

β decay of ^{252}Cf in the transition from the exit point to scissionK. Pomorski,¹ B. Nerlo-Pomorska,¹ and P. Quentin^{2,3}¹*Katedra Fizyki Teoretycznej, Uniwersytet Marii Curie-Skłodowskiej, 20031 Lublin, Poland*²*Université de Bordeaux, CENBG, UMR5797, 33170 Gradignan, France*³*CNRS, IN2P3, CENBG, UMR5797, 33170 Gradignan, France*

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Upon increasing significantly the nuclear elongation, the β -decay energy grows. This paper investigates within a simple yet partly microscopic approach, the transition rate of the β^- decay of the ^{252}Cf nucleus on the way to scission from the exit point for a spontaneous fission process. A rather crude classical approximation is made for the corresponding damped collective motion assumed to be one dimensional. Given these assumptions, we only aim in this paper at providing the order of magnitudes of such a phenomenon. At each deformation the energy available for β^- decay, is determined from such a dynamical treatment. Then, for a given elongation, transition rates for the allowed (Fermi) $0^+ \rightarrow 0^+$ β decay are calculated from pair correlated wave functions obtained within a macroscopic-microscopic approach and then integrated over the time corresponding to the whole descent from exit to scission. The results are presented as a function of the damping factor (inverse of the characteristic damping time) in use in our classical dynamical approach. For instance, in the case of a descent time from the exit to the scission points of about 10^{-20} s, one finds a total rate of β decay corresponding roughly to 20 events per year and per milligram of ^{252}Cf . The inclusion of pairing correlations does not affect much these results.

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I. PURPOSE

Due to the decrease of the Coulomb energy and the increase of the surface energy, upon strongly deforming a nucleus, it may happen that a nucleus which is β^- stable at equilibrium deformation gets a positive $Q(\beta^-) = (M(Z, A) - M(Z + 1, A) - m_e)c^2$ at large deformation. This is exemplified in Fig. 1 within a liquid drop model (LDM) approach [1], where the masses of $A = 252$ isobars (from plutonium to kurchatovium) are plotted for various deformations (defined as the ratio $\rho = R_{12}/R_0$ of the distance R_{12} between the mass centers of nascent fragments and the radius R_0 of the corresponding spherical nucleus). One sees clearly that the β stability valley shifts from californium almost to nobelium when the nuclear shape varies from a sphere to configurations close to the scission point where one has typically in such very heavy nuclei $\rho \approx 2.5$.

This result could be also attained upon considering the opposite behavior of proton and neutron separation energies as could be seen from an extremely crude liquid drop approach. Indeed, let us consider a liquid drop energy formula comprising merely volume, surface, direct Coulomb, and volume symmetry terms. For a spherical shape, one gets for the (positive) separation energies for protons (S_p) and neutrons (S_n) with a usual notation and approximating the energy differences as differentiable quantities (i.e., making a continuous approximation for the variables Z or N):

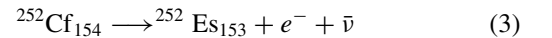
$$\begin{aligned} S_p &= S_p^{(0)} + \frac{1}{3}a_c Z^2 A^{-\frac{4}{3}} - \frac{2}{3}a_s A^{-\frac{1}{3}} - 2a_c Z A^{-\frac{1}{3}}, \\ S_n &= S_n^{(0)} + \frac{1}{3}a_c Z^2 A^{-\frac{4}{3}} - \frac{2}{3}a_s A^{-\frac{1}{3}}, \end{aligned} \quad (1)$$

where $S_p^{(0)}$ and $S_n^{(0)}$ are deformation independent terms given, e.g., for the protons by

