Eur. Phys. J. B **74**, 437–445 (2010)
DOI: 10.1140/epjb/e2010-00100-0

THE EUROPEAN PHYSICAL JOURNAL B

Regular Article

Particle-hole mixing driven by the superconducting fluctuations

T. Domański[a]

Institute of Physics, M. Curie Skłodowska University, 20-031 Lublin, Poland

Received 14 August 2009 / Received in final form 17 December 2009
Published online 18 March 2010 –

Abstract. Development of the STM and ARPES spectroscopy enabled to reach the resolution sufficient for probing the particle-hole entanglement in superconducting materials, even above the critical temperature T_c. On a quantitative level one can characterize such entanglement in terms of the Bogoliubov angle which determines to what extent the particles and holes constitute the effective quasiparticles. In classical superconductors, where the phase transition is related to formation of the Cooper pairs almost simultaneously accompanied by onset of their long-range phase coherence, the Bogoliubov angle is slanted (due to finite particle-hole mixing) all the way up to T_c. In the high temperature superconductors and in superfluid ultracold fermion atoms near the Feshbach resonance the situation is different because the preformed pairs can exist above T_c albeit loosing coherence due to the strong quantum fluctuations. We discuss a generic temperature dependence of the Bogoliubov angle in such pseudogap state indicating a novel, non-BCS behavior. For analysis we use the two-component model describing the pairs coexisting with single fermions and study selfconsistently their feedback effects by the similarity transformation originating from the renormalization group approach.

1 Introduction

Physical systems such as the classical and/or high T_c cuprate superconductors, ultracold superfluid fermion atoms as well as certain cosmological (superfluid neutron stars) and subatomic objects (odd-odd nuclei) can under specific conditions show up appearance of the coherent pairs consisting of fermions from a vicinity of their Fermi surface. What differs one case from another is an underlying mechanism and the energy scale involved in the pairing. They all however share the universal feature of the effective Bogoliubov quasiparticles which represent a superposition of the particles and their absence. It is particularly intriguing that the recent spectroscopic data obtained for the cuprate superconductors [1,2] provide evidence for such particle-hole mixing even above the transition temperature T_c.

The particle-hole (p-h) mixing has a purely quantum nature, to some extent resembling the corpuscular-wave dualism [3]. One of its spectacular manifestations is for instance the mechanism of Andreev reflection in which an incident fermion-particle is converted into Cooper pair with a simultaneous reflection of the fermion-hole. Such particle-hole conversion processes are indeed observed experimentally in the isotropic and/or anisotropic superconductors [4,5], for the relativistic-like fermions [6] or in the nanoscopic systems with quantum dots attached to the superconducting electrodes [7–9].

[a] e-mail: doman@kft.umcs.lublin.pl

On a qualitative level the p-h mixing can be indirectly determined by the STM [3] and ARPES spectroscopy [10] designed to probe either the spatially [11] or the momentum resolved [12] single particle excitation spectra of superconductors. The recent developments allow also for a simultaneous **r**- and **k**-space measurement using the Fourier transformed quasiparticle interference imaging [13]. Roughly speaking, the p-h mixing manifests itself by appearance of two peaks separated around the Fermi level by twice the value of (pseudo)gap and whose spectral weights correspond to the particle/hole contributions to the effective Bogoliubov quasiparticles. In conventional superconductors these contributions are given by the BCS coefficients $u^2_{\mathbf{k}}$ and $v^2_{\mathbf{k}} = 1 - u^2_{\mathbf{k}}$. It is hence convenient to define the, so called, Bogoliubov angle [14]

$$\theta_{\mathbf{k}} = -\frac{\pi}{2} + 2 \arctan\left(\frac{|u_{\mathbf{k}}|}{|v_{\mathbf{k}}|}\right) \tag{1}$$

as a measure of the p-h mixing. Its magnitude can vary between $-\pi/2$ and $\pi/2$ depending on momentum $\mathbf{k}$ and indirectly on temperature. In mathematical terms, $\theta_{\mathbf{k}}$ corresponds to azimuthal angle of the vector $\langle \hat{\mathbf{s}}_{\mathbf{k}} \rangle = \frac{1}{N}\sum_{\mathbf{r}_i} e^{-i\mathbf{k}\cdot\mathbf{r}_i} \langle \hat{\mathbf{s}}(\mathbf{r}_i)\rangle$, where $\hat{\mathbf{s}}(\mathbf{r}_i)$ is the pseudospin operator introduced by Anderson [15] (see Appendix A for details). Restricting to non-magnetic solutions $\langle \hat{c}^\dagger_{\mathbf{k}\uparrow}\hat{c}_{\mathbf{k}\uparrow}\rangle = \langle \hat{c}^\dagger_{-\mathbf{k}\downarrow}\hat{c}_{-\mathbf{k}\downarrow}\rangle$ the pseudospin eventually points down (up) when effective quasiparticles are represented by particles (holes). The upper and bottom panels of Figure 1

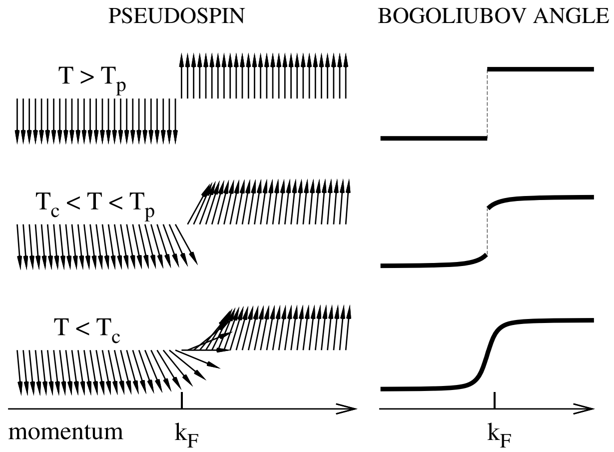

Fig. 1. Variation of the Anderson's pseudospin (the left h.s. column) and the Bogoliubov angle $\theta_{\mathbf{k}}$ (the right h.s. column) against momentum in the normal, pseudogap and superconducting states. Notice that particle-hole mixing is present in the superconducting and pseudogap states, however above T_c the Bogoliubov angle becomes discontinuous at $\mathbf{k}_F$.

illustrate such behavior well known for the normal and superconducting states [15]. In general, the pseudospins $\langle \hat{\mathbf{s}}_{\mathbf{k}} \rangle$ are also governed by a non-trivial dynamics through the Bloch-type equations of motion [15]. Such aspect becomes of a practical importance for the ultracold atoms where time-controlled sweeps through the Feshbach resonance trigger the soliton-like solutions [16,17].

In the high T_c cuprate superconductors the pairing occurs between itinerant holes from the nearest neighbor lattice sites [11]. The Bogoliubov angle is there defined rather locally (see Appendix A). For studying the intrinsic inhomogeneities leading to the amplitude fluctuations of the order parameter (which strongly depend on the doping of charge carriers) one can use the mean-field Bogoliubov de Gennes equations as has been done by Balatsky with coworkers [3]. However, the other very important problem concerns the pseudogap state where the coherence of fermion pairs is restricted to only short spatial and temporal scales. The superconducting state of cuprates is known to obey nearly the BCS-type behavior [18] but it is not yet agreed whether the whole pseudogap regime $T_c < T < T^*$ does or does not correspond to the superconducting fluctuations [19–21]. Nonetheless a number of recent experiments unambiguously indicate that the phase-incoherent pairs exist at temperatures at least up to dozen Kelvin above T_c [1,2,22–26]. The theoretical studies of such precursor pairing conducted independently by several groups pointed out that, remnants of the BCS features can be preserved above T_c [27,28]. Our former analysis [29,30] has also indicated such a possibility. In the present continuation we explore in some more detail the particle and hole contributions and hope that such study could be of real interest for experimentalists.

To analyze the influence of preformed pairs on the Bogoliubov angle we use a phenomenological two-component model where itinerant fermions and their paired counterparts are introduced without referring to any specific microscopic mechanism. The selfconsistent non-perturbative treatment of the paired and single fermions permits us to conclude that the effective p-h mixing, signified by $|\theta_{\mathbf{k}}| \neq \pi/2$, takes place both below and above T_c. In the latter case, absence of the phase coherence causes a characteristic discontinuity of $\theta_{\mathbf{k}}$ at $\mathbf{k}_F$. We thus find that in the pseudogap state the Bogoliubov angle behaves in a manner partly reminiscent of the normal and partly of the superconducting phases (see the middle panel in Fig. 1). We hope that such result can be soon confirmed by other theoretical methods and would be eventually verified/invalidated experimentally. Besides the cuprates the similar effects could also play a role above T_c in the ultracold atoms of Li^6 and K^{40} where near the Feshbach resonance the shallow bound boson molecules are strongly scattered into the large Cooper-like pairs [31].

