

DYNAMICAL QUANTUM PHASE TRANSITIONS IN SUPERCONDUCTING NANOSTRUCTURES

Tadeusz Domański

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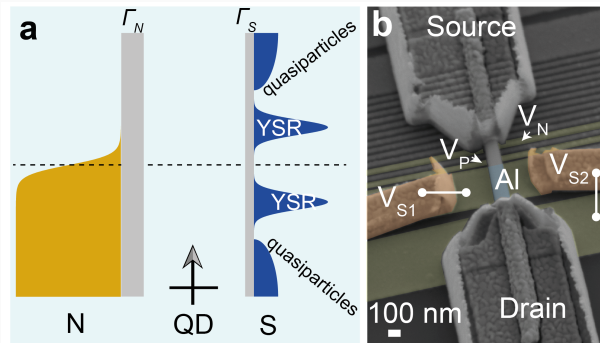
"Seminar on Coherence-Correlations-Complexity"

Wrocław, 13 Jan. 2021

Superconducting nanostructures: **examples**

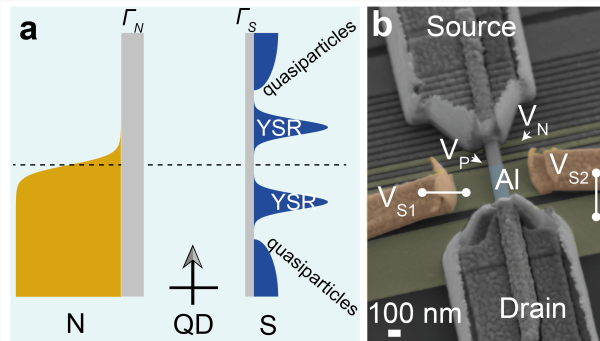
HETEROSTRUCTURES WITH SUPERCONDUCTOR(S)

Small (nanoscopic) objects attached to bulk superconductor



HETEROSTRUCTURES WITH SUPERCONDUCTOR(S)

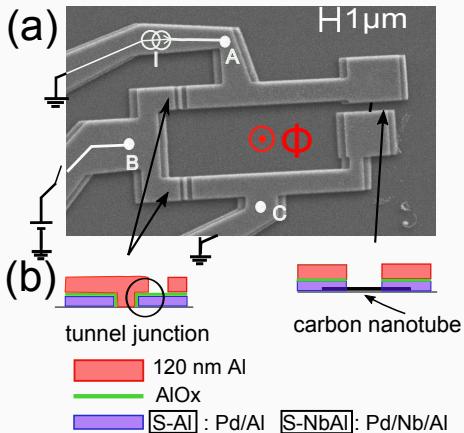
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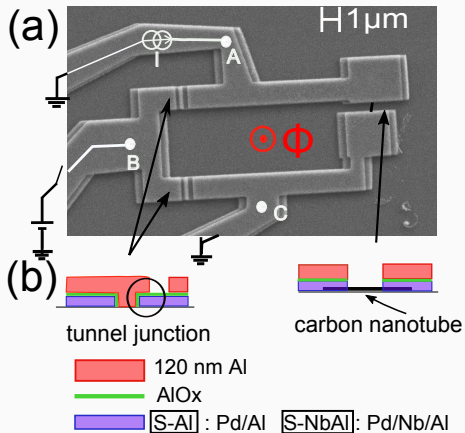
normal metal (N) - quantum dot (QD) - superconductor (S)

J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup,
K. Grove-Rasmussen and J. Nygård, Commun. Phys. **3**, 125 (2020).

HETEROSTRUCTURES WITH SUPERCONDUCTOR(S)



HETEROSTRUCTURES WITH SUPERCONDUCTOR(S)



superconductor (S) - quantum dot (QD) - superconductor (S)

R. Delagrangé, R. Weil, A. Kasumov, M. Ferrier, H. Bouchiat, R. Deblock,
Phys. Rev. B **93**, 195437 (2016).

SUPERCONDUCTING PROXIMITY EFFECT

- Coupling of the localized (QD) to itinerant (SC) electrons induces:

⇒ **on-dot pairing**

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⇒ **in-gap bound states**

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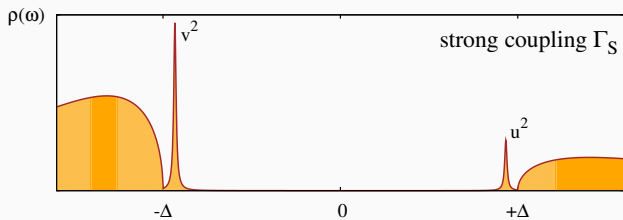
- originating from:

⇒ **leakage of Cooper pairs on QD** (Andreev)

⇒ **exchange int. of QD with SC** (Yu-Shiba-Rusinov)

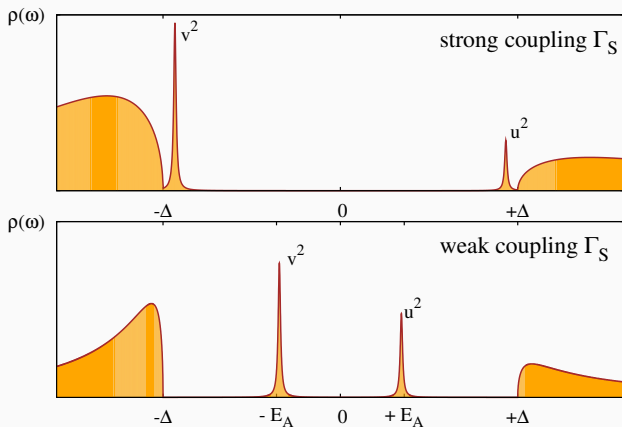
IN-GAP STATES

Spectrum of a single impurity coupled to bulk superconductor:



IN-GAP STATES

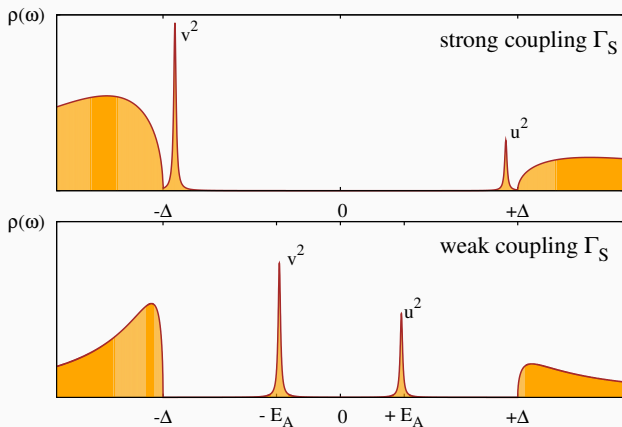
Spectrum of a single impurity coupled to bulk superconductor:



Bound states appearing in the subgap region $-\Delta < \omega < \Delta$.