$$S_p^{(0)} = -a_v + a_{\text{sym}} \frac{(N - Z)(Z + 3N)}{A^2}. \quad (2)$$

It appears clearly that upon deforming the nuclear shape, both the surface (a_s) and Coulomb (a_c) deformation-dependent terms present in S_n will contribute to its decrease. These terms are also present in S_p but are supplemented by a term which increases with the deformation. We thus conclude that the difference $S_p - S_n$ is increasing with deformation from about zero for the ground state of a β stable nucleus to a value which could be of the order of some MeV. Resorting now to state of the art macroscopic-microscopic calculations (using the LDM model of Ref. [1] and the Yukawa folded potential of Ref. [2] with the parameters given in Ref. [3]), including pairing correlations using a seniority pairing force, one gets an inversion of the relative position of neutron and proton Fermi levels of the ^{252}Cf β stable nucleus upon increasing the elongation, the latter becoming larger than the former. This is illustrated in Fig. 2 showing the neutron and proton single particle (s.p.) spectra around the Fermi levels for $\rho \approx 0.9$ and $\rho \approx 2.8$ corresponding, respectively, to the ground state and the scission point solutions.

We are therefore considering the following β^- decay during the spontaneous fission process (beyond the exit point):



In the present approach intending merely to provide order of magnitudes, we limit ourselves to allowed transitions. The spin and parity of the compound nucleus are conserved during the fission process (in the assumed absence of particle emission). We are thus first led into considering $0^+ \rightarrow 0^+$ Fermi transitions. The Gamow-Teller transitions ($0^+ \rightarrow 1^+$) are *a priori* possible. Yet, according to Ref. [4], they are on the average 1.5 orders of magnitude less probable than the Fermi ones. So, in our simplified approach we are thus left with Fermi transitions only.

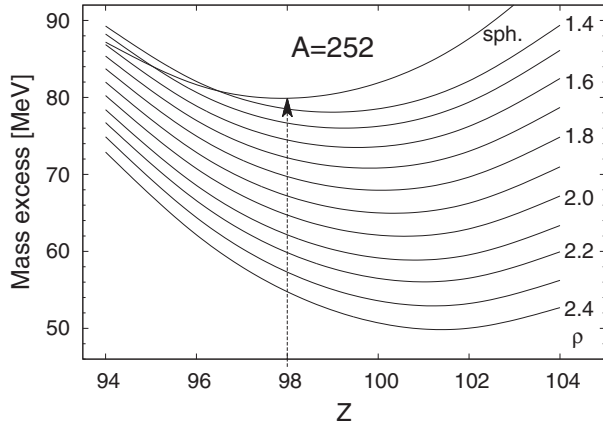


FIG. 1. Mass excess of Pu-Ku nuclei with $A = 252$ evaluated within the LSD model [1]. Each curve corresponds to different elongation of nuclei ρ defined in the text. These values are reported on the r.h.s. of the every second curve. The upper curve (labeled as “sph.”) presents the mass excesses calculated for spherical nuclei.

II. APPROXIMATIONS

The macroscopic-microscopic calculations of the potential energy surface of ^{252}Cf are performed in a four-dimensional deformation parameter space consisting of elongation, neck, nonaxiality, and reflectional asymmetry parameters as described in Ref. [5]. This parametrization approximates very well the optimum in energy shapes of fissioning nuclei [6]. The potential energy surface, the Strutinsky shell, and the pairing energies as well as the liquid drop deformation energy are presented in Fig. 3 on a plane (R_{12}, κ) , where R_{12} is the relative distance between the fragments and κ is the neck parameter. The figure is drawn for the nonaxiality (η) and the reflectional asymmetry (α) parameters equal to zero. All parameters of the macroscopic-microscopic calculations are those of Ref. [7]. One can see that the amplitude of the shell and the pairing energies in the vicinity of the fission path are

