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Quantum fluctuations of ultracold atom–molecule mixtures

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We investigate the evolution of quantum coherence in an ultracold mixture of fermionic atoms and bosonic dimer molecules. Interactions are experimentally controlled via tuning the external magnetic field. Consequently, the fermionic atoms and their bosonic counterparts can be driven to a behavior resembling the usual BCS to BEC crossover. We analyze in some detail how this quantum coherence evolves with respect to time upon a smooth and abrupt sweep across the Feshbach resonance inducing the atom–molecule quantum fluctuations.

1 Atom superfluidity

The recent experimental techniques for trapping and cooling of atomic vapors enabled exploration of the extremely low temperature regions where quantum effects play a crucial role. An example is the Bose–Einstein condensation (BEC) produced out of the bosonic atoms in alkali metals, polarized hydrogen, etc. The phase transition to the BEC state is triggered purely by the quantum-statistical requirements that lead to macroscopic occupancy of the lowest energy level and can occur even in the absence of any interactions. Recent activities in the field of ultracold atomic systems focus on the application of similar techniques to fermionic atoms like ^{6}Li or ^{40}K (besides an even number of nucleons they consist of an odd number of electrons). At ultralow temperatures such quantum effects as the Pauli principle play a considerable role, but eventual quantum phase transitions would be allowed only if fermionic atoms become correlated via interactions.

Interactions between trapped atoms are routinely induced by applying the magnetic field to fermion atoms prepared in several (two or more) hyperfine configurations. From elementary considerations [1] it turns out that the involved hyperfine states experience the effective scattering described by a potential whose magnitude and sign depend on the applied field B. In particular, the various (so-called) Feshbach resonances can take place. On this basis a mechanism was proposed of resonance superfluidity [2] with a transition occurring near the Fermi temperature $T_c \sim T_F$. Besides the isotropic phase there has already been observed also the exotic p-wave superfluidity [3, 4].

A unique possibility of controlling the effective interactions gives the possibility for the experimental realization of the BCS to BEC crossover. The BCS limit corresponds to a case of weakly attracting fermion atoms that get coupled into the large Cooper pairs. In the opposite limit, the tightly bound diatomic molecules are formed that ultimately can undergo transition to the BEC. Experimentalists are able to switch between these limits in a controllable manner. Moreover, the change of interactions can be performed either adiabatically by slowly changing the field [5] or in nonadiabatic way via a sudden sweep [6].

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In this paper we investigate the quantum fluctuations induced by the time-dependent change of the interactions. We focus on a situation when the magnetic field is detuned from the resonant value B_0 towards the far BCS regime at higher field $B > B_0$. We consider the two different processes: a smooth and a sudden switching. The fast sweep has been discussed in the literature but without an unambiguous conclusion concernig the evolution of the order parameters with respect to time [7–9]. From our analysis we find that both parameters would oscillate in a damped way.

2 Heisenberg equations

In close proximity to the Feshbach resonance (i.e. when $B \sim B_0$) the ultracold fermion atoms coexist and interact with the diatomic molecules. On a microscopic basis this situation can be described in terms of the two-component boson–fermion Hamiltonian [2]

$$H = \sum_{\mathbf{k},\sigma} (\varepsilon_{\mathbf{k}}^{\mathrm{F}} - \mu)\, c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} (\varepsilon_{\mathbf{q}}^{\mathrm{B}} + 2\nu(B) - 2\mu)\, b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}} + \frac{g}{\sqrt{N}} \sum_{\mathbf{k},\mathbf{q}} (b_{\mathbf{q}}^{\dagger} c_{\mathbf{q}-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} + c_{\mathbf{k}\uparrow}^{\dagger} c_{\mathbf{q}-\mathbf{k}\downarrow}^{\dagger} b_{\mathbf{q}})\,, \tag{1}$$

