Determinations of Fission Barriers (HFB and Beyond Mean–Field)

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HFB potential energy surfaces with D1S force

- Two–dimensional collective variable space $\{q_2, q_3\}$

$$\hat{Q}_2 = \sqrt{\frac{16\pi}{5}} \sum_{i=1}^{A} r_i^2 \hat{Y}_{20}, \qquad \hat{Q}_3 = \sqrt{\frac{4\pi}{7}} \sum_{i=1}^{A} r_i^3 \hat{Y}_{30}. \tag{1}$$

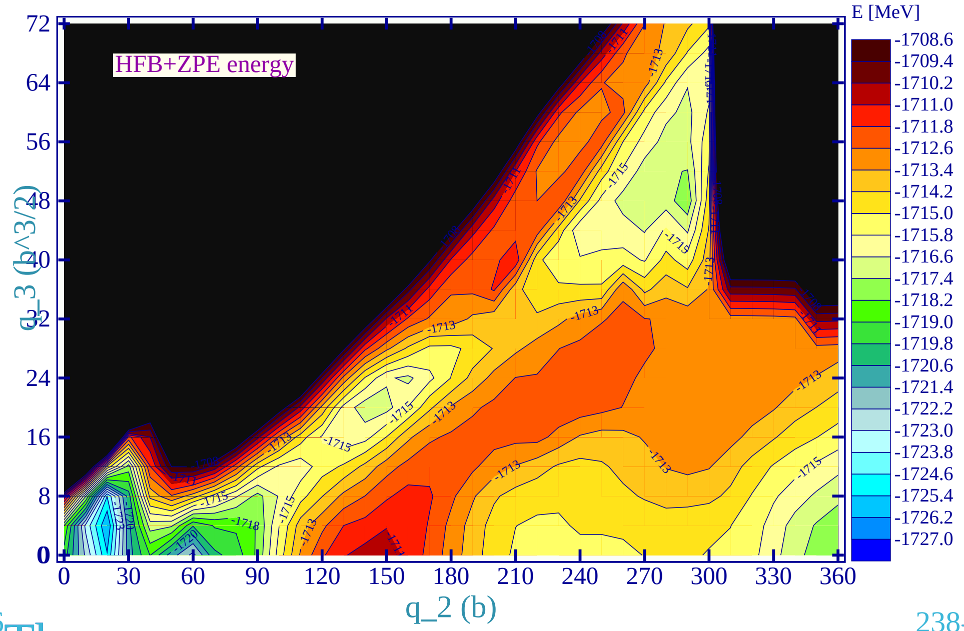
- Nuclear intrinsic axially deformed states $|\Phi(q_2, q_3)\rangle$ are given as the solutions of the constrained HFB variational principle

$$\delta \langle \Phi(q_2, q_3)| \hat{H} - \lambda_N \hat{N} - \lambda_Z \hat{Z} - \sum_i \lambda_i \hat{Q}_i |\Phi(q_2, q_3)\rangle = 0 \tag{2}$$

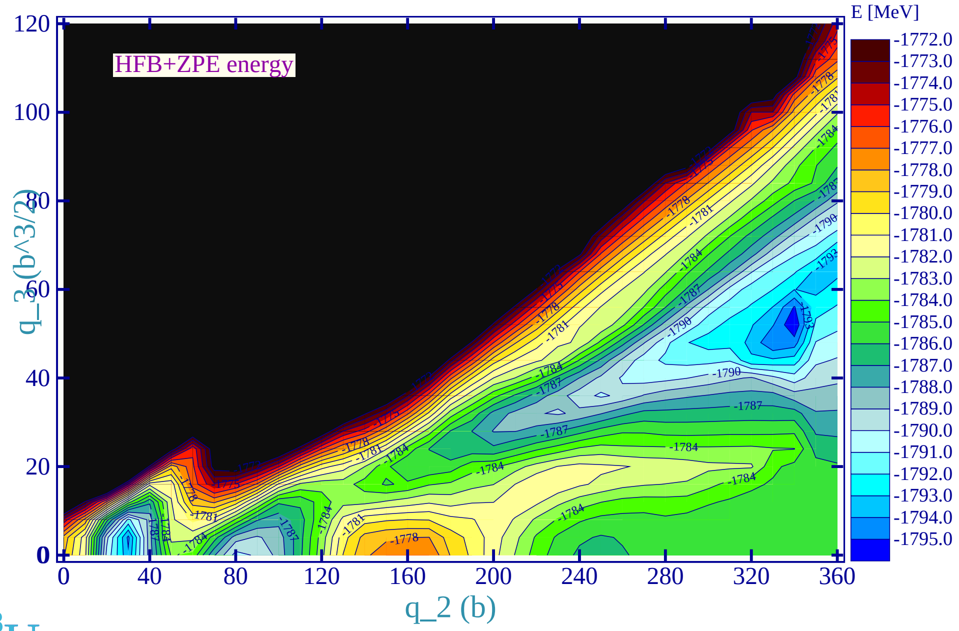
where Lagrange multipliers λ_N, λ_Z, λ_i are calculated from the relations:

