Remnant superfluid collective phase oscillations in the normal state of systems with resonant pairing

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The signature of superfluidity in bosonic systems is a sound-wave-like spectrum of the single particle excitations which in the case of strong interactions is roughly temperature independent. In Fermionic systems, where Fermion pairing arises as a resonance phenomenon between free Fermions and paired Fermionic states (examples are: the atomic gases of ^{6}Li or ^{40}K controlled by a Feshbach resonance, polaronic systems in the intermediary coupling regime, *d*-wave hole pairing in the strongly correlated Hubbard system), remnants of such superfluid characteristics are expected to be visible in the normal state. The single particle excitations maintain a sound-wave-like structure for wave vectors above a certain $q_{\min}(T)$ where they practically coincide there with the spectrum of the superfluid phase for $T<T_c$. Upon approaching the transition from above this region in q space extends down to small momenta, except for a narrow region around $q=0$ where such modes change into damped free particle like excitations.

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I. INTRODUCTION

Approaching the transition to a superconducting or superfluid state from above, one can (under certain conditions) observe incipient macroscopic features which are caused by the emergence of an order parameter. In classical superconductors such features, related to spatial order parameter fluctuations, are restricted to only an extremely narrow temperature region around the superconducting critical temperature T_c, and in practice are hard to detect. Such fluctuations however are visible in systems with real space pairing or, more generally, when the overlap between the pair wave functions is small and we are in the crossover regime between a BCS type superfluidity of Cooperons and a superfluid phase of tightly bound Fermions which behave as bosons. Remnants of superfluidity, sometimes termed *localized superfluidity*, above T_c have been observed[1] in the form of finite range phase correlations in purely bosonic systems such as liquid ^{4}He in porous media of vicors and aerogels, with a characteristic disorder and confinement. Similar features have been seen for Fermionic systems such as ^{3}He in aerogels[2] and superconducting heterostructures.[3] Solution to the theoretical questions raised in this connection lies in a formulation capable of describing on equal footing a BCS-type superconductivity in a system of weakly coupled Fermions and a Bose-Einstein condensation (BEC) of strongly bound Fermion pairs. Early attempts to do that go go back to the work of Leggett[4] and Nozières and Schmitt-Rink[5] and rely on crossover scenarios where electron pairing is given by some unspecified effective attraction between them.

Fermionic systems where the binding between Fermions comes about from an exchange interaction between free itinerant Fermions and two-Fermion bound states, present a different scenario to examine the crossover regime between a BCS-type superfluidity and a condensed states of tightly bound pairs. Such systems have moreover the advantage that sometimes, in real systems, the crossover can be tuned experimentally. An example for such scenarios are Many Polaron systems in the intermediary coupling regime where free itinerant electrons engage in a resonant scattering process with weakly bound bipolaronic states when their respective energy difference is small.[6] This leads to long lived electron pairs which ultimately can condensate. An other example, now widely studied in the literature in connection with their condensation,[7] are gases of Fermionic atomic (such as ^{6}Li and ^{40}K atoms) which can be brought into such resonant Fermionic pair states via a so called Feshbach resonance mechanism[8] which involves hyperfine spin–flip processes between the nuclear and the electronic spins of the atoms together with their molecular counterparts. Finally, also in the highly debated scenarios for the high temperature superconductors (HTSC) resonant pairing between *d*-wave holes has been invoked. It has been suggested that such pairing arises from an exchange between itinerant holes and bound hole pairs in plaquette RVB states on finite clusters.[9]

In all those systems resonant pairing leads to long lived electron pairs which ultimately are driven into a superfluid phase. Furthermore such systems are characterized by strongly interdependent dynamics of single- and two-particle excitations which, upon approaching and passing through the superconducting phase transition, simultaneously undergo qualitative changes. Thus, the opening of a pseudogap in the single particle spectrum, when T_c is approached from above, occurs concomitantly with a changeover from single particle Fermionic transport to one ensured by bosonic molecular entities.[10] The observed transient Meissner effect[11] and a Nernst effect[12] in the normal phase in HTSC can be considered to be signatures of that. In the atomic gases the physics is more involved because of the strong inhomogeneous character of those trapped gases, leading to radial and axial breathing modes[13] instead of the usual sound-wave- like excitation spectrum known in translational invariant homogeneous superfluids. Nevertheless, corresponding manifestation of superfluid fluctuations in the normal state should also be expected in those systems.

