

**COMPETITION BETWEEN AXIAL AND NON-AXIAL OCTUPOLE
DEFORMATIONS IN HEAVY NUCLEI**

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The macroscopic-microscopic method is applied to calculate the energies of heavy nuclei ($A > 220$) in a multidimensional deformation space $\{\alpha_{\lambda,\mu}\}$ including axial and non-axial quadrupole ($\lambda = 2, \mu = 0, 2$), axial and non-axial octupole ($\lambda = 3, \mu = 0, 2$) and axial hexadecapole ($\lambda = 4, \mu = 0$) degrees of freedom. Shell and pairing corrections are calculated from the single-particle energies of the Woods-Saxon potential with the universal parameters and added to the macroscopic energy of the newest Lublin-Strasbourg Drop (LSD) model to obtain the total deformation energy.

1. Introduction

We would like to present some preliminary results related to our systematic investigation of the total nuclear energy including simultaneously the mass-asymmetry and non-axial degrees of freedom. The nuclear shapes are parametrized as usual using the spherical harmonic $Y_{\lambda,\mu}$ basis:

$$\mathcal{R}(\theta, \phi) = R_0 c(\{\alpha_{\lambda,\mu}\}) \left(1 + \sum_{\lambda=2}^{\lambda_{max}} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda,\mu} Y_{\lambda,\mu}(\theta, \phi) \right), \quad (1)$$

where the function $c(\{\alpha_{\lambda,\mu}\})$ is obtained from the volume-conservation condition. The results presented below have the form of two-dimensional plots with the energy minimised with respect to the other deformation variables.

2. Realisation of the Macroscopic-Microscopic Method

The potential energy surfaces are obtained using the macroscopic-microscopic method. The main contribution to the total energy is coming from the macroscopic part obtained with the Lublin-Strasbourg-Drop formula,¹ which has been shown to reproduce the masses of 2766 nuclei with a mean-square deviation of 0.698 MeV when the deformation, shell and pairing corrections of Ref.² are included.

The corresponding macroscopic energy (expressed in MeV) reads:

$$\begin{aligned}
 E_{LSD} = & -15.4920(1 - 1.8601 I^2) A \\
 & + 16.9707(1 - 2.2938 I^2) A^{2/3} B_{surf}(def) \\
 & + 3.8602(1 + 2.3764 I^2) A^{1/3} B_{cur}(def) \\
 & + \frac{3}{5} e^2 \frac{Z^2}{r_0^{ch} A^{1/3}} B_{Coul}(def) - 0.9181 \frac{Z^2}{A} - 10 \exp(-4.2|I|), \quad (2)
 \end{aligned}$$

where $A = Z + N$ and $I = (N - Z)/A$ and where the surface, curvature and Coulomb terms depend on the nuclear deformation.³

The microscopic part contains a shell correction calculated with the Strutinsky method⁴ and a pairing correction⁵

$$E_{pair} = E_{BCS} + \bar{E}_{pc} \quad (3)$$

where

$$E_{BCS} = \sum_{\nu=\mathcal{N}_1}^{\mathcal{N}_2} 2 v_\nu^2 (e_\nu - \lambda) - \frac{\Delta^2}{G} - G \left(\sum_{\nu=\mathcal{N}_1}^{\mathcal{N}_2} v_k^4 - \sum_{\nu=\mathcal{N}_1}^{\mathcal{N}_2} 1 \right) - \sum_{\nu=\mathcal{N}_1}^{\mathcal{N}_2} (e_\nu - \lambda) \quad (4)$$

and

$$\bar{E}_{pc} = -\frac{1}{4} \frac{N^2}{\bar{\rho}} \left\{ \sqrt{1 + \frac{2\bar{\rho}\bar{\Delta}}{N}} - 1 \right\} + \frac{1}{2} \bar{\rho} \bar{\Delta} G \arctan\left(\frac{N}{2\bar{\rho}\bar{\Delta}}\right) \quad (5)$$

where \mathcal{N}_1 and \mathcal{N}_2 are limits of the pairing window, v_ν^2 the single-particle occupation probabilities, $\bar{\rho}$ the average level density and G the pairing strength. We are using the average pairing gap fitted to the experimental masses of Ref.⁶:

$$\bar{\Delta}_n = 9.08/\sqrt{A} \text{ MeV}, \quad \bar{\Delta}_p = 9.85/\sqrt{A} \text{ MeV}. \quad (6)$$

