HFB, SkM* calculations.

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Bariers, mass parameters, rotational energy corrections, vibrational corrections, ..., and fission half-lives for Fermium isotopes in HFB model with MIXED delta-pairing.

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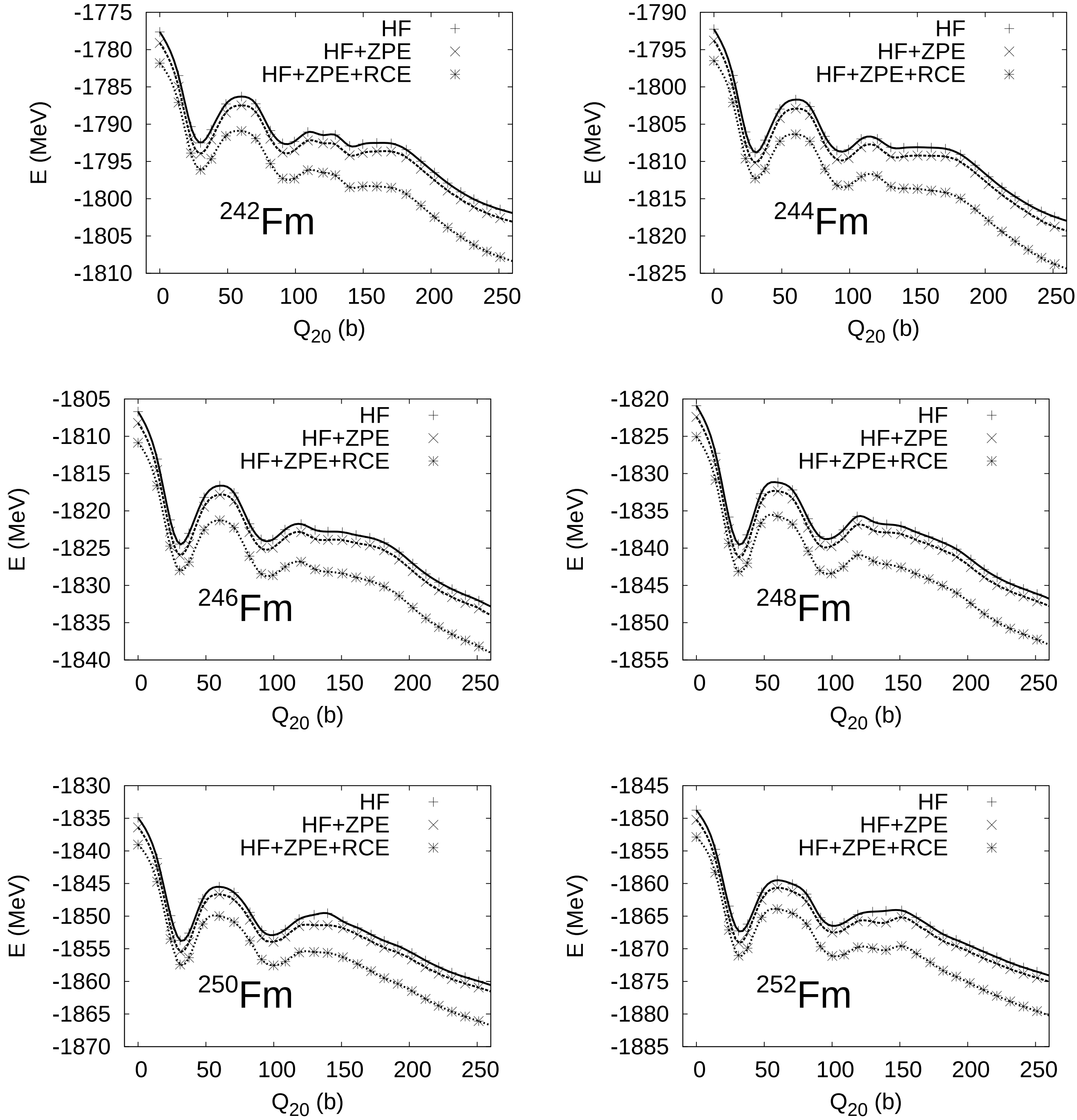
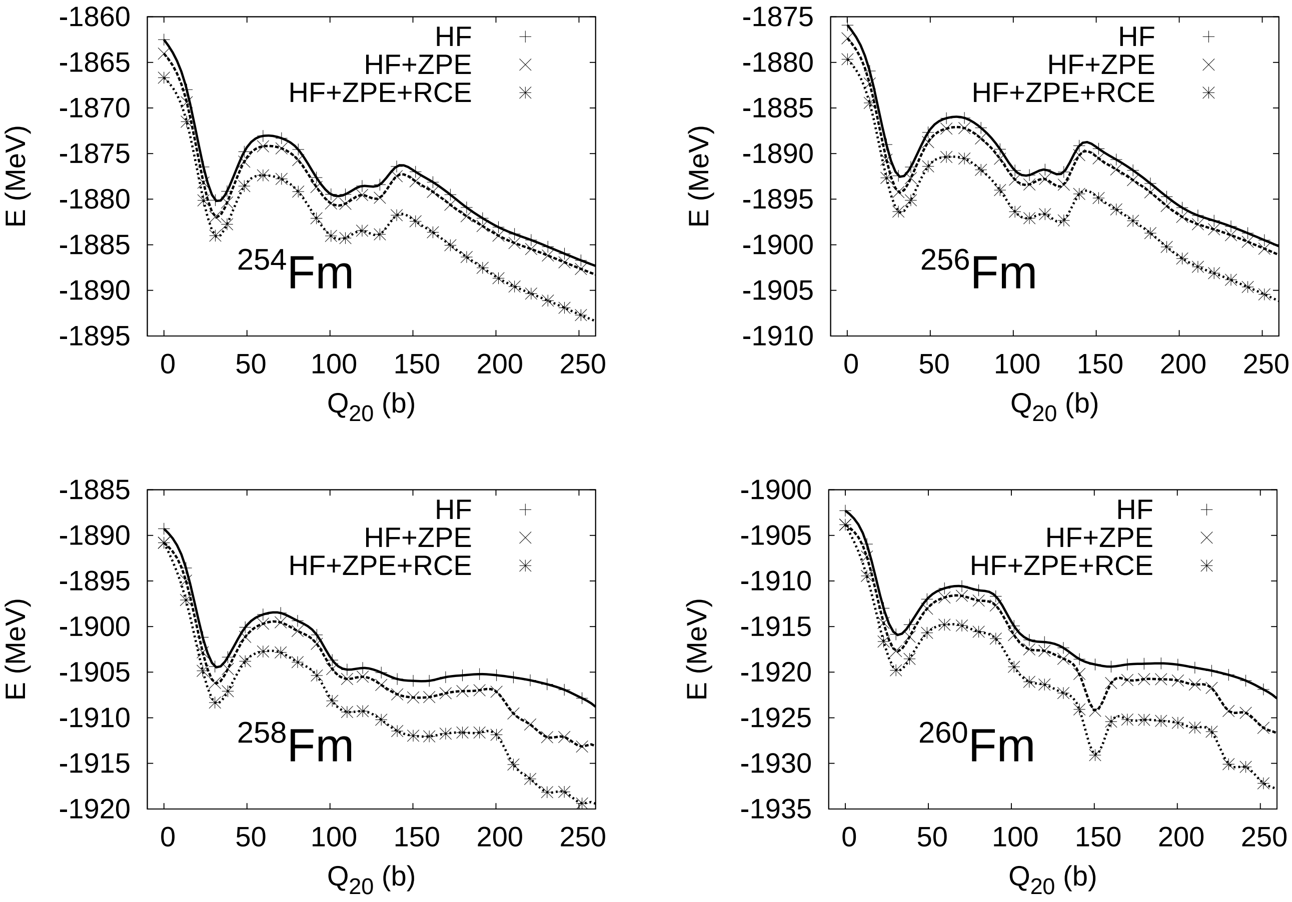
I. SELECTED PROPERTIES

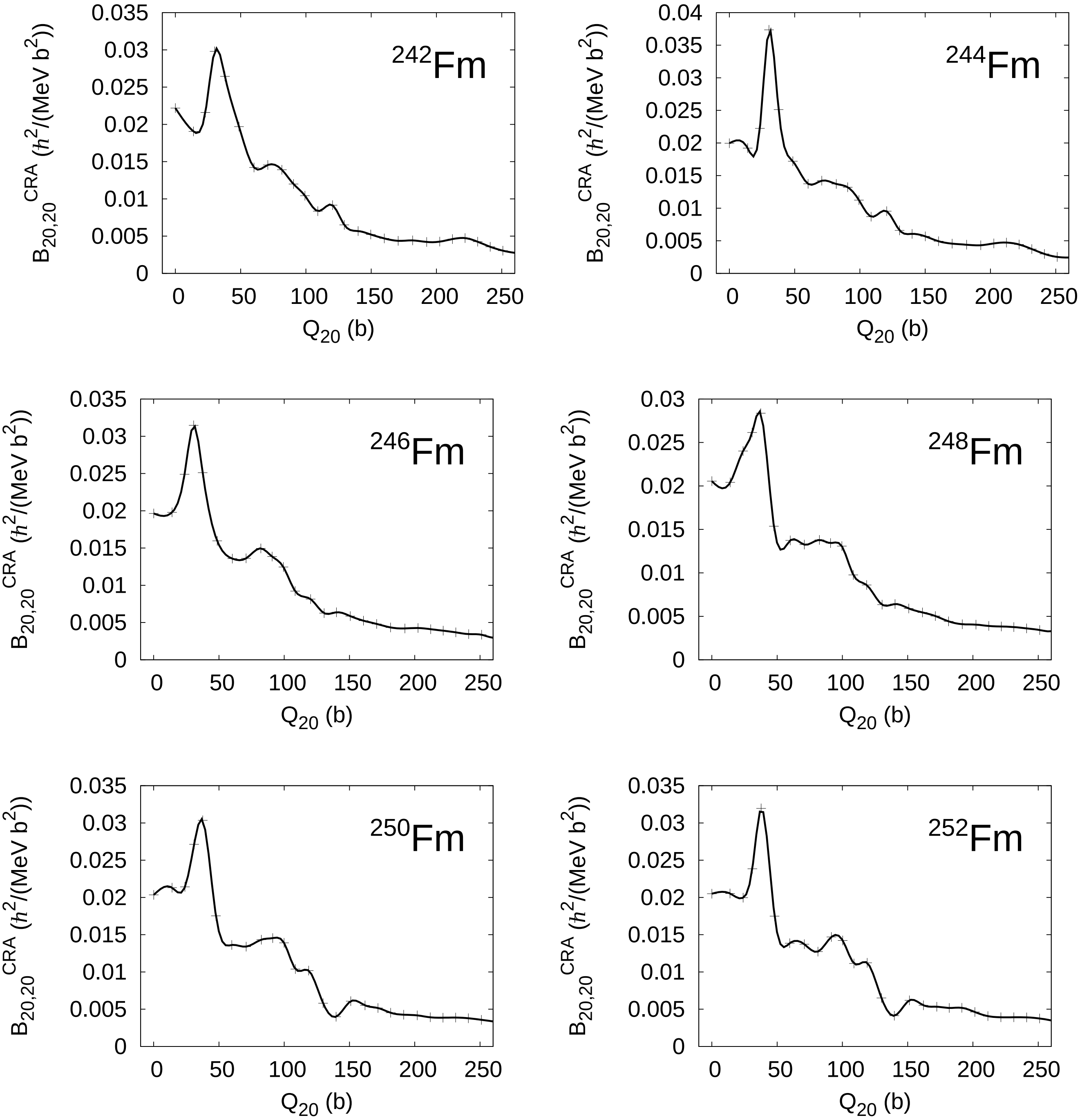
A. Barriers

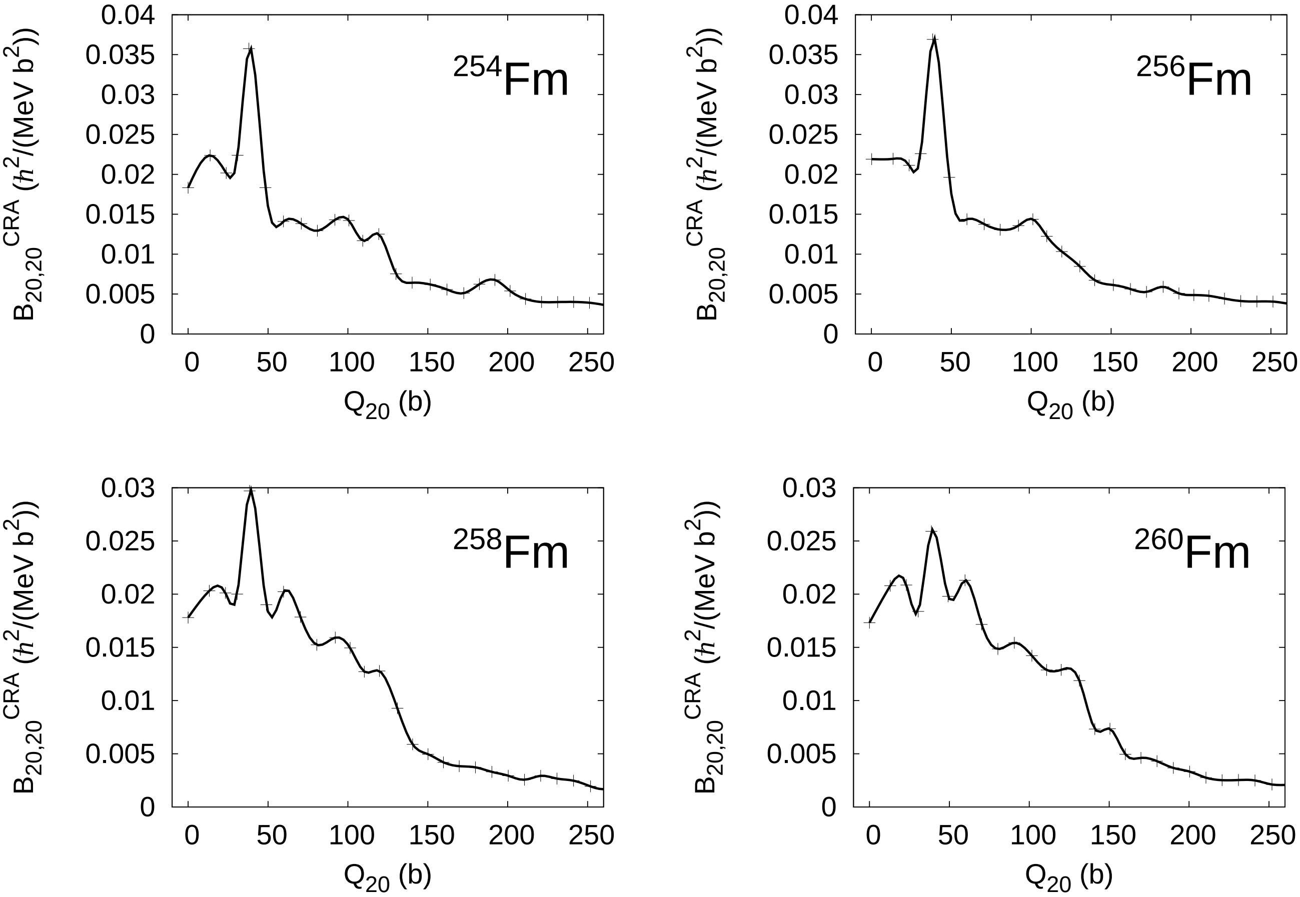




HF
HF+ZPE
HF+ZPE+RCE
E (MeV)
Q20 (b)
254Fm
256Fm
258Fm
260Fm

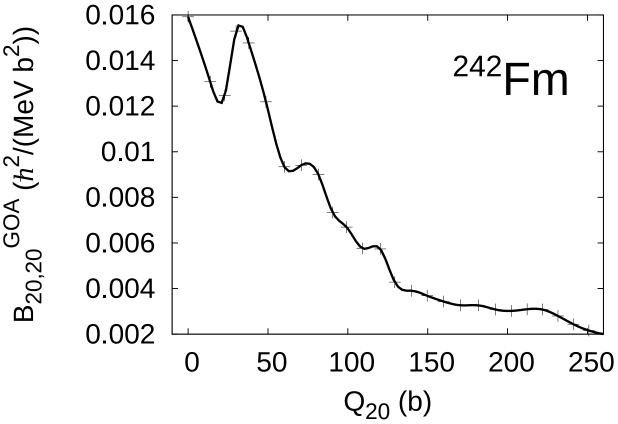
B. Mass parameters - cranking

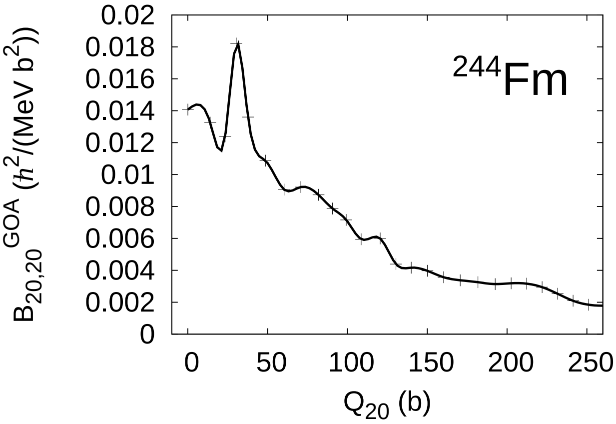


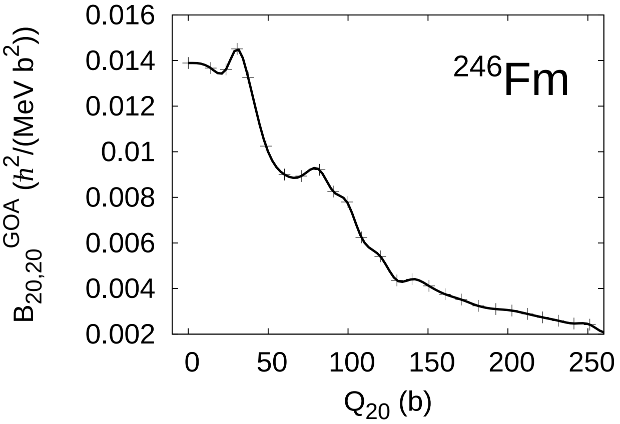


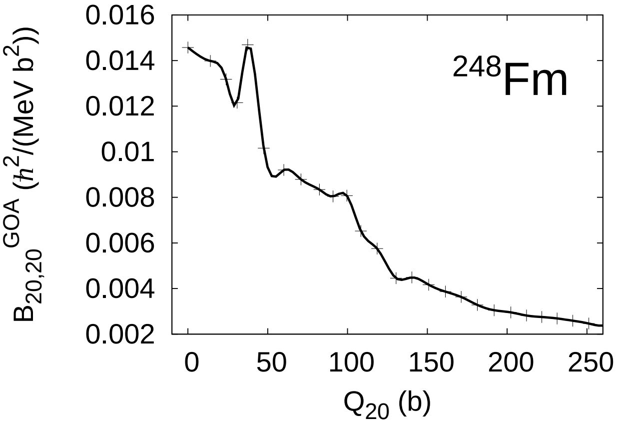
^{254}Fm
^{256}Fm
^{258}Fm
^{260}Fm
$B_{20,20}^{CRA}$ ($\hbar^2$/(MeV b^2))
Q_{20} (b)
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
50
100
150
200
250

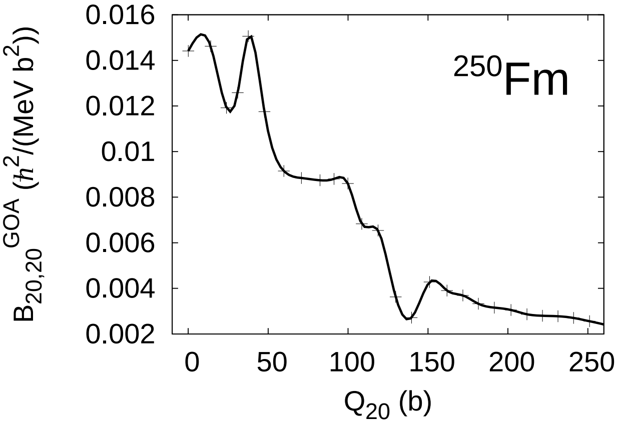
C. Mass parameters - GOA

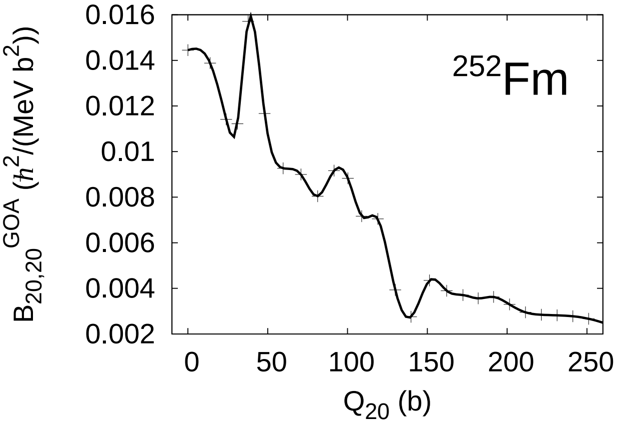


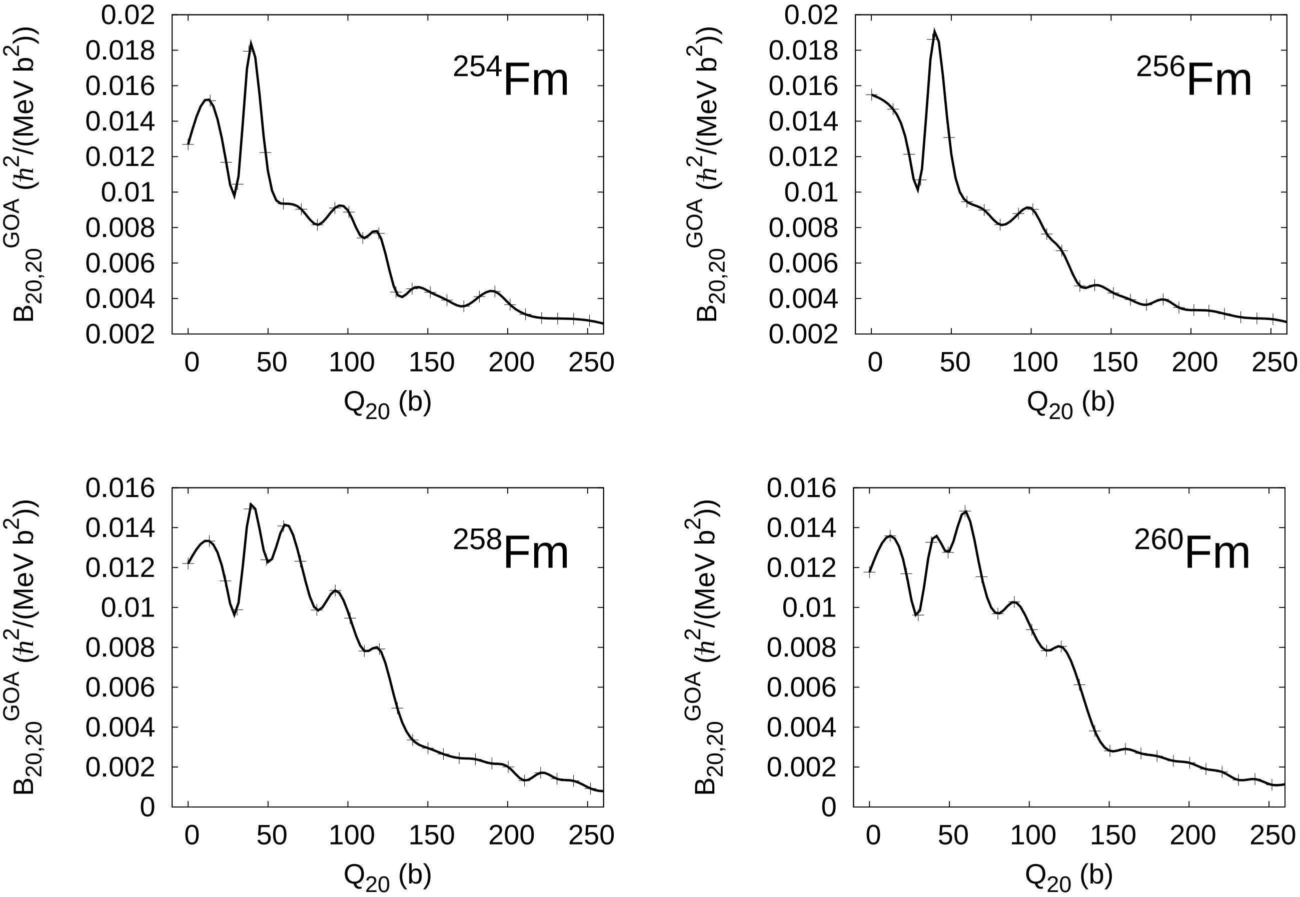




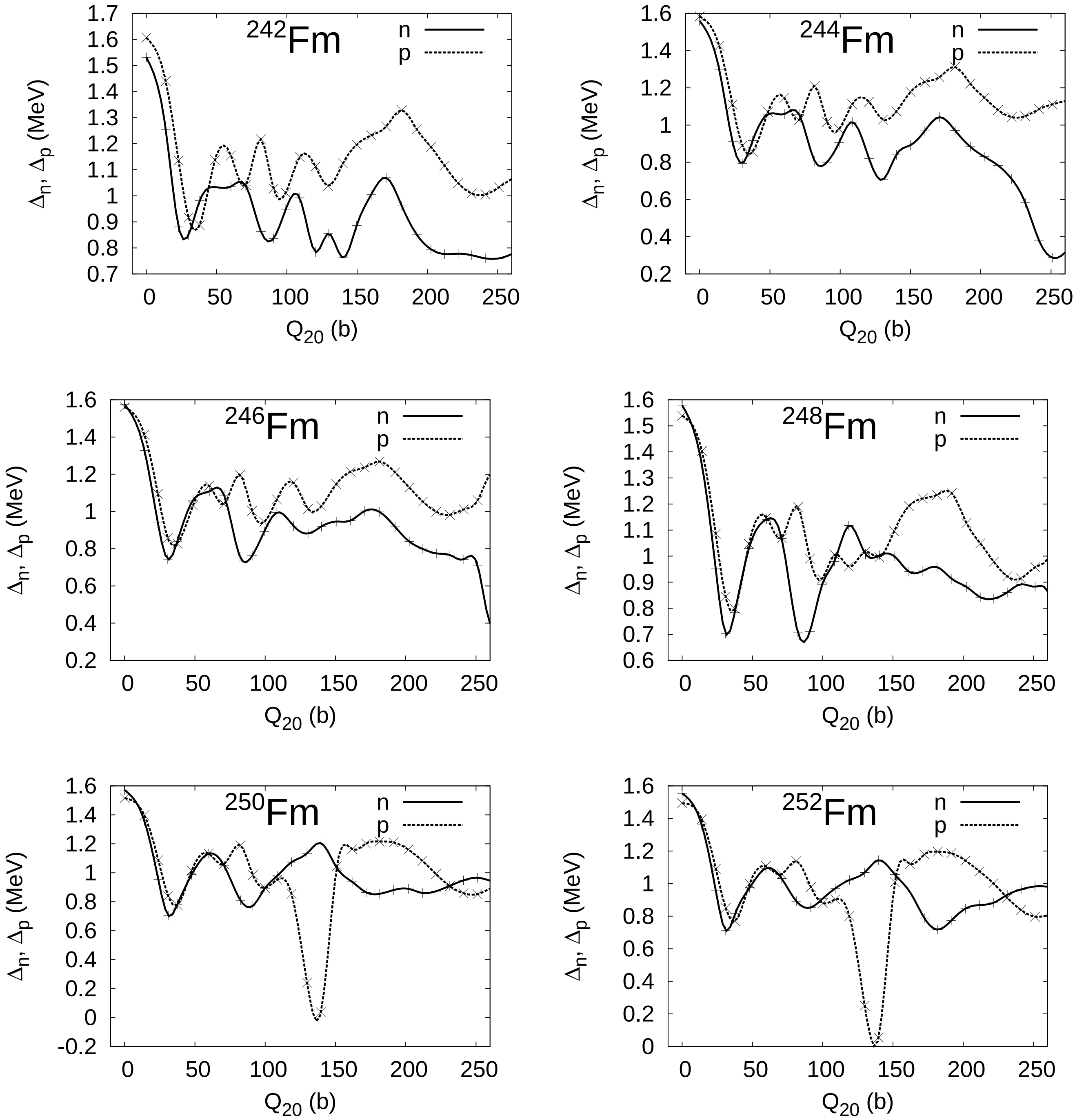


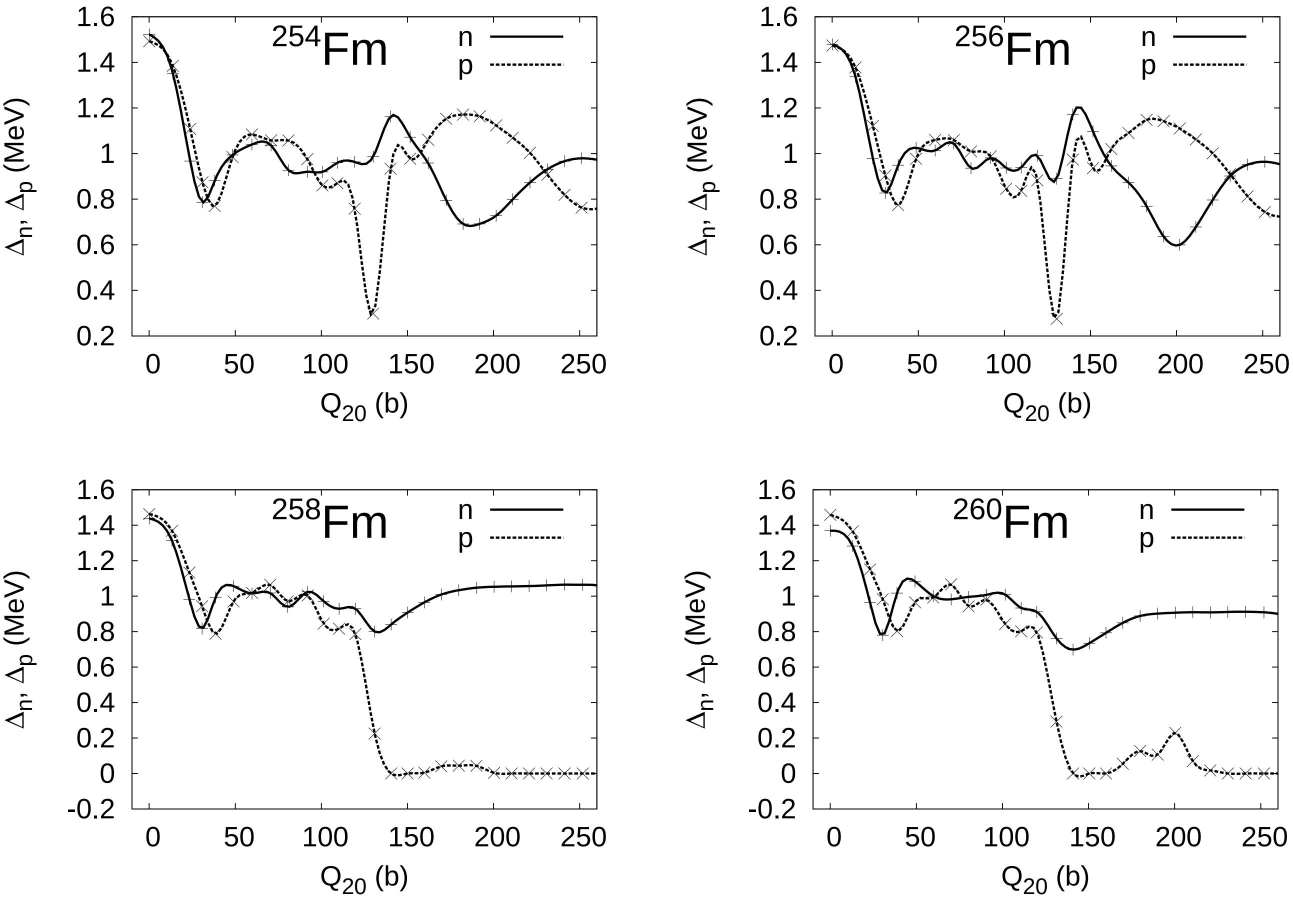





254Fm
256Fm
258Fm
260Fm
B20,20GOA (ħ2/(MeV b2))
Q20 (b)
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02
0
50
100
150
200
250

