Topological states of matter in superconducting heterostructures

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INTiBS PAN, Wrocław

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ISSUES TO BE ADDRESSED

- **quasiparticles of conventional *'trivial'* superconductors**

$\Longrightarrow$ **particle vs hole**

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N. Bogoliubov **J. Bardeen** **E. Majorana**

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II. **On a new method in the theory of superconductivity**

N.N. Bogoliubov, Nuovo Cim. 7, 794 (1958)

I. SUPERFLUIDITY

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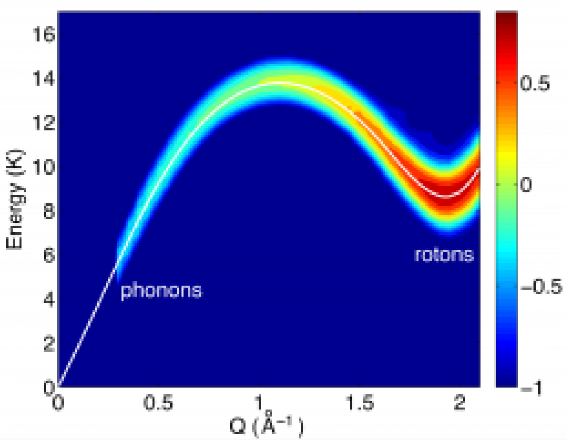

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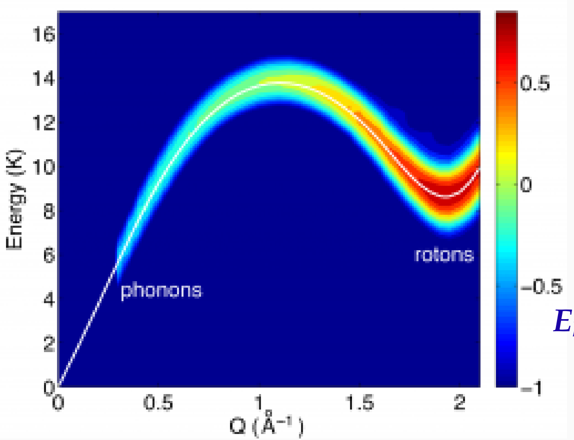

$$E_k = \sqrt{\varepsilon_k^2 + n_0 g \varepsilon_k}$$

where $\varepsilon_k = \frac{\hbar^2 k^2}{2m}$

Effective spectrum of superfluid ^{4}He

$g \leftarrow$ interaction

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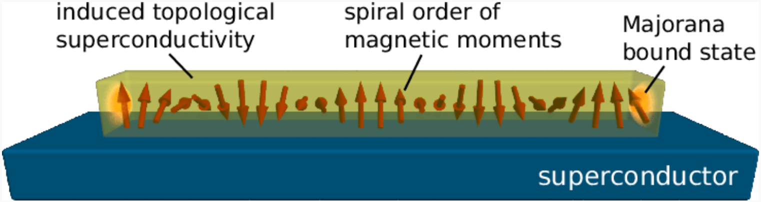
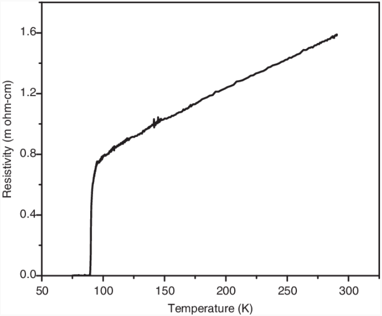


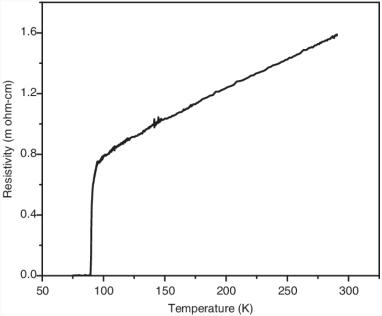
Bulk superconductors

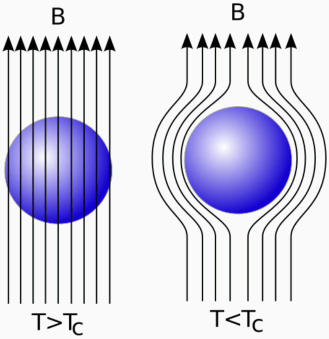
Perfect conductor



SUPERCONDUCTOR

Perfect conductor





Perfect diamagnet

HALLMARKS OF ELECTRON PAIRING

BCS ground state :

$$|\mathbf{BCS}\rangle = \prod_k \left(u_k + v_k \; \hat{c}^\dagger_{k\uparrow} \; \hat{c}^\dagger_{-k\downarrow} \right) \; |\mathbf{vacuum}\rangle$$

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$$\begin{aligned} \hat{\gamma}_{k\uparrow} &= u_k \hat{c}_{k\uparrow} + v_k \hat{c}^\dagger_{-k\downarrow} \\ \hat{\gamma}^\dagger_{-k\downarrow} &= -v_k \hat{c}_{k\uparrow} + u_k \hat{c}^\dagger_{-k\downarrow} \end{aligned}$$

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formally due to

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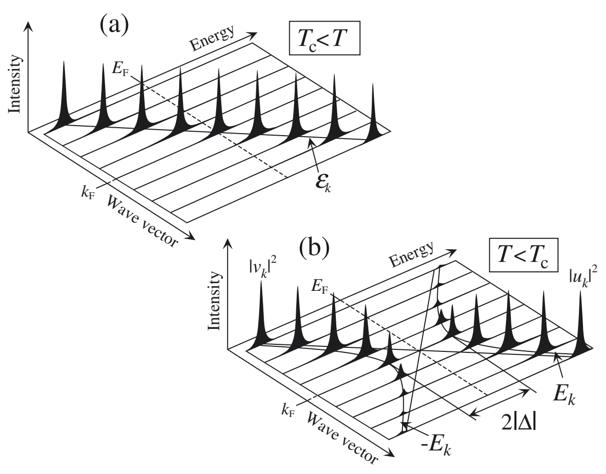
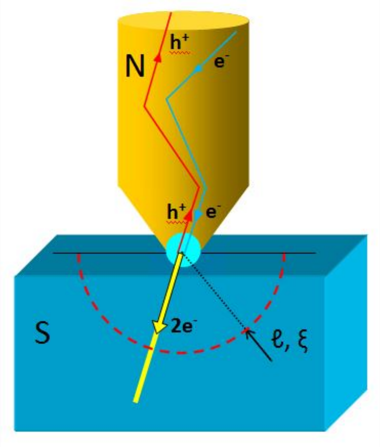
BOGOLIUBOV QUASIPARTICLES

Quasiparticle spectrum of conventional superconductors consists of the Bogoliubov (p/h) branches gaped around E_F



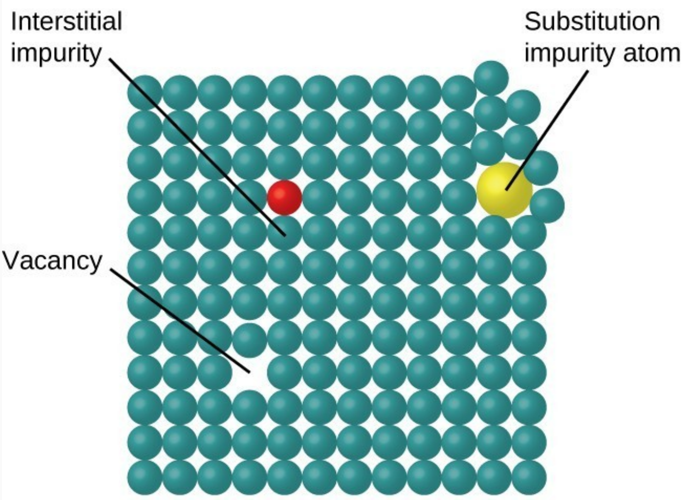
PARTICLE VS HOLE

In superconductors the particle and hole degrees of freedom are mixed via pairing interactions (efficient near the Fermi energy).



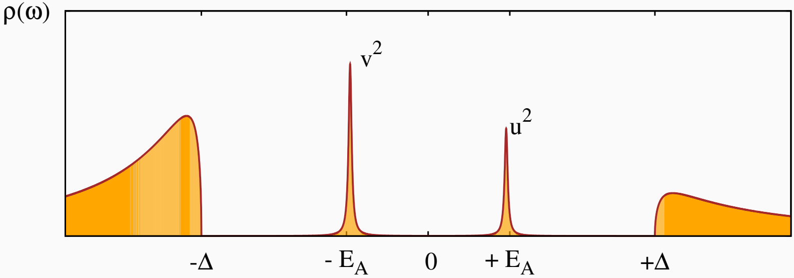
Superconductivity in nanosystems

IMPURITIES IN SOLIDS



IN-GAP STATES

Spectrum of a single impurity hybridized with superconductor:



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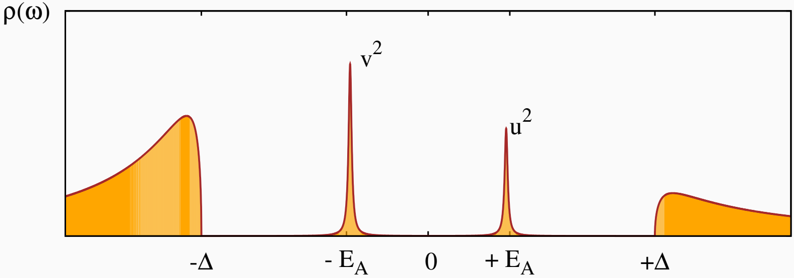
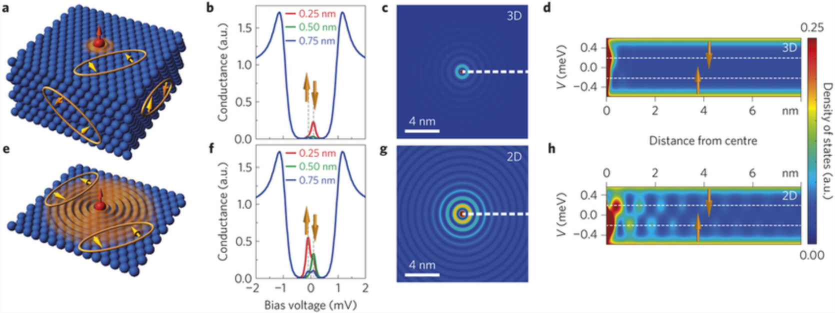


Bound states appearing in the subgap region $E \in \langle -\Delta, \Delta \rangle$

are dubbed Yu-Shiba-Rusinov (or Andreev) quasiparticles.