The spectrum of single-particle energies e_ν is obtained with the Woods - Saxon single particle potential with the universal set of parameters.⁷

3. Results

We use the $\{\alpha_{20}, \alpha_{22}\}$ -plane of the quadrupole deformation parameters and translate it to Cartesian coordinates $\{x, y\}$ defined by

$$x = \beta \cos(\gamma + 30) \quad \text{and} \quad y = \beta \sin(\gamma + 30) \quad (7)$$

where

$$\alpha_{20} = \beta \cos(\gamma) \quad \text{and} \quad \alpha_{22} = \beta \sin(\gamma)/\sqrt{2} \quad (8)$$

In the following we present the total energies as geographical maps in coordinates x and either α_{30} or α_{32} , minimised with respect to y and α_{40} .

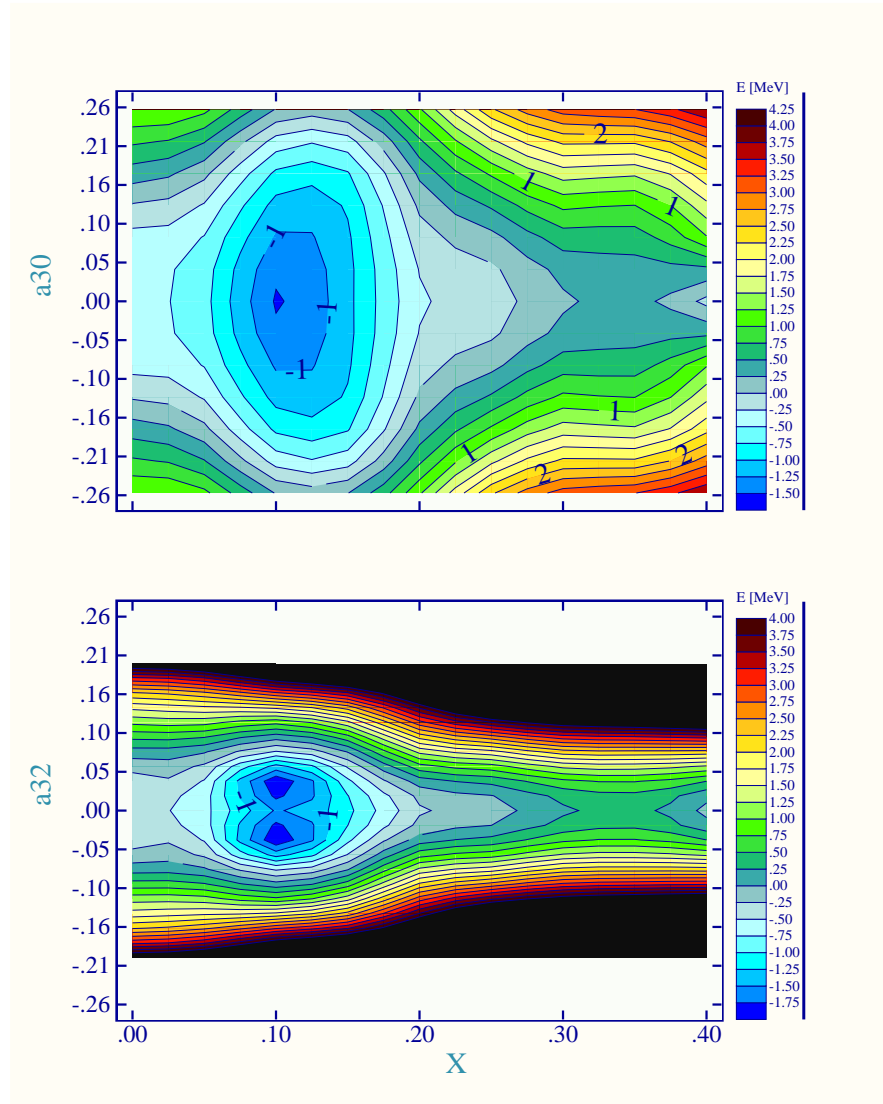


Fig. 1. Comparison of the total nuclear energies for ^{224}Rn as function of the coordinate x (cf. Eqs. (7)-(8)) and the axial α_{30} (top) and non-axial α_{32} (bottom) octupole deformation parameters. Notice that in both cases the equilibrium deformation involves to a non-zero *quadrupole deformation*. The static equilibrium deformation corresponds to a non-axial octupole deformation with a barrier between the two minima of the order of 500 keV.

