International Journal of Modern Physics E

SIMPLE TOOL TO SEARCH QUASI-MAGIC STRUCTURES IN DEFORMED NUCLEI *

Bożena Nerlo-Pomorska and Krzysztof Pomorski

Department of Theoretical Physics, Maria-Curie-Skłodowska University,
20-031 Lublin, Poland

Received (10.10.2008)

Evaluation of shell effects in nuclei plays an important role in studying the nuclear structure. In the Strutinsky method the smooth energy of the nucleus is obtained by a folding procedure of the single-particle (s.p.) energy density in the s.p. energy space e. An alternative way of energy smoothing is obtained by folding the s.p. energy sum in the particle-number space $\mathcal{N}$. For non degenerated s.p. spectra both types of folding yield smooth energies which are close to each other. In the case of strongly degenerated spectra which appear at sphericity or in regions of shape isomers, the smooth energy obtained by the $\mathcal{N}$-folding is a couple of MeV larger than the traditional average Strutinsky energy. It is shown that this smooth energy difference can serve as a simple tool to search for magic or quasi-magic structures in the s.p. spectra, e.g. to find shape isomers in the multidimensional deformation space.

1. Introduction

The shell correction energy, equal to the difference between the single-particle energy sum and the corresponding smoothed energy, can be evaluated in two alternative ways. In the traditional Strutinsky method the s.p. levels are smoothed in the energy space (e–smoothing). In this method the nucleon number is conserved on average only and in some cases it is not easy to fulfill the plateau condition [1]. The prescription for the smoothed energy proposed in Ref. [2] consisting in smoothing the s.p. levels in the nucleon-number space ($\mathcal{N}$-smoothing), allows to avoid these problems, but it gives a few MeV deeper shell corrections (i.e. larger in absolute values) for spherical and other strongly degenerated spectra. This effect is connected with the degeneracy of s.p. levels which appears at points of magic structures, closed sub-shells and locations of low density in the s.p. level scheme. As this information is important especially in regions of exotic or superheavy nuclei the idea of analyzing the s.p. levels scheme by the difference of both types of the smoothed

*This work was partially sponsored by the Polish Ministry of Science and High Education, grant No. N202 179 31/3920.

energy

$$dE(def) = \widetilde{E}(def;\mathcal{N}) - E_{\rm Str}(def;e)\ . \tag{1}$$

seems to give the promising indicator of nuclear shells as function of nuclear deformation. The way of calculating both smoothed energies will be described in Sec. 2.

2. Two shell correction methods

The shell-correction energy can be expressed as the difference between the sum of s.p. energies and the corresponding smoothed energy

$$\delta E^{(q)}_{\rm shell} = \sum_{\nu_{\rm occ}} e^{(q)}_\nu - \widetilde{E}^{(q)}\ \ , \tag{2}$$

where $q = p, n$ denotes protons or neutrons.

We will discuss two methods for calculating this shell correction: the traditional Strutinsky approach [1], consisting of a smearing of the s.p. energy spectrum in energy (e) space and the smoothing in particle number ($\mathcal{N}$) space proposed in Ref. [2].

• The traditional Strutinsky method obtains the shell corrections through Eq. (2) with the smoothed in energy space s.p. energies sum $\widetilde{E}^{(q)}$ equal to

$$E^{(q)}_{\rm Str}(def;e) = \int_{-\infty}^{\widetilde{\lambda}_q} \widetilde{g}(e)\, e\, de\ \ , \tag{3}$$

where the average Fermi energy $\widetilde{\lambda}_q$ is fixed by the particle number condition while the function

$$\widetilde{g}(e) = \frac{1}{\gamma_s}\int_{-\infty}^{\infty} g(e')\, j\left(\frac{e-e'}{\gamma_s}\right)\, de' \tag{4}$$

is obtained from the exact s.p. level density through a smoothing procedure with a Gauss function multiplied by a $6^{\rm th}$ order correctional polynomial [2]

$$j(u) = \frac{1}{\sqrt{\pi}}\, e^{-u^2}\left(\frac{35}{16} - \frac{35}{8}u^2 + \frac{7}{4}u^4 - \frac{1}{6}u^6\right)\ \ . \tag{5}$$

