# Interplay of magnetism and super-conductivity in Josephson junctions


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**UMCS, Lublin**

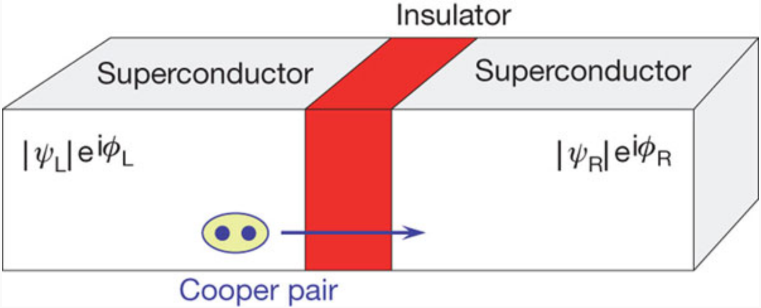

**Institute of Physics PAS (Warsaw)** **7 March 2023**

# Part 1. Josephson effect

(prediction, discovery, advancements)

# JOSEPHSON EFFECT

**Upon contacting two superconductors by a narrow insulating region there is induced a flow of the Cooper pairs whenever their phases are different, $\phi_L \neq \phi_R$.**



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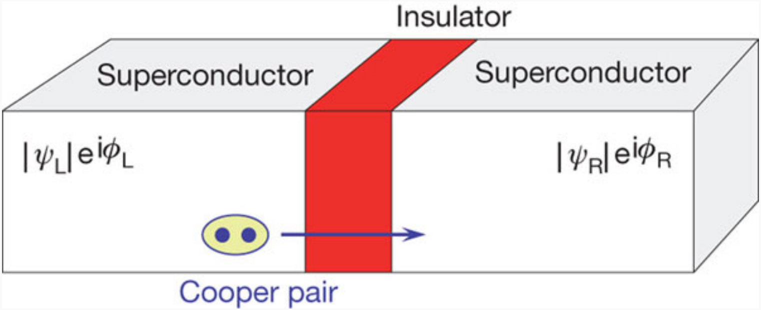



**This effect has been predicted by B.D. Josephson in 1962.**

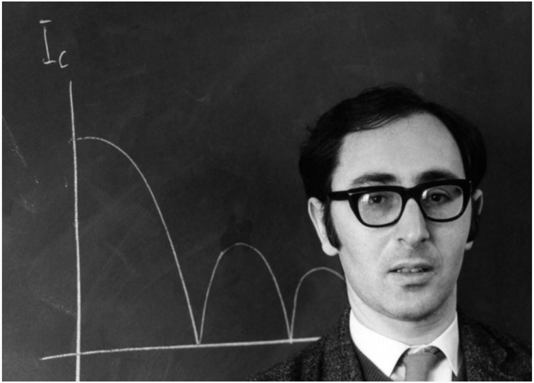
**/ 22-year-old PhD student at Cambridge, England /**

# PREDICTION

**B.D. Josephson, Physics Letters 1, 251 (1962).**

$\Longrightarrow$ **finite current at zero bias (dc Josephson effect)**

$\Longrightarrow$ **current oscillating with frequency $2eV/\hbar$ in biased junction (ac Josephson effect).**



# HYDRODYNAMIC REASONING

**In quantum mechanics the probability current is defined by**

$$\vec{j}(\vec{r},t) = -\frac{i\hbar}{2m}\left[\Psi^\star(\vec{r},t)\nabla\Psi(\vec{r},t) - \Psi(\vec{r},t)\nabla\Psi^\star(\vec{r},t)\right]$$

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**Applying this formalism to the wave-function $\Phi_0$ of Cooper pairs**

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$$\vec{j}_J(\vec{r},t) = -\,q\,\frac{i\hbar}{2m}\left[\Phi_0^\star\nabla\Phi_0 - \Phi_0\nabla\Phi_0^\star\right]$$

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where charge is $q = 2e$ and $\vec{v}(\vec{r},t)$ is the Cooper pairs' velocity.

# EXPERIMENTAL EVIDENCE

VOLUME 10, NUMBER 6 PHYSICAL REVIEW LETTERS 15 MARCH 1963

PROBABLE OBSERVATION OF THE JOSEPHSON SUPERCONDUCTING TUNNELING EFFECT

P. W. Anderson and J. M. Rowell
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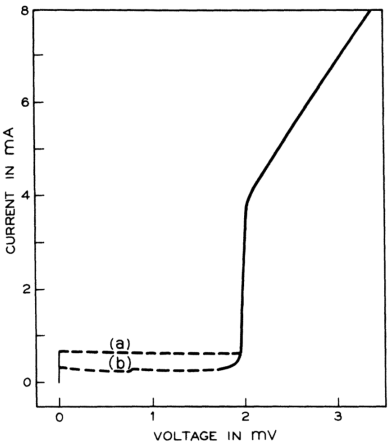



*Authors reported on:*

*„dc tunneling current at or near zero voltage in very thin tin oxide barriers between superconducting Sn and Pb"*

**1973**

**B.D. Josephson (with L. Esaki & I. Giaver)**

# NOBEL PRIZE IN PHYSICS

**1973**

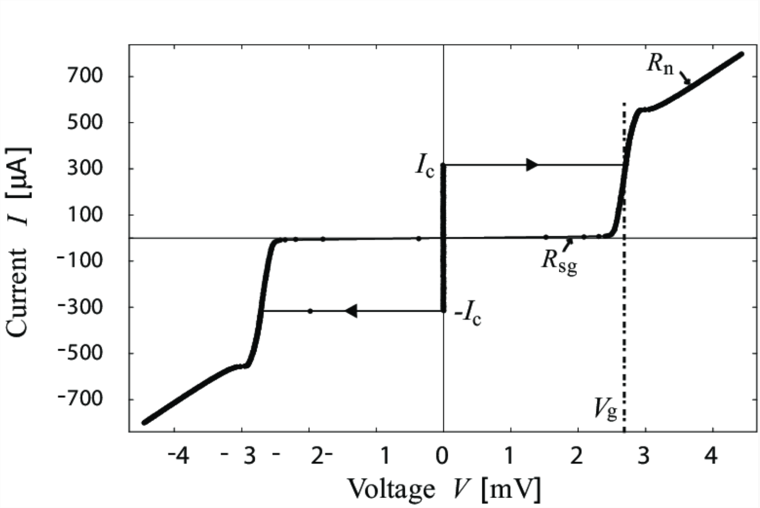
**B.D. Josephson (with L. Esaki & I. Giaver)**

