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Ultracold atom superfluidity induced by the Feshbach resonance

T. Domański[*]

Institute of Physics, Marie Curie-Skłodowska University, 20-031 Lublin, Poland

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We discuss the possible signatures of superfluidity induced by the Feshbach resonance in the ultracold gas of fermion atoms. Optically or magnetically trapped atoms such as ^{6}Li or ^{40}K are used in two hyperfine states where part of them is converted into the diatomic molecules. These fermion and boson entities get coupled in a presence of the external magnetic field. Eventually, at critical T_c, they simultaneously undergo transition to the superfluid state. Approaching this transition from above there appear various signatures manifesting a gradually emerging order parameter, but yet the long range coherence is not established due to the strong quantum fluctuations. Fermion atoms are characterized by the gapped excitation spectrum (*pseudogap*) up to temperature T_p (larger than T_c) while boson molecules exhibit collective features such as *first sound* showing up above a certain critical momentum $q_{\text{crit}}(T)$. Upon lowering temperature to T_c this critical value shifts to zero and hence there appears the Goldstone mode signaling the symmetry broken superfluid state.

1 Introduction

During the last fifteen years or so we observe an increased experimental and theoretical investigation of the atomic gases which, trapped and cooled to ultralow temperatures (~100 nK), manifest quantum effects on the macroscopic scale. These studies were enabled by a progress of the trapping techniques and by unprecedented control of the experimentally adjustable interactions between atoms [1]. From the theoretical point of view it is important to make a distinction whether the number of atom constituents $Z + A$ is odd or even because one is confronted either with the fermion or boson objects which obey different statistical relations.

Available quantum states can be occupied by the arbitrary number of bosons. Statistical rules enforce that, below critical temperature T_c bosons start to populate macroscopically the lowest lying energy level and this fraction is called the Bose Einstein (BE) condensate. Practical realization of such condensates has been obtained in the trapped alkali atoms of ^{87}Rb, ^{23}Na, ^{7}Li and the polarized hydrogen ^{1}H. Another and the only one naturally existing example of BE condensate is ^{4}He below the λ point (i.e. under pressure). Strong interactions between helium atoms lead there moreover to the superfluid behaviour (transport without any observable viscosity). Such unique phenomenon can be probably achieved also in the trapped alkali atoms where interactions can be experimentally varied by tuning the external magnetic field.

On the other hand, alkali atoms with odd Z and even A (such as ^{6}Li or ^{40}K) are fermions an must be described by the asymmetric wave function as required by the Pauli exclusion principle. If the fermion atoms are prepared in two different hyperfine states then by switching on the magnetic field their energy

[*] Corresponding author: e-mail: doman@kft.umcs.lublin.pl, Phone: +48 81 537 61 63, Fax: +48 81 537 61 91

levels appropriately change and part of the atoms is combined into the weakly bound molecules (bosons). These molecules and single atoms interact with each other via the resonant type scattering [2] from which one can eventually obtain the *resonant superconductivity* [3]. Since atoms are neutral in charge we should rather refer to the superfluid instead of superconducting state. In this work we shall investigate this boson-fermion mixture with a purpose to point out such properties which would enable detection of the superfluid transition at T_c. This issue is extremely important because the standard experimental methods of the condensed matter physics cannot be applied to the trapped atoms. Moreover, since the superfluid state is smoothly emerging upon approaching T_c from above we consider also the precursor effects which would possibly show up in the experimental measurements.

2 Microscopic description of the Feshbach resonance

Driving force of the resonant superconductivity/superfluidity is a resonant scattering between fermion atoms. Resonances were for the first time considered in the atomic physics by Ugo Fano and they were later adopted to the nuclear physics by Feshbach [4]. Starting with a realization of such resonances in 1998 [1, 5] they became a powerful experimental tool for controlling the effective scattering potentials ranging between the negative to positive values of an arbitrary magnitude.

