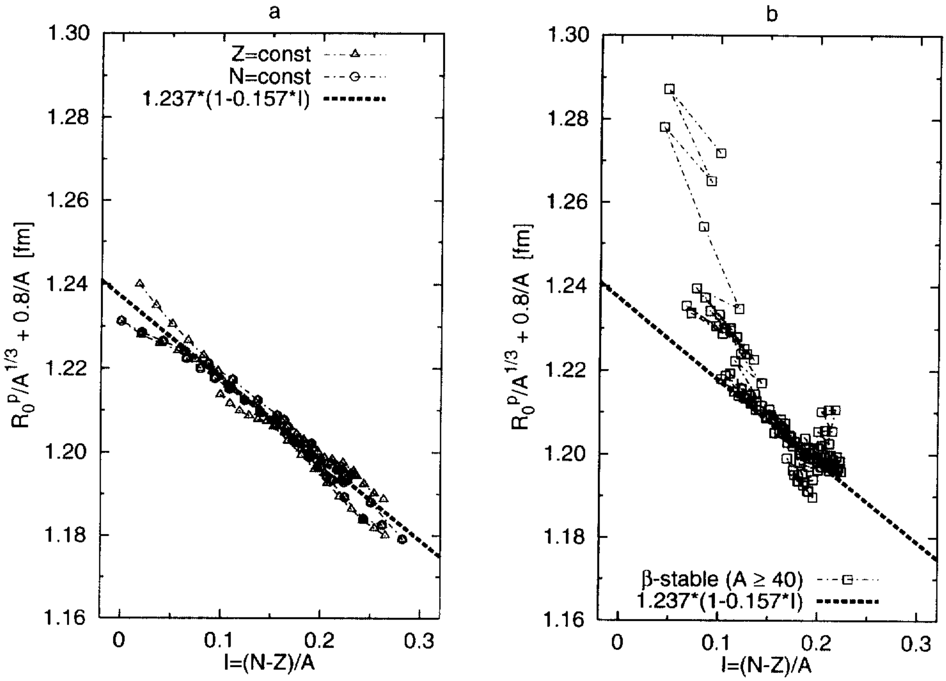

Fig. 3. Renormalized to sphere proton radii constants for discussed chains of isotopes (triangles) and isotones (circles) (a) and for all β stable nuclei with $A \geqslant 40$ (squares) (b) as functions of the relative neutron excess I shifted down by the $-3.3/A$ fm term. Dashed line represents average behavior of calculated values for the chains (Eq. (14)).

by the terms proportional to $1/A$ in order to see better the average I dependence of the radii.

In Fig. 3 we present the proton and in Fig. 4 the neutron renormalized radii constant for all the isotope and isotone chains discussed in this paper (a) and for the β-stable nuclei (b). The results for protons are shifted up by the term $0.8/A$ fm while the neutron estimates by $-3.3/A$ fm.

The dashed lines approximate average behavior of calculated radii constants for nuclei presented in Figs. 3a and 4a. It was found by minimization of the mean square deviations. The following formulae for proton and neutron radii:

$$R_0^p = 1.237\left(1 - 0.157\frac{N-Z}{A} - 0.646\frac{1}{A}\right) A^{1/3}\ \mathrm{fm}, \tag{14}$$

$$R_0^n = 1.176\left(1 + 0.250\frac{N-Z}{A} + 2.806\frac{1}{A}\right) A^{1/3}\ \mathrm{fm}, \tag{15}$$

approximate the results obtained within the RMFT with the NL-3 set of the parameters. One can believe that the parameters of these formulae, found for the representative nuclei all over the periodic system can describe well the average trend of the results obtained within the RMFT.

The following formula approximates the RMFT results for the nuclear charge radii:

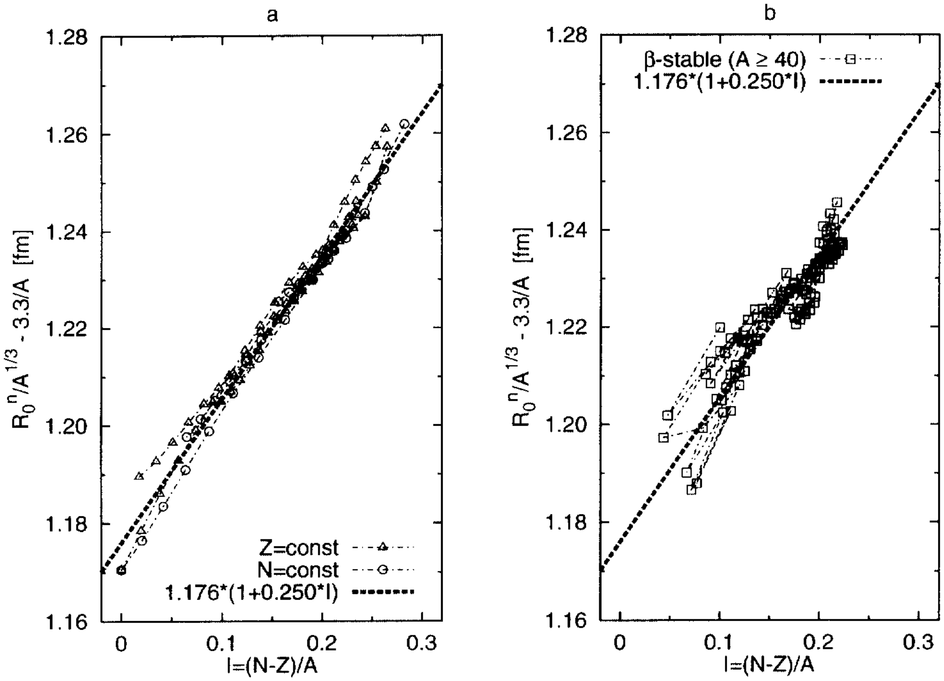

Fig. 4. Same as in Fig. 3 but for neutron radii (shifted up by the 0.8/A fm term).

$$R_0^{\rm ch} = 1.241 \left(1 - 0.154\frac{N-Z}{A} + 0.580\frac{1}{A}\right) A^{1/3}\ \text{fm}\,. \tag{16}$$

This equation, obtained only by the analysis of the RMFT calculation results, has not much different parameters to those from Eq. (4) fitted to all available experimental data. The coefficient of $(N-Z)/A$ term is only slightly smaller than in the phenomenological formula (4). But the parameter at the term $1/A$ in (16) is almost three times smaller than in Eq. (4). It is mostly due to the fact that we have analyzed the nuclei with $A \geqslant 40$ while the Eq. (4) was obtained by fitting the experimental data for all nuclei with $A \geqslant 12$.

From Figs. 3 and 4 we can learn that the average formulae (14,15) work properly only for nuclei with $A \geqslant 60$. Also some nuclei with $I \sim 0.2$ do not fit to the average formulae. It is caused by non-quadrupole deformations (octupole?) of these nuclei which were not included in the present analysis.

As far as the ratio of proton to neutron radii (or RMSR) is concerned there is almost no influence of deformation on the results. It is caused by the similar or identical shapes of proton and neutron density distributions. We have found a short formulae for this ratio similar to those for radii constants (14) and (15). The dependence of proton to neutron radii ratio on I is shown in Fig. 5. We have shifted the ratio by the term $-3.3/A$ in order to remove the influence of the $1/A$ dependence. Dashed lines in Fig. 5 represent the formula:

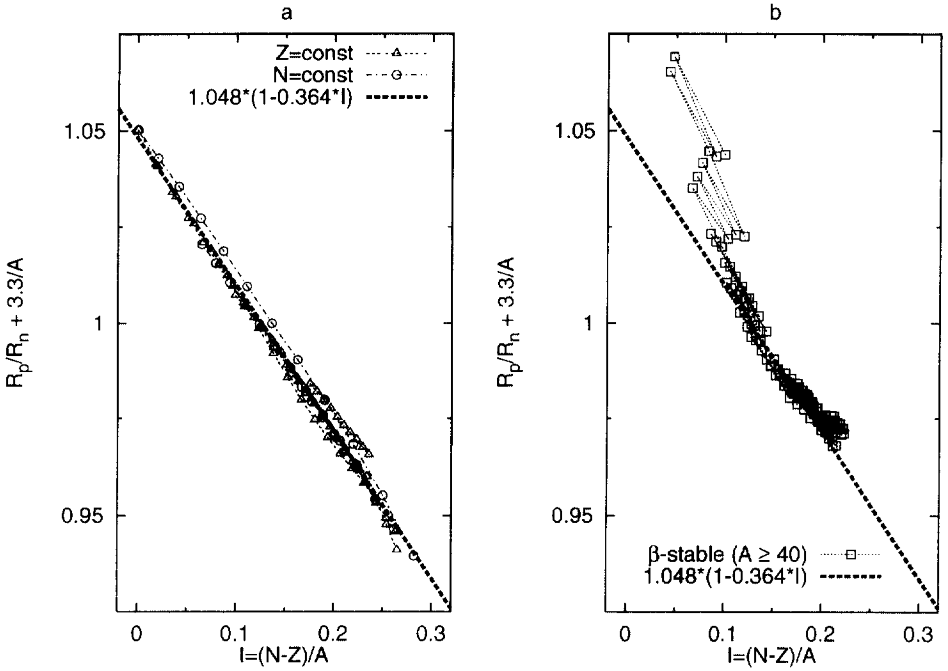

Fig. 5. The ratios of the proton to neutron root mean square radii for discussed chains of isotopes (triangles) and isotones (circles) (a) and for the β stable nuclei with $A \geqslant 40$ (squares) (b) as functions of relative neutron excess I. The ratios are shifted up by $3.3/A$ fm term. Dashed lines represent the average behavior of calculated values (17).

$$\frac{r_\mathrm{p}}{r_\mathrm{n}} = 1.048\left(1 - 0.364\frac{N-Z}{A} - 3.148\frac{1}{A}\right), \tag{17}$$

fitted by the least square fit to the $r_\mathrm{p}/r_\mathrm{n}$ RMFT ratios for all nuclides shown Fig. 1. In Fig. 5a the radii ratios of these chains of isotopes and isotones are presented while in Fig. 5b the corresponding results for β-stable nuclei with $A \geqslant 40$ are plotted. The results for nuclei with $A \geqslant 60$ are reproduced well by formula (17).