DIMENSIONALITY EFFECT

Empirical data obtained from STM measurements for $NbSe_2$



a) very small extent in dim=3

b) much longer extent in dim=2

G.C. Menard et al., Nature Phys. 11, 1013 (2015).

TOPOGRAPHY AND SPATIAL EXTENT

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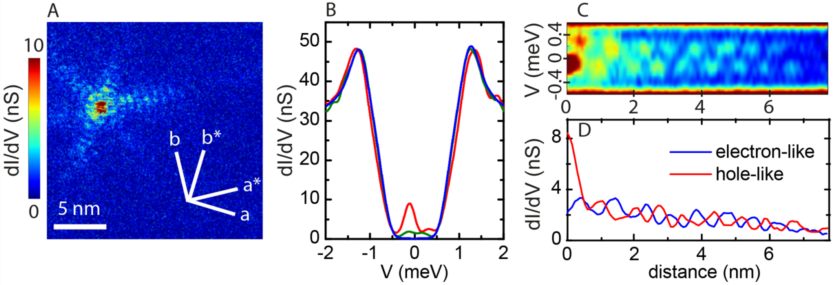

a) bound states extending to 10 nm

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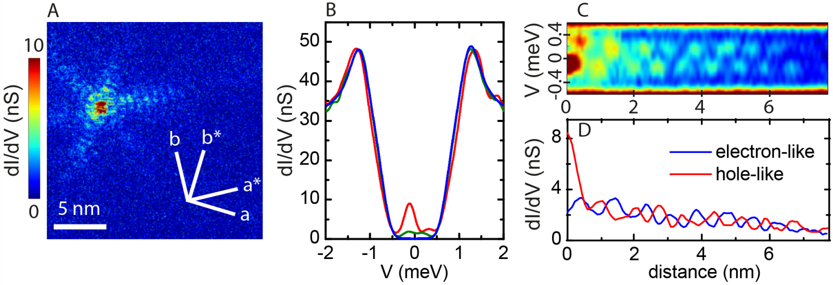

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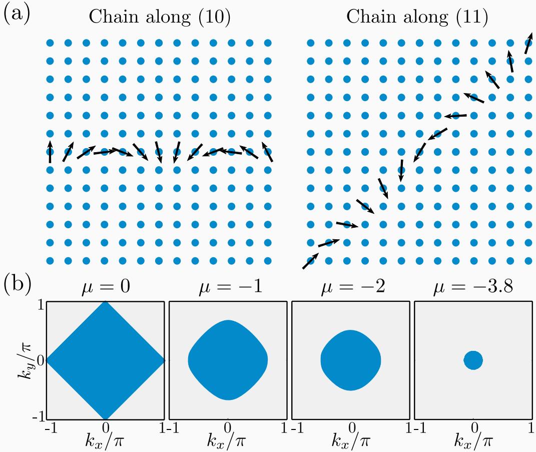
HTTPS://WWW.PKS.MPG.DE/BOSSA19/

7-10 April 2019, M. Planck Inst. (Dresden, Germany)

MAGNETIC CHAINS IN SUPERCONDUCTORS

A chain of magnetic impurities embedded in superconductor:



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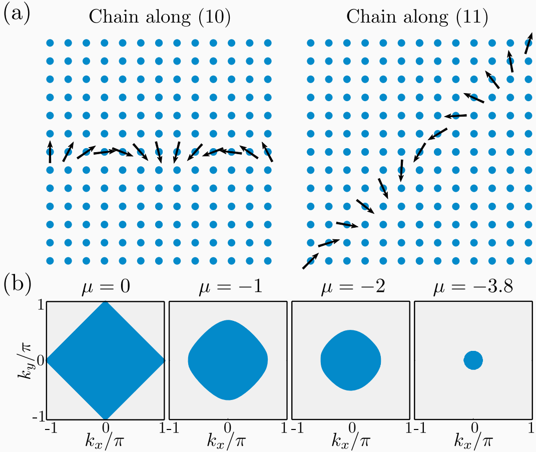

arranges its in-gap bound states into Shiba-band(s).

M.H. Christensen ... J. Paaske, Phys. Rev. B 94, 144509 (2016).

Topological superconductors

KITAEV CHAIN: PARADIGM FOR MAJORANA QPS

Itinerant 1-dimensional fermions with intersite (*p-wave*) pairing

$$\hat{H} = t \sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1} + \text{h.c.} \right) - \mu \sum_i \hat{c}_i^\dagger \hat{c}_i + \Delta \sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1}^\dagger + \text{h.c.} \right)$$

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can be conveniently recast in the Bogolubov/Majorana basis

$$\hat{\gamma}_{j,1} \equiv \frac{1}{\sqrt{2}}\left(\hat{c}_j + \hat{c}_j^\dagger\right)$$

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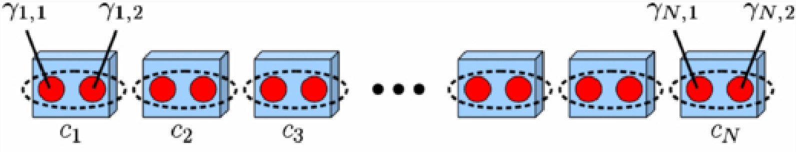

Alexei Y. Kitaev, Phys. Usp. 44, 131 (2001).

KITAEV CHAIN: PARADIGM FOR MAJORANA QPS

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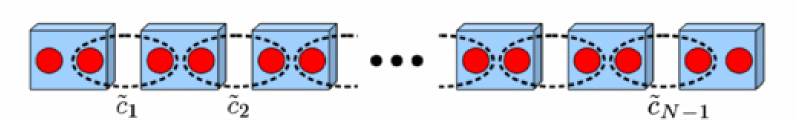

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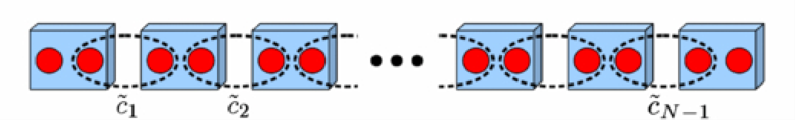

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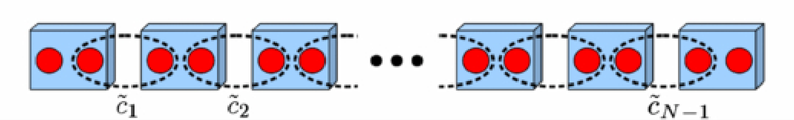

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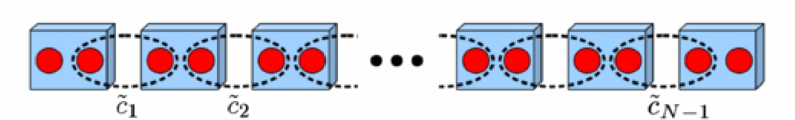

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They manifest themselves by very exotic phenomena !

PROPERTIES OF MAJORANA QPS

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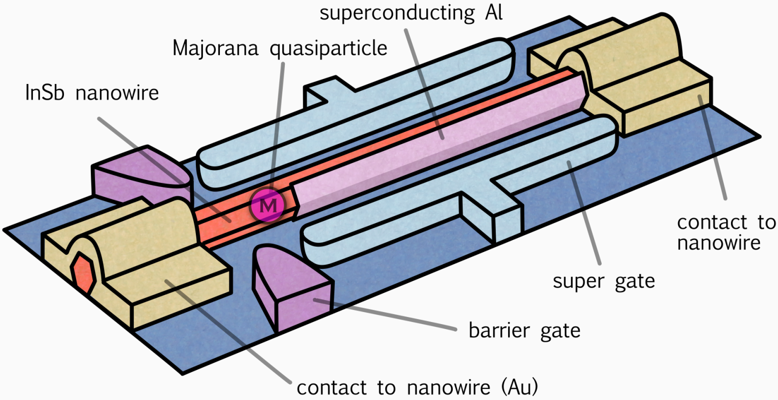
- **topologically protected**

$\Rightarrow$ **immune to dephasing/decoherence**

Scenario 1: Rashba + pairing

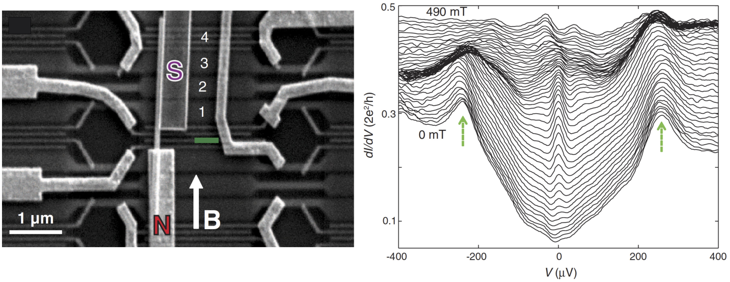
REALIZATIONS OF KITAEV SCENARIO

Intersite pairing of identical spin electrons can be driven e.g. by spin-orbit (Rashba) interaction in presence of external magnetic field, using nanowires proximitized to *s-wave* superconductor.



EMPIRICAL REALIZATION : SCENARIO # 1

Differential conductance dI/dV obtained for InSb nanowire at 70 mK upon varying a magnetic field.