**1972**

**J. Bardeen, L.N. Cooper, J.R. Schrieffer**

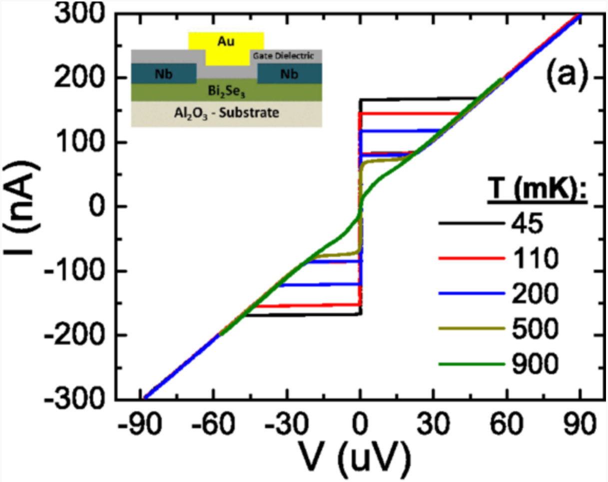
# I(V) CHARACTERISTICS

**Typical current-voltage plot, where $V_g = 2\Delta$**



# TEMPERATURE DEPENDENCE

**The critical dc current $I_c$ diminishes with increasing temperature.**



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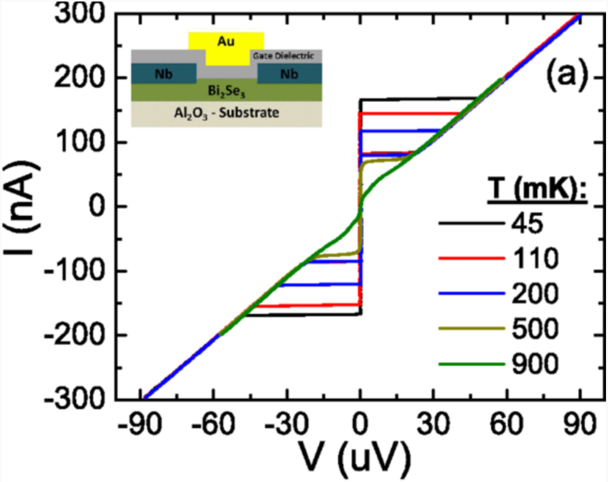
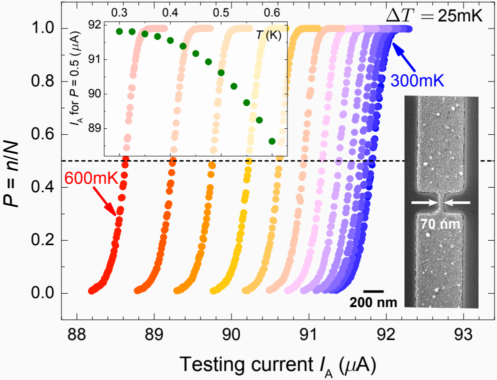



**Switching from superflow to resistive current has stochastic character.**

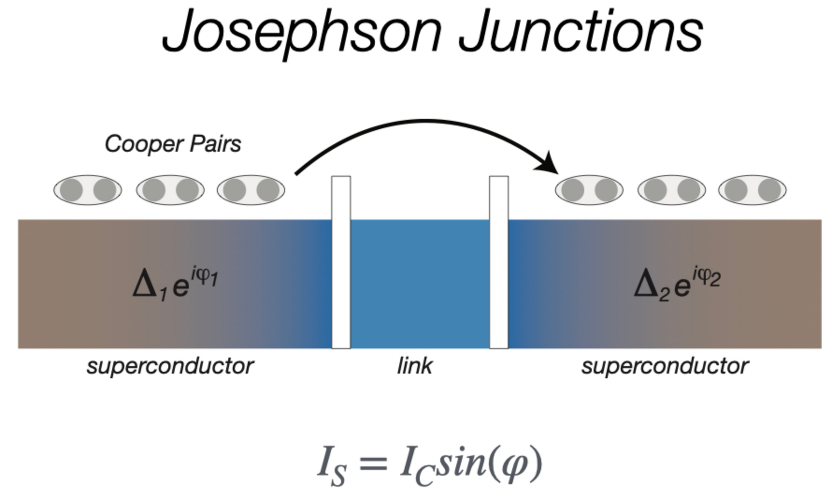
**The temperature dependent Josephson current switching has been recently practically used by Maciej Zgirski for constructing the ultrafast nanoscale thermometry.**




**M. Zgirski et al, Phys. Rev. B 104, 014506 (2021).**

# DC JOSEPHSON CURRENT

**Periodicity:** **Superflow of the Copper pairs depends on phase difference, therefore dc current is (usually) periodic with respect to** $\phi = \phi_L - \phi_R$

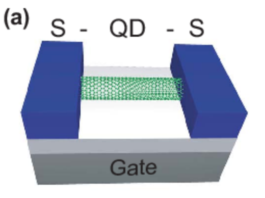


# Recent challenges

## (selected examples)

# 1. JOSEPHSON CURRENT REVERSAL

**Carbon nanotube interconnecting two superconductors, differing in phase.**



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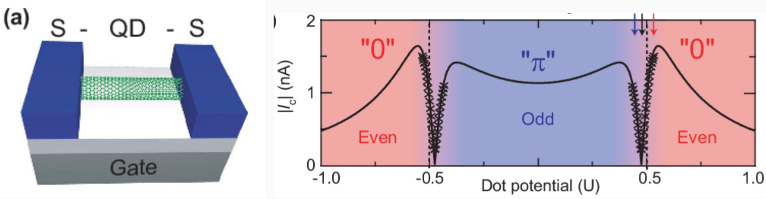
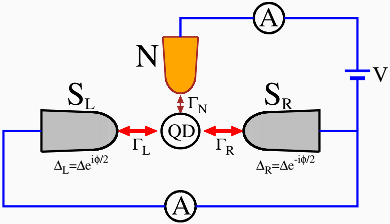



**At certain gate potential the dc Josephson current abruptly changed its magnitude and direction (zero-pi transition).**

H.I. Jorgensen, T. Novotný, K. Grove-Rasmussen, K. Flensberg, P.E. Lindelof, NanoLett. 7, 2441 (2007).