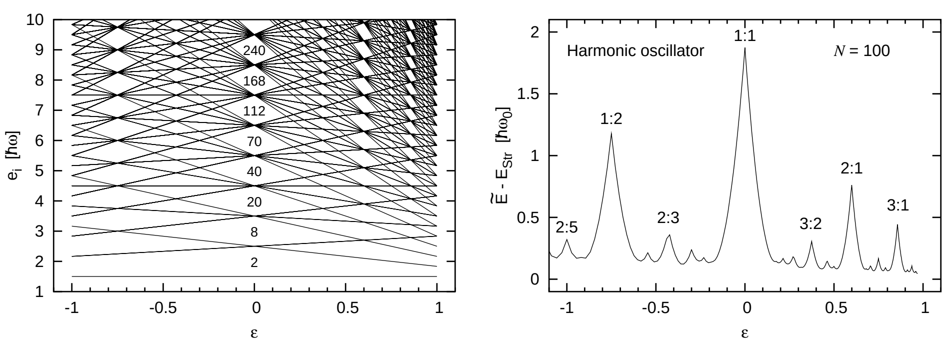

Fig. 1. Harmonic oscillator single particle energy levels (l.h.s.) and differences of the s.p. energies sums (r.h.s.) smoothed in nucleon number ($\widetilde{E}$) and energy ($E_{\rm Str}$) space as function of elongation.

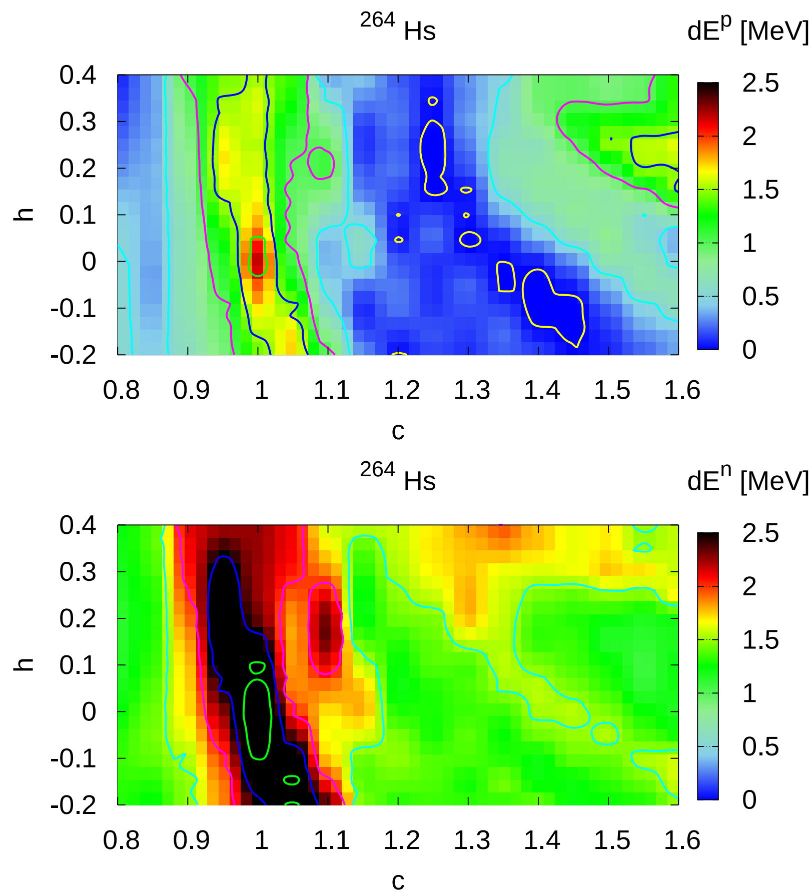

Fig. 2. Differences dE^p (dE^n) between proton (neutron) s.p. energies sums in ^{264}Hs smoothed in nucleon number and energy (e) space as function of elongation and neck parameters c and h respectively obtained with a Yukawa-folded mean field.

It can be shown [2] that the smoothed energy $E^{(q)}_{\rm Str}(def;e)$ calculated in this way is not the average sum of s.p. energies. The plateau condition usually taken at $\gamma_s = 1.2\hbar\omega_0$ is often not very well fulfilled and difficult to establish.

• In the $\mathcal{N}$-averaging method (more precisely one should speak about an averaging in the $\mathcal{N}^{1/3}$ space) the smoothed energy $\widetilde{E}^{(q)}(def;\mathcal{N})$ has the following form as function of the particle number $\mathcal{N}$:

$$\widetilde{E}^{(q)}(def;\mathcal{N}) = \sum_{n=\mathcal{N}_{\rm min}}^{\mathcal{N}_{\rm max}} \frac{2}{3\,n^{2/3}} \left(\sum_{\nu=1}^{n} e_\nu\right) j\left(\frac{\mathcal{N}^{1/3}-n^{1/3}}{\gamma}\right) \quad , \tag{6}$$

where the weight function $j(u)$ is defined by Eq. (5) and $\gamma = 0.78$ is the smearing width [2], for which the plateau condition is almost always nicely fulfilled. The