IN-GAP STATES

Spectrum of a single impurity coupled to bulk superconductor:



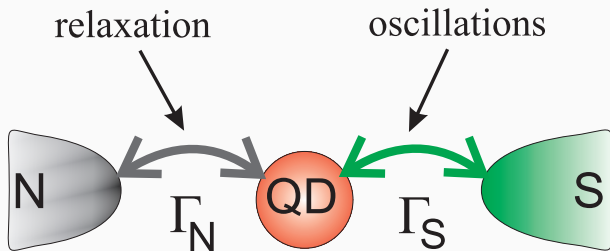
Bound states appearing in the subgap region $-\Delta < \omega < \Delta$.

Yu-Shiba-Rusinov (Andreev) bound states

Characteristic temporal scales

TRANSIENT EFFECTS OF IN-GAP STATES

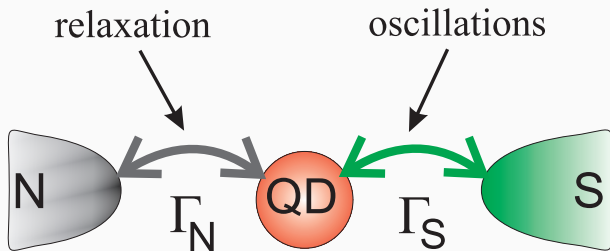
Consider a sudden coupling of QD to external leads



R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

TRANSIENT EFFECTS OF IN-GAP STATES

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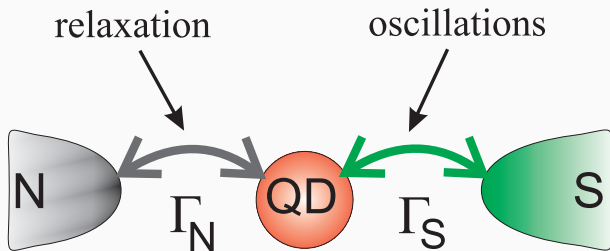


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- how much time does it take to create in-gap states?

TRANSIENT EFFECTS OF IN-GAP STATES

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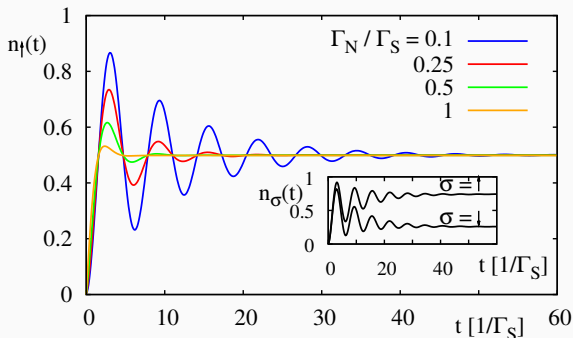


R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

- how much time does it take to create in-gap states?
- are there any characteristic time-scales?

RELAXATION VS QUANTUM OSCILLATIONS

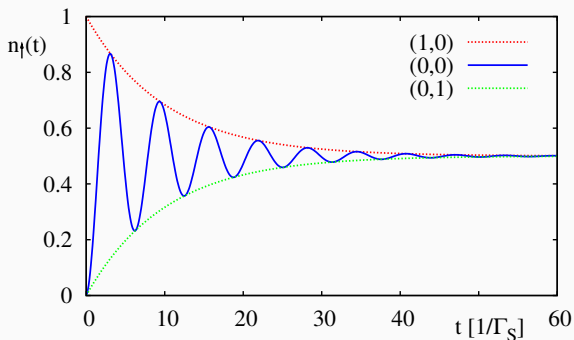
Time-dependent charge of an initially empty QD



- relaxation time is proportional to $1/\Gamma_N$
- oscillations depend on energies of in-gap states

RELAXATION VS QUANTUM OSCILLATIONS

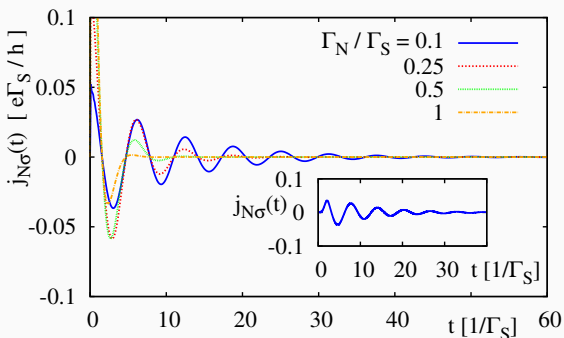
t-dependent charge for various initial fillings ($n_{\downarrow}, n_{\uparrow}$)



- **relaxation time is proportional to $1/\Gamma_N$**
- **oscillations depend on energies of in-gap states**

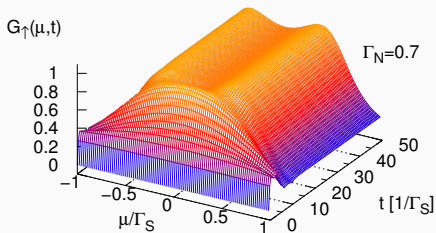
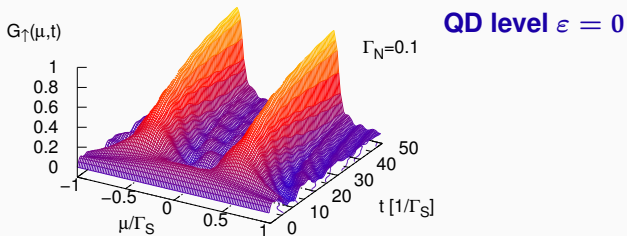
RELAXATION VS QUANTUM OSCILLATIONS

Time-dependent charge current of unbiased junction



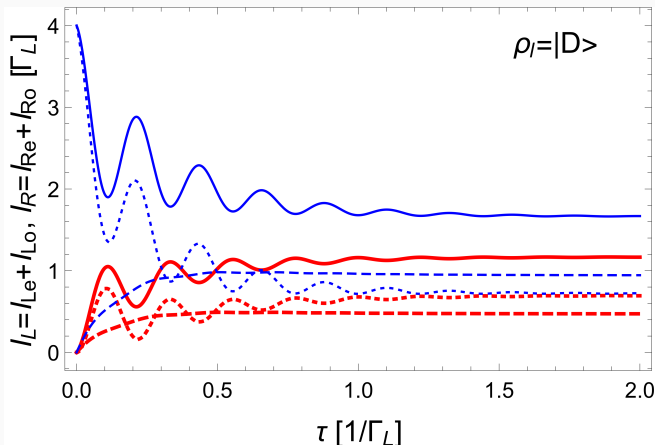
- relaxation time is proportional to $1/\Gamma_N$
- oscillations depend on energies of in-gap states

EXPERIMENTALLY ACCESSIBLE QUANTITIES



Subgap tunneling conductance $G_{\sigma} = \frac{\partial I_{\sigma}}{\partial t}$ vs time (t) and voltage (μ)

STATISTICS OF TUNNELING EVENTS



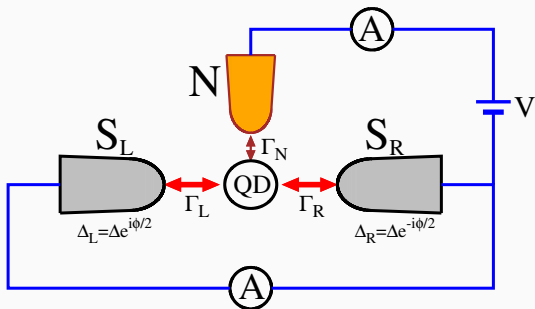
Transient currents from 'Waiting Time Distribution' approach

G. Michałek, B. Bułka, T. Domański & K.I. Wysokiński, Phys. Rev. B 101, 235402 (2020).

Josephson-type structures

JOSEPHSON-TYPE GEOMETRY

Quantum dot embedded into Josephson & Andreev circuits.