## Three-terminal geometry



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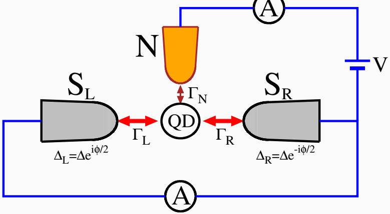
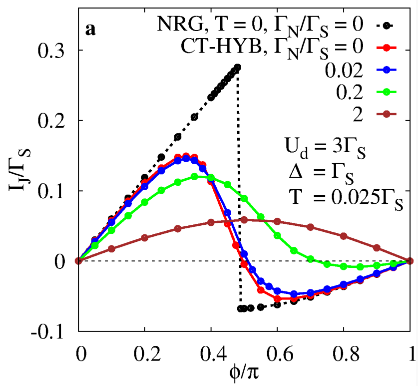


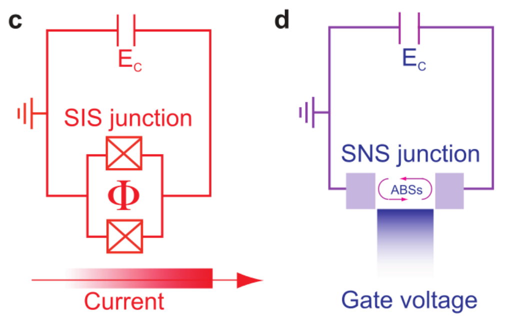




**Reversal of dc Josephson current at certain phase difference $\phi$ is driven by a parity change of the Andreev bound states of QD.**

**T. Domański, M. Žonda, V. Pokorný, G. Górski, V. Janiš, T. Novotný**

**Phys. Rev. B 95, 045104 (2017).**

**Schematical view of the superconducting quantum bits in realization of: transmon (left) and gatemon (right h.s. panel).**

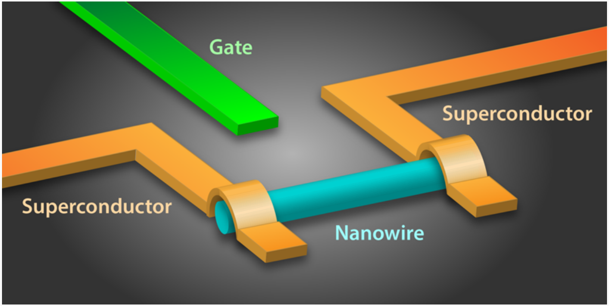



**R. Aguado, Appl. Phys. Lett. 117, 240501 (2020).**

**Superconducting island circuit based on Josephson junction, which is capacitively shunted ($E_C$ is the charging energy).**

# 2. SUPERCONDUCTING QUBITS: GATEMON

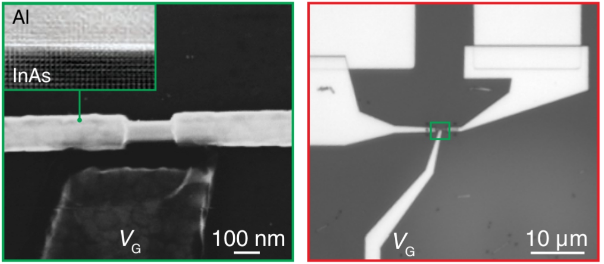
**Idea: Electrical control over the Josephson supercurrent through semiconducting nanowire accomplished by a side-gate potential.**




J.M. Nichol, Physics 8, 87 (2015).

# 2. SUPERCONDUCTING QUBITS: EXPERIMENT

**Semiconducting (InAs) nanowire coupled to superconducting (Al) which is controlled by an electrostatic gate that depletes the carriers in a weak link region.**




T.W. Larsen et al, Phys. Rev. Lett. 115, 127001 (2015).

**Reported relaxation times $\sim$ 0.8 $\mu$s and dephasing times $\sim$ 1 $\mu$s exceeded the gate operation times by 2 orders.**

# The field-free Josephson diode in a van der Waals heterostructure

https://doi.org/10.1038/s41586-022-04504-8

Received: 29 March 2021

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Published online: 27 April 2022

Check for updates

Heng Wu[1,2,3,6], Yaojia Wang[1,3,6], Yuanfeng Xu[1,4], Pranava K. Sivakumar[1], Chris Pasco[5], Ulderico Filippozzi[3], Stuart S. P. Parkin[1], Yu-Jia Zeng[2], Tyrel McQueen[5] & Mazhar N. Ali[1,3]

The superconducting analogue to the semiconducting diode, the Josephson diode, has long been sought with multiple avenues to realization being proposed by theorists[1–3]. Showing magnetic-field-free, single-directional superconductivity with Josephson coupling, it would serve as the building block for next-generation superconducting circuit technology. Here we realized the Josephson diode by fabricating an inversion symmetry breaking van der Waals heterostructure of $NbSe_2$/$Nb_3Br_8$/$NbSe_2$. We demonstrate that even without a magnetic field, the junction can be superconducting with a positive current while being resistive with a negative current.