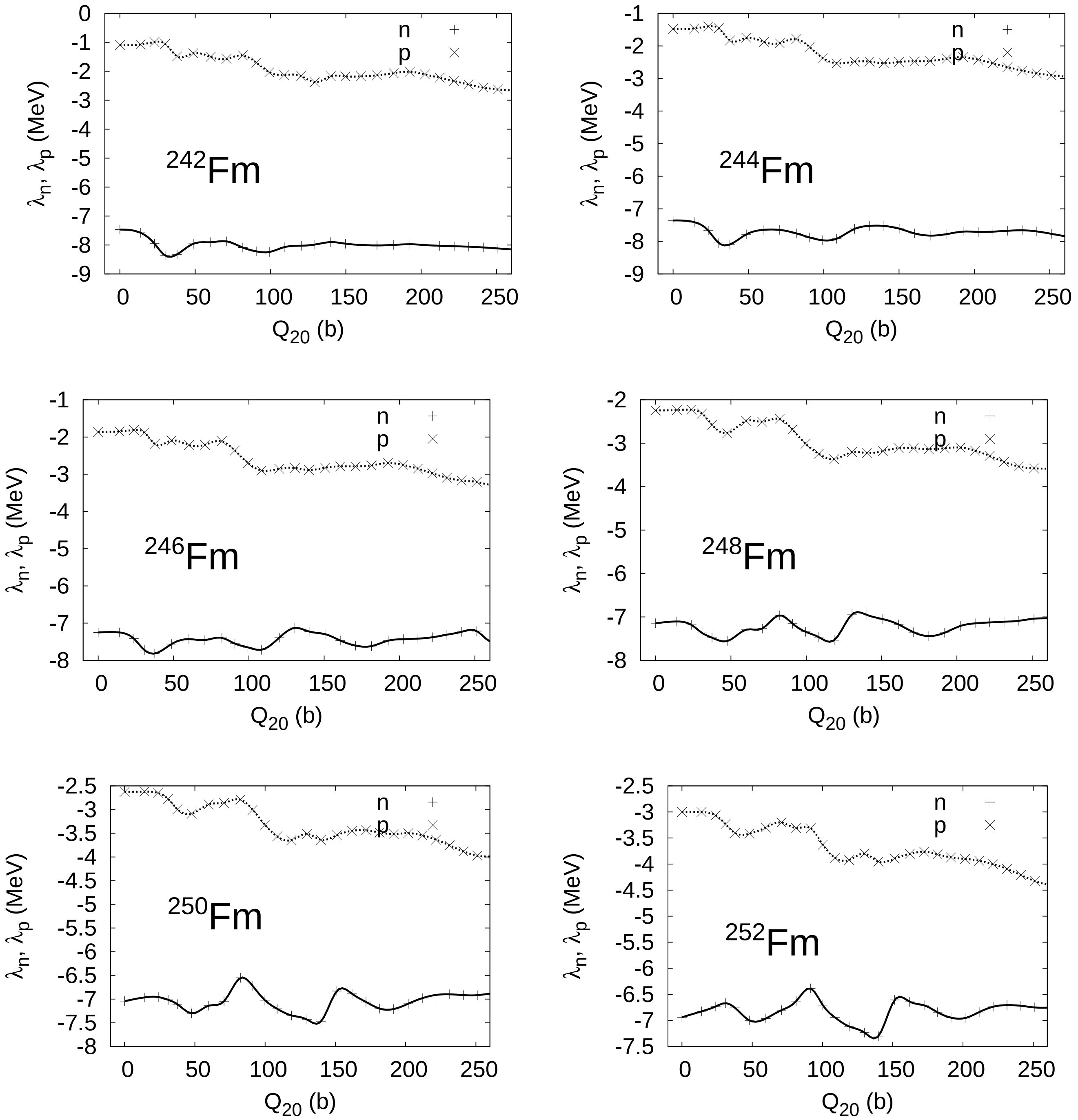
D. Pairing gaps $\Delta_{n,p}$

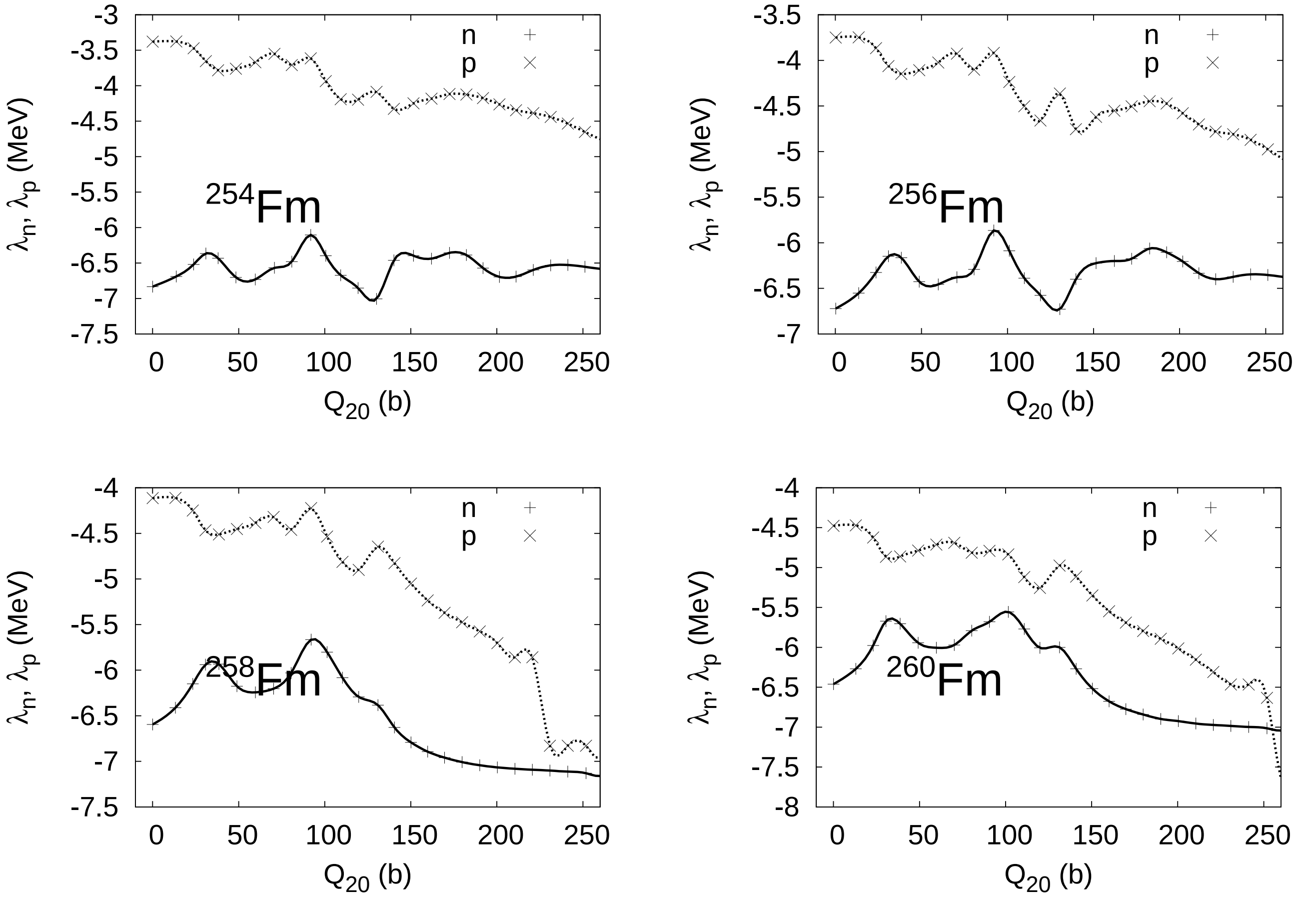




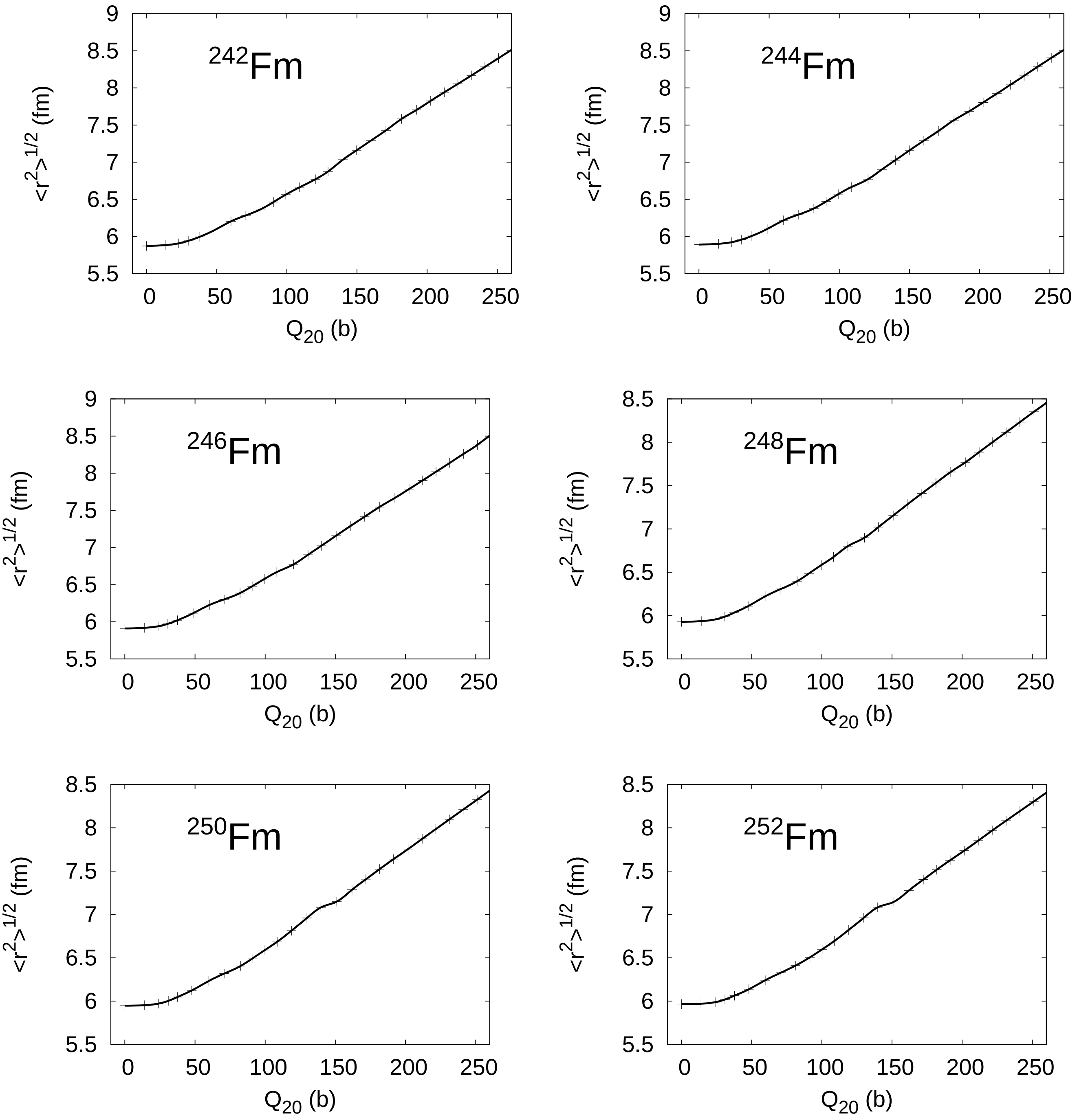
^{254}Fm
^{256}Fm
^{258}Fm
^{260}Fm
n
p
Δ_n, Δ_p (MeV)
Q_{20} (b)

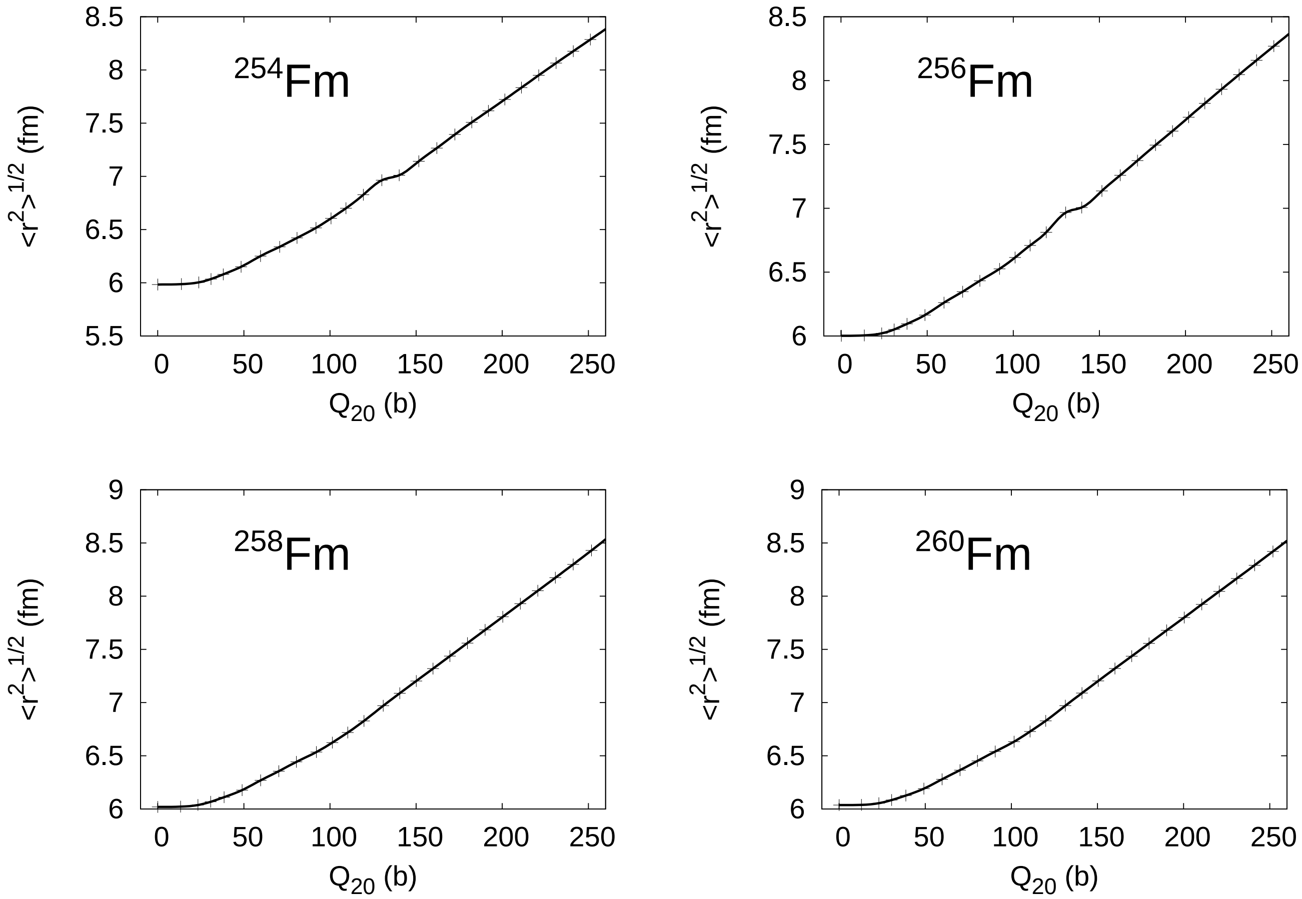
E. Pairing Fermi levels $\lambda_{n,p}$




^{254}Fm
^{256}Fm
^{258}Fm
^{260}Fm
n
p
λ_n, λ_p (MeV)
Q_{20} (b)
0
50
100
150
200
250
-3
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-7.5
-8

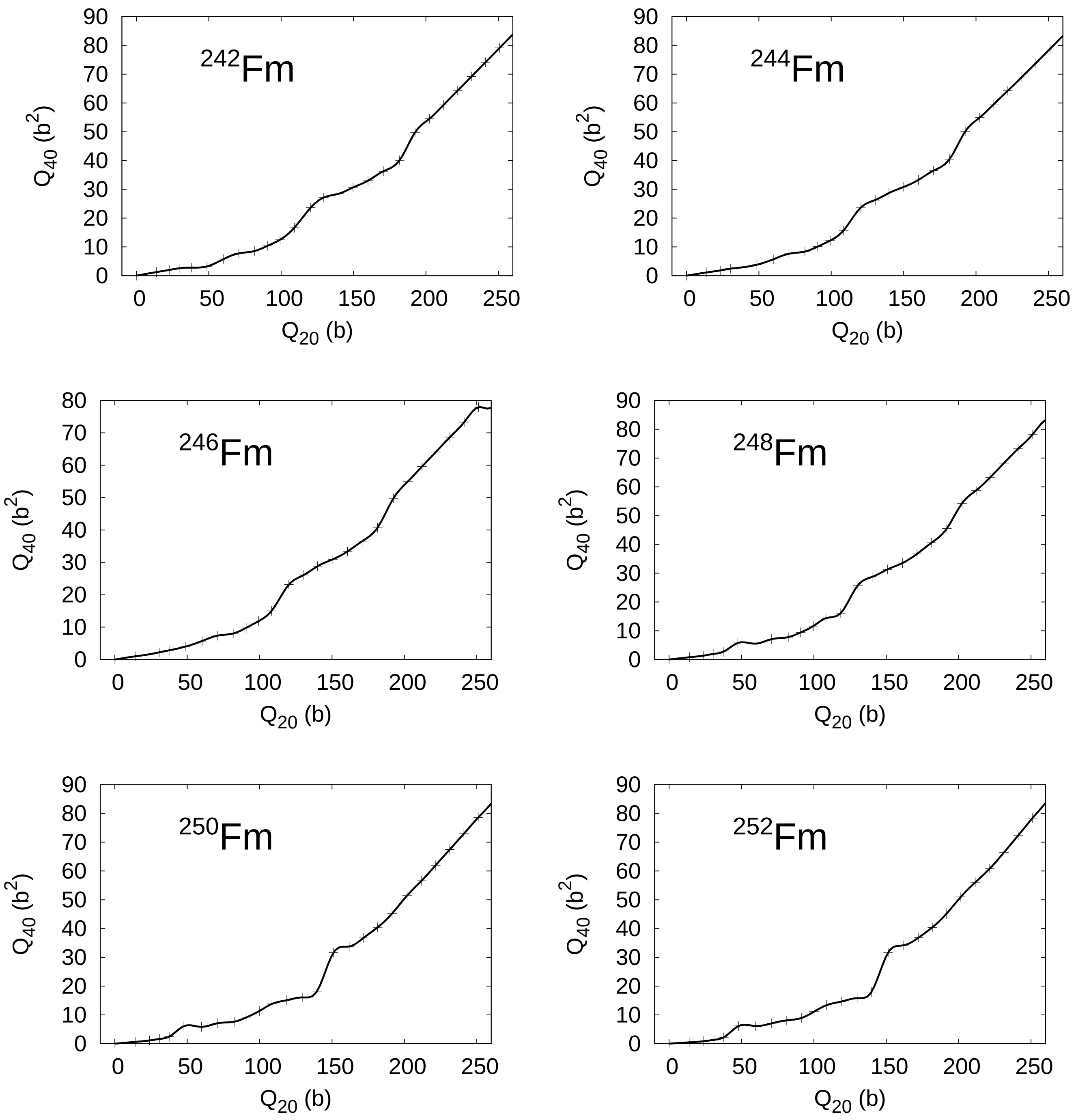
F. Rms radius $\langle r^2 \rangle^{1/2}$

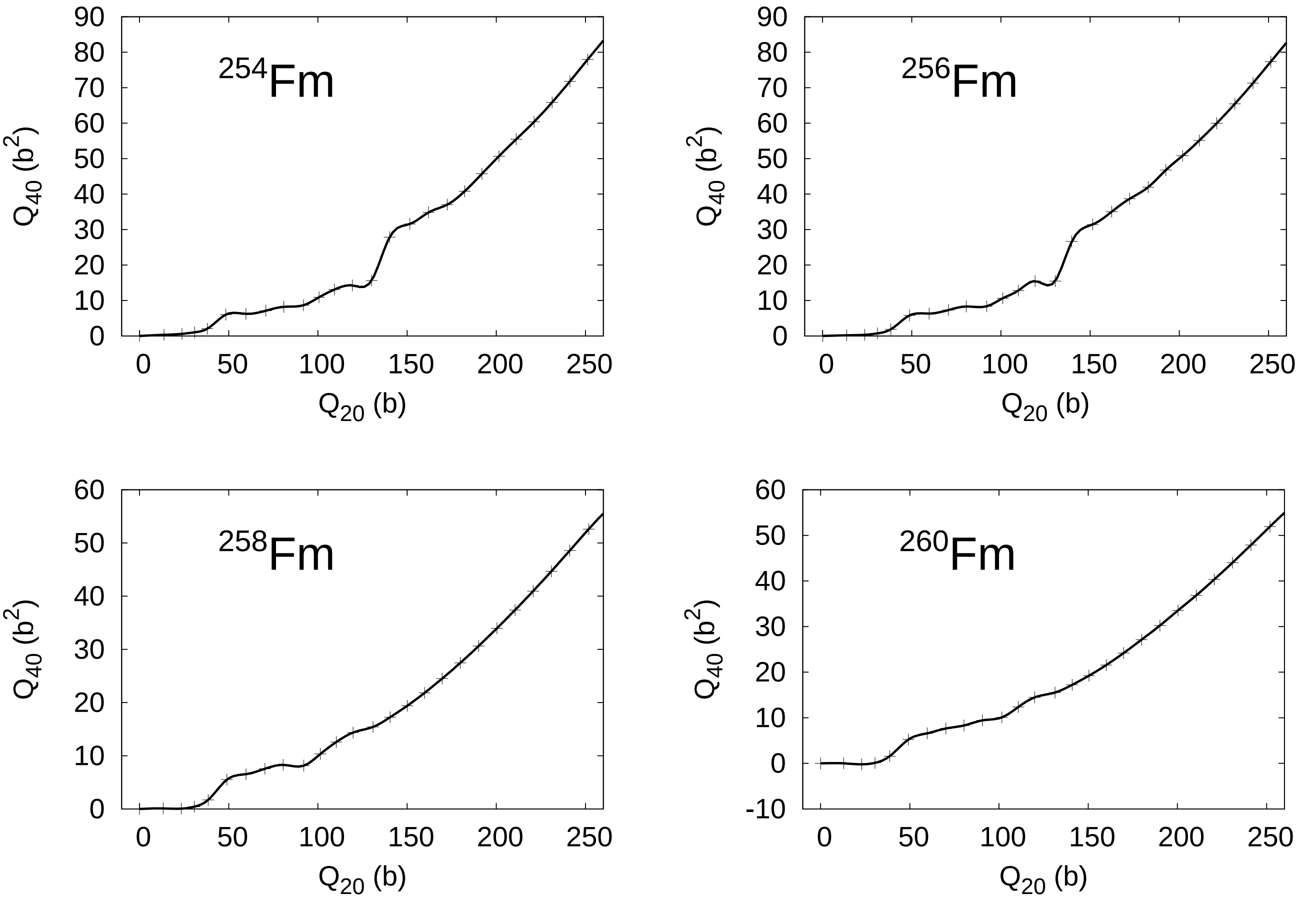




254Fm
256Fm
258Fm
260Fm
<r2>1/2 (fm)
Q20 (b)

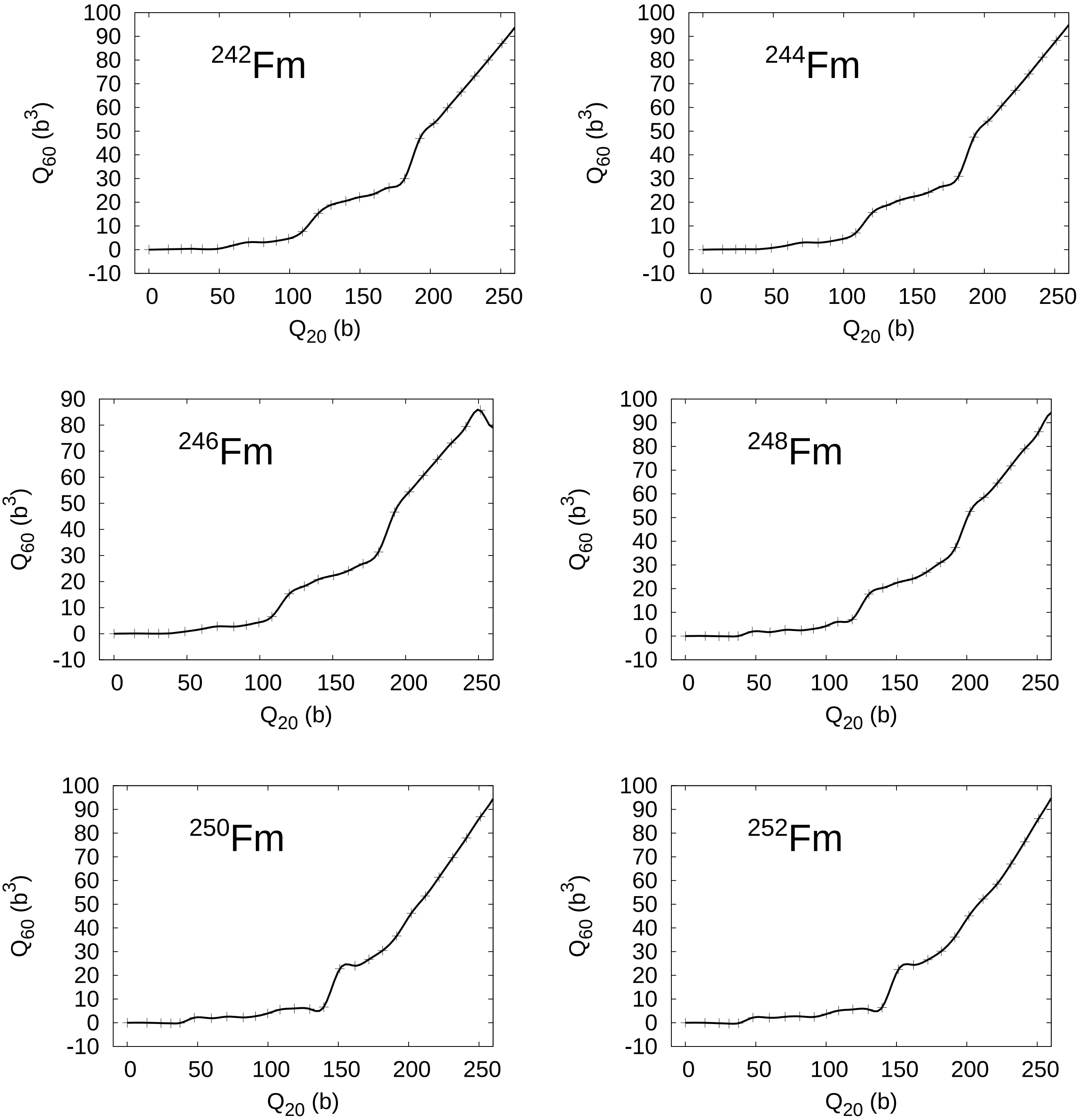
G. Moments Q_{40}

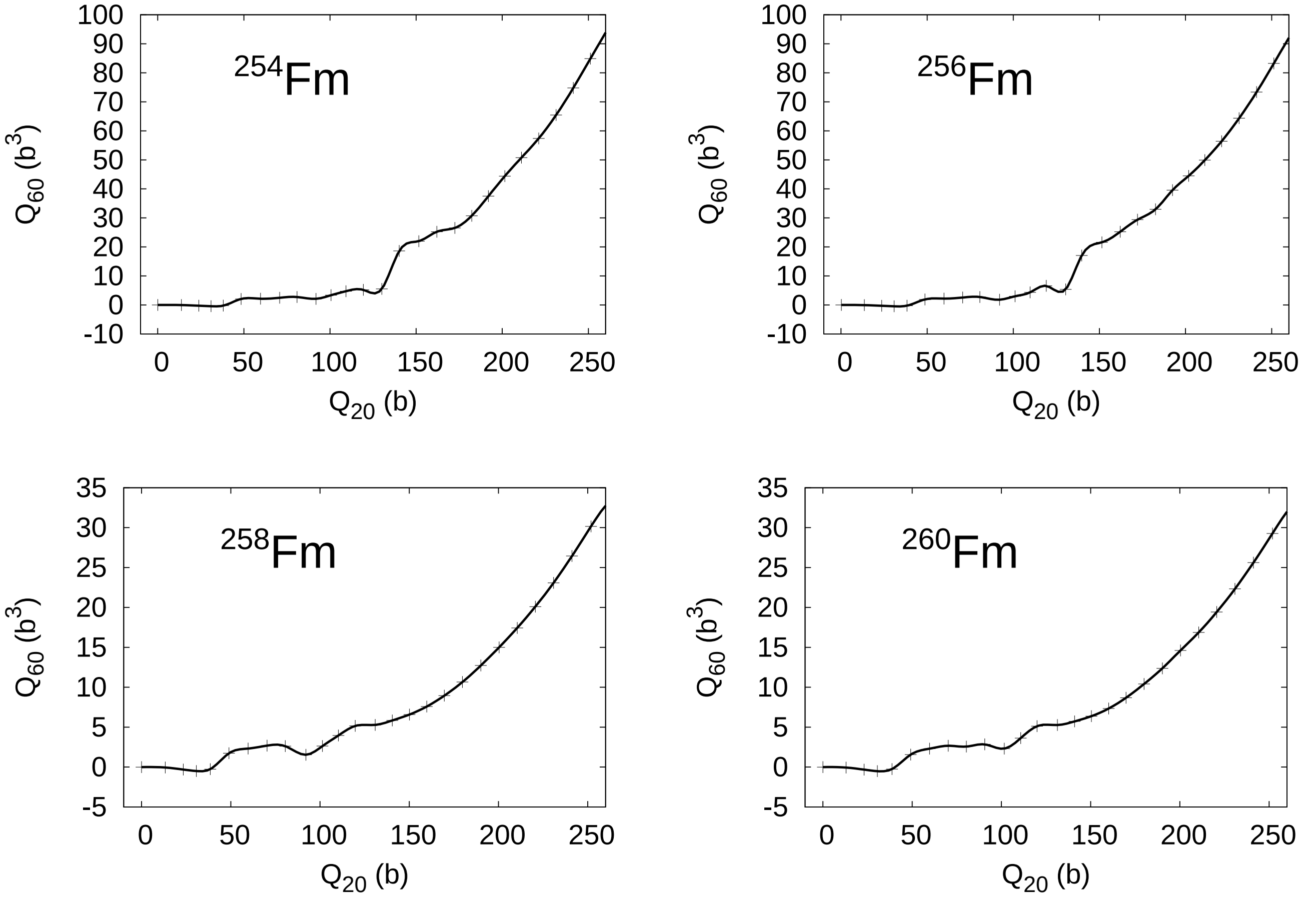




^{254}Fm
^{256}Fm
^{258}Fm
^{260}Fm
Q_{40} (b^2)
Q_{20} (b)

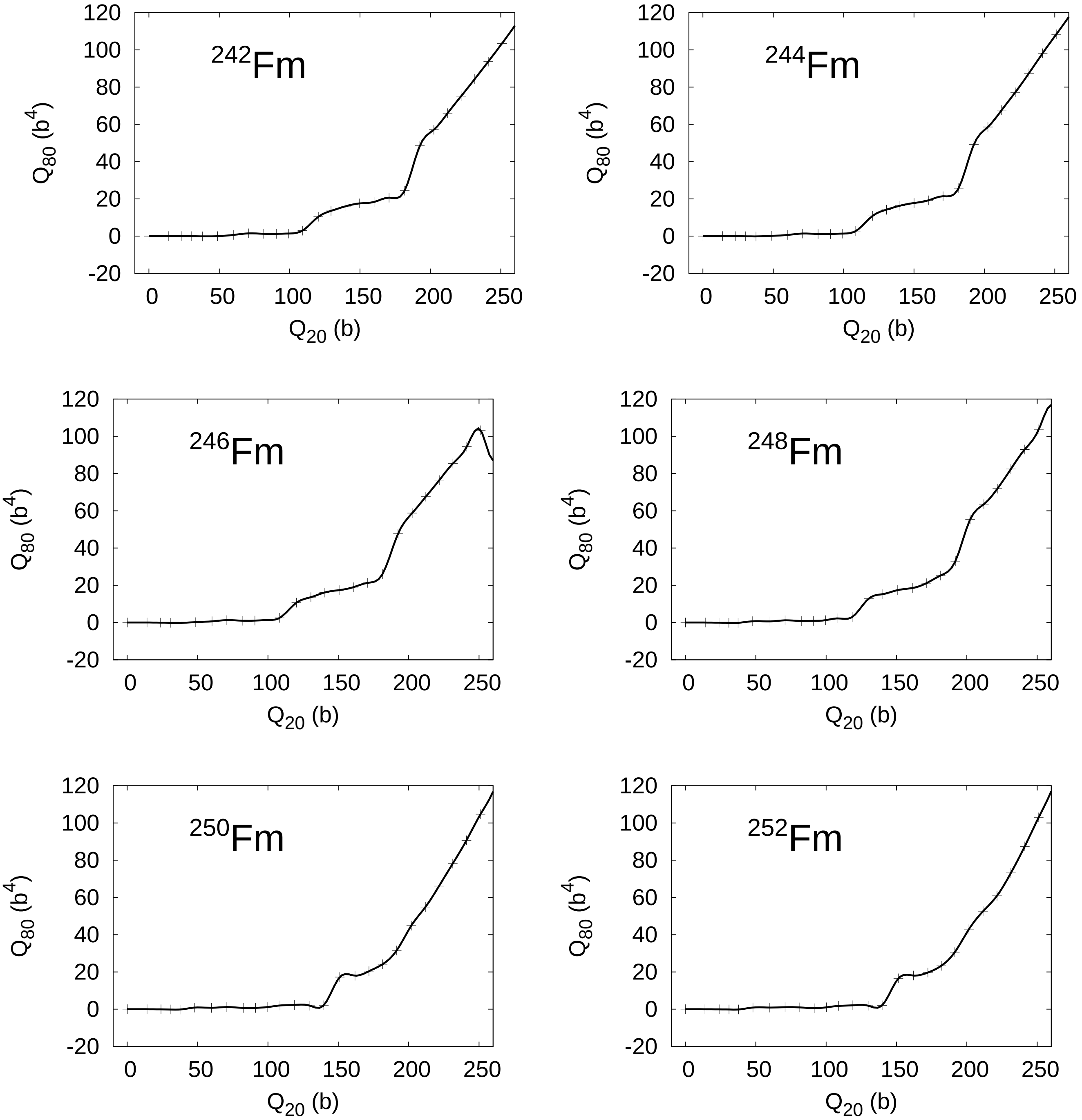
H. Moments Q_{60}

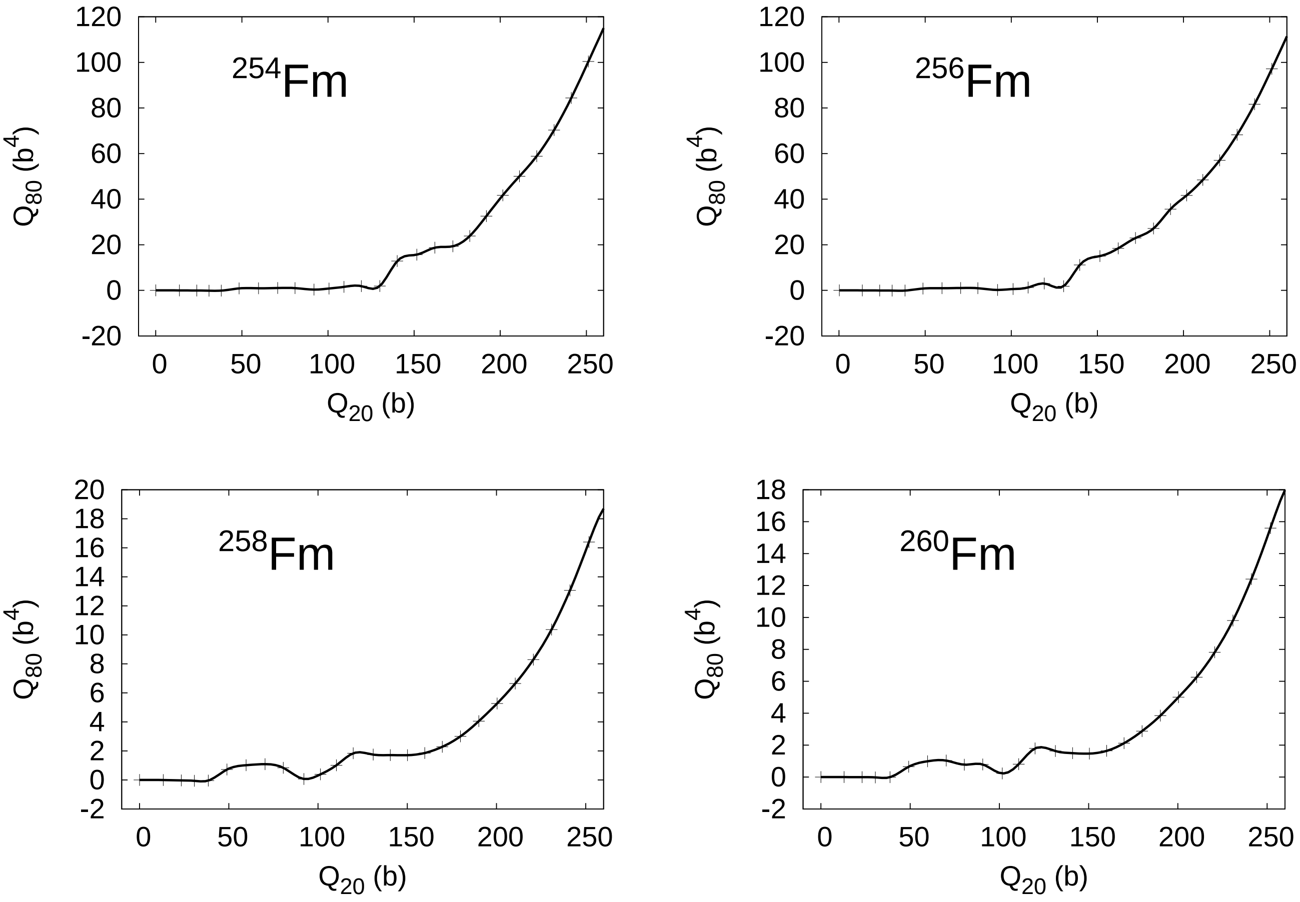




254Fm
256Fm
258Fm
260Fm
Q60 (b3)
Q20 (b)

