Zakopane, 29 Sept. 2016

From superconductivity to electron correlations

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Issues to be addressed:

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- **Superconductivity in nanostructures:**

⇒ **electron pairing** .... **/ due to proximity effect /**

⇒ **subgap quasiparticles** .... **/ Andreev (Shiba) states /**

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$\Rightarrow$ **quantum phase transition** .... **/ spinful $\leftrightarrow$ spinless states /**

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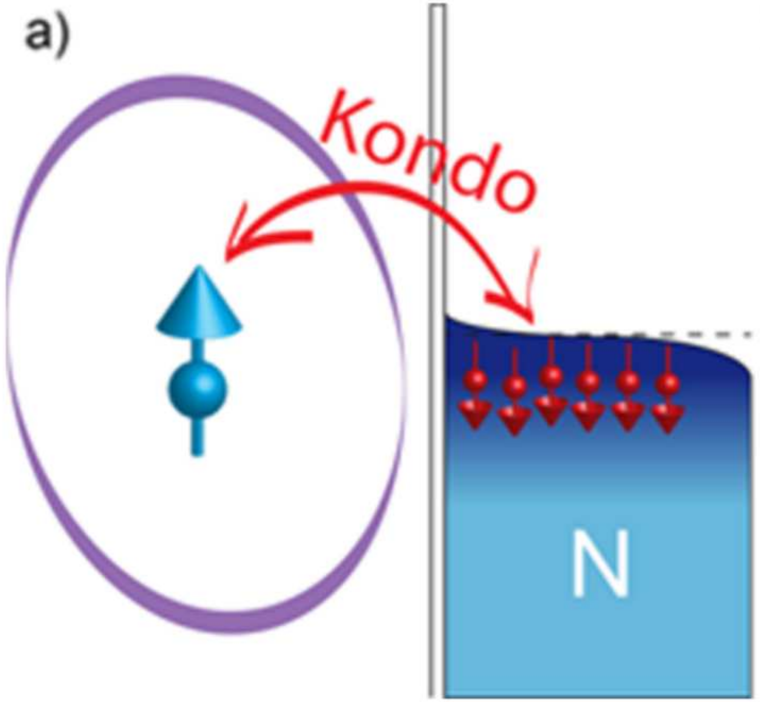
- **Cooper, Kondo, Majorana ...**

Kondo effect – reminder

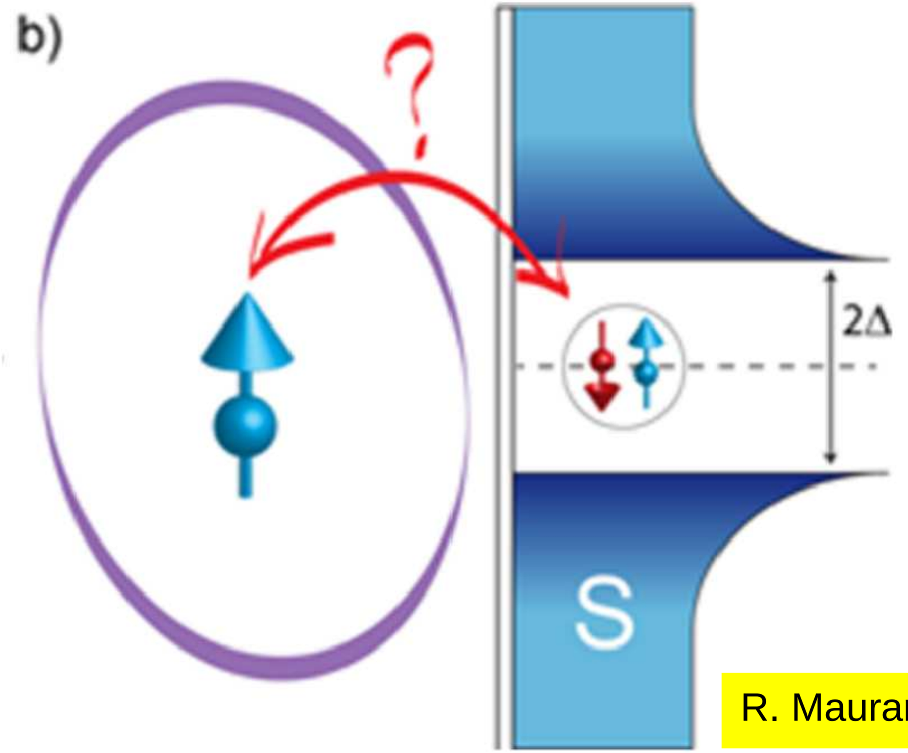
Quantum impurity (dot) embedded in a metallic bath



forms the many-body Kondo state with itinerant electrons thanks to the effective screening (exchange) interactions.

Kondo vs pairing – ′ to screen or not to screen ?′

Quantum impurity (dot) coupled to a superconductor



R. Maurand, Ch. Schönenberger, Physics **6**, 75 (2013).

behaves differently, because :

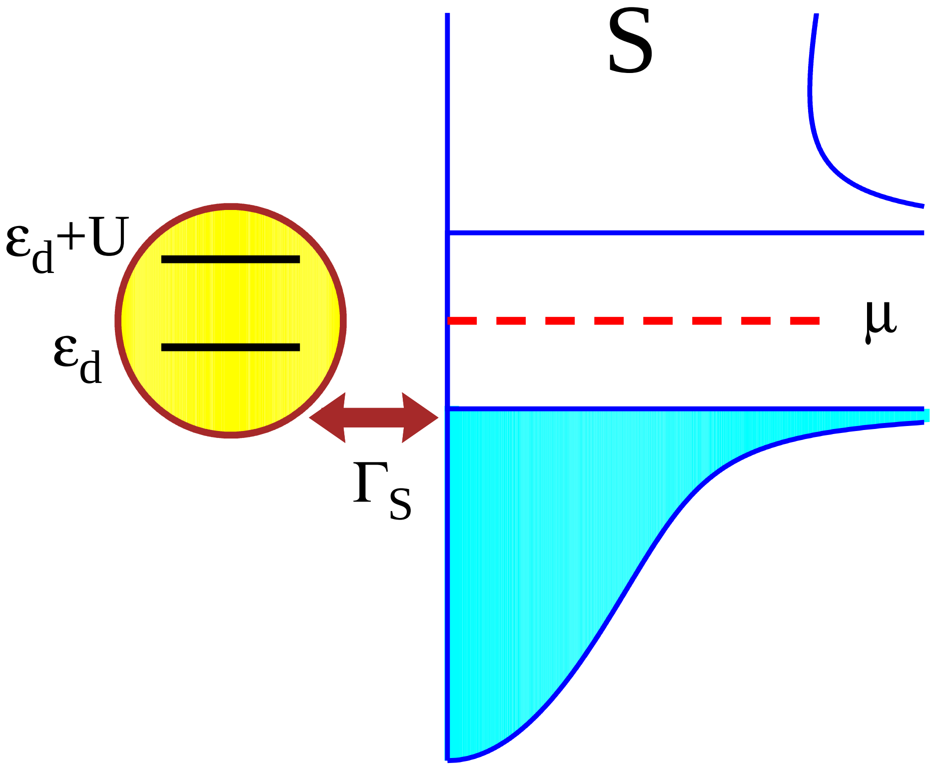
$\Rightarrow$ **electronic states near the Fermi level are missing,**

$\Rightarrow$ **pairing has <u>nontrivial</u> interplay with Kondo state.**

Quantum impurity (dot)

in a superconducting host

Electronic spectrum



Microscopic model

Anderson-type Hamiltonian

Quantum impurity (dot)

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U \; \hat{n}_{d\uparrow} \; \hat{n}_{d\downarrow}$$

coupled with a superconductor

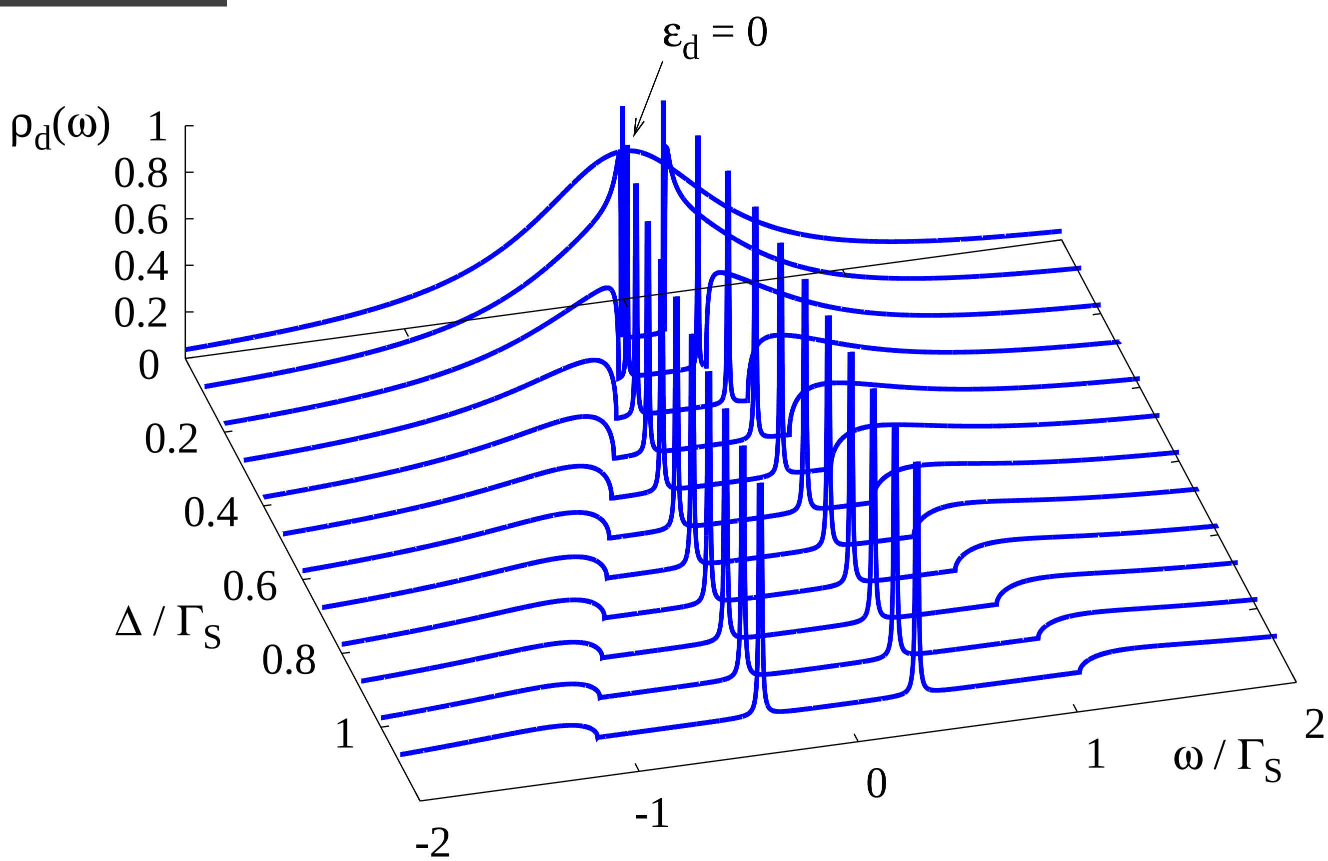
$$\hat{H} = \sum_{\sigma} \epsilon_d \hat{d}^{\dagger}_{\sigma} \hat{d}_{\sigma} + U \; \hat{n}_{d\uparrow} \; \hat{n}_{d\downarrow} + \hat{H}_S$$
$$+ \sum_{\mathbf{k},\sigma} \left(V_{\mathbf{k}} \; \hat{d}^{\dagger}_{\sigma} \hat{c}_{\mathbf{k}\sigma} + V^{*}_{\mathbf{k}} \; \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{d}_{\sigma} \right)$$

where

$$\hat{H}_S = \sum_{k,\sigma} (\varepsilon_k - \mu) \, \hat{c}^{\dagger}_{k\sigma} \hat{c}_{k\sigma} - \sum_{k} \left(\Delta \hat{c}^{\dagger}_{k\uparrow} \; \hat{c}^{\dagger}_{k\downarrow} + \mathrm{h.c.} \right)$$

Uncorrelated QD

$U_d = 0$ (exactly solvable case)

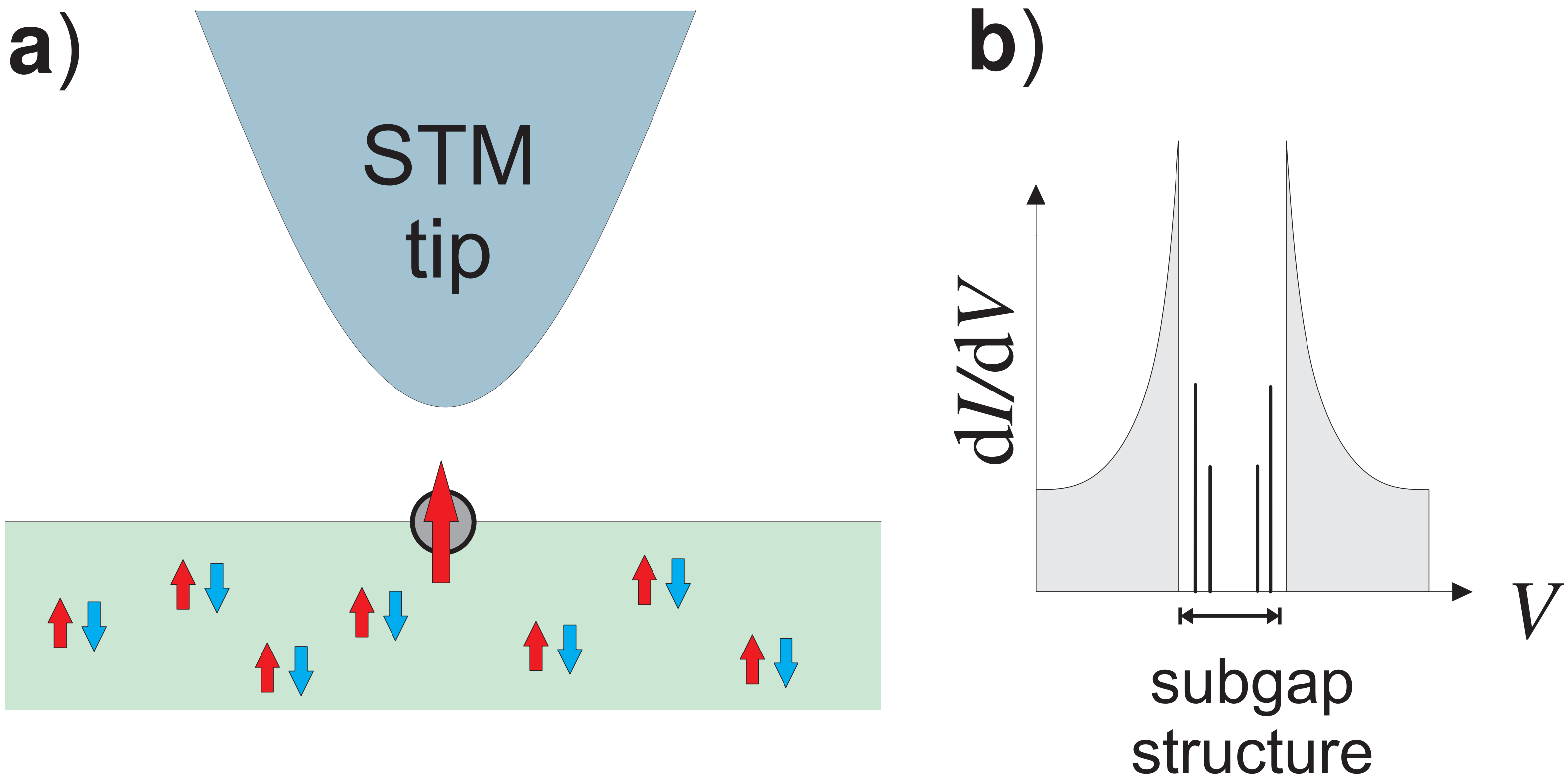


In-gap (Andreev/Shiba) bound states :

⇒ always appear in pairs,

⇒ appear symmetrically at finite energies.

