Kazimierz Dolny, 26 IX 2005

Fluctuations between the BCS and BEC limits in the system of ultracold alkali atoms

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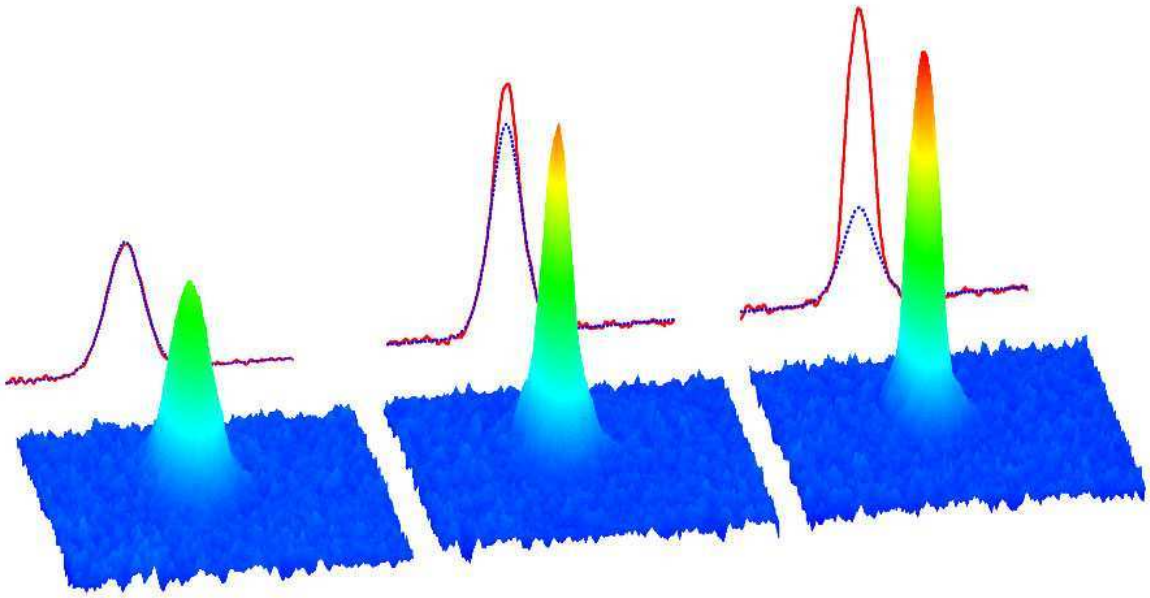

Superfluidity of the ultracold fermion atoms [1-4].

$$T_c \sim 0.15\, T_F \qquad (\sim \mathit{100\ nK})$$

[1] M.W. Zwierlein *et al*, Phys. Rev. Lett. **92**, 120403 (2004).

[2] C.A. Regal *et al*, Phys. Rev. Lett. **92**, 040403 (2004).

[3] J. Kinast *et al*, Phys. Rev. Lett. **92**, 150402 (2004).

[4] C. Chin *et al*, Science **305**, 1128 (2004).

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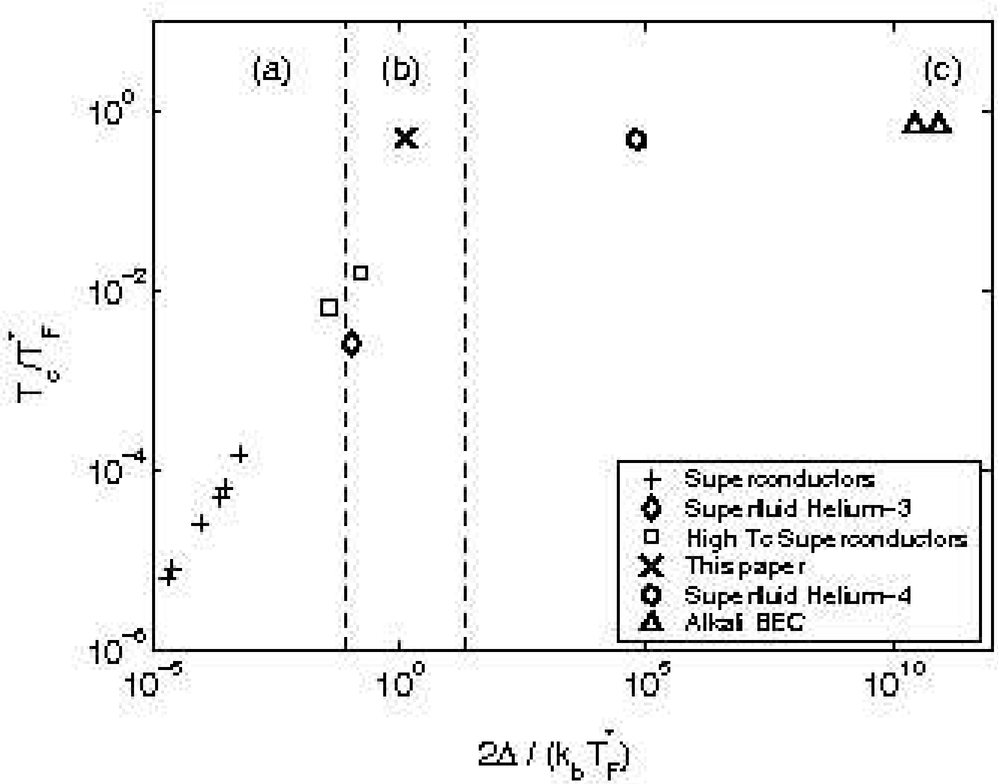
Are such temperatures small or large ?