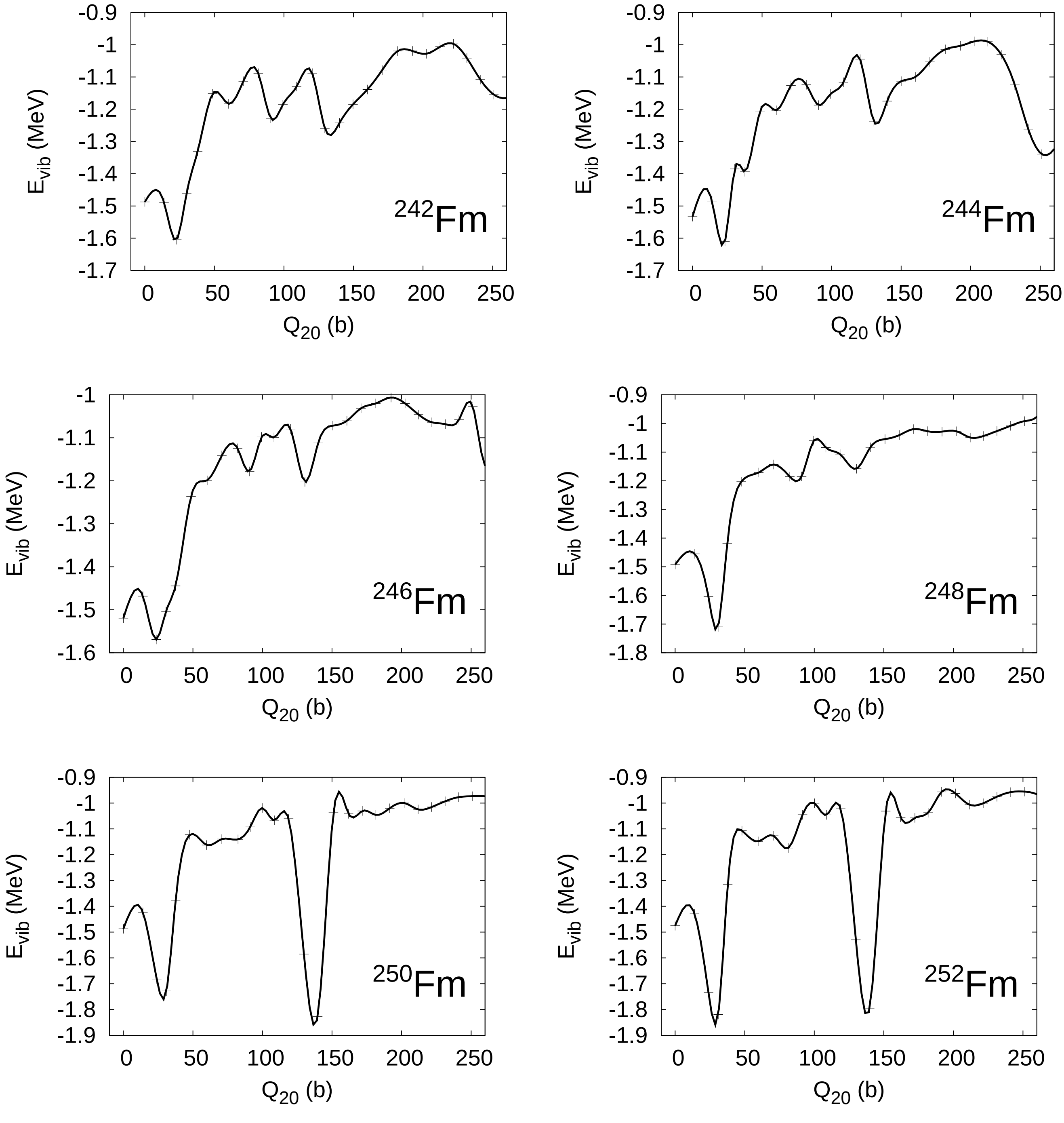
I. Moments Q_{80}

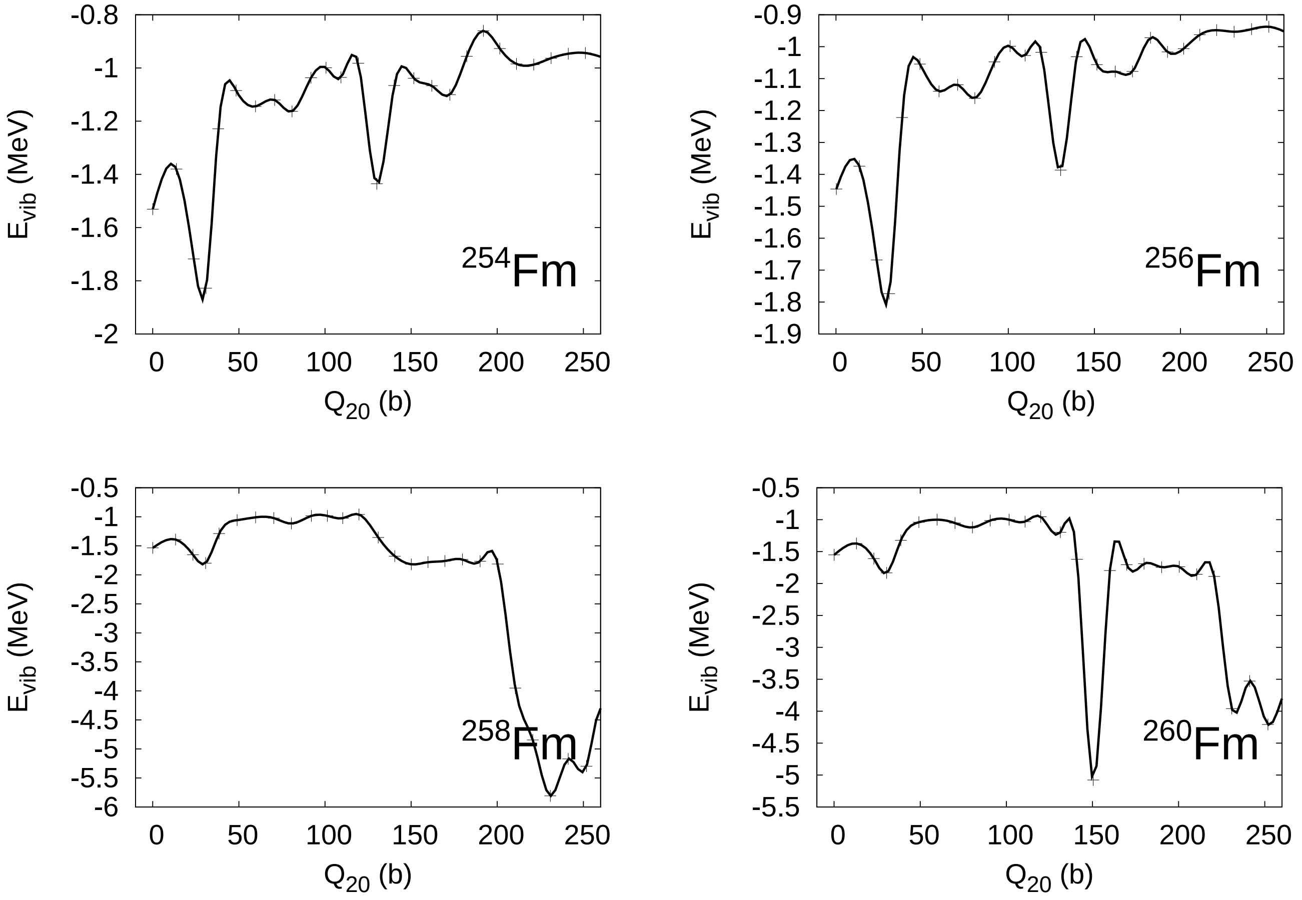




254Fm
256Fm
258Fm
260Fm
Q80 (b4)
Q20 (b)

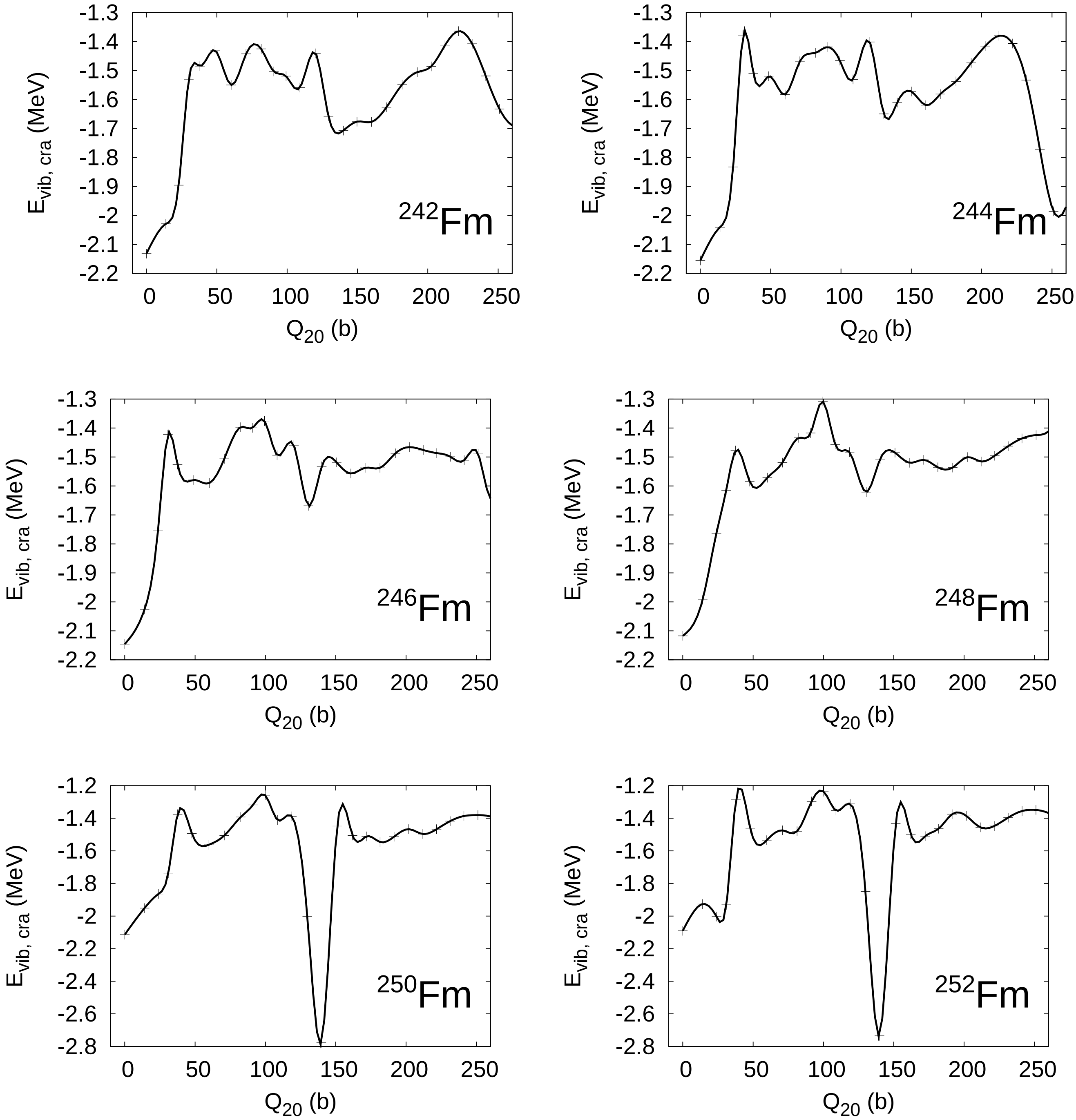
J. Zero point energy correction. CRA

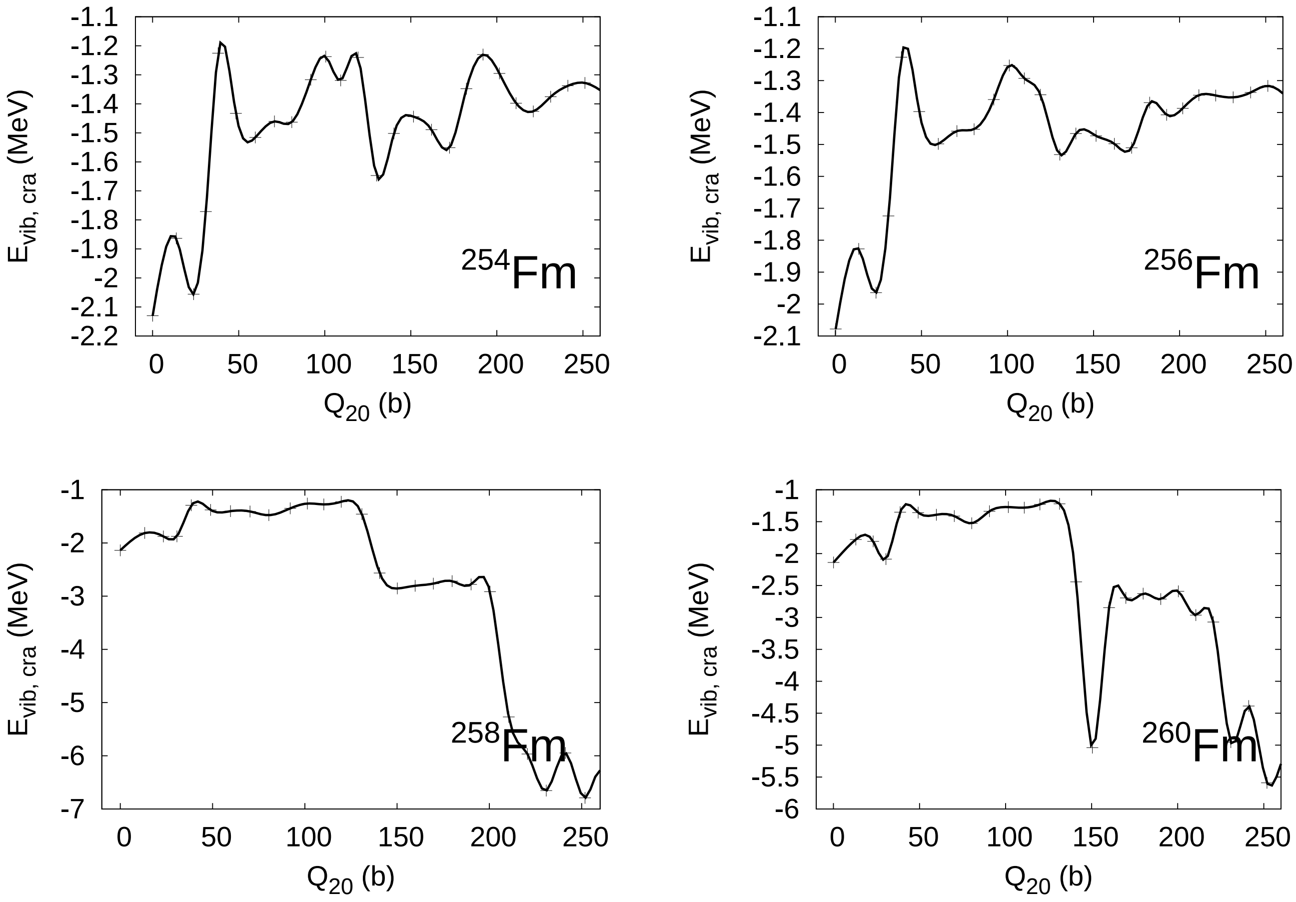




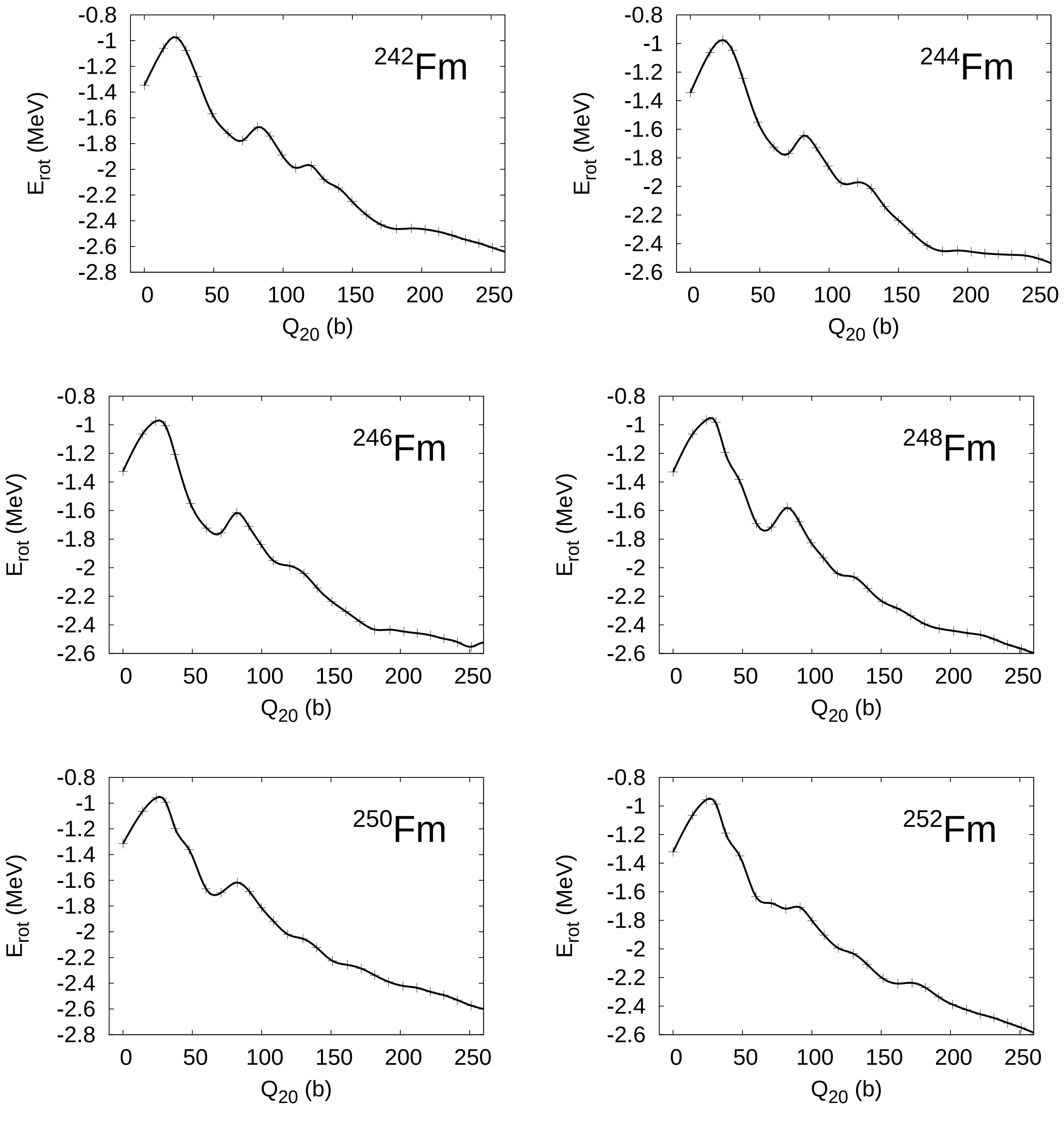
E_vib (MeV)
Q_20 (b)
254Fm
256Fm
258Fm
260Fm

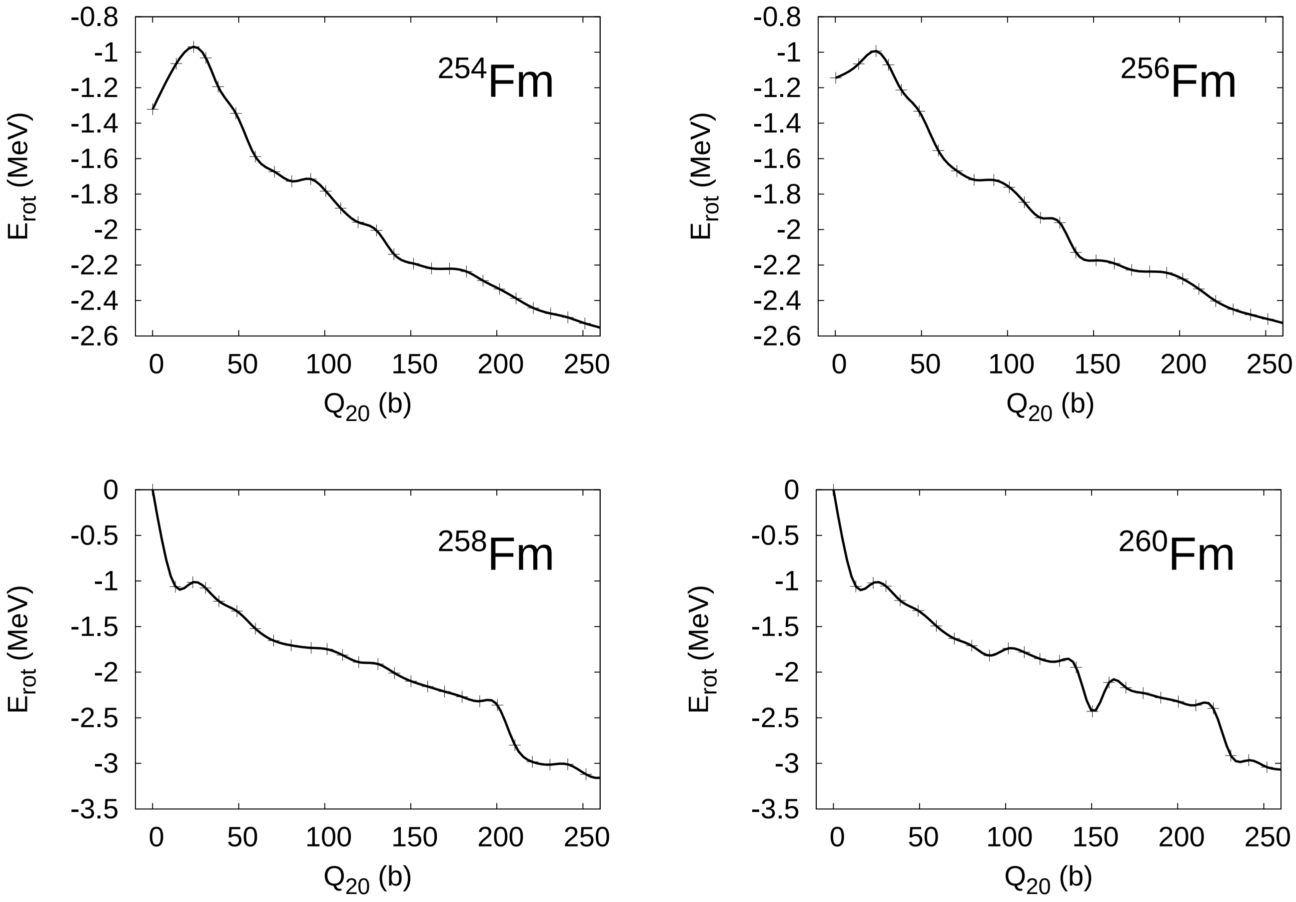
K. Zero point energy correction. GOA




E_vib, cra (MeV)
Q_20 (b)
254Fm
256Fm
258Fm
260Fm

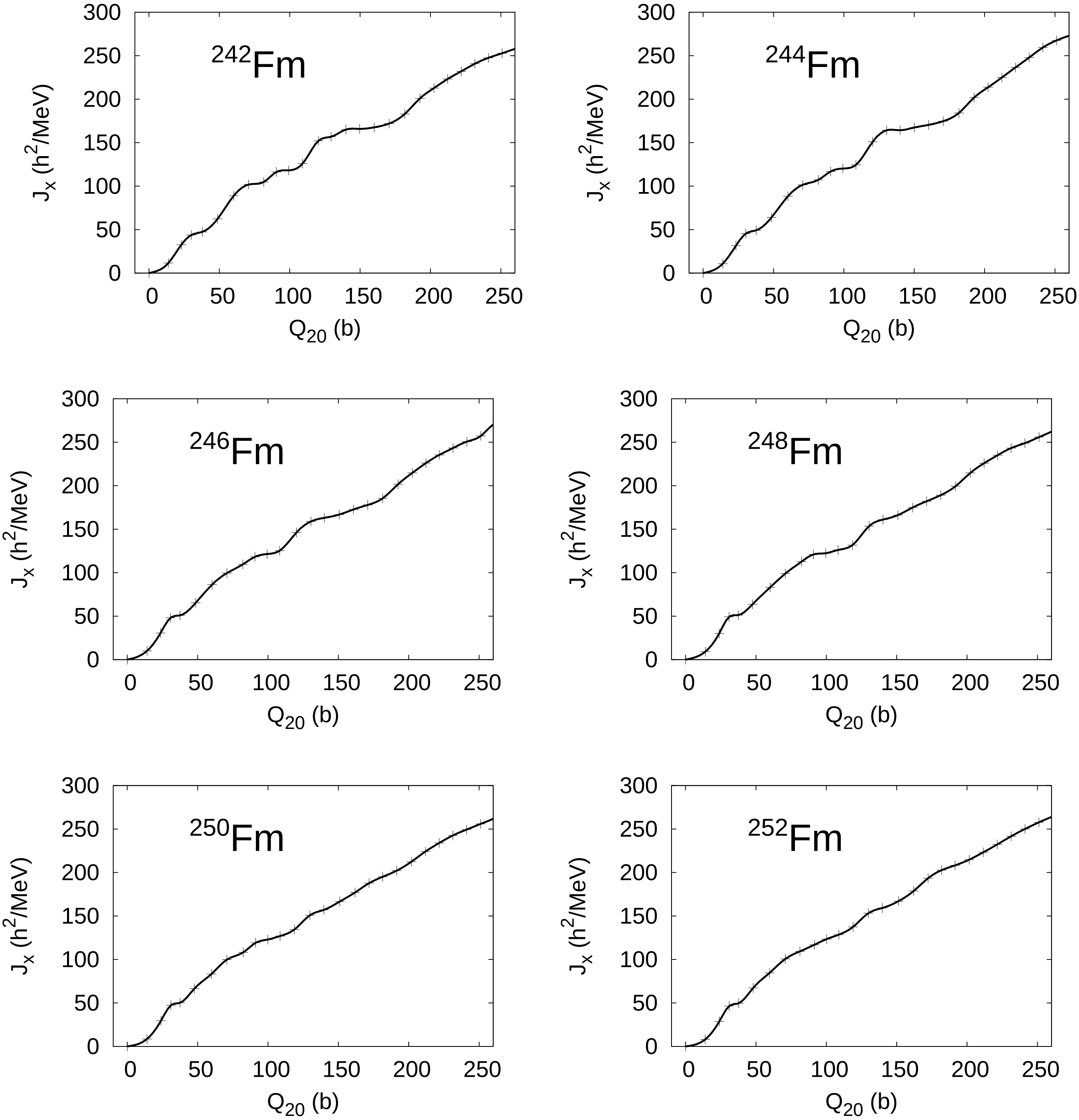
L. Rotational correction E_{rot}

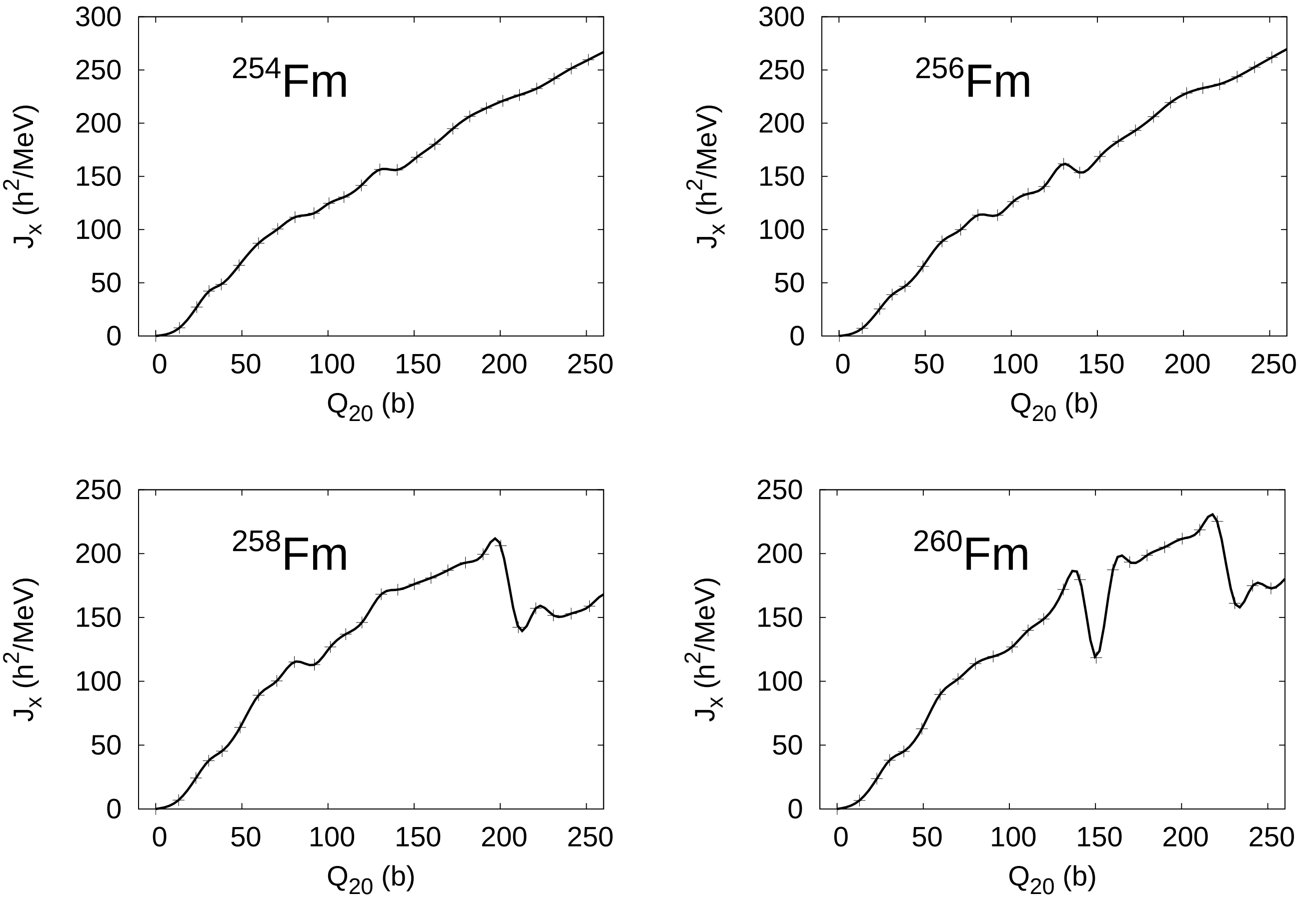




^{254}Fm
^{256}Fm
^{258}Fm
^{260}Fm
E_{rot} (MeV)
Q_{20} (b)

M. Cranking moment of inertia $\mathcal{J}_x$





254Fm
256Fm
258Fm
260Fm
Jx (h2/MeV)
Q20 (b)
0
50
100
150
200
250
300

II. FISSION HALF LIVES: HFB, $\delta - mix$

In the following section, the order of sub-figures corresponds to the following E_0 values: $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 MeV and $E_{0,GOA}$ MeV as calculated in the program.

Meaning of other descriptors:

cra - cranking model mass parameters
goa - Gaussian overlap approximation for mass parameters

noc - no corrections
vib - E_0 (vibrational) correction included
rot - rotational correction included
corr - both corrections (rotational and vibrational) included into the energy

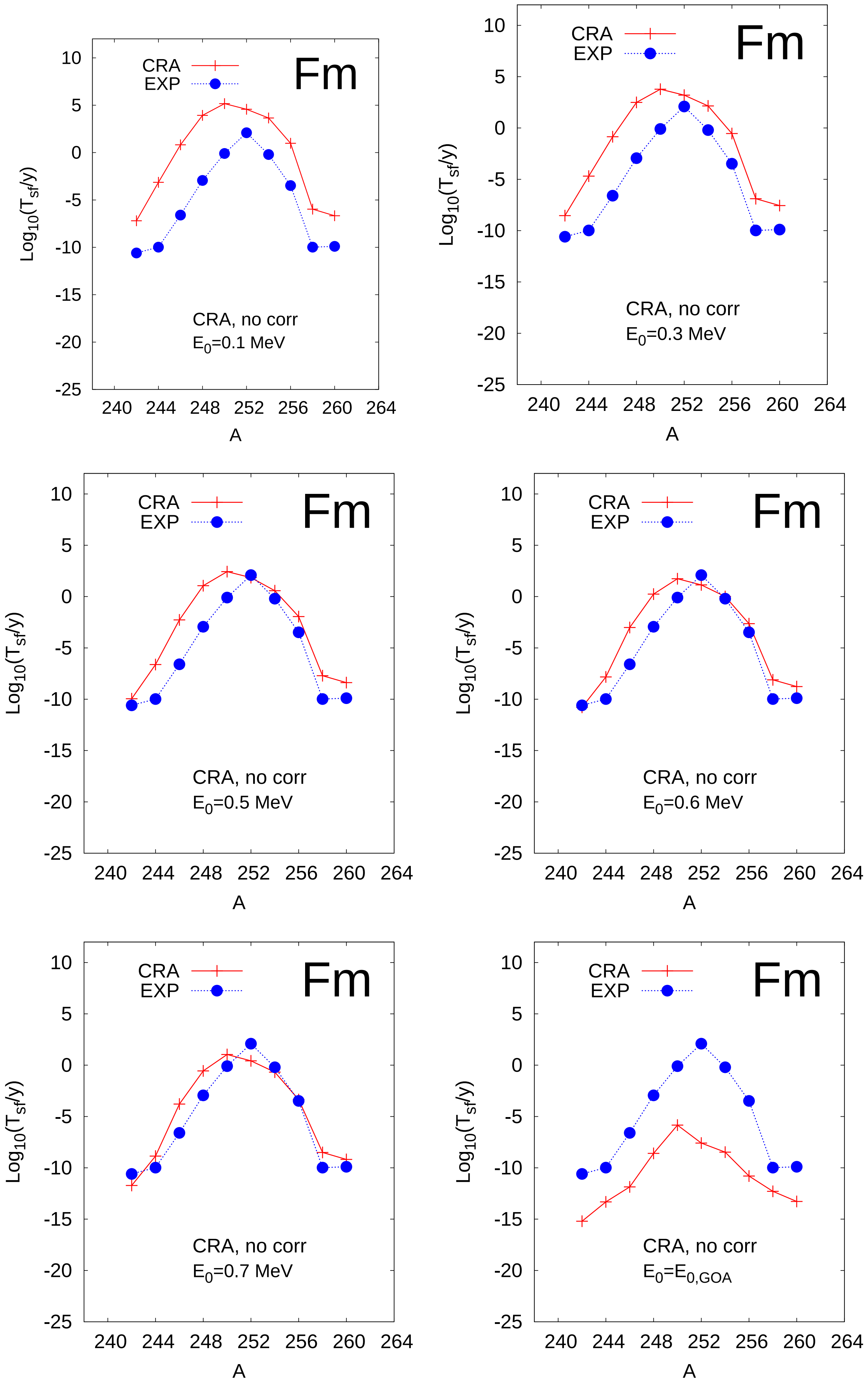


FIG. 1: Spontaneous fission half lives of Fermium isotopes in the case of uncorrected HFB energy barrier and cranking mass parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

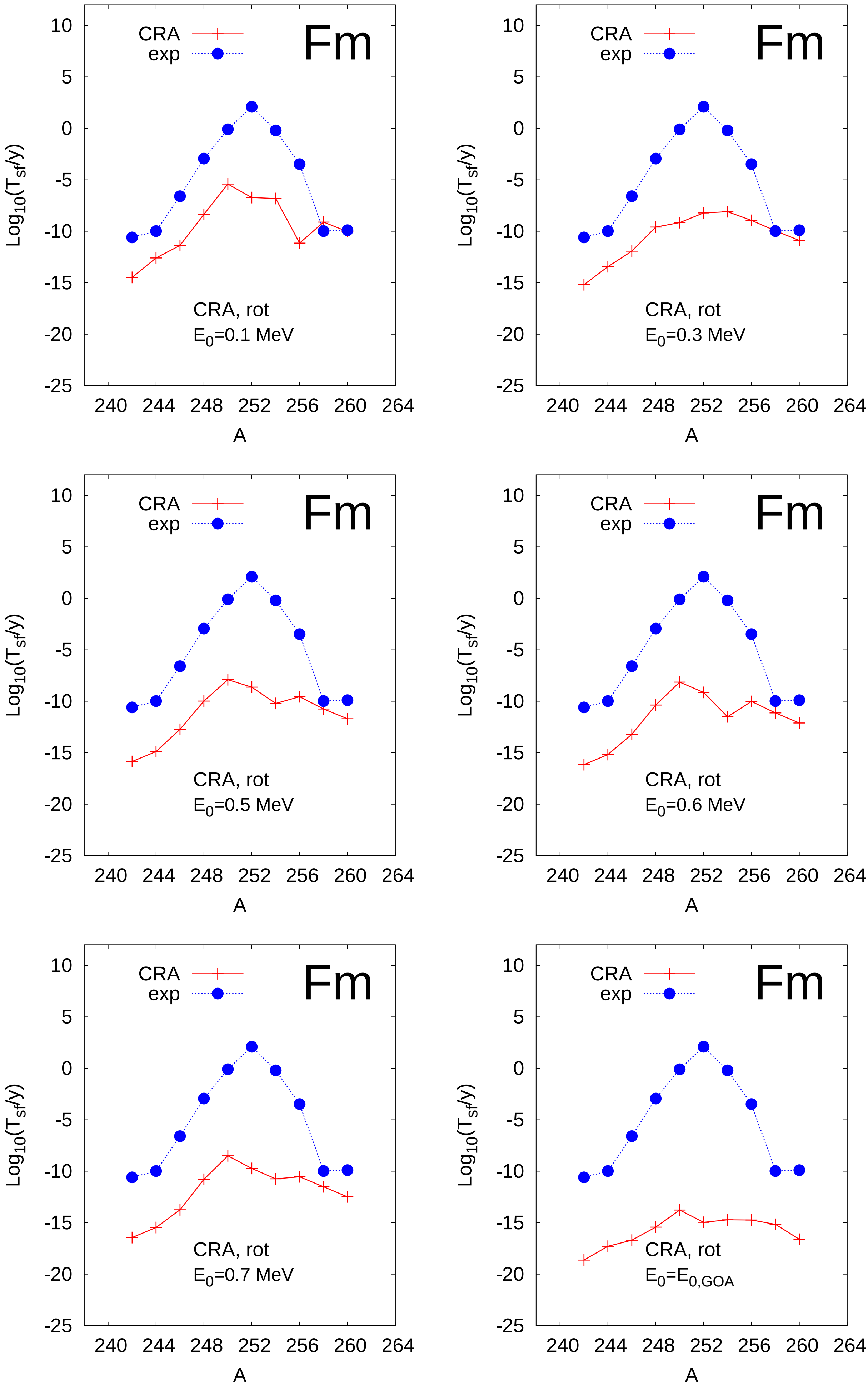


FIG. 2: Spontaneous fission half lives of Fermium isotopes in the case of HFB energy including **rotational** correction and cranking mass parameters. $E_0 = 0.1, 0.3, 0.5, 0.6, 0.7$ and $E_{0,gcm}$.

