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Interplay between the Correlations and Superconductivity in Electron Transport through the Double Quantum Dots

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We study the influence of electron correlations on nonequilibrium transport through the quantum dots coupled between one metallic and one superconducting electrode. Such type of nanodevices are characterized by the induced on-dot pairing spread from the superconducting lead (proximity effect) and effectively responsible for mixing the particle with hole excitations. On the other hand, strong Coulomb repulsion between the opposite spin electrons tends to suppress a double occupancy of the quantum dots competing with the on-dot superconducting order. The Coulomb interactions can also cause a screening of the quantum dot spin by itinerant electrons of the metallic lead giving rise to formation of the Kondo resonance. We analyze interplay of such phenomena for the setup of double quantum dots where the quantum interference (such as Fano) effects influence the subgap electron transport through the Andreev scattering.

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1. Introduction

Correlated quantum dots placed between the metallic and superconducting leads provide a unique possibility for exploring an interplay between the Kondo physics and superconducting correlations [1]. Such many-body effects in the solid state physics are known to compete with one another. Varying the quantum dots' coupling to external electrodes and positioning their discrete energy levels by the gate–voltage enable a controllable changeover from regime dominated by the Kondo effect to another case where the on-dot pairing plays an essential role.

Our previous studies [2] of the single quantum impurity hybridized with the coupling strength Γ_{N} to the metallic and with Γ_{S} to superconducting leads revealed that on-dot pairing and the Kondo effects eventually coexist when $\Gamma_{\mathrm{N}} \sim \Gamma_{\mathrm{S}}$. In particular, their coexistence was shown to enhance the Andreev current near the zero source-drain voltage (reminiscent of enhancing the zero-bias conductance for the quantum dot coupled to both conducting electrodes, which in the Kondo regime approaches the perfect limit $2e^2/h$). As a matter of fact our prediction [2] has been recently confirmed experimentally in the case of self assembled InAs quantum dots deposited between Al and Au electrodes [3].

Present work extends the previous studies onto more complex situation, when superconductivity and the Kondo effect occur in presence of the quantum interference. For this purpose we consider the double quantum dot (DQD) in the T-shape configuration investigating electron transport between the external leads through the interfacial quantum dot (QD).

2. The model

The DQD system hybridized to the metallic (N) and superconducting (S) electrodes can be described by the following Anderson impurity Hamiltonian:

$$\hat{H} = \hat{H}_{\mathrm{N}} + \hat{H}_{\mathrm{N-DQD}} + \hat{H}_{\mathrm{DQD}} + \hat{H}_{\mathrm{S-DQD}} + \hat{H}_{\mathrm{S}}. \quad (1)$$

Reservoirs of the mobile electrons consist of the normal $\hat{H}_{\mathrm{N}} = \sum_{\boldsymbol{k},\sigma} \xi_{\boldsymbol{k}\mathrm{N}} \hat{c}^{\dagger}_{\boldsymbol{k}\sigma\mathrm{N}} \hat{c}_{\boldsymbol{k}\sigma\mathrm{N}}$ and superconducting parts $\hat{H}_{\mathrm{S}} = \sum_{\boldsymbol{k},\sigma} \xi_{\boldsymbol{k}\mathrm{S}} \hat{c}^{\dagger}_{\boldsymbol{k}\sigma\mathrm{S}} \hat{c}_{\boldsymbol{k}\sigma\mathrm{S}} - \sum_{\boldsymbol{k}} \Delta \hat{c}^{\dagger}_{\boldsymbol{k}\uparrow\mathrm{S}} \hat{c}^{\dagger}_{-\boldsymbol{k}\downarrow\mathrm{S}} + \Delta^{*} \hat{c}_{-\boldsymbol{k}\downarrow\mathrm{S}} \hat{c}_{\boldsymbol{k}\uparrow\mathrm{S}}$ where $\xi_{\boldsymbol{k}\beta} = \varepsilon_{\boldsymbol{k}\beta} - \mu_{\beta}$ measures the energies with respect to the chemical potentials. In non-equilibrium conditions the source-drain voltage V can detune the chemical potentials $\mu_{\mathrm{N}} - \mu_{\mathrm{S}} = eV$ and induce the current $I(V)$.

