MACROSCOPIC PROPERTIES OF NUCLEI WITHIN SELF-CONSISTENT AND LIQUID DROP MODELS

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(Received September 29, 2003)

A set of parameters of the relativistic-mean-field theory (RMFT) is obtained by adjusting the macroscopic part of the RMFT binding energies of 142 spherical even-even nuclei to the phenomenological Lublin–Strasbourg–Drop (LSD) model.

PACS numbers: 21.10.Ma, 26.60.Dr, 25.70.Ji, 24.10.Pa

The Lublin–Strasbourg–Drop (LSD) model [1], including shell, deformation and pairing corrections [4] reproduces the experimentally known masses of 2766 nuclei with a mean square deviation of 0.698 MeV. The LSD mass formula for the macroscopic energy of spherical nuclei with \( Z \) protons and \( N \) neutrons is given by

\[
\frac{E_{\text{LSD}}}{\text{MeV}} = -15.4920 \left( 1 - 1.8601 I^2 \right) A + 16.9707 \left( 1 - 2.2938 I^2 \right) A^{2/3} + 3.8602 \left( 1 + 2.3764 I^2 \right) A^{1/3} + \frac{3e^2 Z^2}{5A^{1/3}1.2172} - \frac{0.9181 Z^2}{A},
\]

where \( I = (N - Z)/A \) and \( A = N + Z \). It is now interesting to compare the macroscopic part of self-consistent binding energies obtained in a relativistic-mean-field theory (RMFT) with the estimates of the LSD model in order to test the predictive power of the effective nuclear forces used in the RMFT approach, and eventually adapt its parameters. Subtracting the shell corrections from the selfconsistent energies obtained in this way [2] we have got estimates for the macroscopic RMFT binding energies of 142 spherical

* Presented at the XXVIII Mazurian Lakes School of Physics, Krzyże, Poland, August 31–September 7, 2003.
even–even nuclei [3]. By a least-square-fit procedure, with the LSD energies as a reference, we have found a new optimal set of RMFT parameters: NL4.

The parameters of the selfconsistent RMFT model, which is based on the Dirac equation for the nucleons and the Klein–Gordon equation for the $\rho$, $\omega$ and $\sigma$ mesons, plus the photon, are the masses for the nucleon $m$, the mesons $m_{\rho}$, $m_{\omega}$, $m_{\sigma}$ plus the coupling constants $g_{\omega}$, $g_{\rho}$, $g_{\sigma}$, $g_2$, $g_3$. These parameters should be fitted to the possible largest amount of nuclear observables, but usually the masses and mean-square charge radii of only 8 magic nuclei were used. We have checked the quality of the three commonly used sets of parameters: NL1 [6], NL2, NL3 [7] by comparing the macroscopic energies obtained after subtraction of Strutinsky shell correction energies from the RMFT binding energies, evaluated without taking pairing into account, with the liquid-drop energies given by the LSD approach. In addition, we have generated a new set of RMFT parameters, NL4, by a least-square-fit procedure of the macroscopic part of the RMFT energies of 142 spherical nuclei to the LSD estimates. These parameters are given in the Table together with the commonly used sets: NL1, NL2, NL3. The rms deviation $\langle \delta E \rangle$ of the macroscopic RMFT energies from those of the LSD model for each of these parameter sets is also given.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>NL1</th>
<th>NL2</th>
<th>NL3</th>
<th>NL4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>MeV</td>
<td>938</td>
<td>938</td>
<td>939</td>
<td>938</td>
</tr>
<tr>
<td>$m_{\omega}$</td>
<td>MeV</td>
<td>795.359</td>
<td>780</td>
<td>782.501</td>
<td>782.474</td>
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<tr>
<td>$m_{\sigma}$</td>
<td>MeV</td>
<td>492.25</td>
<td>504.890</td>
<td>508.194</td>
<td>508.194</td>
</tr>
<tr>
<td>$m_{\rho}$</td>
<td>MeV</td>
<td>763</td>
<td>763</td>
<td>763</td>
<td>763.9</td>
</tr>
<tr>
<td>$g_{\omega}$</td>
<td>-</td>
<td>13.285</td>
<td>11.493</td>
<td>12.868</td>
<td>12.867</td>
</tr>
<tr>
<td>$g_{\rho}$</td>
<td>-</td>
<td>4.976</td>
<td>5.507</td>
<td>4.474</td>
<td>4.360</td>
</tr>
<tr>
<td>$g_{\sigma}$</td>
<td>-</td>
<td>10.138</td>
<td>9.111</td>
<td>10.217</td>
<td>10.216</td>
</tr>
<tr>
<td>$g_2$</td>
<td>fm$^{-1}$</td>
<td>-12.172</td>
<td>-2.304</td>
<td>-10.431</td>
<td>-10.432</td>
</tr>
<tr>
<td>$g_3$</td>
<td>-</td>
<td>-36.265</td>
<td>13.783</td>
<td>-28.885</td>
<td>-28.882</td>
</tr>
<tr>
<td>$\langle \delta E \rangle$</td>
<td>MeV</td>
<td>12.7274</td>
<td>33.0981</td>
<td>7.1691</td>
<td>3.2985</td>
</tr>
</tbody>
</table>

The new parameter set NL4 appears to give the results closest to the LSD energies with a mean energy deviation that is more than a factor 2 smaller than in the case of the frequently used NL3 set.

In Fig. 1 these rms deviations are shown as functions of each of the 9 RMFT parameters, keeping the others fixed at their NL4 values.

In Fig. 2 the differences between the LSD energies and the macroscopic RMFT energies obtained with the NL3 and NL4 parameter sets are compared with the results obtained within the Hartree–Fock approach with the Gogny force D1S [8] for a few chains of isotopes (top), isotones (middle) and spherical $\beta$ stable nuclei (bottom) with $A > 40$. One observes that the NL4 set yields binding energies that are very close to the phenomenological LSD values, but for neutron deficient nuclei a significant discrepancy still remains.
Macroscopic Properties of Nuclei within Self-Consistent.

Fig. 1. Rms deviation of the macroscopic NL4 RMFT energies from those of the LSD model as function of each of the RMFT parameters.

Fig. 2. The deviations of the macroscopic energies obtained in the LSD model, and, on the other hand, in the RMFT with the NL3, NL4 parameter sets or with the Gogny DIS effective interaction for 142 spherical even-even isotopes (top), isotones (middle) and β stable nuclei (bottom) as function of mass number $A$. 
It should be mentioned that the rms radii for the neutron and charge distributions obtained with the NL4 RMFT approach are slightly closer to the experimental data than those of the NL3 RMFT or the Gogny D1S results.

In conclusion one may say that the new RMFT NL4 parameter set reproduces the data of spherical even-even nuclei much better than the previous RMFT approach or the D1S Gogny force. The macroscopic RMFT energies obtained with the new set NL4 are very close for most of the 142 investigated nuclei to the LSD data, i.e. an approach that has shown extremely successful. Also the masses and radii obtained for spherical even-even nuclei within the RMFT NL4 approach are closer to the experimental data than for other RMFT forces.

We would like to thank the IN2P3–Polish Laboratories convention 99 95 for the financial support. The fruitful discussions with Johann Bartel and the warm hospitality in Strasbourg are also appreciated.

REFERENCES