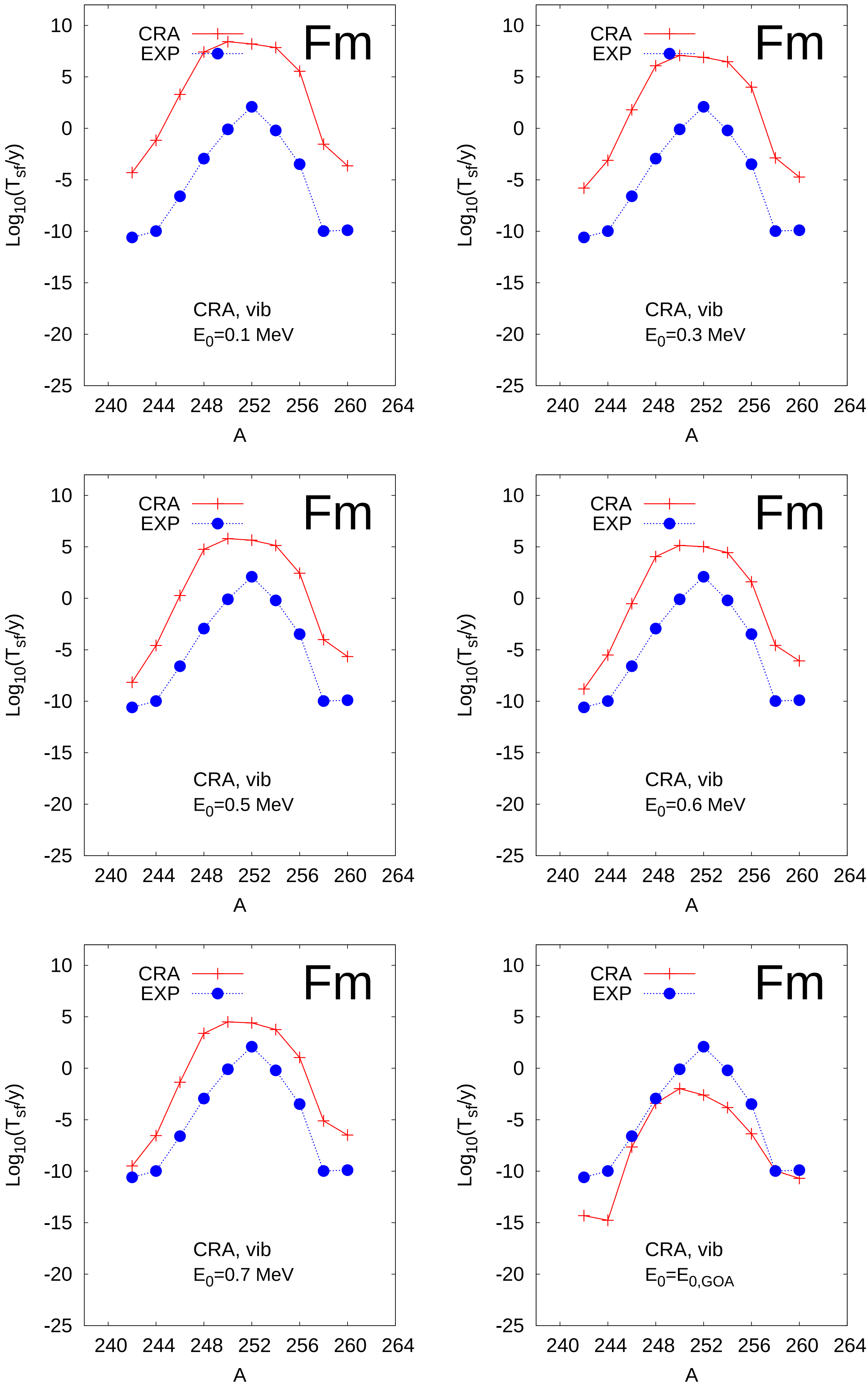


FIG. 3: Spontaneous fission half lives of Fermium isotopes in the case of HFB energy including **vibrational** correction and cranking mass parameters. $E_0 = 0.1, 0.3, 0.5, 0.6, 0.7$ and $E_{0,gcm}$.

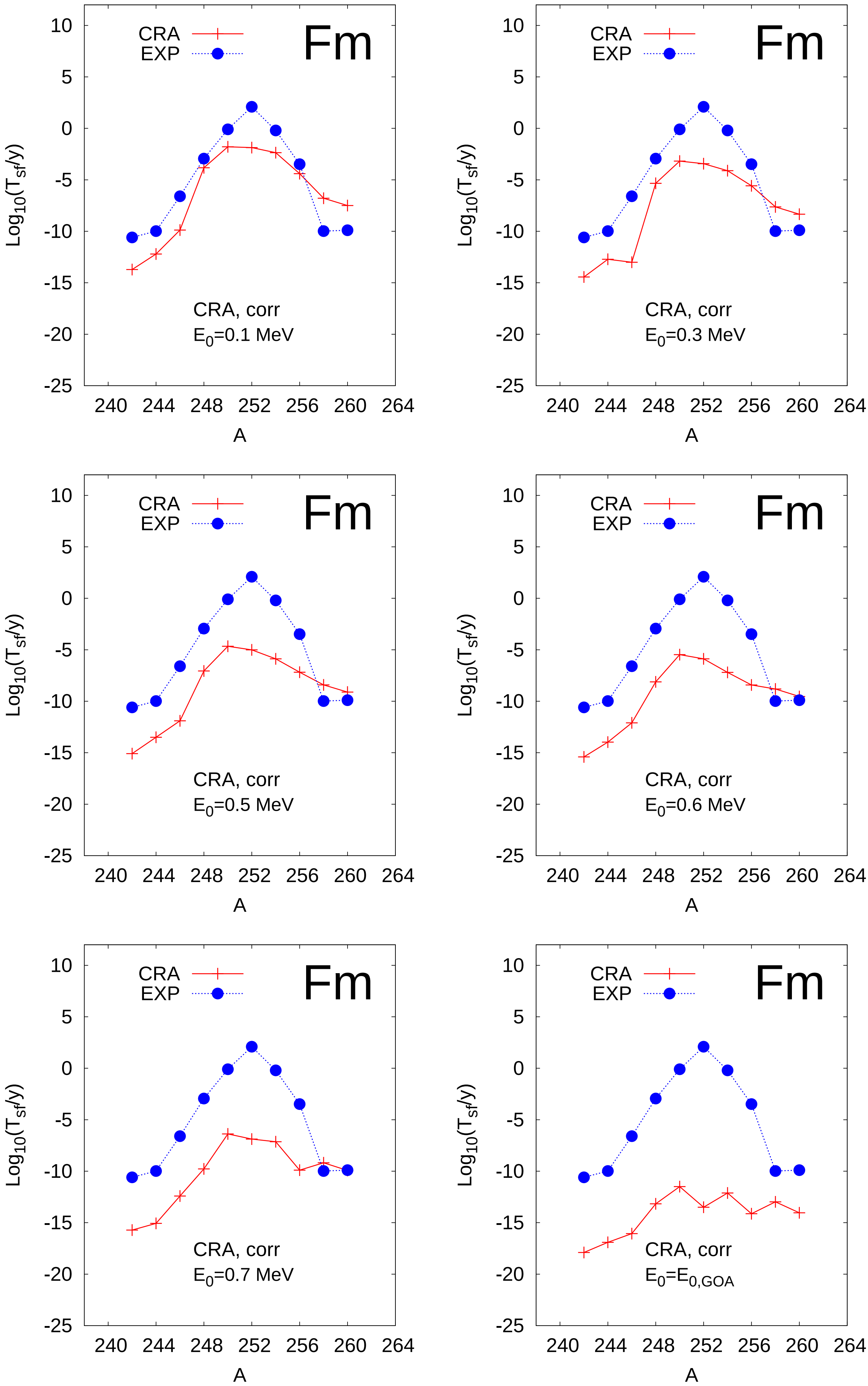


FIG. 4: Spontaneous fission half lives of Fermium isotopes in the case of HFB energy including both **vibrational** and **rotational** energy corrections and GCM-GOA mass parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

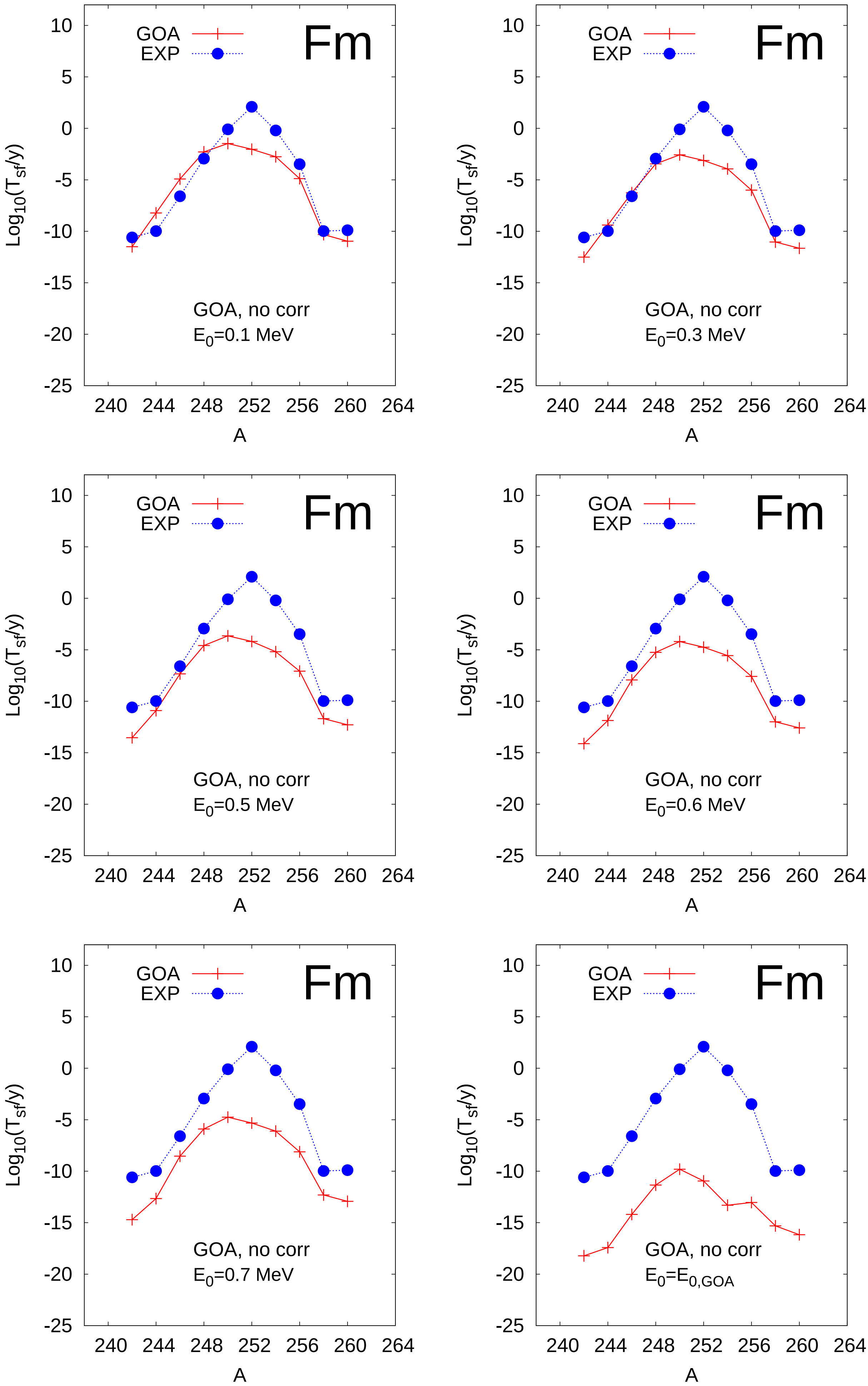


FIG. 5: Spontaneous fission half lives of Fermium isotopes in the case of uncorrected HFB energy and GCM-GOA mass parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

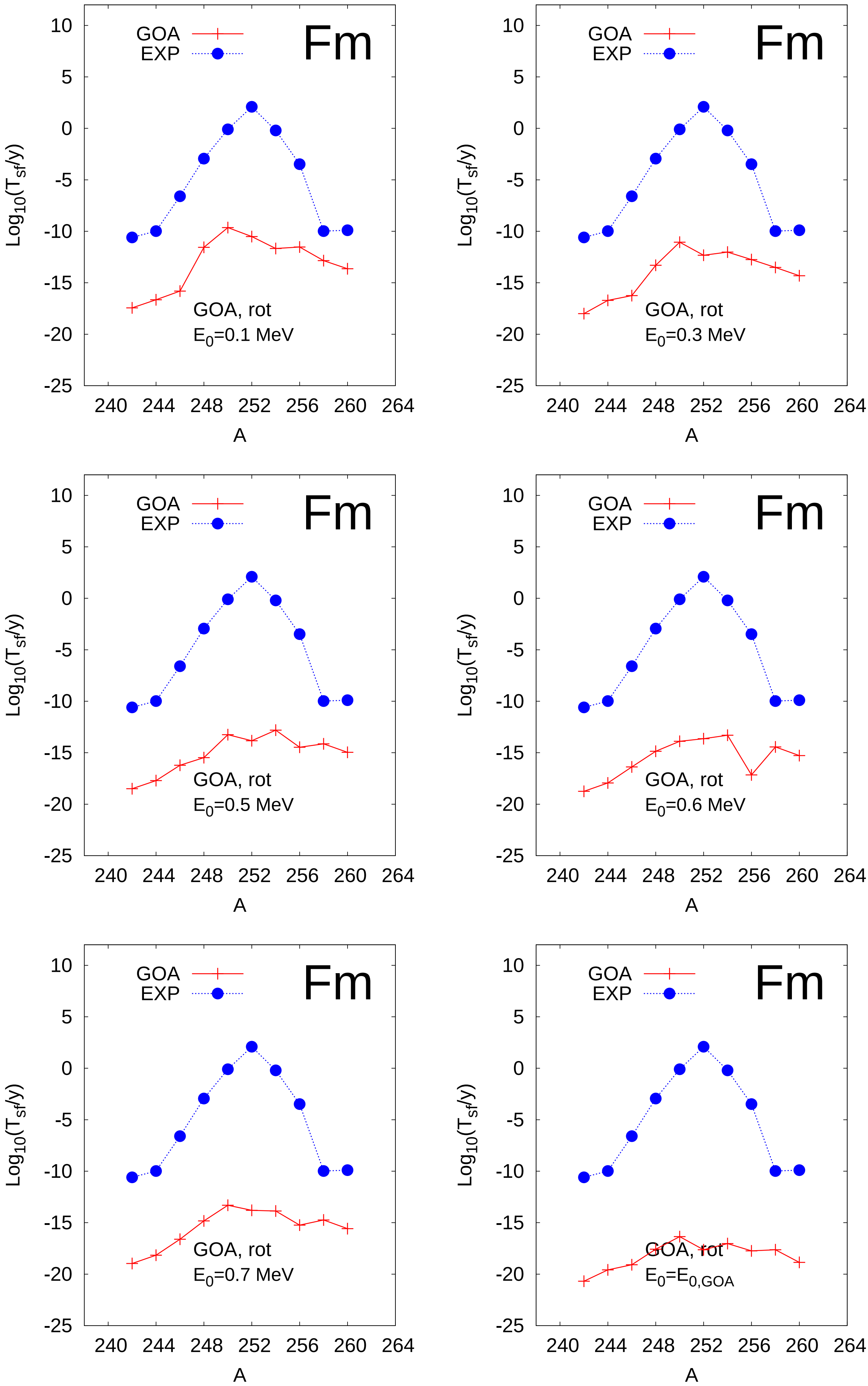


FIG. 6: Spontaneous fission half lives of Fermium isotopes in the case of HFB energy including **rotational** energy correction and GCM-GOA mass parameters. E_0 = 0.1, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

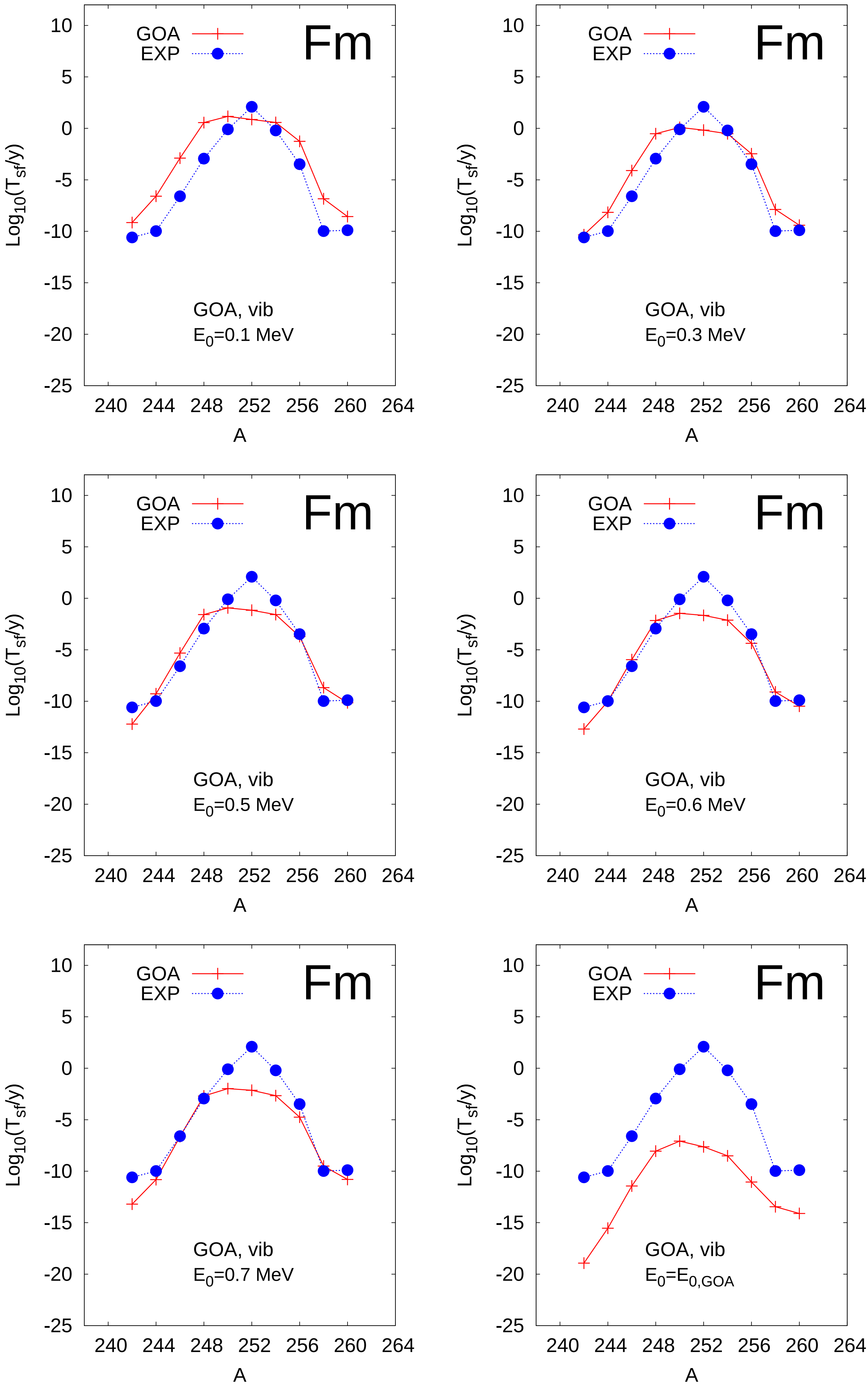


FIG. 7: Spontaneous fission half lives of Fermium isotopes in the case of HFB energy including **vibrational** energy correction within GCM-GOA parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

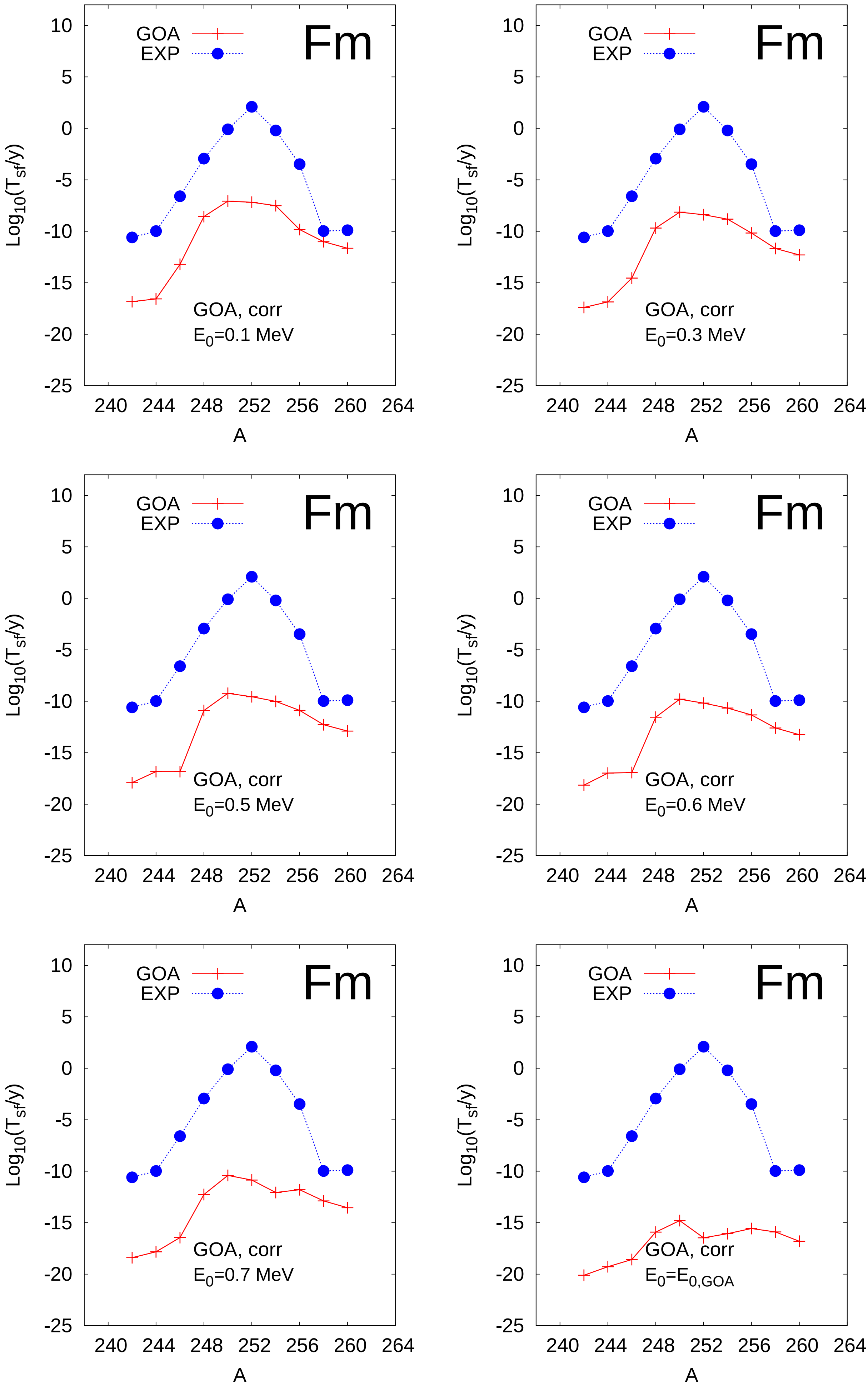


FIG. 8: Spontaneous fission half lives of Fermium isotopes in the case of HFB energy including both **vibrational** and **rotational** energy corrections within GCM-GOA mass parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

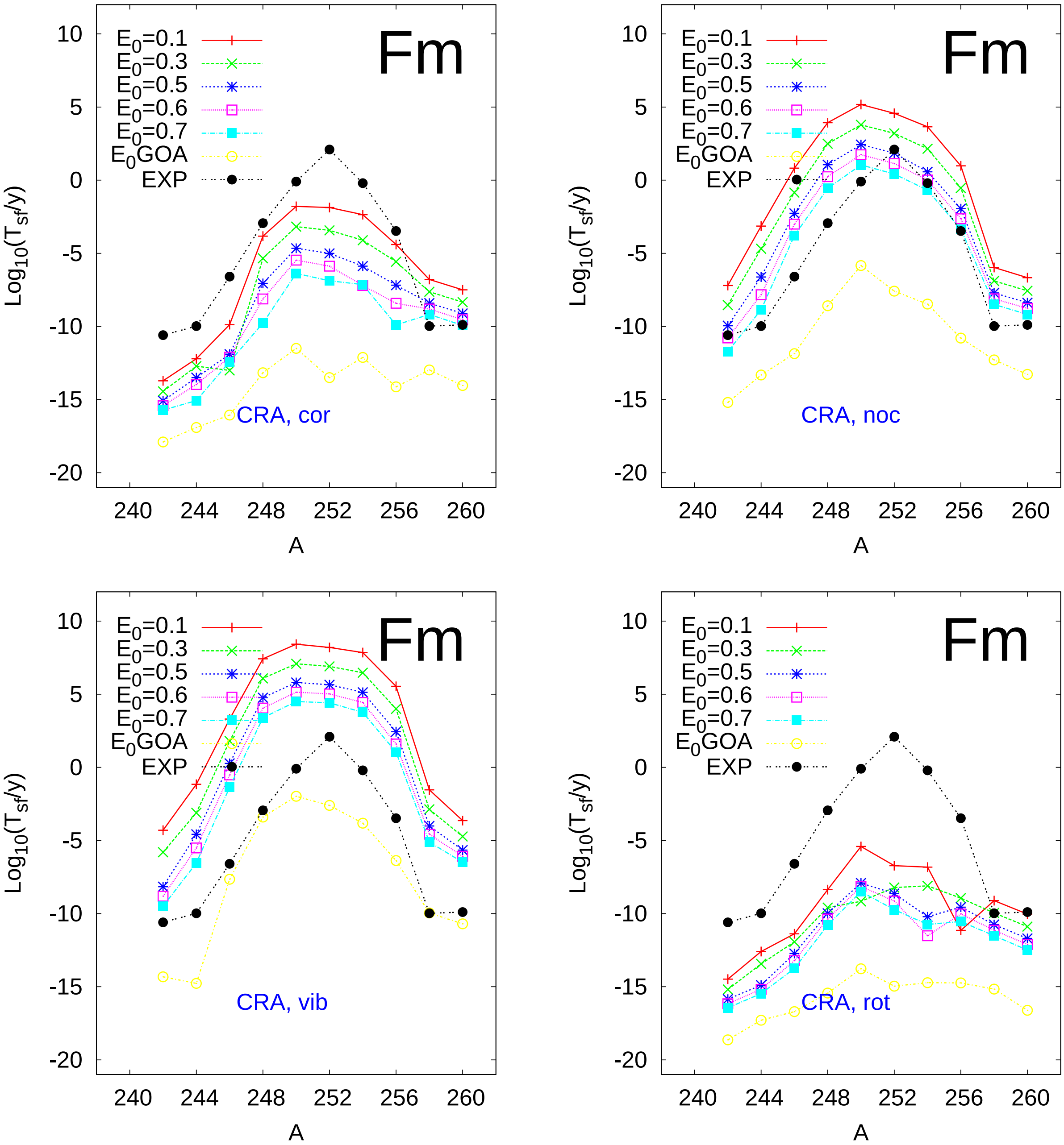


FIG. 9: Fission half lives. Cranking mass.

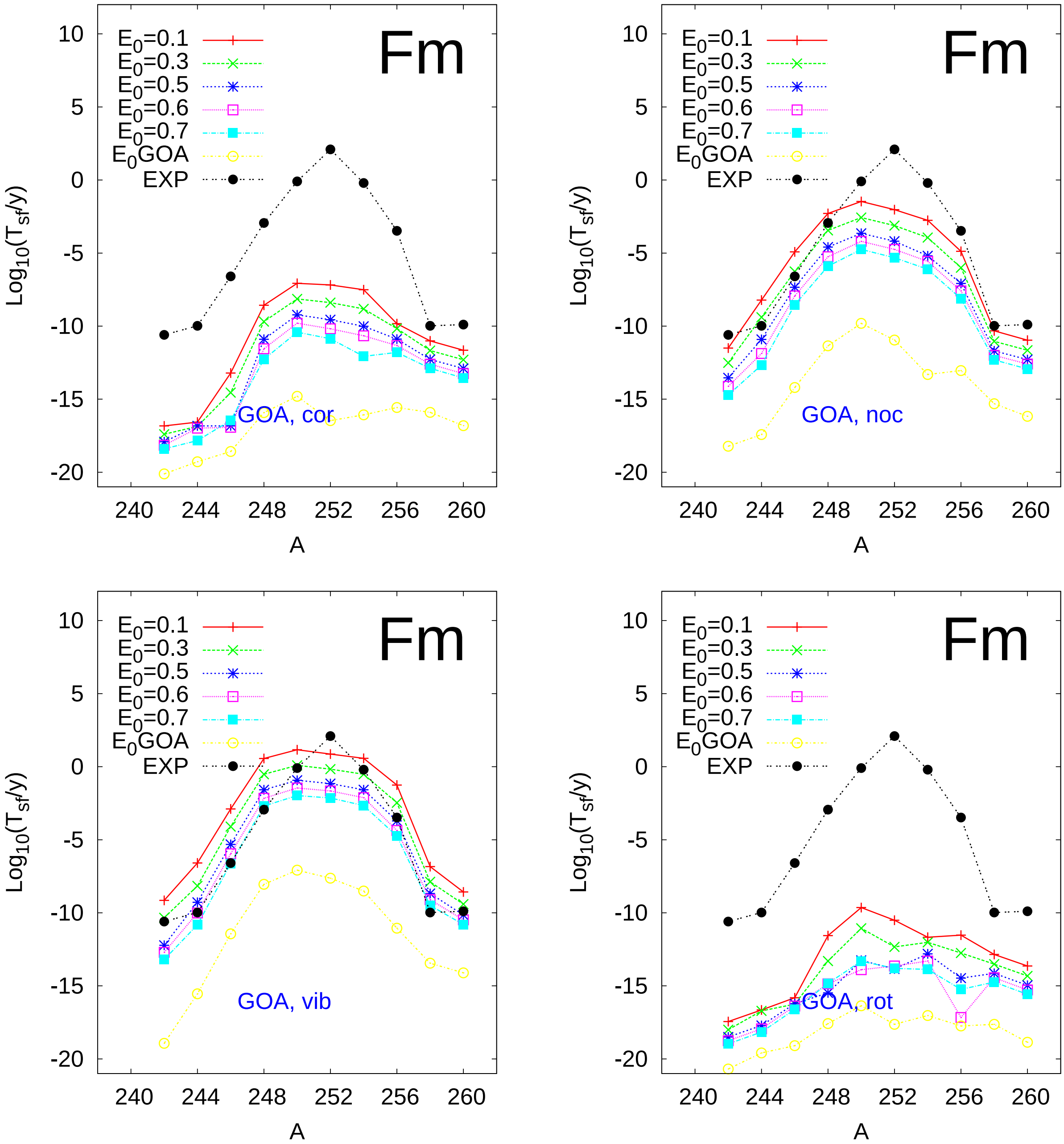


FIG. 10: Fission half lives. GOA mass.

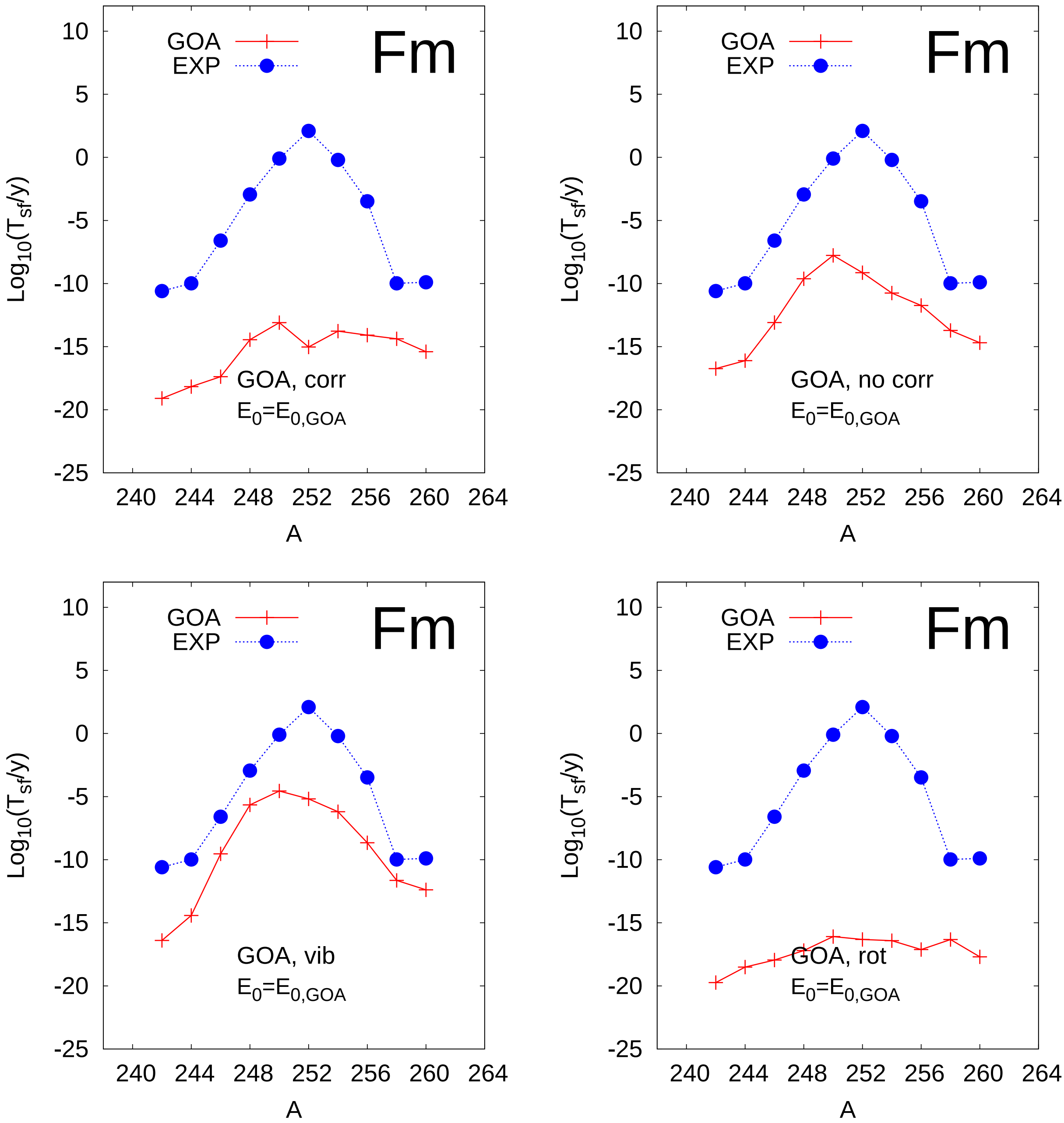


FIG. 11: Spontaneous fission half lives of Fermium isotopes in the case of HFB energy for mass parameter $1.25 \times B_{GOA}$ and $E_0 = E_{0,GOA}$.

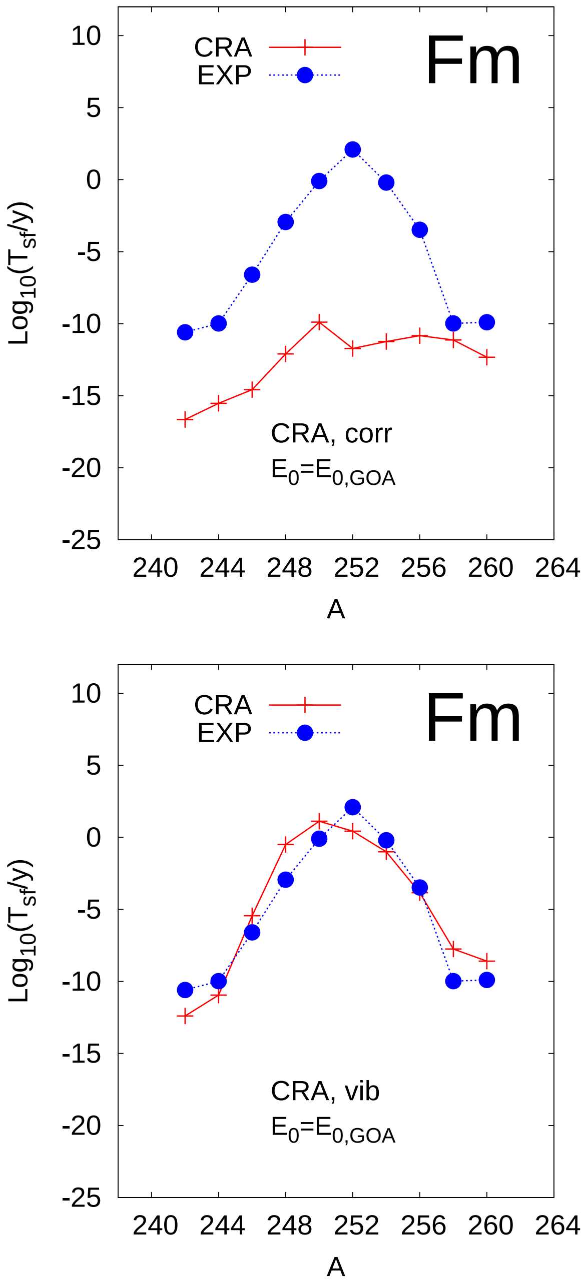


FIG. 12: Spontaneous fission half lives of Fermium isotopes in the case of HFB energy for mass parameter $1.25 \times B_{CRA}$ and $E_0 = E_{0,GOA}$.