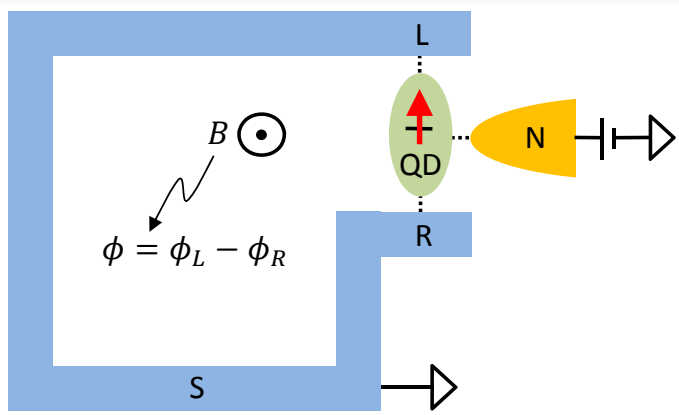


T. Domański ... V. Janiš & T. Novotný, *Phys. Rev. B* **95**, 045104 (2017);

P. Zalom, V. Pokorný & T. Novotný, *Phys. Rev. B* **102**, (2021) *to appear*

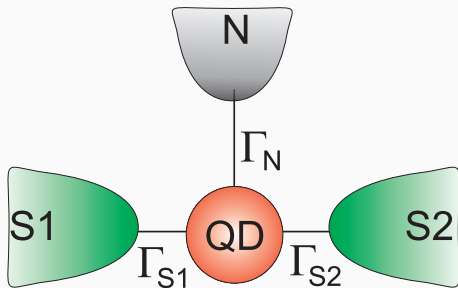
SCHEME FOR EMPIRICAL REALIZATION

Scheme for feasible realization of Josephson & Andreev circuits



G. Kiršanskas, M. Goldstein, K. Flensberg, L.I. Glazman & J. Paaske,
Phys. Rev. B 92, 235422 (2015)

PHASE-CONTROLLED TRANSIENT EFFECTS



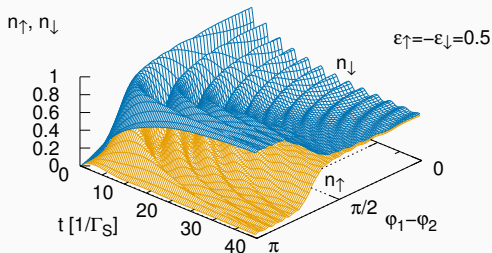
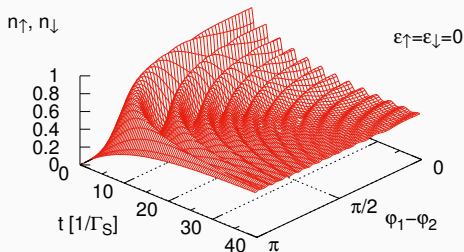
R. Taranko, T. Kwapiński & T. Domański, Phys. Rev. B 99, 165419 (2019).

Physical issue:

- phase tunable evolution of in-gap states

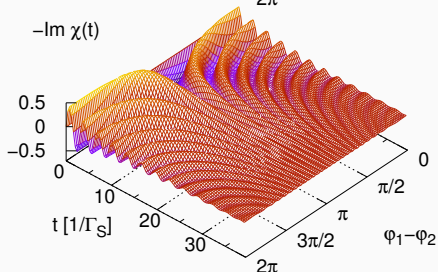
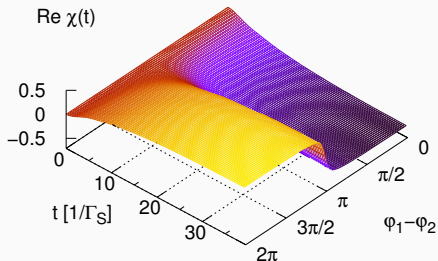
PHASAL TRANSIENT EFFECTS

Phase & time dependent QD occupancy $n_\sigma(t)$



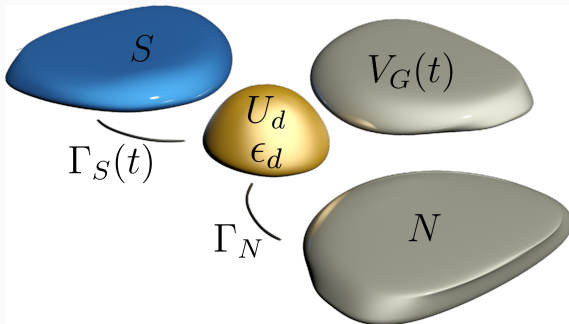
PHASAL TRANSIENT EFFECTS

Proximity induced on-dot pairing $\chi(t) = \langle \hat{d}_\downarrow \hat{d}_\uparrow \rangle$



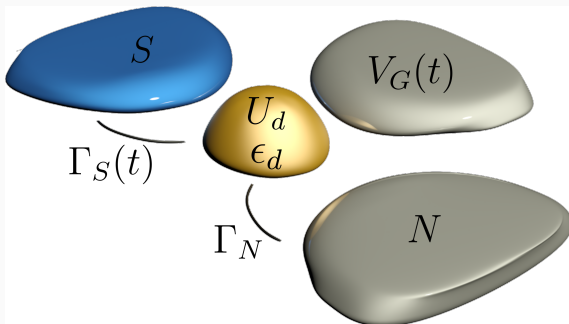
Dynamics of correlated nanosuperconductors

QUENCH DRIVEN DYNAMICS



Quantum quench protocols:

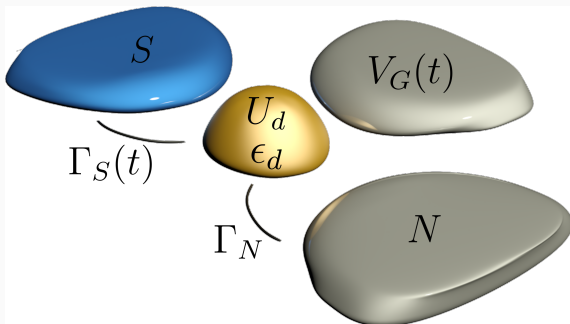
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\Rightarrow sudden coupling to superconductor $0 \rightarrow \Gamma_S$