$$\begin{aligned} \langle \Phi(q_2, q_3)|\hat{N}(\hat{Z})|\Phi(q_2, q_3)\rangle &= N(Z), \\ \langle \Phi(q_2, q_3)|\hat{Q}_i|\Phi(q_2, q_3)\rangle &= q_i, \\ \langle \hat{Q}_1 \rangle = \sum_{i=1}^{A} z_i &= 0. \end{aligned} \tag{3}$$

Potential energy landscapes in $\{q_2, q_3\}$ plane



$^{226}_{90}\mathrm{Th}_{136}$



$^{238}_{92}\mathrm{U}_{146}$

Least–action path

- **The action in the $\{q_2, q_3\}$ space is defined as**

$$S(L) = 2 \int_{s_1}^{s_2} \sqrt{2(V(s) - E_{coll})B_s(s)}ds \tag{4}$$

$$V \equiv V_{HFB} - ZPE_{rot} - ZPE_{vib}$$

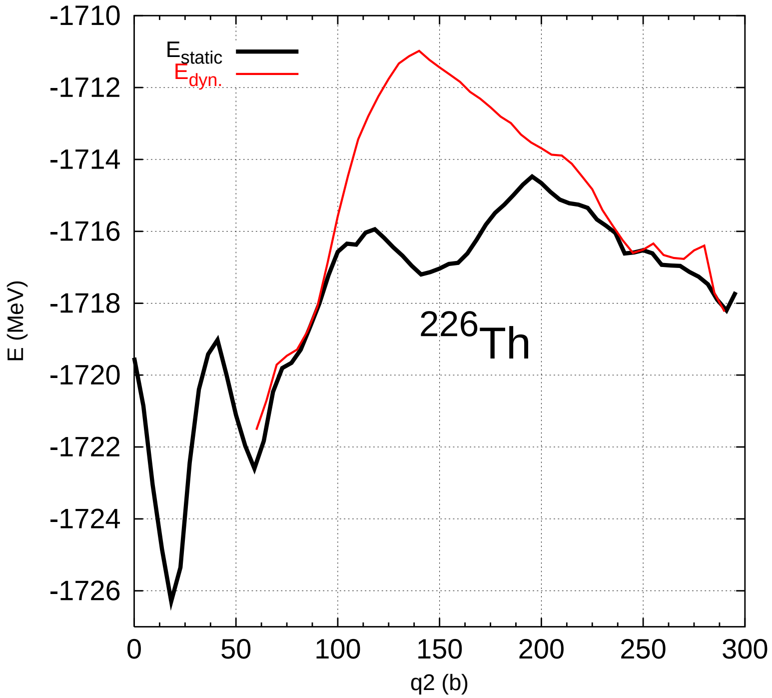
- **The effective inertia $B_s(s)$ along the trajectory L is given as**