III. FISSION HALF LIVES: HF+BCS-MIX

%

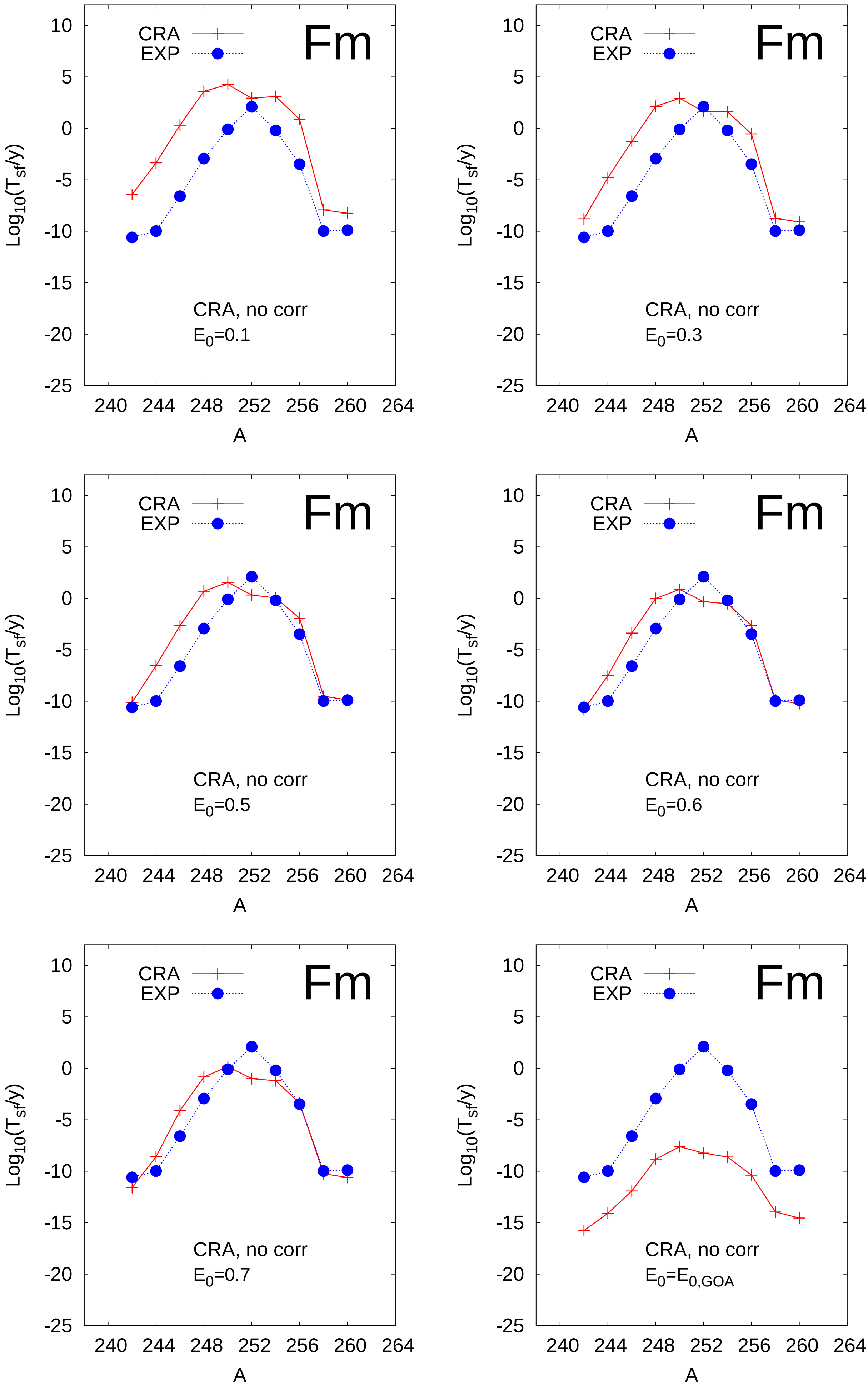


FIG. 13: Spontaneous fission half lives of Fermium isotopes in the case of uncorrected HF+BCS-mix energy barrier and cranking mass parameters. $E_0 = 0.1, 0.3, 0.5, 0.6, 0.7$ and $E_{0,gcm}$.

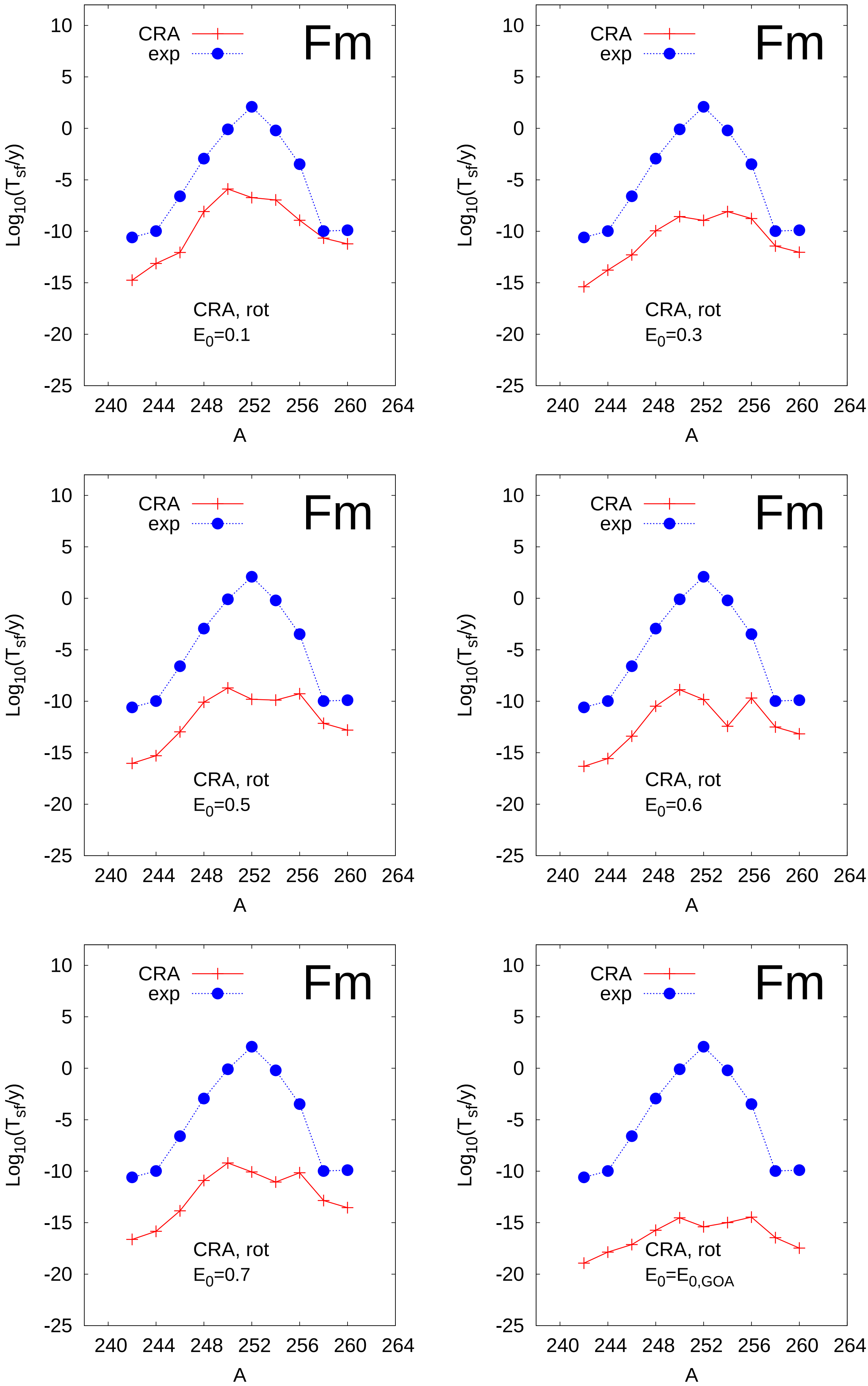


FIG. 14: Spontaneous fission half lives of Fermium isotopes in the case of HF+BCS-mix energy including **rotational** correction and cranking mass parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

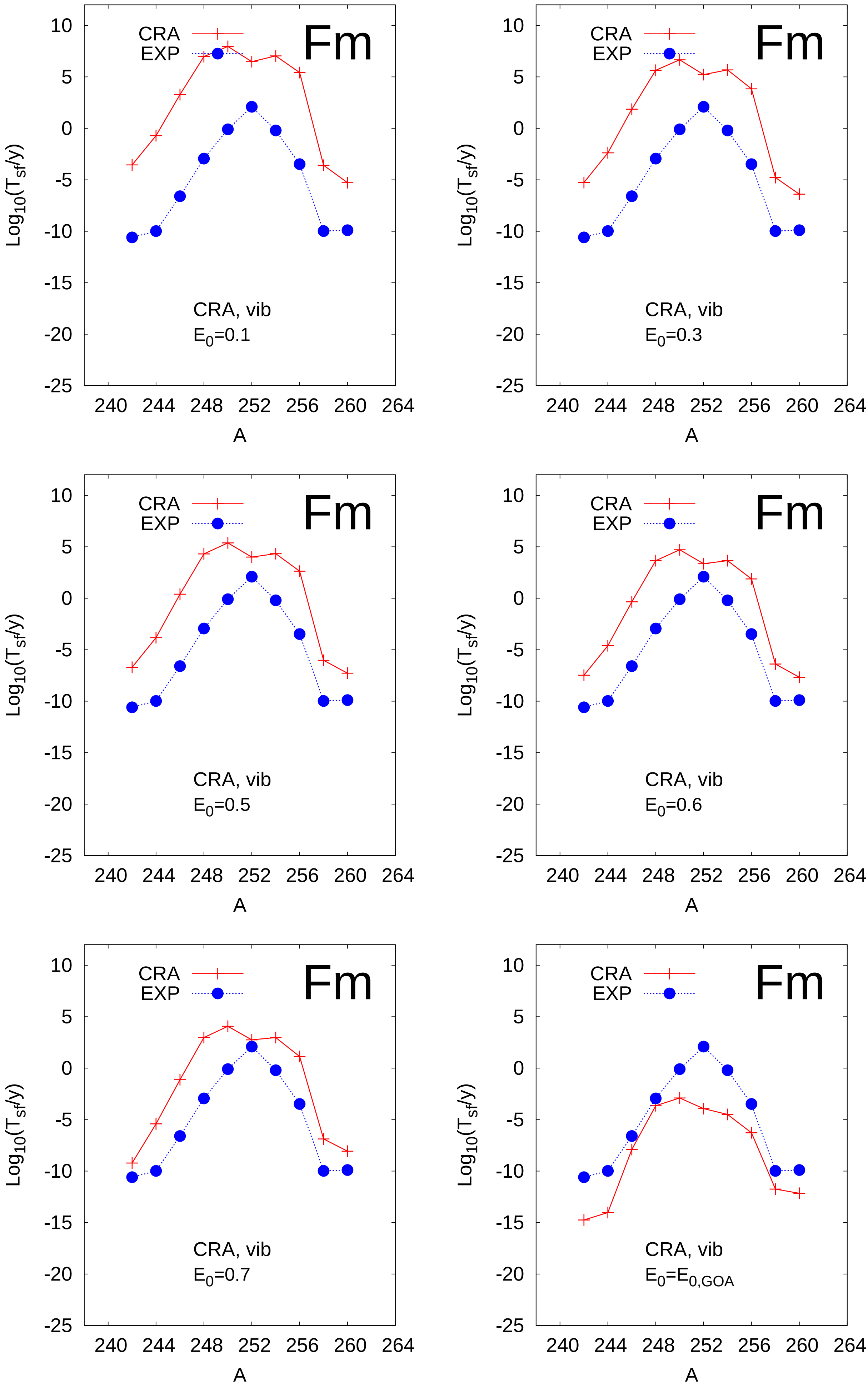


FIG. 15: Spontaneous fission half lives of Fermium isotopes in the case of HF+BCS-mix energy including **vibrational** correction and cranking mass parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

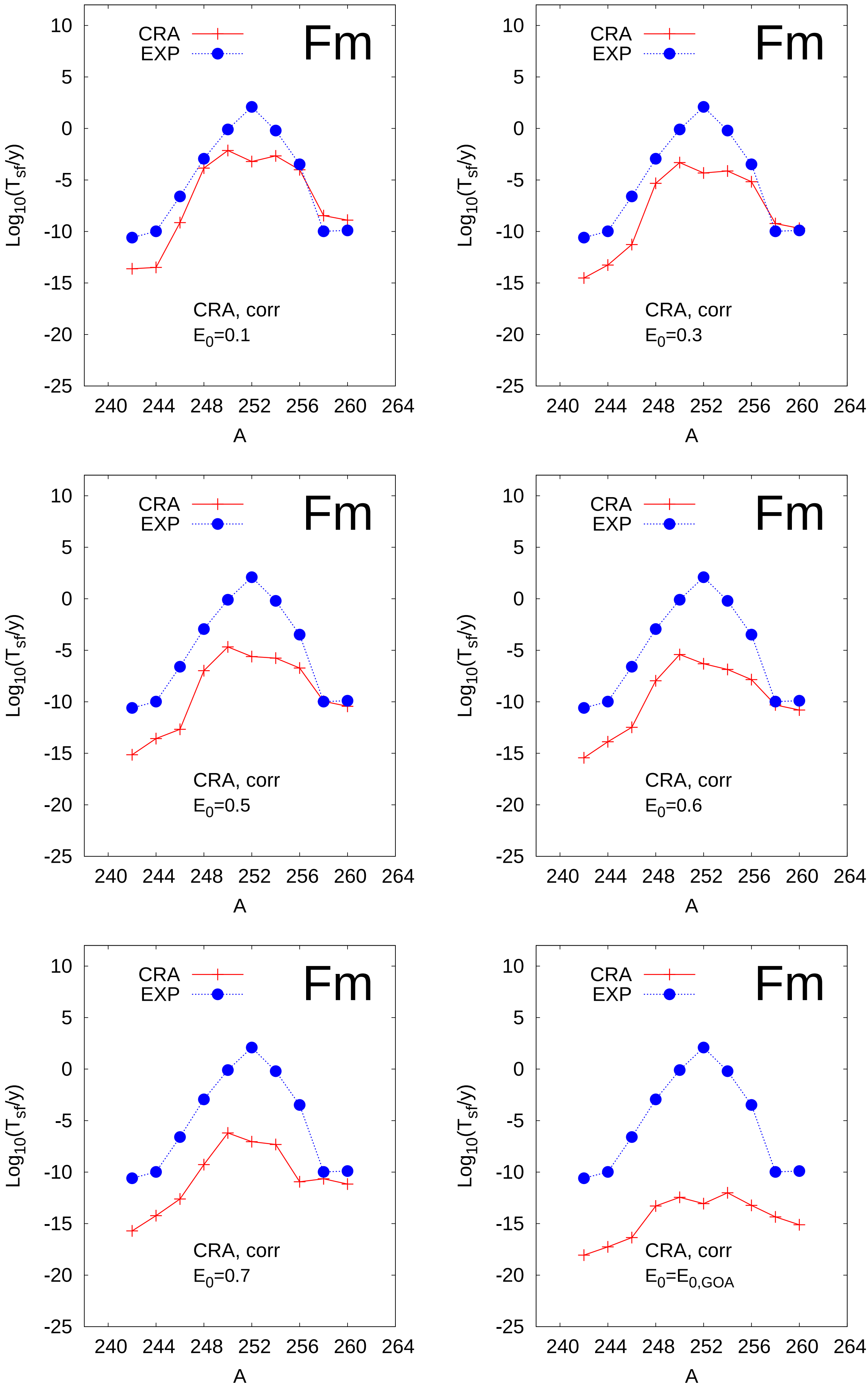


FIG. 16: Spontaneous fission half lives of Fermium isotopes in the case of HF+BCS-mix energy including both **vibrational** and **rotational** energy corrections and GCM-GOA mass parameters. $E_0 = 0.1, 0.3, 0.5, 0.6, 0.7$ and $E_{0,gcm}$.

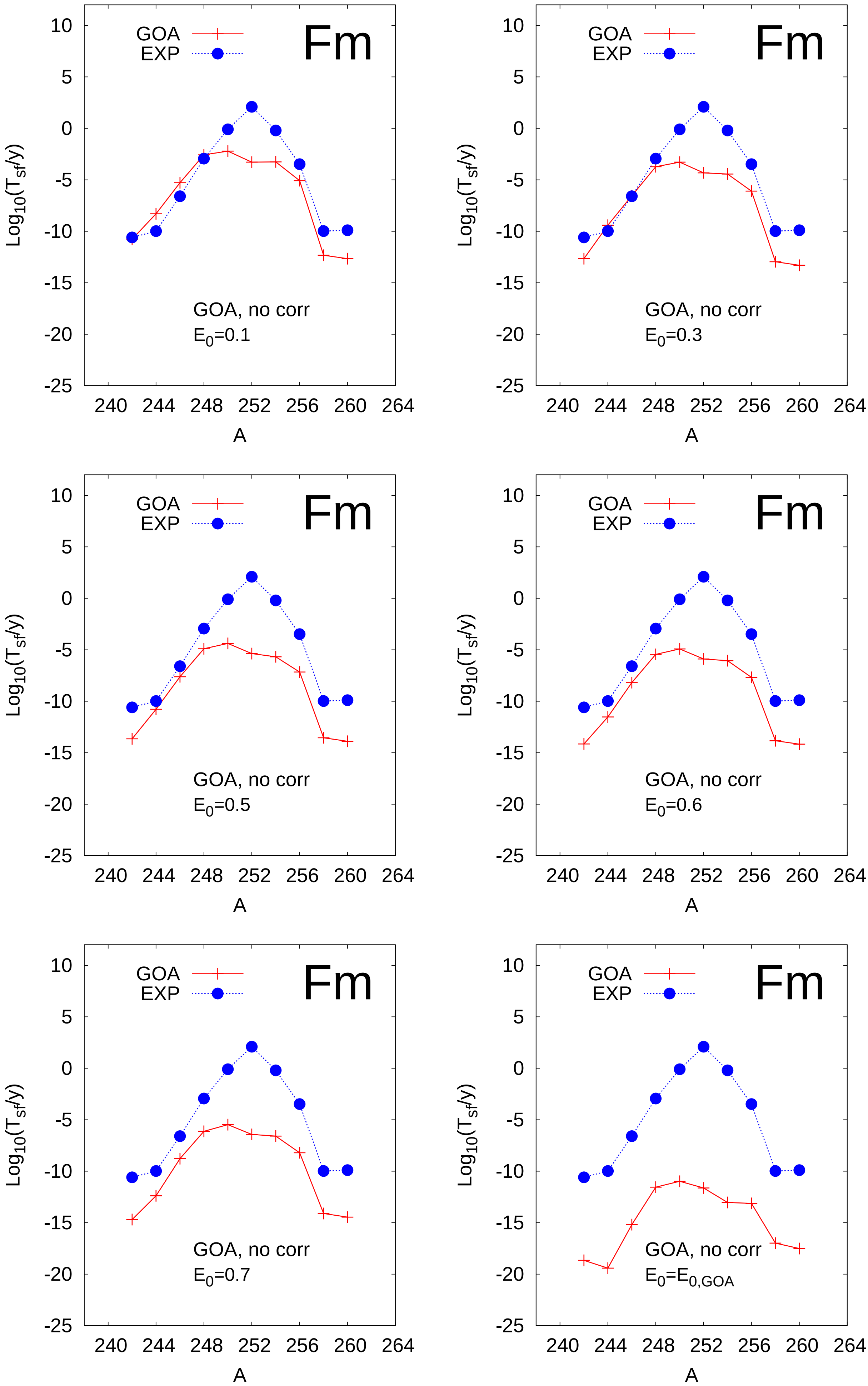


FIG. 17: Spontaneous fission half lives of Fermium isotopes in the case of uncorrected HF+BCS-mix energy and GCM-GOA mass parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

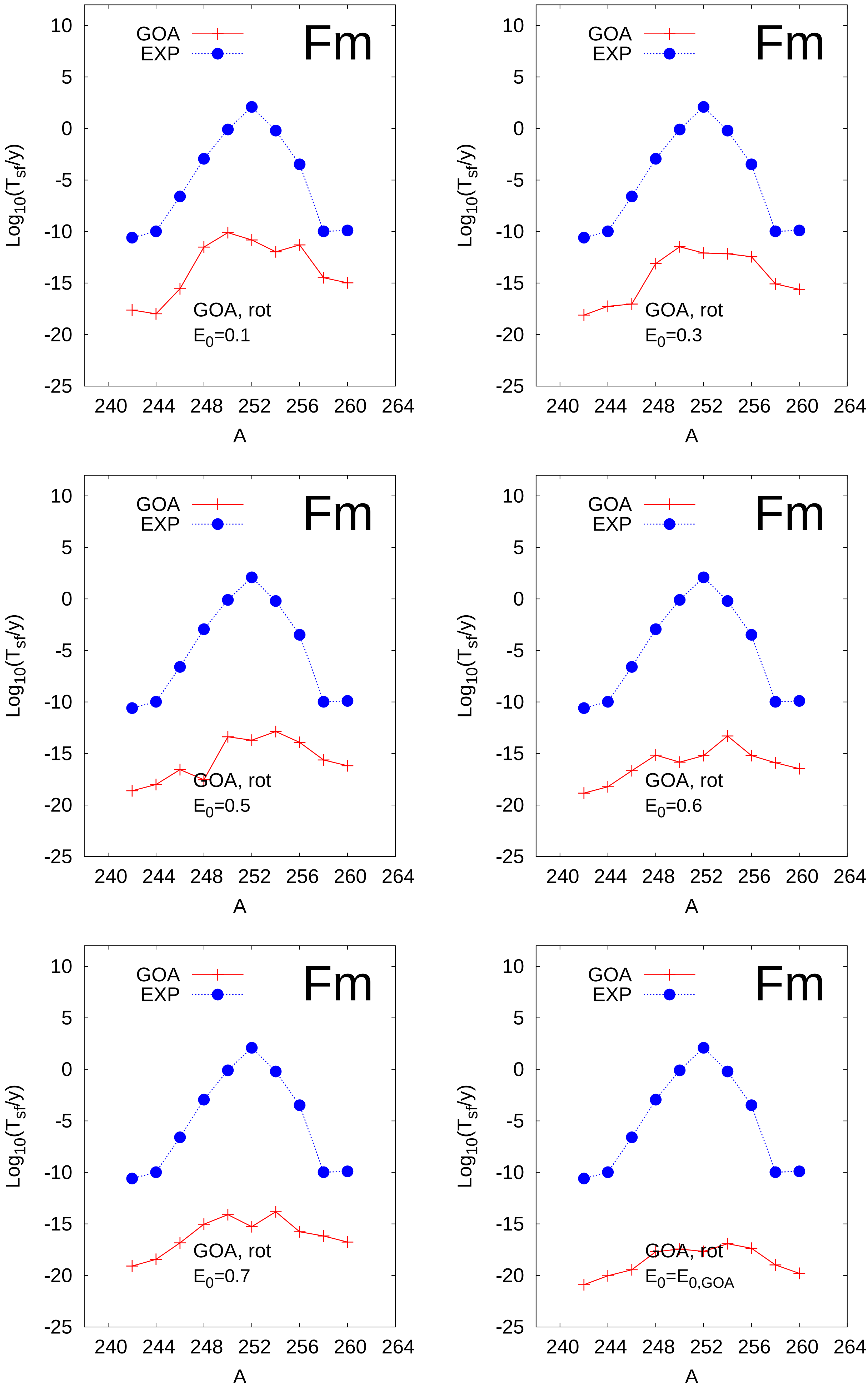


FIG. 18: Spontaneous fission half lives of Fermium isotopes in the case of HF+BCS-mix energy including **rotational** energy correction and GCM-GOA mass parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

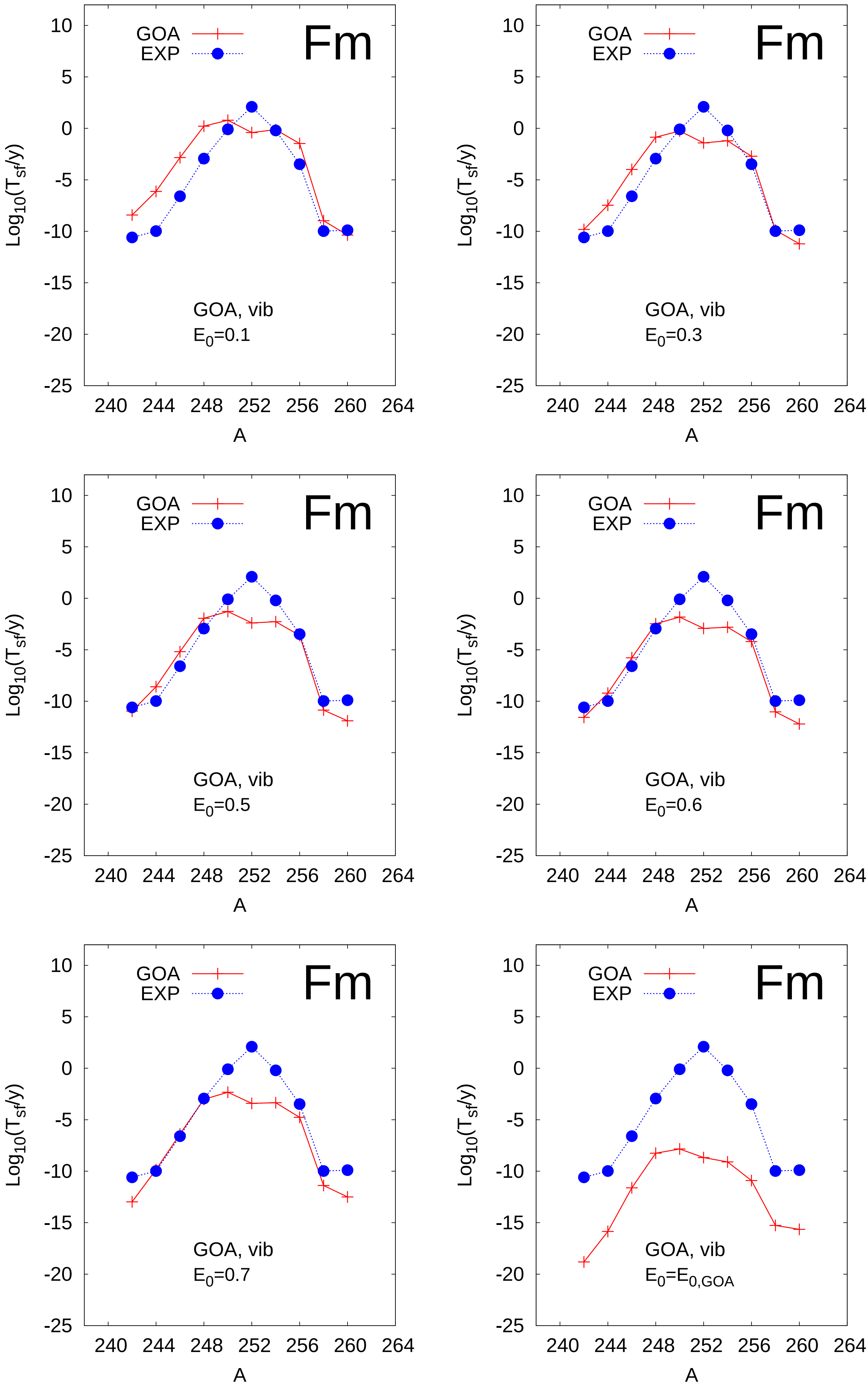


FIG. 19: Spontaneous fission half lives of Fermium isotopes in the case of HF+BCS-mix energy including **vibrational** energy correction within GCM-GOA parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

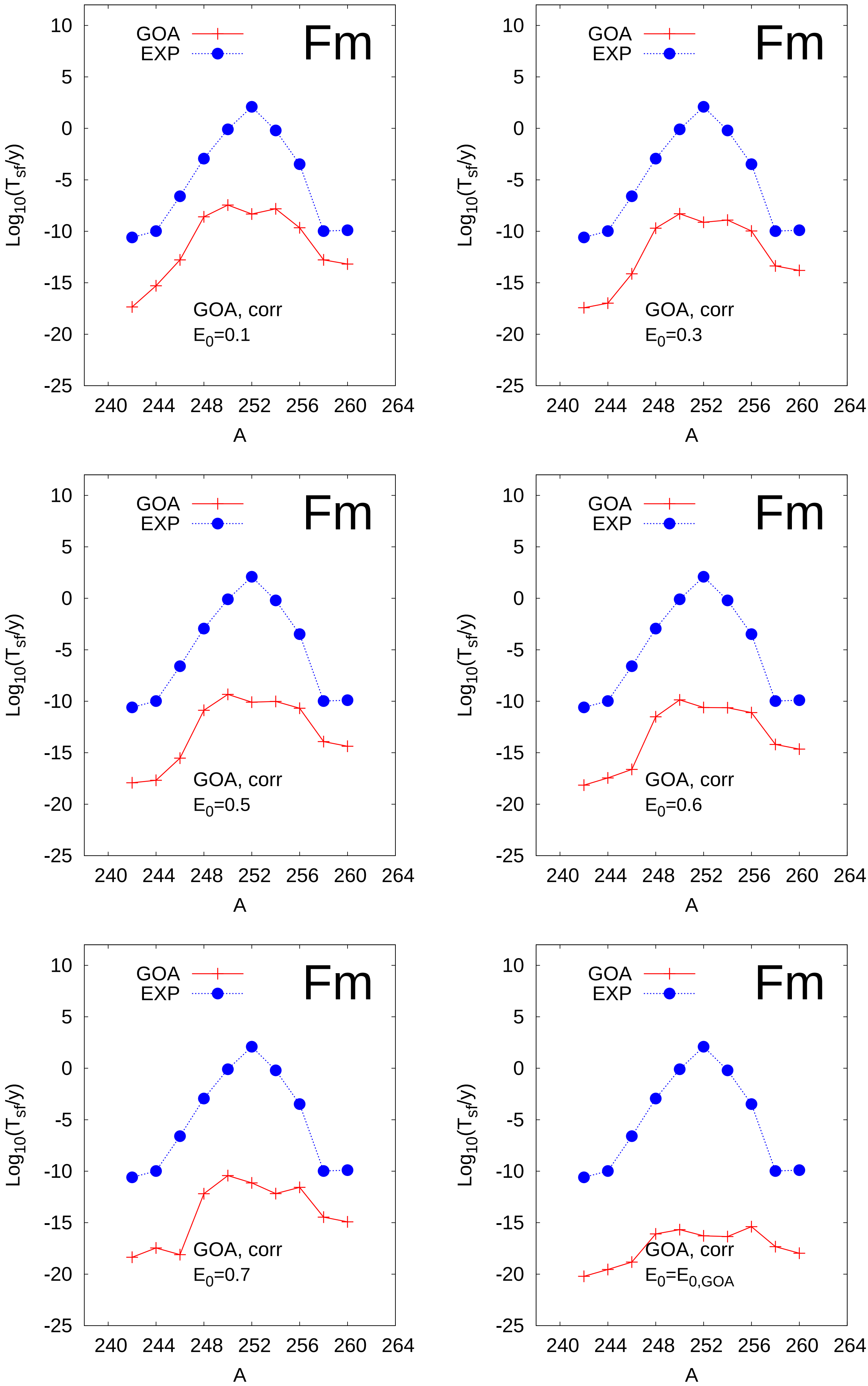


FIG. 20: Spontaneous fission half lives of Fermium isotopes in the case of HF+BCS-mix energy including both **vibrational** and **rotational** energy corrections within GCM-GOA mass parameters. $E_0 = 0.1, 0.3, 0.5, 0.6, 0.7$ and $E_{0,gcm}$.

IV. FISSION HALF LIVES: HF+BCS-G

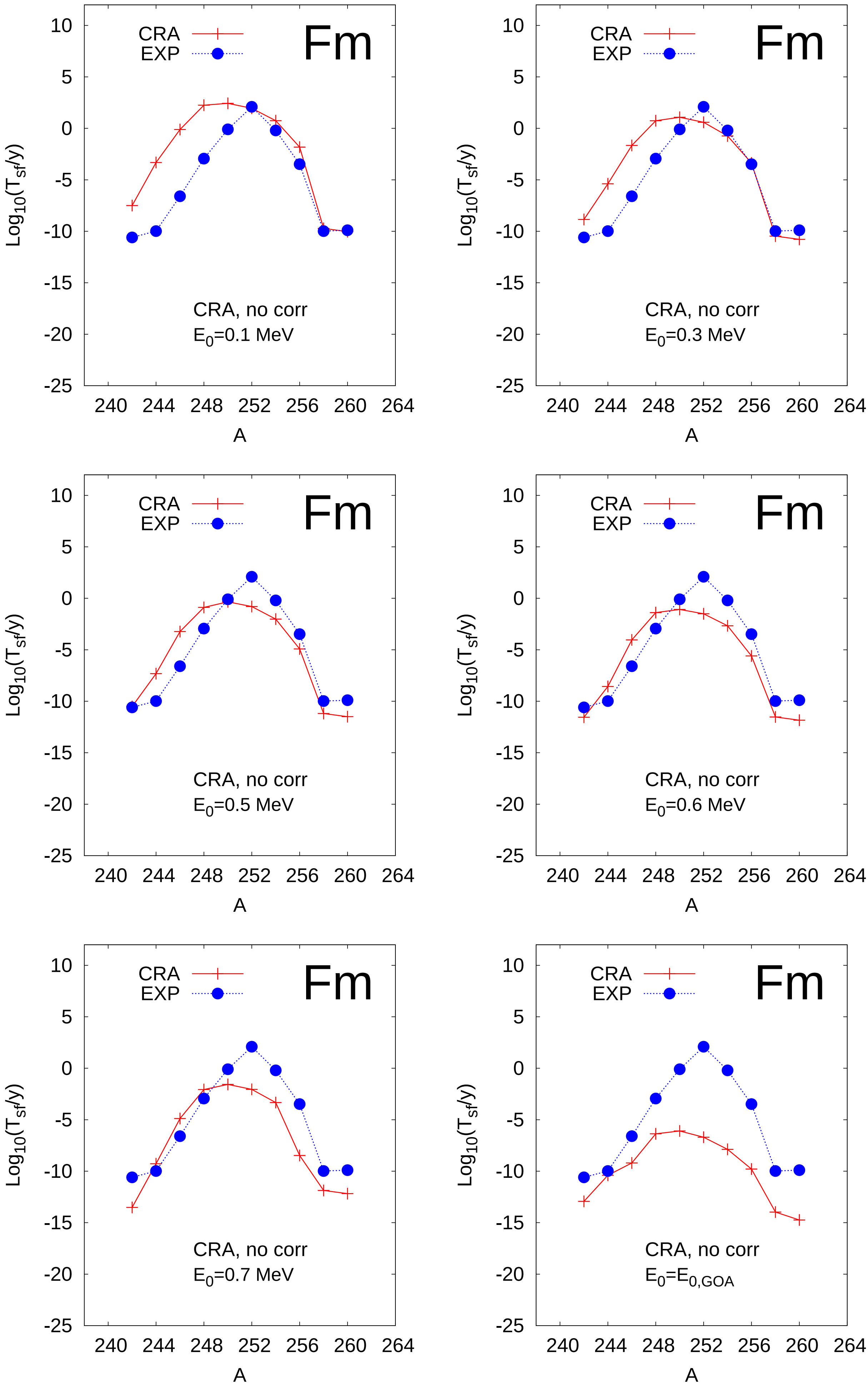


FIG. 21: Spontaneous fission half lives of Fermium isotopes in the case of uncorrected HF+bcs-G energy barrier and cranking mass parameters. $E_0 = 0.1, 0.3, 0.5, 0.6, 0.7$ and $E_{0,gcm}$.