QUENCH DRIVEN DYNAMICS



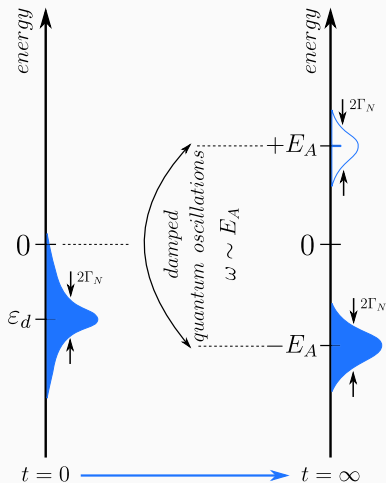
Quantum quench protocols:

\Rightarrow sudden coupling to superconductor $0 \rightarrow \Gamma_S$

\Rightarrow abrupt application of gate potential $0 \rightarrow V_G$

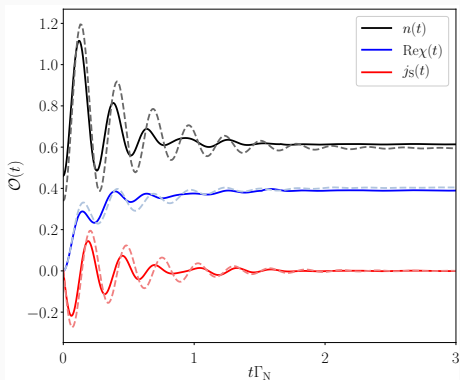
A) BUILDUP OF IN-GAP STATES

Schematics of the Andreev states formation induced by quench $0 \rightarrow \Gamma_5$



A) BUILDUP OF IN-GAP STATES

Time-dependent observables driven by the quantum quench $0 \rightarrow \Gamma_S$

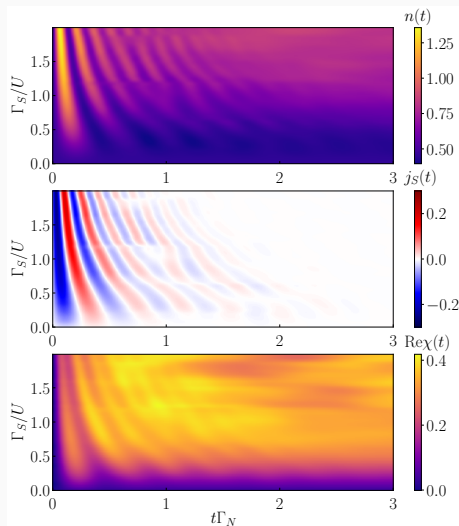


solid lines - time dependent NRG

dashed lines - Hartree-Fock-Bogolubov

A) BUILDUP OF IN-GAP STATES

tNRG results obtained for $\varepsilon_d = 0$



QD charge $n(t)$

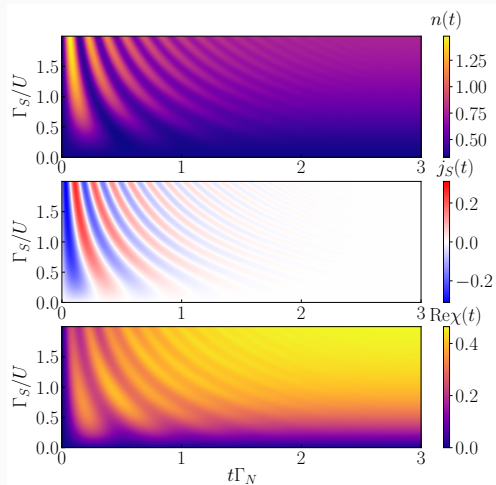
**charge current $j_S(t)$
from supercond. to QD**

on-dot pairing

$\chi(t) \equiv \langle d_\downarrow d_\uparrow \rangle$

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Hartree-Fock-Bogolubov results for $\varepsilon_d = 0$



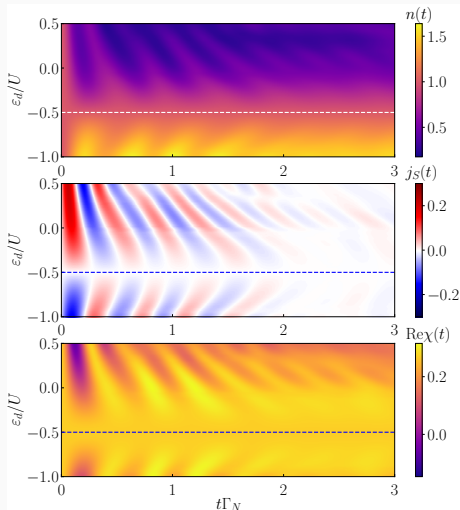
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B) SUDDEN CHANGE OF GATE POTENTIAL

tNRG results for $U = 2\Gamma_S$ and $\varepsilon_d(t < 0) = -U/2$



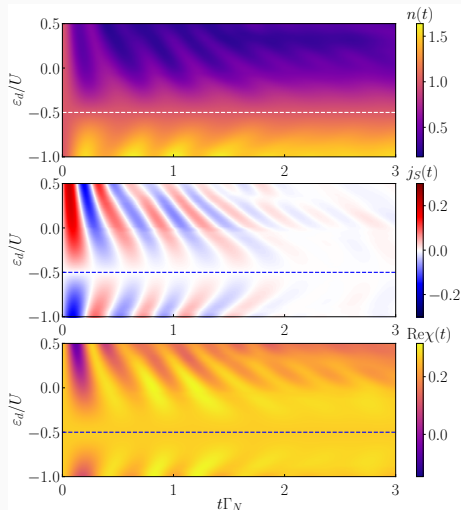
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Quantum dot charge $n(t)$

**Notice reversal of $j_S(t)$!
(dynamical $0-\pi$ transition)**

on-dot pairing

$\chi(t) \equiv \langle d_1 d_2 \rangle$

Quantum phase transition

SINGLY OCCUPIED VS BCS-TYPE CONFIGURATIONS

The proximitized quantum dot can be described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \hat{d}_{\sigma}^{\dagger} \hat{d}_{\sigma} + U_d \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left(\Delta_d \hat{d}_{\uparrow}^{\dagger} \hat{d}_{\downarrow}^{\dagger} + \text{h.c.} \right)$$

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Eigen-states of this problem are represented by:

$$\begin{array}{ll} |\uparrow\rangle \quad \text{and} \quad |\downarrow\rangle & \Leftarrow \quad \text{doublet states (spin } \frac{1}{2} \text{)} \\ \left. \begin{array}{l} u |0\rangle - v |\uparrow\downarrow\rangle \\ v |0\rangle + u |\uparrow\downarrow\rangle \end{array} \right\} & \Leftarrow \quad \text{singlet states (spin 0)} \end{array}$$

