# DISORDER INDUCED CHANGES OF *d*-WAVE PAIRING AMPLITUDE IN THE BOSON FERMION MODEL*

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We investigate a system composed of the itinerant electrons coexisting with the localized pairs (hard-core bosons) whose energies are assumed to be site dependent. The randomness of boson energies induces in turn fluctuations of the effective pairing potential between fermions. We analyze an effect of such randomness on the anisotropic $d$-wave superconducting phase of this model. In particular, we determine transition temperature $T_{\rm c}$ and the amplitude of the order parameter parameter $\chi_{ij} = \langle c_{i\downarrow} c_{j\uparrow} \rangle$ as functions of the boson energies fluctuations.

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The boson fermion model [1] has been recently proposed for a description of the superconducting copper oxide materials [2–5]. It has been argued that in these materials there exists a mixture of local immobile objects of bosonic character coupled to the mobile conduction electrons (fermions). Due to the gauge invariant form of their interaction both subsystems simultaneously undergo a transition to superfluid/superconducting phase.

The order parameter of the high temperature superconductors is known to have $d$-wave symmetry. This requires an appropriate modification of the original isotropic boson fermion model. It has also to be taken into account that these materials are quite disordered. Impurities are introduced either in the doping process, like Sr in $\mathrm{La}_{2-x}\mathrm{Sr}_x\mathrm{CuO}$, or is some other way. They are expected to be effective pair breakers for $d$-wave superconductor. It is the purpose of this work to study how the nonmagnetic impurities affect the properties of the $d$-wave superconducting phase of conduction electrons.

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We assume here that the impurities enter mainly through the random energy levels of the localized bosons. We thus consider the boson fermion model [1, 2, 4] described by the Hamiltonian

$$\begin{aligned} H^{\mathrm{BF}} &= \sum_{i,j,\sigma}(t_{ij} - \delta_{i,j}\mu)c^{\dagger}_{i\sigma}c_{j\sigma} + \sum_{i}\left(\Delta_B + \delta\Delta^i_B - 2\mu\right) b^{\dagger}_i b_i \\ &+ \frac{1}{2}\sum_{i,j} v_{ij}\left(b^{\dagger}_i c_{i\downarrow}c_{j\uparrow} + b_i c^{\dagger}_{i\uparrow}c^{\dagger}_{j\downarrow}\right) , \end{aligned} \tag{1}$$

with the anisotropic interaction potential $v_{ij}$ [5, 6] and the random boson energies $\Delta_B + \delta\Delta^i_B$. We use standard notation for fermion $c^{(\dagger)}_{i,\sigma}$ and boson $b^{(\dagger)}_i$ operators. $t_{ij}$ is the hopping integral of fermions, and $\mu$ stands for the common chemical potential. It is worth mentioning that this boson fermion Hamiltonian with anisotropic (inter-site) exchange coupling $v_{ij}$ has been recently derived from the resonating valence bond state of the $t - J$ model [7].

We consider two dimensional square lattice with a tight binding dispersion $\varepsilon_{\boldsymbol{k}} = 2t\,(\cos k_x a + \cos k_y a)$ and assume the interaction potential to be non-zero only between the nearest neighbor sites such that $v_{ij} = +v\ if\ \vec{r}_i - \vec{r}_j\ ||\ \hat{x}$ and $v_{ij} = +v\ if\ \vec{r}_i - \vec{r}_j\ ||\ \hat{x}$. If this interaction is small we can apply the Hartree–Fock–Bogolubov decoupling $b^{\dagger}_i c_{i\downarrow}c_{j\uparrow} \simeq \langle b_i\rangle^{*} c_{i\downarrow}c_{j\uparrow} + b^{\dagger}_i \langle c_{i\downarrow}c_{j\uparrow}\rangle$ which leads to decomposition of the Hamiltonian into fermion and boson parts $H = H^F + H^B$. Boson part is given by

$$H^B = \sum_i \left[E_i b^{\dagger}_i b_i + x_i\, b^{\dagger}_i + x^{*}_i b_i\right] , \tag{2}$$

where $E_i = \Delta_B + \delta\Delta^i_B - 2\mu$ and $x_i = 1/2\sum_{\langle j\rangle} v_{ij}\,\langle c_{i\downarrow}c_{j\uparrow}\rangle$, the summation goes here only over $\langle j\rangle$ which are the nearest neighbor sites with respect to $i$. For a given random configuration $\delta\Delta^i_B$ we can compute the thermodynamically averaged quantities $\langle\ldots\rangle = \mathrm{Tr}\left\{\ldots e^{-\beta H^B}\right\} / \,\mathrm{Tr}\left\{e^{-\beta H^B}\right\}$ using a suitable canonical transformation [2, 4, 8]. In particular we obtain

$$\left\langle b^{\dagger}_i b_i\right\rangle = \frac{1}{2} - \frac{E_i}{4\gamma_i}\tanh\left(\frac{\gamma_i}{k_{\mathrm{B}}T}\right), \tag{3}$$

$$\langle b_i\rangle = -\frac{x_i}{2\gamma_i}\tanh\left(\frac{\gamma_i}{k_{\mathrm{B}}T}\right), \tag{4}$$

where $\gamma_i = \sqrt{(E_i/2)^2 + |x_i|^2}$.