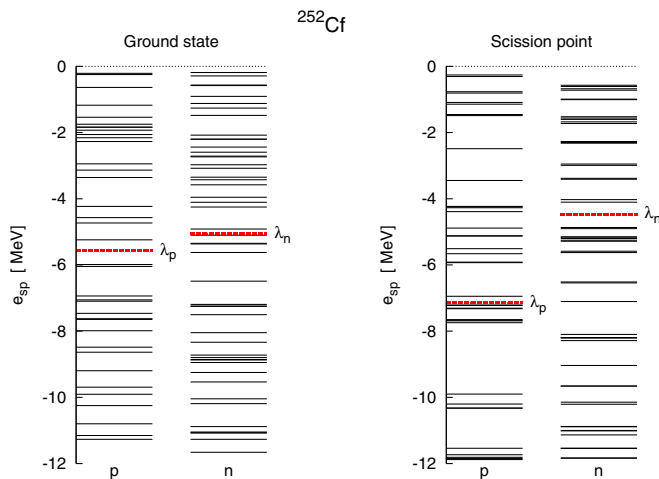


FIG. 2. (Color online) Proton (p) and neutron (n) single-particle levels of ^{252}Cf in the ground state (l.h.s.) and at the scission point (r.h.s.).

of the order of a few MeV but both corrections are almost opposite in phase, so the total microscopic effect modifies only slightly the average behavior of the nuclear energy given by the liquid-drop model. It is also seen in Fig. 3 that in both the liquid-drop and the macroscopic-microscopic models the exit from the barrier appears around an elongation given by $R_{12}/R_0 = 1.5$. The slope of the potential energy towards scission is so large that the scission configuration appears at an energy which is more than 20 MeV below the ground state while the fission barrier height is about 5 MeV only. Such a large energy *a priori* (contingent about the amount of friction in the dynamical collective process) available for the β decay will enhance this process.

At this very preliminary stage of our study, we make the following approximations pertaining to the static and dynamical calculations as well as to the evaluation of transition rates.

(a) We consider only a single fission path assumed to be the most probable one out of static macroscopic-microscopic calculations of the above discussed type. Pairing is included, as it should, to define adequately this optimal path. In Fig. 4, we have plotted the corresponding fission barrier down to the scission point, along with the underlying LDM one.

As a result we find that apart from the ground state region, and in particular at large deformations (namely $R_{12}/R_0 > 1.5$), the LDM energy curve represents a reasonable approximation of the results, where shell and pairing effects are taken into account. However, for the calculated ground state one has definitely to make a choice about its exact definition. One may rule out the LDM solution since shell and pairing effects increase its height. As a good approximation of the macroscopic-microscopic exit point, as can be seen in Fig. 4, is the exit point on the LDM fission barrier (on the side of the descent towards fission) which is located at exactly 4.8 MeV below the saddle point. This value corresponds to the experimental fission barrier height (noting in passing that our macroscopic-microscopic calculations do reproduce it quite well).

In what follows we will use also, instead of the ratio $\rho = R_{12}/R_0$, a collective variable q obtained by a translation of ρ such that $q = 0$ at the exit point. We define in a somewhat arbitrary fashion a single scission point. It corresponds to the point on the LDM fission path, where the nuclear liquid drop ceases to consist of one single piece (it is obtained here for $\rho \approx 2.9$). It corresponds to a gain in energy of $\Delta E_{\text{max}} \approx 28$ MeV with respect to the exit point energy (or the ground state one, since they are of course the same).

In Fig. 4 we have also plotted the LDM $Q(\beta^-)$ value as a function of ρ . It varies from -1.2 MeV at sphericity to enter the instability region for $\rho \approx 1.5$ which happens coincidentally, to be very close to the exit point to reach ≈ 2.5 MeV around the scission point.