Relative scale for the transition temperature



[5] M. Holland *et al*, Phys. Rev. Lett. **87**, 120406 (2001).

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  /experimentally tunable interactions/

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- **Feshbach resonance**
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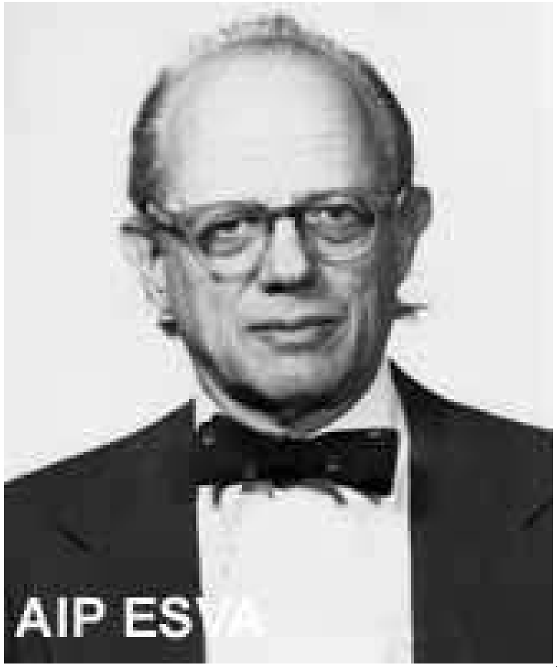
- **BCS to BEC crossover**
  /first experimental realization of Leggett's ideas/

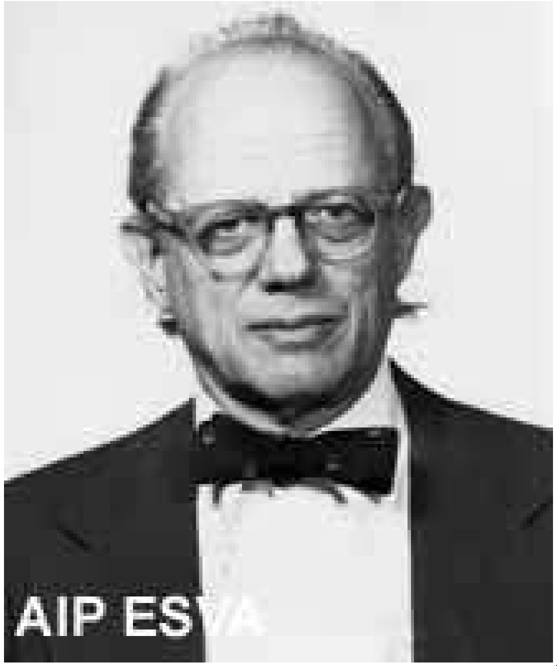
- **Dynamical fluctuations**
  /time-dependence of the order parameters/



Herman Feshbach

(1917-2000)

MIT (USA)



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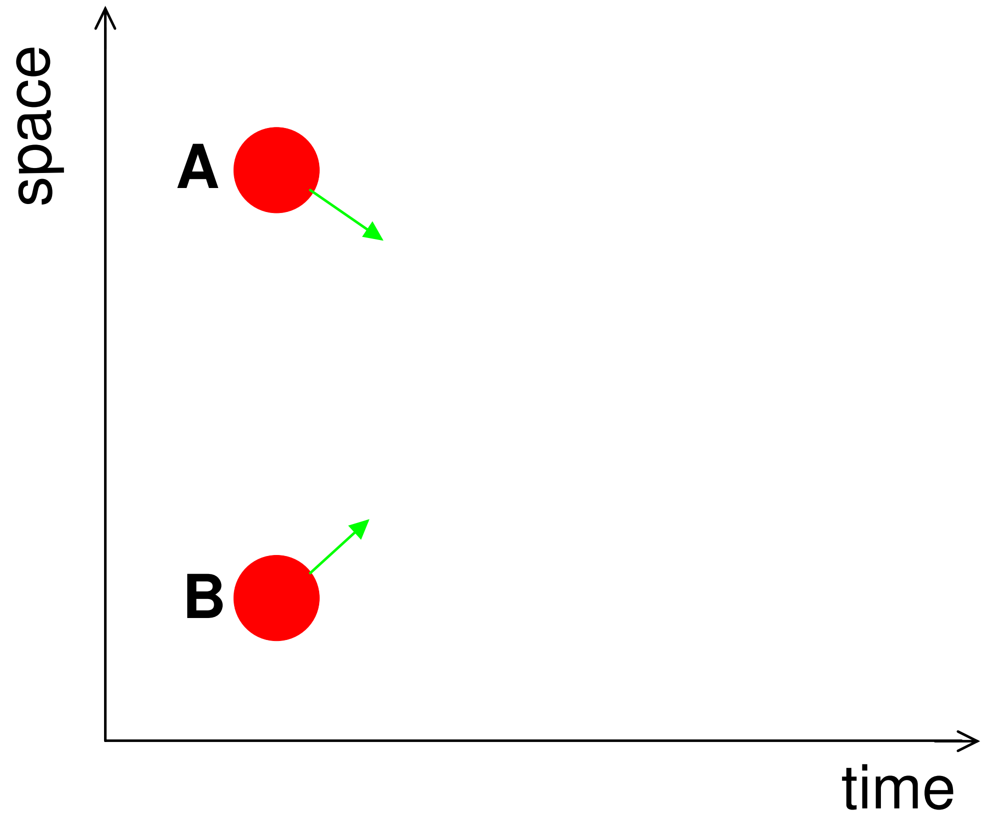
H. Feshbach was regarded as one of the most important scientists in the theory of nuclear physics. Among many contributions he formulated the description of inelastic neutron scattering on atom nucleai introducing the idea of several intermediate steps in such process.

