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FISSION HALF LIVES OF FERMIUM ISOTOPES WITHIN SKYRME HARTREE-FOCK-BOGOLIUBOV THEORY

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Nuclear fission barriers, mass parameters and spontaneous fission half lives of fermium isotopes calculated in a framework of the Skyrme Hartree-Fock-Bogoliubov model with the SkM* force are discussed. Zero-point energy corrections in the ground state are determined for each nucleus using the Gaussian overlap approximation of the generator coordinate method and in the cranking formalism. Results of spontaneous fission half lives are compared to experimental data.

1. Foreword

The field of research of nuclear fission has been turned up more than 70 years ago as a result of the discovery of the process by O. Hahn and F. Strassman explained by L. Meitner and O. Frisch.[1] One of the first creators of fission theories, John Archibald Wheeler, noticed "...we have for the first time in fission a nuclear transformation inescapably *collective* in character."[2] However, nowadays we would like to find the microscopic reasons of the collectivity. Contemporary theories of collective phenomena are based on relatively complex, thought phenomenological, interactions among nucleons leading at some circumstances to large amplitude collective motion (LACM) with fission as an example.

Since the time of the discovery there were proposed plenty of models explaining basic features of the decay. Yet, there is no uniform theory based on fundamental assumptions about nuclear forces which explains in a satisfactory way all of the peculiarities of this complex process of division of many body nuclear system into two fragments. One of the main features which is still a challenge is a variety of observed spontaneous fission half lives (SFHL) of heavy nuclei. The half lives of fermium isotopes can serve as an example.

Experimentally known logarithmic dependence of SFHL of fermium nuclei on the mass or neutron number looks like an inverted parabola showing a maximum at $A = 252$ (see Figure 1). The shortest half-live time corresponds to the isotope

$A = 242$ and the longest one to $A = 252$. The measured SFHL range approximately from 10^{-4} s up to 10^{9} s covering 12–13 decades. Spontaneous fission half lives vary fairly smoothly as one goes from Uranium to Fermium. However, at ^{258}Fm there is a sudden change in all of fission properties, e.g., the fission half-life drops by more than 7 orders of magnitude to 0.370 μs for ^{258}Fm compared to 157.6 m for the previous even-even isotope ^{256}Fm.

To reproduce SFHL in such a broad range of values and to obtain the proper systematics together with its subtleties and peculiarities is especially difficult. Fission half lives are extremely sensitive to details of the shape of fission barriers, on the collective inertia involved and especially on the ground state energy of the nucleus.

In a recent paper[3] an extensive review of methods and results obtained in microscopic-macroscopic approaches and in fully microscopic models is given. The well known macroscopic-microscopic (MM) approach to the problem explains the main features of the process. However, the calculated half lives of the heavier isotopes (with mass numbers $A > 254$) were in all of the approaches few decades too low or too high as compared to experimental data (see Figure 1).

The existing microscopic approaches like self consistent Hartree-Fock (HF) with both the Gogny force Refs.[4,5,6] and the Skyrme effective force Ref.[7] deliver new tools which partly help to overcome the problems mentioned above. The review of basic properties of fission barriers in the framework of the HF+BCS theory with SkM* force was published[8] for isotopes of nuclei ranging from uranium to fermium elements. The paper[5] discusses SFHL of fermium isotopes using the approach with the Gogny force. However, neither the half lives nor the systematics of the half lives are reproduced. The maximal discrepancy is as high as 7 orders of magnitude (see, Ref.[5], Figure 14). A similar approach extended to the isomeric states and to the dynamics of isomeric fission is given in Ref.[6] However, there are no conclusions on the validity of the inertia parameters which were used in the calculations. The SFHL of superheavy nuclei within the Skyrme-HF theory and Skl3, SLy4 and SkP forces have recently been estimated in Ref.[7]. The deviation between theoretical and experimental SFHL of rutherfordium, seaborgium and hassium isotopes exceeds ± 3 decades or more and seems to be too large.

In the case of a relativistic mean field model (RMF) the fission barriers were calculated as well (see e.g., Refs.[9,10]). In the framework of this model the SFHL have not been estimated yet.

The extended discussion of HF+BCS (with the Skyrme SkM* force) fission barriers, possible modes of fission and the half lives of fermium nuclei is performed in our recent paper.[11]

The purpose of the present work is to investigate LACM characteristics of Fermium isotopes in the framework of the pure HFB theory with the SkM* force. We confine our discussion to the dynamic properties related directly to SFHL, namely the mass parameters and dynamical corrections to the fission barriers.

