RENORMALIZATION GROUP APPROACH TO THE INTERACTING BOSON FERMION SYSTEMS*

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We study a pseudo-gap region of the mixed boson fermion system using a recent formulation of the renormalization group technique through the set of infinitesimal unitary transformations. Renormalization of fermion energies gives rise to a depletion of the low energy states (pseudo-gap) for temperatures $T^* > T > T_c$ which foreshadows appearance of the pairwise correlations with their long range phase coherence being missed. With a help of the flow equations for boson and fermion operators we analyze spectral weights and finite life times of these quasi-particles caused by interactions.

Many experimental measurements of *e.g.* the Angle Resolved Photo-Emission (ARPES), tunneling spectroscopy (STM), specific heat as well as the optical and magnetic characteristics (see for an overview [1]) point at a presence of pseudo-gap in the normal phase spectrum of underdoped cuprate superconductors. It is believed that a proper description of this phenomenon would be crucial for understanding of the HTSC mechanism.

Model of itinerant fermions coupled via the charge exchange interaction to boson particles [2], for example of bipolaronic origin, has been shown to posses such a pseudo-gap in the normal phase fermion spectrum. Its occurrence has been obtained on a basis of the self-consistent perturbation theory [3], diagrammatic expansion [4] and also by solving the dynamical mean field equations for this model within the Non-Crossing Approximation

* Presented at the IX School of High T_c Superconductivity, Krynica Czarny Potok, June 10–14, 2001

(NCA) [5]. Unfortunately, these techniques have not been able to study a superconducting phase of the model, so evolution of the pseudo-gap into the true superconducting gap could not be investigated. Only very recently we have reported some prior results for the Boson Fermion (BF) model using the flow equation method [6] where superconducting gap and normal phase pseudo-gap could be treated on equal footing. Both gaps originate from the same source (from the exchange interaction between fermions and bosons) but they are differently scaled against a magnitude of this interaction and the total concentration of charge carriers. Still the most intriguing question concerning the evolution from the pseudo- to the superconducting gap is poorly understood due to technical difficulties (it is a very tough problem to solve the flow equations for $2+\epsilon$ dimensional system). Some attempts in this direction are currently under consideration.

In this paper we address another important issue concerning life time effects of the quasiparticles. The high resolution ARPES data [7] reveal that normal phase of underdoped cuprates has the marginal Fermi liquid type properties where the quasiparticle weight and life time are very sensitive to temperature. We want to check how the life times of fermions and bosons depend on temperature T and momentum using the BF model scenario.

Flow equation for any arbitrary operator, say $\hat{A}(l)$, is given by

$$\frac{d\hat{A}(l)}{dl} = \left[\hat{\eta}(l), \hat{A}(l)\right], \tag{1}$$

where $\hat{\eta}$ is a generating operator of the continuous canonical transformation. The initial BF model Hamiltonian consists of the free part

$$\hat{H}_0 = \sum_{k,\sigma} (\varepsilon_k^\sigma - \mu)\, \hat{c}_{k\sigma}^\dagger \hat{c}_{k\sigma} + \sum_q (E_q - 2\mu) \hat{b}_q^\dagger \hat{b}_q \tag{2}$$

and the interaction

$$\hat{H}_{\rm int} = \frac{1}{\sqrt{N}} \sum_{k,p} \left(v_{k,p}\, \hat{b}_{k+p}^\dagger \hat{c}_{k\downarrow} \hat{c}_{p\uparrow} + v_{k,p}^* \hat{b}_{k+p} \hat{c}_{p\uparrow}^\dagger \hat{c}_{k\downarrow}^\dagger \right) . \tag{3}$$

In order to decouple fermion from boson subsystems we have previously chosen η in a form [6]

$$\hat{\eta}(l) = -\sum_{k,q} \left(\alpha_{k,q}(l) v_{k,q}(l) \hat{b}^\dagger(l) \hat{c}_{k,\downarrow}(l) \hat{c}_{q,\uparrow}(l) - \text{h.c.} \right) \tag{4}$$

with $\alpha_{k,q} = \varepsilon_k^\downarrow + \varepsilon_q^\uparrow - E_{k+q}$. Using the general flow equation (1) we transformed the Hamilton operator $\hat{H}(l)$ so that in the limit $l \to \infty$ the charge exchange interaction got eliminated $v_{k,q}(\infty) = 0$.

In a course of the continuous canonical transformation the initial boson and fermion operators change according to the following flow equations

$$\frac{d\hat{b}_q(l)}{dl} = \sum_k \alpha_{k,q-k}(l)\, v_{k,q-k}(l)\, \hat{c}_{k,\downarrow}(l)\, \hat{c}_{q-k,\uparrow}(l)\,, \tag{5}$$

$$\frac{d\hat{c}_{k,\uparrow}(l)}{dl} = -\sum_q \alpha_{q,k}(l)\, v_{k,q}(l)\, \hat{b}_{q+k}(l)\, \hat{c}^{\dagger}_{q,\downarrow}(l)\,. \tag{6}$$

Similar equations can be derived for their Hermitean conjugates $\hat{b}^{\dagger}_q(l)$, $\hat{c}^{\dagger}_{k,\uparrow}(l)$. By inspecting (5) we notice that initial boson annihilation operator is coupled to the electron pair operator $\hat{c}_{k,\downarrow}(l)\hat{c}_{q-k,\uparrow}(l)$. Thereof one should in a next step determine a corresponding flow equation for the finite momentum pair operator, however, this process will never end (like equations of motion for the Greens functions). There would be the higher and higher order product of operators getting involved into the differential flow equations.