The resonant Feshbach scattering was recently proposed as a mechanism inducing the superconductivity/superfluidity of the trapped fermion atoms [2]. On a microscopic level the underlying mechanism can be described using the following Hamiltonian

$$H = \sum_{\mathbf{k},\sigma} (\varepsilon_{\mathbf{k}} - \mu)\, c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} \left(E_{\mathbf{q}} + 2\nu - 2\mu\right) b^\dagger_{\mathbf{q}} b_{\mathbf{q}} + \frac{v}{\sqrt{N}} \sum_{\mathbf{k},\mathbf{q}} \left(b^\dagger_{\mathbf{q}} c_{\mathbf{q}-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} + \text{h.c.}\right)$$
$$+ \frac{1}{N} \sum_{\mathbf{k},\mathbf{p},\mathbf{q}} U_{\mathbf{k},\mathbf{p}}(\mathbf{q})\, c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}-\mathbf{k}\downarrow} c_{\mathbf{q}-\mathbf{p}\downarrow} c_{\mathbf{p}\uparrow} \,. \qquad (1)$$

Operators $c^{(\dagger)}_{\mathbf{k}\sigma}$ refer to the fermion atoms in two hyperfine configurations (labeled symbolically by $\sigma = \uparrow$ and $\downarrow$) and $b^{(\dagger)}_{\mathbf{q}}$ correspond to the diatomic molecules. First terms of the Hamiltonian (1) describe kinetic energies of fermions and bosons where μ is the chemical potential and ν denotes a *detuning parameter* whose meaning will be explained below. The third term describes the coupling between fermion pairs and diatomic molecules and the last part denotes a small two-body interaction between fermions. The two-body potential is often expressed in the atomic physics in terms of the scattering length a via the following relation $U_{\mathbf{k},\mathbf{p}}(\mathbf{q}) = \frac{2\pi\hbar^2 a}{m}\, n(\mathbf{q})$.

It is the boson-fermion interaction which effectively leads to the resonant scattering. In order to prove it in the simplest way one can treat $H_{B-F} = \frac{v}{\sqrt{N}} \sum_{\mathbf{k},\mathbf{q}} \left(b^\dagger_{\mathbf{q}} c_{\mathbf{q}-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} + \text{h.c.}\right)$ as a perturbation and project out from the Hamiltonian (1) by the canonical transformation e^S. Within the lowest order estimation the transformed Hamiltonian becomes [6]

$$e^S H e^{-S} = \sum_{\mathbf{k},\sigma} (\tilde{\varepsilon}_{\mathbf{k}} - \mu)\, c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} \left(\tilde{E}_{\mathbf{q}} + 2\nu - 2\mu\right) b^\dagger_{\mathbf{q}} b_{\mathbf{q}} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{p},\mathbf{q}} \tilde{U}_{\mathbf{k},\mathbf{p}}(\mathbf{q})\, c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}-\mathbf{k}\downarrow} c_{\mathbf{q}-\mathbf{p}\downarrow} c_{\mathbf{p}\uparrow} \,,$$

where the two-body potential renormalizes to

$$\tilde{U}_{\mathbf{k},\mathbf{p}}(\mathbf{q}) = U_{\mathbf{k},\mathbf{p}}(\mathbf{q}) + \frac{v^2}{2} \left[\frac{1}{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{q}-\mathbf{k}} - (E_{\mathbf{q}} + 2\nu)} + \frac{1}{\varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{q}-\mathbf{p}} - (E_{\mathbf{q}} + 2\nu)} \right]. \qquad (2)$$

Considering for simplicity the $\mathbf{k} = \mathbf{p}$ channel we observe that $\tilde{U}_{\mathbf{k},\mathbf{p}}(\mathbf{q})$ diverges if $\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{q}-\mathbf{k}} - E_{\mathbf{q}} = 2\nu$. In particular, the *detuning parameter* ν measures the relative difference $\frac{1}{2} E_{\mathbf{q}=0} - \varepsilon_{k_F}$. This quantity can be adjusted experimentally by applying the external magnetic field. If one treats H_{B-F} in a better, selfconsistent way the divergence of scattering potential is replaced by a finite resonant-shape jump [6].

Resonant Feshbach interactions were already found for the fermion atoms of ^{6}Li (in two hyperfine states $|1/2, 1/2\rangle$, $|1/2, -1/2\rangle$) [7] and ^{40}K (in two configurations $|9/2, -9/2\rangle$, $|9/2, -7/2\rangle$) [8]. Other possible realizations are searched for in the heterostructural fermion-boson mixtures such as: ^{6}Li (fermion) with ^{7}Li (boson) [9], ^{6}Li with ^{23}Na (boson) [10], ^{40}K (fermion) and ^{87}Rb (boson) [11], etc.