Subgap states of multilevel quantum impurities



a) STM scheme and b) differential conductance for a multilevel quantum impurity adsorbed on a superconductor surface.

R. Žitko, O. Bodensiek, and T. Pruschke, Phys. Rev. B **83**, 054512 (2011).

Correlated QD – spinful / spinless configurations

In a subgap regime $|\omega| \ll \Delta$ the proximized impurity can described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left(\Delta_d \, \hat{d}^{\dagger}_{\uparrow} \hat{d}^{\dagger}_{\downarrow} + \mathrm{h.c.} \right)$$

with the induced pairing potential $\Delta_d = \Gamma_S / 2$.

The true eigen-states of this problem are:

$\vert\uparrow\rangle$ and $\vert\downarrow\rangle$	$\Leftarrow$	doublet (spin $\frac{1}{2}$)
$\left.\begin{array}{l} u\,\vert 0\rangle - v\,\vert\uparrow\downarrow\rangle \\ v\,\vert 0\rangle + u\,\vert\uparrow\downarrow\rangle \end{array}\right\}$	$\Leftarrow$	singlets (spin 0)

Possible quantum phase transition occurs upon varying ε_d, U_d or Γ_S.

Correlated QD – quantum phase transition

Example: ground state of the half-filled QD

$\vert\uparrow\rangle\ ,\ \vert\downarrow\rangle$	when $\Gamma_S < U$	(spinful state)
$u\,\vert 0\rangle - v\,\vert\uparrow\downarrow\rangle$	when $\Gamma_S > U$	(spinless state)

Correlated QD – quantum phase transition

Example: ground state of the half-filled QD

$\vert\uparrow\rangle$, $\vert\downarrow\rangle$	when $\Gamma_S < U$	(spinful state)
$u\,\vert 0\rangle - v\,\vert\uparrow\downarrow\rangle$	when $\Gamma_S > U$	(spinless state)

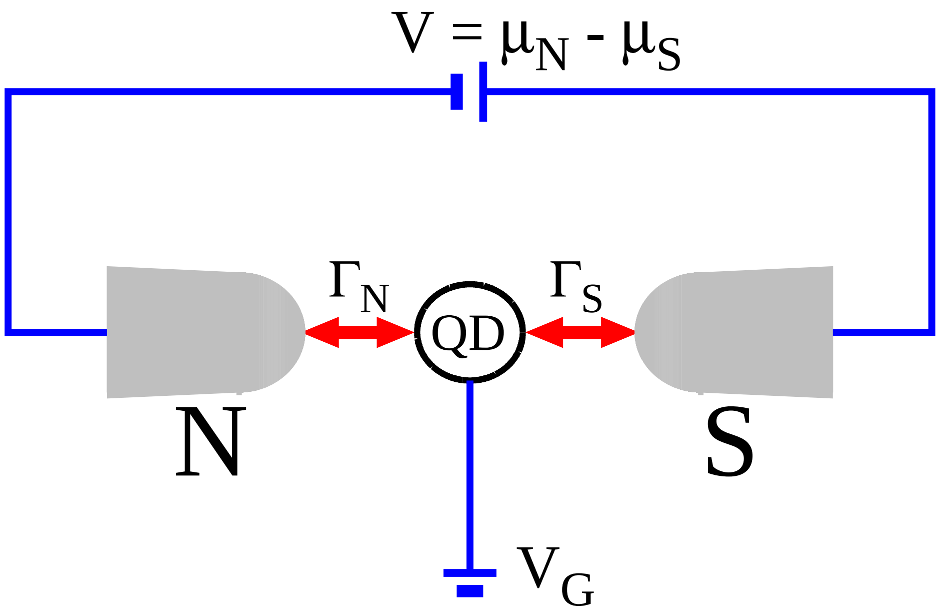
Important remark : the spinless state cannot be screened !

N–QD–S heterojunction

/Cooper vs Kondo /

Physical situation

Subgap states can be probed by measuring the charge transport through a quantum dot (QD) coupled between the normal (N) and superconducting (S) electrodes

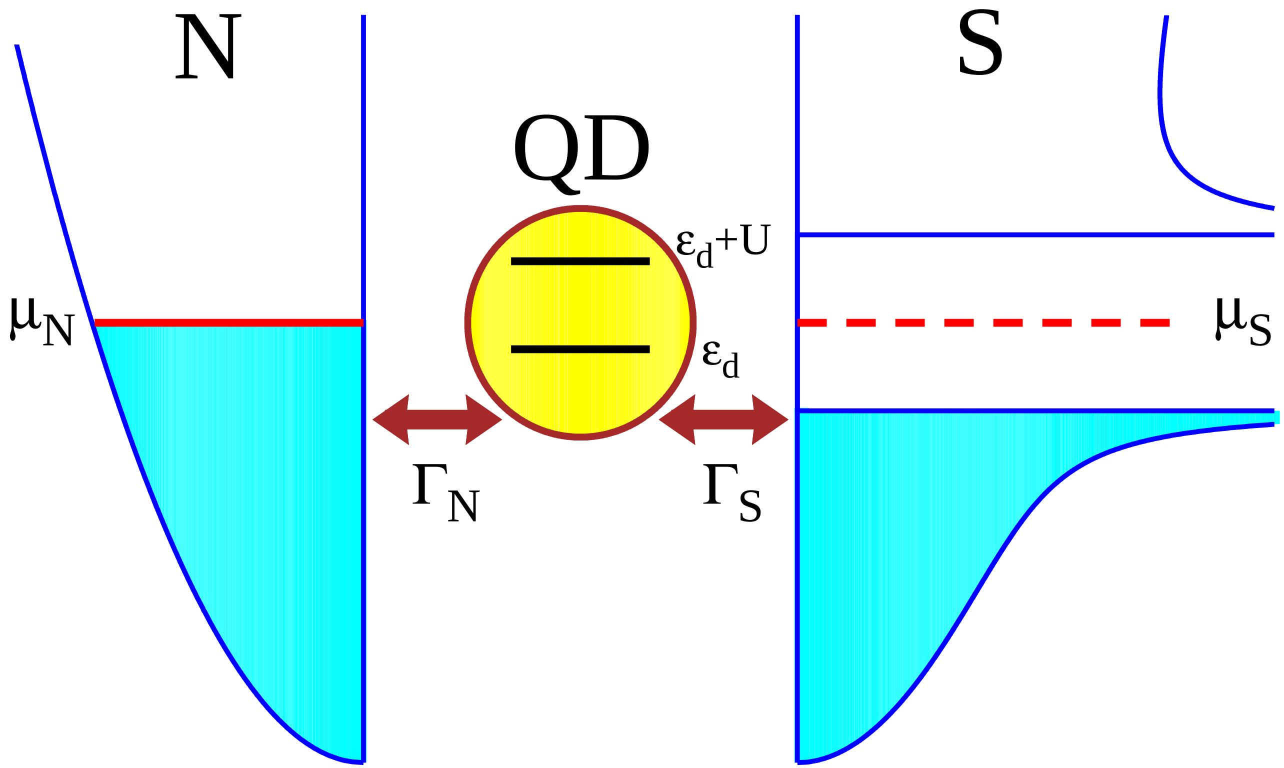


metallic lead | **QD** | **superconductor**

This N–QD–S setup has been practically studied in several recent experiments.

Electronic spectrum

Spectrum of N-QD-S heterostructure

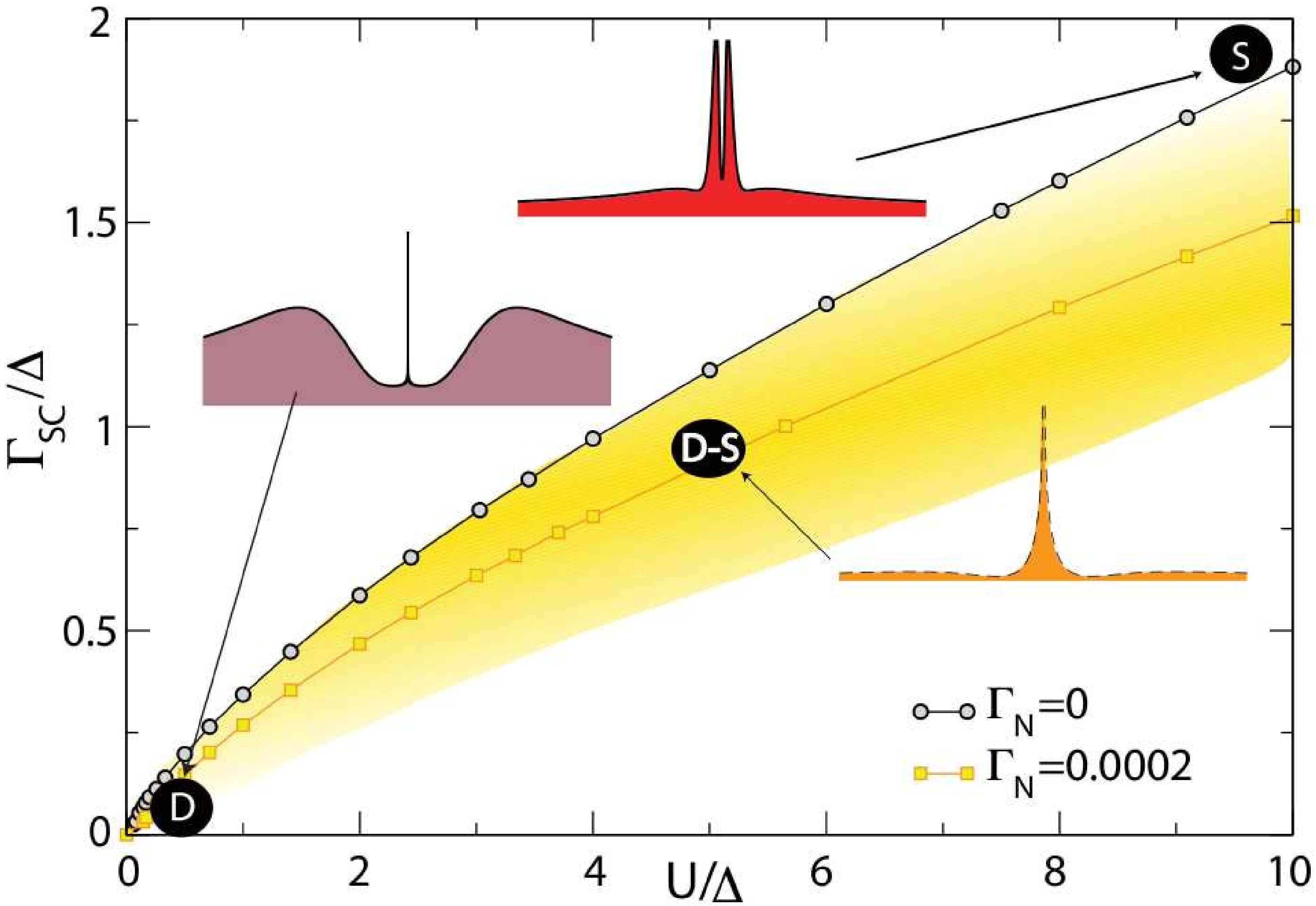


Kondo state in a subgap regime

singlet $\longleftrightarrow$ doublet crossover

Kondo state in a subgap regime

singlet ↔ doublet crossover



Phase diagram obtained by NRG Ljubljana code.