Following the other works using such continuous canonical transformation [8–10], we terminate the flow equations by postulating some (physical) Ansatz. In a context of the BF model it seems natural to postulate the following decompositions

$$\hat{b}_q(l) = X_q(l)\, \hat{b}_q + \sum_k Y_{q,k}(l)\, \hat{c}_{k,\downarrow}\, \hat{c}_{q-k,\uparrow}\,, \tag{7}$$

$$\hat{c}_{k,\sigma}(l) = A_k(l)\, \hat{c}_{k,\sigma} + \sum_q B_{k,q}(l)\, \hat{b}_{q+k}\, \hat{c}^{\dagger}_{q,-\sigma}\,, \tag{8}$$

where $\hat{b}_q \equiv \hat{b}_q(l=0)$ and $\hat{c}_{k,\sigma} \equiv \hat{c}_{k,\sigma}(l=0)$. The initial $l=0$ values are

$$\begin{array}{ll} A_k(0) = 1\,, & B_{k,q}(0) = 0\,, \\ X_q(0) = 1\,, & Y_{k,q}(0) = 0\,. \end{array} \tag{9}$$

Unknown parameters $Y_{q,k}(l)$, $B_{k,q}(l)$ can be found directly from the flow equations (5), (6)

$$\frac{dB_{k,q}(l)}{dl} = -\alpha_{q,k}(l)\, v_{q,k}(l)\, X_{q+k}(l) A_q(l)\,, \tag{10}$$

$$\frac{dY_{q,k}(l)}{dl} = \alpha_{k,q-k}(l)\, v_{k,q-k}(l)\, A_k(l) A_{q-k}(l)\,. \tag{11}$$

There is no unambiguous way for determining the other missing coefficients $A_k(l)$, $X_q(l)$ because with the Ansatz (7), (8) the flow equations still do not close. In what follows bellow we determine these parameters using the constraint

$$\left[b_q(l), b^\dagger_{q'}(l)\right] \;=\; \delta_{q,q'} \;, \tag{12}$$

$$\left\{c_{k,\sigma}(l), c^\dagger_{k',\sigma'}(l)\right\} \;=\; \delta_{k,k'}\delta_{\sigma,\sigma'} \;. \tag{13}$$

These statistical commutation/anticommutation relations yield

$$A_k(l)^2 + \sum_q B_{k,q}(l)^2 \left(f_{q,\downarrow} - b_{k+q}\right) \;=\; 1 \,, \tag{14}$$

$$X_q(l)^2 + \sum_k Y_{q,k}(l)^2 \left(1 - f_{k,\downarrow} - f_{q-k,\uparrow}\right) \;=\; 1 \,. \tag{15}$$

Here $f_{k,\sigma}$, b_q denote the Fermi and Bose distribution functions, respectively. They appear after introducing the normal ordered forms for operators left of the commutation (12) and anticommutation (13) when substituting (7), (8).

We solved numerically a set of the coupled equations (10), (11), (14), (15) simultaneously with the additional flow equations for $\varepsilon^\sigma_k(l)$,$E_k(l)$ and $v_{k,q}(l)$ given in the Ref. [6]. Initially, for $l = 0$, we assumed

(a) the one dimensional tight binding dispersion $\varepsilon^\sigma_k(l = 0) = -2t\cos k$,

(b) localized bosons $E_q(l = 0) = \Delta_B$ and

(c) local exchange interaction $v_{k,q}(l = 0) = v$.

For the sake of comparison with the previous studies of this model we used $D = 4t \equiv 1, \Delta_B = -0.6, v = 0.1$ and total concentration of carriers fixed to be 1.

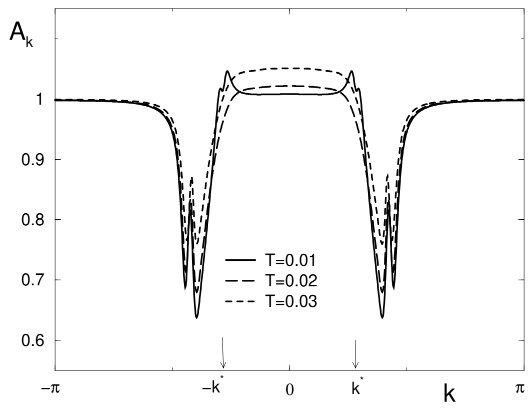

Fig. 1. Momentum dependence of the fermion coefficient $A_k(\infty)$ for temperatures $T = 0.01, 0.02$ and 0.03 (in units of the initial fermions bandwidth). Note a strong renormalization of the coefficient A_k around the characteristic momentum k^*.