The fermion part is more difficult to deal with. Formally the Hamiltonian has the standard BCS-structure

$$H^F = \sum_{i,j,\sigma} (t_{ij} - \delta_{ij}\mu)\, c^\dagger_{i\sigma} c_{i\sigma} + \sum_{i,j} \left( \Delta^*_{ij} c_{i\downarrow} c_{j\uparrow} + \Delta_{ij} c^\dagger_{j\uparrow} c^\dagger_{i\downarrow} \right) , \tag{5}$$

however, $\Delta_{ij} = \frac{1}{2} v_{ji} \langle b_j \rangle$ are random quantities due to (4). This situation corresponds to the random inter-site attractive interactions between fermions. Introducing the Green's function $\boldsymbol{G}(i,j;\omega) = \left\langle\left\langle \Psi_i ; \Psi^\dagger_j \right\rangle\right\rangle_\omega$ in the Nambu representation $\Psi^\dagger_i = (c^\dagger_{i\uparrow}, c_{i\downarrow})$, $\Psi_i = (\Psi^\dagger_i)^\dagger$ we have

$$\sum_l \left[ \begin{array}{cc} (\omega+\mu)\delta_{il} - t_{il} & -\Delta_{il} \\ -\Delta^*_{il} & (\omega-\mu)\delta_{il} + t_{il} \end{array} \right] \boldsymbol{G}(l,j;\omega) = \mathbf{1}\delta_{ij} \, . \tag{6}$$

As a first step we can replace random quantities $\Delta_{ij}$ by their averaged value (over all the possible boson energy configurations) $\Delta_{ij} \simeq \overline{\Delta}_{ij} = \sum_\alpha P_\alpha \Delta^{(\alpha)}_{ij}$ In such case the fermion system becomes homogenous and can be easily solved in the momentum space

$$[\boldsymbol{G}(\boldsymbol{k};\omega)]^{-1} = \left( \begin{array}{cc} \omega - \varepsilon_{\boldsymbol{k}} + \mu & -\Delta_{\boldsymbol{k}} \\ -\Delta^*_{\boldsymbol{k}} & \omega + \varepsilon_{\boldsymbol{k}} - \mu \end{array} \right) , \tag{7}$$

where

$$\Delta_{\boldsymbol{k}} = \frac{1}{2} \sum_{\langle j \rangle} e^{i\boldsymbol{k}(\boldsymbol{r}_i - \boldsymbol{r}_j)} v_{ij} \overline{\langle b_j \rangle} = \overline{\langle b \rangle} v \left(\cos k_x a - \cos k_y a\right) \, . \tag{8}$$

We obtain the standard BCS relations [2, 4] for the number operator $n^{\mathrm{F}} = 1 - \sum_{\boldsymbol{k}} \frac{\varepsilon_{\boldsymbol{k}} - \mu}{E_{\boldsymbol{k}}} \tanh\left(\frac{E_{\boldsymbol{k}}}{2k_{\mathrm{B}}T}\right)$ where $E_{\boldsymbol{k}} = \sqrt{(\varepsilon_{\boldsymbol{k}} - \mu)^2 + \Delta^2_{\boldsymbol{k}}}$ and for the order parameter $\langle c_{-\boldsymbol{k}\downarrow} c_{\boldsymbol{k}\uparrow} \rangle = -\frac{\Delta_{\boldsymbol{k}}}{2E_{\boldsymbol{k}}} \tanh\left(\frac{E_{\boldsymbol{k}}}{2k_{\mathrm{B}}T}\right)$.

For a brief illustration we consider the symmetric boson fermion model with half-filled boson and fermion subsystems ($\Delta_B = 0$, $\mu = 0$). We assume also the symmetric random boson energy distribution in a bimodal form [9] $\delta\Delta^i_B = \pm\delta\Delta_B$ with equal probabilities $\frac{1}{2}$. In this case the value of $\left\langle b^\dagger_i b_i \right\rangle$ (3) and $\langle b_i \rangle$ (4) at site with $\delta\Delta^i_B = \delta\Delta_B$ are equal to that corresponding to $\delta\Delta^i_B = -\delta\Delta_B$. For this particular choice of parameters the average gap $\overline{\Delta}_{ij}$ is identical with its actual value $\Delta_{ij} = \frac{1}{2} v_{ji} \langle b_j \rangle$ at arbitrary site.

Fig. 1 shows the transition temperature $T_{\mathrm{c}}$ calculated for $v/8t = 0.1$ and the zero temperature amplitude of the order parameter both in a function of $\delta\Delta_B$. As can be seen superconductivity is strongly destroyed be the

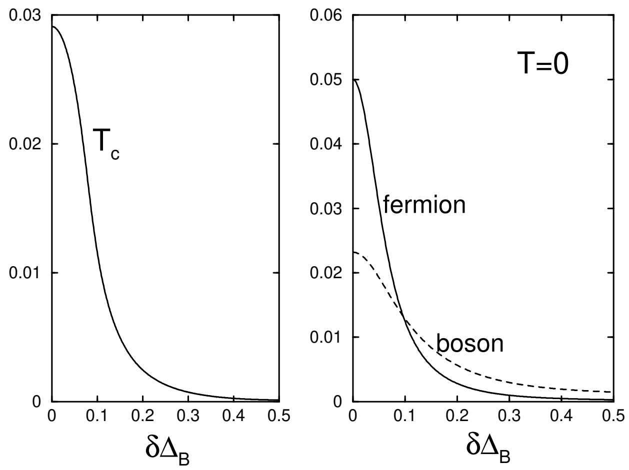


Fig. 1. Transition temperature (left panel) and the zero temperature order parameter (right panel) as functions of the fluctuation $\delta\Delta_B$ of the boson energy level. All quantities are expressed in units of the fermion band $8t \equiv 1$.

randomness of boson energies. This has to be compared with weaker effect of impurities on $s$-wave superconducting phase of the boson fermion model [8, 9].

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