(b) In the present approach, which aims only at providing orders of magnitude, we make the following approximations for the collective dynamics:

- (i) As a function of the collective coordinate q , one may approximate reasonably (see Fig. 4) the potential energy $E_p(q)$ as a linearly decreasing function when q

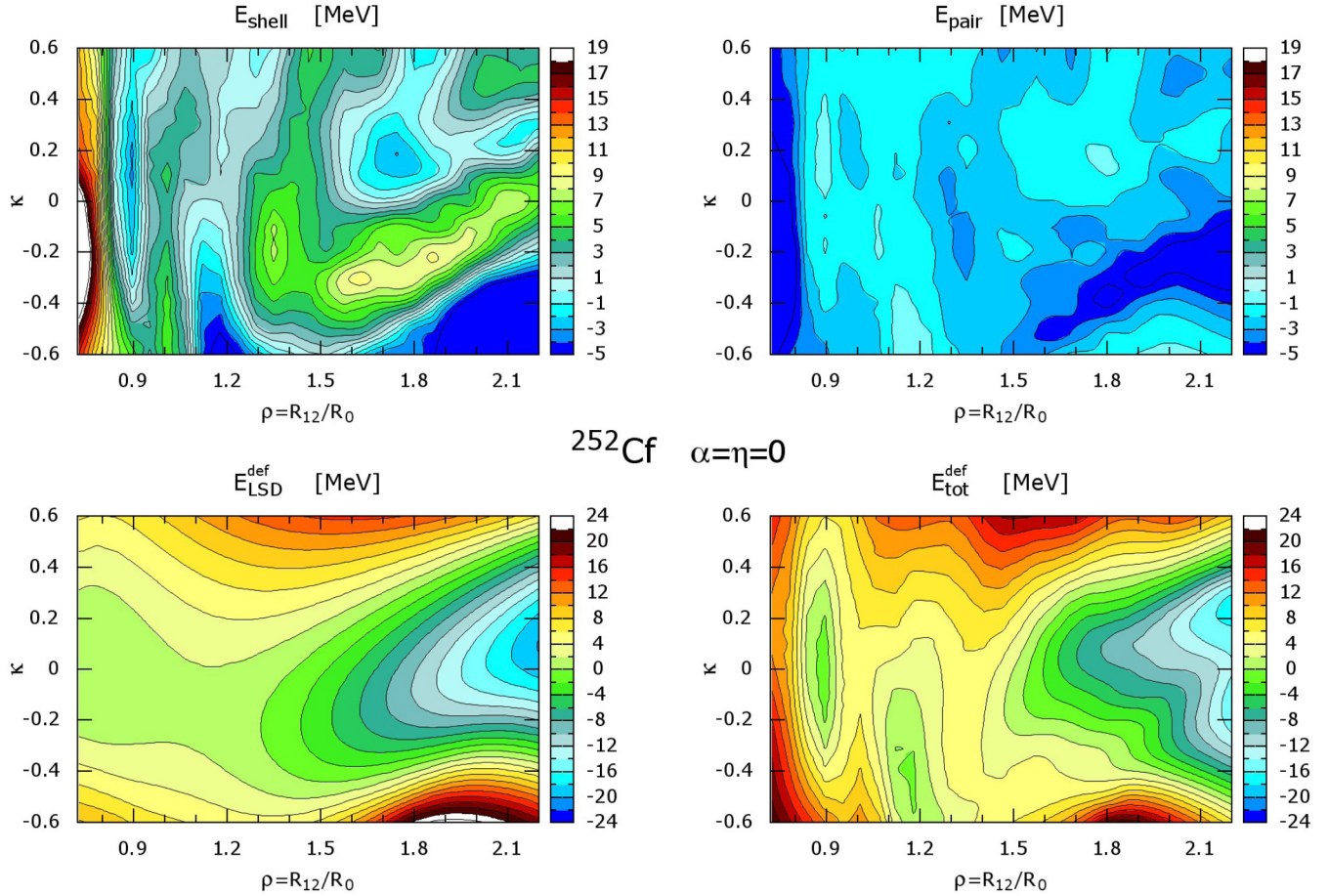


FIG. 3. (Color online) Shell (top, l.h.s), pairing (top, r.h.s.), liquid drop (bottom, l.h.s.), and total deformation energy of ^{252}Cf as function of the relative distance between the fission fragments (ρ) and the neck parameter (κ).

varies from zero to q_s which the value of this parameter at scission. One has thus

$$E_p(q) = -\alpha q \quad \alpha \approx 20 \quad (0 \leq q \leq q_s). \quad (4)$$

- (ii) We approximate the mass parameter $m(q)$ as being constant $m(q) = \mu$ during the descent from the exit

point to scission and equal to the reduced mass between the two final fragments (to assess the relevance of this approximation see Ref. [8]).