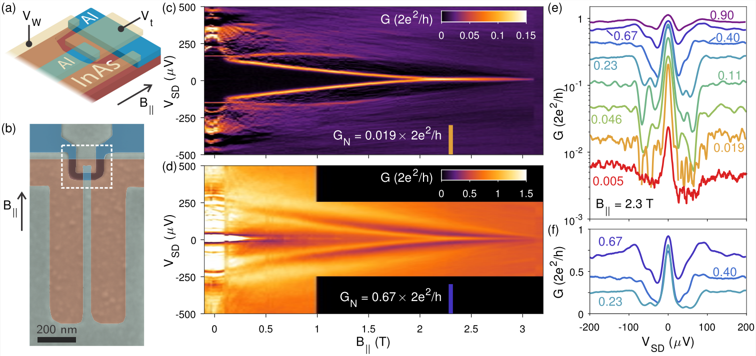


V. Mourik, ..., and L.P. Kouwenhoven, Science **336**, 1003 (2012).

/ Technical Univ. Delft, Netherlands /

EMPIRICAL REALIZATION: SCENARIO # 1

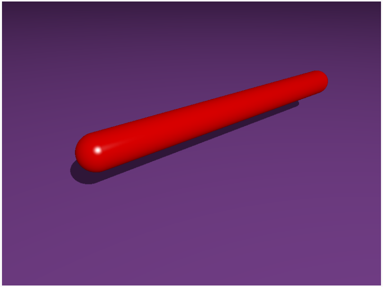
Litographically fabricated Al nanowire contacted to InAs

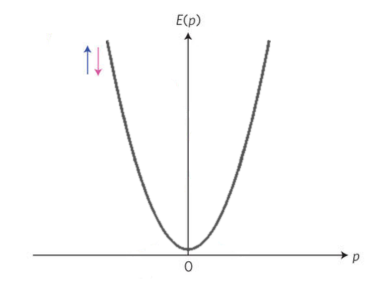


F. Nichele, ..., and Ch. Marcus, Phys. Rev. Lett. **119**, 136803 (2017).

/ Niels Bohr Institute, Copenhagen, Denmark /

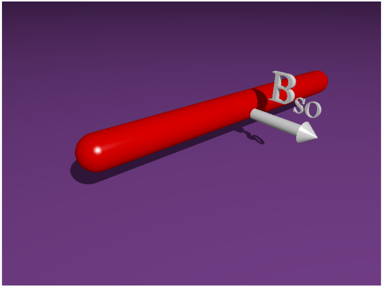
Nanowire

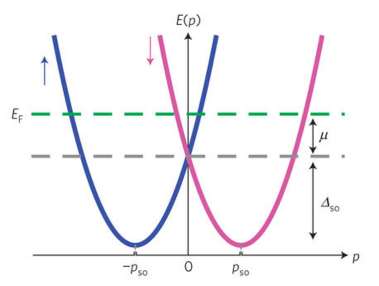




A. Das *et al*, Nature Phys. 8, 887 (2012).

Nanowire + Rashba





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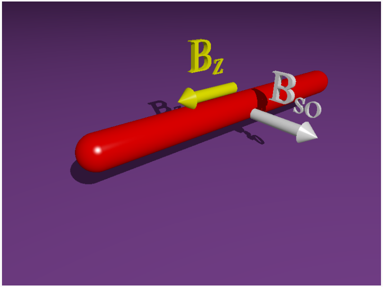
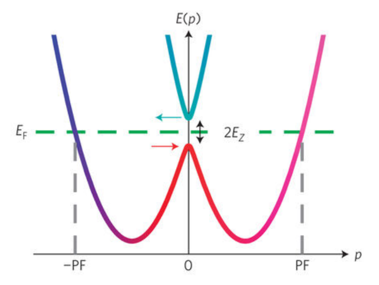
Nanowire + Rashba + magnetic field





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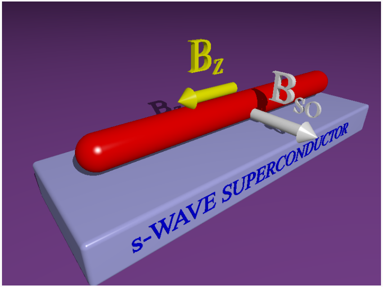
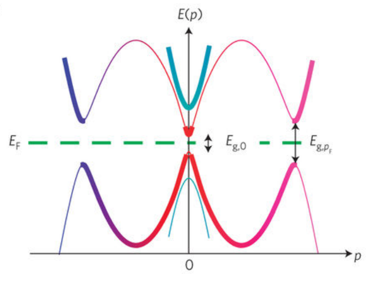
MAJORANA QPS: UNDERLYING MECHANISM

Nanowire + Rashba + magnetic field + superconductor





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$B < B_{cr} \longrightarrow$ **trivial superconducting phase**

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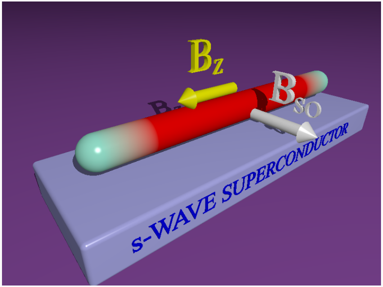

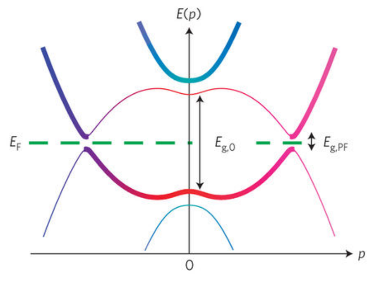

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$B > B_{cr} \longrightarrow$ **<u>nontrivial</u> superconducting phase**

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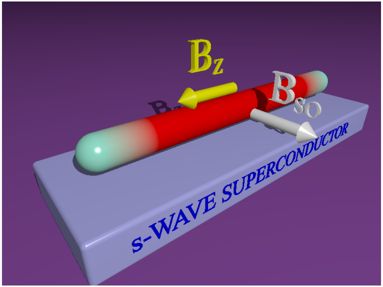
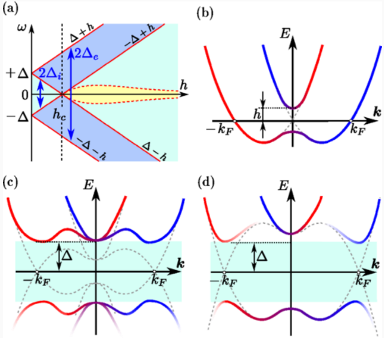
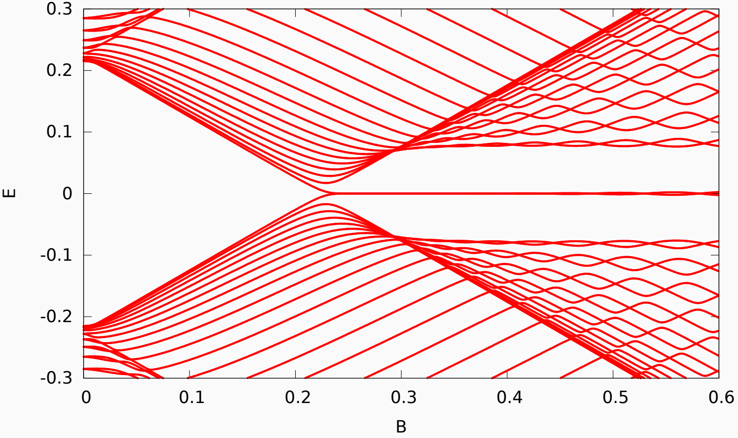




A. Ptok, A. Kobiałka & T. Domański, Phys. Rev. B 96, 195430 (2017).

(transition to <u>nontrivial</u> superconducting phase)

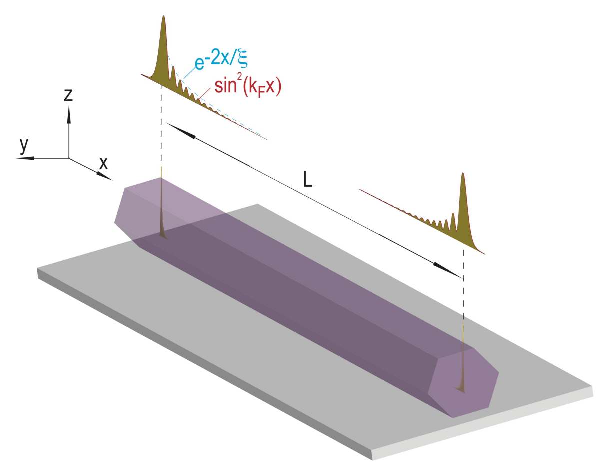
Effective quasiparticle states of the Rashba nanowire



M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

SPATIAL PROFILE OF MAJORANA QPS

Majorana qps are localized near the edges

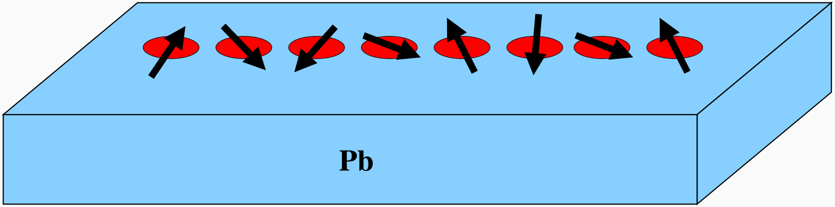


R. Aguado, Riv. Nuovo Cim. 40, 523 (2017).

Scenario 2: selforganization

MAGNETIC CHAINS IN SUPERCONDUCTORS

Nanochain of magnetic impurities embedded in superconductor:

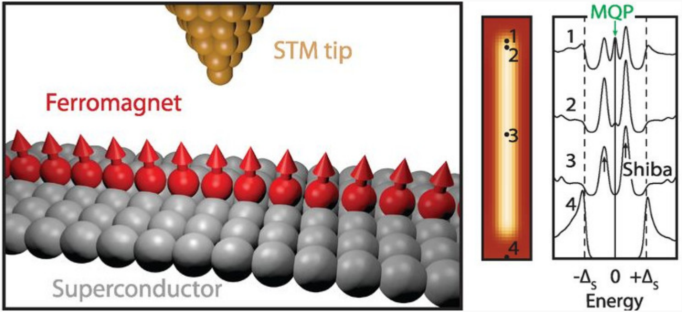


T.-P. Choy, J.M. Edge, A.R. Akhmerov, and C.W.J. Beenakker,

Phys. Rev. B 84, 195442 (2011).