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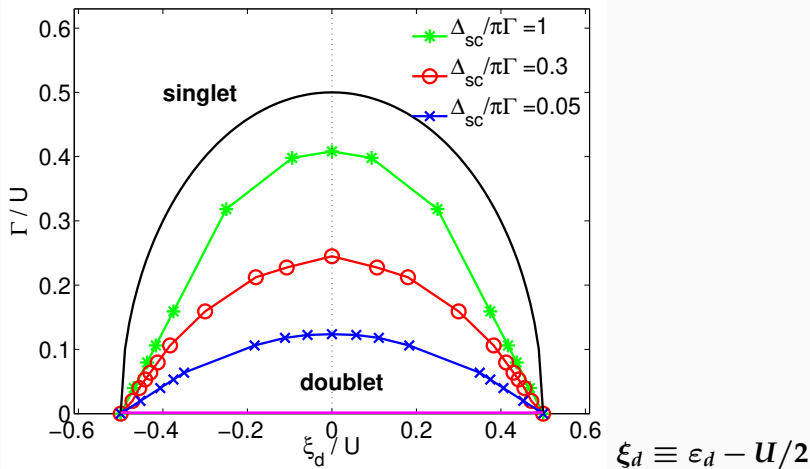
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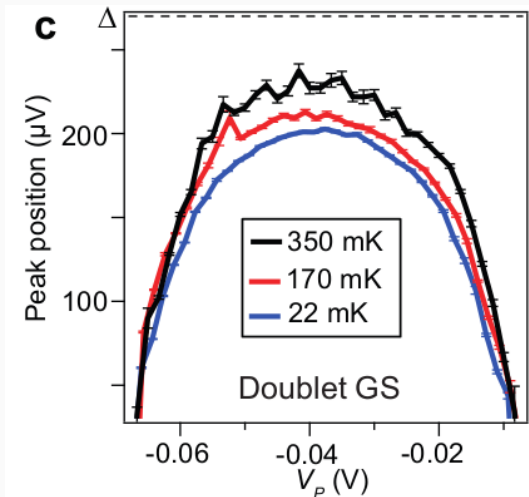
Upon varying the parameters ϵ_d , U_d or Γ_S there can be induced **quantum phase transition** between these doublet/singlet states.

QUANTUM PHASE TRANSITION (STATIC VERSION)

Singlet-doublet quantum phase transition: NRG results



QUANTUM PHASE TRANSITION: EXPERIMENT



J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, *Commun. Phys.* **3**, 125 (2020).

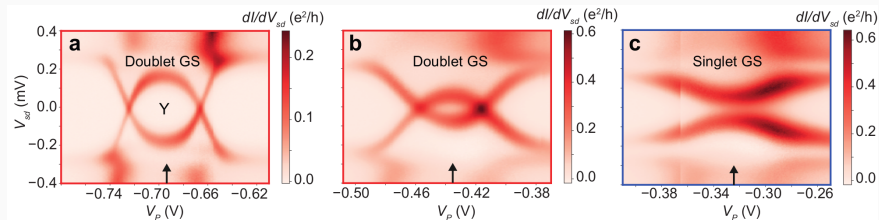
SINGLET VS DOUBLET: EXPERIMENT

Differential conductance vs source-drain bias V_{sd} (vertical axis) and gate potential V_p (horizontal axis) measured for various Γ_s/U

$$U \gg \Gamma_s$$

$$U \geq \Gamma_s$$

$$U < \Gamma_s$$



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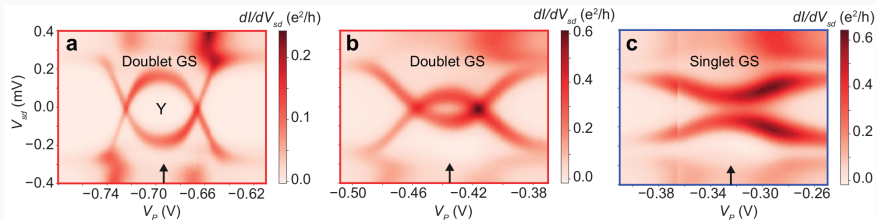
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K. Grove-Rasmussen and J. Nygård, *Commun. Phys.* **3**, 125 (2020).

Crossings of in-gap states correspond to the singlet-doublet QPT.

Dynamical quantum phase transition

QUENCH DYNAMICS

Initially, for $t < 0$:

$$\hat{H}_0 |\Psi_0\rangle = E_0 |\Psi_0\rangle$$

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Loschmidt amplitude

STATIONARY VS DYNAMICAL PHASE TRANSITION

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Loschmidt echo $L(t)$

$$L(t) = \left| \langle \Psi_0 | e^{-it\hat{H}} | \Psi_0 \rangle \right|^2$$

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Critical temperature T_c

nonanalytical $\lim_{T \rightarrow T_c} F(T)$

Loschmidt amplitude

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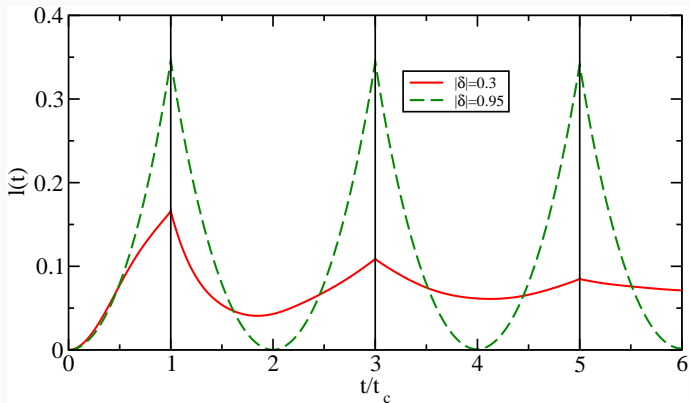
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Critical time t_c

nonanalytical $\lim_{t \rightarrow t_c} \lambda(t)$

EXAMPLE 1: SSH (QUENCH: NONTOPO \rightarrow TOPO)



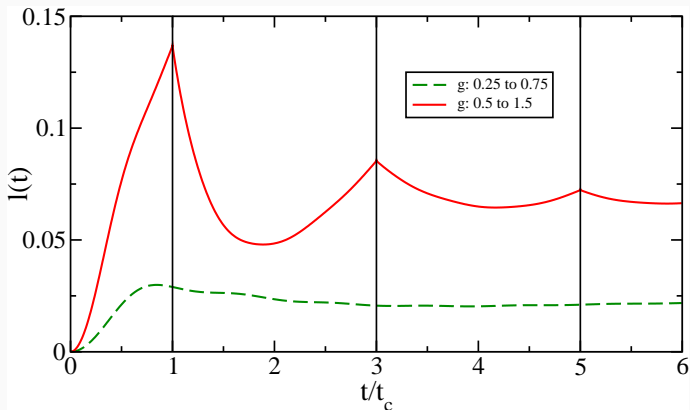
$$\hat{H} = -J \sum_j \left[(1 + \delta e^{i\pi j}) \hat{c}_j^\dagger \hat{c}_{j+1} + \text{h.c.} \right]$$

solid red line $|\delta| = 0.3$

dashed green line $|\delta| = 0.95$

N. Sedlmayr, Acta Phys. Polon. A 135, 1191 (2019).

EXAMPLE 2: ISING MODEL (TRANSITION FOR $g = 1$)