The considered correlated DQD is represented by

$$\hat{H}_{\mathrm{DQD}} = \sum_{\sigma i} \epsilon_i \hat{d}^{\dagger}_{i\sigma} \hat{d}_{i\sigma} + \sum_i U_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + \left(t \hat{d}^{\dagger}_{1\sigma} \hat{d}_{2\sigma} + \mathrm{H.c.} \right), \quad (2)$$

where ε_i denote the energies of each quantum dot $i = 1, 2$ and electrons with opposite spins $\sigma = \uparrow, \downarrow$ interact via the on-dot Coulomb potential U_i. We assume hybridization with the external charge reservoirs $\beta = \mathrm{N}, \mathrm{S}$ only through the interfacial ($i = 1$) quantum dot

$$\hat{H}_{\beta-\mathrm{DQD}} = \sum_{\boldsymbol{k},\sigma} V_{\boldsymbol{k}\beta} \hat{d}^{\dagger}_{1\sigma} \hat{c}_{\boldsymbol{k}\sigma\beta} + \mathrm{H.c.} \quad (3)$$

In the wide-band limit case, we can introduce the useful coupling constants $\Gamma_{\beta} = 2\pi \sum |V_{\boldsymbol{k}\beta}|^2 \delta(\omega - \xi_{\boldsymbol{k}})$. Interference effects originate in the present model (1) from the hopping t to the side-coupled ($i = 2$) quantum dot.

Empirical verification of the interplay between the proximity effect, the Coulomb blockade, the Kondo physics and the quantum interference can be achieved by measuring the differential conductance $\mathrm{d}I(V)/\mathrm{d}V$. If we focus on the low voltage regime then the only channel for the subgap ($|eV| \ll \Delta$) current restricts solely to the processes when electrons from the metallic lead are converted into the Cooper pairs propagating in superconductor with a simultaneous reflection of the electron holes back to the normal lead. This Andreev current [4]:

$$I_{\mathrm{A}}(V)=\frac{2e}{h}\int \mathrm{d}\omega T_{\mathrm{A}}(\omega)\left[f(\omega-eV,T)\right.$$
$$\left.-f(\omega+eV,T)\right], \tag{4}$$

where $f(\omega,T)$ is the Fermi distribution function, depends through the transmittance $T_{\mathrm{A}}(\omega)=\Gamma_{\mathrm{N}}^{2}|G_{12}(\omega)|^{2}$ on off-diagonal terms (in the Nambu representation) of the Green function of interfacial ($i=1$) quantum dot.

3. Proximity versus Fano effects

To get some insight into the physics originating from the quantum interference $t\neq 0$ let us first focus on the noninteracting DQD ($U_i=0$). The single particle Green functions of each quantum dot can be determined exactly. Solving the set of coupled equations of motion we obtain the following conclusions:

a) hybridization $V_{\boldsymbol{k}\mathrm{S}}$ to superconducting lead induces the pair correlations of the interfacial QD $\langle\hat{d}_{1\downarrow}\hat{d}_{1\uparrow}\rangle$,

b) interdot hopping t extends further the pairing on the side-coupled QD $\langle\hat{d}_{2\downarrow}\hat{d}_{2\uparrow}\rangle$ and induces the inter-dot $\langle\hat{d}_{2\downarrow}\hat{d}_{1\uparrow}\rangle$ pairing.

Since transport of the DQD system (1) is determined by the properties of interfacial quantum dot let us inspect in more detail its effective spectrum. The retarded Green function $\boldsymbol{G}_1(\tau)=-\hat{T}_\tau\langle\hat{\Psi}_1(\tau)\hat{\Psi}_1^\dagger\rangle$ in the Nambu spinor notation $\hat{\Psi}_1^\dagger=(\hat{d}_{1\uparrow}^\dagger,\hat{d}_{1\downarrow})$, $\hat{\Psi}_1=(\hat{\Psi}_1^\dagger)^\dagger$ can be expressed by the Dyson equation

$$\boldsymbol{G}_1(\omega)^{-1}=\begin{pmatrix}\omega-\varepsilon_1 & 0\\ 0 & \omega+\varepsilon_1\end{pmatrix}-\boldsymbol{\Sigma}^0_{d1}(\omega)-\boldsymbol{\Sigma}^U_{d1}(\omega). \tag{5}$$