In the next section we briefly introduce the model and discuss its main properties. Some methodological details are outlined in Section 3 and the essential part concerning the p-h mixing of the pseudogap state is described in Section 4. We finally summarize our results and point out some related problems.

2 Phenomenological model

For description of the superconducting and pseudogap states we shall use the following Hamiltonian [32]

$$\hat{H} = \sum_{\mathbf{k},\sigma} (\varepsilon_{\mathbf{k}} - \mu)\, \hat{c}^\dagger_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} (E_{\mathbf{q}} - 2\mu)\, \hat{b}^\dagger_{\mathbf{q}} \hat{b}_{\mathbf{q}} + \frac{1}{\sqrt{N}} \sum_{\mathbf{k},\mathbf{q}} \left(g_{\mathbf{k},\mathbf{q}} \hat{b}^\dagger_{\mathbf{q}} \hat{c}_{\mathbf{q}-\mathbf{k}\downarrow} \hat{c}_{\mathbf{k}\uparrow} + g^*_{\mathbf{k},\mathbf{q}} \hat{c}^\dagger_{\mathbf{k}\uparrow} \hat{c}^\dagger_{\mathbf{q}-\mathbf{k}\downarrow} \hat{b}_{\mathbf{q}} \right), \quad (2)$$

where operators $\hat{c}^{(\dagger)}_{\mathbf{k}\sigma}$ refer to annihilation (creation) of single fermions with energy $\varepsilon_{\mathbf{k}}$ and $\hat{b}^{(\dagger)}_{\mathbf{q}}$ correspond to the local pairs of the corresponding energy $E_{\mathbf{q}}$. Interactions between the single and paired fermions are scaled by the potential $g_{\mathbf{k},\mathbf{q}}$. For simplicity, we assume that concentration of the local pairs per lattice site is small enough so that $\hat{b}^{(\dagger)}_{\mathbf{q}}$ can be treated as the usual bosonic operators (we neglect the hard-core constraint). The boson-fermion model (2) has been introduced by Ranninger [32] and has been intensively studied by several groups [33–35]. Independently of the microscopic scenarios (for instance using the Hubbard model) some authors [36–39] agreeably concluded that essential physics of the strongly correlated cuprates is well captured by the interdependent fermion and boson degrees of freedom given in the Hamiltonian (2). This model turned out to be also useful for description of the Feshbach resonance widely used in the systems of ultracold fermion atoms [28,31].

In the simplest mean-field treatment one can linearize the interaction term so that the decoupled boson and fermion parts become exactly solvable [32]. The fermion spectrum acquires then the BCS structure $A^{MF}(\mathbf{k},\omega) = u^2_{\mathbf{k}}\ \delta(\omega - \xi_{\mathbf{k}}) + v^2_{\mathbf{k}} \delta(\omega + \xi_{\mathbf{k}})$ with the usual quasiparticle energy $\xi_{\mathbf{k}} = \sqrt{(\varepsilon_{\mathbf{k}}-\mu)^2 + \Delta_{\mathbf{k}}^2}$ and the coherence factors $u_{\mathbf{k}}^2, v_{\mathbf{k}}^2 = \frac{1}{2}[1 \pm (\varepsilon_{\mathbf{k}} - \mu)/\xi_{\mathbf{k}}]$ which yield the standard Bogoliubov angle. The energy gap of single particle excitation spectrum is effectively given by $\Delta_{\mathbf{k}} = g_{\mathbf{k},\mathbf{0}}\sqrt{\langle n_{\mathbf{0}}^B\rangle}$ which means that fermions undergo transition to the superconducting state if and only if the Bose-Einstein condensation of bosons takes place [32]. This property is valid exactly [40,41] even beyond the mean field approximation.

The mean-field treatment is not capable to take into account the quantum fluctuations which in turn become more and more efficient upon approaching T_c and above of it. In the next section we shall introduce the selfconsistent method suitable for studying the boson-fermion model in the scheme originating from the renormalization group technique. We shall apply it to determine the superconducting correlations preserved above T_c and obtain the Bogoliubov angle.

3 The procedure

We use the selfconsistent, non-perturbative procedure based on the continuous canonical transformation $\hat{H} \longrightarrow e^{\hat{S}(l)}\hat{H}e^{-\hat{S}(l)}$ [42,43]. Our main goal is to eliminate the interaction part $g_{\mathbf{k},\mathbf{q}}$ by a sequence of infinitesimal steps $l \rightarrow l+\delta l$. In analogy to the Renormalization Group (RG) technique one starts from renormalizing the high energy sector and subsequently turns to the low energy sector (in the present case this corresponds to the fermion states located near μ and the boson states close to 2μ). Below we highlight some technicalities clarifying how the particle and hole contributions can be evaluated within this procedure.

Initially we start by setting $\hat{H}(l) \equiv e^{\hat{S}(l)}\hat{H}e^{-\hat{S}(l)}$, where $\hat{H}(0)$ corresponds to the initial Hamiltonian, and construct the flow equation $\partial_l \hat{H}(l) = [\hat{\eta}(l), \hat{H}(l)]$ with the generating operator $\hat{\eta}(l) \equiv \partial_l \hat{S}(l)$. Following the original proposal of Wegner [42,43] we choose $\hat{\eta}(l) = [\hat{H}_0(l), \hat{H}_{int}(l)]$, where $\hat{H}_0(l)$ denotes the total kinetic energy of fermions and bosons and $\hat{H}_{int}(l)$ stands for their interaction. From this commutator we obtain $\hat{\eta}(l) = -\frac{1}{\sqrt{N}}\sum_{\mathbf{k},\mathbf{q}} \alpha_{\mathbf{k},\mathbf{q}}(l)\left(b_{\mathbf{q}}^\dagger c_{\mathbf{q}-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} - \text{h.c.}\right)$ with $\alpha_{\mathbf{k},\mathbf{q}}(l) = (\varepsilon_{\mathbf{k}}(l) + \varepsilon_{\mathbf{q}-\mathbf{k}}(l) - E_{\mathbf{q}}(l))\, g_{\mathbf{k},\mathbf{q}}(l)$. It has been previously shown [44] that such antihermitean operator $\hat{\eta}(l)$ indeed guarantees the asymptotic disappearance of the boson-fermion coupling $\lim_{l\rightarrow\infty} g_{\mathbf{k},\mathbf{q}}(l) = 0$.

During this continuous transformation the boson-fermion Hamiltonian (2) evolves according to the following set of coupled flow equations $\partial_l g_{\mathbf{k},\mathbf{q}}(l) = -\alpha_{\mathbf{k},\mathbf{q}}^2(l) g_{\mathbf{k},\mathbf{q}}(l)$, $\partial_l \varepsilon_{\mathbf{k}}(l) = \frac{2}{N}\sum_{\mathbf{q}} \alpha_{\mathbf{k},\mathbf{q}}(l)|g_{\mathbf{k},\mathbf{q}}(l)|^2 n_{\mathbf{q}}^{(B)}$ and $\partial_l E_{\mathbf{q}}(l) = \frac{2}{N}\sum_{\mathbf{k}} \alpha_{\mathbf{k}-\mathbf{q},\mathbf{k}}(l)|g_{\mathbf{k}-\mathbf{q},\mathbf{k}}(l)|^2 \left(-1 + n_{\mathbf{k}-\mathbf{q}\downarrow}^{(F)} + n_{\mathbf{k}\uparrow}^{(F)}\right)$ [44]. We have solved them numerically considering itinerant fermions and local bosons placed on the two-dimensional lattice avoiding any need for the infrared cutoffs. The fixed point values

$$\lim_{l\rightarrow\infty} \varepsilon_{\mathbf{k}}(l) \equiv \tilde{\varepsilon}_{\mathbf{k}}, \qquad \lim_{l\rightarrow\infty} E_{\mathbf{q}}(l) \equiv \tilde{E}_{\mathbf{q}} \tag{3}$$

have shown the following features: (a) for $T<T_c$ the renormalized fermion dispersion $\tilde{\varepsilon}_{\mathbf{k}}$ develops a true gap at μ which evolves into a pseudogap for $T_c<T<T_p$, (b) the effective boson dispersion $\tilde{E}_{\mathbf{q}}$ shows the long-wavelength Goldstone mode for $T<T_c$ and its remnants are preserved in the pseudogap state [29,30].