- (iii) We assume that the damping of the motion results from a simple friction force $-k\dot{q}$ with $k > 0$.

Then the equation of motion yields the following trivial solution (introducing of course a vanishing collective velocity at the exit point):

$$q = \frac{\alpha}{k} \left[t - \frac{\mu}{k} (1 - e^{-\frac{kt}{\mu}}) \right], \quad (5)$$

which could be numerically inverted to get the time $t(q)$ as a function of the deformation q . Then the collective kinetic energy will be given by

$$E_K(q) = \frac{1}{2} \frac{\mu \alpha^2}{k^2} (1 - e^{-\frac{kt}{\mu}})^2. \quad (6)$$

The available energy is

$$\Delta E(q) = -E_p(q) = \alpha q, \quad (7)$$

from which one gets the excitation energy $E^*(q)$ as

$$E^*(q) = \frac{\mu \alpha^2}{k^2} \left[x - g(x) - \frac{1}{2} g(x)^2 \right], \quad (8)$$

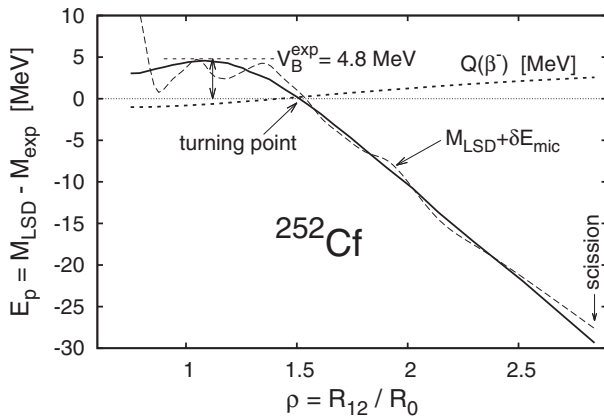


FIG. 4. Macroscopic (solid line) and macroscopic-microscopic (dashed line) fission barriers and the β -decay energy $Q(\beta^-)$ (dotted line) of ^{252}Cf as a function of nuclear elongation.

where the dimensionless quantity x is defined by

$$x = \frac{kt}{\mu} \quad (9)$$

and with

$$g(x) = 1 - e^{-x}. \quad (10)$$

(c) To compute β -decay rates, the effect of pairing correlations will be taken into account. For the sake of clarity we will first discuss our approach without them and in a separate section the modifications brought in by their inclusion.

The conservation of the axial symmetry is assumed along the path. Each s.p. state is thus defined in particular by the usual angular momentum projection K quantum number.

We will assume that there is no polarization, i.e., that the s.p. wave functions in the ^{252}Cf or ^{252}Es mean fields are the same and furthermore that one has, with a transparent notation, the following relation between the binding energies of the parent and daughter nuclei (Koopmans approximation):

$$E(^{252}\text{Es}) = E(^{252}\text{Cf}) + e_p - e_n, \quad (11)$$

where e_p and e_n are the proton and neutron single-particle energies, respectively.