**Nature | Vol 604 | 28 April 2022 | 653.**

# 3. JOSEPHSON DIODE

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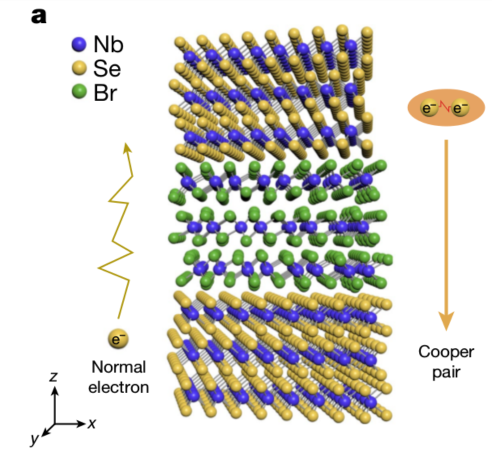
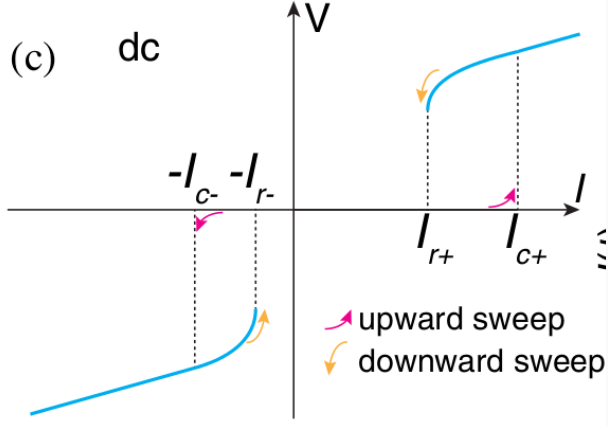
**Discovery of the magnetic field - free superconducting diode in van der Waals heterostructure of $NbSe_2$/$Nb_3Br_8$/$NbSe_2$.**

# 3. JOSEPHSON DIODE

**Niobium bromide (just a few atoms thick) placed between layers of superconducting niobium diselenide does conduct electricity without resistance solely in one direction of the applied voltage.**



**Mechanism behind the Josephson diode effect in not fully understood.**



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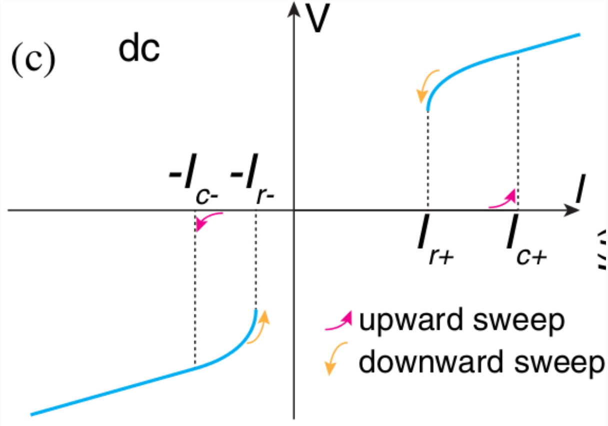



**Speculations:**

**Major role plays the asymmetric sc proximity process inside the tunneling barrier due to: (a) inversion symmetry breaking, (b) time-reversal breaking.**

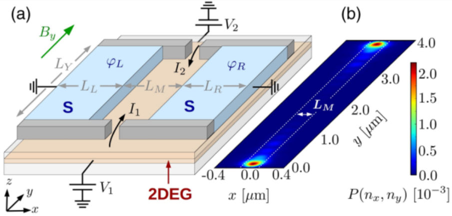
# Part 2. Topological superconductivity

## (in Josephson junctions)

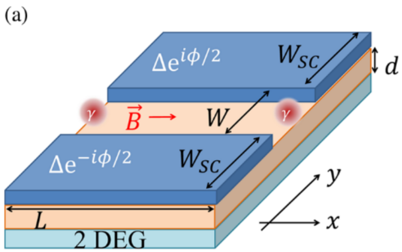
# Theoretical concept (2017)

# PLANAR JOSEPHSON JUNCTIONS

**Idea: Narrow metallic region with the strong spin-orbit interaction and in presence of magnetic field embedded between external superconductors.**




Michael Hell et al., PRL 118, 107701 (2017)




F. Pientka et al., Phys. Rev. X 7,021032 (2017)

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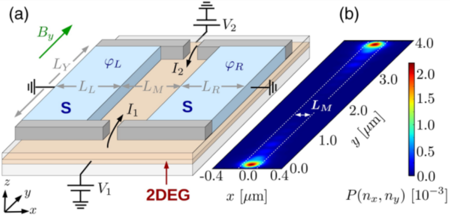
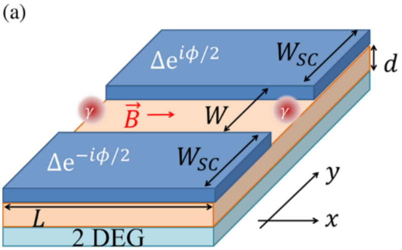



Michael Hell et al., PRL 118, 107701 (2017)




F. Pientka et al., Phys. Rev. X 7,021032 (2017)

**Benefit:**

**Phase-tunable topological superconductivity induced in the metallic stripe.**

# PLANAR JOSEPHSON JUNCTIONS

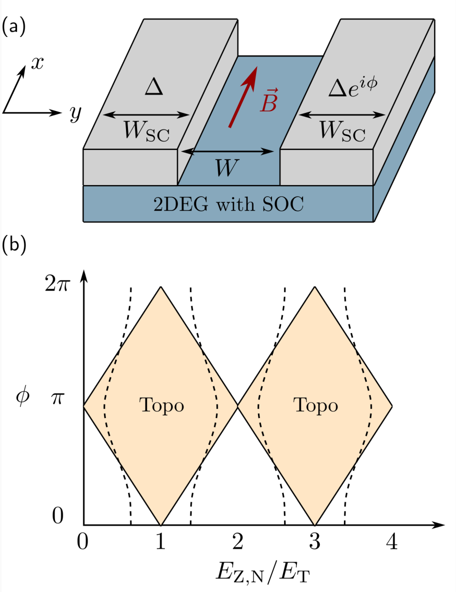
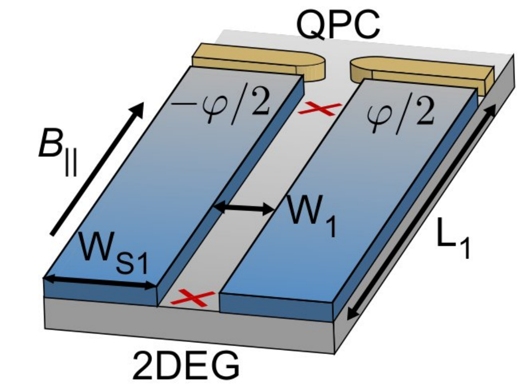