To obtain the needed information about the fermion and boson spectra we have to construct the continuous transformations also for the individual operators $\hat{c}_{\mathbf{k}\sigma}^{(\dagger)}(l) \equiv e^{\hat{S}(l)}\hat{c}_{\mathbf{k}\sigma}^{(\dagger)}e^{-\hat{S}(l)}$ and $\hat{b}_{\mathbf{q}}^{(\dagger)}(l) \equiv e^{\hat{S}(l)}\hat{b}_{\mathbf{q}}^{(\dagger)}e^{-\hat{S}(l)}$. This is a bit difficult task because the operator $\hat{S}(l)$ is not known explicitly. Since we are interested here mainly in the particle-hole mixing of the single particle excitations we can restrict to the fermion operators $\partial_l \hat{c}_{\mathbf{k}\sigma}^{(\dagger)}(l) = [\hat{\eta}, \hat{c}_{\mathbf{k}\sigma}^{(\dagger)}(l)]$. The generating operator $\hat{\eta}(l)$ chosen according to Wegner's prescription [42,43] yields the following ansatz [29,30]

$$\begin{aligned} c_{\mathbf{k}\uparrow}(l) &= u_{\mathbf{k}}(l)\; c_{\mathbf{k}\uparrow} + v_{\mathbf{k}}(l)\; c_{-\mathbf{k}\downarrow}^\dagger \\ &+ \frac{1}{\sqrt{N}}\sum_{\mathbf{q}\neq\mathbf{0}} \left[u_{\mathbf{k},\mathbf{q}}(l)\; b_{\mathbf{q}}^\dagger c_{\mathbf{q}+\mathbf{k}\uparrow} + v_{\mathbf{k},\mathbf{q}}(l)\; b_{\mathbf{q}} c_{\mathbf{q}-\mathbf{k}\downarrow}^\dagger\right], \end{aligned} \tag{4}$$

$$\begin{aligned} c_{-\mathbf{k}\downarrow}^\dagger(l) &= -v_{\mathbf{k}}^*(l)\; c_{\mathbf{k}\uparrow} + u_{\mathbf{k}}^*(l)\; c_{-\mathbf{k}\downarrow}^\dagger \\ &+ \frac{1}{\sqrt{N}}\sum_{\mathbf{q}\neq\mathbf{0}} \left[-v_{\mathbf{k},\mathbf{q}}^*(l)\; b_{\mathbf{q}}^\dagger c_{\mathbf{q}+\mathbf{k}\uparrow} + u_{\mathbf{k},\mathbf{q}}^*(l)\; b_{\mathbf{q}} c_{\mathbf{q}-\mathbf{k}\downarrow}^\dagger\right], \end{aligned} \tag{5}$$

where $u_{\mathbf{k}}(0) = 1$ and all other coefficients initially vanishing at $l=0$. The parametrizations imposed in (4,5) represent the lowest order estimations (beyond the mean field level) which approximately satisfy the formal flow equations for the operators $\partial_l \hat{c}_{\mathbf{k}\sigma}^{(\dagger)}(l) = [\hat{\eta}, \hat{c}_{\mathbf{k}\sigma}^{(\dagger)}(l)]$. In order to satisfy them exactly one should supplement (4, 5) by an infinite set of the higher order terms, but obviously their analysis would become intractable.

Restricting to the lowest order ansatz (4, 5) beyond the mean-field solution we obtain the following differential equations for the appearing l-dependent coefficients [29,30]

$$\begin{aligned} \partial_l u_{\mathbf{k}}(l) &= \sqrt{n_{\mathbf{q}=\mathbf{0}}^B}\; \alpha_{-\mathbf{k},\mathbf{0}}(l)\; v_{\mathbf{k}}(l) \\ &+ \frac{1}{N}\sum_{\mathbf{q}\neq\mathbf{0}} \alpha_{\mathbf{q}-\mathbf{k},\mathbf{q}}(l)\left(n_{\mathbf{q}}^B + n_{\mathbf{q}-\mathbf{k}\downarrow}^F\right) v_{\mathbf{k},\mathbf{q}}(l), \end{aligned} \tag{6}$$

$$\begin{aligned} \partial_l v_{\mathbf{k}}(l) &= -\sqrt{n_{\mathbf{q}=\mathbf{0}}^B}\; \alpha_{\mathbf{k},\mathbf{0}}(l)\; u_{\mathbf{k}}(l) \\ &- \frac{1}{N}\sum_{\mathbf{q}\neq\mathbf{0}} \alpha_{\mathbf{k},\mathbf{q}}(l)\left(n_{\mathbf{q}}^B + n_{\mathbf{q}+\mathbf{k}\uparrow}^F\right) u_{\mathbf{k},\mathbf{q}}(l), \end{aligned} \tag{7}$$

$$\partial_l u_{\mathbf{k},\mathbf{q}} = \alpha_{-\mathbf{k},\mathbf{q}}(l)\; v_{\mathbf{k}}(l), \tag{8}$$

$$\partial_l v_{\mathbf{k},\mathbf{q}} = -\,\alpha_{\mathbf{k},\mathbf{q}}(l) u_{\mathbf{k}}(l). \tag{9}$$

We have explored them numerically along with the equations $\partial_l \varepsilon_{\mathbf{k}}(l)$, $\partial_l E_{\mathbf{q}}(l)$, $\partial_l g_{\mathbf{k},\mathbf{q}}(l)$ setting the initial $(l=0)$ tight-binding dispersion $\varepsilon_{\mathbf{k}}(0) = -2t\,(\cos(k_x a) + \cos(k_y a))$ and $E_{\mathbf{q}}(0) = E_0$. To reproduce d-wave symmetry of the energy gap in the superconducting state we have imposed the prefactor $g_{\mathbf{k},\mathbf{q}}(0)=g\left(\cos(k_x a)-\cos(k_y a)\right)$ and next solved the coupled flow equations iteratively by the Runge-Kutta algorithm. We have chosen $E_0(0) = 0.2t$ whereas the concentration of charge carriers $n_{tot} = \sum_{\mathbf{k}}\left(n^F_{\mathbf{k}\uparrow}+n^F_{\mathbf{k}\downarrow}\right) + 2\sum_{\mathbf{q}} n^B_{\mathbf{q}}$ was fixed at $n_{tot}=2$ in order to reproduce the hole concentration $x\equiv 1-n^F \sim 0.1$ relevant for the underdoped regime. In Figures 2–4 we illustrate the results obtained for several temperatures along the antinodal direction $(\pi,\pi)\leftrightarrow(\pi,0)$ for $g=0.1D$ ($D\equiv 8t$ is used as the unit for energies).

Our ansatz (4, 5) generalizes the standard Bogoliubov-Valatin transformation by including the scattering on finite momentum pairs. The single particle spectral function is given by

$$
\begin{aligned}
A(\mathbf{k},\omega) &= |\tilde{u}_{\mathbf{k}}|^2\delta\left(\omega+\mu-\tilde{\varepsilon}_{\mathbf{k}}\right) \\
&\quad + \frac{1}{N}\sum_{\mathbf{q}\neq\mathbf{0}}\left(n^B_{\mathbf{q}}+n^F_{\mathbf{q}+\mathbf{k}\uparrow}\right)|\tilde{u}_{\mathbf{k},\mathbf{q}}|^2\delta(\omega+\mu-\tilde{\varepsilon}_{\mathbf{q}+\mathbf{k}}+\tilde{E}_{\mathbf{q}}) \\
&\quad + |\tilde{v}_{\mathbf{k}}|^2\delta\left(\omega-\mu+\tilde{\varepsilon}_{-\mathbf{k}}\right) + \frac{1}{N}\sum_{\mathbf{q}\neq\mathbf{0}}\left(n^B_{\mathbf{q}}+n^F_{\mathbf{q}-\mathbf{k}\downarrow}\right) \\
&\quad \times|\tilde{v}_{\mathbf{k},\mathbf{q}}|^2\delta(\omega-\mu+\tilde{\varepsilon}_{\mathbf{q}-\mathbf{k}}-\tilde{E}_{\mathbf{q}}), \qquad (10)
\end{aligned}
$$

where $\tilde{u}_{\mathbf{k}}$, $\tilde{v}_{\mathbf{k}}$ and $\tilde{u}_{\mathbf{k},\mathbf{q}}$, $\tilde{v}_{\mathbf{k},\mathbf{q}}$ denote the asymptotic $l\to\infty$ values. The overall structure (10) indicates that besides the long-lived states (the delta peaks) there is additionally formed a background of the damped (finite life-time) states.

