XVII Nuclear Physics
Workshop
Maria and Pierre Curie,
Kazimierz Dolny, Poland,
September 22 – 26, 2010

FISSION HALF LIVES OF FERMIUM ISOTOPES WITHIN SKYRME HARTREE-FOCK-BOGOLIUBOV THEORY

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Abstract

Nuclear fission mass parameters and spontaneous fission half lives of Fermium isotopes calculated in the framework of Skyrme Hartree-Fock-Bogoliubov (HFB) model with SkM* force are discussed. Zero-point energy corrections (ZPEC) in the ground state are determined for each nucleus using gaussian overlap approximation to generator coordinate method and the cranking formalism. Results of spontaneous fission half lives are compared to the experimental data.

Foreword

Experimentally known logarithmic dependence of SFHL of Fermium nuclei on the mass or neutron (N) number looks like an inverted parabola. The shortest half-live time corresponds to the isotope $A = 242$ and the longest one to $A = 252$. The measured SFHL range approximately from 10^{-4} s up to 10^9 s covering 12–13 orders of magnitude. Spontaneous fission half lives vary fairly smoothly as one goes from Uranium to Fermium. However, at ^{258}Fm there is a sudden changes in all of fission properties, *e.g.*, the fission half-life drops by more then 7 orders of magnitude to 0.370 μs for ^{258}Fm compared to 157.6 m for the previous even-even isotope ^{256}Fm.

The purpose of the present work is to investigate characteristics of Fermium isotopes within HFB theory with SkM* force. We confine our discussion to the dynamic properties related directly to SFHL namely mass parameters, dynamical corrections to the fission barriers and spontaneous fission half lives.

The model

The fission barriers are presented in details in our previous paper.[1] Here we discuss only a zero point energy correection (ZPEC) to the barriers and the inertia tensor.

The modifications of fission barriers related to a zero-point energy (ZPEC) are significant. Using the gaussian overlap approximation (GOA) to GCM the ZPEC reads:[2]

$$E_0 = \frac{1}{4}\mathrm{Tr}(\boldsymbol{\Sigma}^{(2)-1}\Sigma^{(1)})\,, \tag{1}$$

where

$$\boldsymbol{\Sigma}^{(k)} = \frac{1}{4}\boldsymbol{M}^{(1)-1}\boldsymbol{M}^{(k)}\boldsymbol{M}^{(1)-1} \tag{2}$$

is expressed in terms of the inverse energy moments $\boldsymbol{M}^{(k)}$ which are defined by the folowing equation

$$M_{ij}^{(k)}(\boldsymbol{q}) = \sum_{\mu\nu}\frac{\langle\phi|\hat{Q}_i|\mu\nu\rangle\langle\mu\nu|\hat{Q}_j|\phi\rangle}{(E_\mu + E_\nu)^k}\,. \tag{3}$$

The collective coordinates (collective degrees of freedom) are defined as average values of multipole operators.

Approximations to the ATDHFB mass parameters we are using here were already discussed elsewhere.[3, 4, 5] Here we present only a short account of them.

The inertia tensor B is calculated in both GOA and the cranking (C) model. The main task is the calculation of $\boldsymbol{M}^{(k)}$, defined in Eq. 3.

$$\boldsymbol{B}^{\mathrm{GOA}} = \boldsymbol{\Sigma}^{(2)}[\boldsymbol{\Sigma}^{(1)}]^{-1}\boldsymbol{\Sigma}^{(2)}\,, \qquad \boldsymbol{B}^{\mathrm{C}} = \boldsymbol{\Sigma}^{(3)}\,. \tag{4}$$

It can be seen the ZPEC (Eq. 1) can be expressed as

$$E_0 = \frac{1}{4}\mathrm{Tr}(B^{-1}\boldsymbol{\Sigma}^{(1)})\,. \tag{5}$$

The spontaneous fission half life of a nucleus with respect to the decay from its ground state with an energy E_0, can be estimated from the following formula

$$T_{\mathrm{sf}} = \frac{\log 2}{n(E_0)P(E_0)}\,, \tag{6}$$

where E_0 is a ground state energy of the nucleus and $n(E)$ is a number of assaults of the nucleus on the fission barrier at a given energy E. The $n(E)$ is obtained from an expression $n = 1/T(E)$ where T is a classical period of motion in the potential well $V(q)$. The barrier penetrability $P(E)$ for the two peaked barrier is calculated exactly according to Refs.[6, 7, 8]

