1 Runge-Kutta 4th order

On the basis of the article: The PracTEX Journal, 2013, No. 1 Article revision 2013/1/27. *Numerical methods with LuaLATEX* by Juan I. Montijano, Mario Pérez, Luis Rández and Juan Luis Varona.

One of the most popular methods to integrate numerically an initial value problem

$$y'(t) = f(t, y(t)), \qquad y(t_0) = y_0 .$$

is the classical Runge-Kutta method of order 4.

1.1 Equation $y'(t) = y(t)\cos(t + \sqrt{1 + y(t)}$

Here we solve the equation

$$y'(t) = y(t)\cos(t + \sqrt{1 + y(t)}, \qquad y(0) = 1 .$$

using Runge-Kutta method of 4th order. The results are plotted using `tikz` package.

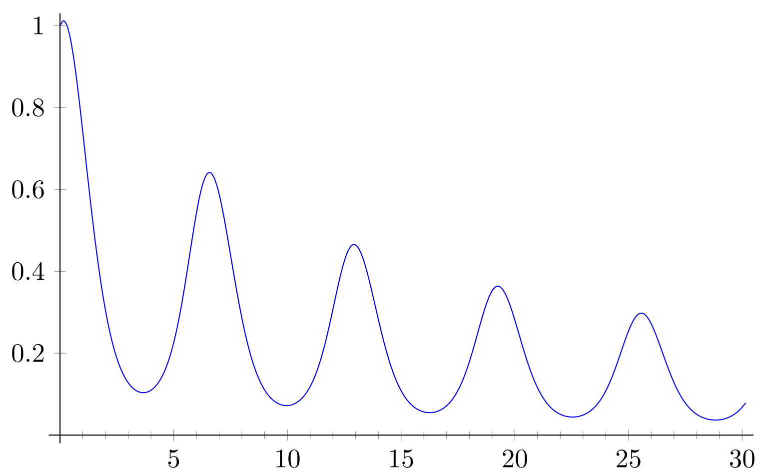

Figure 1: Use of RK4 method.

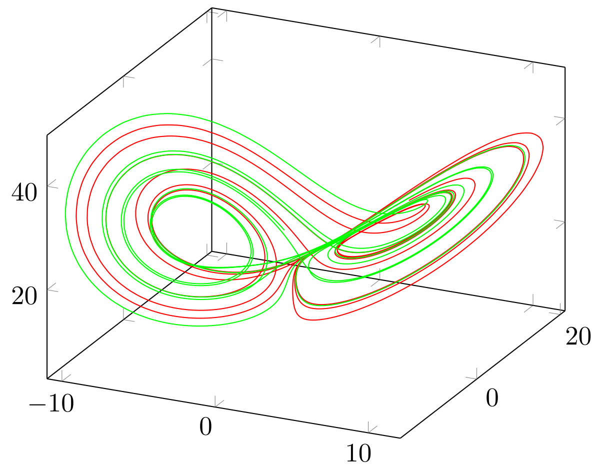

Figure 2: Lorenz attractor (two orbits starting at several initial points).

1.2 Lorenz attractor

The Lorenz attractor is a strange attractor that arises in a system of equations describing the 2-dimensional flow of a fluid of uniform depth, with an imposed vertical temperature difference. In the early 1960s, Lorenz [5] discovered the chaotic behavior of a simplified 3-dimensional system of this problem, now known as the Lorenz equations:

$$\begin{aligned} x'(t) &= \sigma(y(t) - x(t)), \\ y'(t) &= -x(t)z(t) + \rho x(t) - y(t), \\ z'(t) &= x(t)y(t) - \beta z(t). \end{aligned}$$

The parameters σ, ρ, and β are usually assumed to be positive. Lorenz used the values $\sigma = 10$, $\rho = 28$ and $\beta = 8/3$. The system exhibits a chaotic behavior for these values; actually, it became the first example of a chaotic system. A more complete description can be found in [10].

Figure 5 shows the numerical solution of the Lorenz equations calculated with $\sigma = 3$, $\rho = 26.5$ and $\beta = 1$. Six orbits starting at several initial points close to $(0, 1, 0)$ are plotted in different colors; all of them converge to the 3-dimensional chaotic attractor known as the Lorenz attractor.