**Diagram of topological superconducting state vs**
**– phase difference $\phi$,**
**– magnetic field $E_z$.**

# Experimental realization (2019)

# PLANAR JOSEPHSON JUNCTIONS

**Two-dimensional electron gas of InAs epitaxially covered by a thin Al layer**




Width:

$W_1$ = **80 nm**

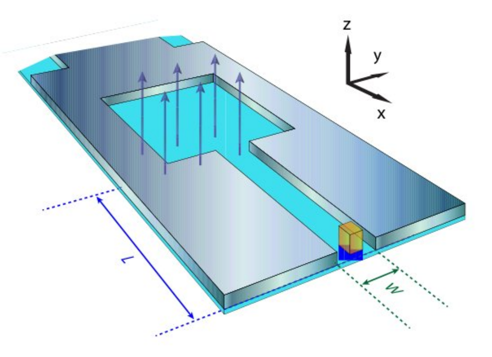
Length:

$L_1$ = **1.6** $\mu$**m**

**A. Fornieri, ..., Ch. Marcus and F. Nichele, Nature 569, 89 (2019).**

**Niels Bohr Institute (Copenhagen, Denmark)**

**Two-dimensional HgTe quantum well coupled to 15 nm thick Al film**




Width:

$W$ **= 600 nm**

Length:

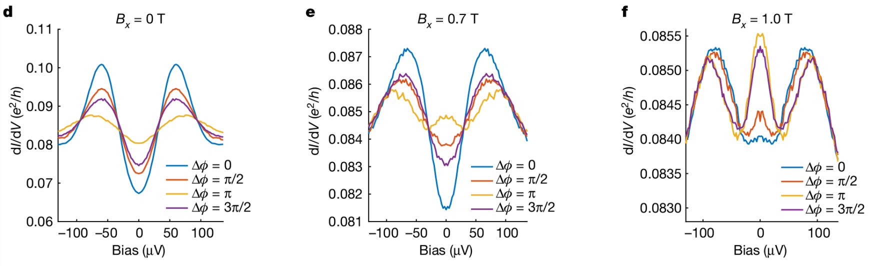
$L$ **= 1.0 $\mu$m**

**H. Ren, ..., L.W. Molenkamp, B.I. Halperin & A. Yacoby, Nature 569, 93 (2019).**

**Würzburg Univ. (Germany) + Harvard Univ. (USA)**

# PLANAR JOSEPHSON JUNCTION: EXPERIMENT

**H. Ren, ..., L.W. Molenkamp, B.I. Halperin & A. Yacoby, Nature 569, 93 (2019).**



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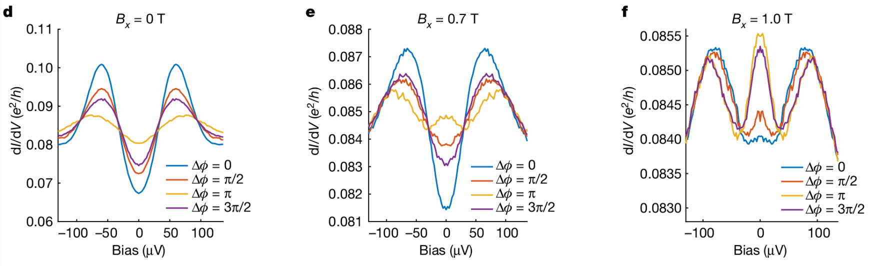
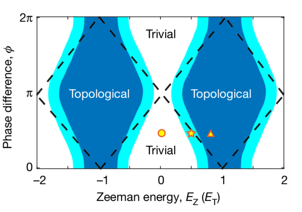


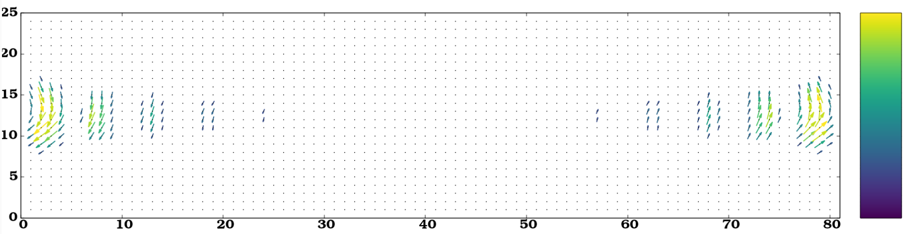

**Experimental data obtained for three different magnetic fields indicated by the symbols in phase diagram ⇒.**



# Topography of Majorana modes

# TOPOGRAPHY OF MAJORANA MODES

**Spatial profile of the zero-energy quasiparticles of a homogeneous metallic strip embedded into the Josephson junction for the phase difference $\phi = \pi$ (which is optimal for topological state).**




**"Majorana polarization" $u_{\uparrow,n}v_{\uparrow,n} - u_{\downarrow,n}v_{\downarrow,n}$ obtained for eigenvalue $E_n = 0$.**