EMPIRICAL REALIZATION: SCENARIO # 2

STM measurements for the nanochain of Fe atoms self-organized on a surface of superconducting Pb.

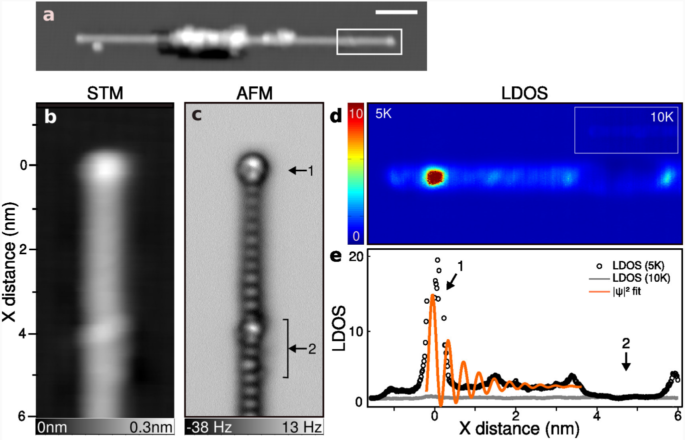


S. Nadj-Perge, ..., and A. Yazdani, Science **346**, 602 (2014).

/ Princeton University, USA /

EMPIRICAL REALIZATION: SCENARIO # 2

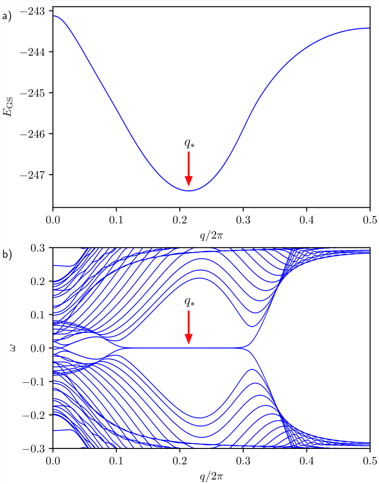
AFM & STM data for Fe chain on Pb(110) surface



R. Pawlak, M. Kisiel *et al*, npj Quantum Information **2**, 16035 (2016).

/ University of Basel, Switzerland /

MAGNETIC CHAINS IN SUPERCONDUCTORS



Ground state energy vs the pitch vector q

Quasiparticle energies

This magnetic chain self-tunes to the *topological phase* / topofilia /

A. Gorczyca-Goraj, T. Domański & M.M. Maśka, Phys. Rev. B 99, 235430 (2019).

SCENARIO 2: SCIENTIFIC HYPOTHESIS

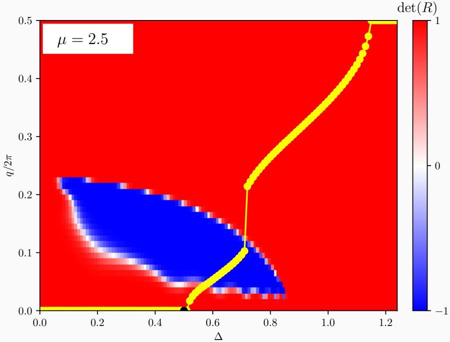
Unresolved questions:

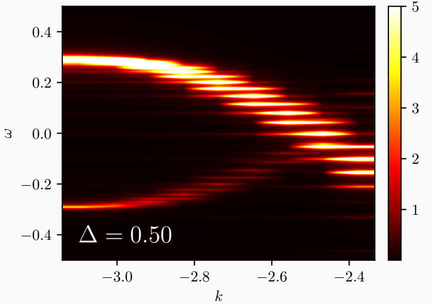
$\Rightarrow$ **is this topofilia always present ?**

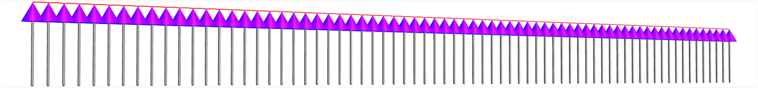
$\Rightarrow$ **does it survive at finite temperatures ?**

SELFORGANISATION AT T=0

A. Gorczyca-Goraj, T. Domański & M.M. Maśka, Phys. Rev. B 99, 235430 (2019).







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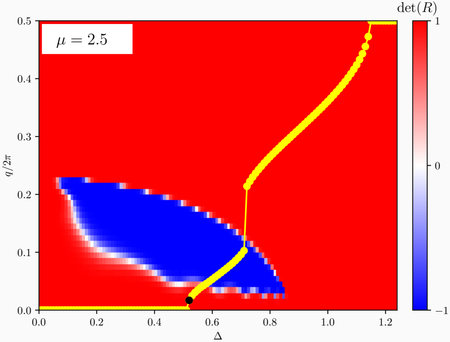

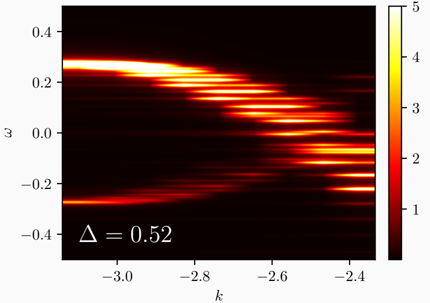

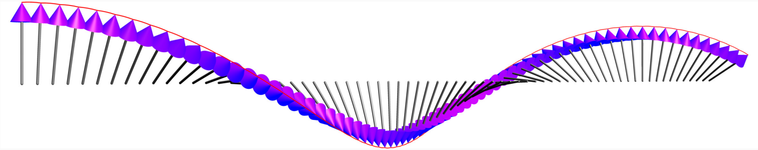

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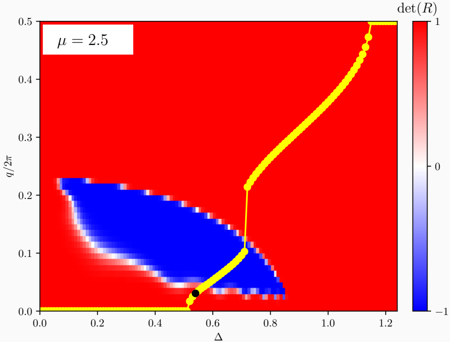
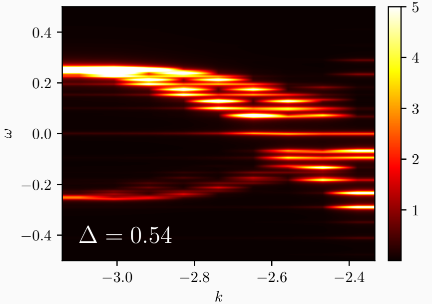
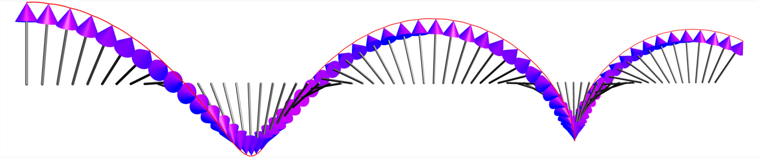
A. Gorczyca-Goraj, T. Domański & M.M. Maśka, Phys. Rev. B 99, 235430 (2019).







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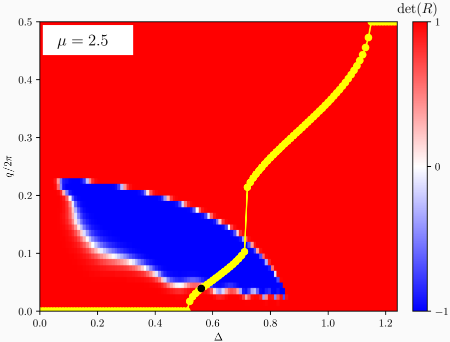

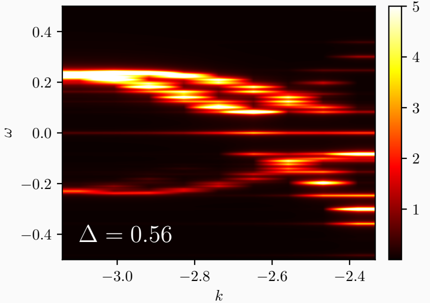

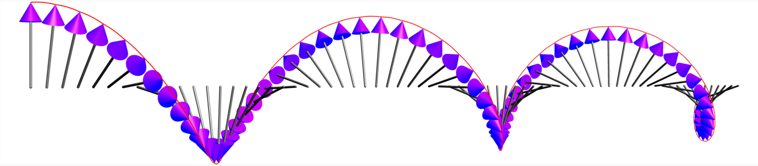

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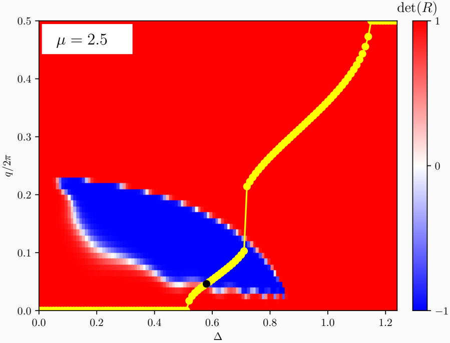

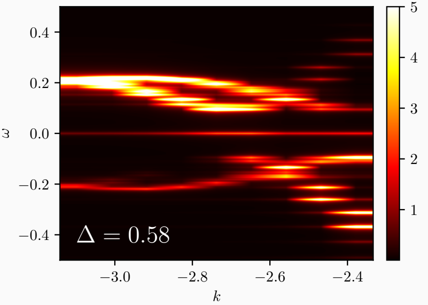

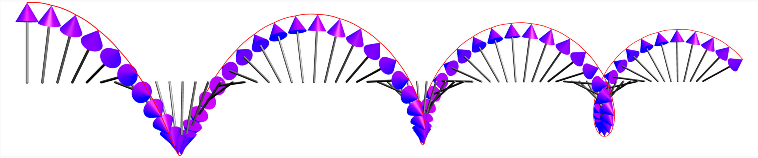

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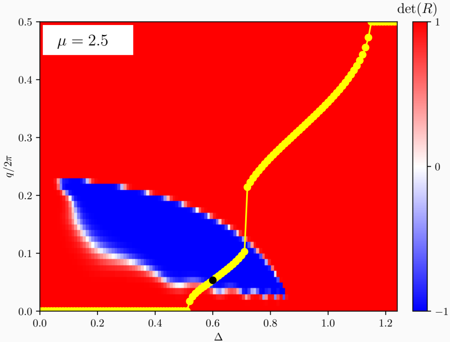