which has been known and studied in solid-state physics by Ranninger and coworkers [10] as a phenomenological model for high-temperature superconductivity. In the present context Eq. (1) describes the atoms in two hyperfine states denoted symbolically by $\sigma = \uparrow$ and $\downarrow$. The second quantization operators $c_{\mathbf{k}\sigma}^{(\dagger)}$, $b_{\mathbf{q}}^{(\dagger)}$ correspond to fermion atoms with energy $\varepsilon_{\mathbf{k}}^{\mathrm{F}} = \hbar^2 \mathbf{k}^2/2m$ and to diatomic molecules with energy $\varepsilon_{\mathbf{k}}^{\mathrm{B}} = \hbar^2 \mathbf{k}^2/2(2m)$. The effect of external magnetic field is included via the *detuning parameter* $\boldsymbol{\nu}$ that shifts the boson energies and hence affects the efficiency of the boson–fermion coupling g [11]. As usual μ is the common chemical potential and we use the grand canonical ensemble to ensure the conservation of the total particle number $\sum_{\mathbf{k},\sigma} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + 2 \sum_{\mathbf{q}} b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}}$.

We are interested here in studying the time-dependent evolution of fermion and boson occupancies together with the corresponding order parameters. For this purpose we derive the Heisenberg equations of motion that for the Hamiltonian (1) are given by

$$i\,\frac{\partial c_{\mathbf{q}-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}}{\partial t} = (\xi_{\mathbf{k}} + \xi_{\mathbf{q}-\mathbf{k}})\, c_{\mathbf{q}-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} + g b_{\mathbf{q}} - g \sum_{\mathbf{q}'} b_{\mathbf{q}'} (c_{\mathbf{q}'-\mathbf{k}\downarrow}^{\dagger} c_{\mathbf{q}-\mathbf{k}\downarrow} + c_{\mathbf{k}+\mathbf{q}'-\mathbf{q}\uparrow}^{\dagger} c_{\mathbf{k}\uparrow})\,, \tag{2}$$

$$i \sum_{\sigma} \frac{\partial c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma}}{\partial t} = 2g \sum_{\mathbf{q}} (b_{\mathbf{q}} c_{\mathbf{k}\uparrow}^{\dagger} c_{\mathbf{q}-\mathbf{k}\downarrow}^{\dagger} - b_{\mathbf{q}}^{\dagger} c_{\mathbf{q}-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow})\,, \tag{3}$$

$$i\,\frac{\partial b_{\mathbf{q}}}{\partial t} = E_{\mathbf{q}} b_{\mathbf{q}} + g \sum_{\mathbf{k}} c_{\mathbf{q}-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}\,, \tag{4}$$

$$i\,\frac{\partial b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}}}{\partial t} = g \sum_{\mathbf{k}} (b_{\mathbf{q}}^{\dagger} c_{\mathbf{q}-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} - b_{\mathbf{q}} c_{\mathbf{k}\uparrow}^{\dagger} c_{\mathbf{q}-\mathbf{k}\downarrow}^{\dagger})\,, \tag{5}$$

where $\xi_{\mathbf{k}} = \varepsilon_{\mathbf{k}}^{\mathrm{F}} - \mu$, $E_{\mathbf{q}} = \varepsilon_{\mathbf{q}}^{\mathrm{B}} + 2\nu(B) - 2\mu$ and we set $\hbar = 1$. In general, Eqs. (2)–(5) are not solvable exactly. In the next section we briefly discuss an approximate method that shall be valid for the ground state and for very low temperatures.

3 The single-mode approach

For temperatures close to the absolute zero we can neglect the excited (finite momentum) boson states. It is sufficient to restrict attention to the $\mathbf{q} = 0$ boson level because it is macroscopically occupied. In such a *single-mode approach* [7, 8] the initial Hamiltonian (1) reduces to

$$H = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} E_0 b_0^{\dagger} b_0 + \frac{g}{\sqrt{N}} \sum_{\mathbf{k}} (b_0^{\dagger} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} + c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger} b_0)\,. \tag{6}$$