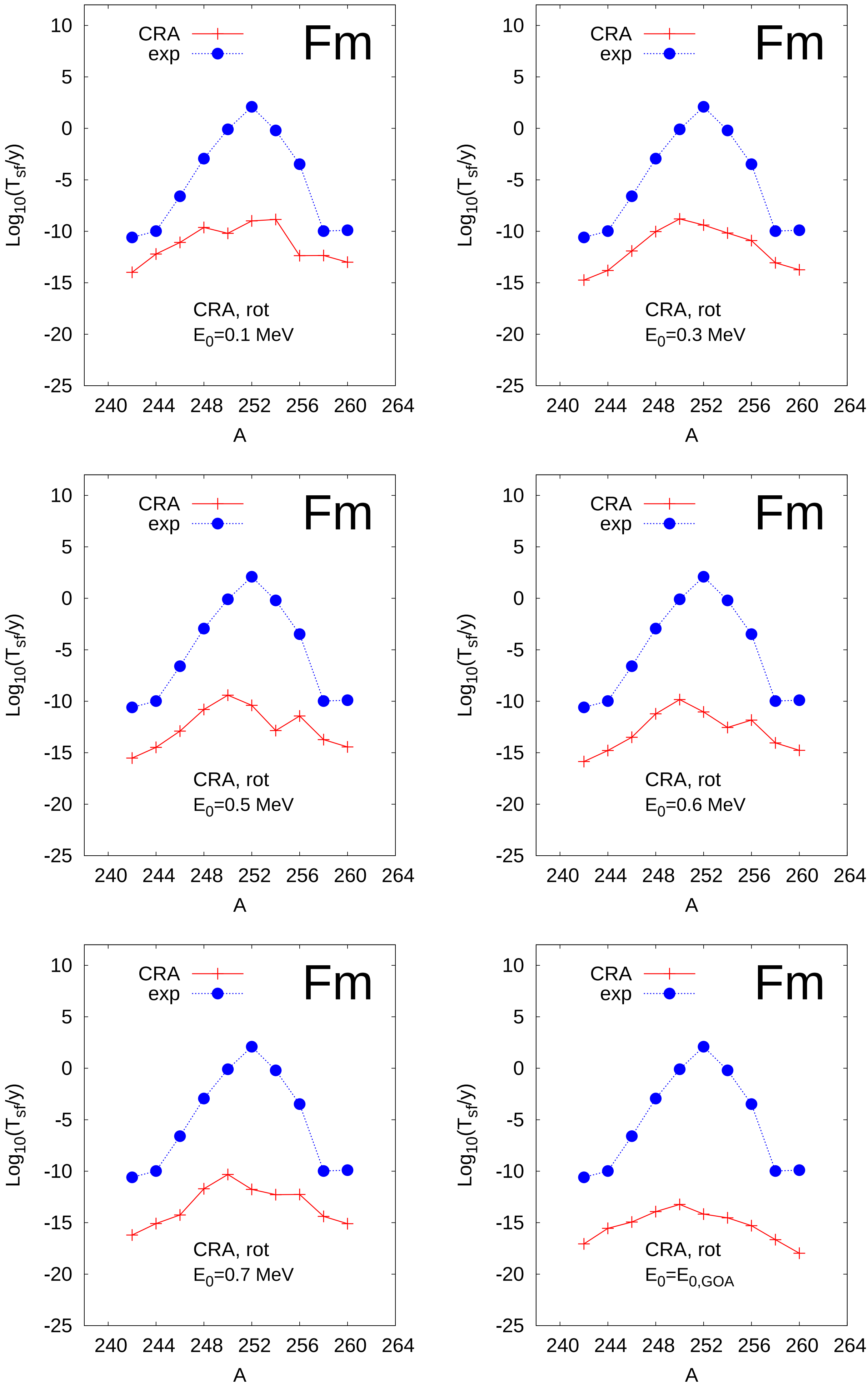


FIG. 22: Spontaneous fission half lives of Fermium isotopes in the case of HF+BCS-G energy including **rotational** correction and cranking mass parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

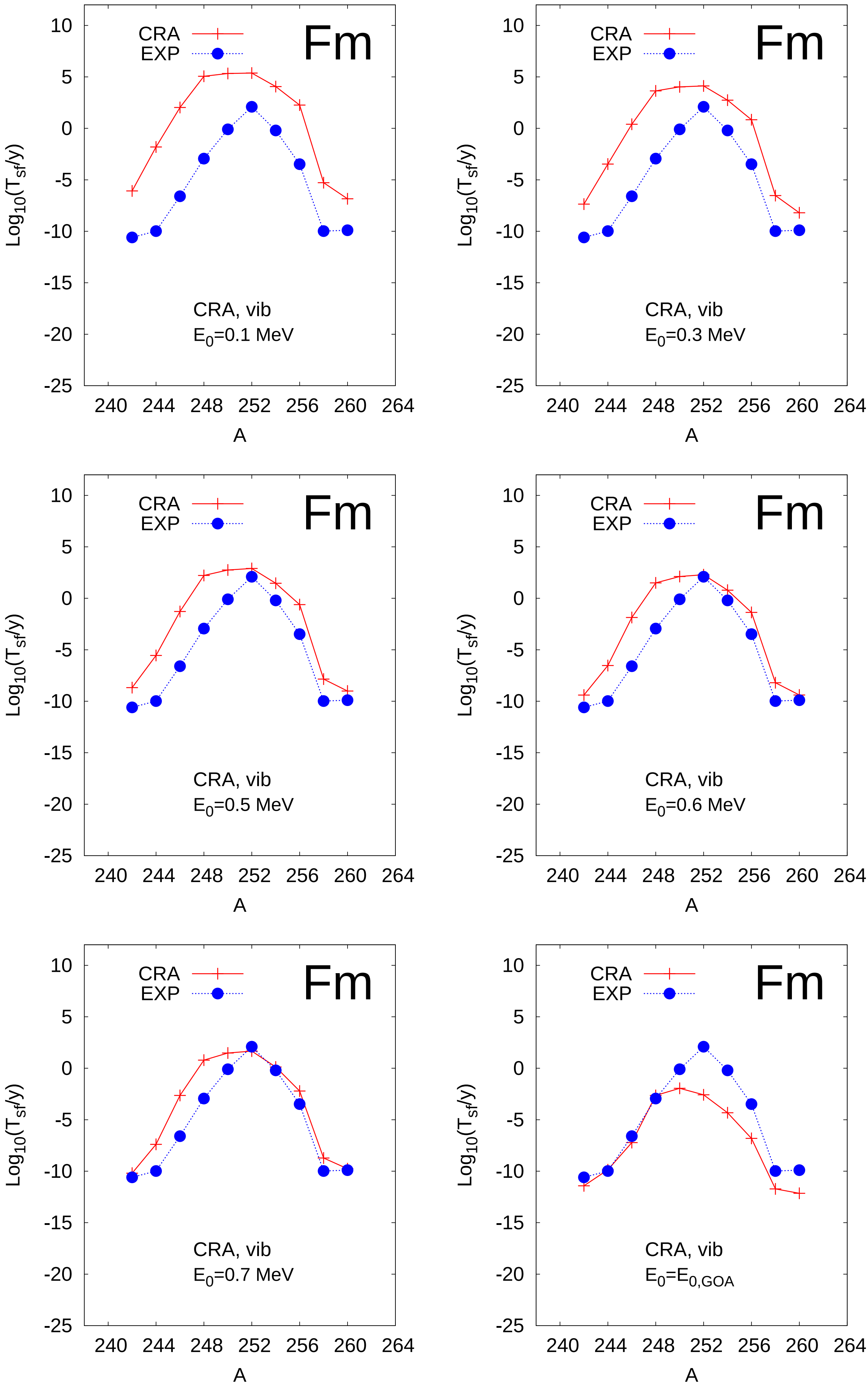


FIG. 23: Spontaneous fission half lives of Fermium isotopes in the case of HF+BCS-G energy including **vibrational** correction and cranking mass parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

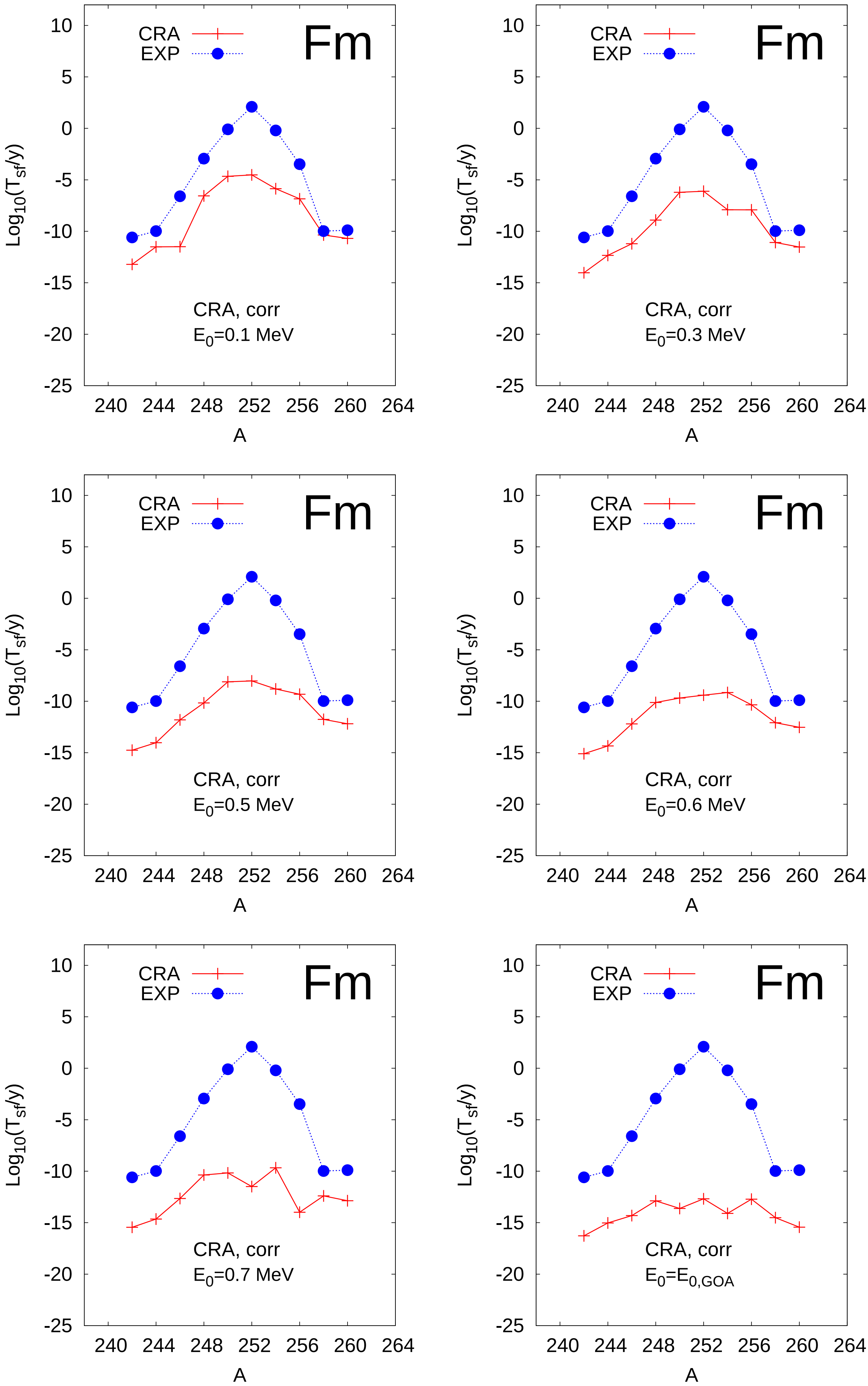


FIG. 24: Spontaneous fission half lives of Fermium isotopes in the case of HF+BCS-G energy including both **vibrational** and **rotational** energy corrections and GCM-GOA mass parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

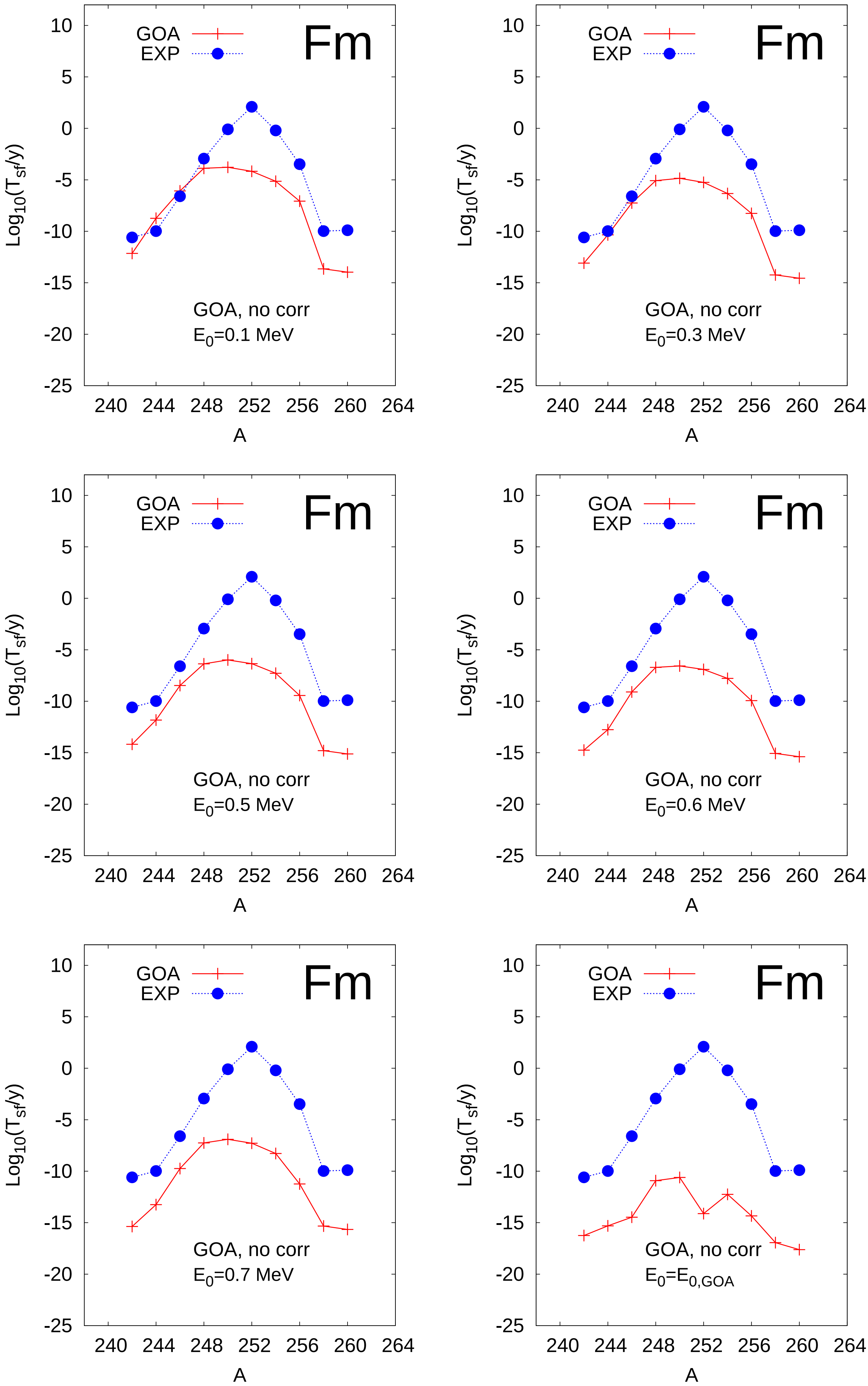


FIG. 25: Spontaneous fission half lives of Fermium isotopes in the case of uncorrected HF+BCS-G energy and GCM-GOA mass parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

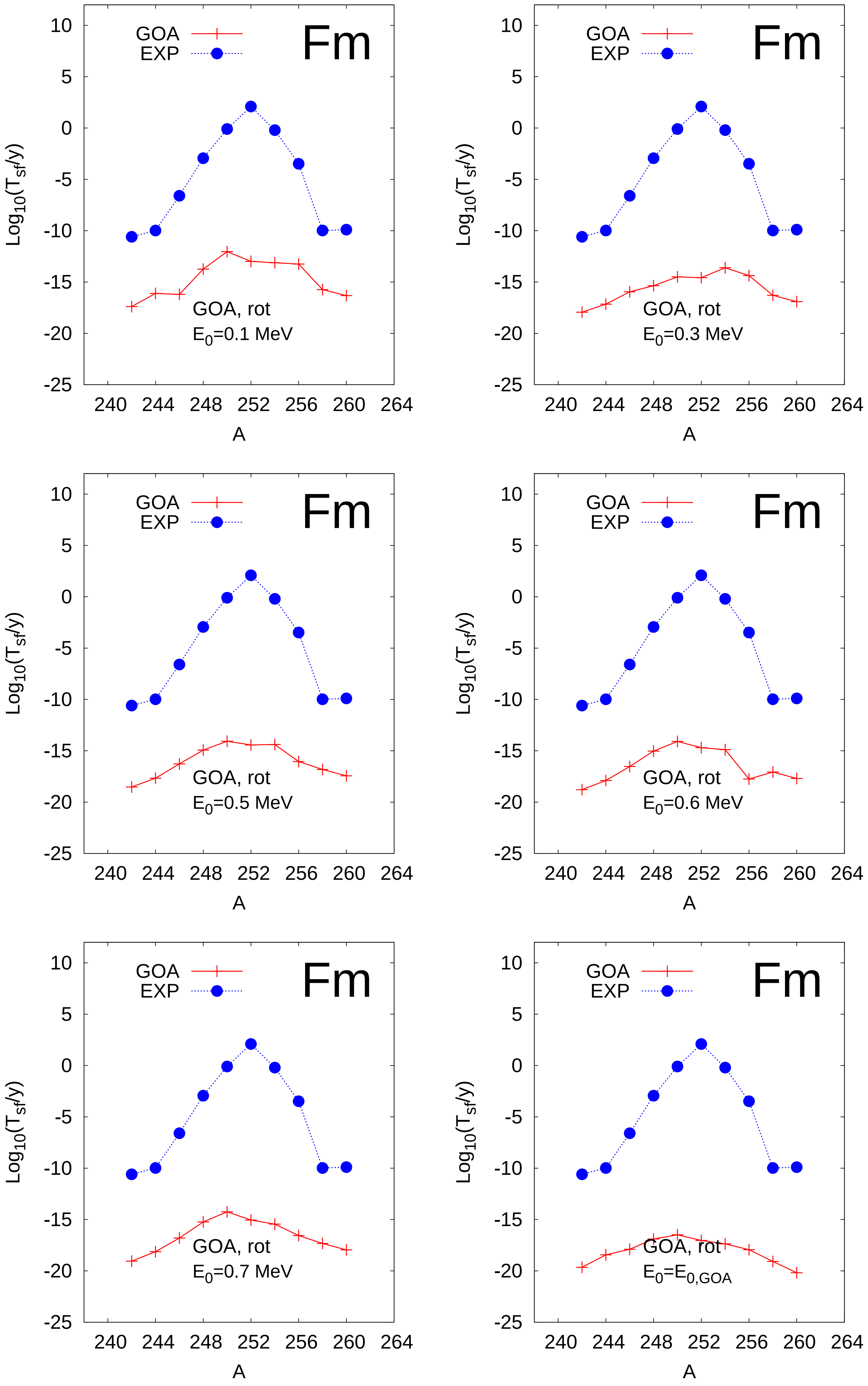


FIG. 26: Spontaneous fission half lives of Fermium isotopes in the case of HF+BCS-G energy including **rotational** energy correction and GCM-GOA mass parameters. $E_0 = 0.1, 0.3, 0.5, 0.6, 0.7$ and $E_{0,gcm}$.

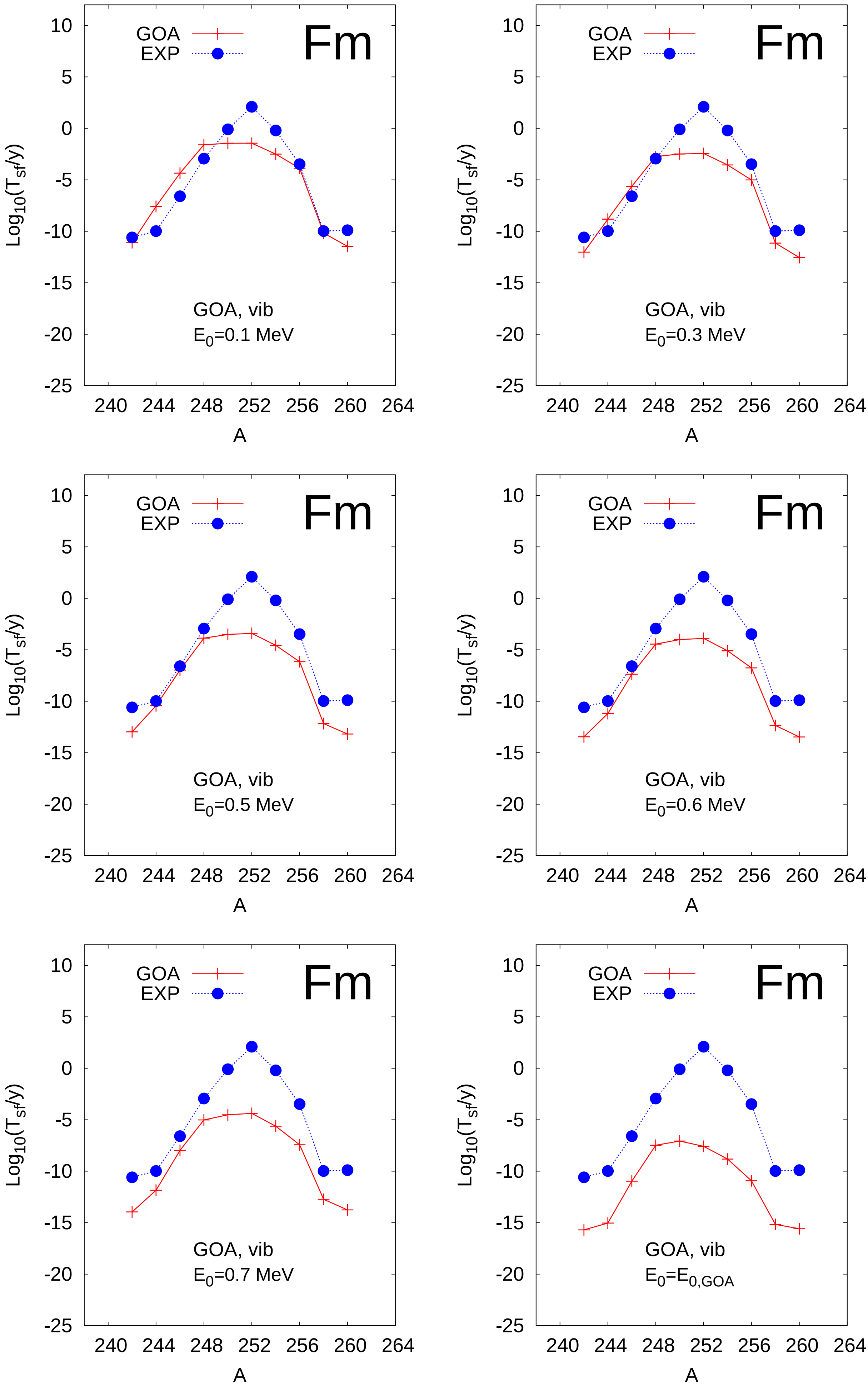


FIG. 27: Spontaneous fission half lives of Fermium isotopes in the case of HF+BCS-G energy including **vibrational** energy correction within GCM-GOA parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

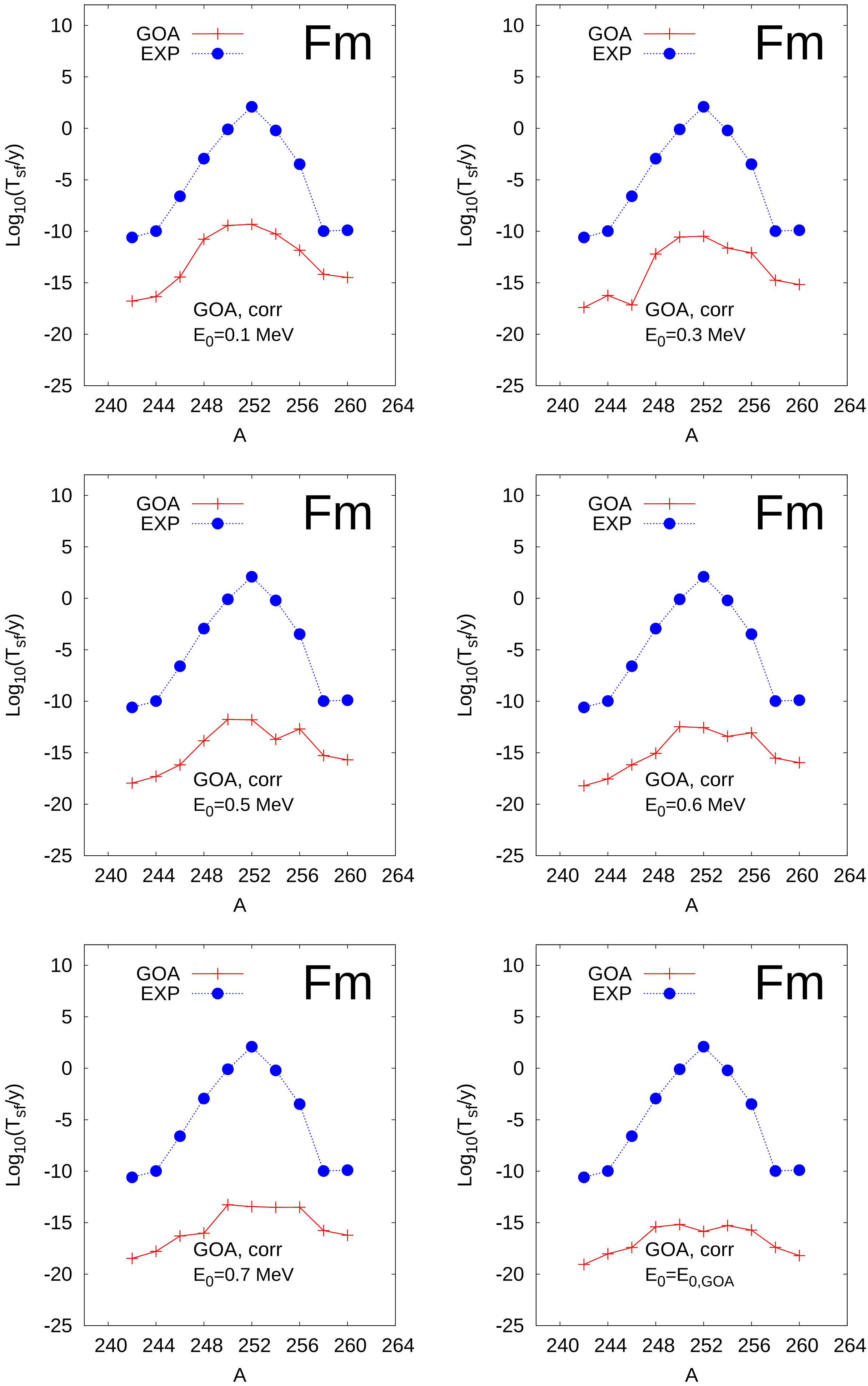
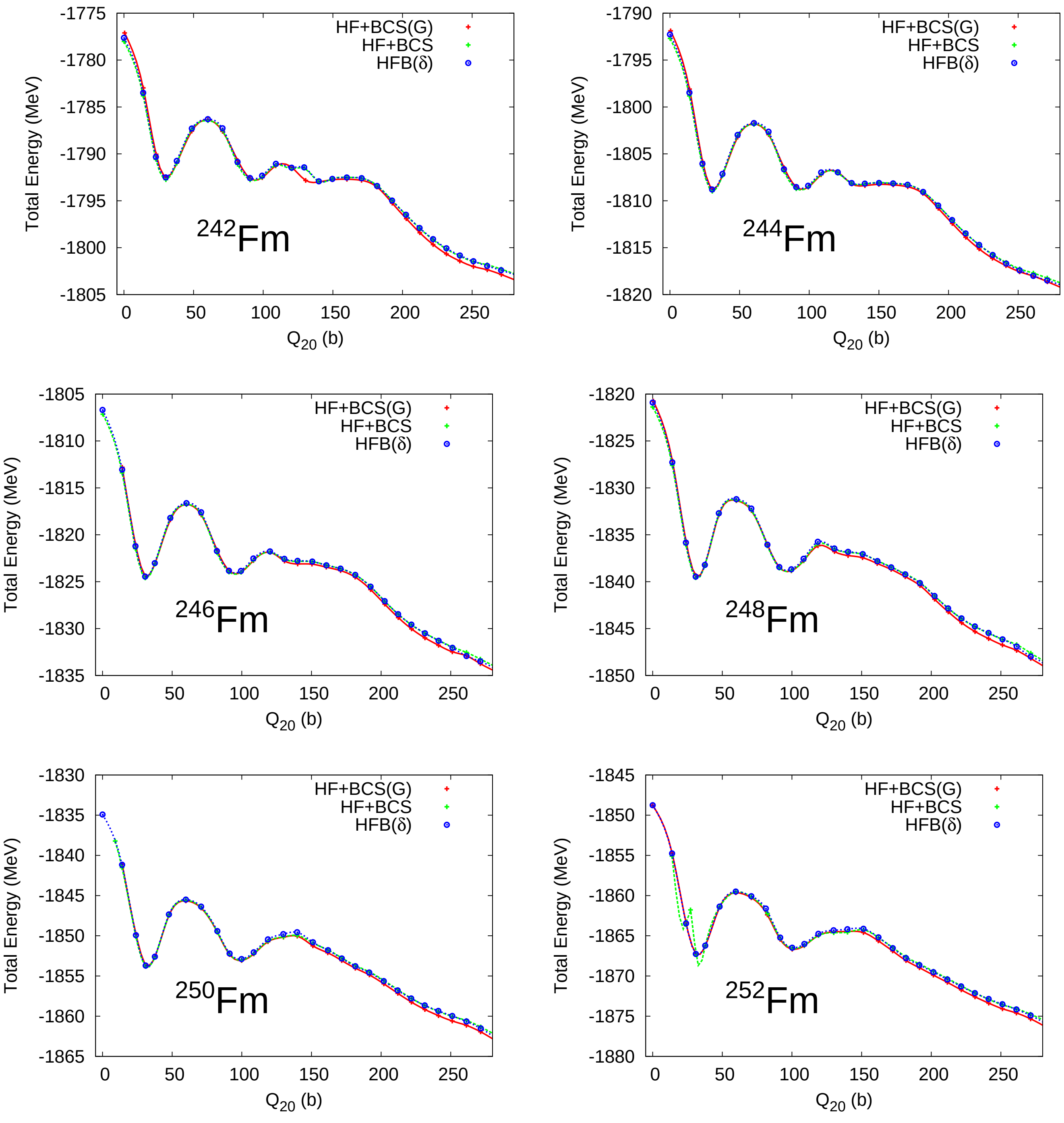
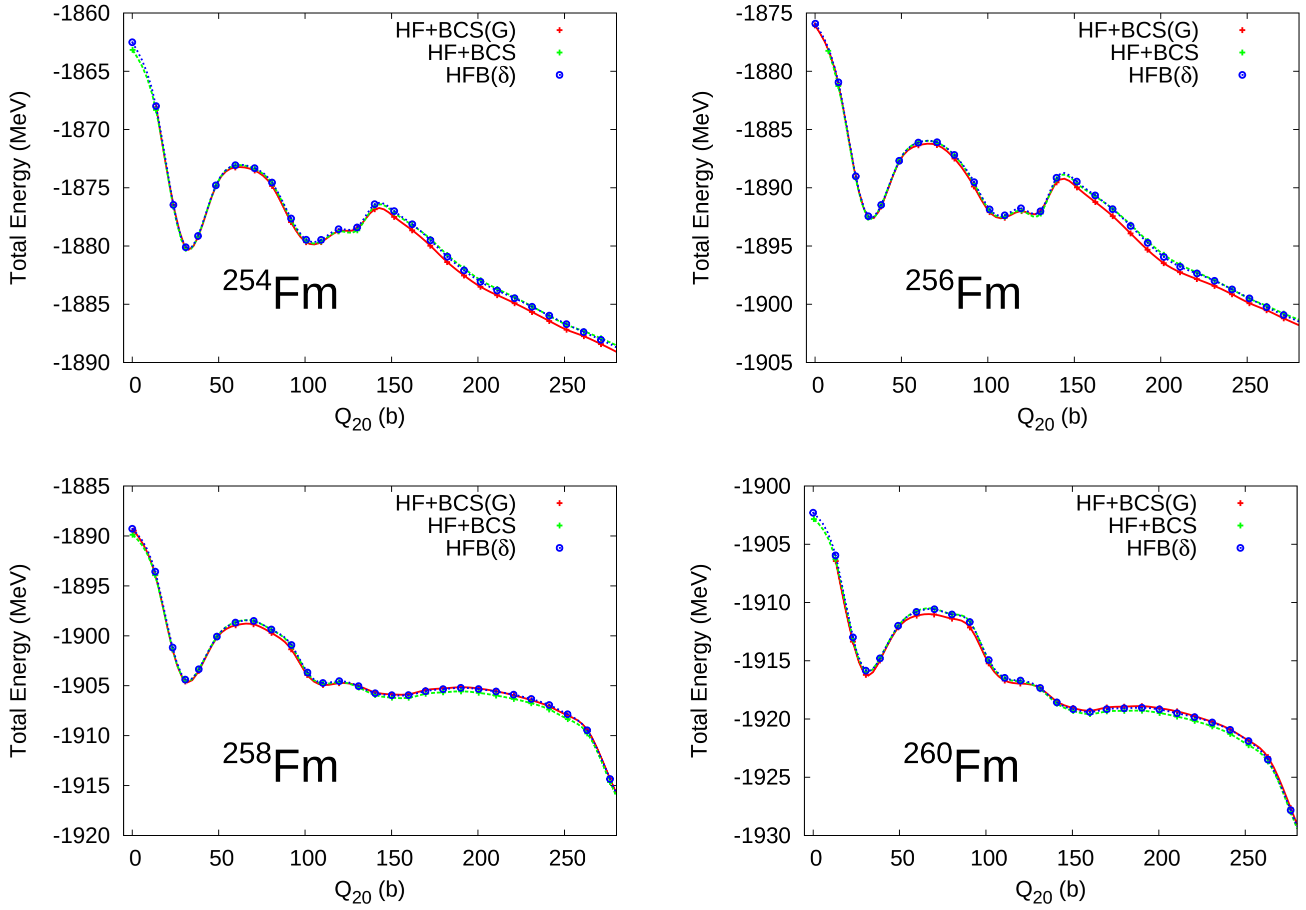


FIG. 28: Spontaneous fission half lives of Fermium isotopes in the case of HF+BCS-G energy including both **vibrational** and **rotational** energy corrections within GCM-GOA mass parameters. $E_0 = 0.1$, 0.3, 0.5, 0.6, 0.7 and $E_{0,gcm}$.