Let us consider two fermion atoms:



singlet states /or the *open channel*/

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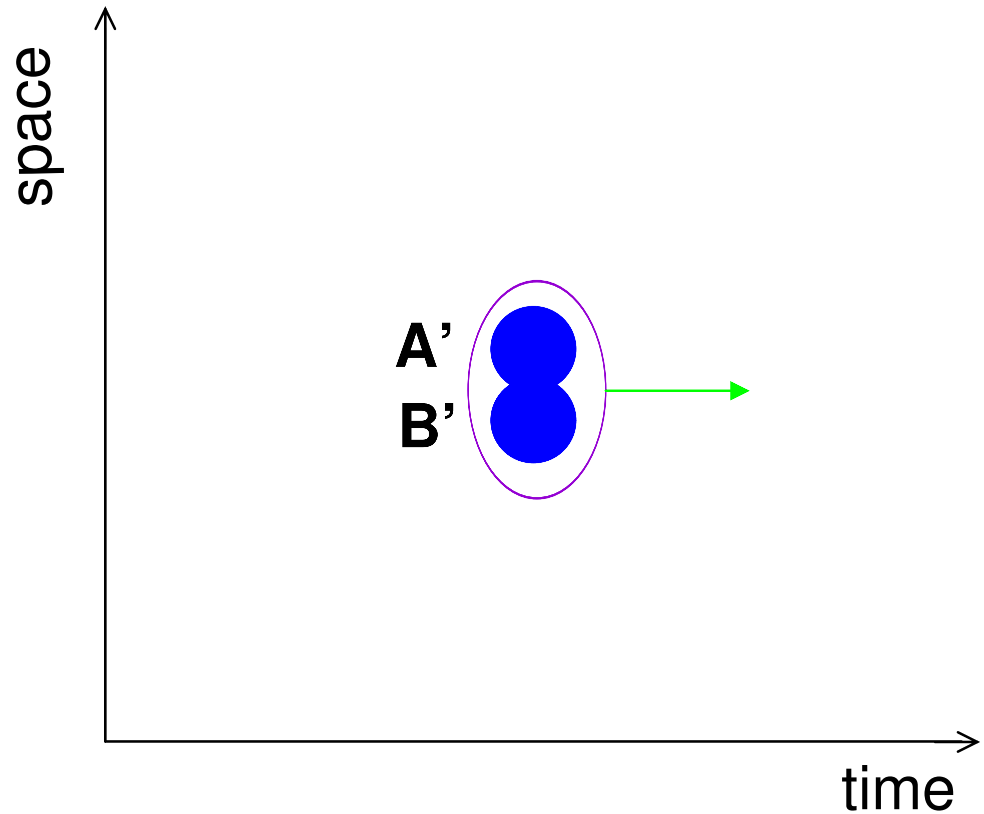
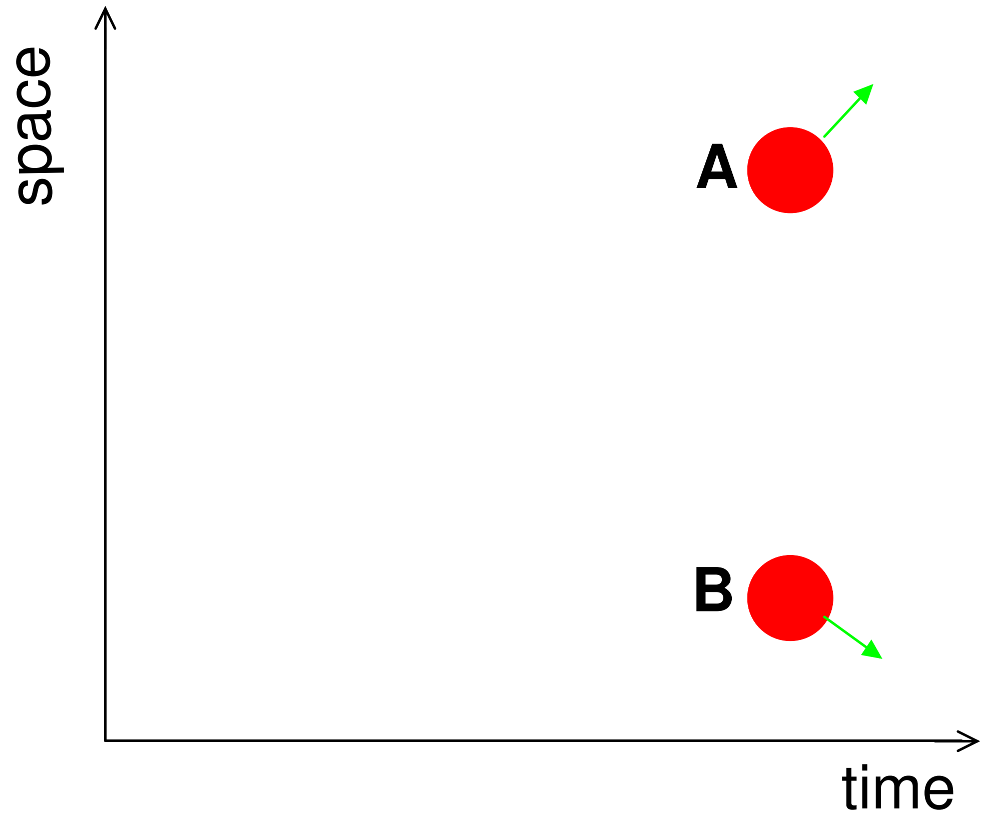