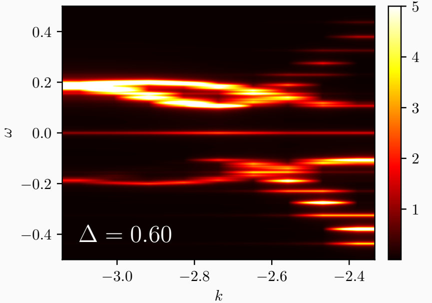

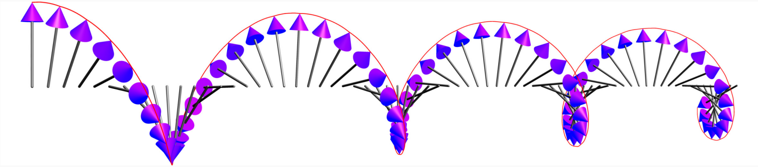

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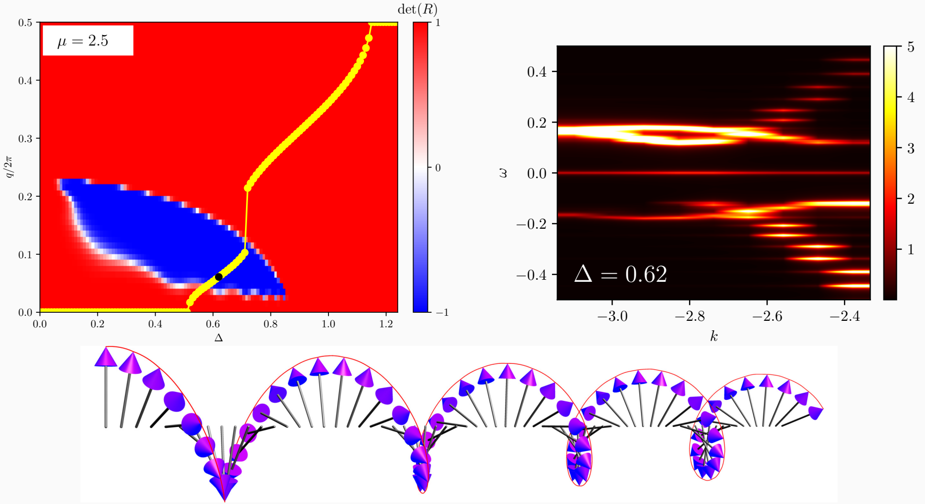

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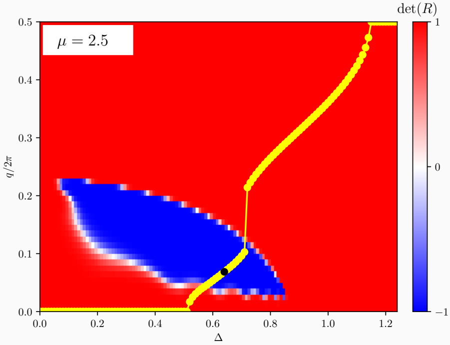

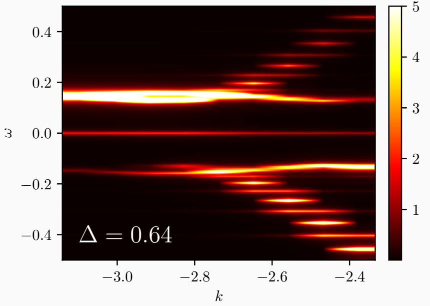

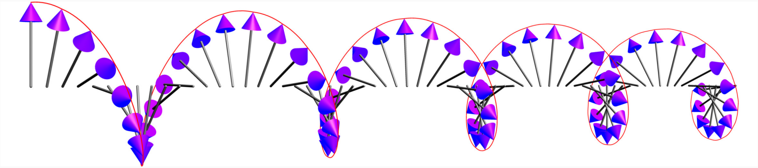

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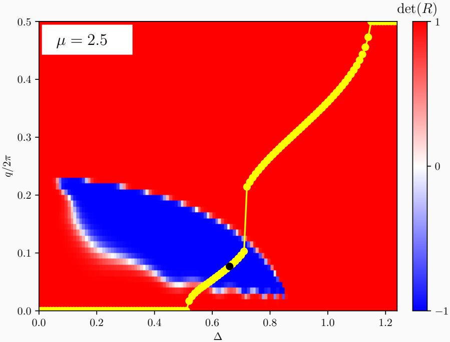

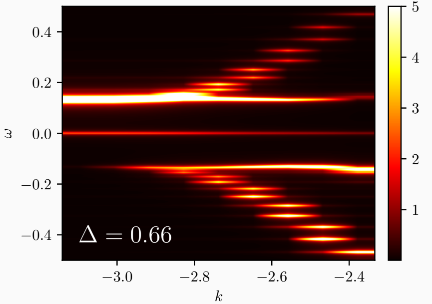

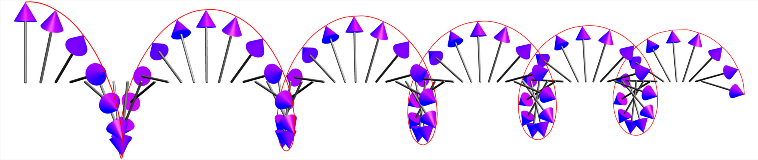

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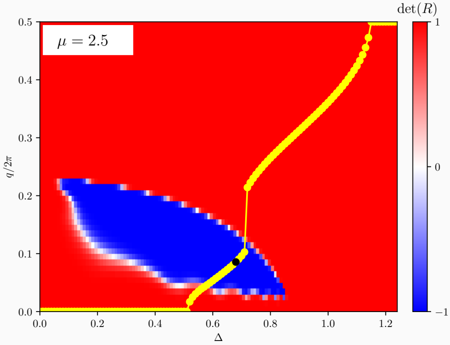

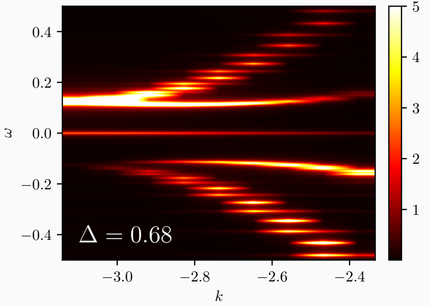

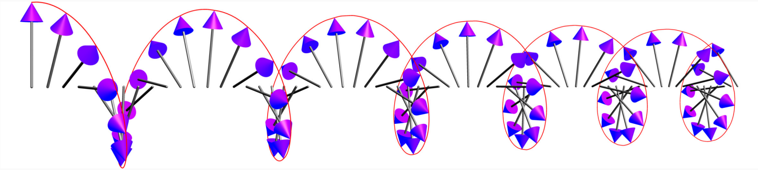

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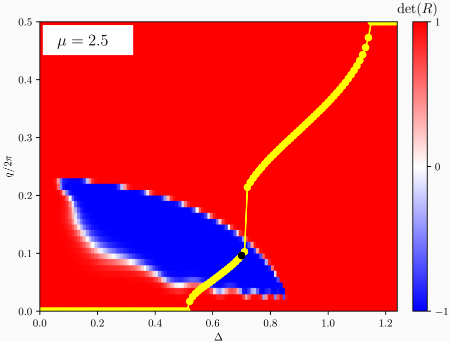

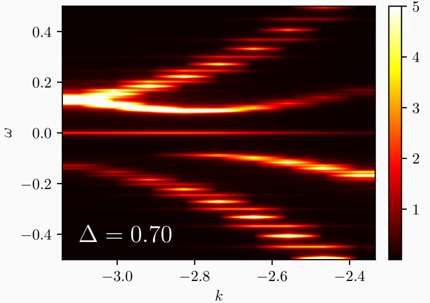

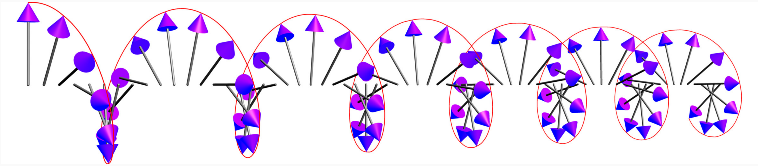

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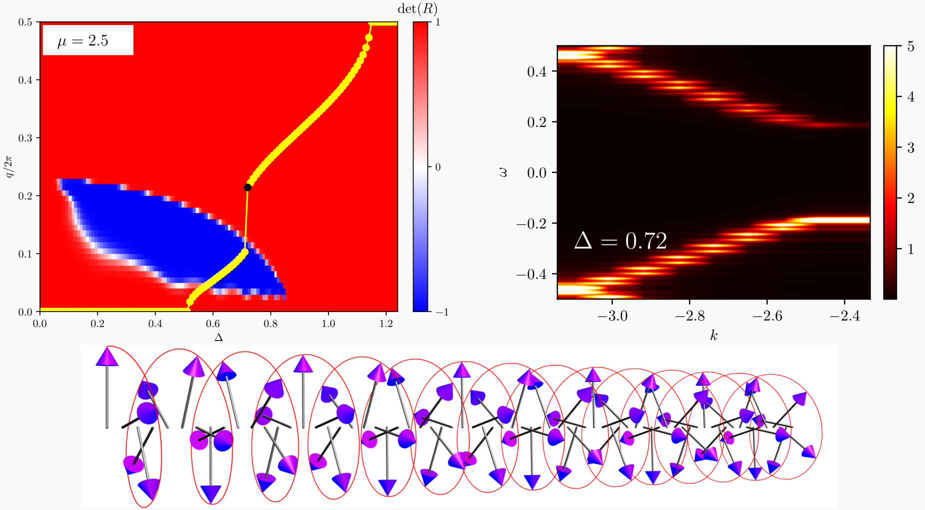
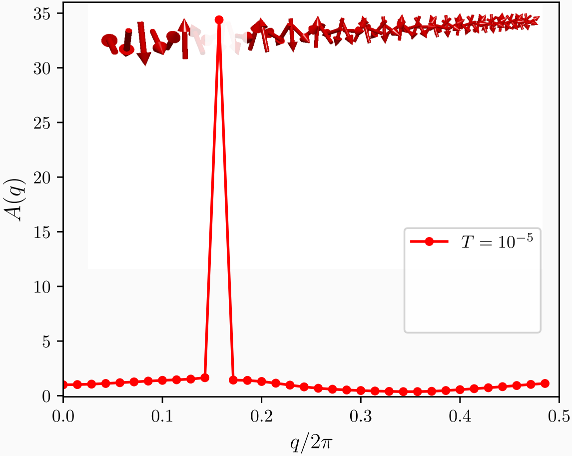