The paper is organized in the following way. In Section 2 we show the way of calculating the fission barriers and mass parameters. In Section 3 we discuss main results of the present work, i.e., the spontaneous fission half lives of even-even fermium isotopes. The short summary is given in the last Section 4.

2. The model

The fission barriers are presented in details in our previous paper.[12] Here we discuss only a zero point energy correction (ZPEC) to the barriers and the inertia tensor.

The calculations were carried out using the code HFODD (v2.245g)[13] that allows for arbitrary symmetry breaking. For the basis, we employed the lowest 1140 single particle states of the deformed harmonic oscillator. This corresponds to 17 oscillator shells at the spherical point. Based on the HFODD self-consistent wave-functions, the collective mass tensor components and ZPEC were computed. The SkM* energy density functional[14] was used in the particle-hole (ph) channel. In the particle-particle (pp) channel we employed the density-dependent pairing interaction in the mixed variant of Refs.[15,16]

$$V_\tau(\vec{r}) = V_{\tau 0}\,(1 - \rho(\vec{r})/2\rho_0)\,\delta(\vec{r})\,, \tag{1}$$

where $\tau = n, p$ and $\rho_0 = 0.16\,\mathrm{fm}^{-1}$. To test the accuracy of various approximations, we carried out both HF+BCS and HFB calculations. The pairing interaction strengths, which were adjusted to reproduce the neutron and proton ground-state pairing gaps in ^{252}Fm, are (in MeV fm^3):

$$V_{n0} = -372.0\,, \quad V_{p0} = -438.0\,, \qquad (\mathrm{HF{+}BCS})\,, \tag{2}$$

$$V_{n0} = -268.9\,, \quad V_{p0} = -332.5\,, \qquad (\mathrm{HFB})\,. \tag{3}$$

2.1. *Zero point energy*

The modifications of fission barriers related to a zero-point energy (ZPEC) are significant. Using the Gaussian overlap approximation (GOA) to GCM the ZPEC reads:[17,18]

$$E_0 = \frac{1}{4}\mathrm{Tr}(\boldsymbol{\Sigma}^{(2)-1}\Sigma^{(1)})\,, \tag{4}$$

where Σ-matrices were defined in Refs[19,20,21].

2.2. *Inertia tensor*

Approximations to the ATDHFB mass parameters used here were already discussed elsewhere.[19,20,21] Here we present only a short account of them.

Both in the GOA and cranking, the inertia tensor can be given by compact expressions[22,17]

$$\boldsymbol{B}^{\mathrm{GOA}} = \boldsymbol{\Sigma}^{(2)}[\boldsymbol{\Sigma}^{(1)}]^{-1}\boldsymbol{\Sigma}^{(2)}\,, \qquad \boldsymbol{B}^{\mathrm{C}} = \boldsymbol{\Sigma}^{(3)}\,. \tag{5}$$

It is easy to see the ZPEC (Eq. 4) can be expressed as

$$E_0 = \frac{1}{4}\mathrm{Tr}(B^{-1}\boldsymbol{\Sigma}^{(1)})\,. \tag{6}$$

2.3. *Half lives*

The following is a short reminder of a method used to calculate the SFHL. Only one dimensional case (one collective degree of freedom) is discussed.

The spontaneous fission half life of a nucleus with respect to the decay from its ground state with an energy E, can be estimated from the following formula[23,24,25]

$$T_{\rm sf} = \frac{\log 2}{n(E)P(E)}\,, \tag{7}$$

where E is a ground state energy of the nucleus and $n(E)$ is a number of assaults of the nucleus on the fission barrier at a given energy E. The $n(E)$ is obtained from an expression $n = 1/T(E)$ where T is a classical period of motion in the potential well $V(q)$. The barrier penetrability $P(E)$ for the two peaked barrier is calculated exactly according to Refs.[26,27,28,29,30]

3. Results

Results of the Skyrme HFB calculation of spontaneous fission half lives are shown in Figure 1. The cranking mass parameters with the perturbative treatment of some derivative terms[21] (ATDHFB-C^p) were used. The ground state energies E_0^C were calculated according to Eq. 6 in the first minimum for each isotope. They are displayed in the lower part of Figure 1. The same Figure shows calculated SFHL (open symbols) as compared to the experimental data (see Refs.[31,32,33,34]) and to microscopic-macroscopic (MM) calculations (crosses: MM-B[25] and pluses[35]). The half lives of heavy isotopes in MM models are not reproduced within a tolerable accuracy. The average deviation of SFHL in present calculations is much less than 1.5 orders of magnitude. Very similar calculations for the zero point energy corrected barriers were also performed. The results differ 0.5 order of magnitude from the uncorrected case. In the case of average impingement energy $E_0 = 0.8$ MeV, common for all nuclei, the disagreement with experiment is similar.