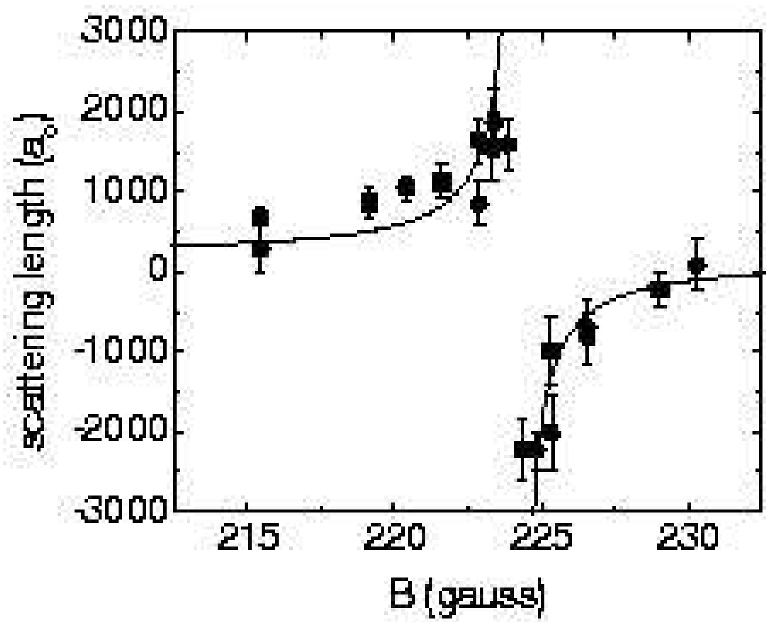
triplet state /or the *closed channel*/

Let us consider two fermion atoms:



singlet states /or the *open channel*/

Effective scattering potential for this process depends on the external magnetic field and has a *resonant character*.



Example:

Feshbach resonance observed experimentally [6] for a mixture of ^{40}K atoms in the Zeeman states $\left|\frac{9}{2},-\frac{9}{2}\right\rangle$ and $\left|\frac{9}{2},-\frac{7}{2}\right\rangle$.

[6] C.A. Regal and D.S. Jin, Phys. Rev. Lett. **90**, 230404 (2003).

On a microscopic level the *resonant interaction* between atoms can be described by a two channel model [7]

$$H = \sum_{\mathbf{k},\sigma} \left(\frac{\hbar^2 \mathbf{k}^2}{2m} - \mu \right) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{4\pi\hbar^2\, a_{bg}}{m} \sum_{\mathbf{k},\mathbf{p},\mathbf{q}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}-\mathbf{k}\downarrow} c_{\mathbf{q}-\mathbf{p}\downarrow} c_{\mathbf{p}\uparrow}$$

$$+ g \sum_{\mathbf{k},\mathbf{q}} \left(b^\dagger_{\mathbf{q}} c_{\mathbf{k},\downarrow} c_{\mathbf{q}-\mathbf{k},\uparrow} + h.c. \right) + \sum_{\mathbf{q}} \left(\frac{\hbar^2 \mathbf{q}^2}{4m} + \delta - 2\mu \right) b^\dagger_{\mathbf{q}} b_{\mathbf{q}}$$

$c^{(\dagger)}_{\mathbf{k}\sigma}$	–	fermion atoms in two states $\sigma = \uparrow$ or $\sigma = \downarrow$
a_{bg}	–	background scattering length
g	–	atom molecule coupling
δ	–	detuning from the resonance
$b^{(\dagger)}_{\mathbf{q}}$	–	diatomic molecules (hard-core bosons)

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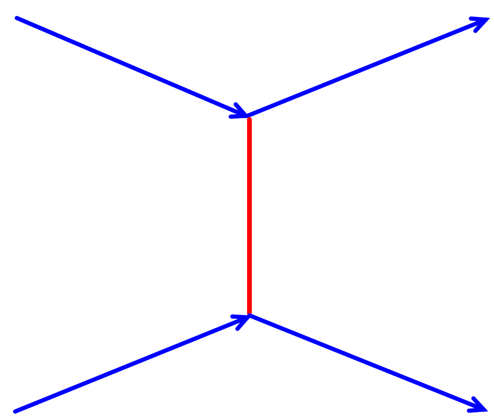
[7] E. Timmermans *et al*, Phys. Rep. **315**, 199 (1999); for a recent review see also R.A. Duine and H.T.C. Stoof, Phys. Rep. **396**, 115 (2004).