SPIRAL ORDER AT FINITE T

Structure factor: $A(q) = \frac{1}{L} \sum_{jk} e^{iq(j-k)} \langle S_j \cdot S_k \rangle$



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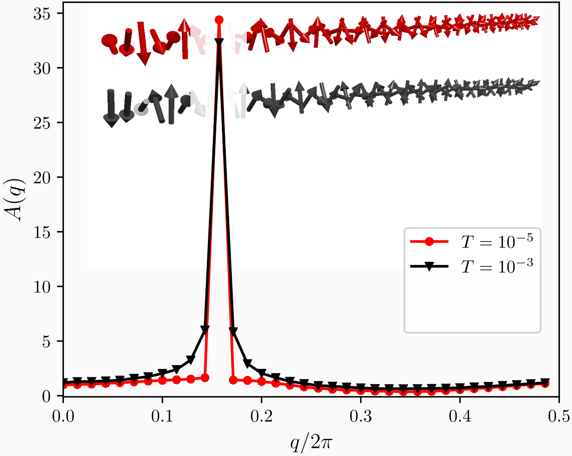

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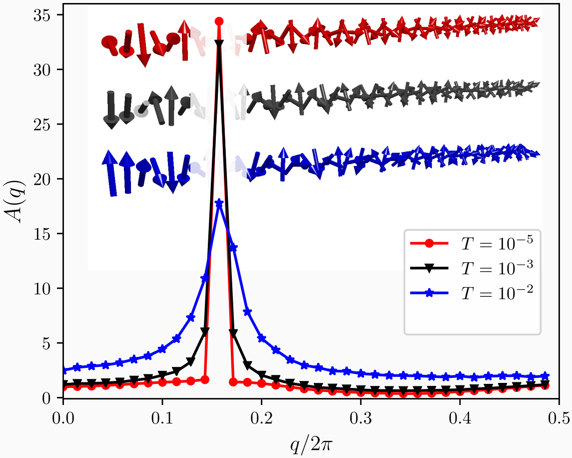

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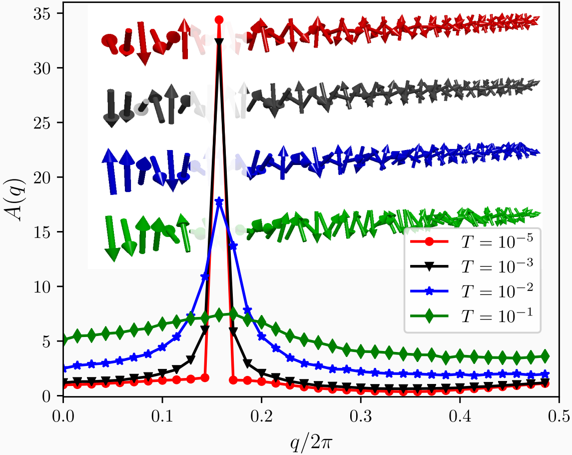
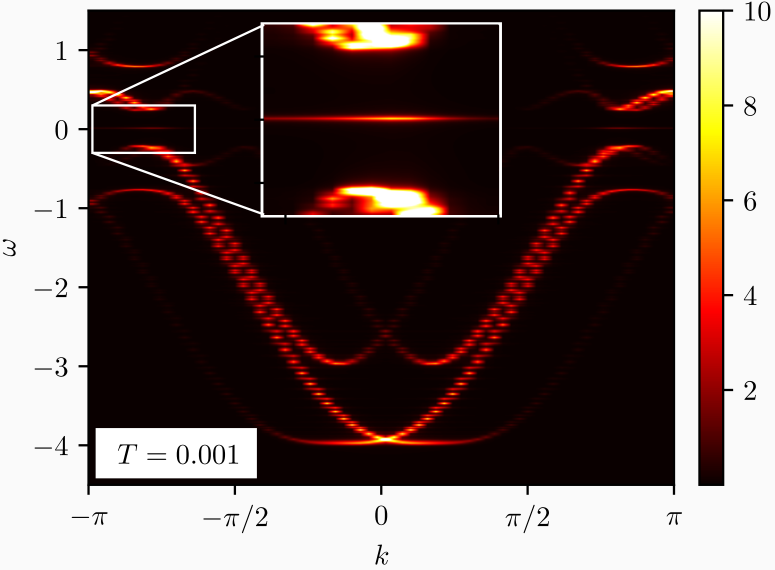
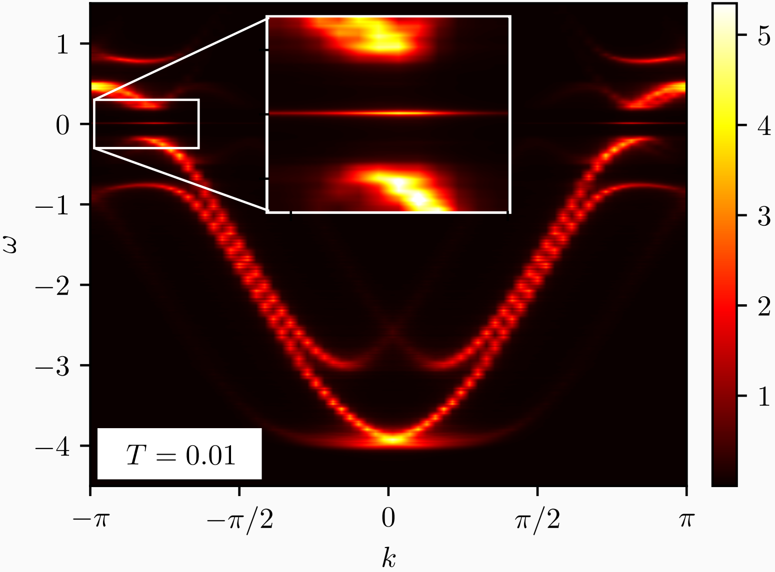


TEMPERATURE EFFECT ON MAJORANA QPS



INFLUENCE OF TEMPERATURE ON MAJORANA QPS



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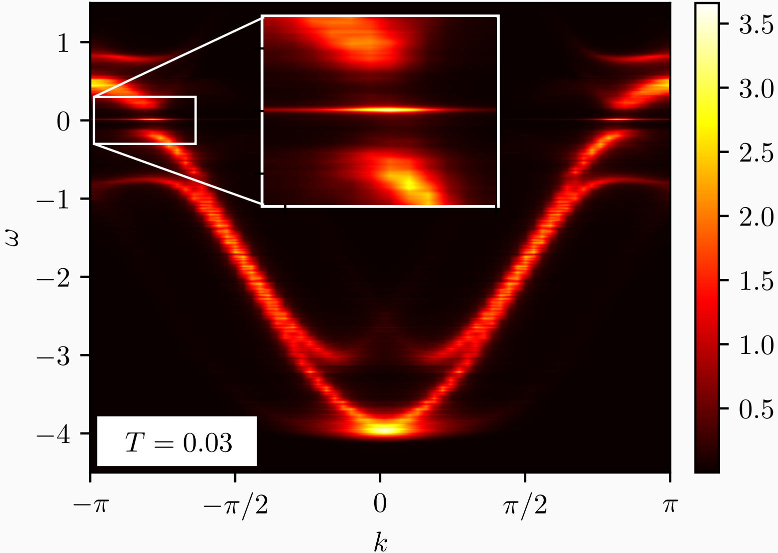

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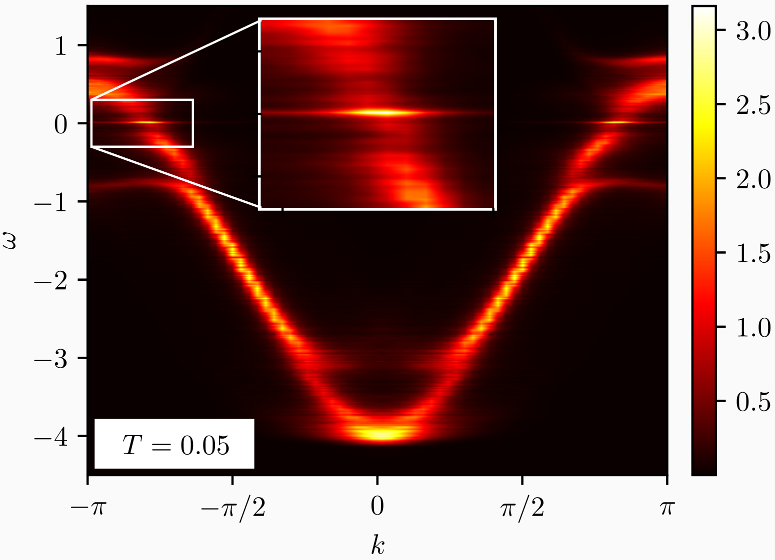

