# BREAKING OF AXIAL AND REFLECTION SYMMETRIES IN SPONTANEOUS FISSION OF FERMIUM ISOTOPES 

A. STASZCZAK ${ }^{a, b, c}$, A. BARAN ${ }^{a, b, c}$, W. NAZAREWICZ ${ }^{b, c, d}$<br>${ }^{a}$ Department of Theoretical Physics, Institute of Physics, Maria Curie-Sktodowska University, pl. M. Curie-Sktodowskiej 1, 20-031 Lublin, Poland<br>${ }^{b}$ Department of Physics and Astronomy, University of Tennessee, Knoxville, TN 37996, USA<br>${ }^{c}$ Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA<br>${ }^{d}$ Institute of Theoretical Physics, University of Warsaw, ul. Ho $\dot{z}$ 69, 00-681 Warsaw, Poland<br>Received (received date)<br>Revised (revised date)


#### Abstract

The nuclear fission phenomenon is a magnificent example of a quantal collective motion during which the nucleus evolves in a multidimensional space representing shapes with different geometries. The triaxial degrees of freedom are usually important around the inner fission barrier, and reduce the fission barrier height by several MeV . Beyond the inner barrier, reflection-asymmetric shapes corresponding to asymmetric elongated fragments come into play. We discuss the interplay between different symmetry breaking mechanisms in the case of even-even fermium isotopes using the Skyrme HFB formalism.


## 1. Introduction

With few exceptions, nuclei have axially symmetric and reflection symmetric mass distributions in their ground states. On its path to fission, between the ground state and the scission point, the nucleus undergoes various shape changes controlled by shell structure. The optimum collective trajectory in a multidimensional space that minimizes the collective action can be associated with a sequence of spontaneously broken symmetries, including axial and mirror symmetries.

Fission half-lives and fragment properties strongly depend on proton and neutron numbers governing configuration changes along the fission path. Variations of underlying mean fields are associated with crossing of single-particle levels having different quantum numbers. Triaxial shapes are most important around the first fission barrier while reflection-asymmetric mean-fields are responsible for asymmetric fission. The competition between various fission pathways characterized by different broken symmetries is behind such phenomena as bimodal ${ }^{1}$ or multimodal ${ }^{2}$ fission.

In this work, we study the competition between axial, triaxial, and reflectionasymmetric shapes on the way to fission within the Hartree-Fock-Bogoliubov (HFB) theory and the Skyrme SkM* energy density functional. ${ }^{3}$ The importance of consecutive symmetry breaking effects in fission due to diabatic configuration changes has been well recognized (see e.g. Refs. ${ }^{4,5}$ ). The rich literature includes microscopic-
macroscopic approaches ${ }^{6,7,8,9,10}$ and self-consistent Hartree-Fock+BCS and HFB theory. ${ }^{11,12,13,14,15,16,17}$ The energy gain due to symmetry breaking strongly depends on the nucleus and the model employed. When comparing various calculations, one should bear in mind that in the case of macroscopic-microscopic models shape degrees of freedom are introduced explicitly through deformation parameters. Consequently, the corresponding mean fields are restricted by this choice. On the other hand, a symmetry-unconstrained self-consistent theory (HFB+BCS or HFB) explores the full collective space through the symmetry-breaking mechanism.

## 2. Results

The HFB calculations were performed for even-even fermium isotopes with neutron numbers $N=136-166$. We used the $\mathrm{SkM}^{*}$ energy density functional ${ }^{3}$ in the particle-hole channel. In the particle-particle channel we employed the densitydependent $\delta$-interaction of Ref., ${ }^{18}$ fitted to experimental pairing gaps of ${ }^{252} \mathrm{Fm}$. The calculations were carried out with the symmetry-free HFB code HFODD (v. 2.43 c ) of Ref. ${ }^{19}$ For the basis, we took the lowest 1140 single-particle states of the deformed harmonic oscillator with $N_{\text {shell }}=26$.

The specific static fission pathways considered in this study have been obtained in the calculations of Refs. ${ }^{20,21}$ Specifically, for the fermium isotopes considered, one predicts both reflection-symmetric (s) and reflection-asymmetric (a) fission valleys. Two kinds of reflection-symmetric paths are predicted; namely, the valley that corresponds to elongated fission fragments (EF) and that with more compact fragments (CF), containing spherical ${ }^{132} \mathrm{Sn}$-like clusters when approaching ${ }^{264} \mathrm{Fm}$. The shorthand notation sEF means symmetric elongated fragments fission path; similarly labels aEF and sCF denote asymmetric elongated and symmetric compact fragments, respectively.

Figure 1 shows the energy curves predicted for the even-even fermium isotopes. The differences between the dashed and solid lines illustrate the barrier reduction due to triaxial shapes. The magnitude of the axial symmetry breaking strongly varies with $A$. For $236 \leq A \leq 240$ the effect is negligible, but for the heavier fermium isotopes the reduction of the inner barrier is about $3-3.5 \mathrm{MeV}$.