Detuning parameter depends on the applied magnetic field

$$\delta = \Delta\mu_{mag}\,(B - B_0)$$

so, within the lowest order perturbation theory [8] the effective scattering length becomes *resonant*

$$a = a_{bg} - \frac{g^2}{\delta}\,\frac{m}{4\pi\hbar^2}$$

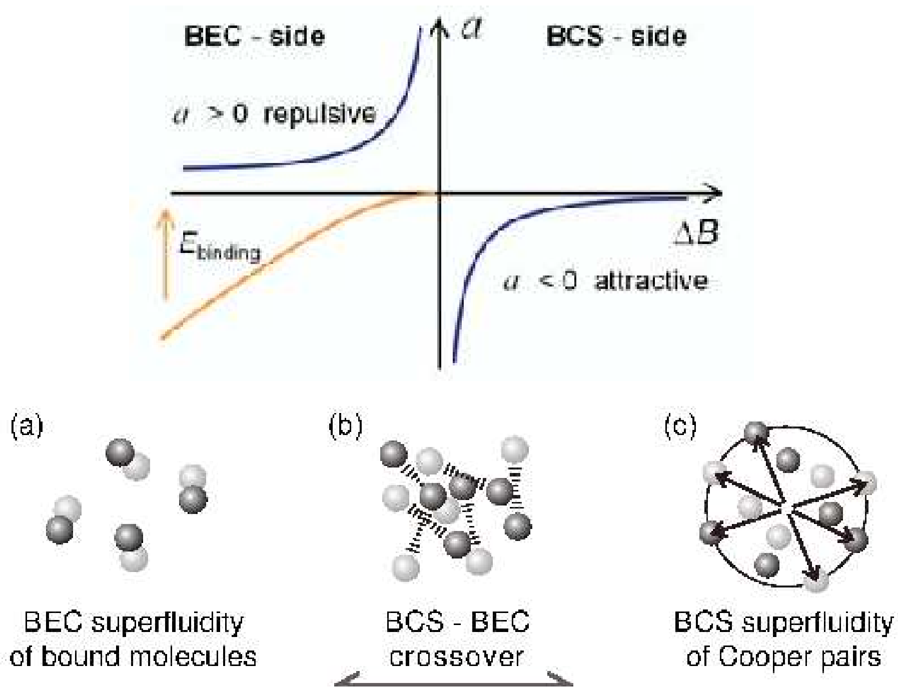


In the selfconsistent treatment this divergence gets smeared [9].

[8] M.W.J. Romans and H.T.C. Stoof, cond-mat/0506282.

[9] T. Domański, Phys. Rev. A **68**, 013603 (2003).

By changing the magnetic field the experimentalists can switch between qualitatively different physical limits:



Such crossover from the BCS to BEC [10] and vice versa [11] has been done performed in various time-dependent sweeping profiles.

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[10] K.E. Strecker *et al*, Phys. Rev. Lett. **91**, 080406 (2003);

M. Greiner *et al*, Nature **426**, 537 (2003).

[11] M. Bartenstein *et al*, Phys. Rev. Lett. **92**, 203201 (2004).

For close proximity to the Feshbach resonance we can:

- omit the weak background scattering a_{bg},
- and neglect the finite momentum molecular states.

In such *single mode approach* Hamiltonian reduces to [12,13]

$$H = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + E_{\mathbf{0}}(t) b^{\dagger}_{\mathbf{0}} b_{\mathbf{0}} + g \sum_{\mathbf{k}} \left(b^{\dagger}_{\mathbf{0}} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} + \textit{h.c.} \right)$$

where $\xi_{\mathbf{k}} = \frac{\hbar^2 \mathbf{k}^2}{2m} - \mu$ and $E_{\mathbf{q}}(t) = \frac{\hbar^2 \mathbf{q}^2}{2m} + \delta(t) - 2\mu$.

[12] A.V. Andreev, V. Gurarie, L. Radzihovsky, Phys. Rev. Lett. **93**, 130402 (2004).

[13] R.A. Barankov and L.S. Levitov, Phys. Rev. Lett. **93**, 130403 (2004).

It is convenient to study the time-dependent Hamiltonian using the pseudospin notation introduced by Anderson [14]

$$\begin{aligned} \sigma_{\mathbf{k}}^{+} &\equiv c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \\ \sigma_{\mathbf{k}}^{-} &\equiv c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger} \\ \sigma_{\mathbf{k}}^{z} &\equiv 1 - c_{\mathbf{k}\uparrow}^{\dagger} c_{\mathbf{k}\uparrow} - c_{\mathbf{k}\downarrow}^{\dagger} c_{\mathbf{k}\downarrow} \end{aligned}$$

such that

$$H = -\sum_{\mathbf{k}} \xi_{\mathbf{k}} \sigma_{\mathbf{k}}^{z} + g \sum_{\mathbf{k}} (b_{\mathbf{0}} \sigma_{\mathbf{k}}^{-} + b_{\mathbf{0}}^{\dagger} \sigma_{\mathbf{k}}^{+}) + E_{\mathbf{0}} b_{\mathbf{0}}^{\dagger} b_{\mathbf{0}}$$