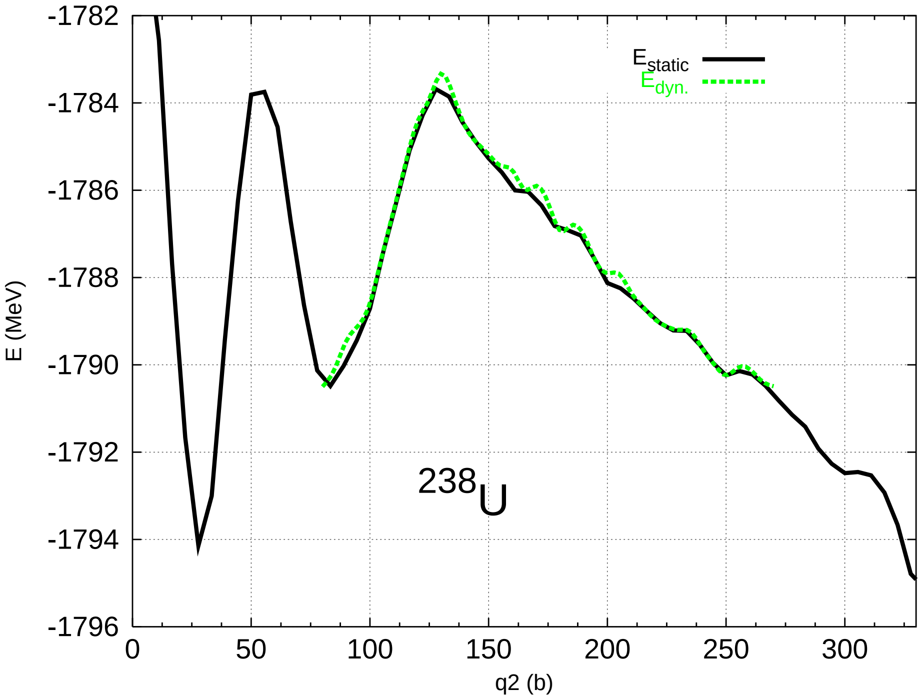
$$B_s(s) = \sum_{ij} B(q_2, q_3) \frac{dq_2}{ds} \frac{dq_3}{ds}, \tag{5}$$

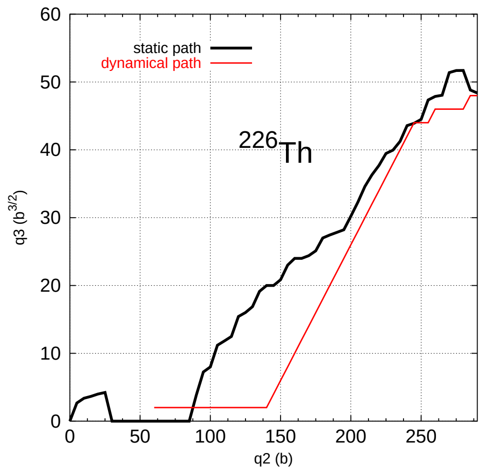
where s is the curvelinear abscisa along the fission path

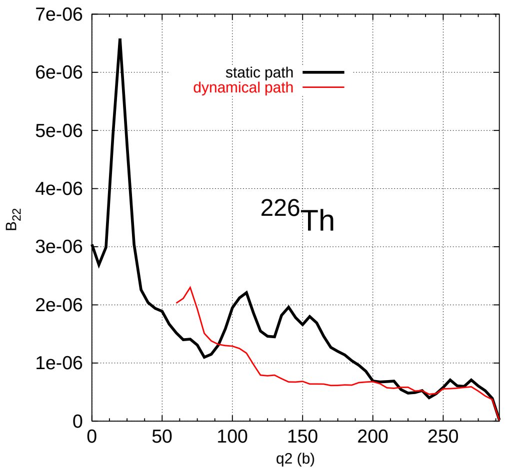
- **Search for the trajectory L_{min} which minimizes the action (4).**