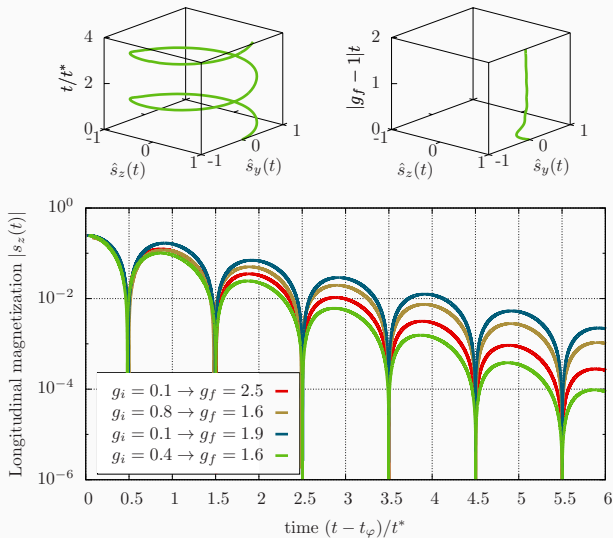


$$\hat{H} = -\frac{1}{2} \sum_{j=1}^{N-1} \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + \frac{g}{2} \sum_{j=1}^N \hat{\sigma}_j^x$$

solid red line - across a phase transition

dashed green line - inside a phase transition

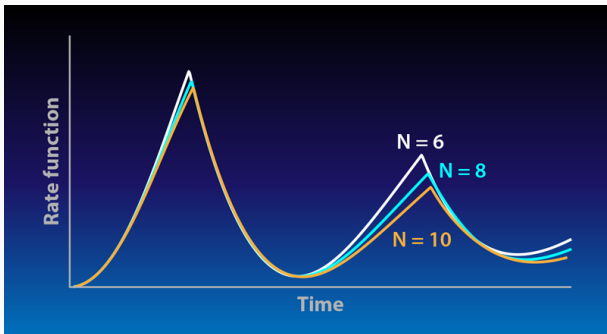
EXAMPLE 2: ISING MODEL (DYNAMICAL ORDER)



M. Heyl, A. Polkovnikov, S. Kehrein, Phys. Rev. Lett. 110, 135704 (2013).

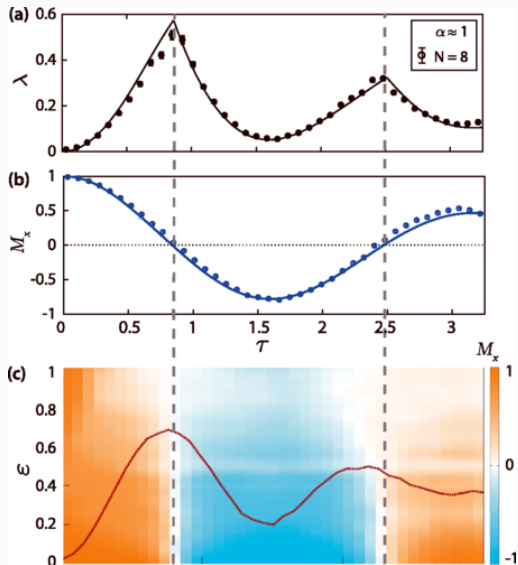
EXAMPLE 2: ISING MODEL (EXPERIMENT)

R. Blatt with coworkers [Austrian Acad. Sciences, Innsbruck] have observed dynamical quantum phase transitions in a linear string of N calcium-40 ions, with N up to 10. They measured the so-called rate function of the system as a function of time for $N=6$, $N=8$, and $N=10$. The singularities at two certain critical times that the measured function exhibited indicate that dynamical quantum phase transitions occurred at these times.



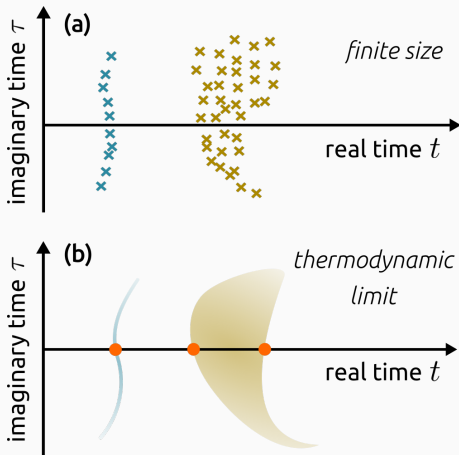
V. Gurarie, *Physics* 10, 95 (2017).

EXAMPLE 2: ISING MODEL (EXPERIMENT)



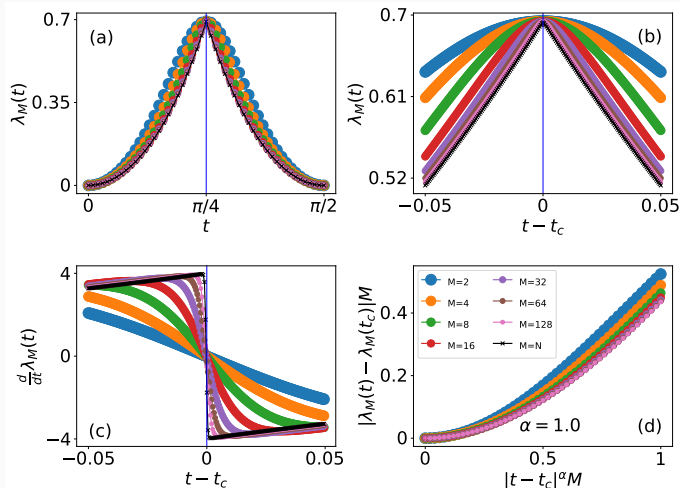
P. Jurcevic et al, Phys. Rev. Lett. 119, 080501 (2017).

WHAT ABOUT FINITE-SIZE SYSTEMS ?



Schematic view of "Fisher zeros" obtained for the Loschmidt amplitude $\langle \Psi_0 | e^{-iz\hat{H}} | \Psi_0 \rangle$ in the complex plane $z = t + i\tau$.