Heisenberg equations of motion for the operators are [12,13]

$$\begin{aligned} i\hbar \frac{\partial \sigma_{\mathbf{k}}^{+}}{\partial t} &= 2\xi_{\mathbf{k}} \sigma_{\mathbf{k}}^{+} + g b_{\mathbf{0}} \sigma_{\mathbf{k}}^{z} \\ i\hbar \frac{\partial \sigma_{\mathbf{k}}^{z}}{\partial t} &= 2g \left(b_{\mathbf{0}}^{\dagger} \sigma_{\mathbf{k}}^{+} - b_{\mathbf{0}} \sigma_{\mathbf{k}}^{-} \right) \\ i\hbar \frac{\partial b_{\mathbf{0}}}{\partial t} &= E_{\mathbf{0}} b_{\mathbf{0}} + g \sum_{\mathbf{k}} \sigma_{\mathbf{k}}^{+} \end{aligned}$$

[14] P.W. Anderson, Phys. Rev. **112**, 1900 (1958).

The time-dependent order parameters

$$b(t) = \langle b_{\mathbf{0}} \rangle$$
$$\chi(t) = \sum_{\mathbf{k}} \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle.$$

were determined by us solving selfconsistently the set of Heisenberg equations of motion.

We started from the stationary case described in detail by R. Micnas et al [15].

[15] R. Micnas, J. Ranninger, S. Robaszkiewicz, Rev. Mod. Phys. **62**, 113 (1990).

In the remaining part we focus on:

a) the sudden sweep [16]

$\delta(t) = \delta_0 \, \theta(-t)$**,**

b) the modulated detuning [17]

$\delta = \delta_0 \sin \omega t$ **.**

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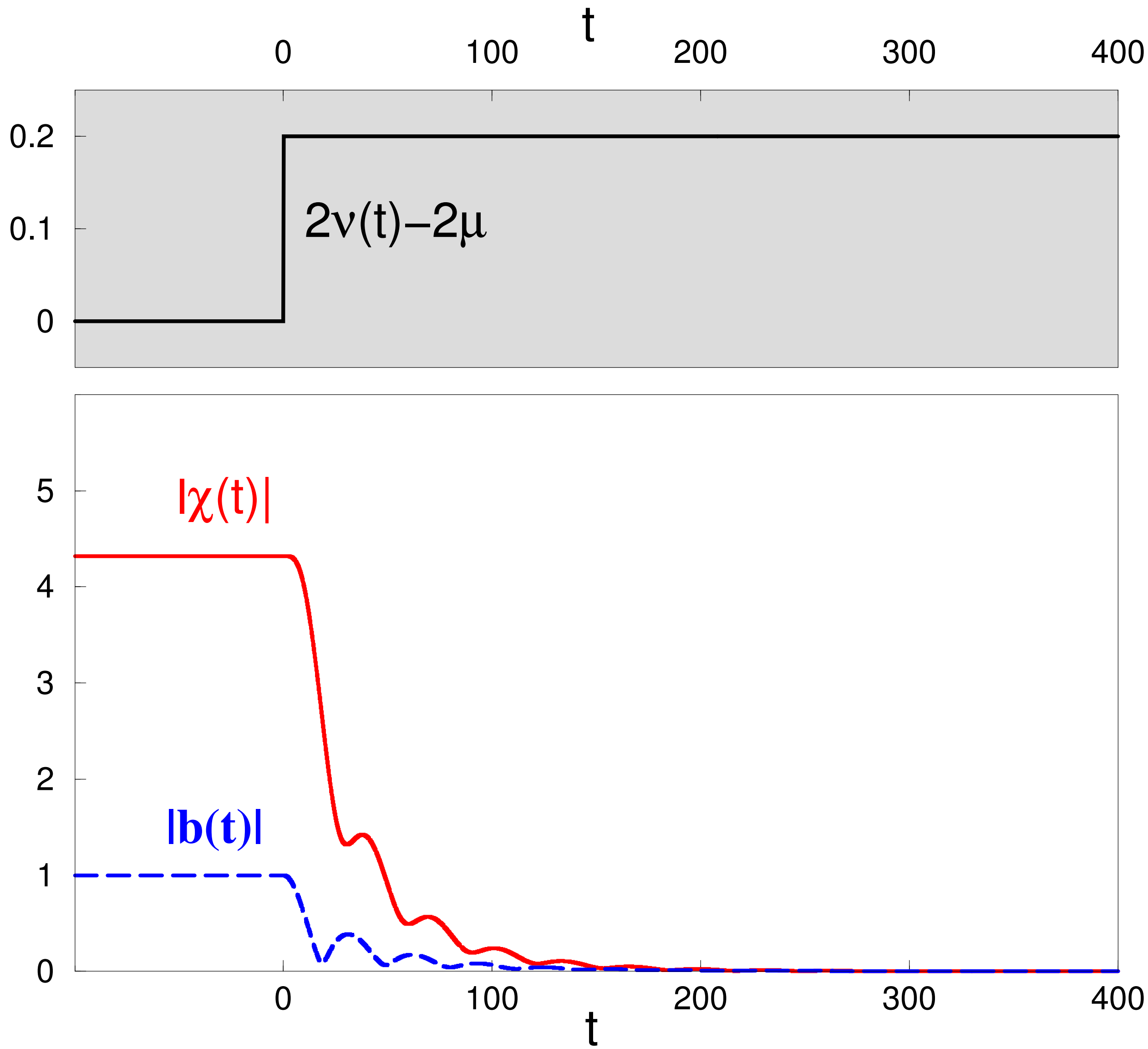
[16] M.H. Szymańska, B.D. Simons, K. Burnet, Phys. Rev. Lett. **94**, 170402 (2005).