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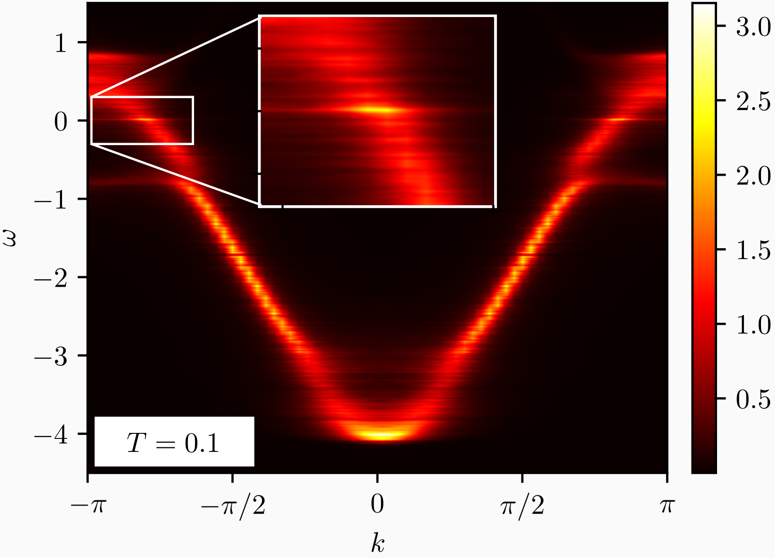

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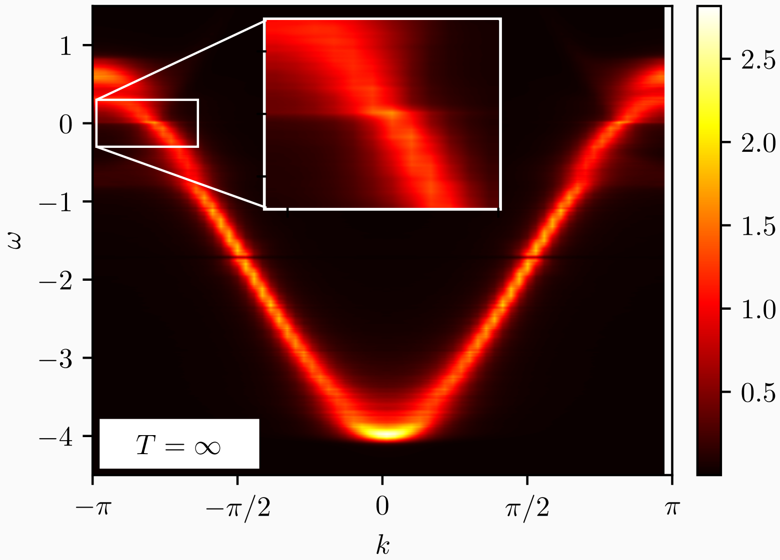

TOPOFILIA: CONCLUSIONS

Thermal effects lead to:

$\Rightarrow$ **closing of the topological energy gap**

$\Rightarrow$ **overdamping of the Majorana qps**

$\Rightarrow$ **changeover of topological $\mathbb{Z}_2$ number**

TOPOFILIA: CONCLUSIONS

Thermal effects lead to:

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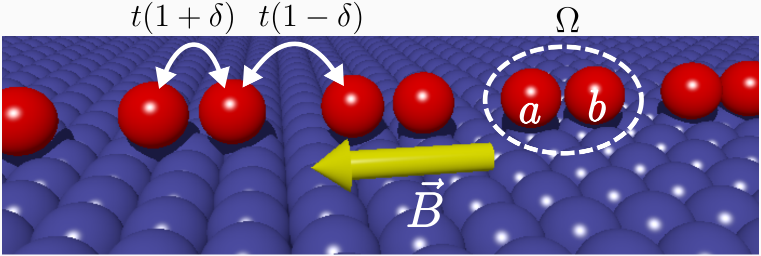
Realistically, topological phase can survive to:

$\Longrightarrow$ $T_c \approx$ **5 K**

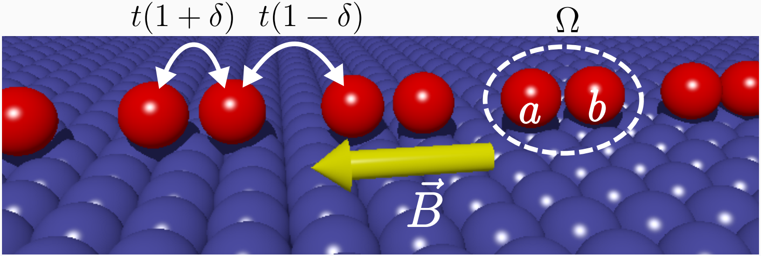
Interplay with dimerization

Dimerization versus topological superconductivity



A. Kobiałka, N. Sedlmayr, M.M. Maśka & T. Domański, Phys. Rev. B (2020), in print.

Dimerization versus topological superconductivity



A. Kobiałka, N. Sedlmayr, M.M. Maśka & T. Domański, Phys. Rev. B (2020), in print.

$\delta \longrightarrow$ **dimerization parameter**

SCIENTIFIC HYPOTHESIS

- **Motivation:**

$\Longrightarrow$ **topological (insulating) state driven dimerization [Su-Schrieffer-Heeger, 1979]**

$\Longrightarrow$ **topological phases of ultracold dimerized atoms [M. Lewenstein et al, 2019]**

SCIENTIFIC HYPOTHESIS

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- **Unresolved questions:**

$\Longrightarrow$ **dimerization vs topological superconductivity ?**

$\Longrightarrow$ **any new (emergent) phenomena ?**

Quasiparticle energies vs dimerization (for $\mu = -2t$).

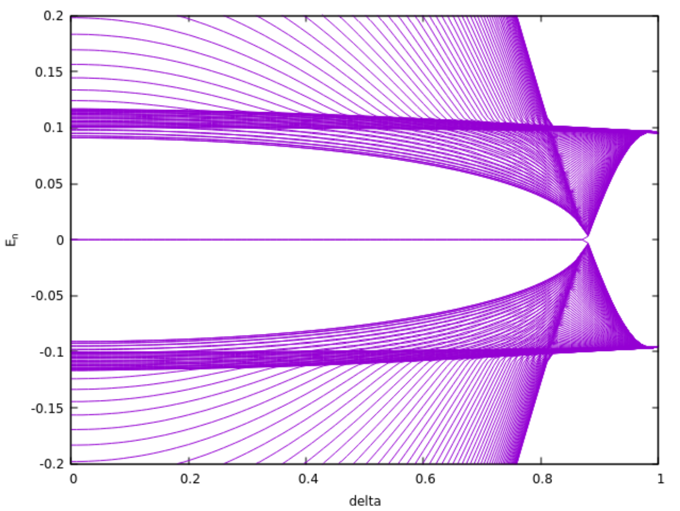
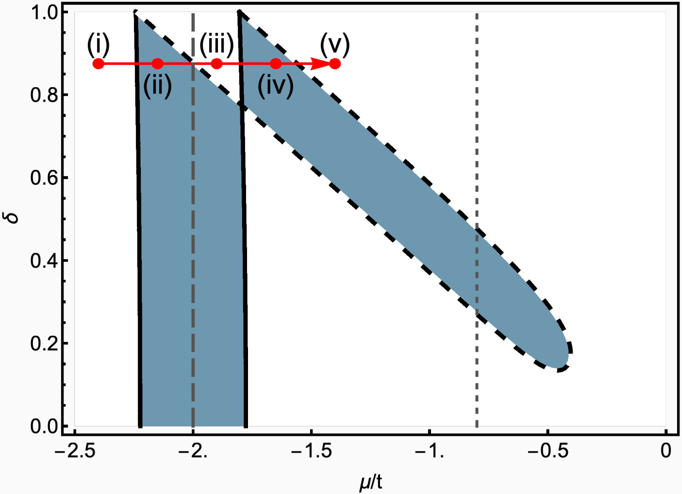
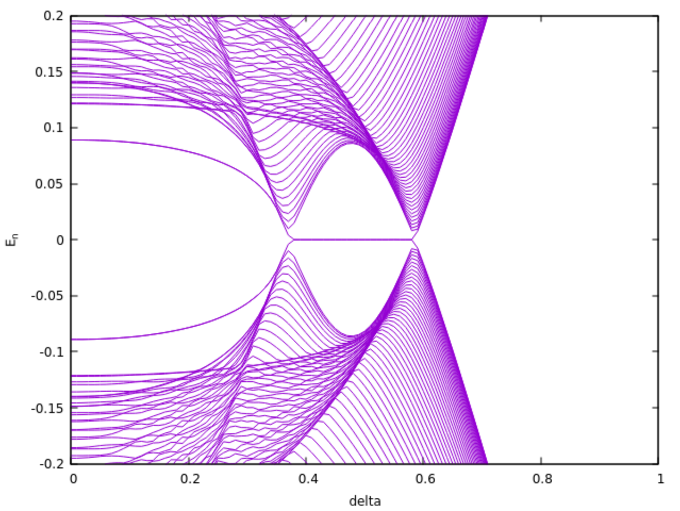


Diagram of the topological superconducting phase.



A. Kobiałka, N. Sedlmayr, M.M. Maśka & T. Domański, Phys. Rev. B (2020), in print.

Quasiparticle energies vs dimerization ($\mu = -0.8t$).



TOPOLOGICAL PHASE DUE TO DIMERIZATION

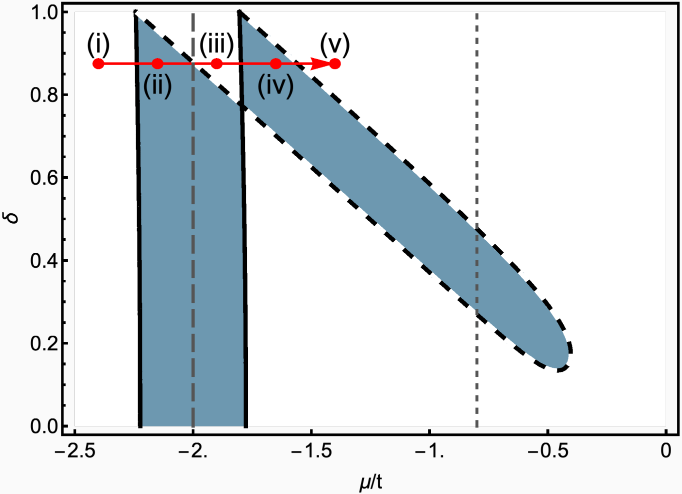
Parity of the dimerized Rashba nanowire.

$$\begin{aligned}(-1)^{\nu} &= \text{Sgn}\Big[\left(h^2-\mu^2\right)^2+\left(4t^2+4\lambda^2\delta^2+\Delta^2\right)^2 \\ &\quad -2\mu^2\left(4t^2+4\lambda^2\delta^2-\Delta^2\right)-2h^2\left(4t^2-4\lambda^2\delta^2+\Delta^2\right)\Big] \\ &\times \text{Sgn}\Big[\left(h^2-\mu^2\right)^2+\left(4\lambda^2+4t^2\delta^2+\Delta^2\right)^2 \\ &\quad -2\mu^2\left(4\lambda^2+4t^2\delta^2-\Delta^2\right)+2h^2\left(4\lambda^2-4t^2\delta^2-\Delta^2\right)\Big]\end{aligned}$$

The first term changes sign when the gap closes at $k=0$, and the second when the band gap closes at $k=\pi$.