To justify a correspondence of this study with the traditional mean field solution let us suppose that the terms $u_{\mathbf{k},\mathbf{q}}$ and $v_{\mathbf{k},\mathbf{q}}$ were absent in (4, 5). In such situation the flow equations (6,7) would simplify to $\partial_l u_{\mathbf{k}}(l) = \sqrt{n^B_{\mathbf{q}=\mathbf{0}}}\;\alpha_{-\mathbf{k},\mathbf{0}}(l)v_{\mathbf{k}}(l)$ and $\partial_l v_{\mathbf{k}}(l) = -\sqrt{n^B_{\mathbf{q}=\mathbf{0}}}\;\alpha_{\mathbf{k},\mathbf{0}}(l)u_{\mathbf{k}}(l)$ yielding the invariance $|v_{\mathbf{k}}(l)|^2+|v_{\mathbf{k}}(l)|^2 = 1$. Rewriting the first equation as $\int_{u_{\mathbf{k}}(0)=1}^{u_{\mathbf{k}}(\infty)=\tilde{u}_{\mathbf{k}}}\frac{du_{\mathbf{k}}(l)}{\sqrt{1-|u_{\mathbf{k}}(l)|^2}} = \sqrt{n^B_{\mathbf{q}=\mathbf{0}}}\;\int_0^\infty \alpha_{-\mathbf{k},\mathbf{0}}(l)dl$ we immediately reproduce the mean-field result $\tilde{u}^2_{\mathbf{k}},\tilde{v}^2_{\mathbf{k}} = \frac{1}{2}\left(1\pm\frac{\varepsilon_{\mathbf{k}}-\mu}{\sqrt{(\varepsilon_{\mathbf{k}}-\mu)^2+n^B_{\mathbf{0}}|g_{\mathbf{k},\mathbf{0}}|^2}}\right)$. In the next section we shall investigate the spectral function (10) taking into account the scattering on finite momentum pairs.

4 Particle-hole mixing above $\mathbf{T_c}$

Any preformed pairs can exist in the normal state only at finite momenta, in other words $\langle\hat{b}_{\mathbf{q}=\mathbf{0}}\rangle = 0$. From the differential equations (7,8) above T_c we infer that $v_{\mathbf{k}}(l)=0$ and $u_{\mathbf{k},\mathbf{q}}(l) = 0$ so, in consequence, the ansatz (4,5) is simplified to

$$
\hat{c}_{\mathbf{k}\uparrow}(l) = u_{\mathbf{k}}(l)\;\hat{c}_{\mathbf{k}\uparrow} \;+\; \frac{1}{\sqrt{N}}\sum_{\mathbf{q}\neq\mathbf{0}} v_{\mathbf{k},\mathbf{q}}(l)\;\hat{b}_{\mathbf{q}}\hat{c}^\dagger_{\mathbf{q}-\mathbf{k}\downarrow} \qquad (11)
$$

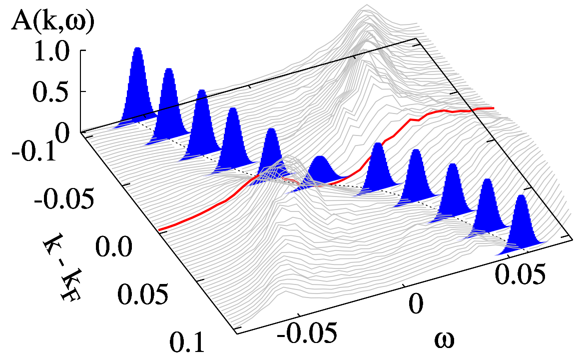

Fig. 2. (Color online) The excitation spectrum of fermions in the pseudogap regime. The narrow quasiparticle peaks (artificially broadened here by the filled Gaussians of the total weight $\tilde{u}_{\mathbf{k}}$) of energy renormalized to $\omega=\tilde{\varepsilon}_{\mathbf{k}}-\mu$ are accompanied by appearance of their mirror reflections representing the Bogoliubov shadow branch which has been indeed observed in the recent ARPES experiments [1,2]. Both branches ultimately merge into a single one at temperatures far above T_c. Close to k_F the quasiparticle peak moves inside the pseudogap however it simultaneously becomes overdamped and its spectral weight undergoes a considerable suppression.

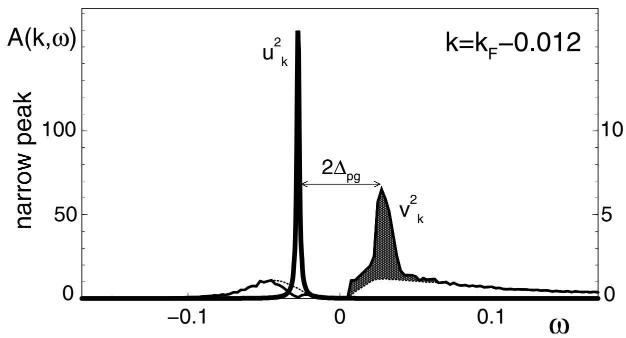

Fig. 3. The spectral function $A(\mathbf{k},\omega)$ consisting of the long-lived states (we have artificially broadened the delta peak using the units marked on the left axis) and the damped fermion states (labels on the right h.s. axis) slightly below $\mathbf{k}_F$ for $k_BT = 0.004D$. Spectral weight of the particle peak is $|u_{\mathbf{k}}|^2 \simeq 0.47$ whereas the total weight of the hole branch formed around $\omega = -(\tilde{\varepsilon}_{\mathbf{k}}-\mu)\simeq\Delta_{pg}$ is estimated to $|v_{\mathbf{k}}|^2\simeq 0.19$ by subtracting the high temperature background (the shaded area).

$$
\hat{c}^\dagger_{-\mathbf{k}\downarrow}(l) = -\frac{1}{\sqrt{N}}\sum_{\mathbf{q}\neq\mathbf{0}} v^*_{\mathbf{k},\mathbf{q}}(l)\;\hat{b}^\dagger_{\mathbf{q}}\hat{c}_{\mathbf{q}+\mathbf{k}\uparrow} \;+\; u^*_{\mathbf{k}}(l)\;\hat{c}^\dagger_{-\mathbf{k}\downarrow}. \qquad (12)
$$

Under such conditions the resulting spectral function takes the following form

$$
\begin{aligned}
A(\mathbf{k},\omega) &= |\tilde{u}_{\mathbf{k}}|^2\delta\left(\omega+\mu-\tilde{\varepsilon}_{\mathbf{k}}\right) + \frac{1}{N} \\
&\quad\times\sum_{\mathbf{q}\neq\mathbf{0}}\left(n^B_{\mathbf{q}}+n^F_{\mathbf{q}-\mathbf{k}\downarrow}\right)|\tilde{v}_{\mathbf{k},\mathbf{q}}|^2\delta(\omega-\mu+\tilde{\varepsilon}_{\mathbf{q}-\mathbf{k}}-\tilde{E}_{\mathbf{q}}). \qquad (13)
\end{aligned}
$$

It consists of the delta peak at the renormalized energy $\tilde{\varepsilon}_{\mathbf{k}}-\mu$ whose spectral weight $|\tilde{u}_{\mathbf{k}}|^2 < 1$ (like in superconductors). The remaining spectral weight is

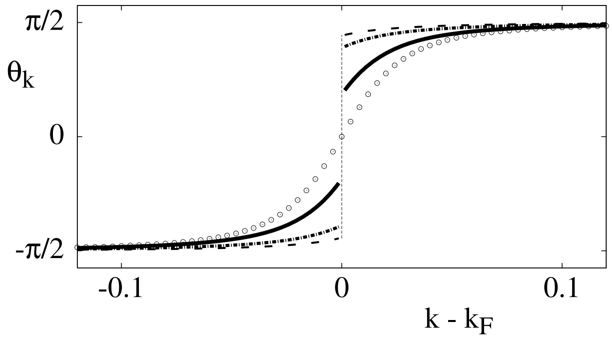

Fig. 4. Variation of the Bogoliubov angle obtained in the pseudogap regime for temperatures $k_BT = 0.004$ (the solid line), 0.007 (the short-dashed curve) and 0.012 (the long-dashed line). For comparison we plot by open circles the characteristics of the superconducting state at $T=0$.

distributed among the finite life-time states given by the second term in (13).