[17] W. Yi and L.M. Duan, to appear in Phys. Rev. A (2005).



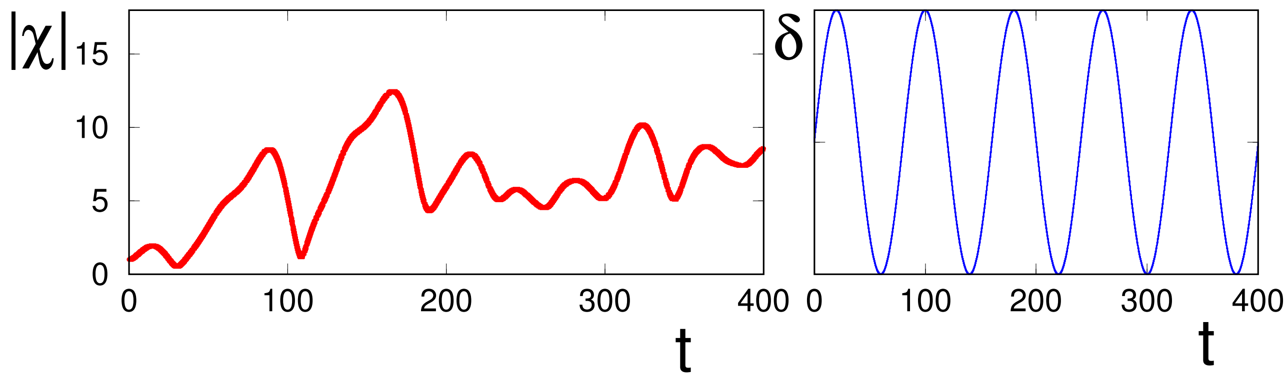
Evolution of the order parameters $\chi(t)$ and $b(t)$ caused by a <u>sudden</u> detuning from the Feshbach resonance

[16] T. Domański, cond-mat/0504588.

Conclusion to the point a):

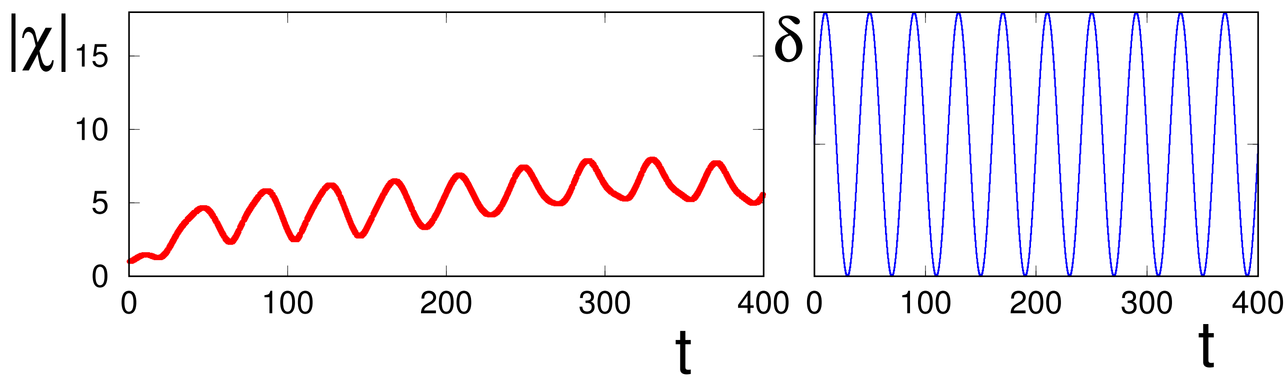
The order parameters decay due to the damping effects.

Modulated detuning



Evolution of the order parameter $|\chi(t)|$ *caused by the modulated detuning* $\delta(t) = \delta_0 \, \sin \omega t$ *around the Feshbach resonance.*