3 Realization of the BCS to BE crossover

Depending on a value of the detuning parameter ν there can occur various kinds of the superconductivity/superfluidity. For negative ν most of the particles are bosons which at critical T_c undergo condensation. The residual interactions (of the order v^4) ultimately induce the superfluid state which resembles the BE condensate of weakly interacting boson systems. In the opposite limit, when ν is positive and large (say $\nu > v$), boson energies are located far above the Fermi level and the system consists predominantly of fermions. Virtual exchange processes via the boson states generate then a kind of the BCS superconductivity of fermions.

The most interesting situation takes place for the intermediate case when ν is small (positive or negative) because fermions and bosons are roughly equally populated and hence their mutual interaction is most effective. From the previous studies (see for example the review paper [12]) it is known that transition temperature is optimal under such circumstances. In addition to high T_c value there arise various symptoms (precursor features) of the superfluid order already in the normal state. Precursor effects result from the quantum fluctuations which, unlike in the usual BCS systems, are very strong. Fluctuations manifest up to characteristic temperature T_p below which fermion pairs are being created, yet of only a short life-time.

Since near the Feshbach resonance (for small ν) precursor effects play considerable role the usual methods for identifying the transition to superconducting/superfluid transition in general fail. So far there are three indirect indications that superfluidity has been already achieved among the trapped fermion atoms. These indications are: (a) the resonance condensation of fermion pairs [13], (b) a qualitative change of the radial and axial modes of the trapping potential [14], and (c) a double peak structure observed in the radio-frequency spectroscopy [15]. First of them is only a necessary condition and is not sufficient to confirm the superfluidity. The second point emphasizes the role of hydrodynamic changes for the cigar shaped trapping potential [16]. In a remaining part of this work we focus on discussing the third indication and eventually also other related experiments whih can help to infer the superfluid transition.

4 The RF spectroscopy

Most of the condensed matter techniques investigating the superconducting materials rely upon detecting a gap in the single particle spectrum. Temperature at which such gap appears is regarded as T_c. Certainly this criterion is not valid here because of pseudogap.

One of feasible tunneling methods used on the trapped atoms is the radio-frequency (RF) spectroscopy [17]. The main idea is to excite selectively one of the fermion species by the appropriately adjusted short time (~ 1 s) laser impulses. Let us assume that laser is tuned to excite atoms from the state $|\downarrow\rangle$ to another hyperfine state which we denote by $|e\rangle$. Perturbation caused by the laser pulse can be described via

$$H_{RF} = \sum_{k} \frac{\delta_{RF}}{2}\left(c_{ke}^{\dagger} c_{ke} - c_{k\downarrow}^{\dagger} c_{k\downarrow} - b_{k}^{\dagger} b_{k}\right) + \sum_{k,p}\left[(M_{k,p} c_{k\downarrow}^{\dagger} c_{pe} + \sum_{q} D_{k,p,q} b_{q}^{\dagger} c_{k\uparrow} c_{pe}) + \text{h.c.} \right],$$

where the matrix elements $M_{k,p}$, $D_{k,p,q}$ are both proportional to the Rabi frequency and the RF detuning parameter is $\delta_{\rm RF} = E_{\rm RF} - \varepsilon_{ke} - \varepsilon_{k\downarrow}$ with $E_{\rm RF}$ being the photon energy. In the experimental setup one is measuring the single particle tunneling current of atoms transfered to $|e\rangle$ state what can be expressed by the expectation value $I(\delta_{\mathrm{RF}}) = \langle \dot{N}_e \rangle$. Because of lack of space we do not present the specific expressions for $I(\delta_{\mathrm{RF}})$ but it can be obtained within the linear response theory in a straightforward manner [18].

For physical understanding it is enough to explain that current $I(\delta_{\mathrm{RF}})$ is proportional to the density of single particle states $|\downarrow\rangle$. It seems that the recent measurements of Innsbruck group on ^{6}Li indeed provide the first evidence for observation of the pairing gap [15]. One should keep in mind however, that due to quantum fluctuations the gap itself is expected to exist even above T_c [19] thereof it cannot serve as a proof of the superfluid state. Some authors claimed that asymmetry of the tunneling current would be the needed indication of superfluidity. We have shown previously [20] that asymmetry is present also in the normal spectrum so this argument does not work either.