In this paper we will analyze the molecular (and/or Fermion-pair) excitation spectrum of such a general class of systems which can be described in terms of resonating pairs of Fermions and discuss how, on a finite length scale, superfluid phase fluctuations can emerge upon approaching T_c from above. We restrict ourselves here to the study of homogeneous systems, leaving the more complex structures to be expected for remnant collective modes in inhomogeneous atomic gases in optical traps to a future work. The simplest, and generally adopted approach to study such systems, is on the basis of a phenomenological Boson–Fermion model.

Pairing in such a model can be viewed as the Andreev scattering processes between itinerant carriers and bosonic bound pairs on small clusters. One then is faced on one hand with local intracluster phase correlations between pairs of itinerant Fermions and Bosonic bound Fermion pairs and on the other hand with nonlocal intercluster phase correlations.[14,15] The first ones play the role of local density fluctuations and the second ones of effective intersite Josephson coupling. This physics, which is an intrinsic ingredient of the various representative examples cited above and which are effectively realized in nature, is qualitatively different from that of the standard crossover scenarios based on effective attractive interparticle interactions. It leads to features such as superfluid–insulator transitions, and lets one envisage the possibility of normal state bose metals and exotic elementary as well as collective excitations which remain to be fully explored.[15]

II. THE MODEL

The following boson Fermion model (BFM) Hamiltonian for resonant pairing

$$H=\sum_{\mathbf{k},\sigma}\varepsilon_{\mathbf{k}}c^{\dagger}_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}+v\sum_{\mathbf{k},\mathbf{q}}(b^{\dagger}_{\mathbf{q}}c_{\mathbf{q}-\mathbf{k}\downarrow}c_{\mathbf{k}\uparrow}+\text{h.c.})+\sum_{\mathbf{q}}(E_{\mathbf{q}}+2\nu)b^{\dagger}_{\mathbf{q}}b_{\mathbf{q}}. \qquad (1)$$

is currently employed in studies of the abovementioned systems. The operators $c^{\dagger}_{\mathbf{k}\sigma}(c_{\mathbf{k}\sigma})$ correspond, according to the physical system we are studying, to the creation (annihilation) of either free electrons, or free itinerant holons or Fermionic atoms in one of two possible hyperfine configurations, denoted symbolically by $\sigma=\uparrow$ and $\sigma=\downarrow$. The energy $\varepsilon_{\mathbf{k}}$ of those Fermions is measured with respect to the chemical potential μ. Correspondingly, $b^{\dagger}_{\mathbf{q}}(b_{\mathbf{q}})$ refer to bound diatomic molecules of bosonic character (either localized bipolarons, or bound hole pairs on plaquette RVB states or weakly bound pairs of atoms in a triplet configuration), having an energy $E_{\mathbf{q}}$ being measured with respect to 2μ. The parameter $2\nu=E_{q=0}-2\varepsilon_{\mathbf{k}_F}$ (where $\mathbf{k}_F$ is the Fermi momentum), denotes the difference in energy of the weakly bound Fermion pairs and the single Fermion scattering states. If ν is small, pairing will be introduced among the uncorrelated fermions via resonance scattering, tantamount to a Boson-Fermion pair exchange with coupling strength v. Tuning the value of ν, one can cover the whole regime between Cooper pairs and locally bound pairs and their corresponding condensed phases.

Such a BFM (1) has been introduced originally in solid state theory many years ago, in an attempt to describe the situation of intermediary electron-lattice coupling[16] and has been intensively studied over the last decade, mainly in connection with the pseudogap phenomenon in the HTSC. As shown recently,[17] this model does indeed capture the resonant-type scattering between Fermions due to the Feshbach mechanism and has been widely studied in connection with several issues of the atomic gas superfluidity.[18]

Our main objective here is to study the two-Fermion dynamical correlation functions when the *detuning* ν from the resonance is small, thus putting ourselves in the center of the crossover regime between a superfluid ground state of BCS characteristics and one corresponding to tightly bound Fermion pairs of bosonic character. The Green's function describing the Fermion pairs $G^{\text{pair}}(\mathbf{q},\omega)$ is related to the single particle boson propagator via $G^{B}(\mathbf{q},\omega)=G^{B}_{0}(\mathbf{q},\omega)+v^{2}G^{B}_{0}(\mathbf{q},\omega)G^{\text{pair}}(\mathbf{q},\omega)G^{B}_{0}(\mathbf{q},\omega)$, where $G^{B}_{0}(\mathbf{q},\omega)=[\omega-E_{\mathbf{q}}-2\nu]^{-1}$. This implies that the excitation energies of the bound molecules and Fermionic diatomic pairs are *identical*. Only the spectral weights differ as can be seen from the relation between their spectral functions, i.e., $A^{\text{pair}}(\mathbf{q},\omega)=v^{-2}(\omega-E_{\mathbf{q}}-2\nu)^{2}A^{B}(\mathbf{q},\omega)$. It is thus sufficient to determine one of these functions in order to derive the excitation spectra for both.