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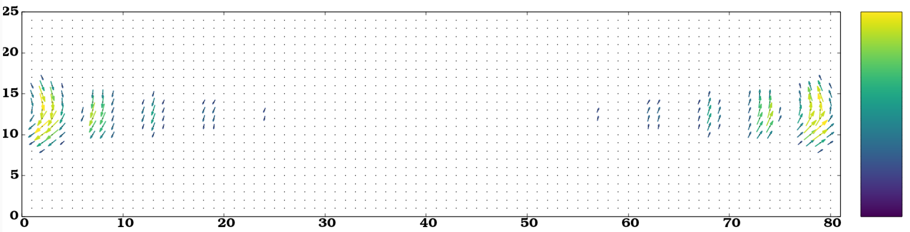
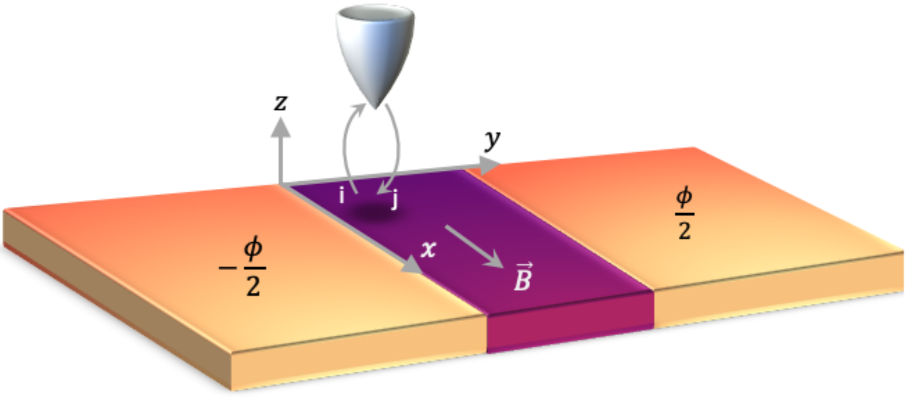



**"Majorana polarization" $u_{\uparrow,n} v_{\uparrow,n} - u_{\downarrow,n} v_{\downarrow,n}$ obtained for eigenvalue $E_n = 0$. Magnitude of this quantity is measurable by the conductance of SESAR spectroscopy. For details see:**

**Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).**

**Selective Equal Spin Andreev Reflection (SESAR) spectroscopy:**

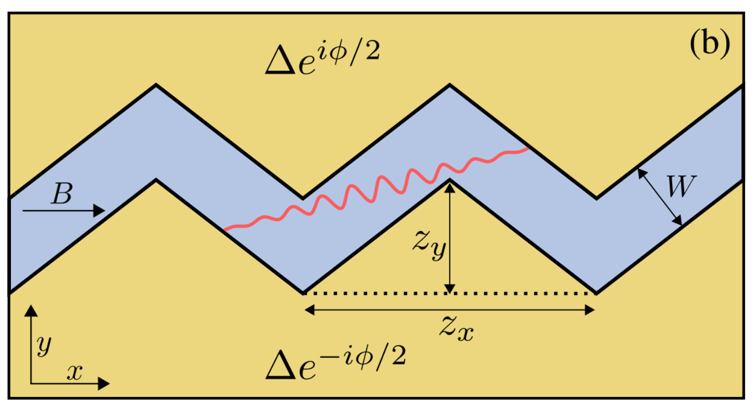



Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

# Means to localize Majoranas

# I. DESHAPED JOSEPHSON JUNCTION

**To reduce spatial extent of the Majorana modes and increase the topological gap one can use zigzag-shape metallic stripe.**




**T. Laeven, B. Nijholt, M. Wimmer & A.R. Akhmerov, PRL 102, 086802 (2020).**

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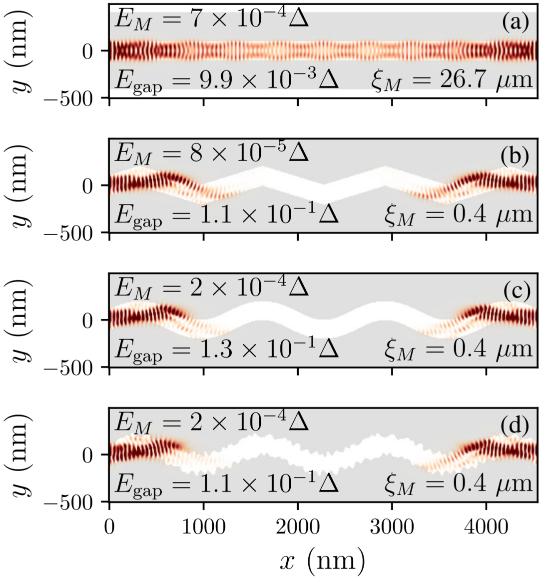
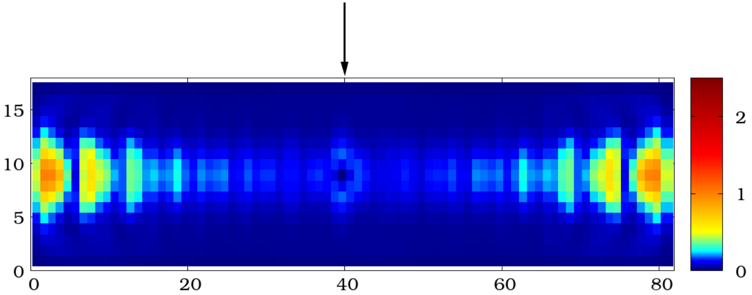



**T. Laeven, B. Nijholt, M. Wimmer & A.R. Akhmerov, PRL 102, 086802 (2020).**

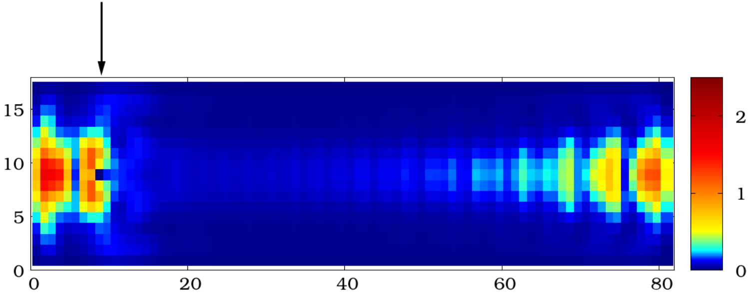
**Spatial profile of the Majorana modes in presence of the strong electrostatic defect placed in the center.**




**Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).**

# II. LOCAL DEFECT IN JOSEPHSON JUNCTION

**Spatial profile of the Majorana modes in presence of the strong electrostatic defect placed near the edge.**




**Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).**

# III. RANDOM DISORDER

**"Benefits of Weak Disorder in dim=1 Topological Superconductors"**

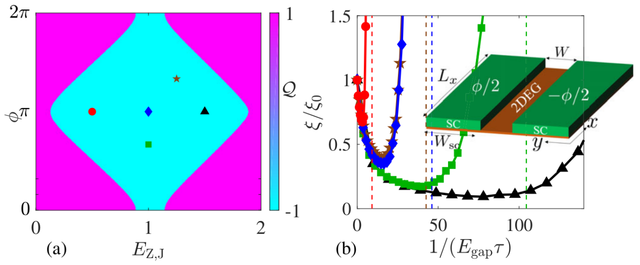



FIG. 1. (a) Phase diagram of the planar Josephson junction Eq. (1) in the clean limit. In the topological phase ($\mathcal{Q} = -1$), the system supports zero-energy MBSs at each end of the junction. (b) The Majorana localization length $\xi$ versus the disorder-induced inverse mean free time $\tau^{-1}$ for different points inside the topological phase [see markers in (a)].

**A. Haim & A. Stern, Phys. Rev. Lett. 122, 126801 (2019).**

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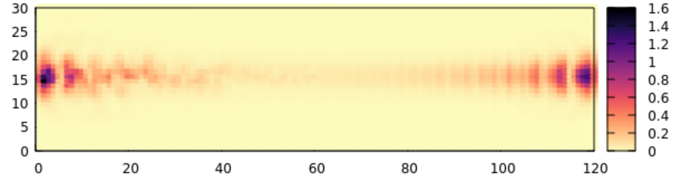
**The left-hand-side part of the metallic stripe is randomly disordered**

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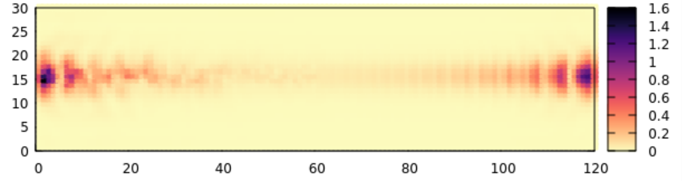
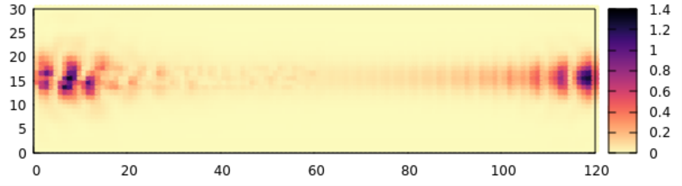



**weak disorder**



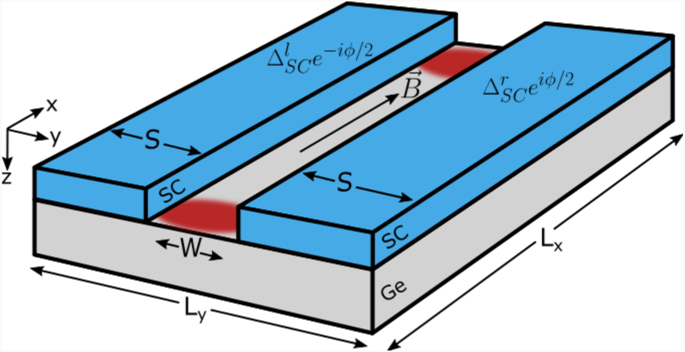

**moderate disorder**

**Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).**

# New proposals

# 1. GERMANIUM BASED PLANAR JJ

**2D hole gas of germanium (Ge) exhibits strong and tunable spin-orbit interaction (cubic in momentum). Such Ge structures can be compatible with the existing metal-oxide-semiconductor (CMOS) technology.**

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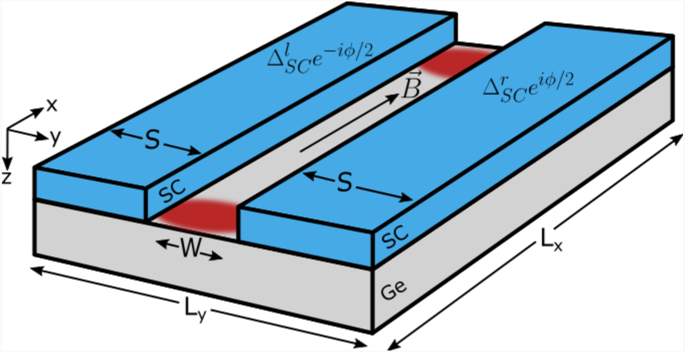


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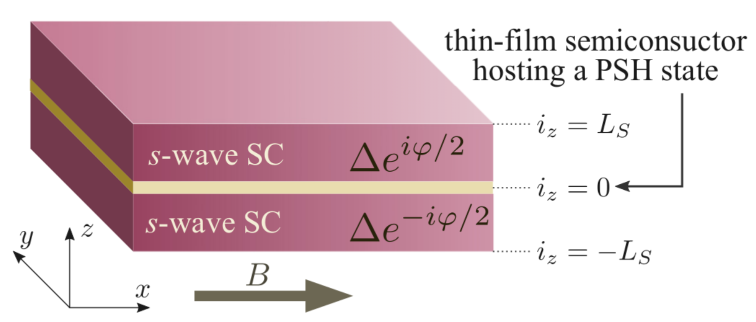
$\Longrightarrow$ **topological phase is asymmetric on phase reversal $\phi \longrightarrow -\phi$**

# 2. VERTICAL JOSEPHSON JUNCTION

**Josephson junction, comprising a thin semiconducting film sandwiched between conventional s-wave superconductors.**

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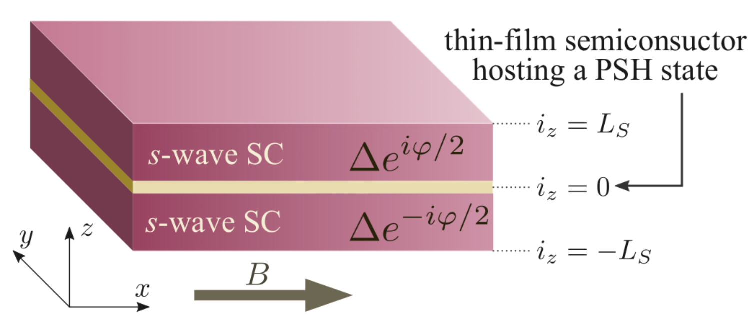