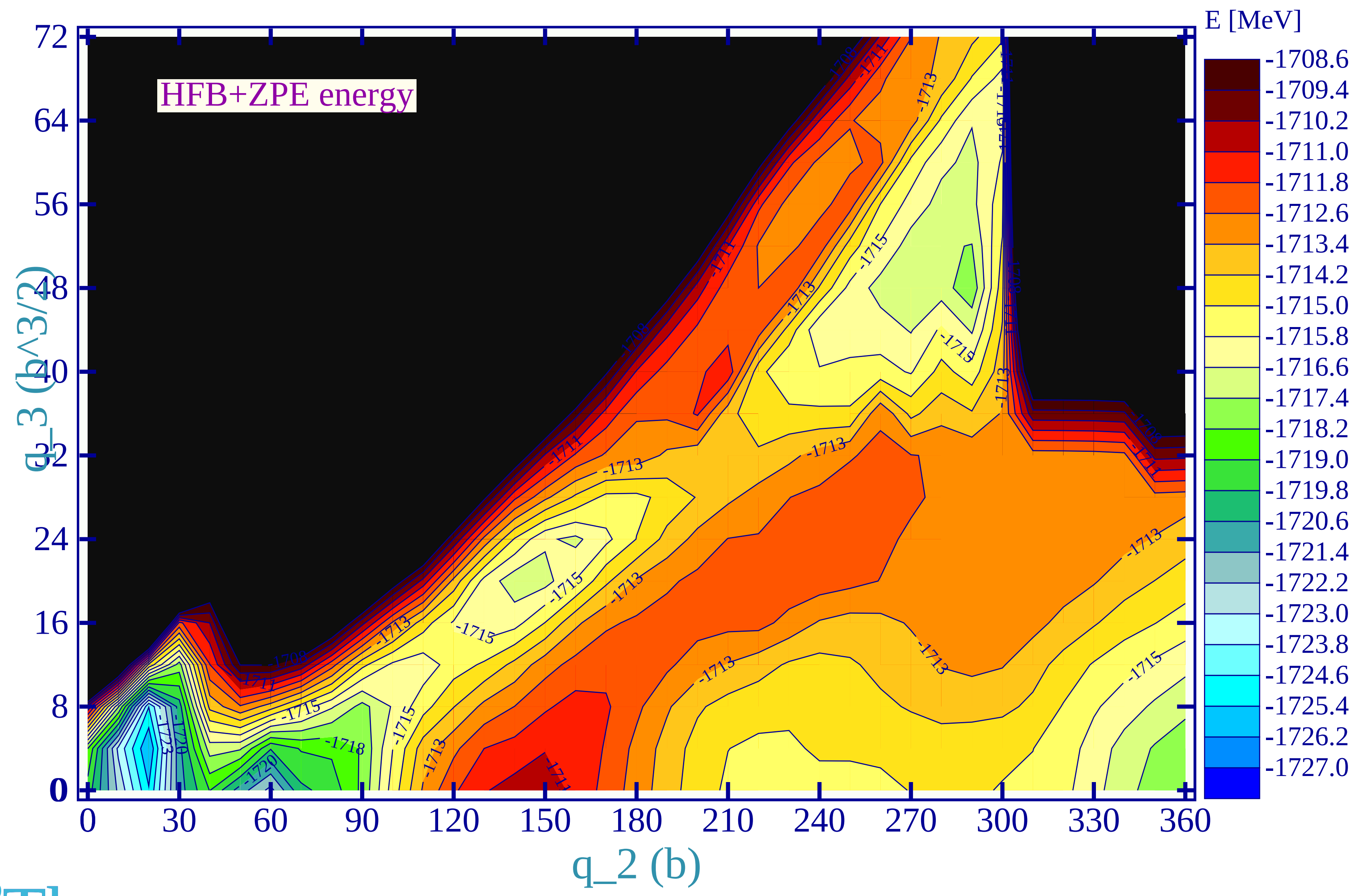
Static versus least–action fission barriers