III. THE PROCEDURE

The interdependence between the single- and two-particle correlations is required to treat them on equal footings. For that purpose we employ a continuous renormalization group procedure[19] which, through a set of infinitesimal canonical transformations, reduces the initial Hamiltonian (1) to an essentially diagonalizable form, containing the relevant physics which we want to describe, plus additional terms which can be treated as small perturbations. Contrary to standard renormalization group techniques, where one integrates out the high energy states and subsequently derives an effective low energy Hamiltonian, in this method both, the high and low energy sectors, are renormalized and kept throughout the whole transformation process.

The specific construction of such a procedure for the BFM was given previously,[20] where also the single particle spectrum of the Fermionic atoms, pointing to Bogoliubov-like excitations below as well as above T_c was studied.[21] We apply here this procedure for the study of the boson spectral function. In the course of diagonalizing the Hamiltonian, the boson operators evolve toward a form given by

$$\tilde{b}_{\mathbf{q}}=\tilde{\mathcal{A}}_{\mathbf{q}}b_{\mathbf{q}}+\frac{1}{\sqrt{N}}\sum_{\mathbf{k}}\tilde{\mathcal{B}}_{\mathbf{q},\mathbf{k}}c_{\mathbf{k}\downarrow}c_{\mathbf{q}-\mathbf{k}\uparrow},\ \tilde{b}^{\dagger}_{\mathbf{q}}=(\tilde{b}_{\mathbf{q}})^{\dagger}. \qquad (2)$$

The two complex coefficients appearing in Eq. (2) are calculated in the limit of the convergence of the renormalization flow procedure $\lim_{l=\infty}\boldsymbol{\mathcal{A}}_{\mathbf{q}}(l)=\tilde{\boldsymbol{\mathcal{A}}}_{\mathbf{q}}$ and $\lim_{l=\infty}\boldsymbol{\mathcal{B}}_{\mathbf{q},\mathbf{k}}(l)=\tilde{\boldsymbol{\mathcal{B}}}_{\mathbf{q},\mathbf{k}}$, where l denotes the continuous flow parameter. We base ourselves on the general relations which describe the evolution of operators[21]

$$dO(l)/dl = [\eta(l), O(l)] \tag{3}$$

with η being judiciously chosen[19] as

$$\eta(l) = \frac{1}{\sqrt{N}} \sum_{\mathbf{k},\mathbf{p}} \alpha_{\mathbf{k},\mathbf{p}}(l)(c^\dagger_{\mathbf{p}\uparrow} c^\dagger_{\mathbf{k}\downarrow} b_{\mathbf{p}+\mathbf{k}} - \text{h.c.}), \tag{4}$$

and $\alpha_{\mathbf{k},\mathbf{p}}(l)=[\varepsilon_{\mathbf{k}}(l)+\varepsilon_{\mathbf{p}}(l)-E_{\mathbf{k}+\mathbf{p}}(l)]v_{\mathbf{k},\mathbf{p}}(l)$.[20] Coefficients $A_{\mathbf{q}}(l)$ and $B_{\mathbf{q},\mathbf{k}}(l)$ satisfy the renormalization equations

$$\frac{d\mathcal{A}_{\mathbf{q}}(l)}{dl} = -\frac{1}{N}\sum_{\mathbf{k}} \alpha_{\mathbf{k},\mathbf{q}-\mathbf{k}}(l) f_{\mathbf{k},\mathbf{q}-\mathbf{k}} \mathcal{B}_{\mathbf{q},\mathbf{k}}(l), \tag{5}$$

$$\frac{d\mathcal{B}_{\mathbf{q},\mathbf{k}}(l)}{dl} = \alpha_{\mathbf{k},\mathbf{q}-\mathbf{k}}(l)\mathcal{A}_{\mathbf{q}}(l), \tag{6}$$

with the initial conditions $\mathcal{A}_{\mathbf{q}}(0)=1$, $\mathcal{B}_{\mathbf{q},\mathbf{k}}(0)=0$, and $f_{\mathbf{k},\mathbf{p}}=1 - n^F_{\mathbf{k}\downarrow} - n^F_{\mathbf{p}\uparrow}$.