Let us emphasize that mass asymmetric deformations lead to different pictures in both cases: while in ^{224}Rn the minimum deformation corresponds to a non-axial mass-asymmetric shape, the situation is just the opposite in the neighbouring isotope ^{226}Ra . This signifies two different classes of shapes given the fact that the non-zero *quadrupole* defor-

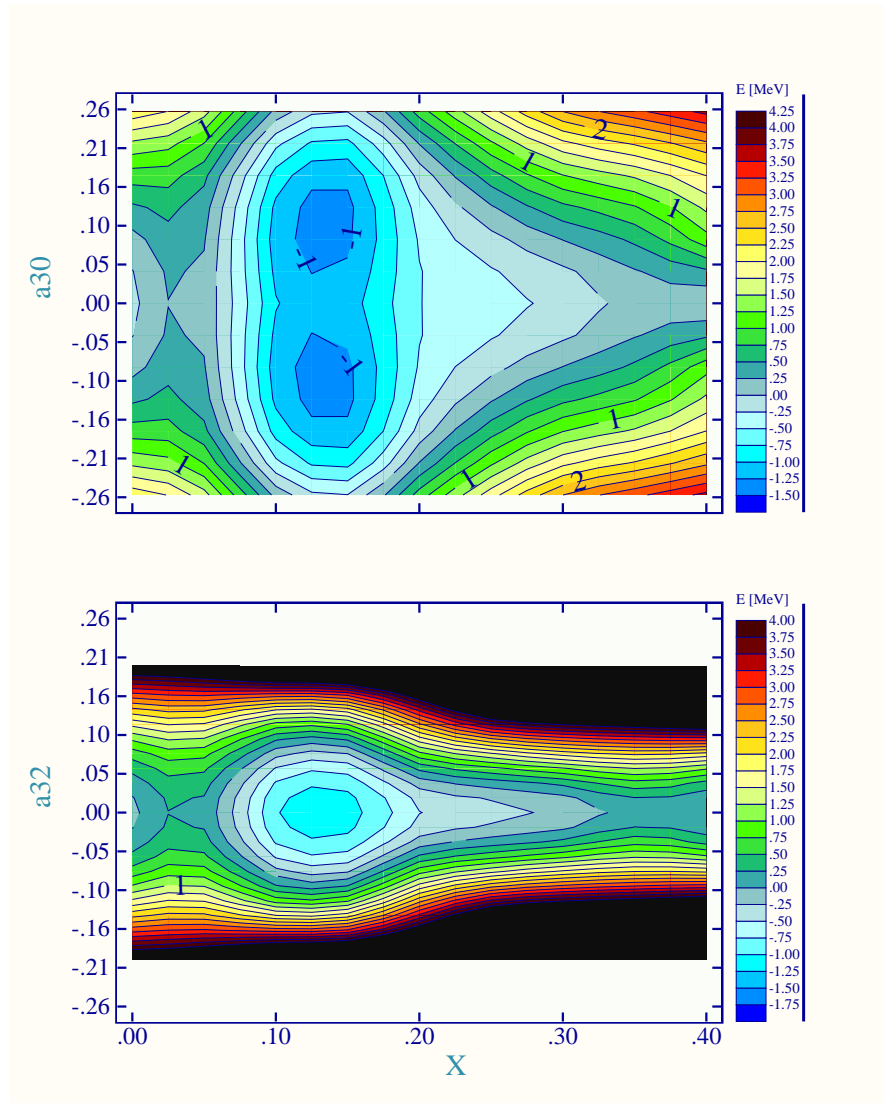


Fig. 2. Similar to Fig. (1) but for the isotone of ^{224}Rn , the ^{226}Ra nucleus. Observe that in this case the axially symmetric octupole deformation leads to static mass asymmetric minima separated by a barrier of over a 400 keV height while the α_{32} degree of freedom does *not* lead to a static equilibrium deformations.

mations are superposed with the non-zero axial in one case and the non-axial one in the other. Consequently, the analysis of the octupole degrees of freedom strongly suggests a much more complicated picture as compared to the earlier analysis of Ref.⁸ and several others where only the axial-octupole deformations have been used.

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