III. EFFECTIVE Q_β VALUES AND AVAILABLE PHASE-SPACE

At a given elongation q or at given time $t(q)$, the parent nucleus has an excitation energy of $E^*(q)$. The effective $Q_\beta(q)$ value for a transition from a neutron single particle state i to a proton single particle state f is thus given by

$$Q_\beta^{(i,f)}(q) = E^*(q) + e_n(i) - e_p(f) + 0.78 \text{ MeV}, \quad (12)$$

where the last constant originates from the mass difference of the neutron and proton plus the electron mass. Defining the neutron Fermi level λ_n and similarly for protons λ_p , one will consider an effective neutron Fermi level as $\lambda_n^{\text{eff}}(q)$ as

$$\lambda_n^{\text{eff}}(q) = \lambda_n + E^*(q) + 0.78 \text{ MeV}. \quad (13)$$

Possible transitions between neutron s.p. states and proton s.p. states will require the following conditions to be satisfied:

$$K_n(i) = K_p(f), \quad \lambda_p \leq e_p(f) \leq e_n(i) \leq \lambda_n^{\text{eff}}(q). \quad (14)$$

IV. TRANSITIONS RATES

The nuclear matrix element for such a (i, f) Fermi transition is given with a usual notation by

$$M_{i,f} = \iiint \phi_{p(f)}^*(\vec{r}) \phi_{n(i)}(\vec{r}) d^3r. \quad (15)$$

A special note is to be made here, about the intrinsic parity breaking. Let us call $|\Psi\rangle$ the intrinsic wave function breaking the left-right symmetry, either for the parent or the daughter states. One should project a positive parity state out of it, as

$$|\Psi^{(+)}\rangle = \frac{1}{\sqrt{2}}(|\Psi\rangle + \hat{\Pi}|\Psi\rangle), \quad (16)$$

where $\hat{\Pi}$ is the parity operator.

The above nuclear matrix element will thus comprise two parts corresponding to the two overlaps $\langle\Psi(i)|\Psi(f)\rangle$ and $\langle\Psi(i)|\hat{\Pi}|\Psi(f)\rangle$. However it is known from parity projection calculations of the fission barrier of heavy nuclei, see Ref. [9], that somewhere before the second fission barrier and beyond, the intrinsic parity breaking deformation is so large that the overlap between the corresponding wave function and its parity image is negligible. We are just left here with the $\langle\Psi(i)|\Psi(f)\rangle$ overlap as in the nonparity breaking case. Yet, in this case one cannot, of course, apply a selection rule on the quantum number π which is not conserved but only on the angular momentum projection K which we have assumed to be a good quantum number.

The associated transition rate is [10]

$$R_{i,f} = G^2 \frac{|M_{i,f}|^2 m_e^5 c^4}{2\pi^3 \hbar^7} f(Z=99, Q_\beta^{i,f}), \quad (17)$$

which after inserting of the weak interaction coupling constant (G) and other constants values takes the following form:

$$R_{i,f} = 1.105 \cdot 10^{-4} |M_{i,f}|^2 f(Z=99, Q_\beta^{i,f}). \quad (18)$$

A rough estimate of the function f as a function of the effective Q value $Q_\beta^{(i,f)}(q)$ is given for $Z=99$ by

$$\log_{10}(f) = 3.5 \log_{10}(Q_\beta^{(i,f)}(q)) + 3, \quad (19)$$

where $Q_\beta^{(i,f)}(q)$ is expressed in MeV.

V. INCLUDING PAIR CORRELATED NUCLEAR STATES

When pairing correlations are included in both the parent and daughter nuclear states (assuming again no polarization effects, i.e., taking the s.p. wave functions, occupation probabilities, and quasiparticle energies as obtained in the parent nucleus) one has just to multiply the overlaps $M_{i,f}$ by the BCS factor $u_{p(f)} v_{n(i)}$ while the effective $Q_\beta^{i,f}(q)$ becomes

$$Q_\beta^{(i,f)}(q) = E^*(q) - [E_n^{qp}(i) + E_p^{qp}(f)] + (\lambda_n - \lambda_p) + 0.78 \text{ MeV}, \quad (20)$$

where $E_n^{qp}(i)$ and $E_p^{qp}(f)$ are the initial neutron and the final proton quasiparticle energies.