This procedure leads finally to the following form of the spectral function for the bosonic molecules

$$A^B(\mathbf{q},\omega) = |\tilde{\mathcal{A}}_{\mathbf{q}}|^2 \delta(\omega - \tilde{E}_{\mathbf{q}}) + \frac{1}{N}\sum_{\mathbf{k}} f_{\mathbf{k},\mathbf{q}-\mathbf{k}} |\tilde{\mathcal{B}}_{\mathbf{q},\mathbf{k}}|^2 \times \delta(\omega - \tilde{\varepsilon}_{\mathbf{k}} - \tilde{\varepsilon}_{\mathbf{q}-\mathbf{k}}). \tag{7}$$

The first term of Eq. (7) describes long-lived quasiparticles with the renormalized energy $\tilde{E}_{\mathbf{q}}$[20] and whose spectral weight is $|\tilde{\mathcal{A}}_{\mathbf{q}}|^2$. The second term describes the incoherent background extending over the region determined by the renormalized Fermion energies $\tilde{\varepsilon}_{\mathbf{k}}$.[20] From Eqs. (5) and (6) we derive the following sum rule $|\tilde{\mathcal{A}}_{\mathbf{q}}|^2 + 1/N\Sigma_{\mathbf{k}}|\tilde{\mathcal{B}}_{\mathbf{q},\mathbf{k}}|^2 f_{\mathbf{k},\mathbf{q}-\mathbf{k}}=1$ which correctly preserves the total spectral weight $\int_{-\infty}^{\infty} d\omega A^B(\mathbf{q},\omega) = \langle [b_{\mathbf{q}}, b^\dagger_{\mathbf{q}}]\rangle = 1$.

IV. THE PAIR EXCITATION SPECTRUM BELOW T_c

At a certain critical temperature T_c the static pair susceptibility $\Sigma_{\mathbf{k},\mathbf{p}}\int_0^\beta d\tau e^{\tau\omega}\langle c^\dagger_{\mathbf{k}\uparrow}(\tau) c^\dagger_{\mathbf{q}-\mathbf{k}\downarrow}(\tau) c_{\mathbf{q}-\mathbf{p}\downarrow} c_{\mathbf{p}\uparrow}\rangle_{|\omega\to 0}$ becomes divergent for $\mathbf{q}=\mathbf{0}$ and, due to the Thouless criterion, the system undergoes a phase transition to a superfluid state. For $T<T_c$ two order parameters appear which are proportional to each other: $\chi_F \equiv \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}\rangle$ for the Fermions and $\chi_B \equiv \langle b_{\mathbf{q}=\mathbf{0}}\rangle$ for the bosons (atom molecules).

Near the Fermi energy, the single particle Fermionic excitations become gaped: $\tilde{\varepsilon}_{\mathbf{k}} = \text{sgn}\{\varepsilon_{\mathbf{k}}\}\sqrt{(\varepsilon_{\mathbf{k}})^2 + (v\chi_B)^2}$. In consequence, no Fermionic states, neither coherent nor incoherent, exist within the energy window $|\omega| \leqslant v\chi_B$.[21] This simultaneously affects the incoherent part of bosonic spectrum, as can be seen from Eq. (7). For the long wavelength limit $\mathbf{q}\to\mathbf{0}$ the incoherent background is pushed up to energies $|\omega| > 2v\chi_B$ and thus permits long-lived excitations, which correspond to collective modes, known as *first sound* for interacting bosonic systems in the superfluid state (see Fig. 1). The temperature dependence of these modes has previously been studied for this BFM[23] in the superfluid phase within a framework of the dielectric formalism with use of the Ward identities, currently employed in the theory of interacting bose gases. A behavior similar to that of the strong