ISING MODEL: DQPT OF FINITE-SIZE SYSTEM

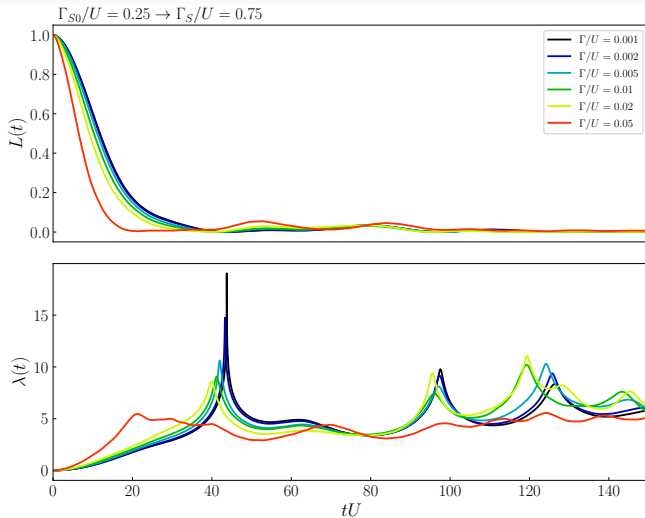


"Local measures of dynamical quantum phase transitions"

J.C. Halimeh, D. Trapin, M. Damme & M. Heyl, arXiv:2010.07307 (2020).

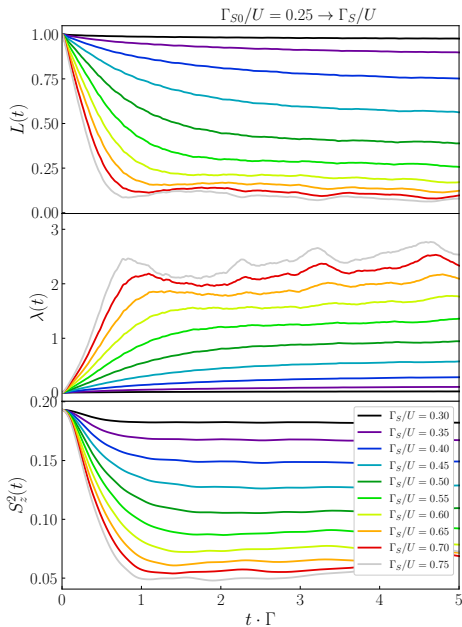
Singlet-doublet DQPT

t NRG RESULTS: ABRUPT CHANGE OF Γ_S



Loschmidt ampl. $L(t)$ and return rate $\lambda(t)$ obtained for various $\Gamma_N \equiv \Gamma$

t NRG RESULTS: ABRUPT CHANGE OF Γ_S



Loschmidt echo

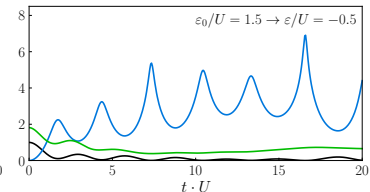
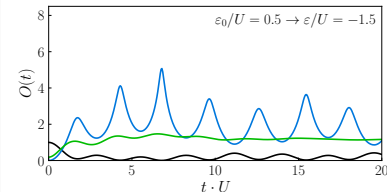
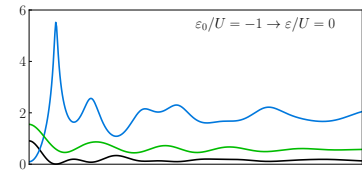
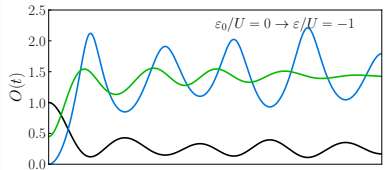
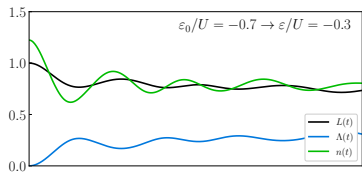
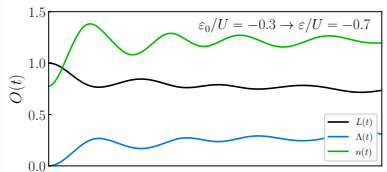
$$L(t) \equiv |\langle \Psi(t) | \Psi(0) \rangle|^2$$

Return rate

$$|L(t)| \equiv e^{-N\lambda(t)}$$

**The squared magnetic
moment $\langle S_z^2(t) \rangle$**

t NRG RESULTS: ABRUPT CHANGE OF ε_d



back

forth

QUESTIONS

(OF THIS ONGOING PROJECT)

Issues to be clarified:

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- what physical quantity does control t_c
- is critical time t_c really periodic ?
- is there any dynamical order ?
- empirical means to detect dynamical QPT ?

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- and ...
- **can anybody help us ?**

CONCLUSIONS

Quenching N-QD-S (and S_L -QD- S_R) nanostructures:

- **rescales/develops in-gap quasiparticles**

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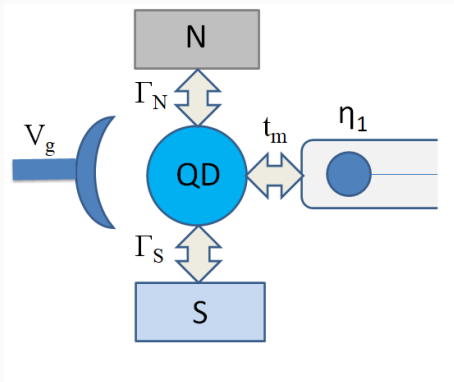
- rescales/develops in-gap quasiparticles
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⇒ These phenomena are manifested in transport properties.

Other related topics

DYNAMICS OF TOPOLOGICAL SUPERCONDUCTORS

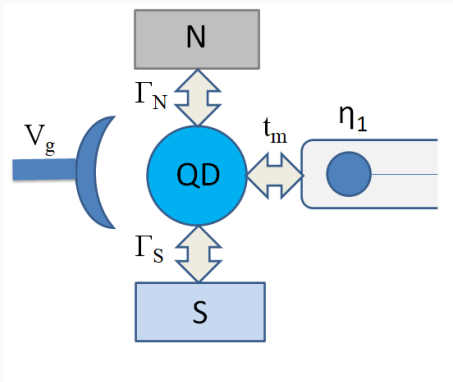
Abrupt coupling (t_m) of quantum dot to topological SC nanowire



J. Barański, ... & T. Domański,
arXiv:2012.03077 (2020).

DYNAMICS OF TOPOLOGICAL SUPERCONDUCTORS

Abrupt coupling (t_m) of quantum dot to topological SC nanowire

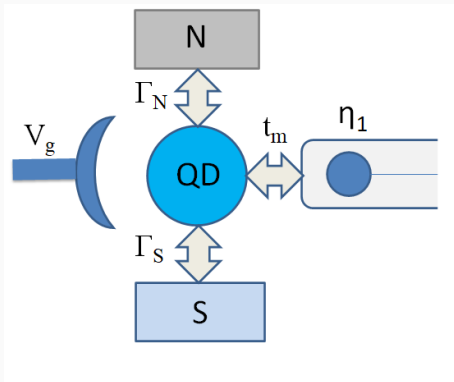


J. Barański, ... & T. Domański,
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- time needed for Majorana leakage on QD,