Following Anderson [12] we introduce the pseudospin notation $\sigma_k^+ \equiv c_{-k\downarrow} c_{k\uparrow}$, $\sigma_k^- \equiv c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger$ and $\sigma_k^z \equiv 1 - c_{k\uparrow}^\dagger c_{k\uparrow} - c_{k\downarrow}^\dagger c_{k\downarrow}$ such that $\sigma_k^\pm = \frac{1}{2}(\sigma_k^x \pm i\sigma_k^y)$ and $\sigma_k^z = [\sigma_k^+, \sigma_k^-]$ are the usual Pauli operators. In the single mode approach we rewrite the Heisenberg Eqs. (2)−(5) using the pseudospin notation

$$i\,\frac{\partial \sigma_k^+}{\partial t} = 2\xi_k \sigma_k^+ + g b_0 \sigma_k^z, \qquad i\,\frac{\partial \sigma_k^z}{\partial t} = 2g(b_0^\dagger \sigma_k^+ - b_0 \sigma_k^-)\,, \tag{7}$$

$$i\,\frac{\partial b_0}{\partial t} = E_0 b_0 + g\sum_k \sigma_k^+, \qquad i\,\frac{\partial b_0^\dagger b_0}{\partial t} = g\sum_k (b_0^\dagger \sigma_k^+ - b_0 \sigma_k^-)\,, \tag{8}$$

which are identical with expressions (5) and (6) in Ref. [8]. One next replaces the boson operators by their time-dependent expectation values $b(t) = \langle b_0 \rangle$ and $b^*(t) = \langle b_0^\dagger \rangle$.

In the stationary case when all parameters in Eq. (6) are time independent we can derive various expressions for the static expectation values [10]. Hamiltonian (6) has formally the following structure

$$H = -\sum_k \boldsymbol{h}_k \boldsymbol{\sigma}_k + \text{const}\,, \tag{9}$$

so the pseudospin $\boldsymbol{\sigma}_k$ behaves as though affected by a fictitious magnetic field $\boldsymbol{h}_k = (-\Delta', \Delta'', \xi_k)$ where $\Delta' + i\Delta'' \equiv g\langle b_0 \rangle$. Following Anderson [12] we can solve this problem (9) for arbitrary temperature. In analogy to the Weiss theory of ferromagnetism we obtain that the magnitude of the pseudospin expectation value is $|\langle \boldsymbol{\sigma}_k \rangle| = \tanh\left\{\sqrt{\xi_k^2 + |\Delta|^2}/2k_\text{B}T\right\}$. Determining the angle between the z and xy components of the vector $\boldsymbol{h}_k$ we finally arrive at the stationary equations [10]

$$\langle \sigma_k^+ \rangle = \langle c_{-k\downarrow} c_{k\uparrow} \rangle = \frac{-g\Delta^*}{2\sqrt{\xi_k^2 + |\Delta|^2}} \tanh\left\{\frac{\sqrt{\xi_k^2 + |\Delta|^2}}{2k_\text{B}T}\right\}, \tag{10}$$

$$\langle \sigma_k^z \rangle = 1 - \sum_\sigma \langle c_{k\sigma}^\dagger c_{k\sigma} \rangle = \frac{\xi_k}{\sqrt{\xi_k^2 + |\Delta|^2}} \tanh\left\{\frac{\sqrt{\xi_k^2 + |\Delta|^2}}{2k_\text{B}T}\right\}. \tag{11}$$

4 Dynamic fluctuations of the order parameters

In the symmetry broken state (for $T < T_\text{c}$) the two-component model (1) is characterized by two order parameters: $b(t)$ and another one of the fermion subsystem is defined as $\chi(T) = \sum_k \langle c_{-k\downarrow} c_{k\uparrow} \rangle$. These quantities are complex. In the stationary case they are proportional to each other as can be seen from Eq. (10). However, this relation is no longer valid when the Hamiltonian (6) depends on time. The evolution of the order parameters $b(t)$ and $\chi(t)$ with respect to time must be determined by solving the Heisenberg Eqs. (7) and (8) subject to some boundary conditions.

We analyze here such dynamics assuming that initially, for $t \le 0$, the system is at the Fesbach resonance (i.e. $\nu = \mu$). We assume the initial value of the boson order parameter to be $b(t < 0) = 1$ and determine the fermion order parameter $\chi(t \le 0)$ by solving Eq. (10). For simplicity we focus on the ground state and set the boson fermion coupling g as a unit or all the energies appearing in our study.