Figure 2 shows the representative results obtained above T_c for $k_BT = 0.007D$. Damped states are spread over the large energy regime and most of them are insensitive to temperature except the certain fraction (very important to us) accumulating around $\omega = -(\tilde{\varepsilon}_{\mathbf{k}} - \mu)$. This broadened excitation branch, being a mirror reflection of the quasiparticle dispersion $\tilde{\varepsilon}_{\mathbf{k}} - \mu$, corresponds to the hole (particle) contribution of the Bogoliubov quasiparticles for momenta below (above) $\mathbf{k}_F$. Its total spectral weight completes the needed information about the Bogoliubov angle (1) for the pseudogap regime (the technical procedure used for estimating the particle and hole weights is illustrated in Fig. 3). Signatures of such quasiparticle branches appearing above T_c at $\omega = \pm(\tilde{\varepsilon}_{\mathbf{k}} - \mu)$ have been recently detected experimentally by ARPES measurements for $Bi_2Sr_2CaCu_2O_8$ [1] and $La_{1.895}Sr_{0.105}CuO_4$ [2] compounds. These facts unambiguously confirm that the Bogoliubov type quasiparticles (along with the p-h mixing) survive above T_c even-though the off-diagonal-long-range-order is completely lost.

Strictly speaking, we find that at $T > T_c$ the quasiparticle dispersion $\tilde{\varepsilon}_{\mathbf{k}}$ evolves into an S-like shape near the chemical potential μ [44]. Absence of its sharp discontinuity is directly related to disappearance of the Bose Einstein condensed pairs (physically such loss of the superfluid fraction leads to non-vanishing resistance above T_c). Nevertheless, the low energy states are still considerably depleted because of a strong suppression of the spectral weight $|\tilde{u}_{\mathbf{k}}|^2$ [29,30]. In a narrow momentum regime around k_F we thus observe the in-gap state which at first glance seems to be in a contradiction with the ARPES experimental data. This is however not the case. Such peak must exist (for all temperatures above T_c) as has been recently emphasized by Senthil and Lee [45,46] in their 'synthesis of the phenomenology of the underdoped cuprates'. In-gap states result from the interference of the paired fermions (which above T_c represent only the diffusive modes) with the unpaired fermions (into which they decay). In particular, such states are expected [45–47] to be responsible for the magnetooscillations observed experimentally by Doiron-Leyraud et al. [48]. We shall discuss this issue separately in a future paper, let us only comment here that in-gap states are heavily overdamped so ARPES measurements [1,2] apparently could not resolve them. In analysis of the p-h mixing we have determined the Bogoliubov angle over momenta distant from k_F where only two excitation branches are present, and we then extrapolated our data numerically for the region $\tilde{\varepsilon}_{\mathbf{k}} \sim \mu$.

Figure 4 shows the Bogoliubov angle as a function of momentum calculated along the antinodal direction $(\pi, 0) \longrightarrow (\pi, \pi)$ for several temperatures. Presence of the shadow Bogoliubov branch above T_c yields the non-vanishing p-h mixing which finally fades away for higher temperatures when the pseudogap closes. We moreover notice that upon approaching $\mathbf{k}_F$ the Bogoliubov angle is discontinuous. Physically it means that particles and holes not equally participate in the effective Bogoliubov quasiparticles, even near the Fermi surface. Similar suggestion has been also previously stressed within the RVB theory (see Eq. (2) in Ref. [49]). In the present study we observe that the BCS-type behavior is finally recovered at temperatures $T \leq T_c$ as shown by the open circles in Figure 4. Since the magnitude of superconducting gap does not much vary below T_c [50] the Bogoliubov angle is there practically frozen, i.e. temperature-independent.

5 Concluding remarks

We have analyzed the superconducting fluctuations [51,52] above the transition temperature T_c for the system where itinerant fermions coexist and interact with the preformed pairs. Interaction between the paired and single fermions has been studied within the selfconsistent similarity transformation [42,43]. We have found that the single particle excitation spectrum is depleted near the chemical potential over the energy region $|\omega| \leq \Delta_{pg}$. Additionally there emerge the Bogoliubov-type branches which signify the particle-hole mixing above T_c. We have numerically estimated such particle and hole spectral weights determining the Bogoliubov angle $\theta_{\mathbf{k}}$ in the pseudogap regime.

We have found that momentum dependence of the Bogoliubov angle of the pseudogap state differs qualitatively from its behavior known for the normal and superconducting phases. In the normal state (where no particle-hole mixing exists) $\theta_{\mathbf{k}}$ changes abruptly at $\mathbf{k}_F$ from $-\pi/2$ to $\pi/2$. On the other hand in the superconducting state below T_c the Bogoliubov angle smoothly evolves between these extreme values over the energy regime $|\omega| \leq \Delta_{sc}$ where the particle and hole excitations are mixed with one another. Our present study shows that in the pseudogap regime $|\theta_{\mathbf{k}}| \neq \pi/2$ (what is similar to the superconducting state) but at the Fermi surface the Bogoliubov angle is discontinuous (like in the normal state). We hope that STM and ARPES techniques could verify whether such unconventional relation between the particle and hole weights does really occur in the systems with strong superconducting fluctuations.

Author acknowledges discussions with J. Ranninger, R. Micnas and F. Wegner. This work is partly supported by the Ministry of Science and Education under the grant NN202187833.

Appendix A: Concept of the BA angle in pseudogap state

Following Anderson [15] let us introduce the local operators

$$\hat{s}_+(\mathbf{r}_i,t) = \hat{c}_\downarrow(\mathbf{r}_i,t)\;\hat{c}_\uparrow(\mathbf{r}_i,t), \tag{A.1}$$

$$\hat{s}_-(\mathbf{r}_i,t) = \hat{c}^\dagger_\uparrow(\mathbf{r}_i,t)\;\hat{c}^\dagger_\downarrow(\mathbf{r}_i,t), \tag{A.2}$$

$$\hat{s}_z(\mathbf{r}_i,t) = \frac{1}{2} - \frac{1}{2}\sum_{\sigma=\uparrow,\downarrow} \hat{c}^\dagger_\sigma(\mathbf{r}_i,t)\;\hat{c}_\sigma(\mathbf{r}_i,t), \tag{A.3}$$

which obey the spin-$\frac{1}{2}$ algebra where the corresponding $x(y)$ components are defined by $\hat{s}_{x(y)}(\mathbf{r}_i,t) = \frac{1}{2(i)}\left(\hat{s}_+(\mathbf{r}_i,t)+(-)\hat{s}_-(\mathbf{r}_i,t)\right)$. In the superconducting and pseudogap states the Bogoliubov angle $\theta(\mathbf{r}_i,t)$ refers to azimuthal orientation of the statistically averaged vector-operator $\mathbf{s}(\mathbf{r}_i,t) \equiv \langle \hat{\mathbf{s}}(\mathbf{r}_i,t)\rangle$ located on the Bloch sphere. The other planar angle $\phi(\mathbf{r}_i,t)$ of the spherical coordinates corresponds to phase of the complex order parameter

$$\langle \hat{c}^\dagger_\uparrow(\mathbf{r}_i,t)\hat{c}^\dagger_\downarrow(\mathbf{r}_i,t)\rangle \equiv \chi(\mathbf{r}_i,t)\; e^{i\phi(\mathbf{r}_i,t)}. \tag{A.4}$$

In physical systems where pairing occurs in the real space (which is the case in cuprates) both the azimuthal $\theta(\mathbf{r}_i,t)$ and planar angles $\phi(\mathbf{r}_i,t)$ can fluctuate over some characteristic spatial and temporal scales. Intrinsic inhomogeneities might furthermore generate also the amplitude $\chi(\mathbf{r}_i,t)$ fluctuations.

In general, the Bogoliubov angle $\theta(\mathbf{r}_i,t)$ is convoluted with the amplitude $\chi(\mathbf{r}_i,t)$ and phase $\phi(\mathbf{r}_i,t)$ of the local order parameter (A.4). Effect of the amplitude fluctuations has been already investigated within the single [3] as well as two-component pairing models [53] by means of the Bogoliubov de Gennes equations solved on the finite size lattice clusters.

As concerns the planar angle its fluctuations depend on the superfluid stiffness hence they can be expected to play a role in the underdoped regime of HTSC materials. To analyze the phase fluctuations and study their impact on $\theta(\mathbf{r}_i,t)$ one must go beyond the mean field framework. In this work we have done it using the renormalization group-like approach but certainly one can also try some alternative methods. To convince ourselves that apart of any particular technique the Bogoliubov angle is indeed a reasonable concept above T_c let us consider the case of a uniform amplitude $\chi(\mathbf{r}_i,t)=\chi$. The pseudogap state $\chi\neq 0$ can then be envisioned as a randomly oriented planar angle $\phi(\mathbf{r}_i,t)$ with vanishing planar components of the pseudospin vector averaged over all lattice sites $\mathbf{r}_i$ (and/or eventually over some time interval Δt). Such situation where pairing of the *incoherent local pairs* does not establish the off-diagonal-long-range-order (ODLRO) has been previously discussed by Györffy et al. [54]. In Figure 5 we illustrate orientation of the pseudospin in the superconducting and pseudogap states. Notice, that the latter case with a completely random planar phase does still describe a well defined azimuthal (Bogoliubov) angle.

