

NILSSON SINGLE PARTICLE POTENTIAL PARAMETERS REPRODUCING THE GROUND STATE AND *K*-ISOMERS RADII* **

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The optimal parameters of the Nilsson single particle potential in order to reproduce the experimental radii of the ground states of nuclei and the *K*-isomers Lu, Hf, Ta were found. The isospin dependence of harmonic oscillator frequency and the new parameters set of the Seo correction term in the single-particle potential make able to use the macroscopic–microscopic model with free quasiparticle BCS pairing approximation to describe the properties of nuclear ground and rotational *K*-isomeric states.

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1. Introduction

The macroscopic–microscopic method has served for years as the practical tool to describe the properties of the ground states of nuclei, their energies, deformations and radii. Though the Hartree–Fock self-consistent models with various two-body nuclear forces as Gogny [1], Skyrme [2], relativistic mean field theory [3] were developed as the better description of the nuclear properties it is still interesting to check how do the quick simple methods work. They have usually less free parameters and give quickly results for the broad regions of nuclei. Large progress in fitting the parameters of the single particle potentials like Woods–Saxon [4] or Nilsson [5] one makes their applications reasonable and practical.

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The deformed harmonic oscillator Nilsson [5] potential has got broader application after introducing the Seo [6] correction term with shell dependent κ , μ parameters of the spin orbit and square momentum terms.

This single particle base was used successfully to describe the deformations and radii of even–even [4] and odd–even nuclei, specially after including the dynamical effects by generator coordinate method [7]. Though the single-particle levels set of Seo [6] reproduced well the Woods–Saxon’s one for spherical nuclei, it failed in description of the rotation excited states for example the long living K isomers.

It was our aim to improve the parameters of Nilsson–Seo single particle potential in order to reproduce charge radii of the ground states of experimentally known nuclei and the proper energies of excited K isomeric states of Lu, Hf and Ta to foresee their radii and shapes.

2. Theory

The potential energies of nuclei are found by the minimization of the potential energy surface $E(\varepsilon, \varepsilon_4)$ on the quadrupole ε , and hexadecapole ε_4 deformations parameters plane. The energy E consists of the two macroscopic and microscopic parts

$$E = E_{LD} + \Delta E_{\text{shell}} . \quad (1)$$

The first term is the liquid drop energy E_{LD} [8] and the second one is the microscopic shell correction of Strutinsky. The calculation is done for proton and neutron sets separately, the proton–neutron pairing forces are not taken into account yet. The sum of protons and neutrons microscopic energies together with common liquid drop part is minimized over $\varepsilon, \varepsilon_4$ deformation giving the equilibrium deformations $\varepsilon^{\text{eq}}, \varepsilon_4^{\text{eq}}$ of every nucleus in the ground or excited state.

The nuclear mean square radii or quadrupole moment are found in this equilibrium point from the formula

$$\langle r^2 \rangle = 2 \sum_{\nu > 0, \nu \neq \nu_i} \langle \nu | \hat{r}^2 | \nu \rangle V_\nu^2 + \sum_i \langle \nu_i | \hat{r}^2 | \nu_i \rangle , \quad (2)$$

where V_ν^2 are the BCS occupation factors [9], $|\nu_i\rangle$ the single particle states occupied by the quasiparticles. The single-particle levels scheme e_ν and $|\nu\rangle$ states are obtained by diagonalization of the single particle Hamiltonian consisting of the kinetic energy and the Nilsson–Seo [6] potential V

$$H = \frac{1}{2} \hbar \omega_0(\varepsilon, \varepsilon_4) \hat{\Delta}_t + \frac{1}{2} \hbar \omega_0(\varepsilon, \varepsilon_4) \rho^2 \left[1 - \frac{2}{3} \varepsilon P_2(\cos \vartheta_t) + 2 \varepsilon_4 P_4(\cos \vartheta_t) \right] - \hbar \hat{\omega}_0 (2 \kappa_{Nl} \vec{l} \cdot \vec{s} + \nu_{Nl} \vec{l}^2 - \langle \nu_{Nl} l^2 \rangle_N) , \quad (3)$$

where the laplacian $\hat{\Delta}_t$, radius ρ , $\cos \vartheta_t$, and angular momentum \vec{l} are expressed in stretched coordinates.