R. Žitko, J.S. Lim, R. López, and R. Aguado, Phys. Rev. B **91**, 045441 (2015).

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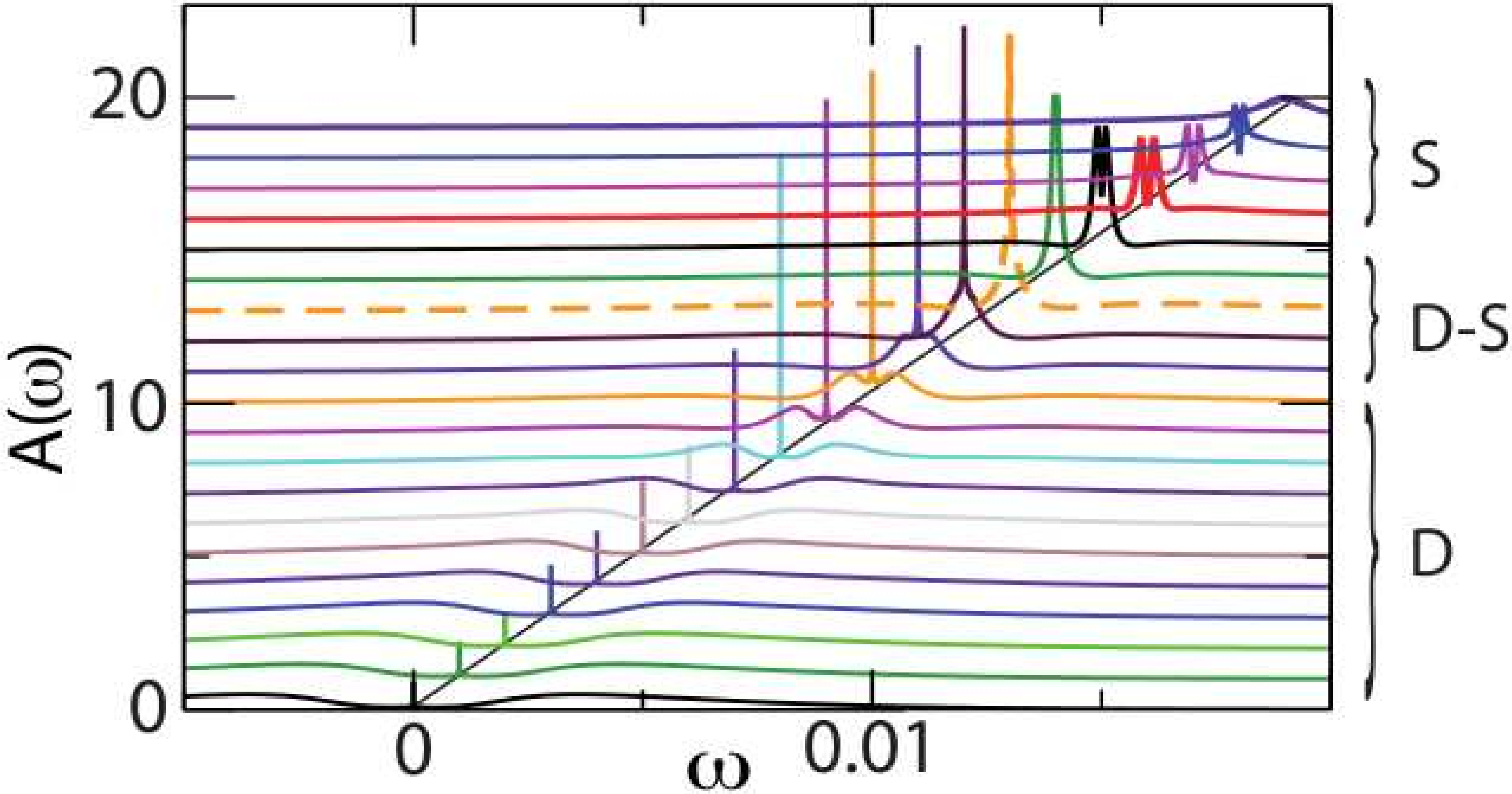


QD spectrum obtained by NRG Ljubljana code.

R. Žitko, J.S. Lim, R. López, and R. Aguado, Phys. Rev. B **91**, 045441 (2015).

Strategy

/ motivated by R. Žitko's results /

To clarify such weird evolution of the Kondo state we have investigated this problem by several complementary methods:

$\Longrightarrow$ **perturbative treatment of $\hat{V}_{QD-N}$ . . . .(a lá Schrieffer and Wolff),**

$\Longrightarrow$ **NRG calculations . . . .(using the Budapest code),**

$\Longrightarrow$ **2^{nd} order pert. treatment of U . . . . (with anomalous diagrams).**

T. Domański, I. Weymann, M. Barańska & G. Górski, Scientific Reports **6**, 23336 (2016).

Generalized S-W approach

canonical transformation

Proximized quantum impurity $\hat{H}_{QD}^{prox} \equiv \hat{H}_{QD} + \hat{H}_S + \hat{V}_{QD,S}$

$$\begin{aligned} \hat{H}_{QD}^{prox} &= \sum_{\sigma} \epsilon_d \hat{d}_{\sigma}^{\dagger} \hat{d}_{\sigma} + U \; \hat{n}_{d\uparrow} \; \hat{n}_{d\downarrow} \\ &- \left(\Delta_d \; \hat{d}_{\uparrow}^{\dagger} \; \hat{d}_{\downarrow}^{\dagger} + \Delta_d^{*} \; \hat{d}_{\downarrow} \hat{d}_{\uparrow} \; \right) \end{aligned}$$

weakly coupled to a normal metal

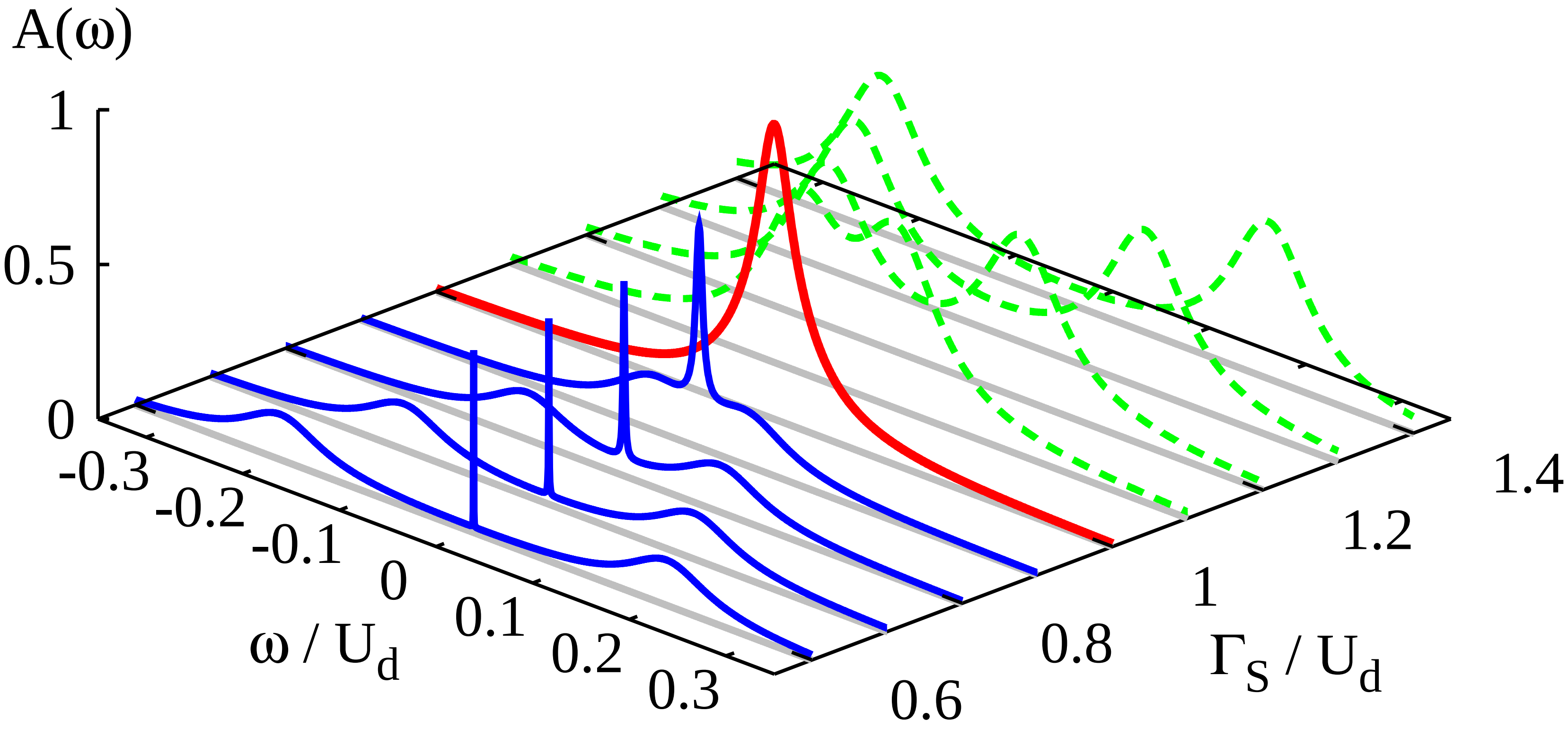
$$\hat{H} = \hat{H}_{QD}^{prox} + \sum_{k,\sigma} \xi_k \hat{c}_{k\sigma}^{\dagger} \hat{c}_{k\sigma} + \hat{V}_{QD-N}$$

where

$$\hat{V}_{QD-N} = \sum_{k,\sigma} \left(V_k \hat{c}_{k\sigma}^{\dagger} \hat{d}_{\sigma} + h.c. \right)$$

Correlated quantum dot – $\Gamma_N \ll \Gamma_S$

The half-filled quantum dot ($\varepsilon_d = -\frac{1}{2}U_d$)

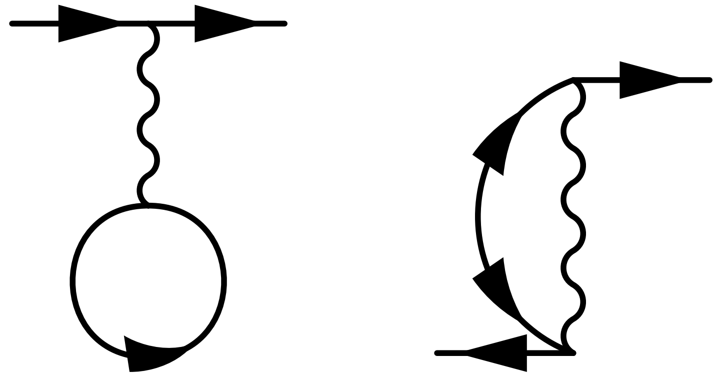


Kondo peak is present only in the spinful (doublet) state

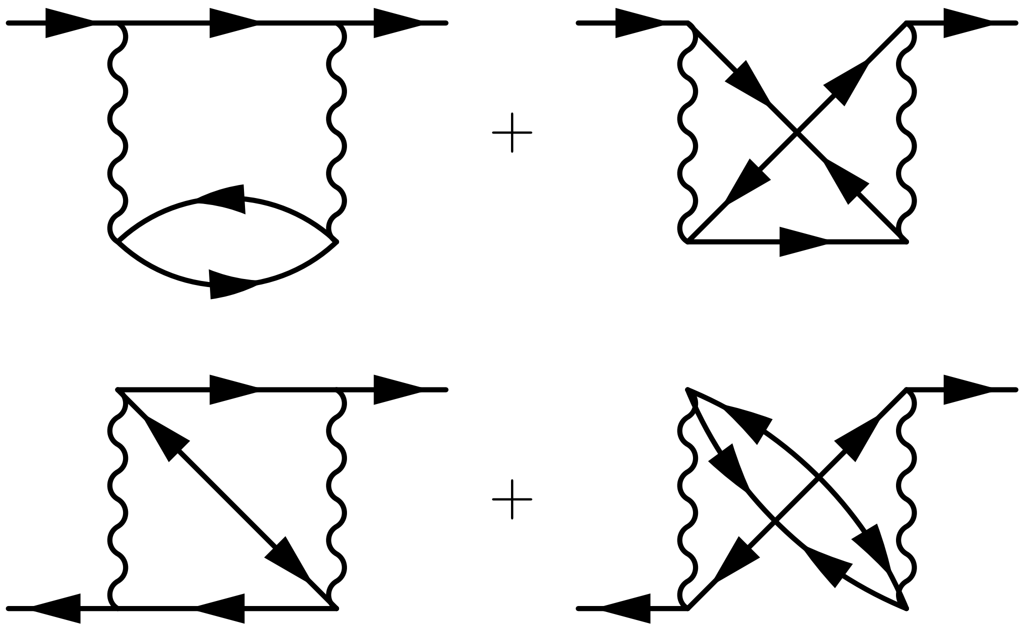
Results of the generalized Schrieffer-Wolff transformation

SOPT treatment – **/ normal & anomalous channels /**

1-st order diagrams:

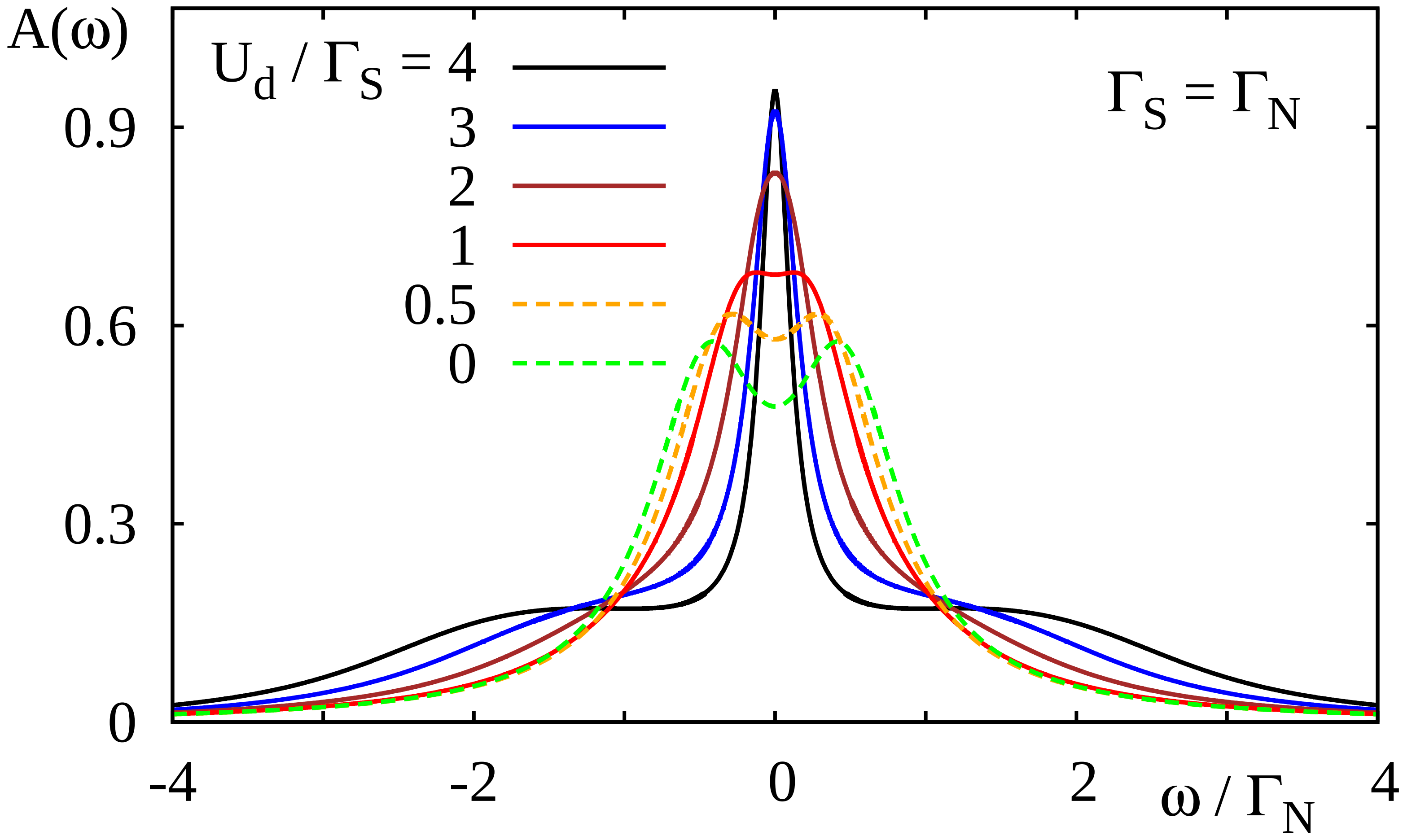


2-nd order diagrams:



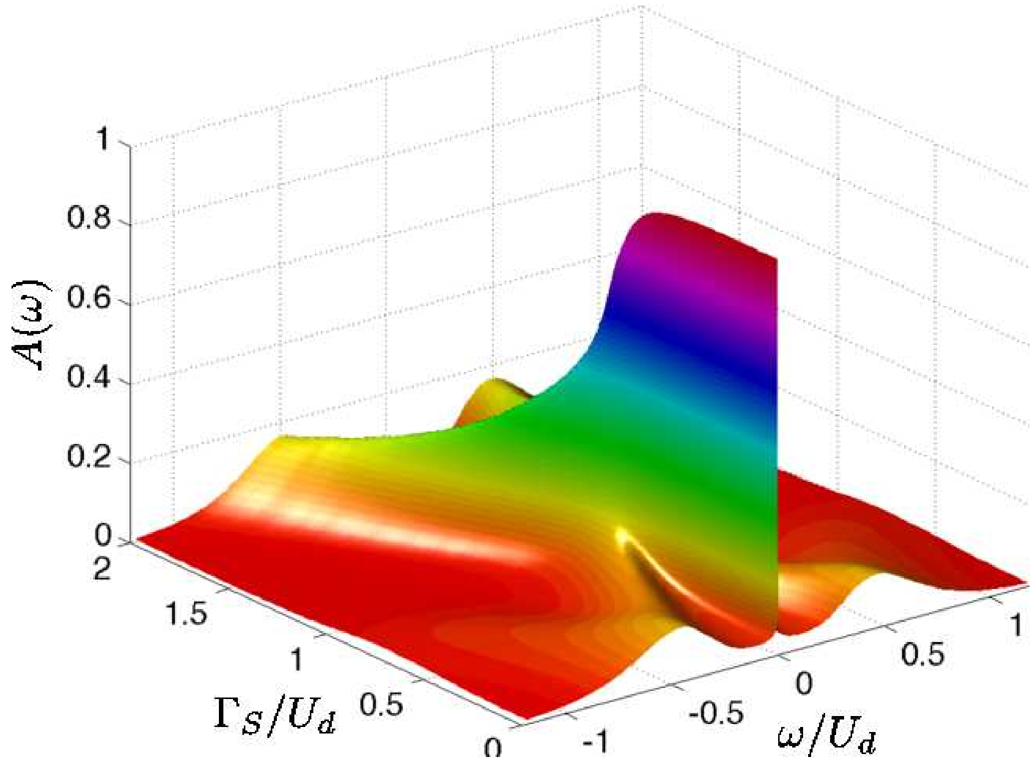
Correlated quantum dot – **Kondo meets Cooper**

The half-filled quantum dot ($\varepsilon_d = -\frac{1}{2}U_d$)



Results obtained from the 2-nd order perturbative treatment of U_d.

Correlated quantum dot – NRG results

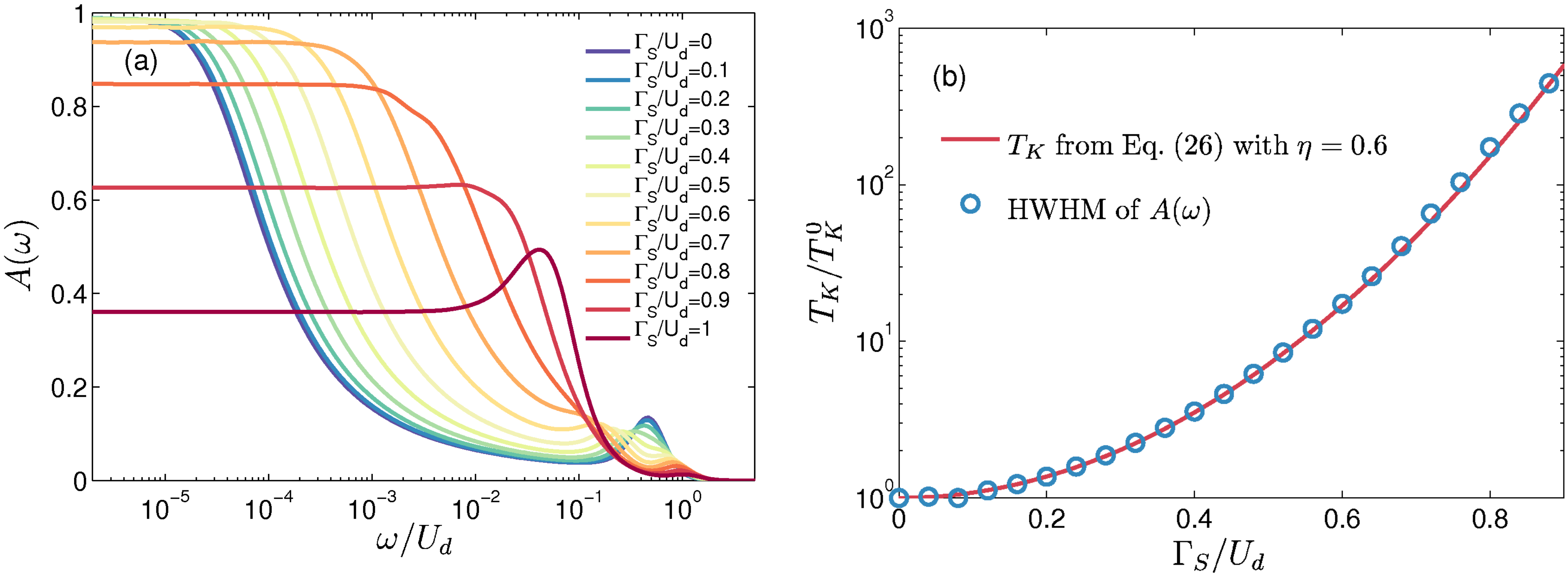


Observation: T_K is enhanced with increasing Γ_S

Results obtained using NRG calculations (Budapest code).

T. Domański, I. Weymann, M. Barańska & G. Górski, Scientific Reports **6**, 23336 (2016).

Correlated quantum dot – Kondo temperature T_K



T_K estimated from the NRG and the extended Schrieffer-Wolff transformation

$$T_K \simeq 0.3 \sqrt{\Gamma_N U_d} \exp\left[\frac{\pi\varepsilon_d(\varepsilon_d+U_d)+(\Gamma_S/2)^2}{\Gamma_N U_d}\right]$$

T. Domański, I. Weymann, M. Barańska & G. Górski, Scientific Reports **6**, 23336 (2016).

Related issues

$\Rightarrow$ **Josephson phase-tunable Kondo effect**

Josephson and Andreev circuits

phase-tunable effects

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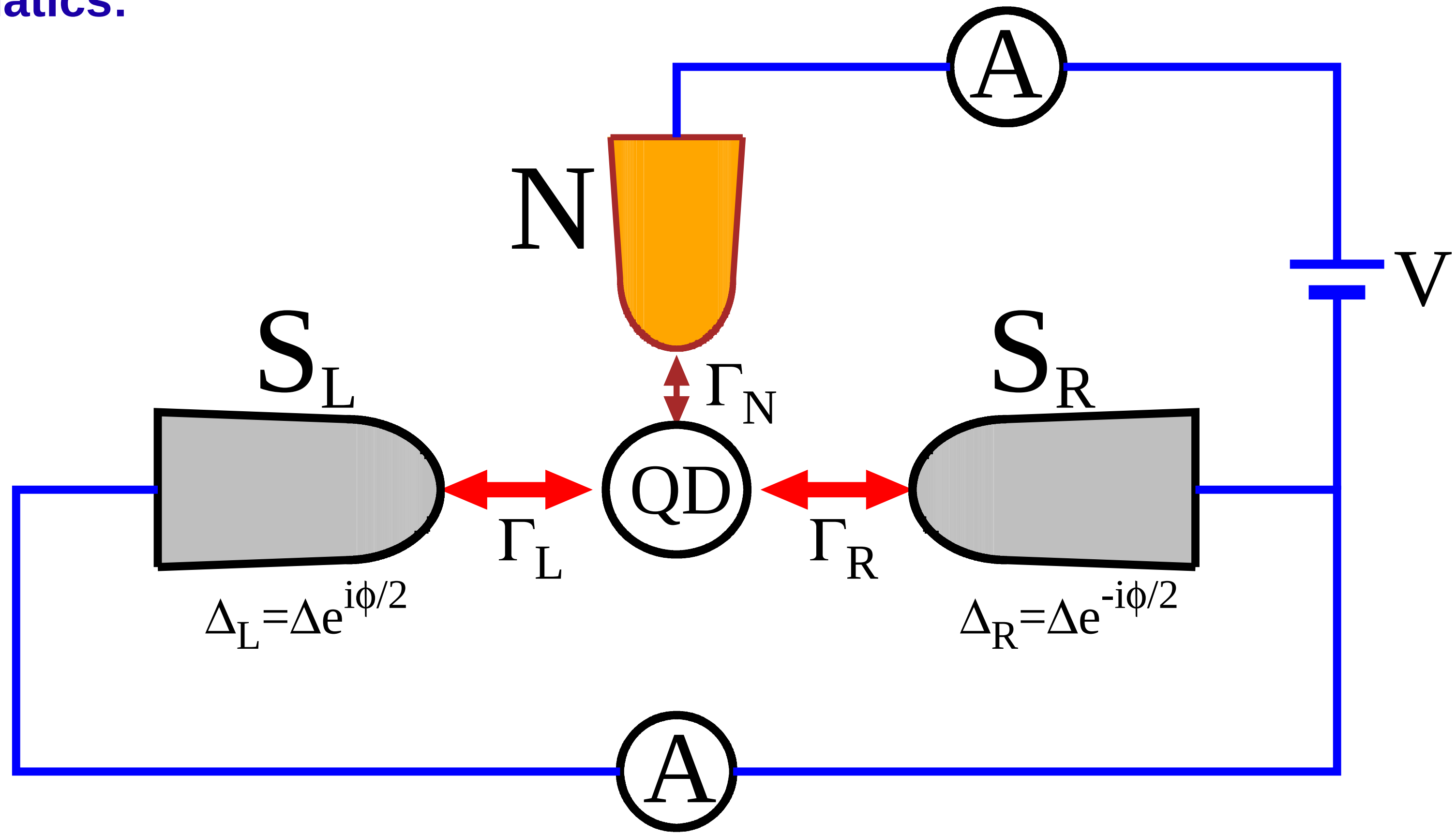
phase-tunable effects

Schematics:

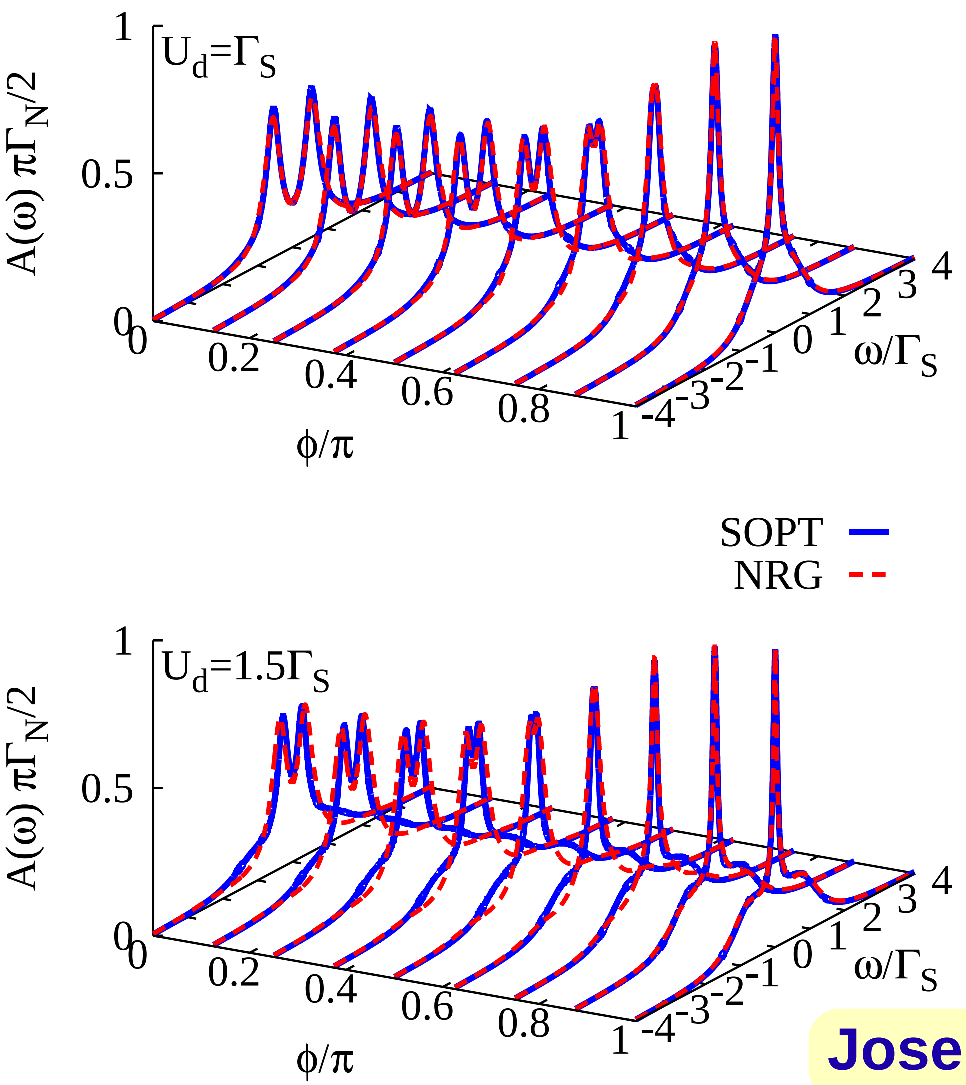


L, R **– superconducting electrodes,** N **– normal lead,** QD **– quantum dot**

T. Domański, M. Žonda, G. Górski, V. Pokorný, V. Janiš, T. Novotný, (2016) submitted.

Josephson and Andreev circuits

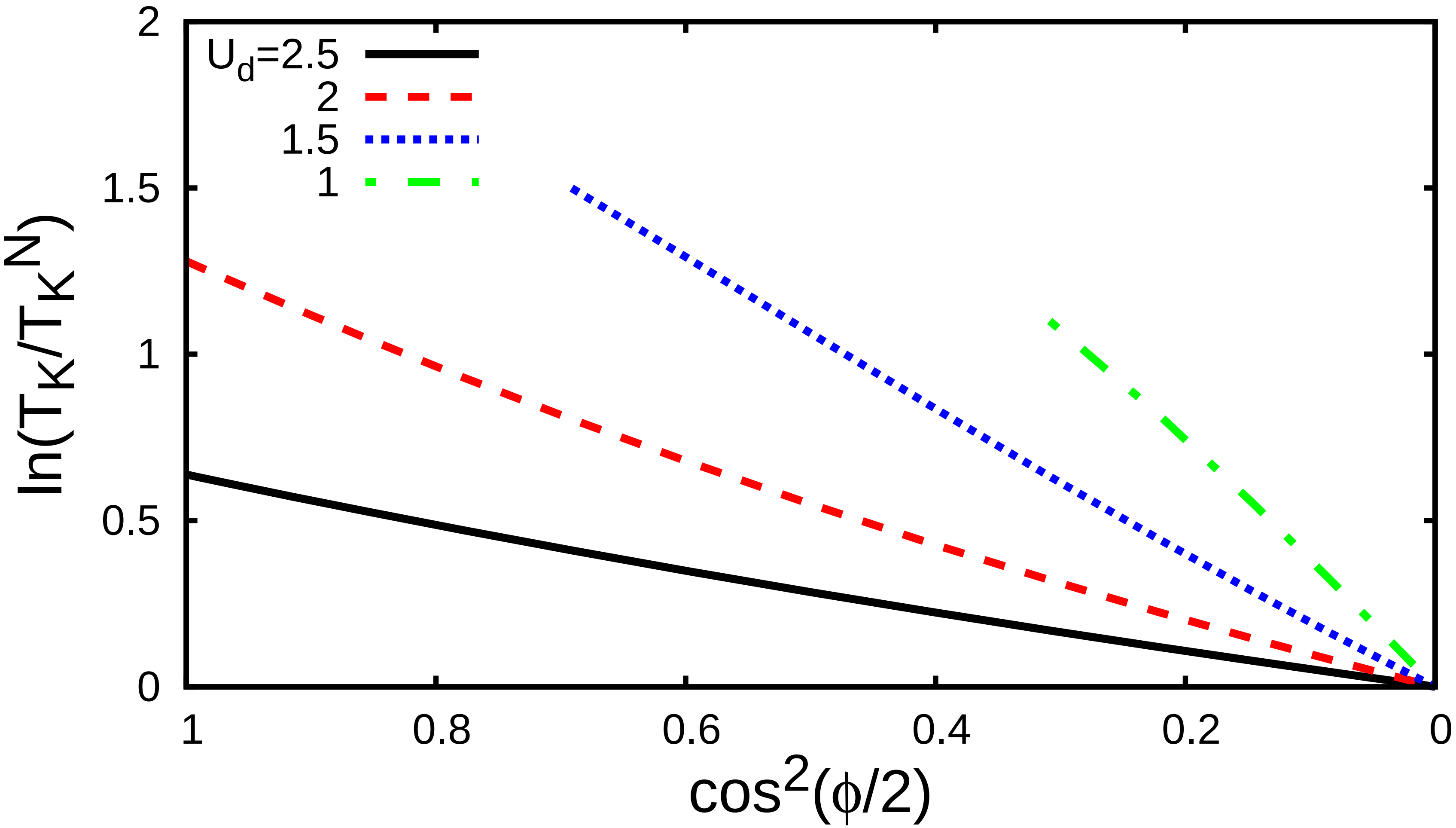
phase-tunable effects



Josephson phase-dependent spectrum

Josephson and Andreev circuits

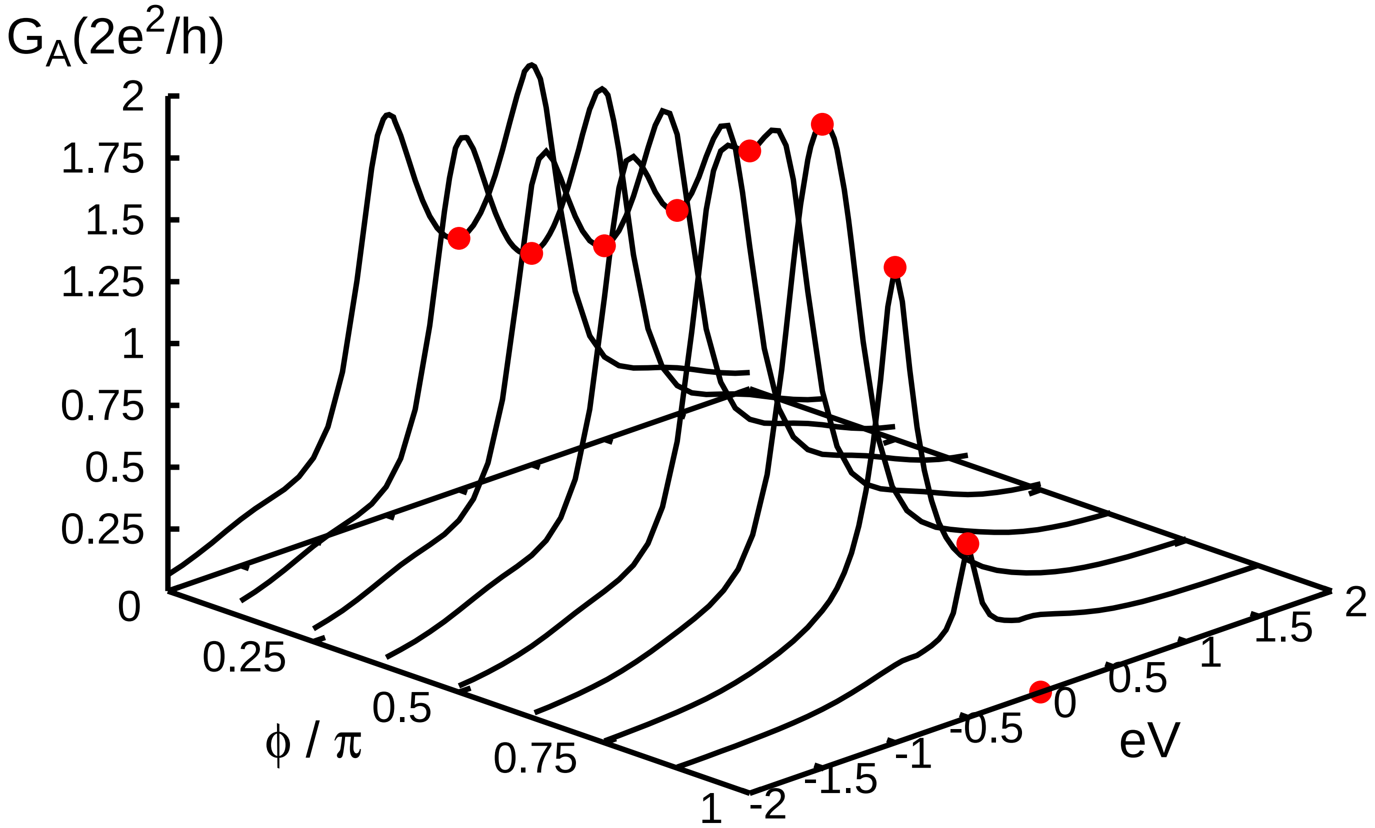
phase-tunable effects



Universal scaling of the Kondo temperature.

Josephson and Andreev circuits

phase-tunable effects



Phase-tunable conductance of the Andreev current.

Conclusions

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Electron pairing in nanosystems:

$\Rightarrow$ **induces the subgap (Andreev/Shiba) states**

$\Rightarrow$ **has unusual relationship with correlations**

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moreover it also enables realization of:

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Part 2

$\Rightarrow$ **Majorana quasiparticles**

Experimental evidence – for Majorana quasiparticles

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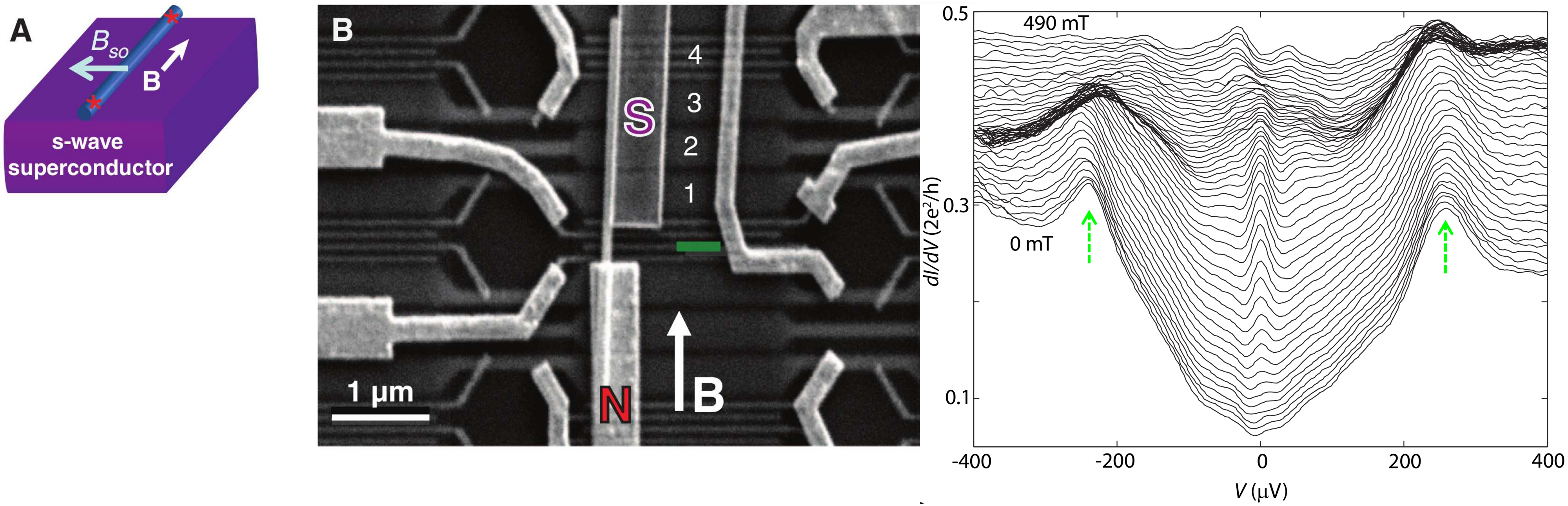
InSb nanowire between a metal (gold) and a superconductor (Nb-Ti-N)



dI/dV **measured at 70 mK for varying magnetic field B indicated:**

$\Rightarrow$ **a zero-bias enhancement due to Majorana state**

V. Mourik, ..., and L.P. Kouwenhoven, Science **336**, 1003 (2012).

/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /

Experimental evidence – for Majorana quasiparticles

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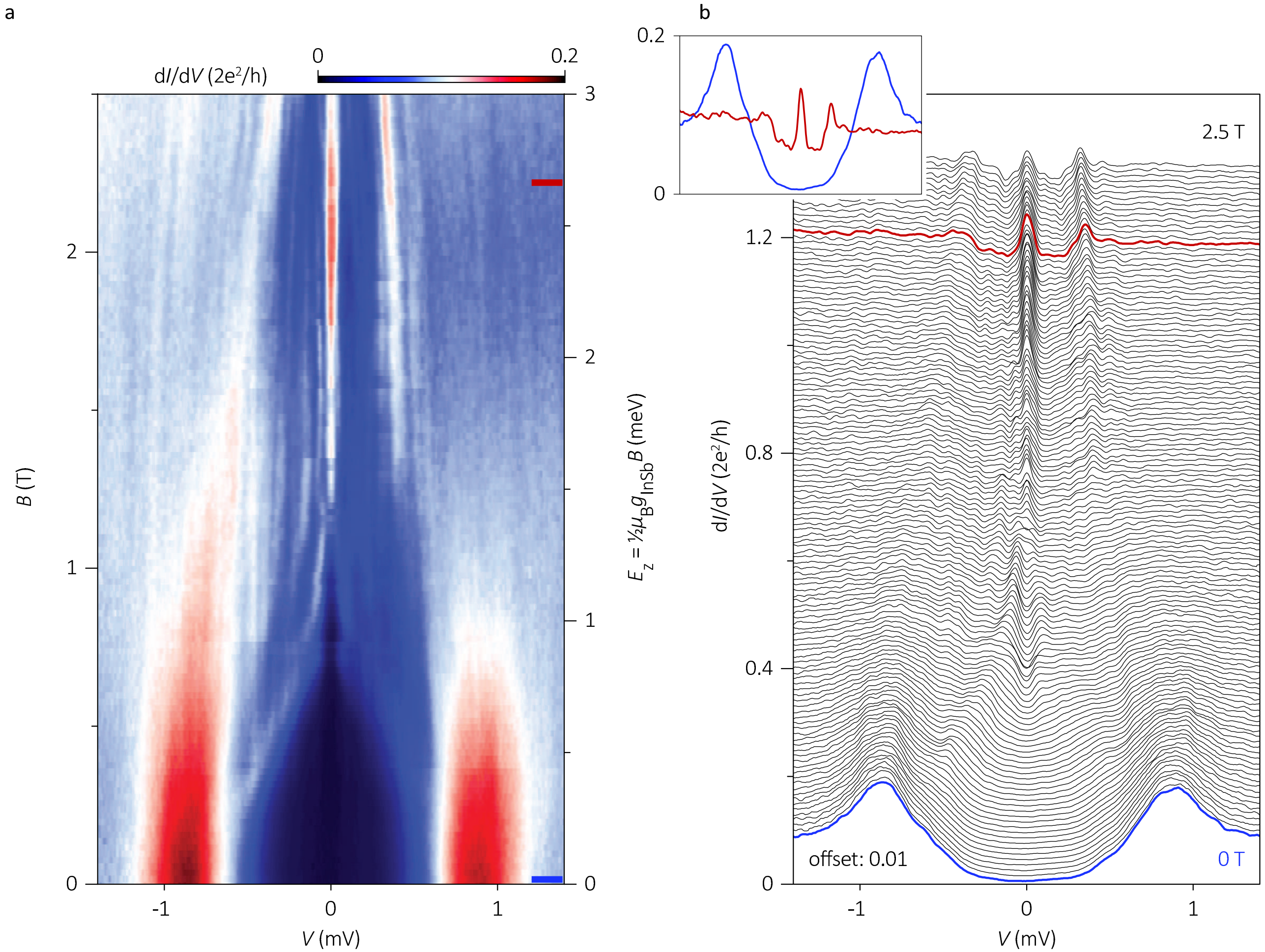
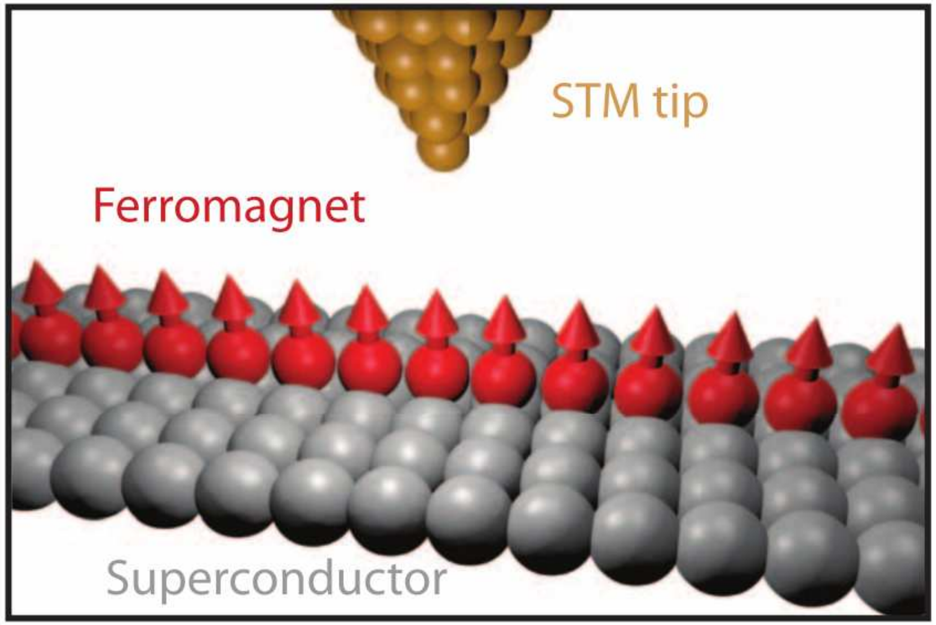