In this case, one may consider all transitions from single neutron to single proton states which satisfy the following conditions beyond the angular momentum component matching rule [$K_n(i) = K_p(f)$]:

- (i) the proton state should not be fully occupied (i.e., lying below the valence space),
- (ii) the neutron state should not be fully unoccupied (i.e., lying above the valence space),
- (iii) the final state energy should be lower or equal to the initial state energy, i.e., $Q_\beta^{i,f}(q) \geq 0$.

VI. TOTAL TRANSITION RATES

At a given time (or at a given elongation) the total rate of decay will be obtained by summing all individual rates $R_{i,f}$ to get $R(q)$. For the whole descent from exit to scission, one will

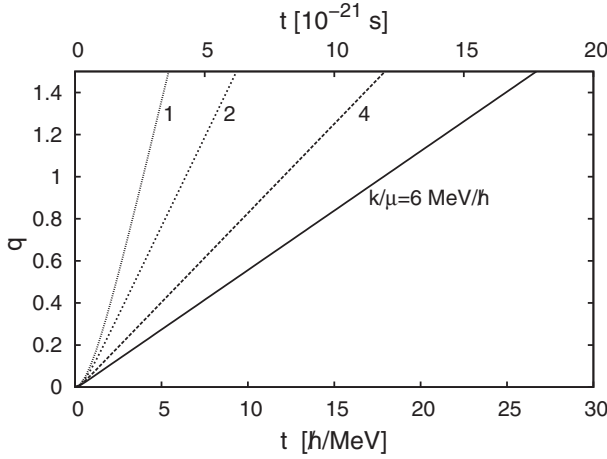


FIG. 5. Collective elongation $q = (R_{12} - R_{12}^{\text{exit}})/R_0$ as function of time [Eq. (5)] for different values of the nuclear damping k/μ .

get the number of decay for a single fission, i.e., the probability of decay, by integrating $R(q)$ over the whole time of descent from the exit to the scission points

$$P_{\text{decay}} = \int_0^{q_s} R(q) \frac{k}{\alpha} \left(1 - e^{-\frac{k}{\mu} t}\right)^{-1} dq. \quad (21)$$

This probability, and similarly the pre-scission kinetic energy $E_K^0 = E_K(q_s)$, are of course strongly dependent on the retained value for the friction parameter k . In the following we will use our estimates of P_{decay} as a function of the time t_{sc} to evaluate the number of β decays from a sample of ^{252}Cf in a given time period.

VII. RESULTS

Let us first use the results of our simple dynamical calculations to describe the collective motion from the exit to the scission points. Our results will be presented for a few values of the reduced friction parameter k/μ , of which the

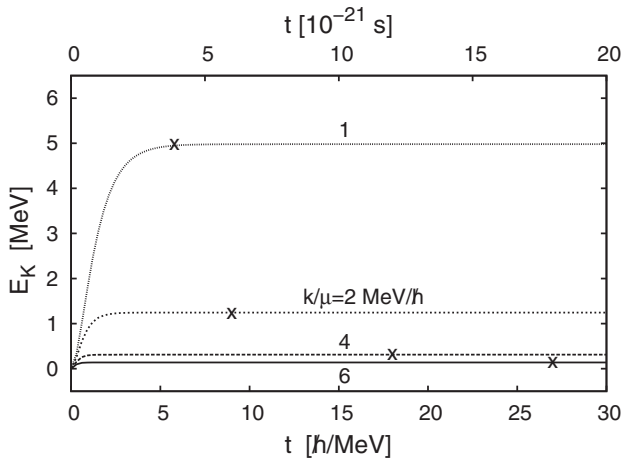


FIG. 6. Pre-scission kinetic energy for a few values of the reduced friction (k/μ) as function of time. The time at which scission configuration is reached for a given k/μ value is marked by a cross.