The self-energy $\boldsymbol{\Sigma}^0(\omega)$ corresponds to the noninteracting case ($U_i=0$) and the second contribution $\boldsymbol{\Sigma}^U(\omega)$ accounts for the correlation effects. In far subgap regime $|\omega|\ll|\Delta|$ the noninteracting contribution simplifies to

$$\boldsymbol{\Sigma}^0_{d1}(\omega)=\begin{pmatrix}-\frac{\mathrm{i}\,\Gamma_{\mathrm{N}}}{2}+\frac{t^2}{\omega-\varepsilon_2} & -\frac{\Gamma_{\mathrm{S}}}{2}\\ -\frac{\Gamma_{\mathrm{S}}}{2} & -\frac{\mathrm{i}\,\Gamma_{\mathrm{N}}}{2}+\frac{t^2}{\omega+\varepsilon_2}\end{pmatrix}. \tag{6}$$

Figure 1 illustrates the resulting spectral function $\rho_1(\omega)=-\frac{1}{\pi}\mathrm{Im}\boldsymbol{G}_{11}(\omega+\mathrm{i}0^+)$ for $\varepsilon_1=0$, $\varepsilon_2=0.3\Gamma_{\mathrm{N}}$, $\Gamma_{\mathrm{S}}=5\Gamma_{\mathrm{N}}$ and several values of the interdot hopping t. The coupling strength Γ_{S} is responsible for the particle-hole splitting whereas Γ_{N} controls the level broadening. On top of this known spectrum there appear additional features at $\pm\varepsilon_2$ caused by hopping to the side-coupled QD.

Interference effects due to the side-coupled QD have a qualitative effect on the Andreev current. In Fig. 2 we show gate–voltage V_{G} dependence of the differential Andreev conductance obtained in a small interval around $-\varepsilon_2$ (let us remark that the subgap conductance G_{A} is even function of V_{G} because particle and hole states equally participate in the Andreev reflections). In some analogy to the T-shape DQD coupled to both metallic

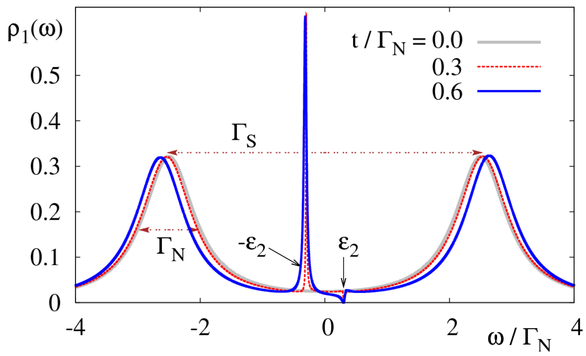

Fig. 1. Density of states $\rho_1(\omega)$ of the interfacial $i=1$ QD for $\varepsilon_1=0$, $\varepsilon_2=0.3\Gamma_{\mathrm{N}}$, $\Gamma_{\mathrm{S}}=5\Gamma_{\mathrm{N}}$ and few representative values of the interdot hopping t.

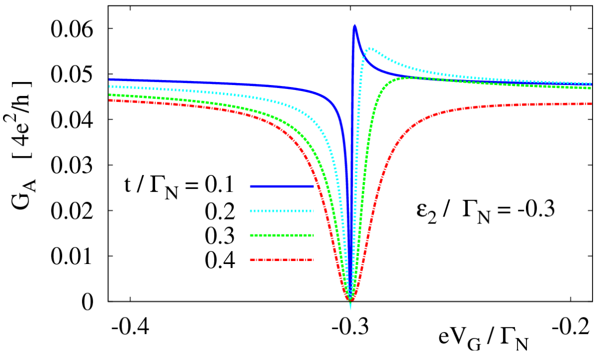

Fig. 2. The gate voltage V_{G} dependence of the subgap Andreev conductance G_{A} with the characteristic Fano resonance at $eV_{\mathrm{G}}=\pm\varepsilon_2$ obtained for the same set of parameters as in Fig. 1.

leads [5, 6] we notice the characteristic Fano line-shape $G_{\mathrm{A}}=G_0\frac{(x+q)^2}{x^2+1}+G_1$ with $x=|eV_{\mathrm{G}}-\varepsilon_2|/\Gamma_{\mathrm{N}}$ and the asymmetry parameter q gradually decreasing with respect to the hopping integral t.

4. Correlation effects

Certain aspects concerning the interrelation between the Kondo and Fano effects in the Andreev current of the nanosystems coupled to metallic and superconducting leads have been so far analyzed considering the molecular [7] and double quantum dot [8, 9] structures by means of the density functional and the numerical renormalization group method, respectively. We outline here the main issues related to such problematics — a detailed study will be discussed elsewhere.