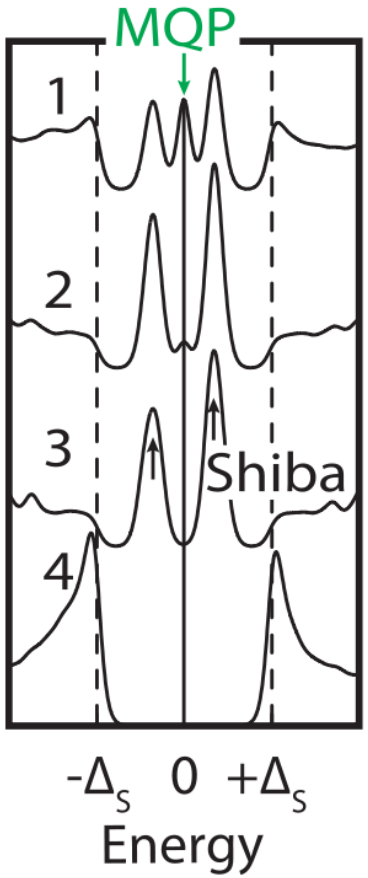
H. Zhang, ..., and L.P. Kouwenhoven, arXiv:1603.04069 (2016).

/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /

Experimental evidence – for Majorana quasiparticles

A chain of iron atoms deposited on a surface of superconducting lead





STM measurements provided evidence for:

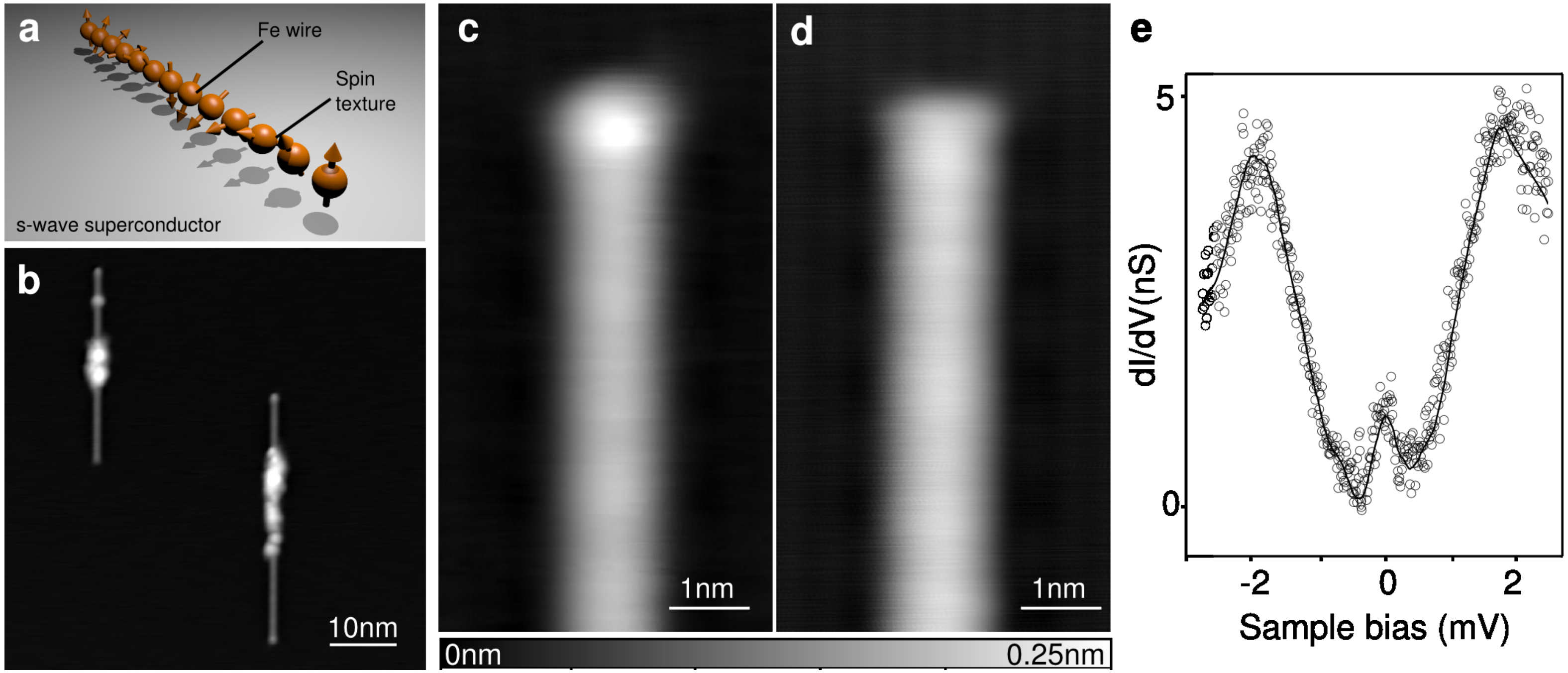
⇒ **Majorana bound states at the edges of a chain.**

S. Nadj-Perge, ..., and A. Yazdani, Science **346**, 602 (2014).

/ Princeton University, Princeton (NJ), USA /

Experimental evidence – for Majorana quasiparticles

Self-assembled Fe chain on superconducting Pb(110) surface



AFM combined with STM provided evidence for:

⇒ **Majorana bound states at the edges of a chain.**

R. Pawlak, M. Kisiel , ..., and E. Meyer, arXiv:1505.06078 (2015).

/ University of Basel, Switzerland /

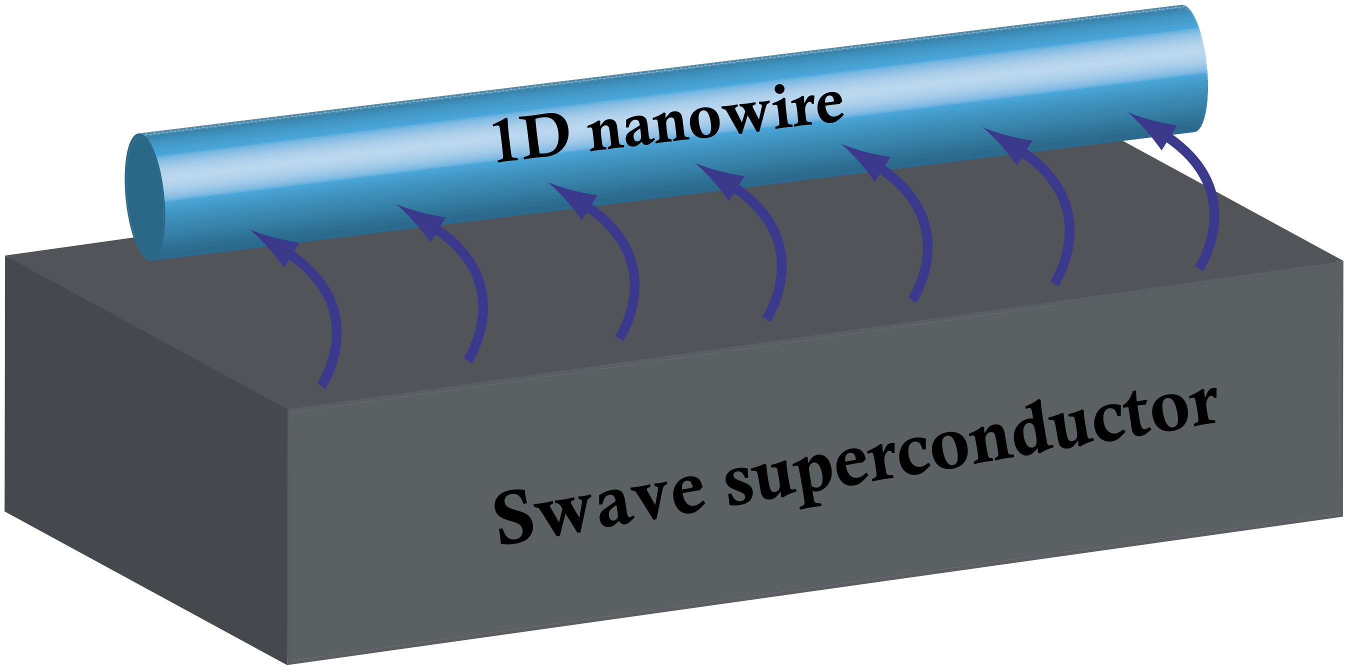
Question:

$\Rightarrow$ **where does Majorana come from ?**

Andreev vs Majorana states – a story of mutation

Andreev vs Majorana states – **a story of mutation**

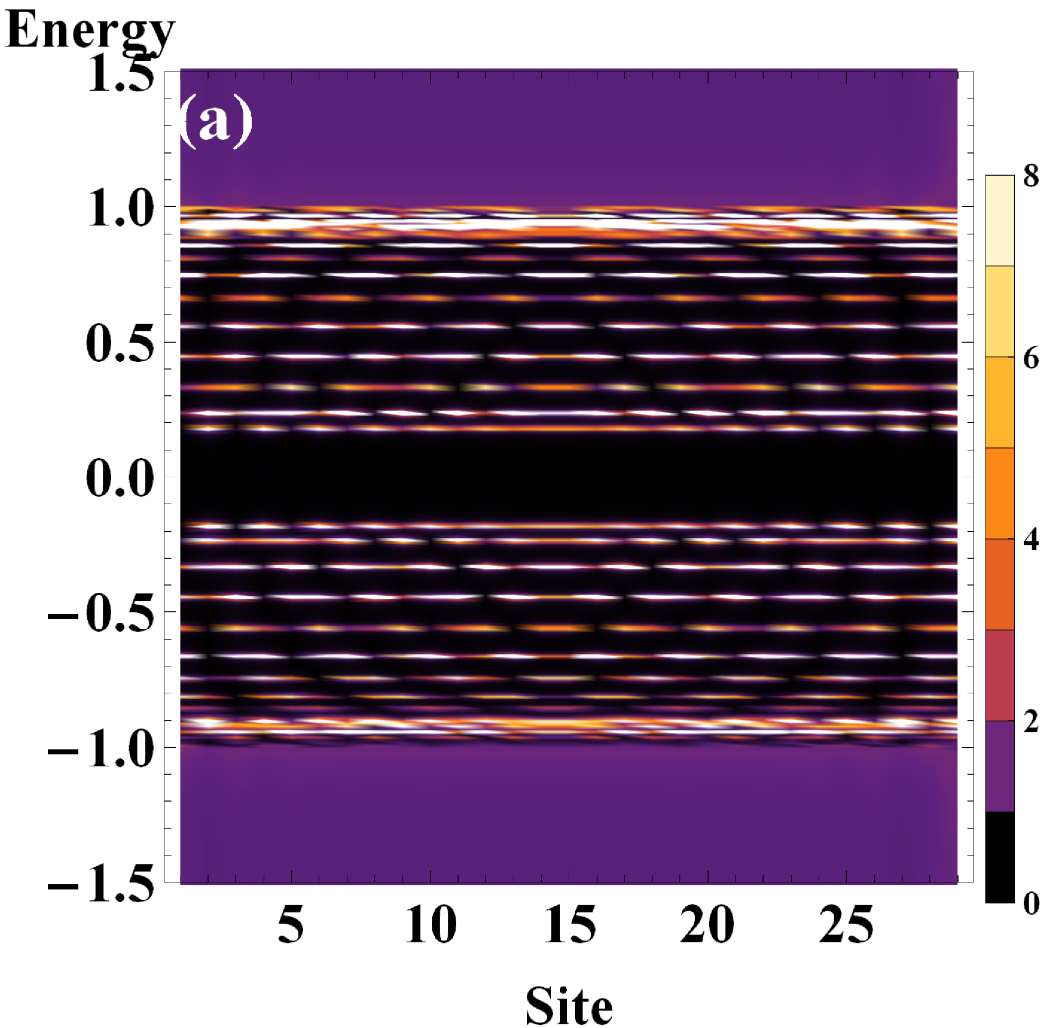
Let us consider:



1D quantum wire deposited on s-wave superconductor

D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B ***88****, 165401 (2013).*