Modulated detuning



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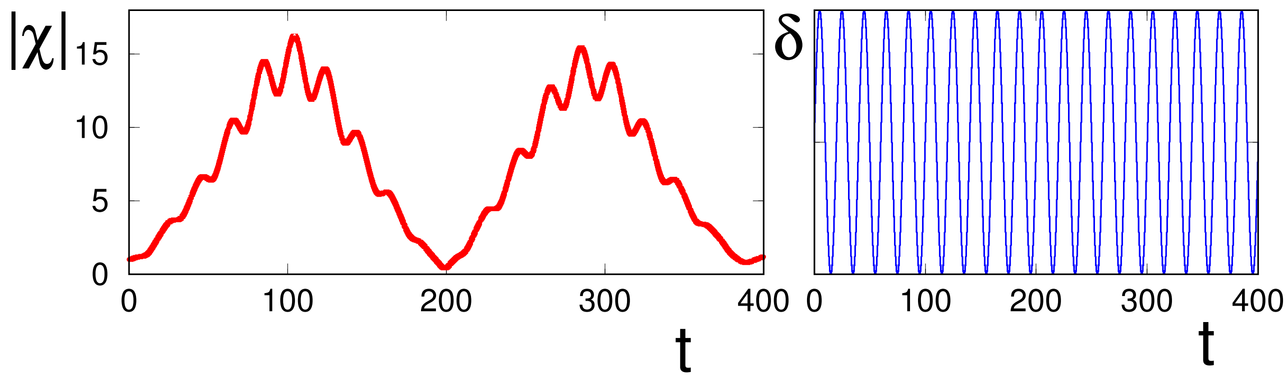

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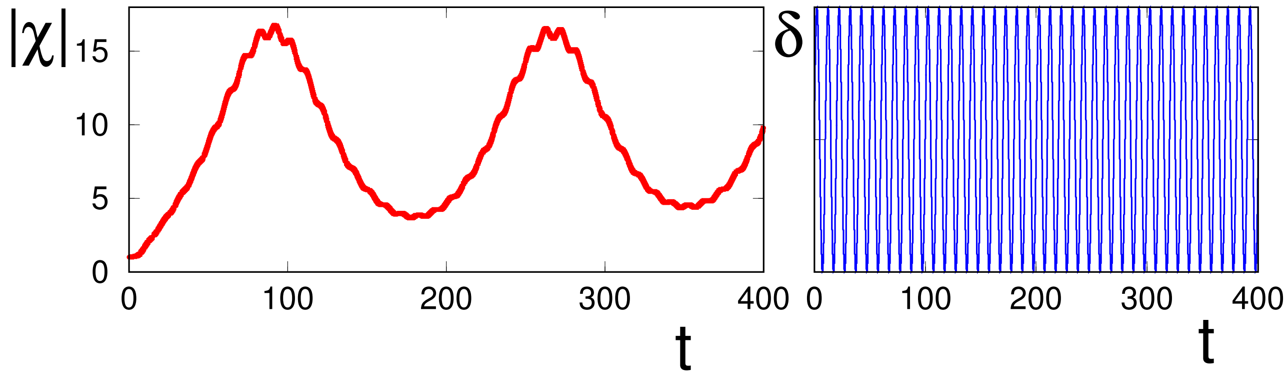

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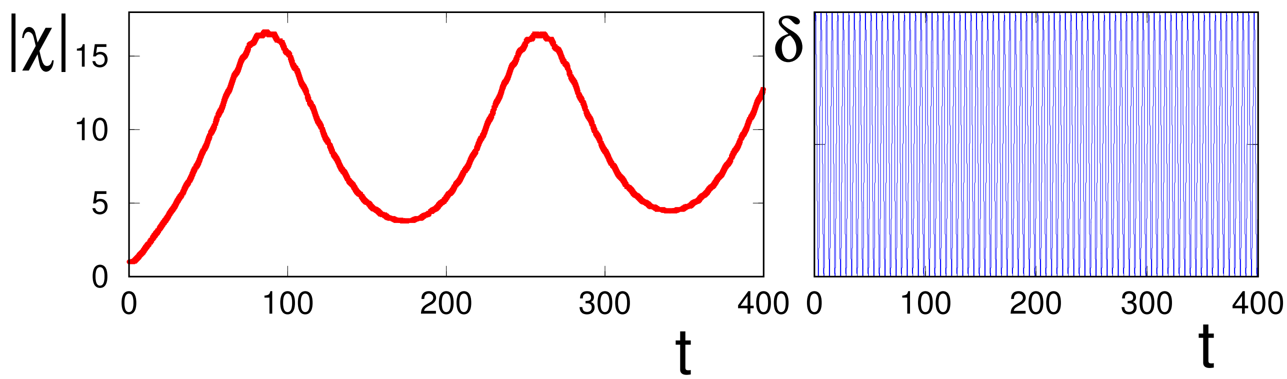

Evolution of the order parameter $|\chi(t)|$ *caused by the modulated detuning* $\delta(t) = \delta_0 \ \sin\omega t$ *around the Feshbach resonance.*

Conclusion to the point b):

For slow modulations the order parameter behaves in a non-regular manner.

For fast modulations ($\omega \geq 2\Delta$) the order parameter starts oscillationg.

CONCLUSIONS

$\Rightarrow$ **Any time-dependent changes of the magnetic field lead to the fluctuations of the order parameters.**

- In case of a sudden detuning from the resonance the order parameter decays due to damping effects.
- For modulated detunings there appear the quantum oscillation (at sufficiently fast frequencies).
- Experimental techniques e.g. the RF spectroscopy should be able to detect such oscillations.

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CONCLUSIONS

- Any time-dependent changes of the magnetic field lead to the fluctuations of the order parameters.
- In a case of sudden detuning from the resonance the order parameter decays due to damping effects.
- Phase fluctuations depend on a specific profile of the detuning process (can be regular or not).
- Experimental techniques e.g. the RF spectroscopy should be able to detect such oscillations.

T h a n k y o u .

http://kft.umcs.lublin.pl/doman/lectures