V. COMPARISON OF HFB, HF+BCS, AND HF+BCS-MIX RESULTS

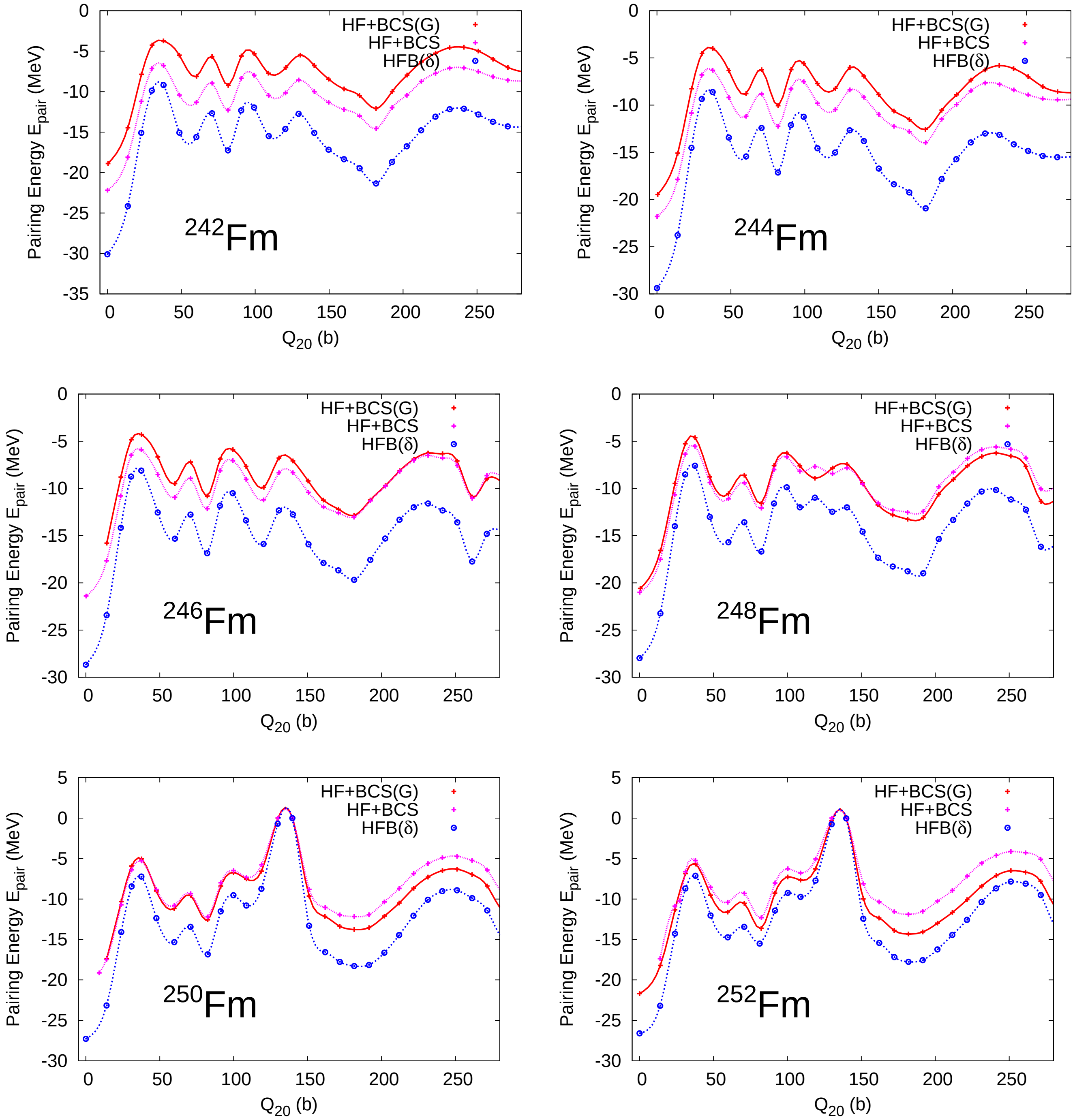
A. Barriers

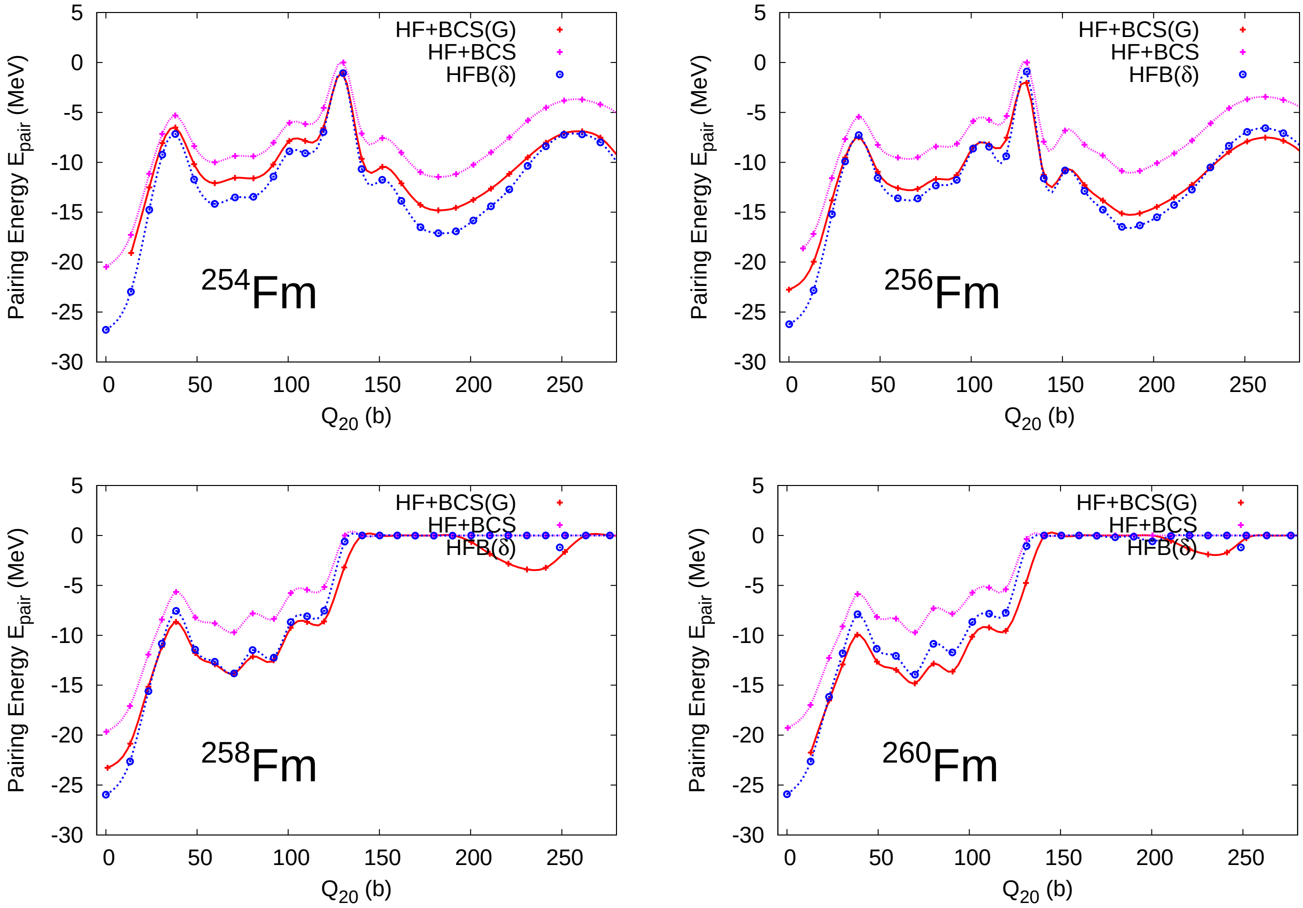




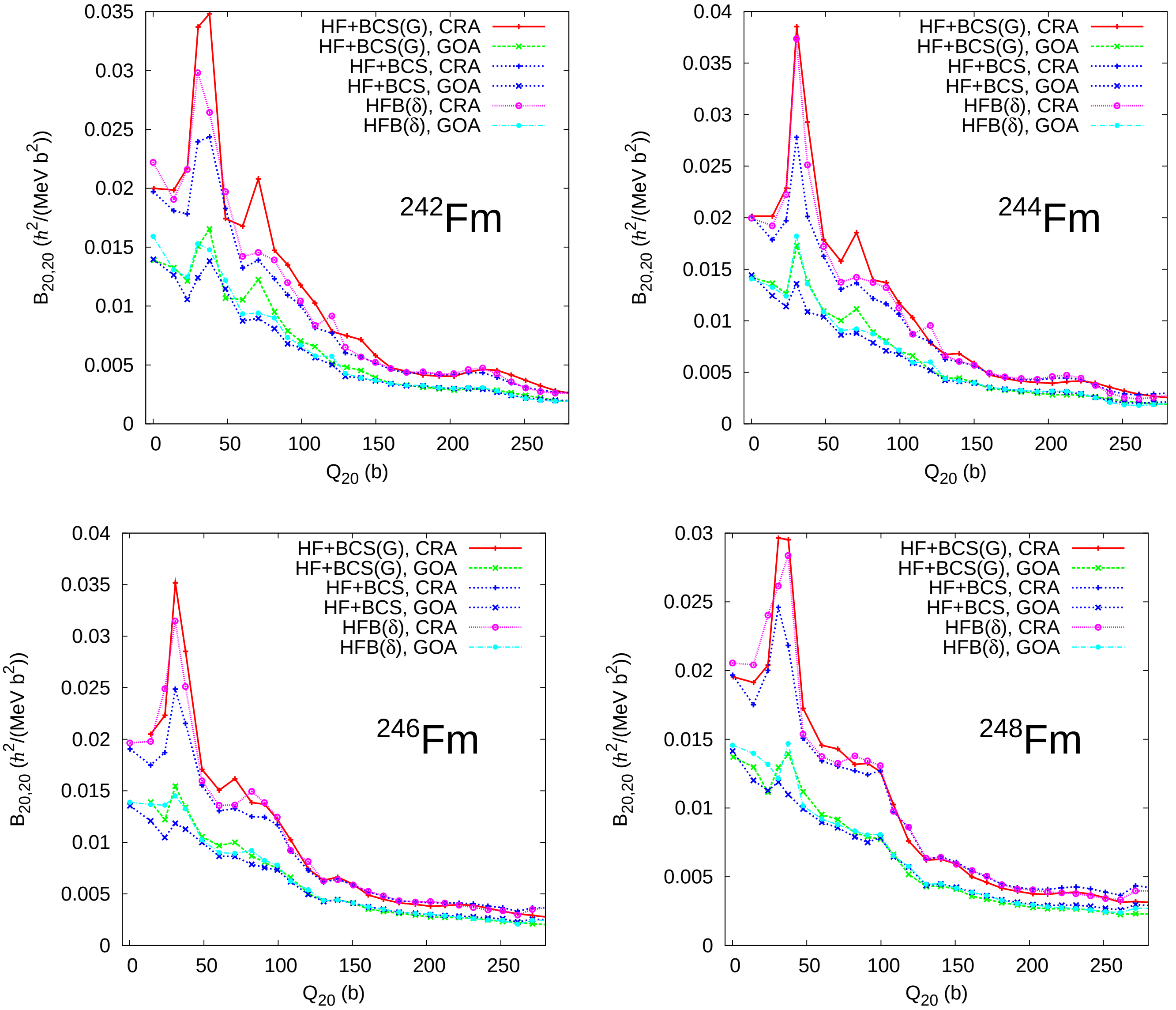
HF+BCS(G)
HF+BCS
HFB(δ)
Total Energy (MeV)
Q20 (b)
254Fm
256Fm
258Fm
260Fm

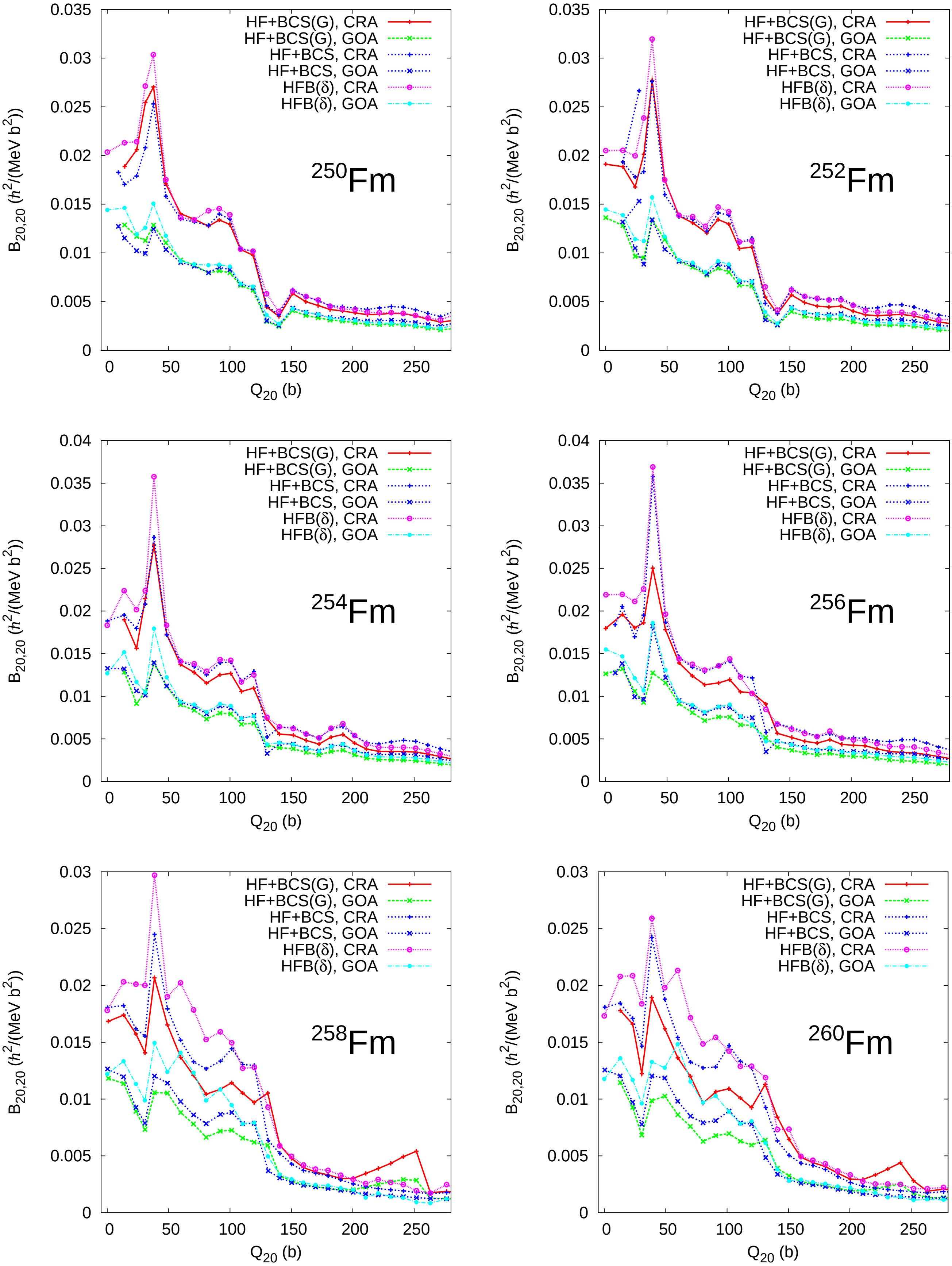
B. Pairing Energy



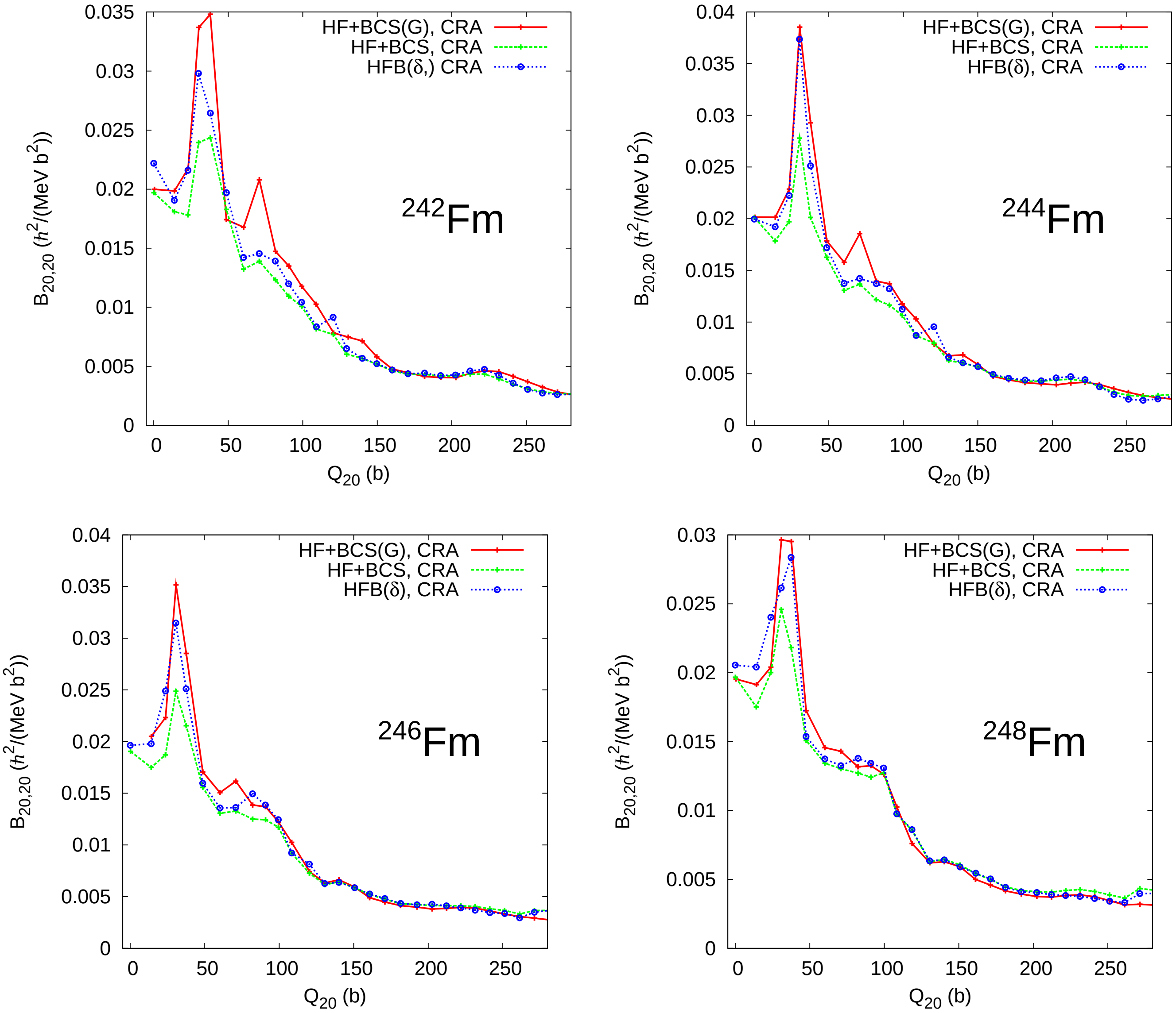

Pairing Energy E_pair (MeV)
Q_20 (b)
HF+BCS(G)
HF+BCS
HFB(δ)
254Fm
256Fm
258Fm
260Fm

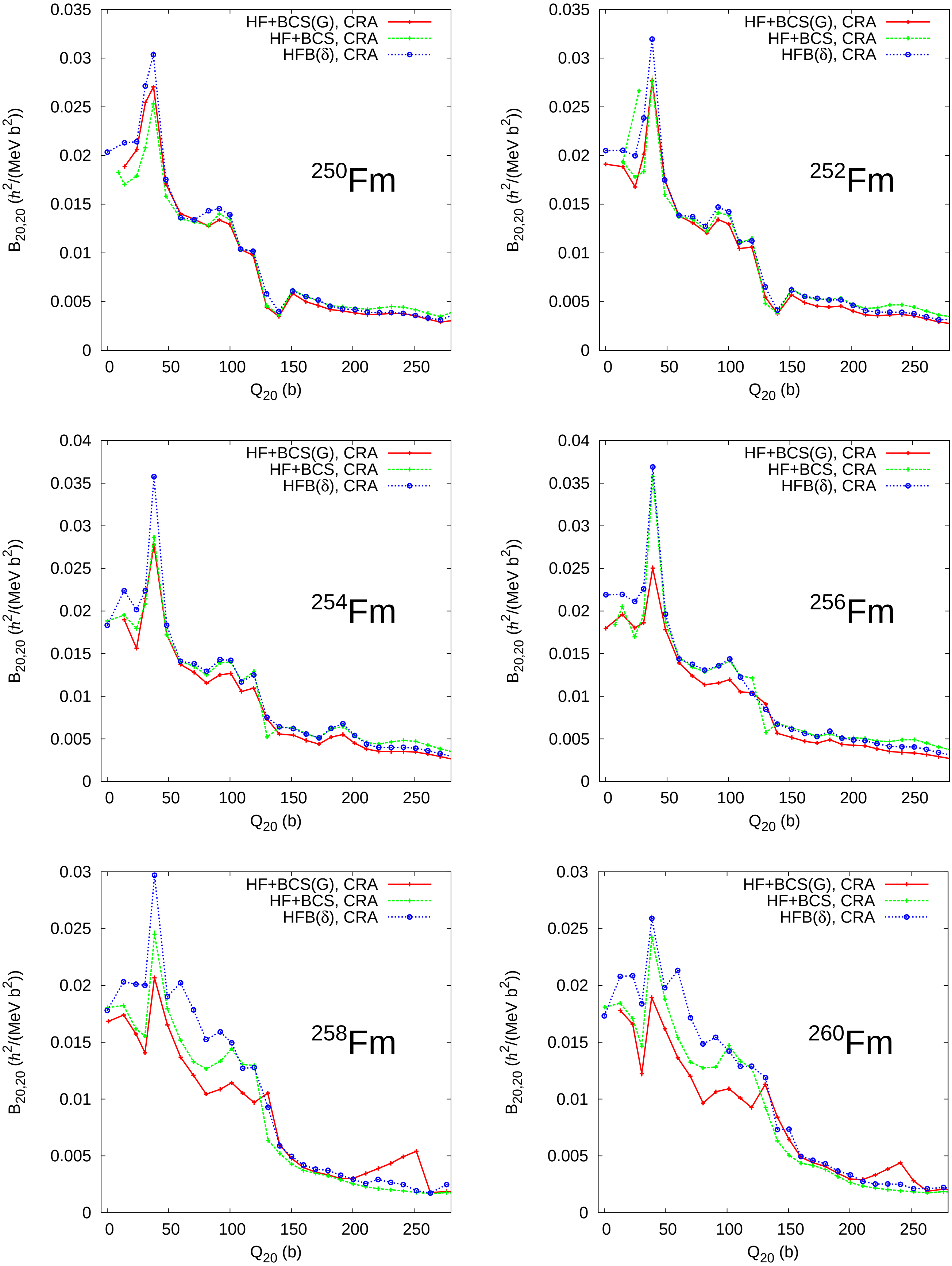
C. Mass parameters - all



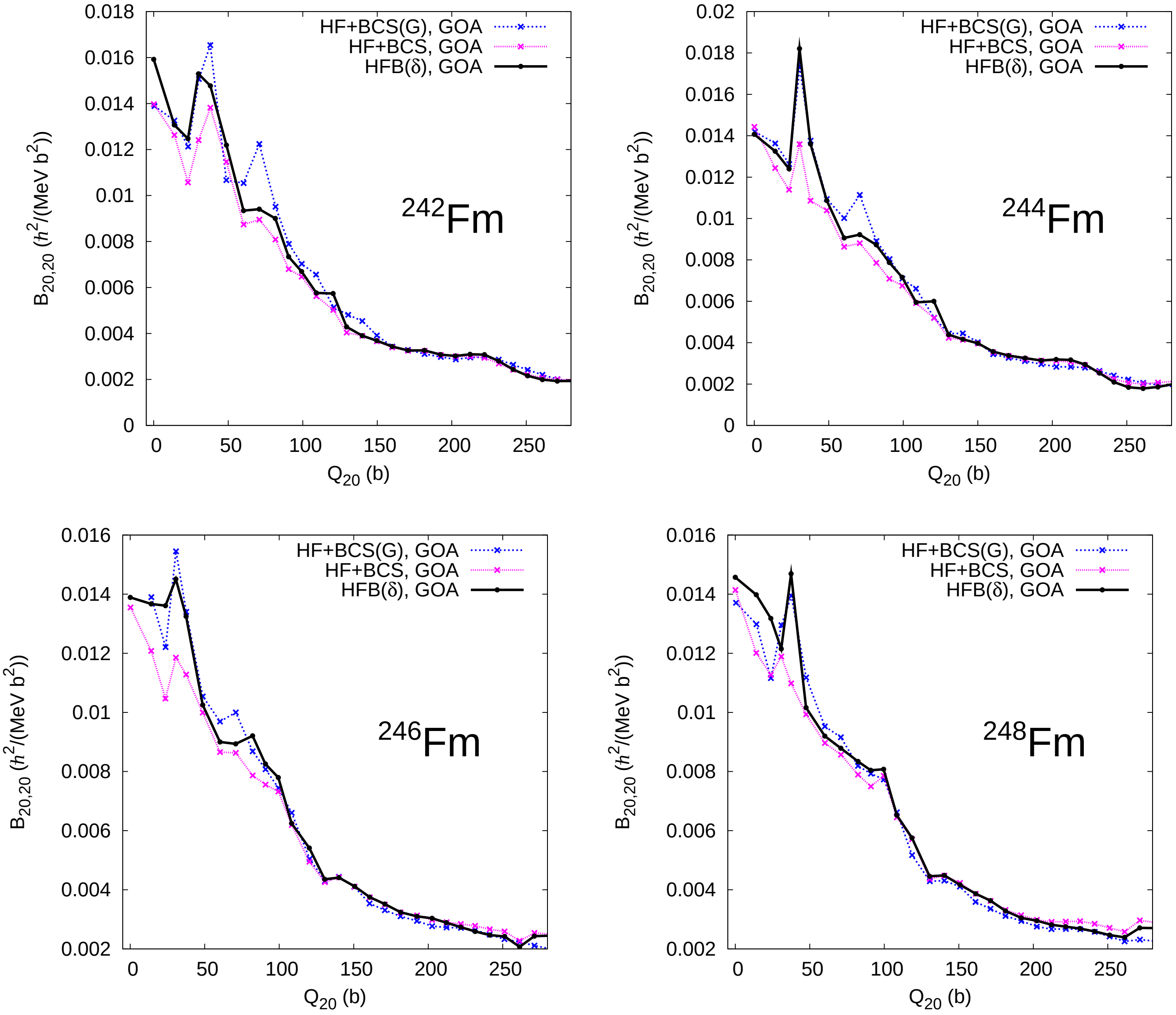

HF+BCS(G), CRA
HF+BCS(G), GOA
HF+BCS, CRA
HF+BCS, GOA
HFB(δ), CRA
HFB(δ), GOA
$B_{20,20}$ ($\hbar^2$/(MeV b^2))
Q_{20} (b)
^{250}Fm
^{252}Fm
^{254}Fm
^{256}Fm
^{258}Fm
^{260}Fm

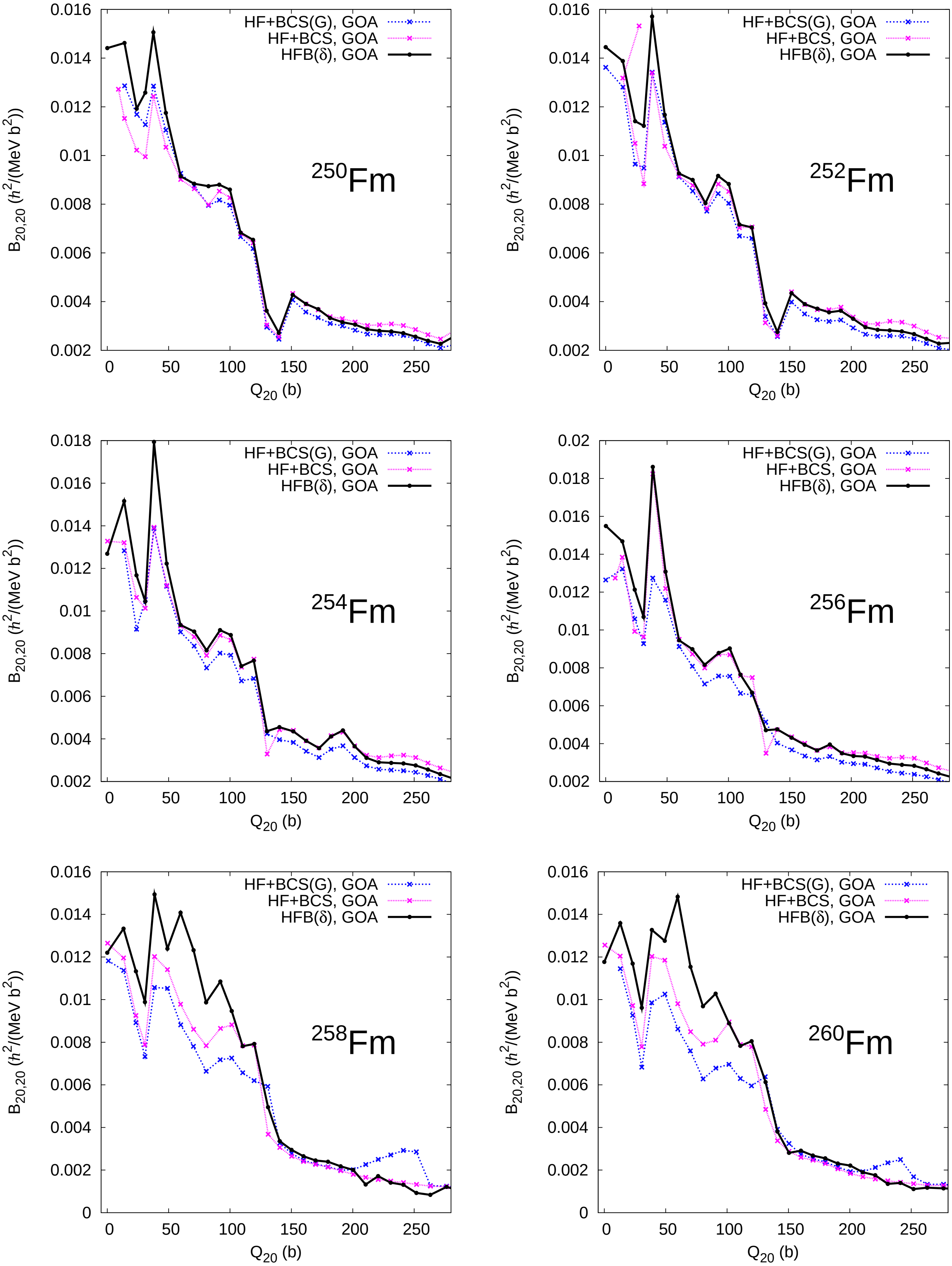
D. Mass parameters - CRANKING



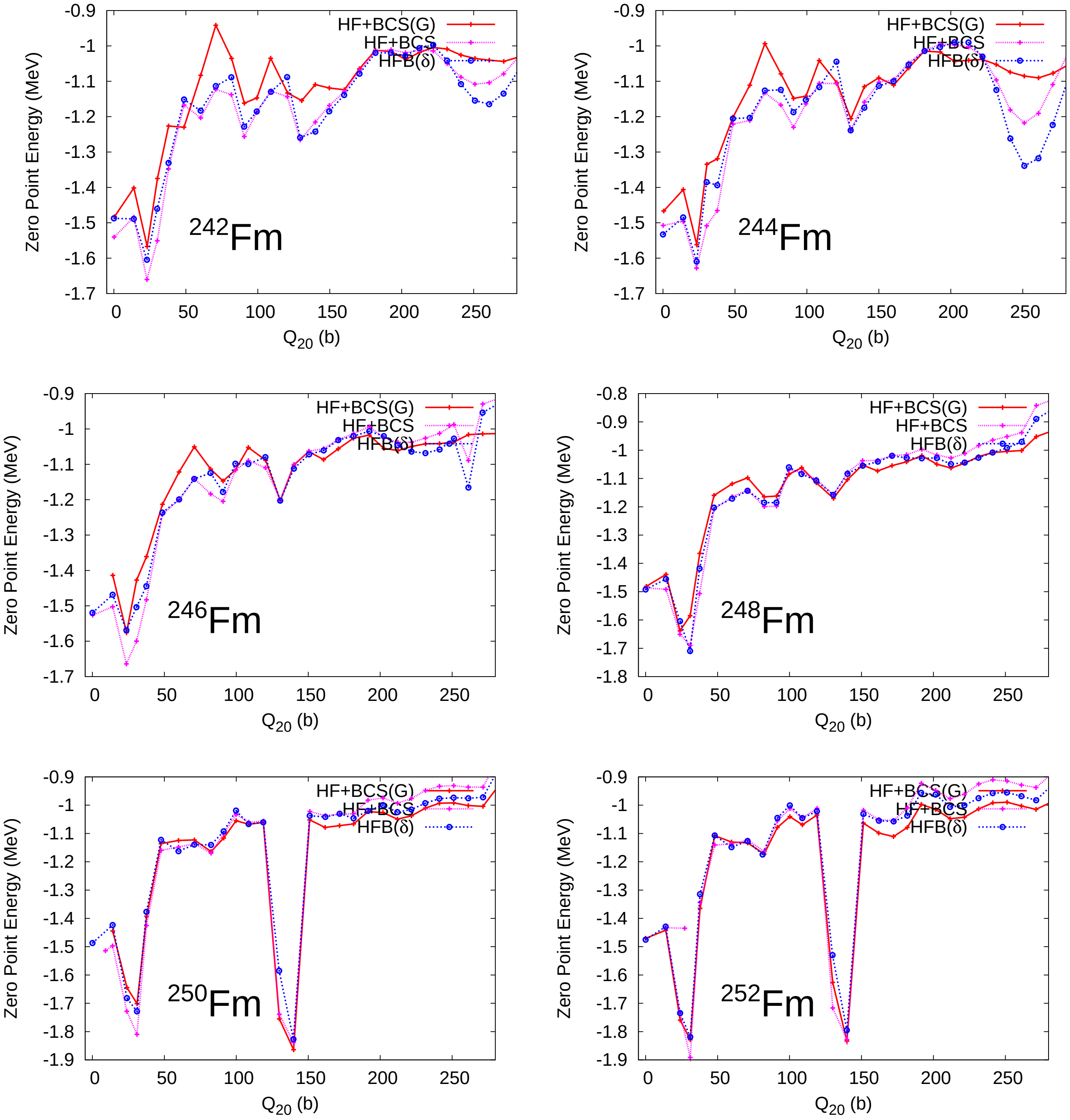

HF+BCS(G), CRA
HF+BCS, CRA
HFB(δ), CRA
$B_{20,20}$ ($\hbar^2$/(MeV b^2))
Q_{20} (b)
^{250}Fm
^{252}Fm
^{254}Fm
^{256}Fm
^{258}Fm
^{260}Fm

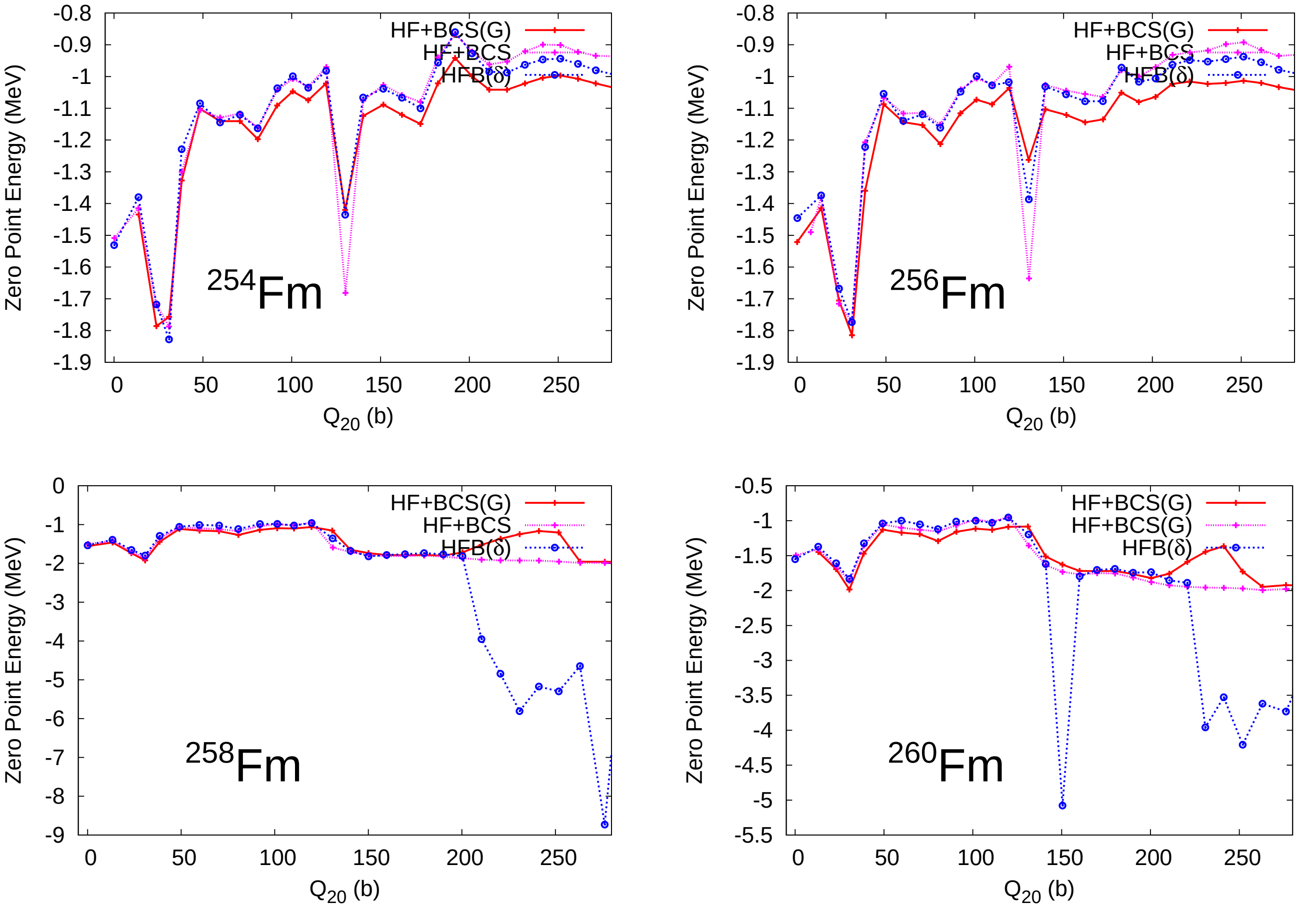
E. Mass parameters - GOA



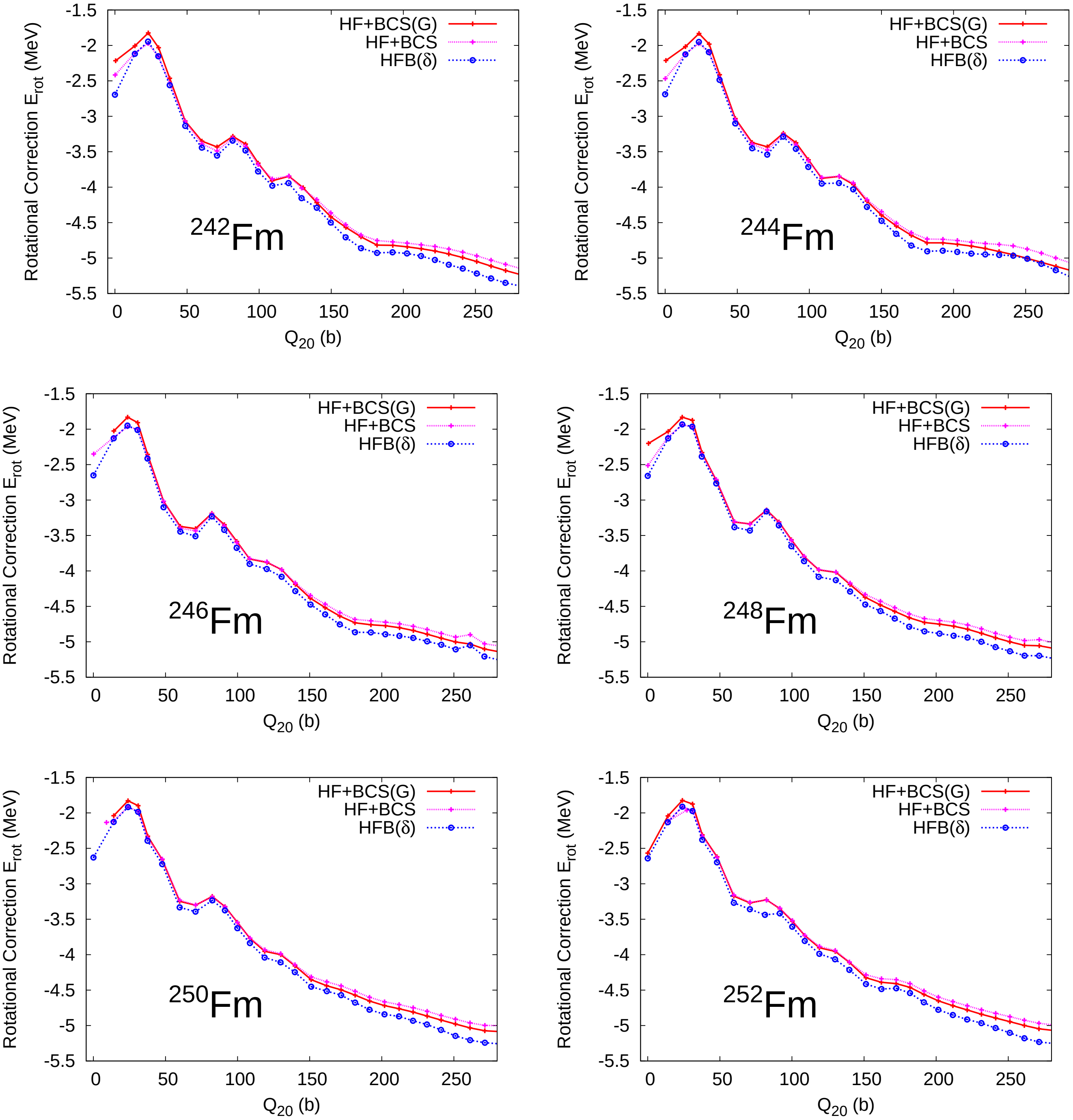

HF+BCS(G), GOA
HF+BCS, GOA
HFB(δ), GOA
B20,20 (ℏ2/(MeV b2))
Q20 (b)
250Fm
252Fm
254Fm
256Fm
258Fm
260Fm