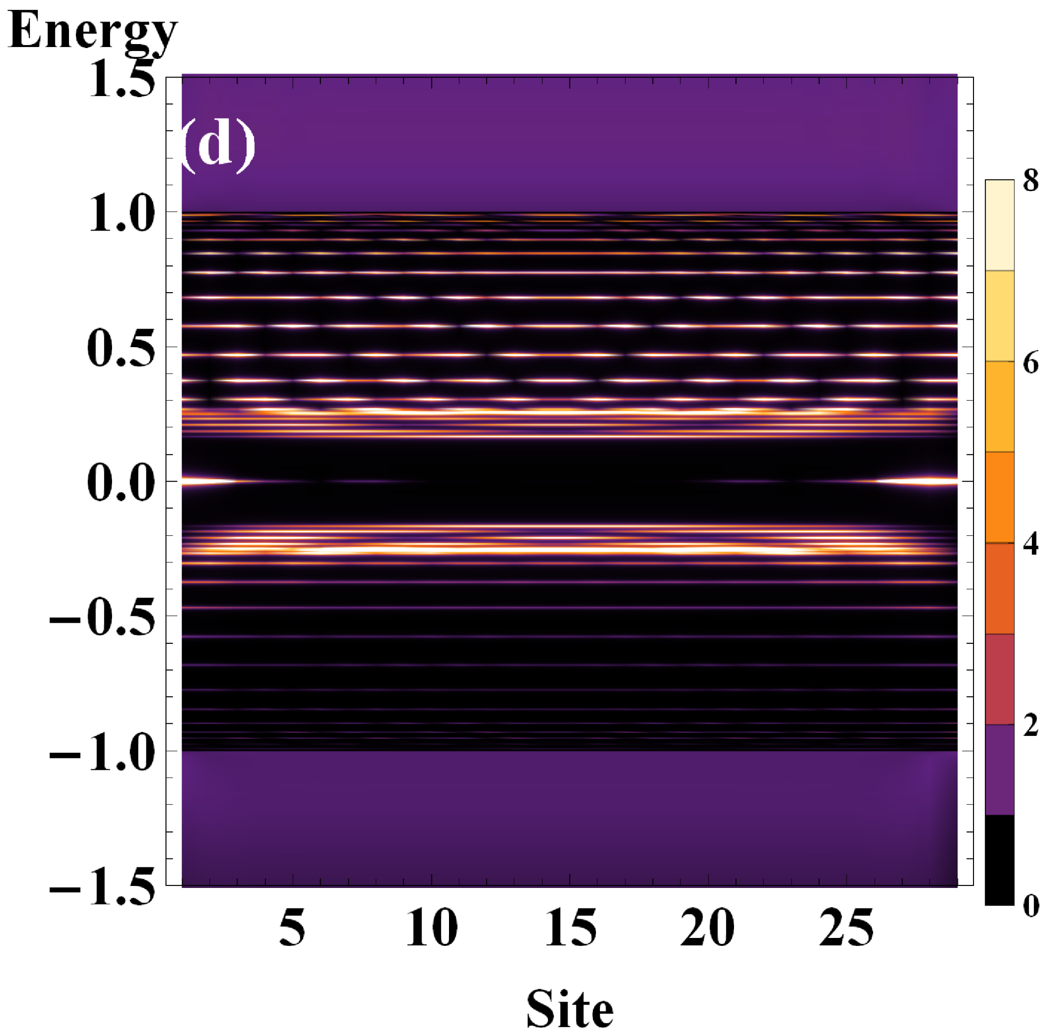
Andreev vs Majorana states – a story of mutation



Spectrum of a quantum wire has a series of Andreev states.

D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B **88***, 165401 (2013).*

Andreev vs Majorana states – a story of mutation



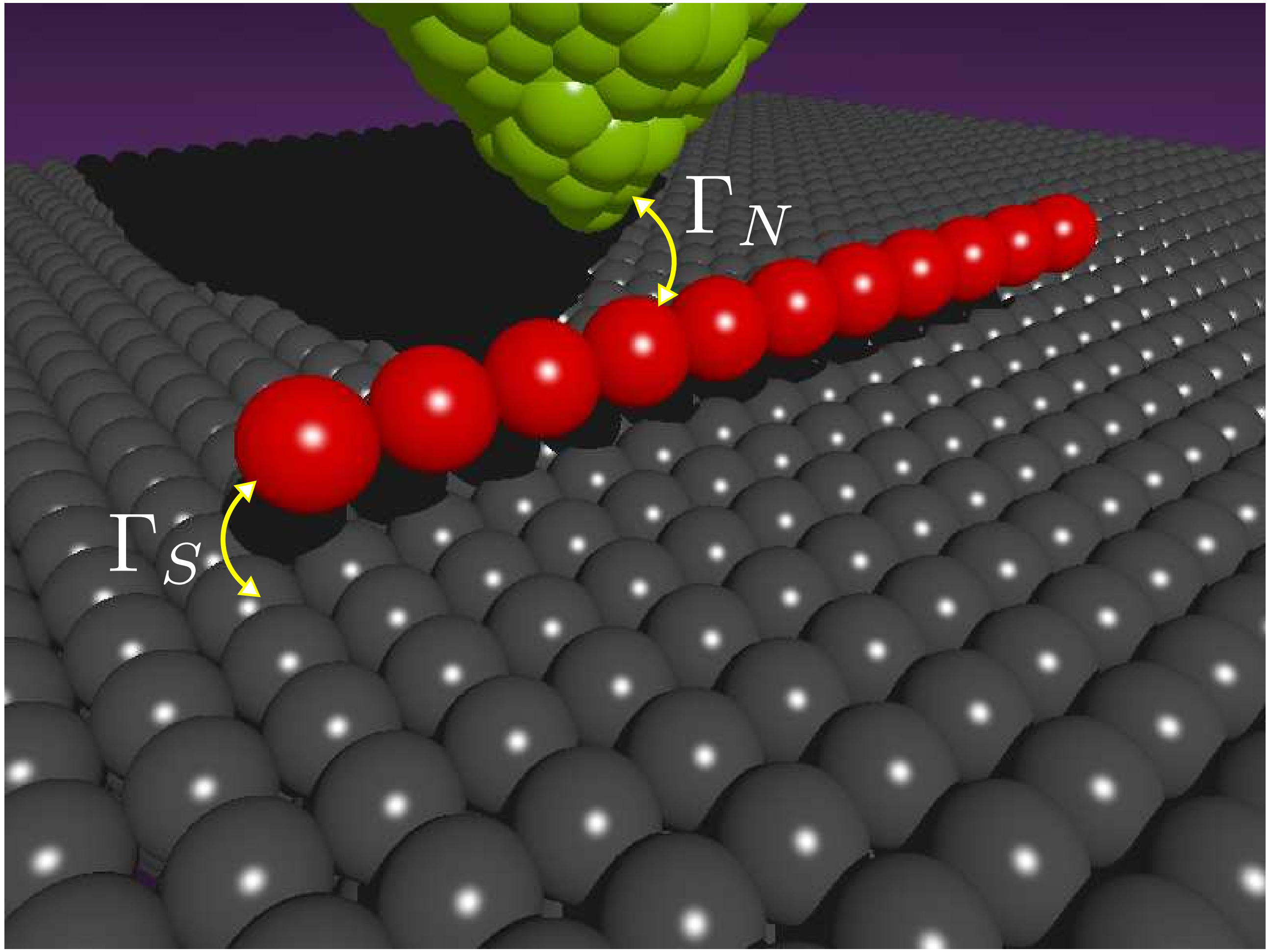
Spin-orbit coupling induces the Majorana-type quasiparticles.

D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B ***88****, 165401 (2013).*

Majorana states – of the Rashba chain

Majorana states – of the Rashba chain

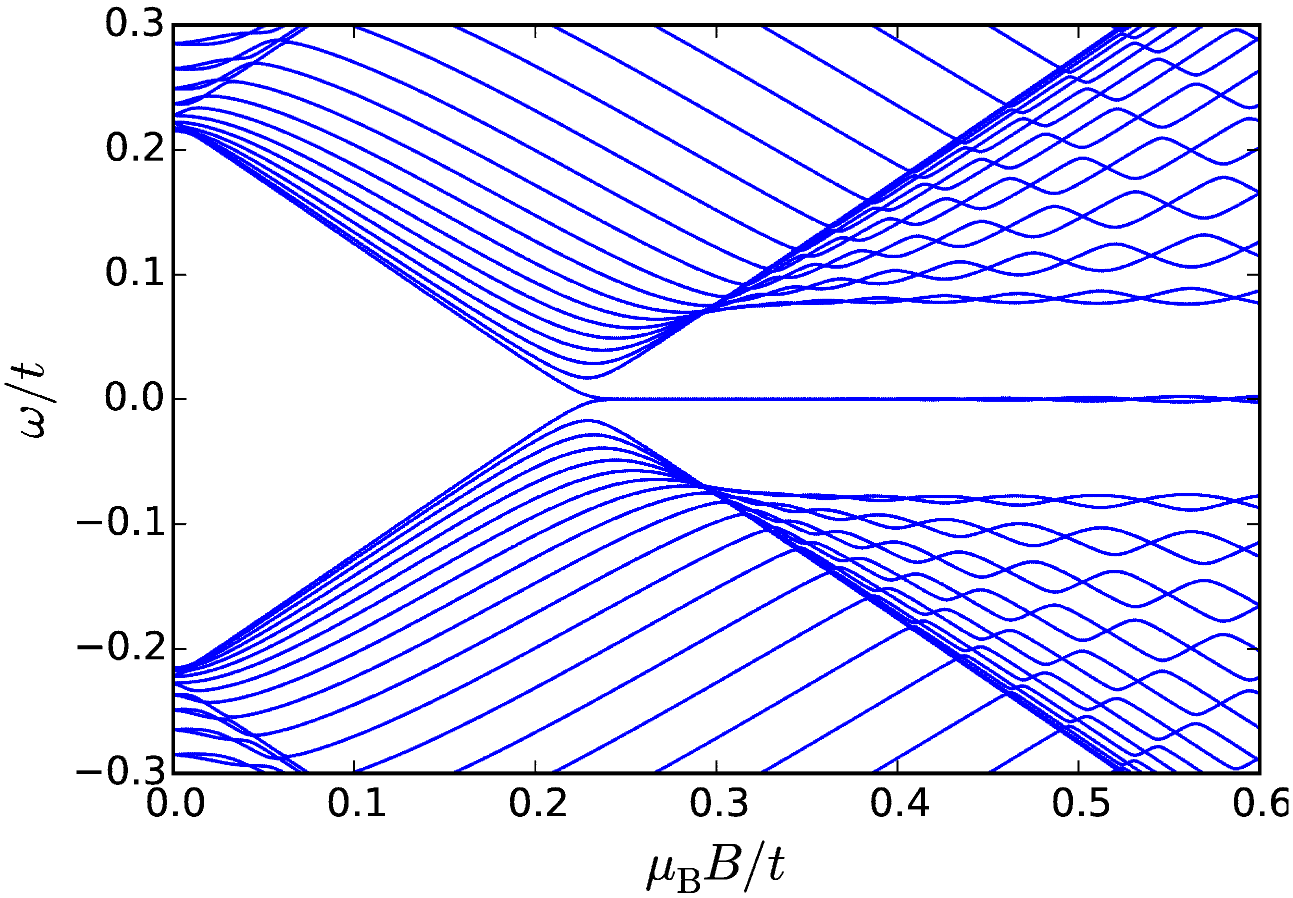
STM setup:



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv:1609.00685 (2016).

Majorana states – of the Rashba chain

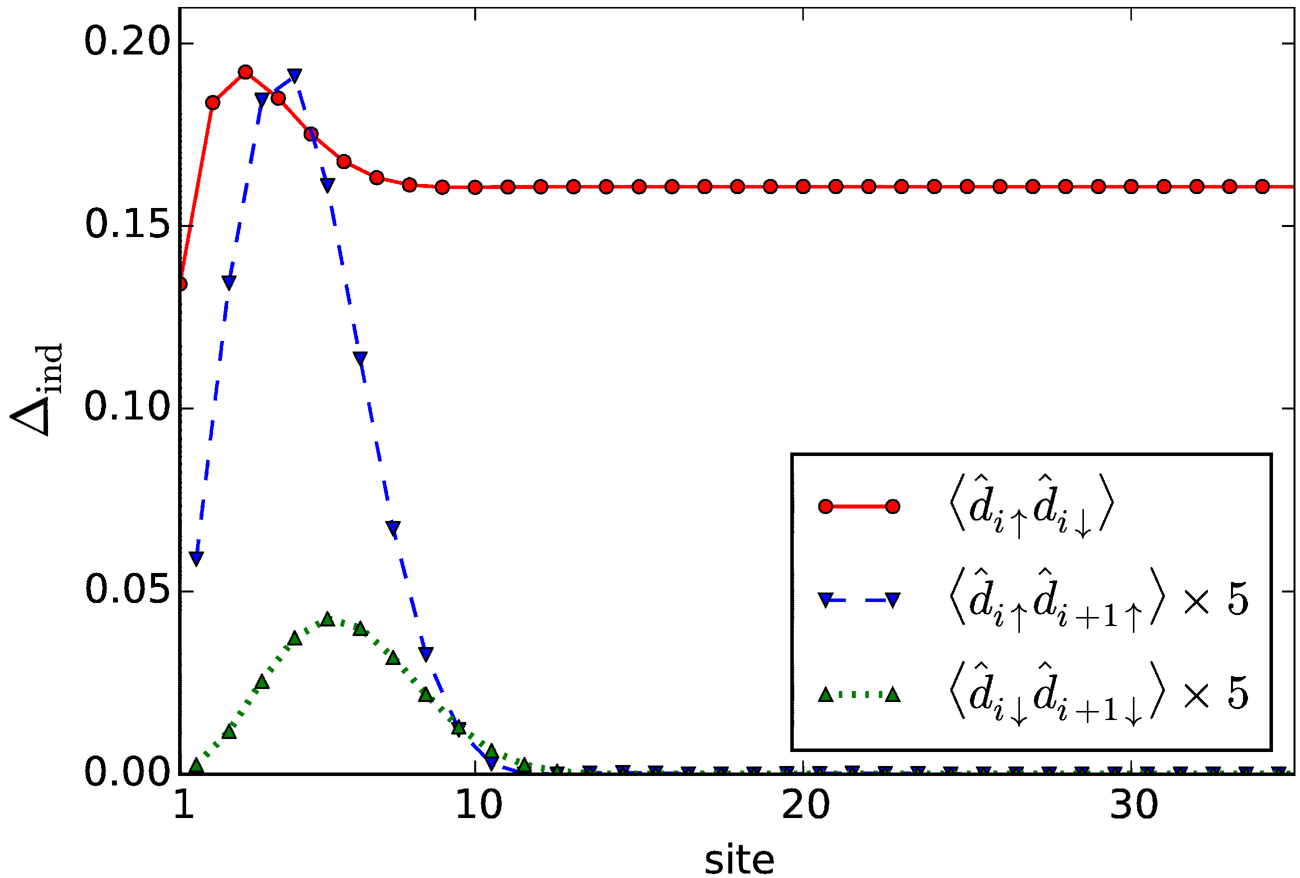
Mutation of Andreev states into zero-energy (Majorana) mode



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv:1609.00685 (2016).