Results

The calculations were carried out using the code HFODD (v2.245g).[9] For the basis, we employed the lowest 1140 single particle states of the deformed harmonic oscillator. This corresponds to 17 oscillator shells at the spherical point. Based on the HFODD self-consistent wave-functions, the collective mass tensor components and ZPEC were computed. The SkM* energy density functional[10] was used in the particle-hole (ph) channel. In the particle-particle (pp) channel we employed the density-dependent pairing interaction

$$V_\tau(\vec{r}) = V_{\tau 0}\left(1 - \rho(\vec{r})/2\rho_0\right)\delta(\vec{r})\,, \tag{7}$$

where $\tau = n, p$ and $\rho_0 = 0.16\,\mathrm{fm}^{-1}$. The pairing interaction strengths, which were adjusted to reproduce the neutron and proton ground-state pairing gaps in ^{252}Fm, are (in MeV fm^3):

$$V_{n0} = -372.0\,, \quad V_{p0} = -438.0\,, \qquad (\mathrm{HF{+}BCS})\,, \tag{8}$$

$$V_{n0} = -268.9\,, \quad V_{p0} = -332.5\,, \qquad (\mathrm{HFB})\,. \tag{9}$$

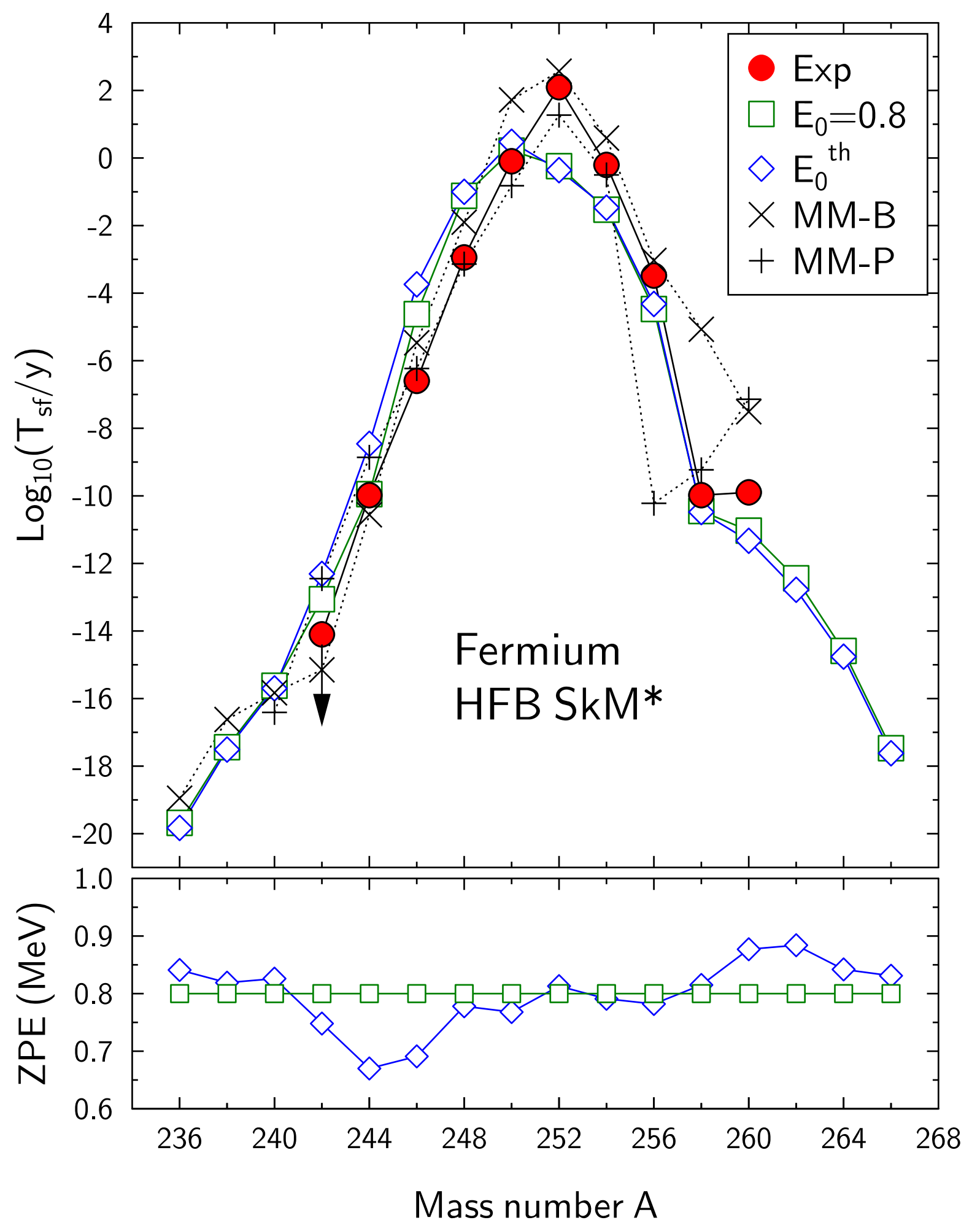

Figure 1: Spontaneous fission half lives of Fermium isotopes in HFB SKM* theory (upper panel). Calculated data are pointed by open symbols. The experimental data (filled symbols) is taken from Ref.[11, 12, 13] The cranking model of inertia parameters (ATDHFB-C^p) were used. Ground state energies (the impingement energies) E_0^C are shown in the lower panel.

Results for SFHL are shown in Figure 1. The cranking mass parameters (ATDHFB-C^p) were used. The ground state energies E_0^C were calculated according to Eq. 5 and they are shown in the upper part of Figure 1. The same Figure shows calculated SFHL (open symbols). The experimantal data are taken from Refs.[11, 14, 13]). Similar calculations for the ZPEC barriers were performed as well. The results differ 0.5 order of magnitude from the presented ones. In the case of impingement energy $E_0 = 0.8$ MeV common for all nuclei the disagreement with experiment is one order of magnitude.