5 The Bragg scattering

The other method applicable for probing the single particle gap as well as the correlations between trapped fermions is the Bragg spectroscopy. Typical procedure is based on weak scattering of the ultracold atoms by a moving potential of the form $V_0 \cos(\boldsymbol{qr} - \omega t)$. Bragg potential can be formed, for instance, by *ac* Stark shift arising from two interfering laser fields [21]. Such spectroscopy was pioneered by NIST and MIT groups [22] who used it for investigation of the BE condensed boson atoms.

Usually the two laser beams used in the experiment are polarized and tuned in such a way to allow excitations from the state $|\downarrow\rangle$ to $|e\rangle$. The laser-atom interaction can be described by the following Hamiltonian

$$H_{\mathrm{Bragg}} = \tfrac{1}{2} V_0 \left(\rho_{\boldsymbol{q}}^{\dagger}\, e^{-i\omega t} + \rho_{\boldsymbol{q}}\, e^{i\omega t} \right), \tag{3}$$

where $\rho_{\boldsymbol{q}} = \sum_{\boldsymbol{k}} c^{\dagger}_{\boldsymbol{k}+\boldsymbol{q},\downarrow} c_{\boldsymbol{k},\downarrow}$ is the usual density operator of $\downarrow$ atoms. Bragg spectroscopy is sensitive to the density-density correlation function analyzed by the time-of-flight techniques. Thus measured *dynamic structure factor* is given by $S_\rho(\boldsymbol{q}, \omega) = \frac{1}{Z} \sum_{i,j} e^{-E_i/k_B T} \left| \langle i | \rho_{\boldsymbol{q}} | j \rangle \right|^2 \delta\left(\omega - E_j + E_i\right)$, where Z is the partition function and E_i denote eigenenergies of the unperturbed Hamiltonian (1). For the ground state of BCS superconductor [23] one finds [24]

$$S_\rho^{\mathrm{BCS}}(\boldsymbol{q}, \omega) = \sum_{\boldsymbol{k}} \left| u_{\boldsymbol{k}+\boldsymbol{q}}\, v_{\boldsymbol{k}} \right|^2 \delta\left(\omega - E_{\boldsymbol{k}+\boldsymbol{q}} - E_{\boldsymbol{k}}\right), \tag{4}$$

where $|u_{\boldsymbol{k}}|^2, |v_{\boldsymbol{k}}|^2 = \frac{1}{2}[1 \pm \xi_{\boldsymbol{k}}/E_{\boldsymbol{k}}]$ with $\xi_{\boldsymbol{k}} = \varepsilon_{\boldsymbol{k}} - \mu$ and $E_{\boldsymbol{k}} = \sqrt{\xi_{\boldsymbol{k}}^2 + \Delta^2}$. Bragg pulses are thus absorbed only at frequencies $\omega \geq 2\Delta$ so this method can measure value of the gap. In practice, expression (4) should be modified taking into account finite temperature $T \neq 0$ and the effect of spatial variation of the trapping potential $V_{\mathrm{trap}} = \frac{m}{2}\left[\omega_\perp (x^2 + y^2) + \omega_\parallel z^2\right]$. Gap parameter becomes then space-dependent $\Delta(\boldsymbol{r})$ and thereof the sharp absorption edge evolves into a kink occuring in the function $S_\rho(\boldsymbol{q}, \omega)$ at energy $\omega = 2 \max\left\{\Delta(\boldsymbol{r})\right\}$ (see figure 1 in Ref. [24]).

Besides measuring a value of the single particle gap Bragg spectroscopy can also detect some collective features. Density-density correlation function $\langle \rho_{\boldsymbol{q}}(t)\, \rho_{\boldsymbol{q}}(0) \rangle$ was shown [25, 26] to be convoluted with the phase and amplitude fluctuations of the order parameter. Collective phase oscillations (*phasons*) are characterized below T_c by the famous Goldstone mode [27]. This mode shows up in the dynamic structure factor $S_\rho(\boldsymbol{q}, \omega)$ as a narrow peak appearing in the long wave-length limit $\boldsymbol{q} \to 0$. Such property could be used in the future studies as a tool for identifying the superfluid state. Theoretical studies focused so far mainly on the temperature regime $T \leq T_c$ but in our recent paper [28] we also presented calculations both for the pseudogap $T_p > T > T_c$ and superconducting/superfluid states.