For time $t > 0$ we change the detuning parameter ν in the following ways: a) via the sudden detuning as previously discussed in Refs. [7−9] and b) through gradually increasing $\nu(t) - \mu \propto t$. Avoiding any constraint solutions we solved numerically the Heisenberg Eqs. (7) and (8) using the Runge−Kutta algorithm.

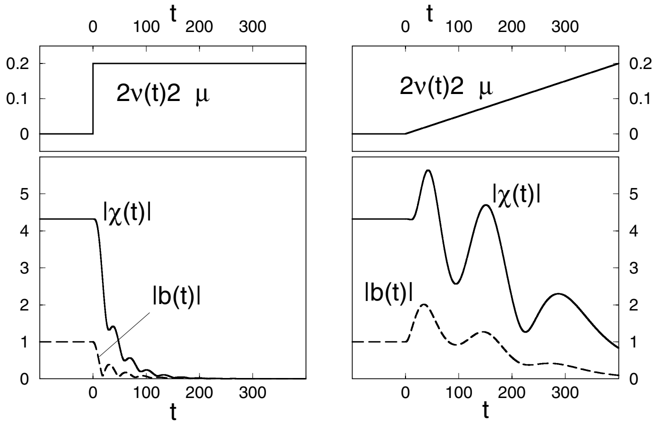

Fig. 1 Variation of the fermion $|\chi(t)|$ and boson $|b(t)|$ order parameters caused by the detuning from the Feshbach resonance. Initially, for $t < 0$, the system is in the stationary state with $\nu(t < 0) = \mu$ where the magnitudes of both order parameters are static. Upon lifting the molecule level there appear the damped oscillations. In the left h.s. panel we illustrate the case of an abrupt detuning $\nu(t > 0) = 0.1g$ and in the right h.s. a smooth detuning $\nu(t) \propto t$.

By increasing ν the boson fermion system is pushed to the far BCS regime. Choosing $\nu = 0.1g$ for $t \to \infty$ both the order parameters ought to decrease asymptotically down to negligibly small values. Figure 1 shows that such evolution occurs after several oscillations. In both cases the oscillations are clearly damped in agreement with the previous study by Burnet and coworkers [9]. However, the process of damping is sensitive to the particular profile of the time-dependent detuning. This can be seen from Fig. 1 and also in Fig. 2, where we plot the phase θ of the boson order parameter $b(t) = |b(t)|\, e^{i\theta(t)}$. For a smooth switching the oscillations do not appear regular at all.

5 Summary

We studied the dynamics of the ultracold fermion atoms upon the sudden and gradual detuning from the Feshbach resonance. Such situation can be experimentally realized by switching the external magnetic field from B_0 to the higher values. From the self-consistent numerical solution of the equations of motion we find that the order parameters start oscillating with the amplitude decaying in time. Such damped oscillations depend on the specific form in which the detuning $\nu(t)$ is carried out. The more specific explanation of the time-dependent fluctuations of both order parameters will be given in a forthcoming paper. In conclusion, quantum oscillations of the order parametrs turn out to be considerably damped even on the level of the single-mode approach (without scattering to the finite boson momenta).

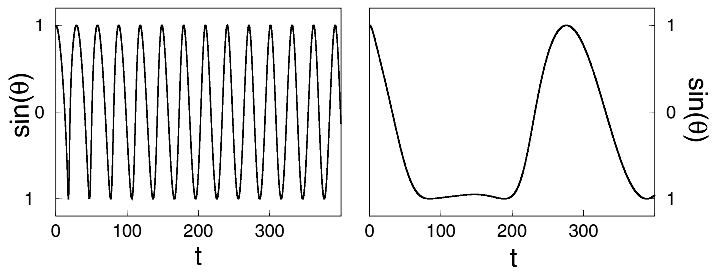

Fig. 2 Evolution of the phase θ of the boson order parameter $b(t)$ with respect to time t for an abrupt detuning (left) and for a smoothly increasing detuning $\nu(t) \propto t$ (right). Instead of the bare angle θ we plot the function $\sin\theta = \text{Im } b(t)/|b(t)|$.

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