DYNAMICS OF TOPOLOGICAL SUPERCONDUCTORS

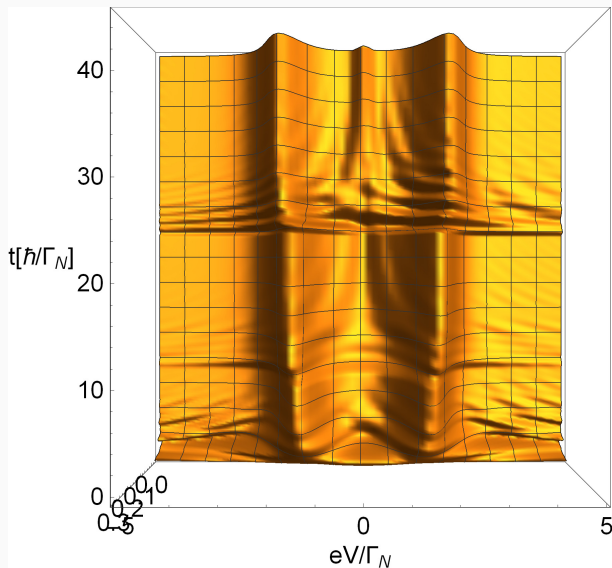
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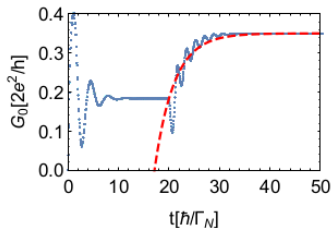
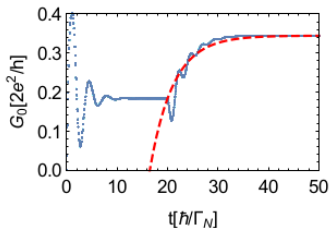
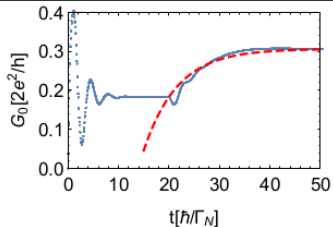
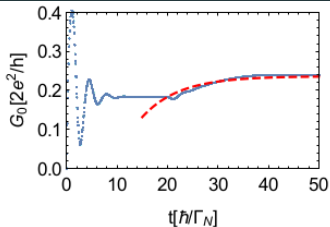
- time needed for Majorana leakage on QD,
- time-resolved zero bias conductance.

TIME-RESOLVED MAJORANA LEAKAGE



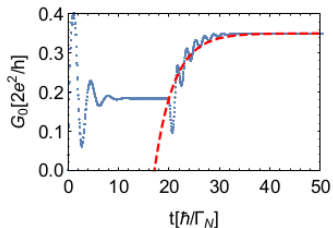
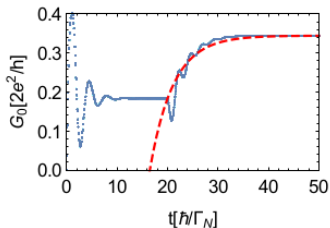
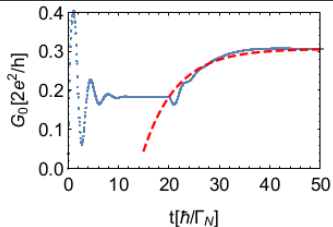
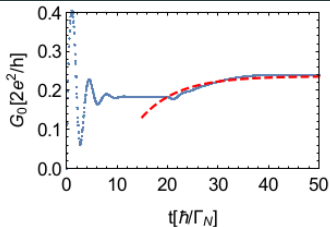
The differential Andreev conductance vs bias voltage V and time

TIME-RESOLVED ZERO BIAS CONDUCTANCE



The zero-bias differential conductivity obtained for $\Gamma_S = 3\Gamma_N$ and $\epsilon_d = \Gamma_N$, assuming: $t_m = 0.25$ (upper left), 0.5 (upper right), 1 (lower left), 1.5 (lower right) Γ_N . QD is abruptly connected to Majorana mode at time $t = 20\hbar/\Gamma_N$.

TIME-RESOLVED ZERO BIAS CONDUCTANCE

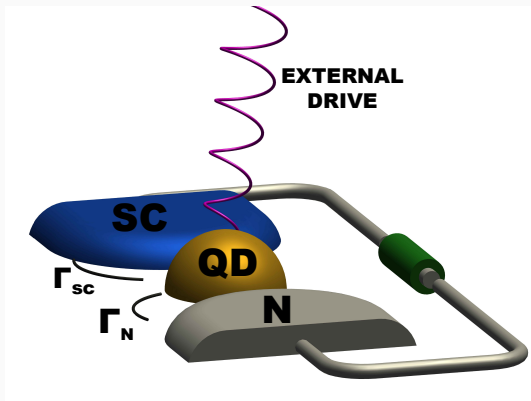


The zero-bias differential conductivity obtained for $\Gamma_S = 3\Gamma_N$ and $\epsilon_d = \Gamma_N$, assuming: $t_m = 0.25$ (upper left), 0.5 (upper right), 1 (lower left), 1.5 (lower right) Γ_N . QD is abruptly connected to Majorana mode at time $t = 20\hbar/\Gamma_N$. For realistic systems such leakage time takes 2-20 nanoseconds.

Floquet description of in-gap states

PERIODIC DRIVING

Quantum impurity with periodically oscillating energy level



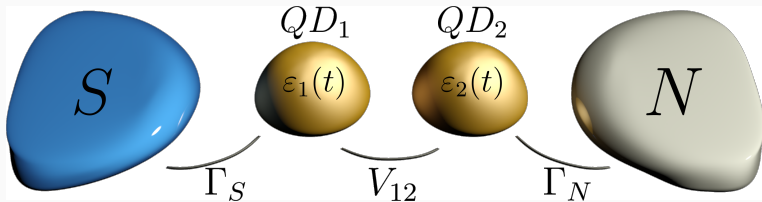
$$\varepsilon(t) = \varepsilon_0 + V \times \cos(\omega t)$$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).

Proximitized double QDs (qubits)

NONEQUILIBRIUM DYNAMICS

Quantum quench scenarios: gating, biasing & driving



Methods:

- Floquet approach,
- Keldysh technique,
- machine learning

ACKNOWLEDGEMENTS

- **dynamics of in-gap states (transients, periodic driving, etc.)**

⇒ R. Taranko (Lublin), B. Baran (Lublin),

- **dynamical singlet-doublet phase transition**

⇒ K. Wrześniewski (Poznań), I. Weymann (Poznań),

N. Sedlmayr (Lublin),

- **time-dependent leakage of Majorana qps**

⇒ J. Barański (Dęblin), M. Barańska (Dęblin),

- **dynamical topological phase transitions**

⇒ A. (Kobiałka), G. Wlazłowski (Warsaw).