The cranking mass parameters $B_{Q_{20}Q_{20}}(Q_{20})$ as a function of Q_{20} are shown in Figure 2. There are shown the following approximations to an adiabatic time dependent Hartre-Fock inertia: ATDHFB-C^p, ATDHFB$^{\mathrm{GOA}}$, ATDBCS-C^p and ATDBCS$^{\mathrm{GOA}}$.[5]

Summary

In this study we performed calculations of collective inertia, zero-point quadrupole correlation energy, fission barriers and spontaneous fission half lives of Fermium isotopes.

- **The half-lives of Fermium chain show a right, observed systematics and approximate the experimental data with an accuracy which is better then 1 order and a half.**
- **The zero-point correlation energy is important to include as it modifies the action integrals. In the present approach an impingement energies on the barrier were assumed equal to ZPEC at the ground states. They depend on the isotope considered. However, the detailed fluctuations of theoretical E_0 on calculated half lives are not essential.**
- **The collective quadrupole inertia calculated with the force SkM* within HFB theory have a very similar deformation pattern to those in the HF+BCS model.**
- **In both HF+BCS and HFB approaches the collective inertia obtained in the cranking approximation are about twice as large as the GOA results.**

The perspectives are the following:

- **The calculation of time odd terms in ATDHFB inertia parameters. (see discussion in Ref.[5])**
- **A systematics of fission half lives of actinide nuclei in the framework of full HFB theory.**
- **All of this investigations are very important steps in predicting properties of superheavy nuclei - the goal of our future research.**