$^{226}_{90}\mathrm{Th}_{136}$

Effective fission barriers including transverse vibrations

- **The way we do:**
  - **We take the least–energy path**
  - **We determine the transverse direction to this path for each q_2–value**
  - **We derive a 2-D collective Hamiltonian from the GCM + GOA approximation**

$$|\Psi\rangle = \int\int dq_2\, dq_3 f(q_2, q_3)\, |\Phi(q_2, q_3)\rangle,$$

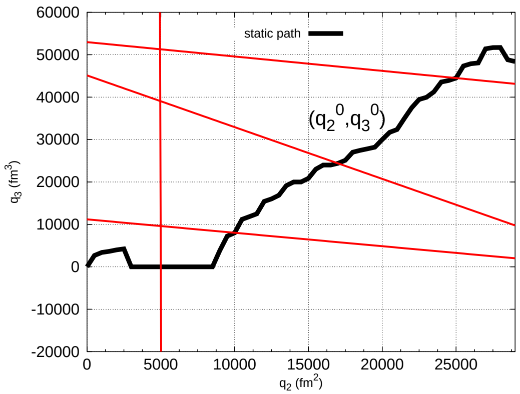
$$\hat{H}_{coll} = -\frac{\hbar^2}{2}\sum_{ij=2}^{3}\frac{\partial}{\partial q_i}\frac{1}{B_{ij}(q_2, q_3)}\frac{\partial}{\partial q_j} + \Big[V(q_2, q_3) - ZPE(q_2, q_3)\Big]$$

mass term **collective potential**

Collective Hamiltonian across fission path

- **We transform the 2-D collective Hamiltonian into a 1-D one in the transverse (here x) direction for each q_2 using the relations**

$$x = \begin{cases} (q_3 - q_3^0)\dfrac{1}{\cos\phi}, \\ (q_2^0 - q_2)\dfrac{1}{\sin\phi}, \end{cases}$$



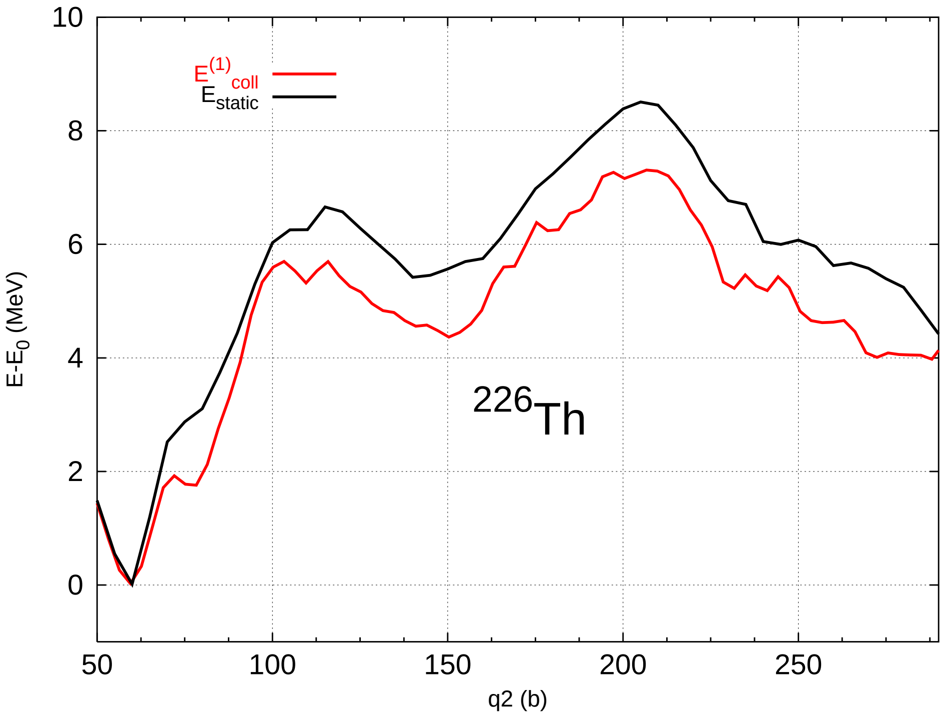
where ϕ is the angle between q_2−axis and the x−direction, $\{q_2^0, q_3^0\}$-coordinates of the fission path