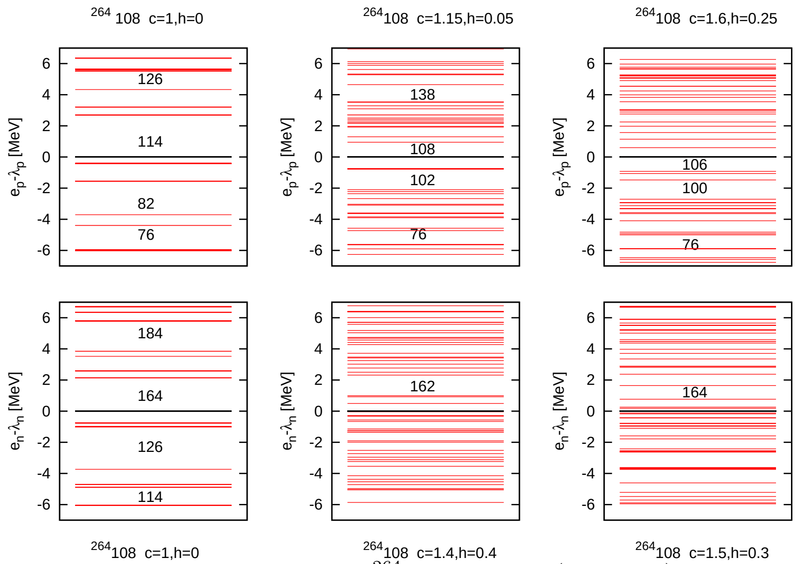

Fig. 3. Single particle levels schemes of ^{264}Hs for protons (upper row) and neutrons (lower row) in the magic points of deformation c, h obtained with the Yukawa folded mean field .

summation limits are: $\mathcal{N}_{\min} = (\mathcal{N}^{1/3} - 3\gamma)^3$ and $\mathcal{N}_{\max} = (\mathcal{N}^{1/3} + 3\gamma)^3)$.

3. Results

To illustrate the influence of s.p. levels degeneration on the both smoothed energies difference, the simple example of the ellipsoidally deformed harmonic oscillator potential is shown in Fig. 1. On the left hand side the s.p. energies are plotted versus nuclear quadrupole deformation ε. The magic numbers corresponding to the spherical oscillator are marked. The points in which the degeneracy of the s.p. levels appears correspond to the picks in the right side of Fig. 1, where the difference of the both smoothed energies is plotted. Every pick is marked by the corresponding ratio of the length of the ellipsoid semi-axes. This example proves that the difference dE (1) can be used for searching local degeneracies in spectra of various s.p. potentials in the multidimensional deformation space.

The calculations were performed for nuclei around ^{240}Pu and ^{264}Hs with s.p. energies of a Yukawa folded potential [3] on a deformation grid defined in Ref. [4] with elongation and neck parameters c and h.

In Fig. 2 the difference of the smoothed energies is shown for protons (dE^p) and neutrons (dE^n) for ^{264}Hs. One notices the strong peak at the spherical point: $c = 1$, h=0 and the appearance of a magic structure around the top of the fission barrier: c=1.6, h=0.25 with sub-shells Z=76, 100, 108 for protons and c=1.36, h=0.4 with N=162 for neutrons. There is also some indication for a *magic* deformation at c=1.15, h=0.05 with Z=76, 102, 108, 138 for protons and for neutrons at c=1.5,

h=0.3. The corresponding levels schemes for these points are shown in Fig. 3.

4. Conclusions

The following conclusions can be drawn from our analysis:
• The Strutinsky shell-correction obtained by the e-smearing are in places of deformation with magic structure of s.p. energies up to 10 MeV deeper than those calculated by the new $\mathcal{N}$-folding procedure.
• The magnitudes of the both shell corrections evaluated for non degenerate spectra are close each other.
• Shell corrections obtained by smoothing the s.p. energies in the nucleon number space fulfill the plateau condition better than those calculated traditionally by smearing in the s.p. energy space.
• Significant differences of the shell energies calculated by e- and $\mathcal{N}$- appears in the deformation points at which magic or quasi-magic shell structure is observed.

We have shown that difference dE (Eq. (1)) as the quantity weakly dependent on the number of nucleons can serve as a simple practical tool for searching of the quasi-magic structures in the multidimensional deformation parameter space.

References

1. V.M. Strutinsky, Nucl. Phys. **A122**, 1 (1968).
2. K. Pomorski, Phys. Rev. **C70**, 044306 (2004).
3. K.T.R. Davies and J.R. Nix, Phys. Rev. **C14**, 1977 (1976).
4. J. Bartel, A. Dobrowolski, and K. Pomorski, Int. Journ. Mod. Phys. **E16**, 459 ((2007).