In the case of ATDHFB-GOA mass parameters the shape of the systematics does not change significantly. However, the half lives obtained with the ATDHFB-GOA mass parameters are few orders of magnitude shorter than in the case of ATDHFB-C^p.

The cranking mass parameters $B_{Q_{20}Q_{20}}(Q_{20})$ as a function of Q_{20} are shown in Figure 2. According to Ref.[21] there are shown the following approximations to an adiabatic time dependent Hartree-Fock inertia: ATDHFB-C^p, ATDHFB-GOA, ATDBCS-C^p and ATDBCS-GOA. The p-superscript means the perturbative treatment of some derivative terms in mass formulae.[21] The mass displays an interesting structure which on the average (in the large scale of deformation) is a monotonically decreasing function of the collective deformation Q_{20}. In the region of the first minimum and at the first barrier (30-50 b) it shows a peak which value is 30-35% larger than the average one. The structure is rather typical for all of considered nuclei. The calculated GOA parameters are usually twice as small as cranking ones. On the average the ATDHFB-C^p mass parameters are larger than the other ones.

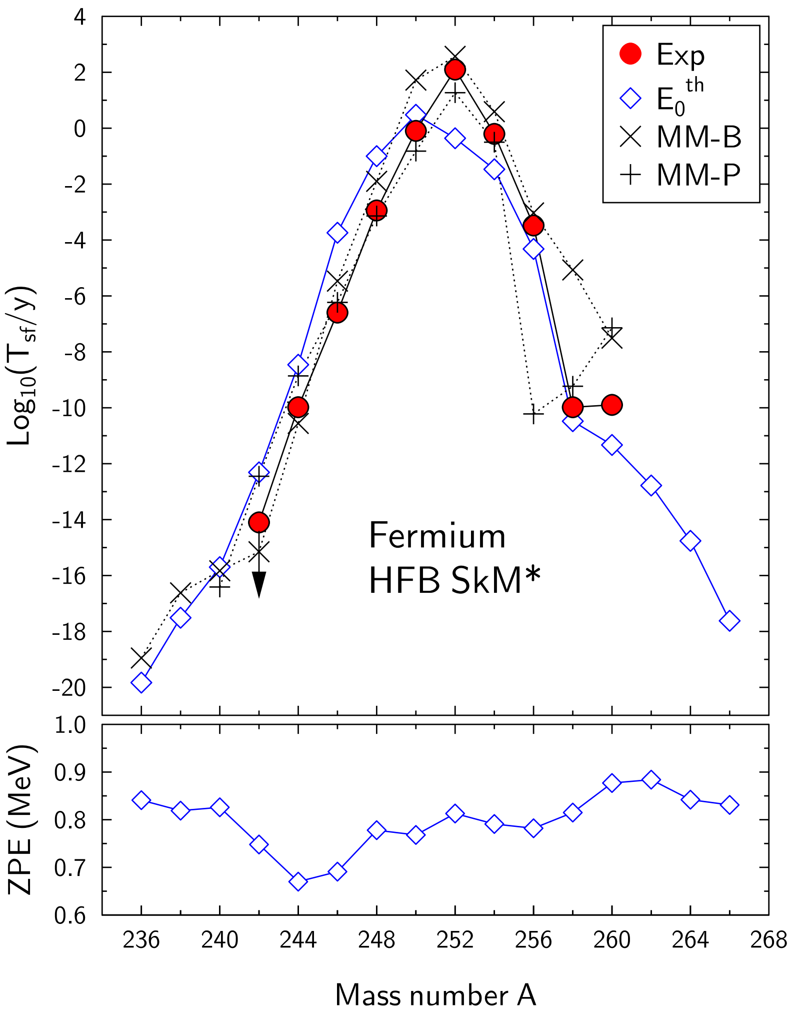

Fig. 1. (Color online) Spontaneous fission half lives of fermium isotopes in the HFB SkM* theory (upper panel). Calculated data are pointed by open symbols. The experimental data (filled symbols) are taken from Refs.[31,32,33,34] Microscopic-macroscopic calculations are shown as crosses (MM-B)[25] and pluses (MM-P)[35]. The cranking model of inertia parameters (ATDHFB-C^p) were used. The corresponding ground state energies (the impingement energies) E_0 and its average value are shown in the lower panel.