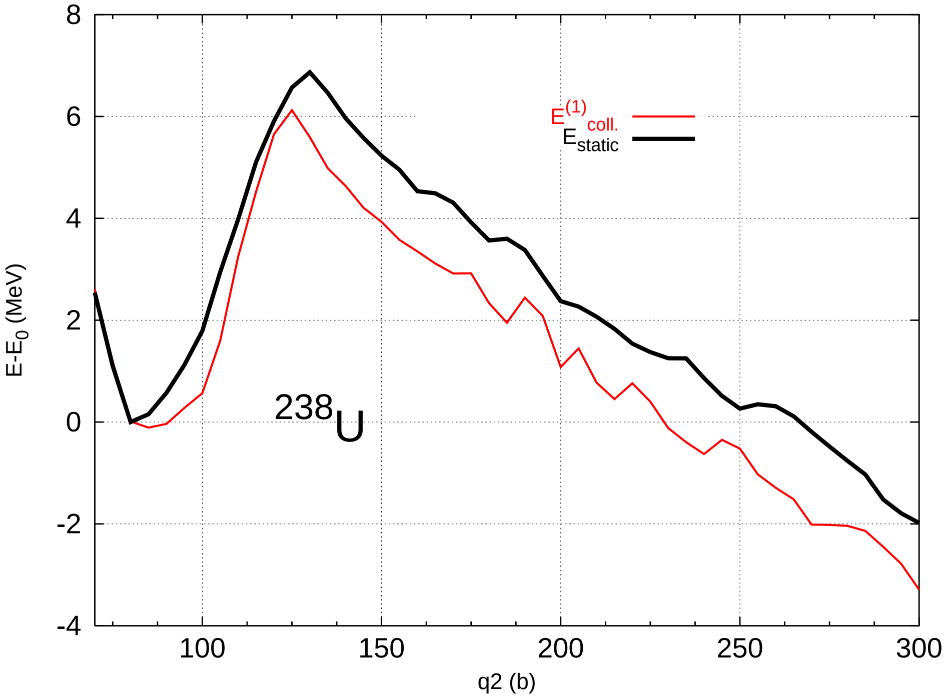
- **1-D GCM + GOA collective Hamiltonian**

$$\hat{H}'_{coll}\left(x, \frac{\partial}{\partial x}\right) = -\frac{\hbar^2}{2}\left(\frac{\partial}{\partial x}\frac{1}{B_x}\frac{\partial}{\partial x}\right) + \left[V(x) - ZPE(x)\right] \qquad (6)$$

$$B_x = \left(\frac{\partial q_2}{\partial x}\right)^2 B_{22} + 2\frac{\partial q_2}{\partial x}\frac{\partial q_3}{\partial x}B_{23} + \left(\frac{\partial q_3}{\partial x}\right)^2 B_{33}, \qquad \frac{\partial}{\partial x} = \cos(\phi)\frac{\partial}{\partial q_3} \qquad (7)$$

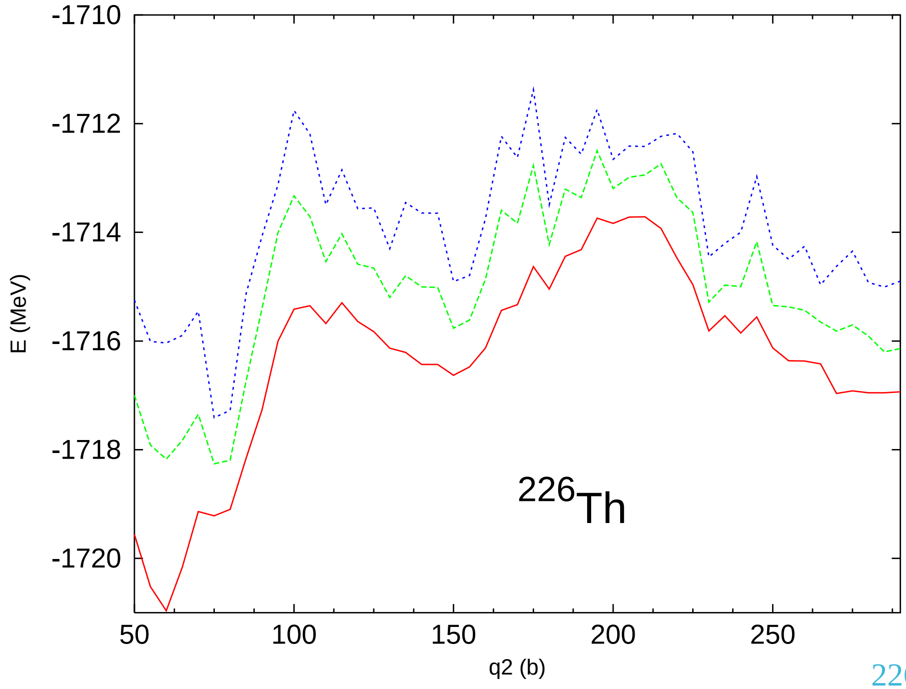
Dynamical vs static effective fission barriers