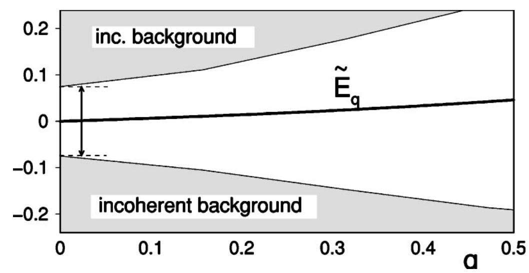

FIG. 1. The characteristic sound-wave mode $\tilde{E}_{\mathbf{q}}=v|\mathbf{q}|$ of the long-lived boson and/or Fermion pair excitation spectrum in the superfluid state at $T=0$. The shaded regions show the incoherent background, which is energetically separated from the collective excitation branch. At $\mathbf{q}=\mathbf{0}$ the incoherent background exists for energies larger than $2v\chi_B$ (twice the value of the single particle Fermion gap). We used the following dispersions $\varepsilon_{\mathbf{k}}=-D/2\cos(a|\mathbf{k}|)$ and $E_{\mathbf{q}}=-D/4\cos(a|\mathbf{q}|)$ such that the mass $m_B=2m_F$ and the potential $v=0.1D$. We further set the lattice constant $a=1$ and use the bandwidth D as a unit for energies.

coupling limit of interacting bose gases[24] was found, showing a sound velocity being little dependent on temperature as one traverses the superfluid transition, but whose spectral weight in the boson single particle spectral function disappears upon approaching T_c.

Such sound wave-like modes are not realized in charged superconducting systems because of the long range Coulomb interaction which pushes them up to the generally huge plasma frequency.[25] For electrically neutral atoms, such as the trapped atomic gases, this is no longer the case and hence one can realistically expect collective sound-wave-like modes, although appropriately modified due to the inhomogeneous structure of the gas density.[13]

V. THE PAIR EXCITATION SPECTRUM ABOVE T_c

Decreasing the temperature in the normal state below a certain $T^*(>T_c)$ one expects precursor pairing effects which show up in the single particle Fermionic excitations spectrum in the form of a pseudogap which opens up near the chemical potential.[20–22] Above T^* the low energy part of the pair excitations has the usual parabolic dispersion. However, upon decreasing the temperature and approaching T_c, phase coherence gradually sets in on a finite length and time scale, which becomes visible in form of a linear in q dispersion of the single boson (respectively, Fermion pair) excitation for small q vectors, in an interval $[q_{\min}(T), q_{\max}(T)]$ (see Fig. 2). There, the derivative of the effective Bose single particle energy spectrum $d\tilde{E}_q/dq$ shows a flat portion, which, when extrapolated to $q=0$, practically coincides with the corresponding quantity in the superfluid phase at $T=0$. We observe that, as the temperature is decreased, $q_{\min}(T)$ decreases toward zero, but always leaving a small interval in q space $[0, q_{\min}(T)]$ where one clearly observes a free particle like spectrum with an effective mass which decreases as T decreases. This is in accordance with an earlier study on this subject using self-consistent perturbation theory.[22] For $T \geqslant T^*$ the coherent boson mode overlaps with an incoherent

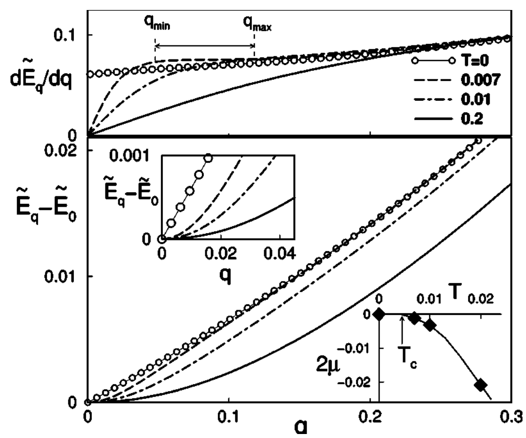

FIG. 2. Comparison of the dispersion $\tilde{E}_{\mathbf{q}}$ of the coherent part of the boson spectral function at temperatures corresponding to the superfluid ($T=0$), pseudogap (0.007, 0.01) and the normal phase above T^* (0.02). Upper panel shows the derivative $d\tilde{E}_{\mathbf{q}}/dq$ of these curves. The insets contain correspondingly: the low momentum q limit of the dispersion $\tilde{E}_{\mathbf{q}}$ and the temperature dependence of chemical potential (the marked points correspond to four temperatures $T=0.02$, 0.01, 0.007, and 0 chosen in this work).