Majorana states – **of the Rashba chain**

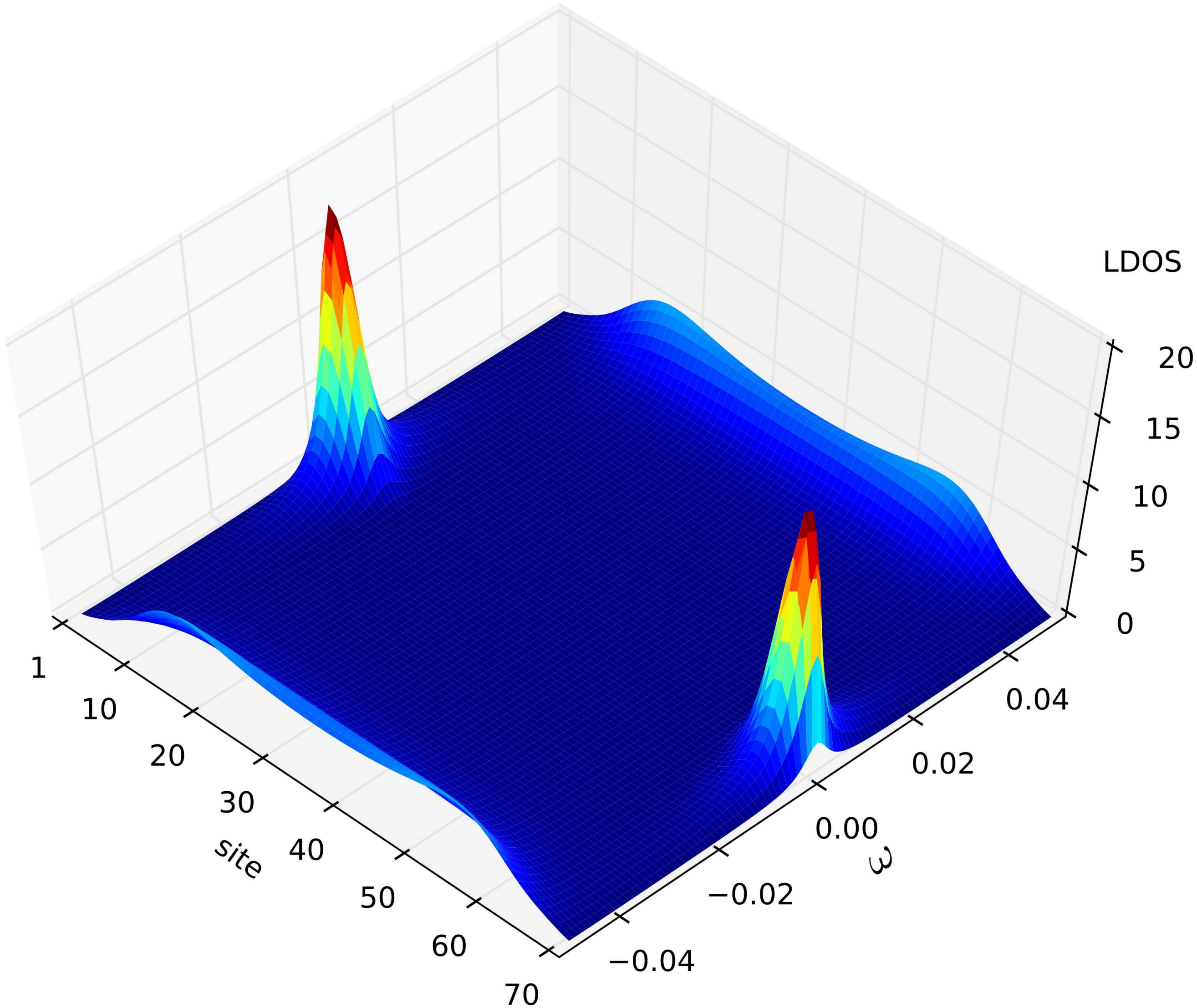
Spatial variation of the trivial and nontrivial pairings



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv:1609.00685 (2016).

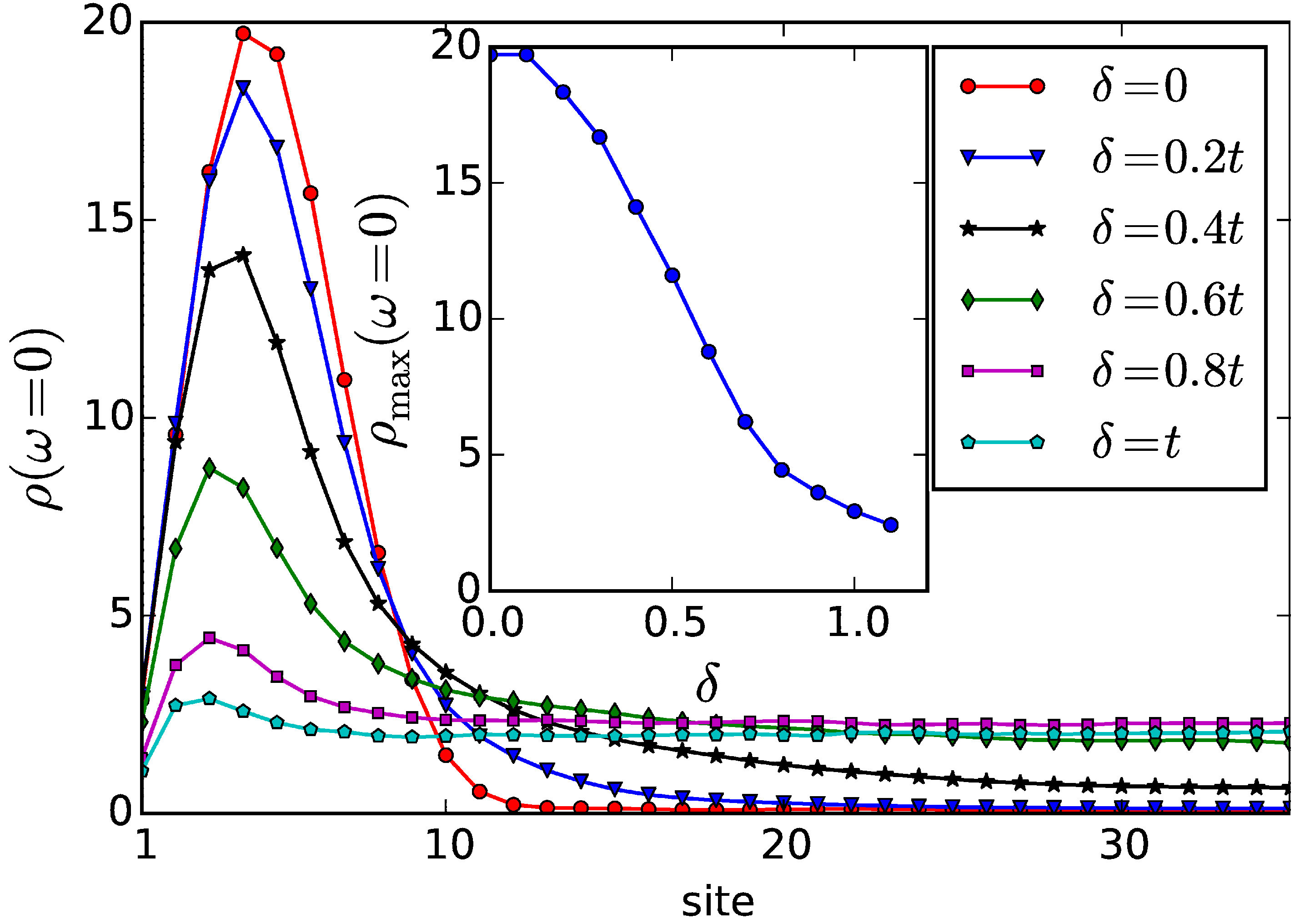
Majorana states – of the Rashba chain

In-gap spectrum with the edge Majorana quasiparticles



Majorana states – **of the Rashba chain**

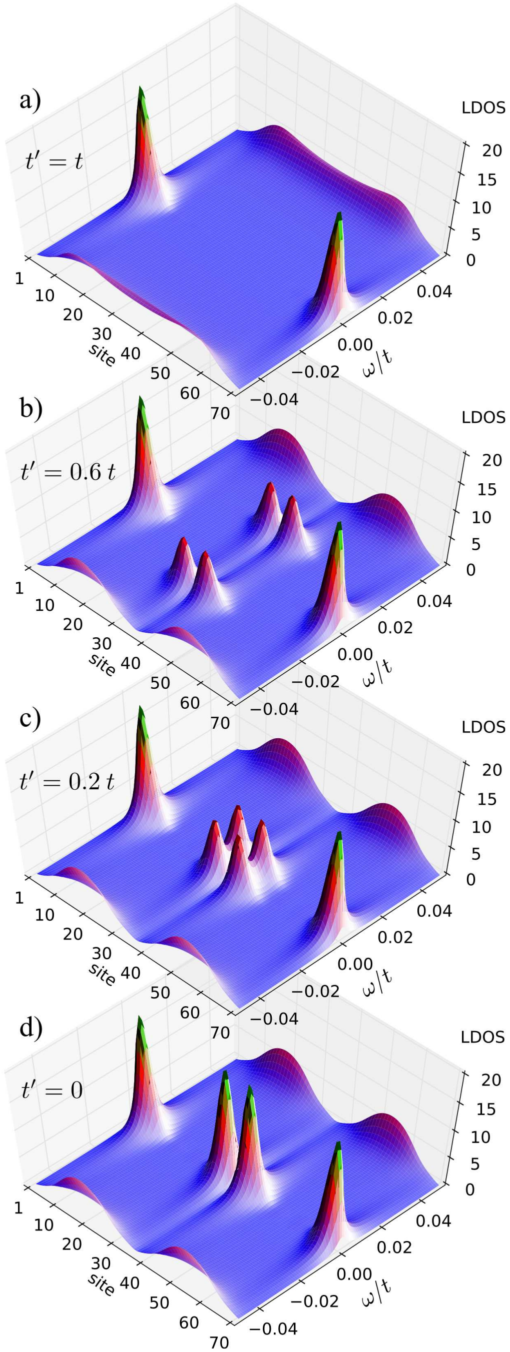
Are Majorana quasiparticles really immune to disorder ?



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv:1609.00685 (2016).

Majorana states – of the Rashba chain

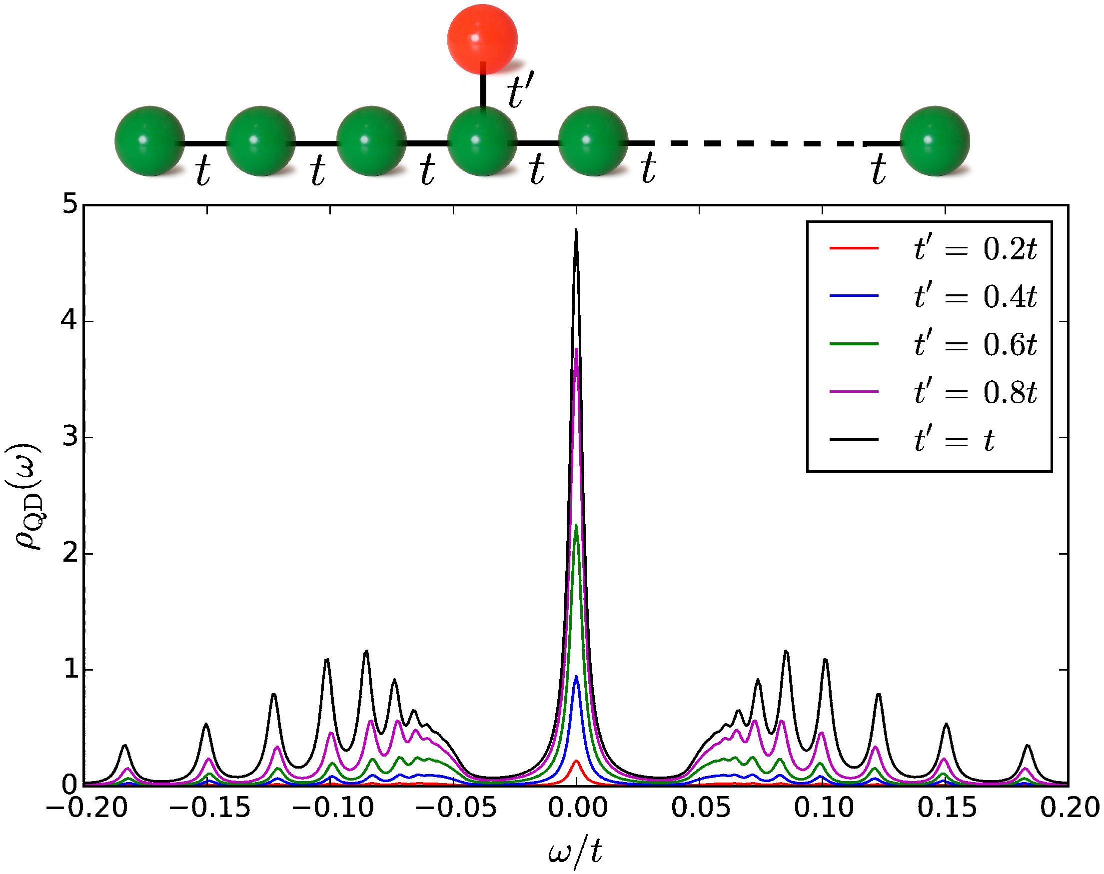
Gradual partitioning of the Rashba chain into separate pieces by reducing hopping t_j at site $j = N/2$



M. Maśka et al, arXiv:1609.00685 (2016).

Majorana states – of the Rashba chain

Majorana quasiparticle 'leaking' into a normal quantum impurity



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv:1609.00685 (2016).

Conclusions (part 2)

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Majorana-type quasiparticles:

⇒ **evolve out of the Andreev/Shiba states**

⇒ **are not completely immunue to disorder**

⇒ **are strictly non-local entities**

⇒ **can ’leak’ into normal quantum impurities**

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