In Fig. 1 we plot the momentum dependence of A_k coefficient for $l = \infty$ (when fermion subsystem is decoupled from the boson one). Considerable variation of the parameter A_k from its initial value (being 1) can be observed for such momenta which are located near $k^* \simeq 0.89$. As discussed in Ref. [6] this situation corresponds to the resonant scattering processes when $\varepsilon_{k*} + \varepsilon_{-k*} = \Delta_B$. It is worth mentioning that for $n_{\rm tot} = 1$ the Fermi momentum approaches this k^* from bellow when temperature decreases. We could think of the coefficient A_k as a measure of the spectral weight for fermions. The missing part $|1-A_k|$ is transfered towards some composite objects (mixtures of fermions and bosons or fermion pairs). Taking into account the fact that *for a decreasing temperature* $k_{\rm F}$ *shifts closer and closer to* k^**, we shall thus observe a diminishing spectral weight of fermions.* This is in agreement with interpretation of the ARPES experimental results [7].

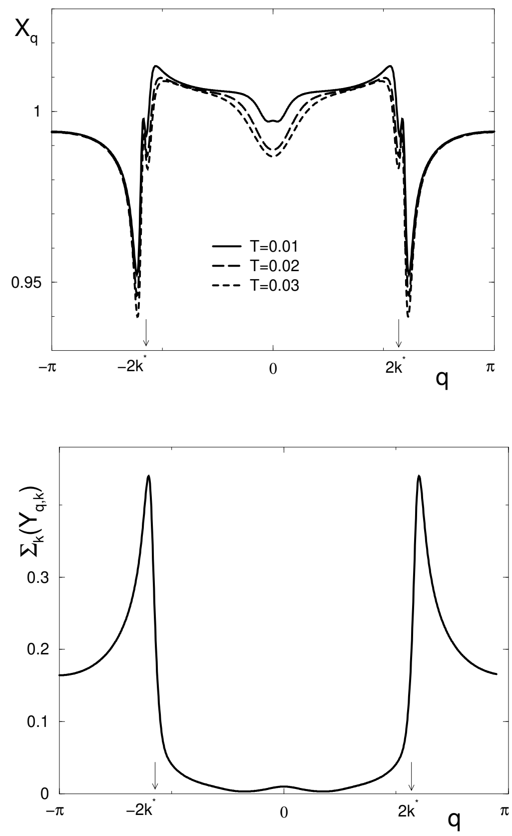

Fig. 2. Momentum dependence of the boson coefficients $X_q(\infty)$ and $\sum_k Y_{q,k}(\infty)$ for the same temperatures as in Fig. 1. The most efficient renormalization of boson coefficients occurs near $q \sim 2k^*$.

For a completeness we illustrate also the resulting ($l = \infty$) values of the boson coefficients given in (7). Parameter X_q shows only a small deviation from its initial value 1. This time such a change is most effective for boson momenta close to $2k^*$. It is exactly the same region of the Brillouin zone where the effective boson dispersion shows a kink (see Fig. 3).

The missing part of the boson spectral weight is transferred to the fermion pairs, as given by the Ansatz (7). Bottom part of Fig. 2 shows that $\sum_k Y_{q,k}$ (which roughly measures the fermion pairs spectral weight) changes a lot for $q \sim 2k^*$. Again, if we recall that for a decreasing temperature $k_{\rm F} \to k^*$ we can conclude that there is an increasing probability of finding more and more fermion pairs, even though our system is not in the superconducting phase. This is one of possible ways for observing the precursor effect.

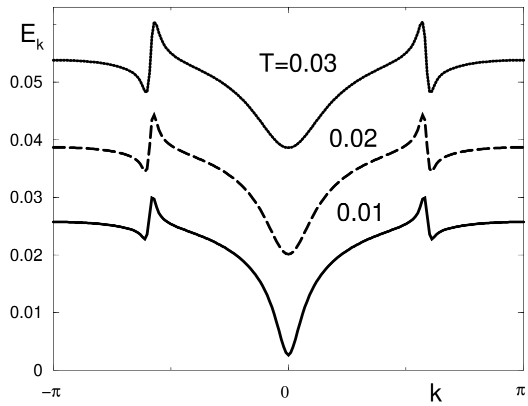

Fig. 3. Effective boson dispersion $E_k(l = \infty)$ as measured from 2μ level for the same set of temperatures as in Fig. 1. Recall that initialy (for l=0) bosons were localized.

In conclusion we studied evolution of fermion and boson operators in the BF model. During a continuous transformation which eliminated the exchange interaction we observe transformation of the initial fermion and boson particles into the composite objects. In particular we find that fermion (boson) spectral weights at $k_{\rm F}$ ($2k_{\rm F}$) are reduced when temperature decreases. This effect is in an agreement with the recent interpretation of the ARPES measurements for underdoped Bi2212 cuprates. To clarify a problem of the quasiparticles life time we need a more detailed study of the correlation functions. Such an analysis is currently under investigation and the results will be published elsewhere.

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