Eq. (17) can be used to estimate the neutron RMSR from the charge one. It could be useful because it exists over 250 measured charge RMSR and only few experimental ones for the neutron RMSR. One can rearrange Eq. (17) using the relation between the charge and proton mean square radii

$$r_{ch}^2 = r_\mathrm{p}^2 + 0.64\ \mathrm{fm}^2. \tag{18}$$

Here the term 0.64 fm^2 represents the average correction for the mean square radii of the charge distribution in a proton and a neutron (see also the discussion in Ref. [11]). Finally one gets:

$$r_\mathrm{n} = \frac{(r_{ch}^2 - 0.64)^{1/2}}{1.048\left(1 - 0.364(N-Z)/A - 3.148/A\right)}\ \mathrm{fm}. \tag{19}$$

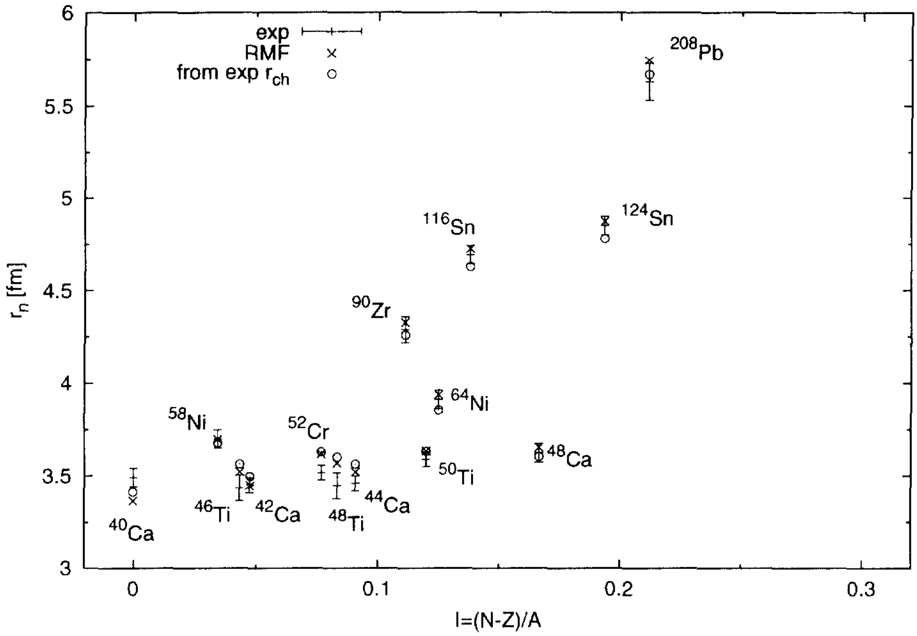

Fig. 6. Comparison of the experimental neutron root mean square radii [4] (errorbars) with the RMFT results (crosses) and the values obtained from experimental charge radii [4] through the new formula (19) (circles).

In Fig. 6 all known experimental RMSR (errorbars) [4] for neutrons are compared with the RMFT predictions (crosses) and with the results estimated from experimental charge RMSR [4] using Eq. (19) (circles). All results agree very well with each other, so we hope that the formula (19) can be used to foresee the neutron distribution radii for other nuclei as well.

4. Conclusions

The following conclusions can be drawn from our calculation

(i) The renormalized to sphere RMFT proton and neutron distributions radii $R_0^{p(n)}$ depend almost linear on neutron excess $I = (N-Z)/A$. R_0^p decrease with I, while R_0^n increase what was suggested in Ref. [10] and obtained in Ref. [11] by the analysis of the results obtained within the HFB calculation with the Skyrme forces.

(ii) The parameters of the formulae for the charge radii are similar as in the phenomenological formula in Ref. [10]. However, the dependence of the RMFT radii on I is about 20% weaker than in the phenomenological formula which describes the global experimental trend. It means that the parameters NL-3 of the RMFT should be slightly changed.

(iii) The term $\sim A^{-1}$ in the formula for $R_0^{p(n)}$ is needed in order to reproduce the average MSR values obtained within the RMFT. Its role is especially important for lighter nuclei.

(iv) The ratio of the proton radius to the neutron radius is a smooth function of I and A^{-1} and it could be very well described by the simple formula (17). Using this global dependence we have written the phenomenological formula (19) which allows to foresee the magnitude of the neutron radius when the experimental charge radius is known. The prediction power of the formula (19) is not worse than that of very advanced microscopical calculations based on the RMFT or the HFB-Skyrme model [11].

The obtained formulae for the R_0^n and R_0^p radii will be used to develop the liquid drop like model, which will depend on the different proton and neutron density distributions, i.e. different radii and deformations. Such investigations are in progress now [18].

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