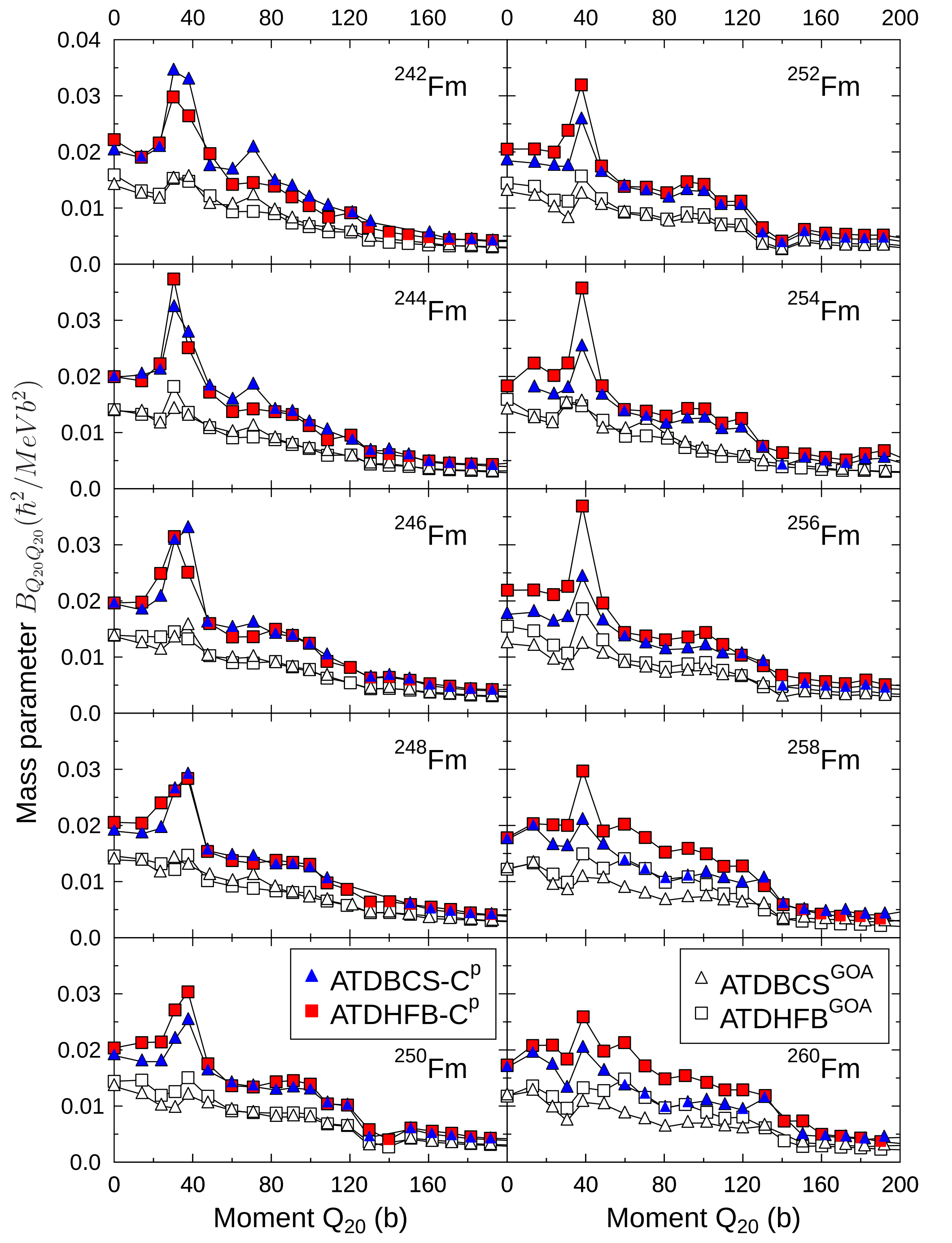

Figure 2: Cranking mass parameter $B_{Q_{20}Q_{20}}$ (in units $\hbar^2$/MeV b^2) for Fermium isotopes vs. mass quadrupole moment Q_{20}. Both GOA (open symbols) and cranking (filled symbols) mass are shown for two types of calculations: HF+BCS (triangles) and HFB (squares). In all considered cases the SkM* force was used.

Acknowledgements

This work was supported in part by the National Nuclear Security Administration by the National Nuclear Security Administration under the Stewardship Science Academic Alliances program through DOE Grant DE-FG52-09NA29461; by the U.S. Department of Energy under Contract Nos. DE-FG02-96ER40963 (University of Tennessee), and DE-FC02-09ER41583 (UNEDF SciDAC Collaboration); by the NEUP grant DE-AC07-05ID14517 (sub award 00091100); by the Polish Ministry of Science under Contracts No. N N202 231137. Computational resources were provided by the National Center for Computational Sciences at Oak Ridge National Laboratory and the National Energy Research Scientific Computing

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