4. Summary and perspectives

In this study we performed calculations of collective inertia, zero-point quadrupole correlation energy, fission barriers and spontaneous fission half lives of Fermium isotopes. The HFODD code employed allows for an arbitrary symmetry breaking; this feature is of crucial importance when discussing spontaneous fission where the

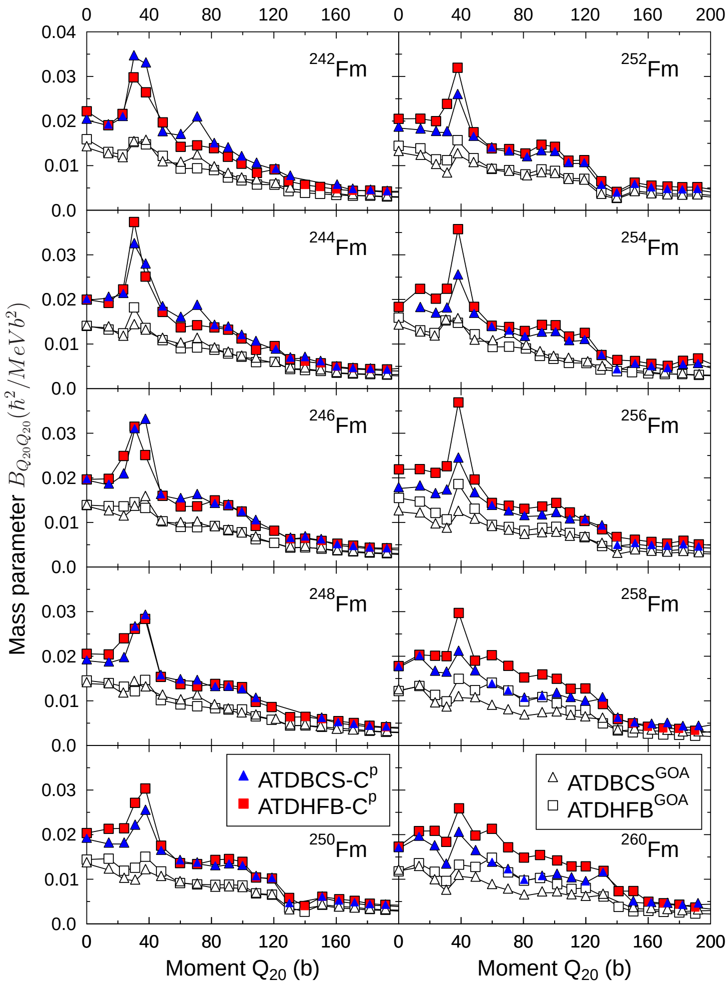

Fig. 2. (Color online) Cranking mass parameter $B_{Q_{20}Q_{20}}$ (in units $\hbar^2$/MeV b^2) for Fermium isotopes vs. mass quadrupole moment Q_{20}. Both GOA (open symbols) and cranking (filled symbols) mass are shown for two types of calculations: HF+BCS (triangles) and HFB (squares). In all considered cases the SkM* force was used.

reflection-asymmetric and triaxial shapes can play a role. The main conclusions of this work can be summarized as follows:

- The collective quadrupole inertia calculated with the force SkM* within the HFB theory have a very similar deformation pattern to those in the HF+BCS model.
- In both the HF+BCS and HFB approaches the collective inertia obtained

in the cranking approximation are about twice as large as the GOA results.

- The half-lives of Fermium chain show a right, observed systematics and approximate the experimental data with an accuracy which is better than 1 order and a half.
- The zero-point correlation energy is important to include as it modifies the action integrals. In the present approach the impingement energies on the barrier were assumed equal to ZPEC at the ground states. They depend on the isotope considered. However, the detailed fluctuations of theoretical E_0 on calculated half lives are not essential.

The perspectives are the following:

- The calculation of time odd terms in ATDHFB inertia parameters. (see discussion in Ref.[21])
- A systematics of fission half lives of actinide nuclei in the framework of the full HFB theory.
- All of this investigations are very important steps in predicting properties of superheavy nuclei - a goal of our future research.

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