Nilsson harmonic oscillator frequency $\omega_0(\varepsilon, \varepsilon_4)$ is obtained from the volume conservation condition inside the equipotential surface

$$\omega_0^{n(p)}(\varepsilon, \varepsilon_4) = \overset{\circ}{\omega}_0^{n(p)} \left\{ \frac{1}{\left(1 + \frac{\varepsilon}{3}\right) \left(1 - \frac{2}{3}\varepsilon\right)^{1/2}} \int_0^1 \frac{dx}{\left(1 - \frac{2}{3}\varepsilon P_2(x) + 2\varepsilon_4 P_4(x)\right)^{3/2}} \right\}^{1/3}. \tag{4}$$

The spherical oscillator frequency, usually taken as

$$\hbar \overset{\circ}{\omega}_0 = 40A^{-1/3} \text{ MeV} \tag{5}$$

is in our calculation isospin dependent $\overset{\circ}{\omega}_0^{n(p)}$. This dependence was found from the condition

$$\hbar \overset{\circ}{\omega}_0^{n(p)} = \hbar \overset{\circ}{\omega}_0 \frac{\langle r^2 \rangle^{n(p)}}{\langle r^2 \rangle_{\text{exp}}^{n(p)}}, \tag{6}$$

where neutron $\langle r^2 \rangle_{\text{exp}}^n$ are calculated from [10] RMFT formula, the proton ones taken from Ref. [11].

The results are approximated by the formulae

$$\hbar \overset{\circ}{\omega}_0^p = 39.12 \left(1 + 0.09 \frac{N - Z}{A} \right) A^{-1/3} \text{ MeV}, \tag{7}$$

$$\hbar \overset{\circ}{\omega}_0^n = 40.45 \left(1 + 0.11 \frac{N - Z}{A} \right) A^{-1/3} \text{ MeV}. \tag{8}$$

In formula (3) the correction term contains the following functions

$$\kappa_{Nl} = \kappa_0 \left[1 + 8\nu_{Nl} \left(N + \frac{3}{2} \right) \right] + \kappa_1 A^{-1/3} p_{s,Nl}, \tag{9}$$

where

$$p_{s,Nl} = \int_{\overset{\circ}{R}_0 - \frac{a}{2}}^{\overset{\circ}{R}_0 + \frac{a}{2}} R_{Nl}^2(r) r^2 dr \tag{10}$$

and

$$\nu_{Nl} = \nu_0 P_{i,Nl}^2, \tag{11}$$

where the probability of finding a nucleon inside a nucleus is

$$P_{i,Nl} = \int_0^{\overset{\circ}{R}_0 - \frac{a}{2}} R_{Nl}^2(r) r^2 dr. \tag{12}$$

The average orbital momentum square within N state is

$$\langle \nu_{Nl} \vec{l}^2 \rangle_N = \frac{\sum_l (2l+1)l(l+1)\nu_{Nl}}{\sum_l (2l+1)}. \quad (13)$$

$R_{Nl}(r)$ are the radial functions of harmonic oscillator potential, the spherical radius $\overset{\circ}{R}_0$ is

$$\overset{\circ}{R}_0 = 1.2 \cdot A_{\text{av}}^{1/3} \text{ fm}. \quad (14)$$

A_{av} denotes the average mass number of nuclear region, $a = 0.7$ is the smooth parameter of nuclear skin.

The phenomenological parameters reproducing Woods–Saxon levels scheme were [6] $\kappa_0 = 0.021$, $\kappa_1 = 0.90$, $\nu_0 = 0.062$. They are not sufficient for the description of the rotational K isomeric excited states of nuclei. Investigating the single particle levels schemes of the 9 K -isomers: $^{176,177,178}\text{Lu}$, $^{177,178,179}\text{Hf}$ and $^{178,179,180}\text{Ta}$ and the possible quasiparticle configurations of odd nucleons systems and excited states we have found the new set, better reproducing the experimental energies of 56 states $\kappa_0 = 0.025$, $\kappa_1 = 0.88$, $\nu_0 = 0.062$. The energies, radii and quadrupole moments of K -isomers with the new harmonic oscillator frequencies (7), (8) and correction term parameters were calculated. They agree better with the experimental data, then the previous ones.

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