6 Pair excitations

Probably the best and unambiguous way for investigating the transition to superfluid state might be achieved by analysis of the pair excitation spectrum. Spectral function of the fermion pair operator $\pi_{\mathbf{q}}^{\dagger} = \frac{1}{N}\sum_{\mathbf{k}} c_{\mathbf{q}-\mathbf{k}\uparrow}^{\dagger} c_{\mathbf{k}\downarrow}^{\dagger}$ is defined as $S_{\pi}(\mathbf{q},\omega) = \frac{1}{Z}\sum_{i,j} e^{-E_i/k_B T} \left|\langle i|\,\pi_{\mathbf{q}}\,|j\rangle\right|^2 \delta\left(\omega - E_j + E_i\right)$. Close to the Feshbach resonance (i.e. for $\nu \sim 0$) this spectral function was shown [28] to become

$$S_{\pi}(\mathbf{q},\omega) = W_{\mathbf{q}}^{\mathrm{coh}}\,\delta\left[\omega - (\tilde{E}_{\mathbf{q}} + 2\nu - 2\mu)\right] + \frac{1}{N}\sum_{\mathbf{k}} W_{\mathbf{q},\mathbf{k}}^{\mathrm{inc}}\,\delta\left[\omega - (\tilde{\varepsilon}_{\mathbf{k}} - \mu) - (\tilde{\varepsilon}_{\mathbf{q}-\mathbf{k}} - \mu)\right]. \tag{5}$$

$W_{\mathbf{q}}^{\mathrm{coh}}$ and $\sum_{\mathbf{k}} W_{\mathbf{q},\mathbf{k}}^{\mathrm{inc}}$ are the weights of coherent and incoherent parts in the fermion pair spectrum which along with renormalized energies $\tilde{E}_{\mathbf{k}}$, $\tilde{\varepsilon}_{\mathbf{k}}$ were obtained by the continuous diagonalization procedure [19].

In the superfluid state the single particle fermion energies $\tilde{\varepsilon}_{\mathbf{k}}$ become gapped around the Fermi level and hence the incoherent part of the pair spectrum (5) forms only outside the energy window $|\omega| \geq 2\Delta_{sc}$. The coherent part on the other hand is characterized by a gapless linear (*first sound*) mode $\lim_{\mathbf{q}\to 0} \tilde{E}_{\mathbf{q}} + 2\nu - 2\mu \propto |\mathbf{q}|$ which appears at small energies as a narrow peak in $S_{\pi}(\mathbf{q},\omega)$. Since incoherent background is expelled to $\omega > 2\Delta_{sc}$ this coherent branch becomes well detectable. Existence of such Goldstone mode signifies the broken symmetry caused by the order parameter $\langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}\rangle \neq 0$ but in the charged superconducting systems could not be observed due to the long range Coulomb interactions lifting it to plasma frequencies [27]. The neutral fermion atom gases are very good candidates for this mode to be observed. Its observation would prove achievement of the atom superfluidity.

In the normal state the Goldstone mode fades away. For low momenta the pair dispersion $\tilde{E}_{\mathbf{q}}$ becomes parabolic (massive) and moreover overlaps with the incoherent background. Thereof it is hardly visible at all. However, at sufficiently large momenta exceeding critical value $q_{\mathrm{crit}}(T)$ the coherent part splits off from the incoherent background and again there appears a remnant of the linear branch [28]. Sound velocity of such branch seems to be rather independent of temperature what is typical for the strongly interacting boson systems [29].

7 Summary

We studied some signatures of superfluidity possible to induce in the trapped fermion atoms by the Feshbach resonance. We claim that, unlike in the usual BCS systems, one cannot rely upon appearance of the single particle gap as a criterion for T_c because such gap exists there even in the normal state. Experimental methods should focus, in our opinion, on analysis of other many-body effects. For instance the pair excitation spectrum is expected to undergo qualitative changes near transition temperature. Appearance of pseudogap in the normal state is usually accompanied by emergence of the fermion pairs [19] whose life-time gradually increases for temperature approaching T_c. The pair excitations reveal a remnant of the collective first sound at sufficiently large momenta $q > q_{\mathrm{crit}}(T)$ and this feature can be detected in practice by the Bragg spectroscopy. At $T \leq T_c$ fermion pairs become the infinite life-time objects and their coherence spreads over the large distance. In consequence the collective mode extends down to the zero $q_{\mathrm{crit}}(T < T_c) = 0$. This property is one of possible ways for determination of the transition temperature.

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