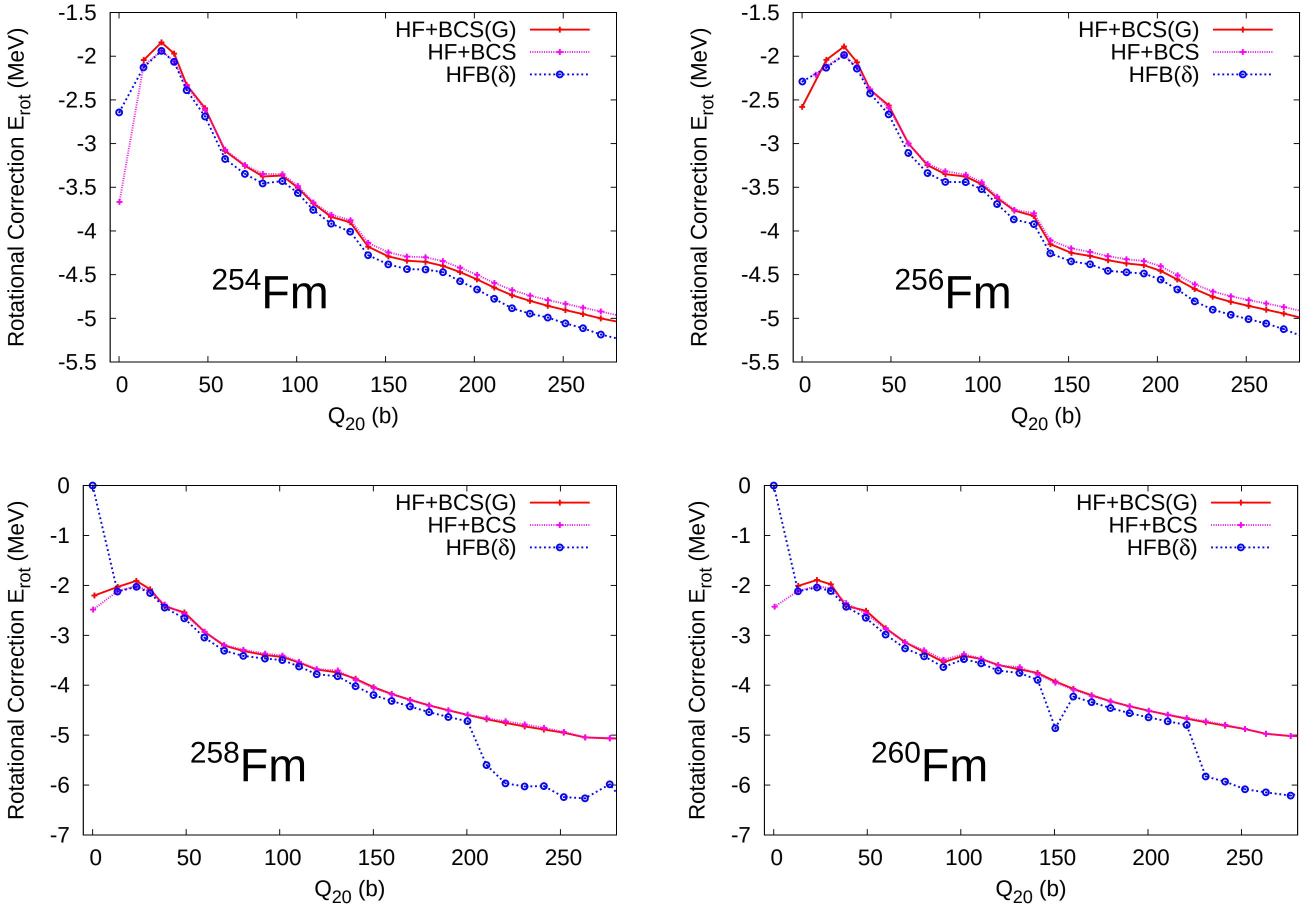
F. Zero point vibrational correction E_{vib} (GCM)

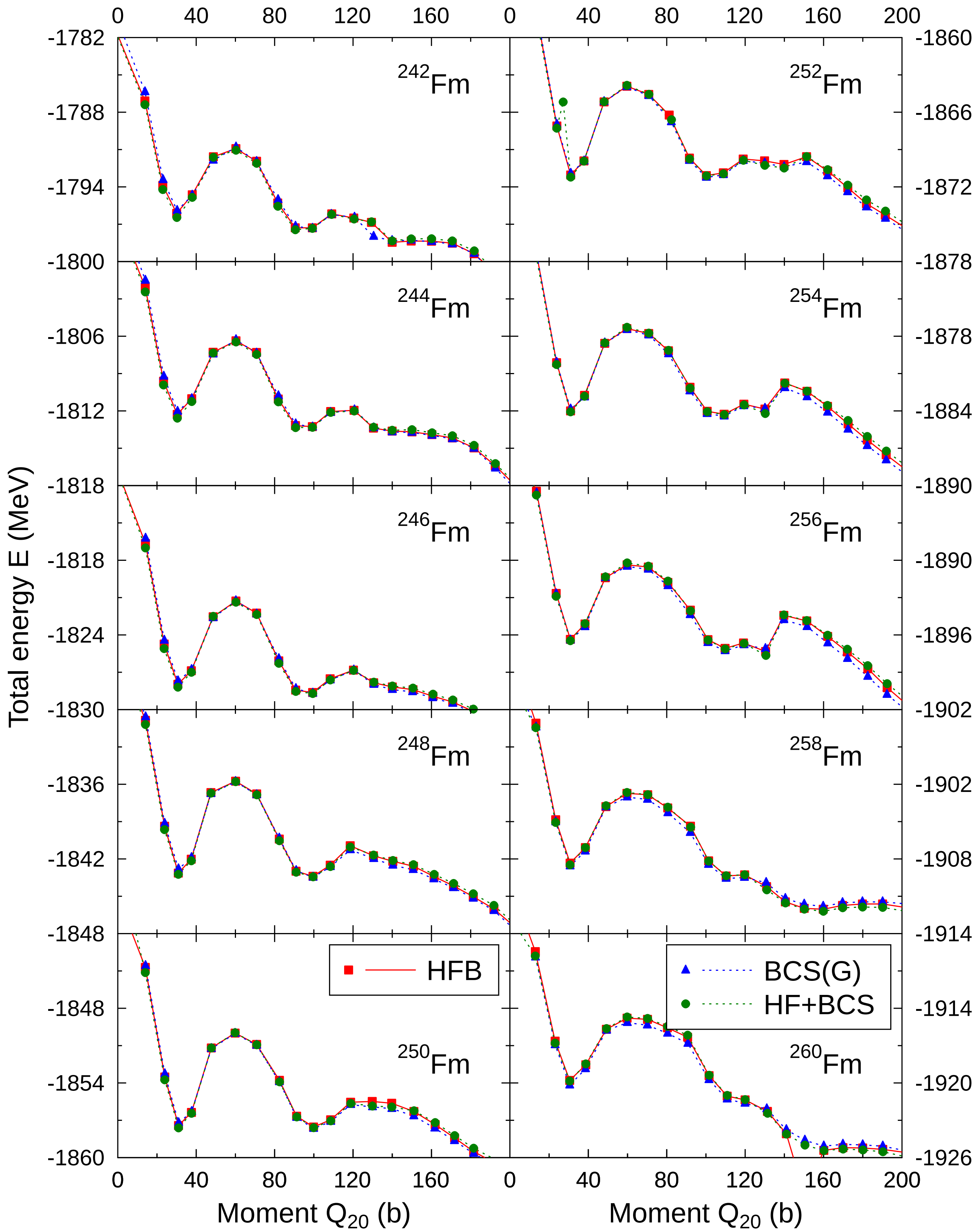



HF+BCS(G)
HF+BCS
HFB(δ)
HF+BCS(G)
Zero Point Energy (MeV)
Q_{20} (b)
^{254}Fm
^{256}Fm
^{258}Fm
^{260}Fm

G. Rotational correction E_{rot}

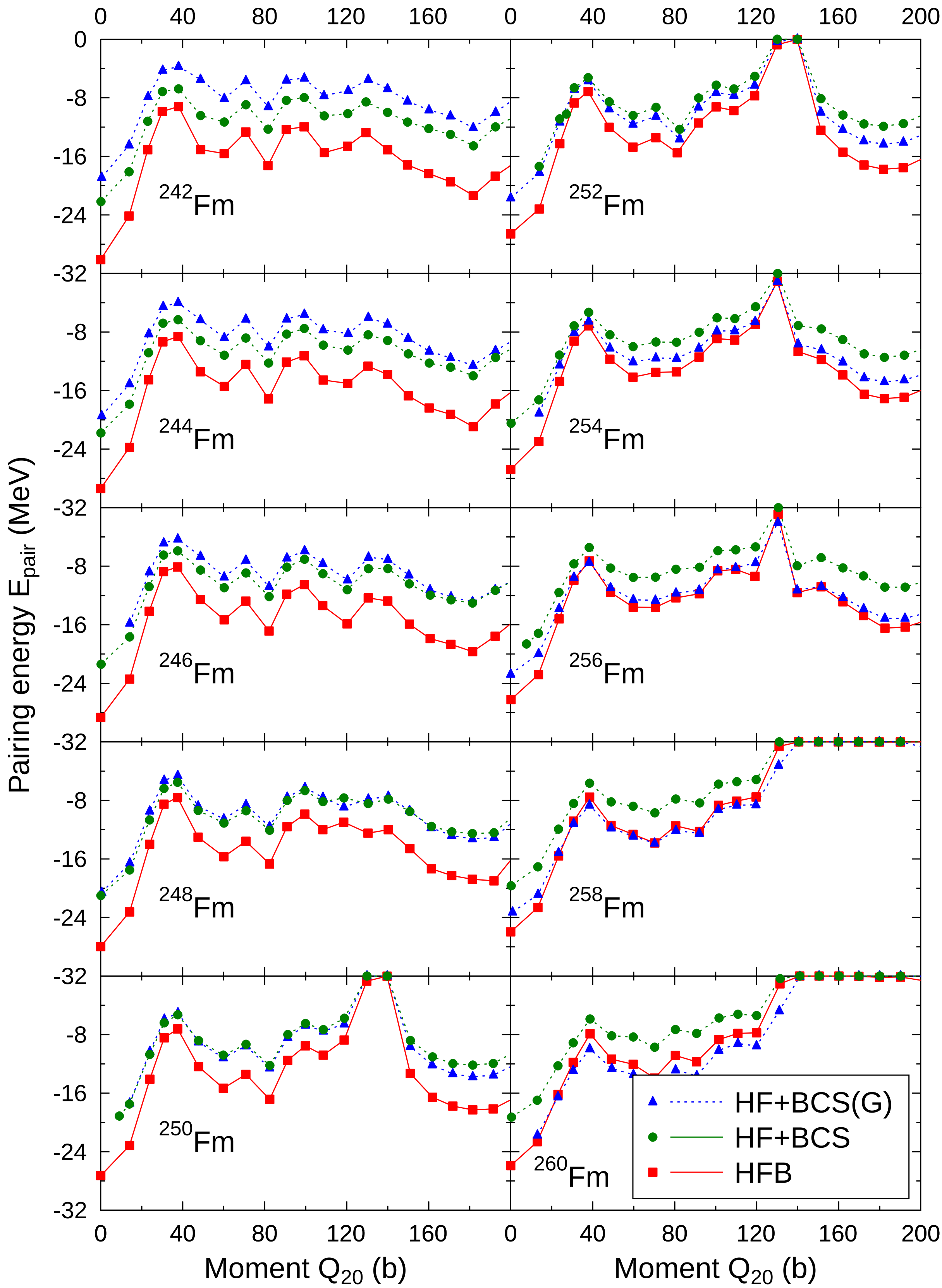


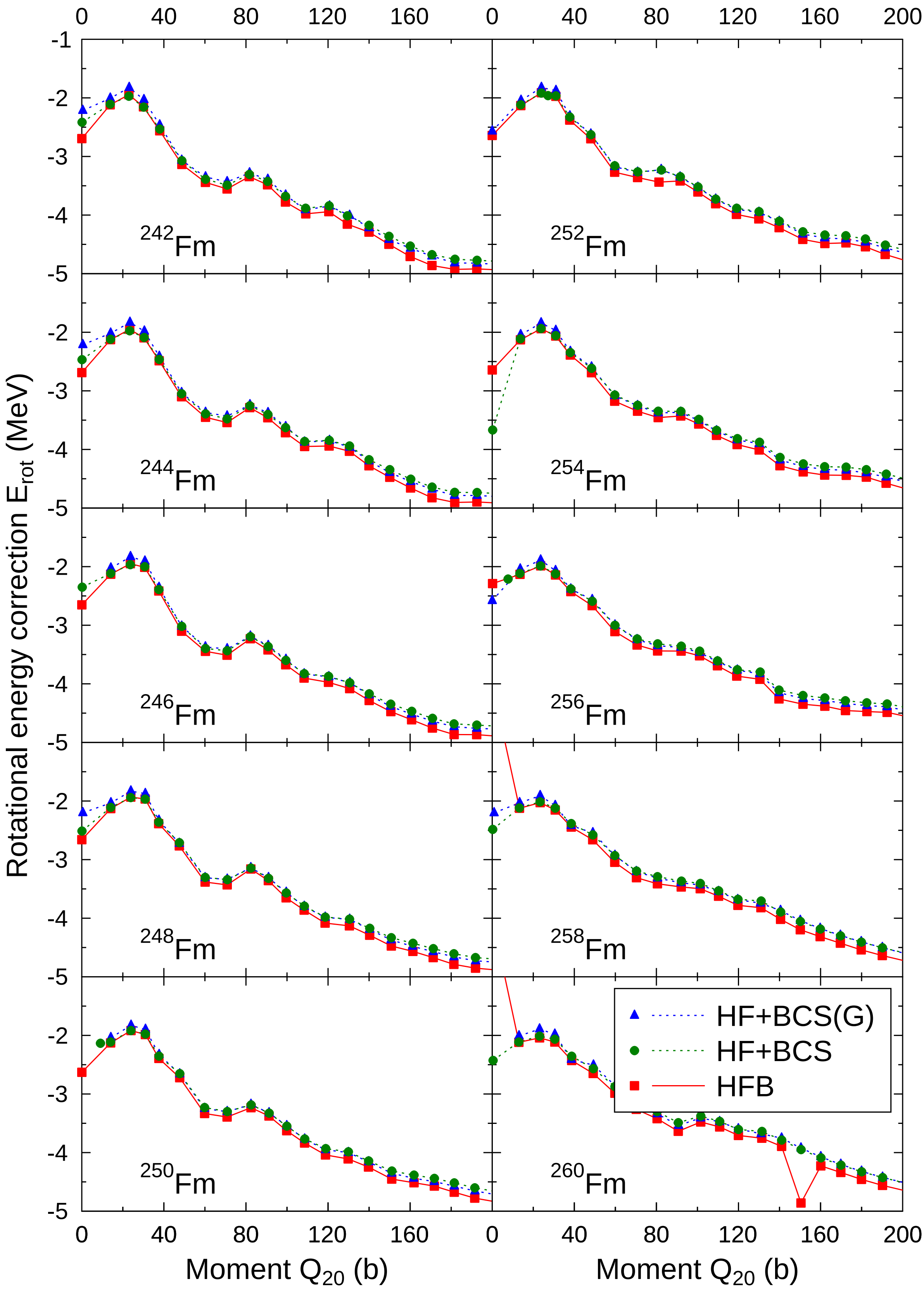

Rotational Correction E_rot (MeV)
Q_20 (b)
HF+BCS(G)
HF+BCS
HFB(δ)
254Fm
256Fm
258Fm
260Fm



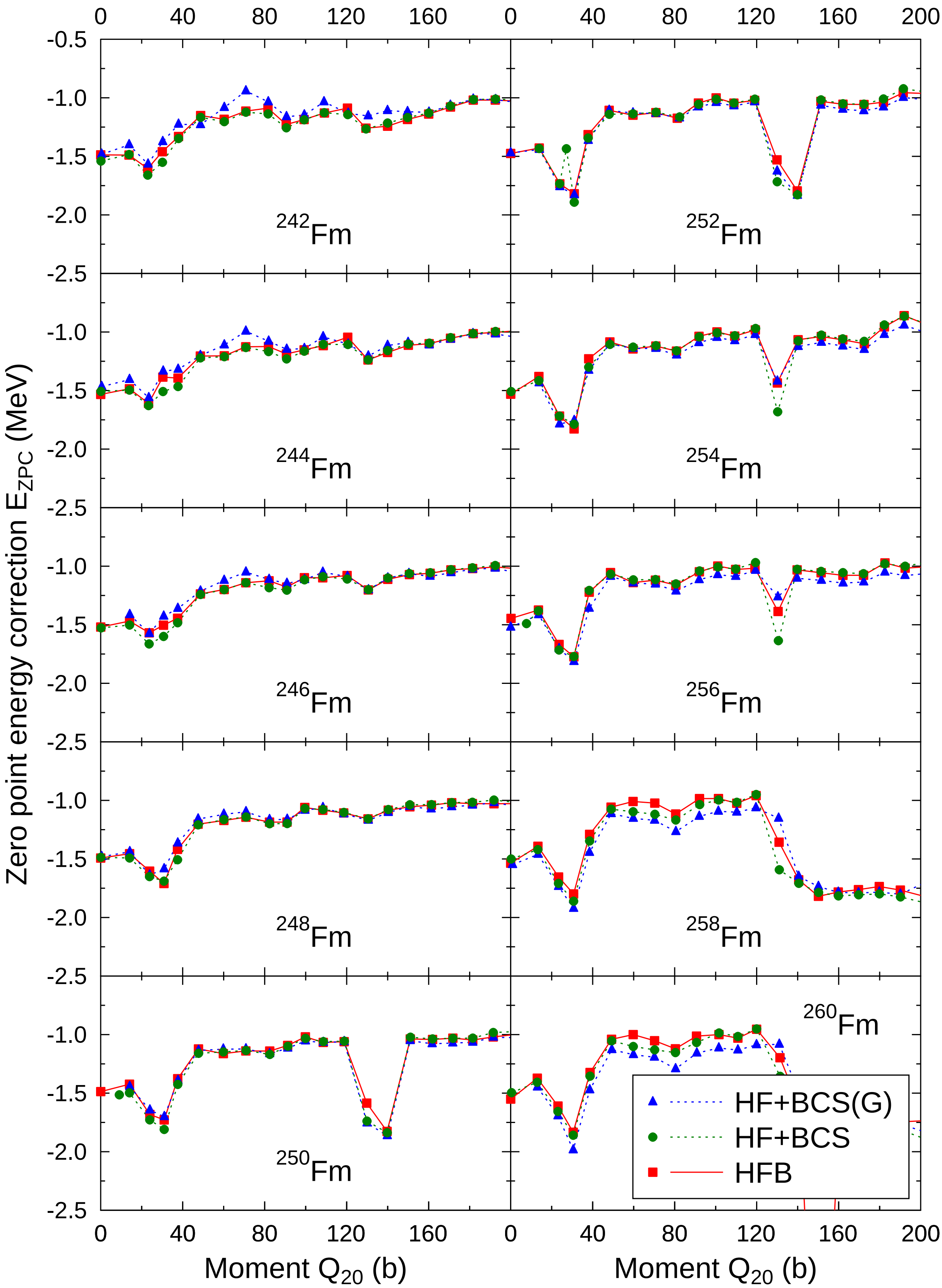
Total energy E (MeV)
Moment Q_{20} (b)
^{242}Fm
^{244}Fm
^{246}Fm
^{248}Fm
^{250}Fm
^{252}Fm
^{254}Fm
^{256}Fm
^{258}Fm
^{260}Fm
HFB
BCS(G)
HF+BCS

I. Pairing energy

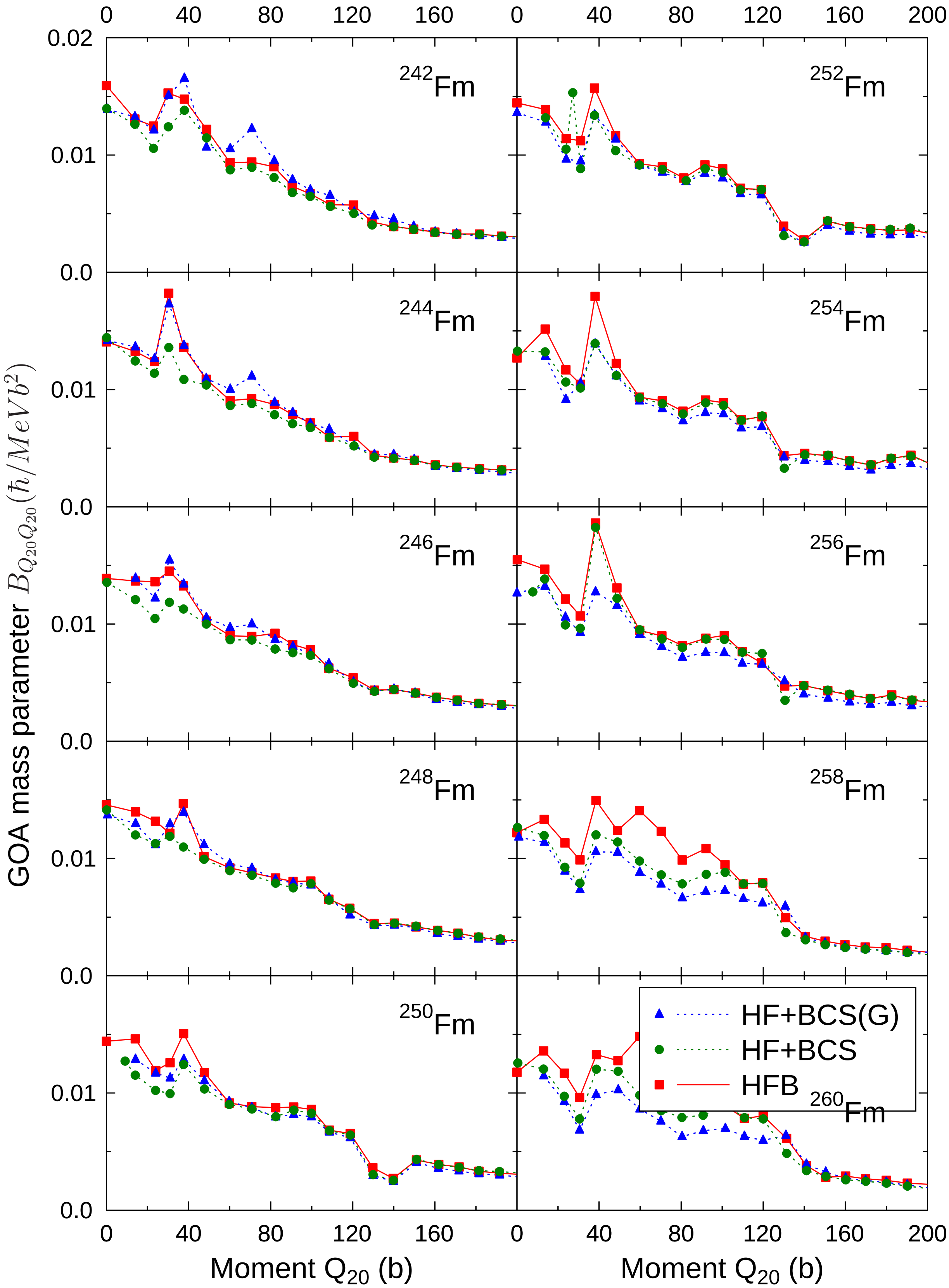



Rotational energy correction E_{rot} (MeV)
Moment Q_{20} (b)
^{242}Fm
^{244}Fm
^{246}Fm
^{248}Fm
^{250}Fm
^{252}Fm
^{254}Fm
^{256}Fm
^{258}Fm
^{260}Fm
HF+BCS(G)
HF+BCS
HFB

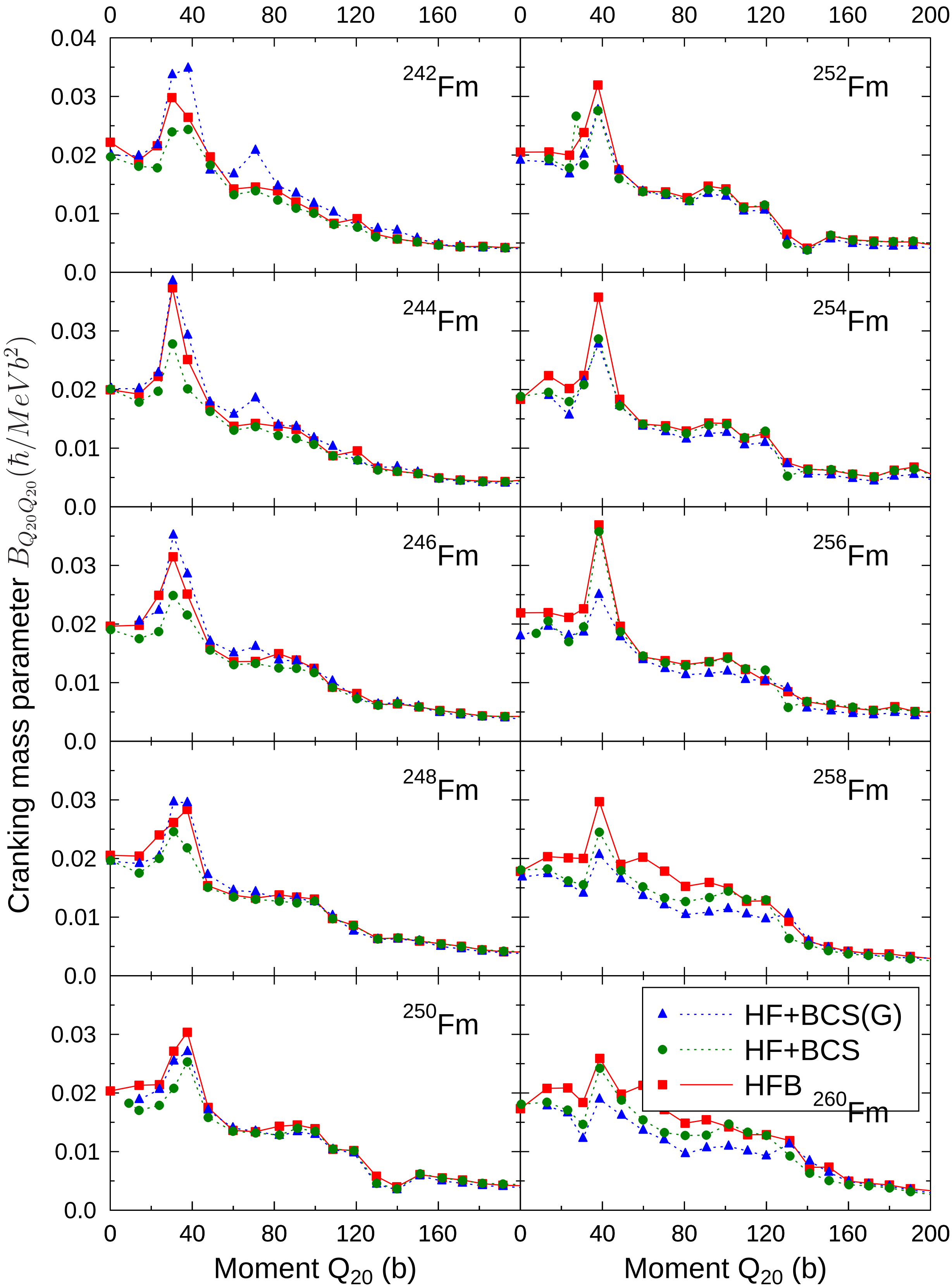
K. Zero point energy correction



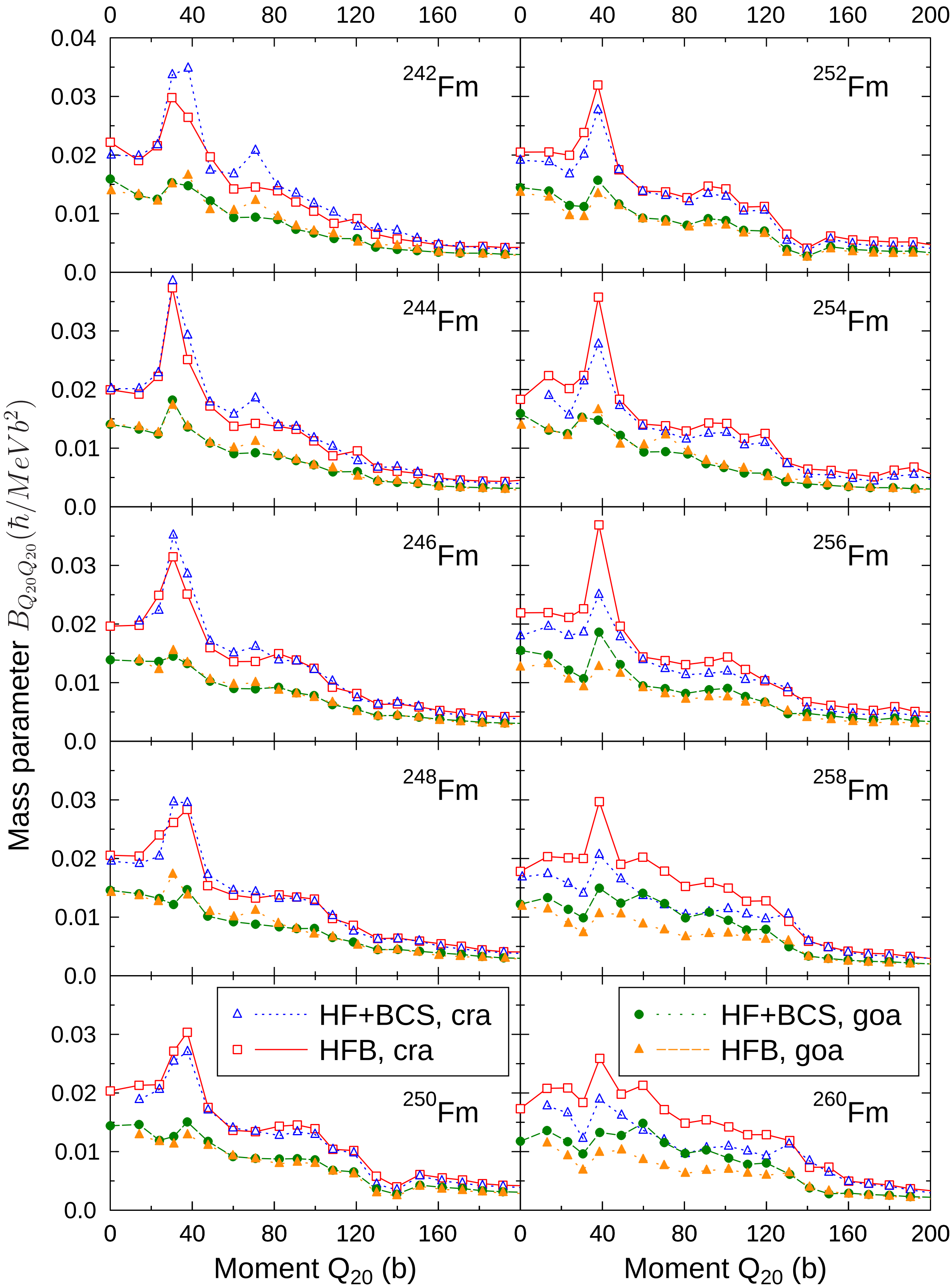
L. GOA mass parameters



M. Cranking mass parameters



N. All mass parameters



VI. BARRIER PENETRABILITY

A. WKB approximation – *old* method

In the case of the method named “old” one uses the following formulae for the penetrability (*see e.g., Funny Hills* [?]):

$$P(E) = 1/(1 + e^{2K}) , \tag{1}$$

where

$$K = \int_{\text{entry}}^{\text{exit}} \left(\frac{2\mu}{\hbar^2} |V(x) - E| dx \right)^{1/2} . \tag{2}$$

B. JWKB approximation

Following relations were obtained for the case of two peaked barrier (first peak A, second peak B): (Ignatiuk *et al. (1969)* and Gai *et al. (1969)*):

$$P(E) = 64 P_A P_B [(P_A P_B + 16) \cos^2 \phi + 16 (P_A + P_B)^2 \sin^2 \phi]^{-1} \tag{3}$$

where

$$\phi(E) = \int_a^b \left(\frac{2\mu}{\hbar^2} [E - V(x)] \right)^{1/2} \tag{4}$$

C. FJWKB formula - *new* method

In the case of the “new” method using the Frøman’s formalism [?] one has the following relations (*see:* Leboeuf and Sharma (1973) [? ?]) and also Cramer, Nix (1970)).

a. Well separated entrance and exit points In the case where both entry and exit points (c, d) are well separated (energy below the top of the barrier B) one has

$$P(E) = P_A P_B / 4 \cos^2 \phi , \tag{5}$$

where

$$P_A = \exp \left(-2 \int_a^b |q(x)| dx \right) \equiv \exp(-2K_A), \qquad P_B = \exp \left(-2 \int_c^d |q(x)| dx \right) \equiv \exp(-2K_B) . \tag{6}$$

b. Entry and exit points close together If the points (c, d) are close, the energy is very close to the top of the second barrier the penetrability formula reads

$$P(E) = P_A P_B [(2 - P_B) + 4 \cos\phi \sin\phi (1 - P_B)^{1/2}]^{-1}] , \tag{7}$$

where

$$P_A = e^{-2K_A} , \qquad P_B = 1/(1 + e^{2K_B}) . \tag{8}$$

c. Both entry and exit point coincide In the limiting case where c and d coincide one has: $K_B = 0$, $P_B = 1/2$ and

$$P(E) = P_A / (3 + 2 \cos\phi \sin\phi) . \tag{9}$$