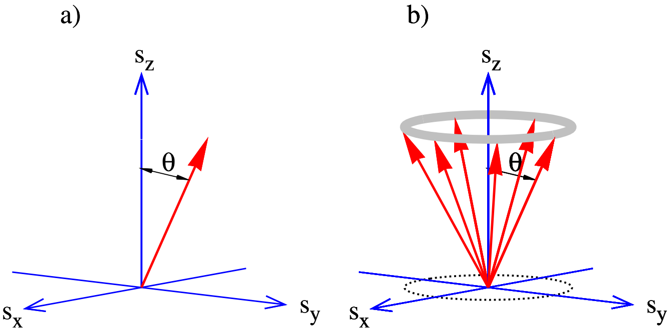

Fig. 5. (Color online) Visualization of the local pseudospin vector for (a) the superconducting and (b) the pseudogap states. In both cases the azimuthal (Bogoliubov) angle is well established although in the pseudogap state the ODLRO does not exist due to the random planar phase.

Since ODLRO does not exist above T_c we have to search for some possible fingerprints of the pairing fluctuations (and the related p-h mixing) either in the single particle excitation spectrum or the two-particle correlation functions. Preformed pairs have then a total finite momentum $\mathbf{q}\neq\mathbf{0}$ and represent the overdamped (short life-time) objects. Nevertheless they still affect unpaired electrons through the scattering processes. Perturbative treatment within the single component pairing model gives usually the following type selfenergy [55–58]

$$\Sigma(\mathbf{k},\omega) = \frac{\Delta^2_{pg}(\mathbf{k})}{\omega + (\varepsilon_{\mathbf{k}}-\mu) + i\Gamma_0} - i\Gamma_1, \tag{A.5}$$

where parameters Γ_0, Γ_1 correspond to the lifetime effects and $\Delta_{pg}(\mathbf{k})$ is a magnitude of pseudogap. The single particle spectral function $A(\mathbf{k},\omega) = -\frac{1}{\pi}\mathrm{Im}\left[\omega-(\varepsilon_{\mathbf{k}}-\mu)-\Sigma(\mathbf{k},\omega)\right]^{-1}$ is then composed of two broadened QP peaks. By integrating their spectral weights one eventually can estimate the coefficients $u^2_{\mathbf{k}}$, $v^2_{\mathbf{k}}$ and through (1) this determines the Bogoliubov angle. Our procedure based on the RG equations described in Sections 3 and 4 relies practically on the same idea. However, instead of (A.5) for $T>T_c$ we obtain the spectral function (13) which contains one delta-peak and another QP peak being broadened (this particular result is typical for the flow equation procedure [42,43] whose scheme resembles the projective methods where the long-lived modes are well separated from damped states). Thus $u^2_{\mathbf{k}}$ is directly determined from the flow equations whereas we derive the other QP coefficient numerically (see illustration in Fig. 3) using the algorithm described above.

Appendix B: Pairing in the single component scenario

Let us consider electron pairing using the flow equation scheme applied to the usual one component system described by the Hamiltonian

$$\hat{H} = \sum_{\mathbf{k},\sigma} (\varepsilon_{\mathbf{k}} - \mu)\, \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} + \frac{1}{N} \times \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} g_{\mathbf{k},\mathbf{k}'}(\mathbf{q}) \hat{c}^{\dagger}_{\mathbf{k}'\uparrow} \hat{c}^{\dagger}_{\mathbf{q}-\mathbf{k}'\downarrow} \hat{c}_{\mathbf{q}-\mathbf{k}\downarrow} \hat{c}_{\mathbf{k}\uparrow}. \quad \text{(B.1)}$$

We assume a separable form $g_{\mathbf{k},\mathbf{k}'}(\mathbf{q}) = g \eta_{\mathbf{k}} \eta_{\mathbf{k}'}$ of the attractive potential $g < 0$, so that the prefactor $\eta_{\mathbf{k}} = \frac{1}{2}\left[\cos(k_x) + \cos(k_y)\right]$ can eventually yield d-wave symmetry of the order parameter. By introducing the pair operators

$$\hat{b}_{\mathbf{q}} = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} \eta_{\mathbf{k}} \hat{c}_{\mathbf{q}-\mathbf{k}\downarrow} \hat{c}_{\mathbf{k}\uparrow} \quad \text{(B.2)}$$

and $\hat{b}^{\dagger}_{\mathbf{q}} = (\hat{b}_{\mathbf{q}})^{\dagger}$ we rewrite the Hamiltonian (B.1) in a more compact form as

$$\hat{H} = \sum_{\mathbf{k},\sigma} (\varepsilon_{\mathbf{k}} - \mu)\, \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} g\, \hat{b}^{\dagger}_{\mathbf{q}} \hat{b}_{\mathbf{q}}. \quad \text{(B.3)}$$

On the level of mean field approach one simplifies (B.1) to the bilinear structure

$$\hat{H} \simeq \hat{H}^{MF} = \sum_{\mathbf{k},\sigma} (\varepsilon_{\mathbf{k}} - \mu)\, \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} + g \left(\hat{b}^{\dagger}_{\mathbf{0}} \langle \hat{b}_{\mathbf{0}} \rangle + \langle \hat{b}^{\dagger}_{\mathbf{0}} \rangle \hat{b}_{\mathbf{0}} - \langle \hat{b}^{\dagger}_{\mathbf{0}} \rangle \langle \hat{b}_{\mathbf{0}} \rangle \right) \quad \text{(B.4)}$$

neglecting the contributions from: (a) the finite momentum pairs $\delta \hat{H}' = g \sum_{\mathbf{q}\neq\mathbf{0}} \hat{b}^{\dagger}_{\mathbf{q}} \hat{b}_{\mathbf{q}}$, and (b) quantum fluctuations of the Bose-Einstein (BE) condensed pairs $\delta \hat{H}_0 = g\; \delta \hat{b}^{\dagger}_{\mathbf{0}}\; \delta \hat{b}_{\mathbf{0}}$, where $\delta \hat{b}_{\mathbf{0}} = \hat{b}_{\mathbf{0}} - \langle \hat{b}_{\mathbf{0}} \rangle$. Such reduced BCS Hamiltonian (B.4) is exactly diagonalizable using the standard Bogoliubov-Valatin transformation

$$\hat{\tilde{c}}_{\mathbf{k}\uparrow} = u_{\mathbf{k}}\, \hat{c}_{\mathbf{k}\uparrow} + v_{\mathbf{k}}\, \hat{c}^{\dagger}_{-\mathbf{k}\downarrow}, \quad \text{(B.5)}$$

$$\hat{\tilde{c}}^{\dagger}_{-\mathbf{k}\downarrow} = -v_{\mathbf{k}}\, \hat{c}_{\mathbf{k}\uparrow} + u_{\mathbf{k}}\, \hat{c}^{\dagger}_{-\mathbf{k}\downarrow}. \quad \text{(B.6)}$$

Quasiparticles defined by equations (B.5, B.6) imply the p-h mixing of initial fermions for momenta $\mathbf{k}$ located near the Fermi surface. The effective single particle excitation spectrum

$$A(\mathbf{k},\omega) = u^2_{\mathbf{k}} \delta(\omega - \xi_{\mathbf{k}}) + v^2_{\mathbf{k}} \delta(\omega + \xi_{\mathbf{k}}), \quad \text{(B.7)}$$

consists of two quasiparticle branches $\xi_{\mathbf{k}} = \pm\sqrt{(\varepsilon_{\mathbf{k}} - \mu)^2 + \Delta^2_{\mathbf{k}}}$ separated by the energy gap $\Delta_{\mathbf{k}} = |g \eta_{\mathbf{k}} \langle \hat{b}_{\mathbf{0}} \rangle|$ and the corresponding spectral weights given by $u^2_{\mathbf{k}} = \frac{1}{2}\left[1 + (\varepsilon_{\mathbf{k}} - \mu)/\xi_{\mathbf{k}}\right]$, $v^2_{\mathbf{k}} = 1 - u^2_{\mathbf{k}}$. In classical superconductors (which are fairly well described by mean field theory) the electron pairs extend over the large spatial scales and exist only below T_c. The order parameter, related to BEC of the Cooper pairs $\langle \hat{b}_{\mathbf{q}=\mathbf{0}} \rangle$, vanishes then at the transition temperature and so does the p-h mixing.