The Coulomb interactions trigger the Kondo singlet state when the QD spin is screened by the mobile electrons. In the present context the Kondo peak eventually forms at μ_{N} of the metallic lead because in superconductor the single particle states are prohibited (for $|\omega|<\Delta$). Interference due to the side-coupled quantum dot is hence expected to have non-trivial effect on the Kondo resonance whenever the gate voltage shifts the energies to $\pm\varepsilon_2$ close to $\mu_{\rm N}$.

To treat these issues on a formal level we adopt the method previously used for studying the single QD [2]. We assume that the correlation effects are manifested in the diagonal terms of the self-energy

$$\boldsymbol{\Sigma}^U_{d1}(\omega) \simeq \begin{pmatrix} \Sigma_{\rm N}(\omega) & 0 \\ 0 & -\Sigma^*_{\rm N}(-\omega) \end{pmatrix}. \tag{7}$$

This assumption is justified as long as occupancy of the QD is close to half-filling [10]. To be specific we estimate the role of correlations by the equation of motion method procedure [11] which yields

$$\Sigma_{\rm N}(\omega) = \omega - \varepsilon_d - \frac{[\tilde\omega - \varepsilon_d][\tilde\omega - \varepsilon_d - U - \Sigma_3(\omega)] + U\Sigma_1(\omega)}{\tilde\omega - \varepsilon_d - [\Sigma_3(\omega) + U(1-n_{d,\sigma})]}, \tag{8}$$

where $\Sigma_{\nu=1,3}(\omega) = \sum_{\boldsymbol{k}} |V_{\boldsymbol{k}\rm N}|^2 [f(\omega,T)]^{\frac{3-\nu}{2}} [(\omega - \xi_{\boldsymbol{k}\rm N})^{-1} + (\omega - U - 2\varepsilon_d + \xi_{\boldsymbol{k}\rm N})^{-1}]$ and $\tilde\omega = \omega + \frac{{\rm i}\Gamma_{\rm N}}{2}$.

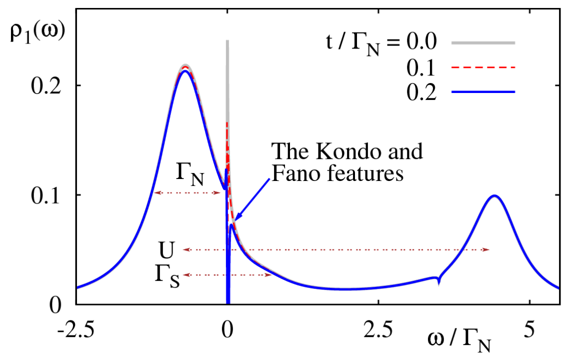

Fig. 3. Density of states $\rho_1(\omega)$ of the correlated interfacial QD in the Kondo regime. Numerical data are obtained for the following set of model parameters: $\varepsilon_1 = -1.5\Gamma_{\rm N}$, $\varepsilon_2 = 0$, $\Gamma_{\rm S} = 1.5\Gamma_{\rm N}$, $U = 5\Gamma_{\rm N}$ and temperature $k_{\rm B}T = 0.001\Gamma_{\rm N}$.

The Kondo resonance formed at $\omega = \mu_{\rm N}$ is strongly reshaped by interference with the side-coupled QD. We show such influence in Fig. 3 for $\varepsilon_2 = 0$, $\varepsilon_1 = -1.5\Gamma_{\rm N}$ for the weak on-dot pairing $\Gamma_{\rm S} = 1.5\Gamma_{\rm N}$ (otherwise for $\Gamma_{\rm S} \gg \Gamma_{\rm N}$ the Kondo resonance is suppressed [2]) therefore particle–hole splitting is hardly visible. The upper peak in Fig. 3 represents the Coulomb satellite.

For $t = 0$ at sufficiently low temperatures (below $T_{\rm K}$) the Kondo resonance is known to enhance the zero bias conductance [2]. In the present case of T-shaped double quantum dot (1) the additional quantum interference due to $t \neq 0$ contributes qualitative modifications in the spectral function $\rho_1(\omega)$ and the Andreev conductance (not shown here), both of which indicate the characteristic Fano line-shapes.

Summarizing, the electron correlations and quantum interference have a strong influence on the differential conductance of the Andreev current. By tuning the gate voltage with a suitable choice of the ratio $\Gamma_{\rm S}/\Gamma_{\rm N}$ it would be possible to experimentally observe the interplay between these many-body quantum effects.

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