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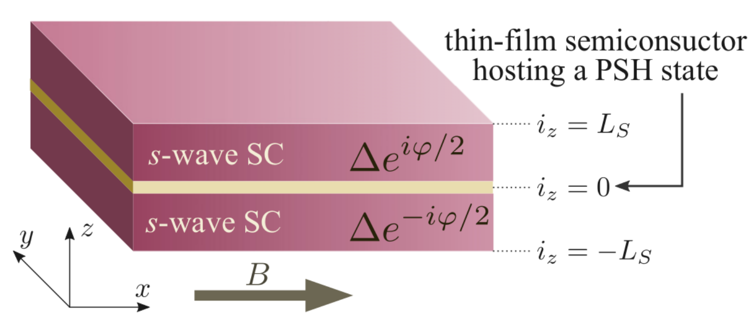


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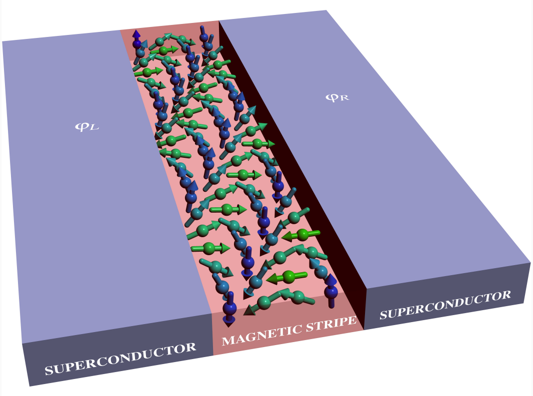
$\Longrightarrow$ semiconducting region hosts a Persistent Spin-Helix state

$\Longrightarrow$ a weak Zeeman field can induce $p_x$-wave superconductivity

# 3. JJ WITH SELFORGANIZED MAGNETIC STRIPE

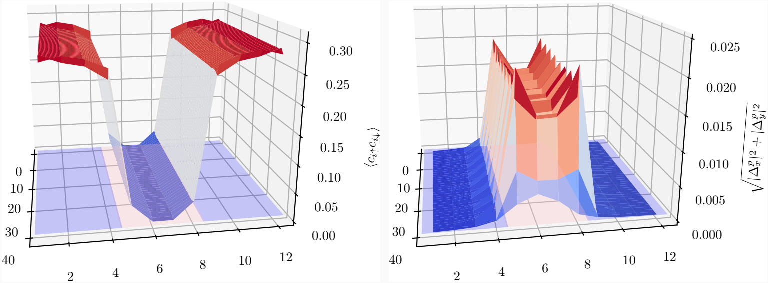
**Narrow metallic stripe with the classical magnetic moments placed between two s-wave superconductors, differing in phase $\phi_L \neq \phi_R$.**




**M.M. Maśka, M. Dziurawiec, M. Strzałka & T.D. – work in progress**

**/ Technical University (Wrocław) & UMCS (Lublin) /**

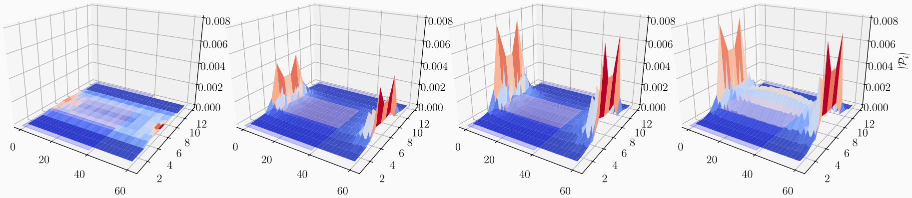
**The effective s-wave (left) and induced p-wave (right) pairings.**




**Width:** **left superconductor (sites 1-4),**
**metallic stripe (sites 5-8),**
**right superconductor (sites 9-12),**

**Length:** **40 sites.**

**Spatial profiles of the Majorana (zero-energy) quasiparticles for selected values of the Josephson phase difference $\phi_R - \phi_L$.**




$\phi_R - \phi_L = 0$ $\phi_R - \phi_L = 0.2\pi$ $\phi_R - \phi_L = 0.4\pi$ $\phi_R - \phi_L = 0.6\pi$

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## PART 2: SUMMARY

**Magnetism in Josephson-type geometries:**

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**http://kft.umcs.lublin.pl/doman/lectures**

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⇒ **Maciek Maśka & coworkers**

**(Technical University, Wrocław)**

⇒ **Nick Sedlmayr**

**(M. Curie-Skłodowska University, Lublin)**

⇒ **Aksel Kobiałka**

**(University of Basel, Switzerland)**


⇒ **Szczepan Głodzik**

**(University of Ljubljana, Slovenia)**

# SINGLY OCCUPIED VS BCS-TYPE CONFIGURATIONS

**The proximitized quantum dot can described by**

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left( \Delta_d \, \hat{d}^{\dagger}_{\uparrow} \hat{d}^{\dagger}_{\downarrow} + \mathrm{h.c.} \right)$$

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**Eigen-states of this problem are represented by:**

$|\uparrow\rangle$ **and** $|\downarrow\rangle$ $\Leftarrow$ **doublet states (spin $\frac{1}{2}$)**

$$\left. \begin{array}{l} u\,|0\rangle - v\,|\uparrow\downarrow\rangle \\ v\,|0\rangle + u\,|\uparrow\downarrow\rangle \end{array} \right\} \Leftarrow$$ **singlet states (spin 0)**

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**Upon varrying the parameters $\varepsilon_d$, $U_d$ or $\Gamma_S$ there can be induced quantum phase transition between these doublet/singlet states.**