In order to go beyond this usual BCS framework one should consider the effects arising from the quantum fluctuations $g\; \delta \hat{b}^{\dagger}_{\mathbf{0}}\; \delta \hat{b}_{\mathbf{0}}$ and the finite momentum pairs $\hat{b}^{(\dagger)}_{\mathbf{q}\neq\mathbf{0}}$. So far there have been implemented various methods taking into account the superconducting-type fluctuations (via the Maki-Thompson or Aslamazov-Larkin diagrams) to the transport [59], Gaussian corrections around the saddle point solution [60,61], and several other [62–64]. Here we proceed guided by the RG-like treatment [42,43]. Our main idea is to redefine the Bogoliubov-Valatin transformation (B.5, B.6) by introducing new terms arising from scattering of electrons on the finite momentum pairs (these local pairs can originate from an arbitrary mechanism). Following the general scheme outlined in Section 3 we obtain

$$\begin{aligned} \hat{\tilde{c}}_{\mathbf{k}\uparrow} = {} & u_{\mathbf{k}}\, \hat{c}_{\mathbf{k}\uparrow} + v_{\mathbf{k}}\, \hat{c}^{\dagger}_{-\mathbf{k}\downarrow} + \frac{1}{\sqrt{N}} \\ & \times \sum_{\mathbf{q}\neq\mathbf{0}} \left[u_{\mathbf{k},\mathbf{q}}\, \hat{b}^{\dagger}_{\mathbf{q}} \hat{c}_{\mathbf{q}+\mathbf{k}\uparrow} + v_{\mathbf{k},\mathbf{q}}\, \hat{b}_{\mathbf{q}} c^{\dagger}_{\mathbf{q}-\mathbf{k}\downarrow} \right] \\ & + \frac{1}{N} \sum_{\mathbf{q},\mathbf{q}'\neq\mathbf{0}} \left[u_{\mathbf{k},\mathbf{q},\mathbf{q}'}\, \hat{b}^{\dagger}_{\mathbf{q}} \hat{b}_{\mathbf{q}'} \hat{c}_{\mathbf{q}-\mathbf{q}'+\mathbf{k}\uparrow} \right. \\ & \left. + v_{\mathbf{k},\mathbf{q},\mathbf{q}'}\, \hat{b}_{\mathbf{q}} \hat{b}^{\dagger}_{\mathbf{q}'} c^{\dagger}_{\mathbf{q}-\mathbf{q}'-\mathbf{k}\downarrow} \right] + \mathcal{O}(\hat{b}^3), \end{aligned} \quad \text{(B.8)}$$

$$\begin{aligned} \hat{\tilde{c}}^{\dagger}_{-\mathbf{k}\downarrow} = {} & -v^*_{\mathbf{k}}\, \hat{c}_{\mathbf{k}\uparrow} + u^*_{\mathbf{k}}\, \hat{c}^{\dagger}_{-\mathbf{k}\downarrow} + \frac{1}{\sqrt{N}} \\ & \times \sum_{\mathbf{q}\neq\mathbf{0}} \left[-v^*_{\mathbf{k},\mathbf{q}}\, \hat{b}^{\dagger}_{\mathbf{q}} \hat{c}_{\mathbf{q}+\mathbf{k}\uparrow} + u^*_{\mathbf{k},\mathbf{q}}\, \hat{b}_{\mathbf{q}} \hat{c}^{\dagger}_{\mathbf{q}-\mathbf{k}\downarrow} \right] \\ & + \frac{1}{N} \sum_{\mathbf{q},\mathbf{q}'\neq\mathbf{0}} \left[-v^*_{\mathbf{k},\mathbf{q},\mathbf{q}'}\, \hat{b}^{\dagger}_{\mathbf{q}} \hat{b}_{\mathbf{q}'} \hat{c}_{\mathbf{q}-\mathbf{q}'+\mathbf{k}\uparrow} \right. \\ & \left. + u^*_{\mathbf{k},\mathbf{q},\mathbf{q}'}\, \hat{b}_{\mathbf{q}} \hat{b}^{\dagger}_{\mathbf{q}'} c^{\dagger}_{\mathbf{q}-\mathbf{q}'-\mathbf{k}\downarrow} \right] + \mathcal{O}(\hat{b}^3), \end{aligned} \quad \text{(B.9)}$$

where the terms $\mathcal{O}(\hat{b}^3)$ involve three and more pair operators $\hat{b}^{(\dagger)}_{\mathbf{q}}$. Let us remark that $\hat{b}^{(\dagger)}_{\mathbf{0}}$ simplify to the complex numbers so formally we could interpret $v_{\mathbf{k}}$ as $v_{\mathbf{k},\mathbf{0}}\, \frac{\hat{b}_{\mathbf{0}}}{\sqrt{N}}$, however we retain $v_{\mathbf{k}}$ to have a clear comparison between (B.8, B.9) and the mean-field ansatz (B.5, B.6).

To determine all the appearing coefficients u and v we have to fulfill the anticommutation relations between the operators $\hat{\tilde{c}}_{\mathbf{k}\sigma}$ and $\hat{\tilde{c}}^{\dagger}_{\mathbf{k}\sigma}$ along with further constraints which naturally come out from the flow equation procedure. Since this is a rather cumbersome issue itself we merely focus on the generic outcome of our proposal (B.8, B.9). The leading contributions in the single particle spectral function are given by

$$A(\mathbf{k},\omega) = |u_{\mathbf{k}}|^2\delta\left(\omega-\tilde{\xi}_{\mathbf{k}}\right) + \frac{1}{N}\sum_{\mathbf{q}\neq\mathbf{0}}|u_{\mathbf{k},\mathbf{q}}|^2 n^F_{\mathbf{q}+\mathbf{k}\uparrow} \times \left(1+n^F_{-\mathbf{k}\downarrow}\right)\delta(\omega-\tilde{\xi}_{\mathbf{q}+\mathbf{k}}+\tilde{E}^{pair}_{\mathbf{q}}) + |v_{\mathbf{k}}|^2\delta\left(\omega+\tilde{\xi}_{-\mathbf{k}}\right) + \frac{1}{N}\sum_{\mathbf{q}\neq\mathbf{0}}|v_{\mathbf{k},\mathbf{q}}|^2 n^F_{\mathbf{q}-\mathbf{k}\downarrow} \times \left(1+n^F_{\mathbf{k}\uparrow}\right)\delta(\omega+\tilde{\xi}_{\mathbf{q}-\mathbf{k}}-\tilde{E}^{pair}_{\mathbf{q}}) + \mathcal{O}(\hat{b}^2). \tag{B.10}$$

As before, $\tilde{\xi}_{\mathbf{k}}$ denotes the effective fermion energy measured from the chemical potential μ and $\tilde{E}^{pair}_{\mathbf{q}}$ corresponds to the energy of fermion pairs. Specific information about the coefficients u and v is not crucial and will be presented elsewhere.

In the superconducting state below T_c the effective dispersion $\tilde{\xi}_{\mathbf{k}}$ is gaped. Under such circumstances besides the QP peaks (whose weights determine p-h mixing) the spectral function (B.10) develops an additional background arising from the scattering on the finite momentum pairs. Such damped states exist only outside the superconducting gap. For increasing temperature a number finite momentum pairs is growing thereby such scattering becomes more efficient. Ultimately in the pseudogap state above T_c the term $|v_{\mathbf{k}}|^2\delta\left(\omega+\tilde{\xi}_{-\mathbf{k}}\right)$ completely vanishes (because the BE condensate $\langle\hat{b}_{\mathbf{q}=\mathbf{0}}\rangle$ no longer exists) but in its place there emerges a well pronounced branch of the damped states located along the energy $\omega \simeq -\tilde{\xi}_{\mathbf{k}}$. They arise from the scattering of fermions on the low momentum $\mathbf{q} \sim \mathbf{0}$ pairs and formally are contributed by the terms containing $|v_{\mathbf{k},\mathbf{q}}|^2$ in expression (B.10). Such remnants of the Bogoliubov-type quasiparticles above T_c seem thus to be a universal feature of the models describing pre-existing pairs. Their existence above T_c has been recently confirmed experimentally by the ARPES measurements [1,2].