Comparing the results for ${ }^{256} \mathrm{Fm}$ and ${ }^{258} \mathrm{Fm}$, one can see the disappearance of a second (outer) barrier in ${ }^{258} \mathrm{Fm}$ and the heavier isotopes. This is related to the change of the most dominant fission mode for these nuclei, from aEF for $240 \leq A \leq$ 256 to sCF for $258 \leq A \leq 266$. The ground-state zero-point energies $E$ marked in Fig. 1 were obtained in the gaussian overlap approximation to the generator coordinate method. ${ }^{21}$ They vary from 0.67 MeV in ${ }^{244} \mathrm{Fm}$ to 0.89 MeV in ${ }^{260,262} \mathrm{Fm}$. These variations have a substantial impact on predicted fission half-lives. ${ }^{21}$

To better illustrate different fission modes predicted in our calculations, we present in Fig. 2 the total density distributions corresponding to pre-scission configurations. The sEF mode prevails in ${ }^{236,238} \mathrm{Fm}$, the aEF mode predominates for ${ }^{240-256} \mathrm{Fm}$, and the sCF mode is most important for ${ }^{258-266} \mathrm{Fm}$. The quadrupole


Fig. 1. Potential energy curves in $\mathrm{HFB}+\mathrm{SkM}^{*}$ for even-even fermium isotopes $(A=236-266)$ plotted against the mass quadrupole moment $Q_{20}$. The results of calculations with axial symmetry are marked by dashed lines. The zeropoint energies $E$ shown for each nucleus were calculated in the Gaussian overlap approximation to the generator coordinate method.


Fig. 2. Pre-scission shapes of fermium isotopes shown in Fig. 1. The values of $Q_{20}$ are indicated.
moments of pre-scission configurations in the heavy fermium isotopes ${ }^{258-266} \mathrm{Fm}$ $\left(Q_{20} \approx 260 \div 270 \mathrm{~b}\right)$ are well below those in ${ }^{240-256} \mathrm{Fm}\left(Q_{20} \approx 400 \div 410 \mathrm{~b}\right)$ and ${ }^{236,238} \mathrm{Fm}\left(Q_{20} \approx 450 \mathrm{~b}\right)$.

## 3. Summary

The HFB calculations performed here with the HFODD computer code allow for an arbitrary symmetry breaking; this feature is essential for the microscopic description of the fission process where triaxial and reflection-asymmetric effects come into play.

The main conclusions of this work can be summarized as follows:

- For the heavy fermium isotopes $(A \geq 256)$, triaxiality lowers the inner barrier by about $3-3.5 \mathrm{MeV}$.
- In ${ }^{236,238} \mathrm{Fm}$, the symmetric elongated path sEF is predicted to be the lowest.
- The isotopes ${ }^{240-256} \mathrm{Fm}$ fission along the asymmetric elongated fission path aEF.
- In ${ }^{260-266} \mathrm{Fm}$, the symmetric compact fission pathway sCF dominates.
- For $A=258$ our calculations predict the bimodal fission (sEF +sCF ), see Ref. ${ }^{2}$


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## References

1. E. K. Hulet et al., Phys. Rev. C40, 770 (1989).
2. A. Staszczak, A. Baran, J. Dobaczewski, and W. Nazarewicz, Phys. Rev. C80, 014309 (2009).
3. J. Bartel et al., Nucl. Phys. A386, 79 (1982).
4. J. W. Negele, Nucl. Phys. A502, 371 (1989).
5. W. Nazarewicz, Nucl. Phys. A557, 489 (1993).
6. S. E. Larsson, I. Ragnarsson, and S. G. Nilsson, Phys. Lett. 38B, 269 (1972).
7. S. E. Larsson and G. Leander, in Physics and Chemistry of Fission 1973, vol. 1, p. 177. IAEA, Vienna, Austria, 1974.
8. V. V. Pashkevich, Nucl. Phys. A477, 1 (1988).
9. S. Cwiok and A. Sobiczewski, Z. Phys. A342, 203 (1992).
10. P. Möller, D. G. Madland, A. J. Sierk, and A. Iwamoto, Nature 409, 785 (2001).
11. S. Ćwiok, P.-H. Heenen, and W. Nazarewicz, Nature 433, 705 (2005).
12. L. Bonneau, P. Quentin, and D. Samsoen, Eur. J. Phys. A21, 391 (2004).
13. L. Bonneau, Phys. Rev. C74, 014301 (2006).
14. M. Warda, J. L. Egido, L. M. Robledo, and K. Pomorski, Phys. Rev. C66, 014310 (2002).
15. J.-P. Delaroche, M. Girod, H. Goutte, and J. Libert, Nucl. Phys. A771, 103 (2006).
16. M. Bender et al., Phys. Rev. C58, 2126 (1998).
17. H. Abusara, A. V. Afanasjev, and P. Ring, Phys. Rev. C82, 044303 (2010).
18. M. Warda, A. Staszczak, and L. Próchniak, Int. J. Mod. Phys. E19, 787 (2010).
19. J. Dobaczewski et al., Comput. Phys. Commun. 180, 2361 (2009).
20. A. Staszczak, J. Dobaczewski, and W. Nazarewicz, Acta Phys. Pol. B38, 1589 (2007).
21. A. Baran, A. Staszczak, and W. Nazarewicz, Int. J. Mod. Phys. E, this volume (2011).