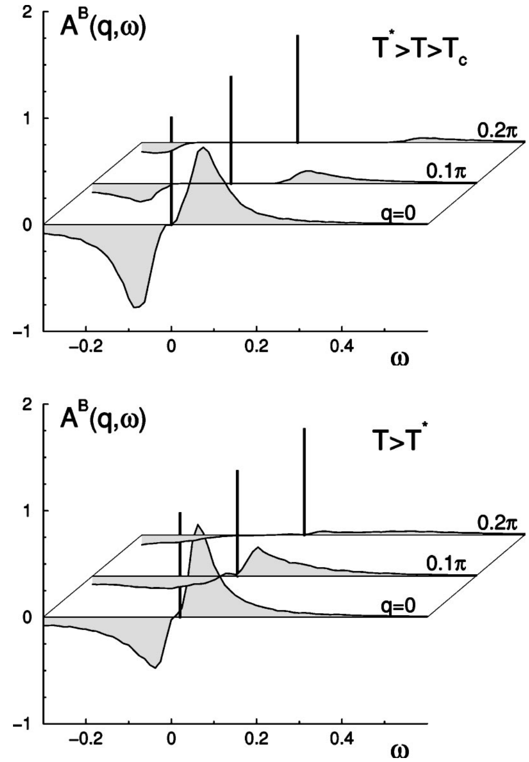

FIG. 3. The boson spectral function $A^B(\mathbf{q},\omega)$ for the low energy pair excitations. The upper panel corresponds to $T=0.007$ being close to T_c in the pseudogap region $T^*>T>T_c$. The bottom panel refers to the normal state $T=0.02$ (above T^*). In the pseudogap phase a propagating coherent contribution given by the δ-function peak and an incoherent background, given by the shaded regions, get separated above some relatively small critical momentum $q_{\min}$. This is no longer the case for $T>T^*$.

background in the single particle boson spectral function (see Fig. 3, bottom panel). However, upon decreasing the temperature to below T^*, we observe that this incoherent background moves away from the position of the coherent contribution (upper panel of Fig. 3) which ensures that a linear in q branch of the boson spectrum is well defined in the corresponding interval of q vectors. This strongly suggests that remnants of the first sound still can exist as part of the single particle boson spectrum above the superfluid phase transition for a limited region of wave vectors due to a persistence of superfluid phase correlations above T_c on a finite length and time scale.

VI. CONCLUSIONS

We studied the qualitative changes of the excitation spectrum for the resonant Fermion pairs which occur upon varying the temperature. We found that quantum fluctuations play a crucial role when detuning ν from the Feshbach resonance is small. Fluctuations manifest themselves in the pseudogap regime $T^*>T>T_c$.

Far above T_c the off-diagonal long range order is not established. The pair excitation spectrum for small q vectors is then characterized by a parabolic branch (see Fig. 2) and overlaps with the incoherent background (see the bottom panel of Fig. 3) such as to effectively destroy any bosonic quasiparticle features.

This situation changes dramatically when the temperature drops below T^* where resonant pairing sets in. Phase correlations start to build up on a finite spatial and temporal scale as the temperature decreases and approaches T_c.[26] The single particle Fermion spectrum reveals then a partial suppression of states (pseudogap) around the Fermi energy,[20–22] which is accompanied by qualitative changes in the pair excitation spectrum. Quantum fluctuations lead to emergence of the collective sound-wave mode which above T_c exists in a finite momentum interval $[q_{\min}(T), q_{\max}(T)]$. Upon decreasing the temperature the long-lived branch of the pair spectrum gradually splits off from the incoherent background (upper panel in Fig. 3) and spreads over a wider and wider momentum region, with $q_{\min}(T)$ steadily decreasing as we approach T_c. We note however that invariably the linear in q dispersion changes into a damped free particle like behavior in the close vicinity of $q=0$.

The sound-wave mode has been so far measured above T_c in the liquid helium by ultrasonic techniques[27] as well as by neutron scattering measurements.[1] In the case of the trapped Fermionic atoms the corresponding mode is expected to be compressional density waves and, similar to the present study, one should expect remnants of those modes in the normal state. In principle, such modes can be experimentally checked by the Bragg spectroscopy.[28] Indirect methods for detecting the collective modes which rely on measuring the magnetic susceptibility and density–density correlation functions have been discussed (although only for $T<T_c$) in Ref. 29. In some future work we shall discuss how collective modes can possibly be observed in measurements of the magnetic susceptibility in the pseudogap regime above T_c.

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