References

1. A. Kanigel, U. Chatterjee, M. Randeria, M.R. Norman, G. Koren, K. Kadowaki, J.C. Campuzano, Phys. Rev. Lett. **101**, 137002 (2008)
2. M. Shi, J. Chang, S. Pailhes, M.R. Norman, J.C. Campuzano, M. Mansson, T. Claesson, O. Tjernberg, A. Bendounan, L. Patthey, N. Momono, M. Oda, M. Ido, C. Mudry, J. Mesot, EPL **88**, 27008 (2009)
3. K. Fujita, I. Grigorenko, J. Lee, W. Wang, J.X. Zhu, J.C. Davis, H. Eisaki, S. Uchida, A.V. Balatsky, Phys. Rev. B **78**, 054510 (2008)
4. G. Deutscher, Rev. Mod. Phys. **77**, 109 (2005)
5. G.E. Blonder, M. Tinkham, T.M. Klapwijk, Phys. Rev. B **25**, 4515 (1982)
6. C.W.J. Beenakker, Rev. Mod. Phys. **80**, 1337 (2008)
7. T. Domański, A. Donabidowicz, Phys. Rev. B **78**, 073105 (2008)
8. T. Domański, A. Donabidowicz, K.I. Wysokiński Phys. Rev. B **76**, 104514 (2007)
9. G.E. Graber, M. Tinkham, T.M. Klawijk, Notechnology **15**, 479 (2004)
10. A.V. Balatsky, W.S. Lee, Z.X. Shen, Phys. Rev. B **79**, 020505 (2009)
11. O. Fisher, M. Kugler, I. Maggio-Aprile, Ch. Berthod, Ch. Renner, Rev. Mod. Phys. **79**, 353 (2007)
12. A. Damascelli, A. Hussain, Z.X. Shen, Rev. Mod. Phys. **75**, 473 (2003)
13. Y. Kohsaka, C. Taylor, P. Wahl, A. Schmidt, J. Lee, K. Fujita, J.W. Alldredge, K. McElroy, Jinho Lee, H. Eisaki, S. Uchida, D.-H. Lee, J.C. Davis, Nature **454**, 1072 (2008)
14. In reference [3] the Bogoliubov angle is defined as $\theta_{\mathbf{k}} = \arctan\left|\frac{u_{\mathbf{k}}}{v_{\mathbf{k}}}\right|$ but we use a different notation adopted from the Anderson's pseudospin representation [15]
15. P.W. Anderson, Phys. Rev. **112**, 1900 (1958)
16. R.A. Barankov, L.S. Levitov, Phys. Rev. A **73**, 033614 (2006)
17. A.V. Andreev, V. Gurarie, L. Radzihovsky, Phys. Rev. Lett. **93**, 130402 (2004)
18. H. Matsui, T. Sato, T. Takahashi, S.-C. Wang, H.-B. Yang, H. Ding, T. Fujii, T. Watanabe, A. Matsuda, Phys. Rev. Lett. **90**, 217002 (2003)
19. P.A. Lee, N. Nagaosa, X.G. Wen, Rev. Mod. Phys. **78**, 17 (2006)
20. M. Eschrig, Adv. Phys. **55**, 47 (2006)
21. M. Norman, D. Pines, C. Kallin, Adv. Phys. **54**, 715 (2005)
22. O. Yuli, I. Asulin, Y. Kalchaim, G. Koren, O. Millo, Phys. Rev. Lett. **103**, 197003 (2009)
23. N. Bergeal, J. Lesueur, M. Aprili, G. Faini, J.P. Contour, B. Leridon, Nature Phys. **4**, 608 (2008)
24. Z.A. Xu, N.P. Ong, Y. Wang, T. Takeshita, S. Uchida, Nature **406**, 486 (2000)
25. J. Corson, R. Mallozzi, J. Orenstein, J.N. Eckstein, I. Bozovic, Nature **398**, 221 (1999)
26. L. Li, J.G. Checkelsky, S. Komiya, Y. Ando, N.P. Ong, Nature Phys. **3**, 311 (2007)
27. V.J. Emery, S.A. Kivelson, Nature **374**, 434 (1995)
28. Q. Chin, J. Stajic, S. Tan, K. Levin, Phys. Rep. **412**, 1 (2005)
29. T. Domański, J. Ranninger, Phys. Rev. Lett. **91**, 255301 (2003)
30. T. Domański, J. Ranninger, Phys. Rev. B **70**, 184503 (2004)
31. W. Ketterle, M.W. Zwierlein, *Making, probing and understanding ultracold Fermi gases*, in *Proc. Int. School Phys. Enrico Fermi, course CLXIV*, edited by M. Inguscio, W. Ketterle, C. Salomon (IOS Press, Amsterdam, 2008)
32. J. Ranninger, S. Robaszkiewicz, Physica B **135**, 468 (1985)
33. R. Micnas, J. Ranninger, S. Robaszkiewicz, Rev. Mod. Phys. **62**, 113 (1990)
34. for a recent review see also the paper by J. Ranninger, A. Romano, Phys. Rev. B **78**, 054527 (2008)
35. R. Friedberg, T.D. Lee, Phys. Rev. B **40**, 423 (1989)
36. J.A. Wilson, Philos. Mag. **84**, 2183 (2004)
37. E. Altman, A. Auerbach, Phys. Rev. B **65**, 104508 (2002)
38. M. Franz, Z. Tesanovic, Phys. Rev. Lett. **87**, 257003 (2001)
39. V.B. Geshkenbein, L.B. Ioffe, A.I. Larkin, Phys. Rev. B **55**, 3173 (1997)
40. T. Kostyrko, J. Ranninger, Phys. Rev. B **54**, 13105 (1996)
41. Y. Ohashi, A. Griffin, Phys. Rev. Lett. **89**, 130402 (2002)
42. F. Wegner, Ann. Physik (Leipzig) **3**, 77 (1994)

43. S. Kehrein, *Flow Equation Approach to Many Particle Systems*, Springer Tracts in Modern Physics **217** (Berlin, 2006)
44. T. Domański, J. Ranninger, Phys. Rev. B **63**, 134505 (2001)
45. T. Senthil, P.A. Lee, Phys. Rev. Lett. **103**, 076402 (2009)
46. T. Senthil, P.A. Lee, Phys. Rev. B **79**, 245116 (2009)
47. T. Micklitz, M.R. Norman, Phys. Rev. B **80**, 220513 (2009)
48. N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallais, J.-B. Bonnemaison, R. Liang, D.A. Bonn, W.N. Hardy, L. Taillefer, Nature **447**, 565 (2007)
49. P.W. Anderson, Phys. Rev. Lett. **96**, 017001 (2006)
50. A. Kanigel, U. Chatterjee, M. Randeria, M.R. Norman, S. Souma, M. Shi, Z.Z. Li, H. Raffy, J.C. Campuzano, Phys. Rev. Lett. **99**, 157001 (2007)
51. A. Schmid, Z. Phys. **231**, 324 (1970)
52. A. Schmid, J. Phys. Condens. Matt. **5**, 302 (1966)
53. J. Krzyszczak, T. Domański, K.I. Wysokiński, R. Micnas, S. Robaszkiewicz, Phys. Rev. B (submitted)
54. B.L. Györffy, J.B. Staunton, G.M. Stocks, Phys. Rev. B **44**, 5190 (1991)
55. E. Abrahams, M. Redi, J.W.F. Woo, Phys. Rev. B **1**, 208 (1970)
56. M.R. Norman, M. Randeria, H. Ding, J.C. Campuzano, Phys. Rev. B **57**, 11093 (1998)
57. D. Wulin, Y. He, C.-C. Chien, D.K. Morr, K. Levin, Phys. Rev. B **80**, 134504 (2009)
58. M. Shi, J. Chang, S. Pailhes, M.R. Norman, J.C. Campuzano, M. Mansson, T. Claesson, O. Tjernberg, A. Bendounan, L. Patthey, N. Momono, M. Oda, M. Ido, C. Mudry, J. Mesot Phys. Rev. Lett. **101**, 047002 (2008)
59. A.I. Larkin, A.A. Varlamov, *Theory of fluctuations in superconductors* (Oxford University Press, Oxford, 2009)
60. H.T.C. Stoof, Phys. Rev. B **47**, 7979 (1993)
61. C.A.R. Sá de Melo, M. Randeria, J.R. Engelbrecht, Phys. Rev. Lett. **71**, 3202 (1993)
62. R. Haussmann, Phys. Rev. B **49**, 12975 (1994)
63. C. Lannert, M.P.A. Fisher, T. Senthil, Phys. Rev. B **64**, 014518 (2001)
64. P. Nikolić, S. Sachdev, Phys. Rev. A **75**, 033608 (2007)