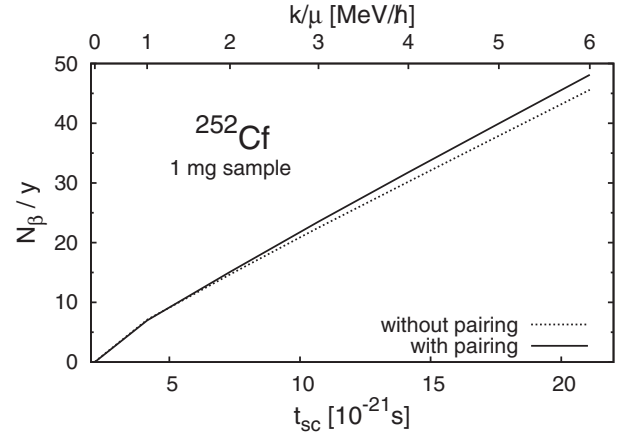


FIG. 7. Number of β decays per year for a 1 mg sample of ^{252}Cf as a function of the descent time t_{sc} from the exit to the scission points.

inverse is the damping time (5). The damping parameter k/μ is given in units $\text{MeV}/\hbar \approx 1.5 \cdot 10^{21} \text{s}^{-1}$.

As seen in Fig. 5, the collective descent from $q = 0$ to $q \approx 1.4$ takes from $\approx 3.5 \cdot 10^{-21} \text{s}$ for a large damping time ($k/\mu = 1$ in the above discussed units) to $\approx 18 \cdot 10^{-21} \text{s}$ for a small damping time ($k/\mu = 6$). In all considered cases the collective motion is completely damped at scission as exemplified on Fig. 6. The pre-scission kinetic energy E_K values range from a couple hundred keV for the large damping case ($k/\mu = 6$) to $\approx 5 \text{ MeV}$ for the low damping case ($k/\mu = 1$).

The resulting number of β decays for a 1 mg sample of ^{252}Cf is given in Fig. 7, as a function of the descent time t_{sc} from the exit to the scission points (or equivalently as a function of the damping parameter k/μ). It ranges from ≈ 6 for the low damping case ($k/\mu = 1$) to ≈ 46 for the large damping case ($k/\mu = 6$). The range of considered values of the damping parameter k/μ is taken following estimates made in Ref. [11] (see Fig. 10 of this reference).

The previous figures are given for an evaluation which does not take into account pairing correlations effects. As seen of Fig. 7, including these correlations increases slightly the above given numbers (only by $\approx 2\%$ for $k/\mu = 6$), this correction being an increasing function of the damping parameter k/μ .

VIII. CONCLUSIONS

This paper aimed at pointing out the strong dependence of the β -decay stability as a function of the nuclear deformation. From state of the art macroscopic-microscopic calculations it has been claimed that a very heavy nucleus like ^{252}Cf , while β stable as is well known in its ground state becomes unstable near the fission exit point. Through a very simple description of the collective dynamics, the transition rates of the Fermi β decay during the spontaneous fission process of this nucleus up to the scission point, have been calculated. They depend of course on the amount of damping of the fission collective mode which regulates the excitation energy available at each time, for such a decay. In so far as these rates could prove to be experimentally reachable, they would provide a much needed source of information on the

pre-scission dynamics, as, e.g., the average pre-scission kinetic energy or the value of the reduced friction parameter. Taking pairing correlations into account or not is not an important factor.

As a result a relatively weak number of β decays are expected from our results during the descent from the exit to the scission points for standard values of the damping parameter. The capacity of existing experimental devices to assess this conclusion for tractable amounts of ^{252}Cf is questionable. Improving these so far very crude estimates by a better treatment of the static and dynamic parts of these calculations as well as exploring this phenomenon for other

nuclei (involving possibly the inclusion of Gamow-Teller decays as well) will be undertaken.

ACKNOWLEDGMENTS

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