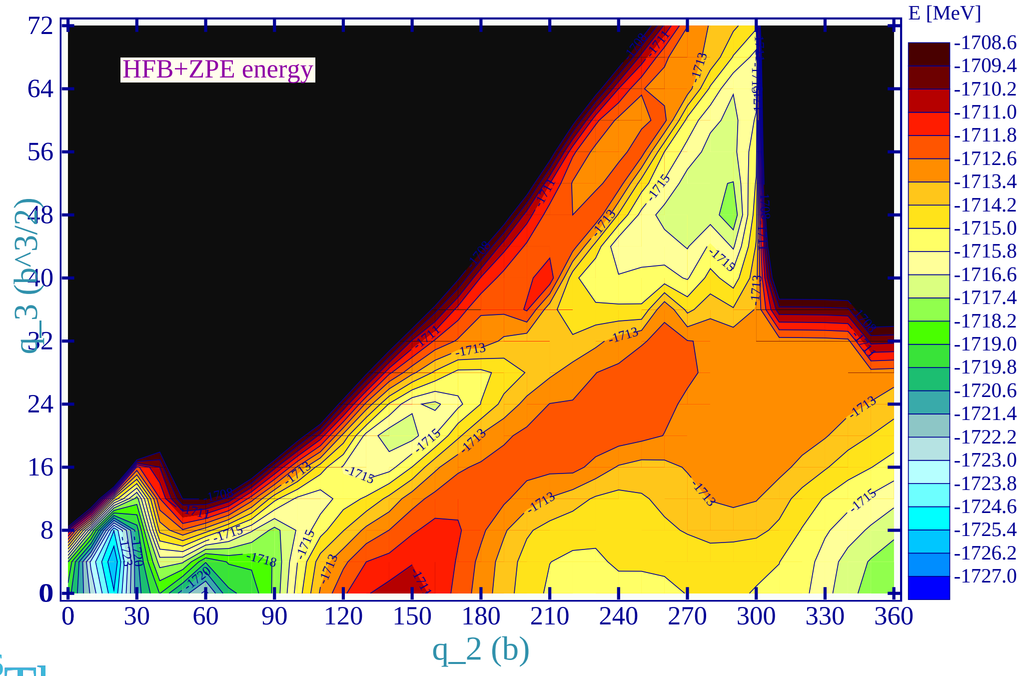




Black line – static fission barrier, **red** line – the barrier obtained matching the first vibrational states of the transverse mode for given q_2–value .

Higher collective states





$^{226}_{90}\mathrm{Th}_{136}$

Conclusions and perspectives

- Least–energy, least–action and „collective" paths are calculated for ^{226}Th and ^{238}U starting from the isomeric minima,
- Dynamical treatment of fission process in $\{q_2, q_3\}$ plane can lead to a substantial change of nuclear paths to fission and effective fission barrier heights (fission half–lives), (see ^{226}Th),
- The effect of collective vibrations in the transverse to the fission path direction plays quite a substantial role for the fission properties lowering the fission barrier on average by ≈ 1 MeV for both ^{226}Th and ^{238}U nuclei,
- Dynamics and colletive motions in an extended deformation space are also expected to play a crucial role for estimates of fission half–lives and fission cross sections,
- Calculations of fission half–lives for Fm isotopes.