A. Kobiałka, N. Sedlmayr, M.M. Maśka & T. Domański, Phys. Rev. B (2020), in print.

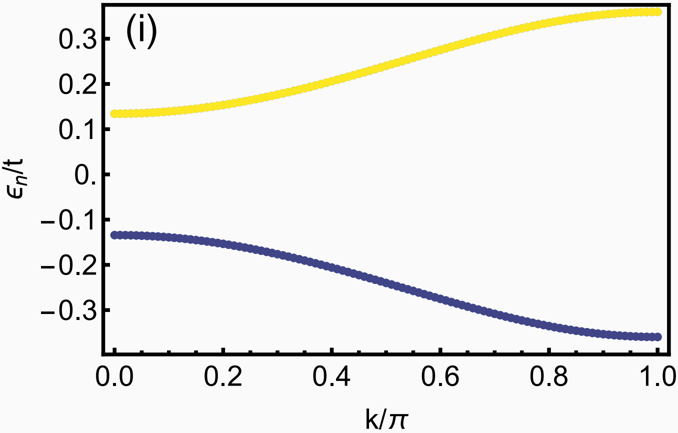
Diagram of the topological superconducting phase.



A. Kobiałka, N. Sedlmayr, M.M. Maśka & T. Domański, Phys. Rev. B (2020), in print.

TOPOLOGICAL PHASE DUE TO DIMERIZATION

Band dispersion of the infinite dimerized Rashba chain.



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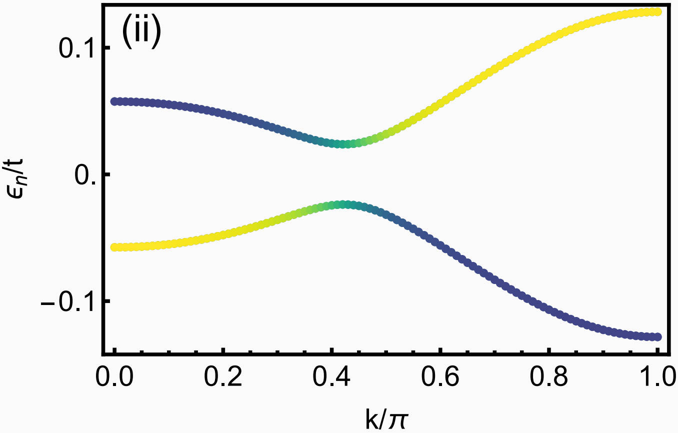
TOPOLOGICAL PHASE DUE TO DIMERIZATION

Band dispersion of the infinite dimerized Rashba chain.



A. Kobiałka, N. Sedlmayr, M.M. Maśka & T. Domański, Phys. Rev. B (2020), in print.

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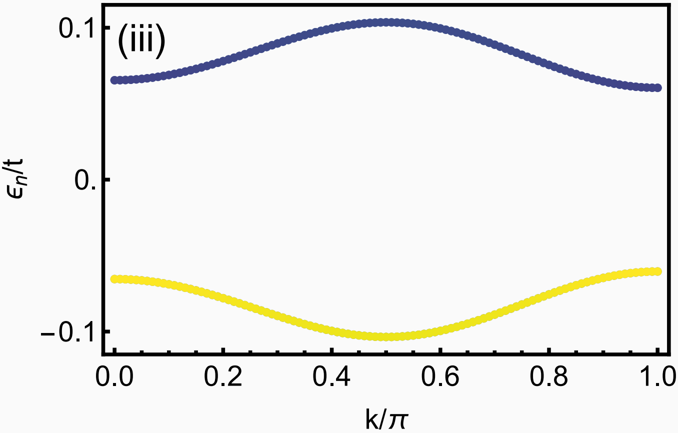

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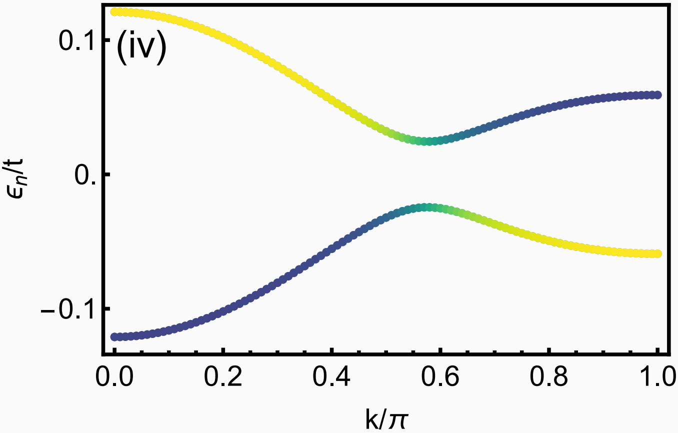

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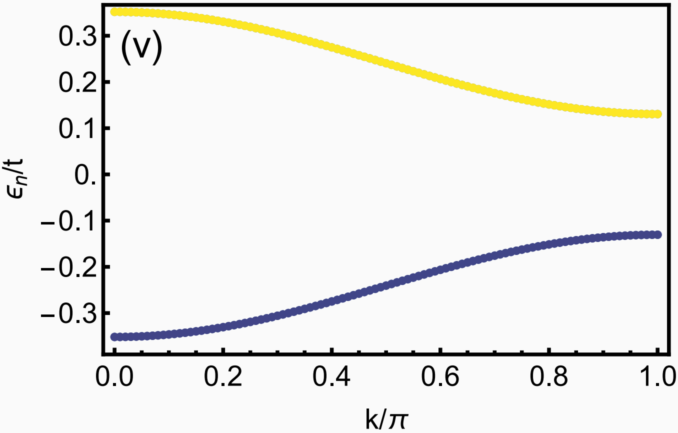

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DIMERIZATION: CONCLUSIONS

⇒ **dimerization can lead to the band-inversion**

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⇒ inducing transition to/from topological phase

⇒ novel topological regions emerge

⇒ solely due to dimerization

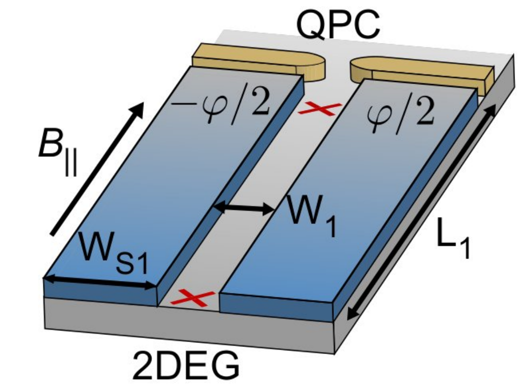
⇒ topological SSH phase does not coincide

⇒ with topological superconductivity

Localized Majorana modes in dim=2

PLANAR JOSEPHSON JUNCTIONS

Two-dimensional electron gas of InAs epitaxially covered by a thin Al layer



Width:

W_1 = 80 nm

Length:

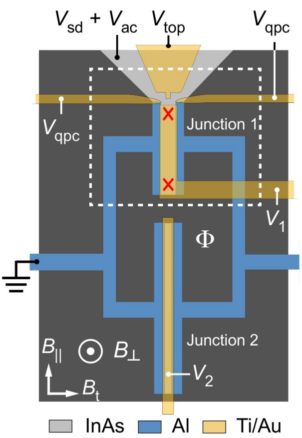
L_1 = 1.6 μm

A. Fornieri, ..., Ch. Marcus and F. Nichele, Nature 569, 89 (2019).

Niels Bohr Institute (Copenhagen, Denmark)

PLANAR JOSEPHSON JUNCTIONS

Majorana qps at the ends of 2DEG depend on the phase-difference Φ



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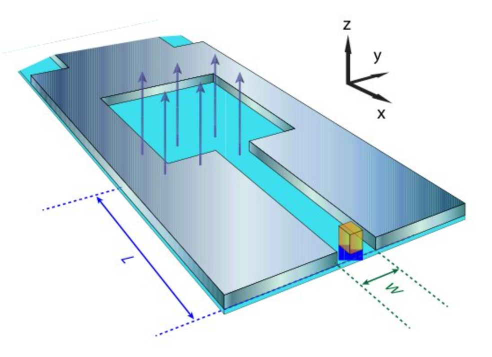
Two-dimensional HgTe quantum well coupled to 15 nm thick Al film



Width:

W = **600 nm**

Length:

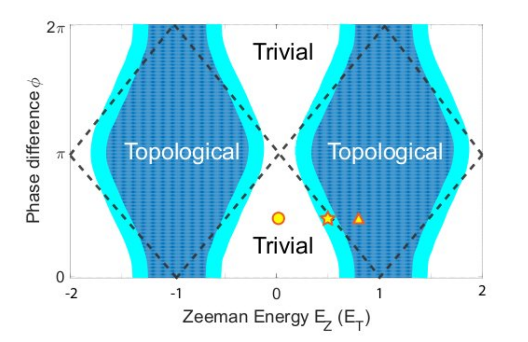
L = **1.0 μm**

H. Ren, ..., L.W. Molenkamp, B.I. Halperin & A. Yacoby, Nature 569, 93 (2019).

Würzburg Univ. (Germany) + Harvard Univ. (USA)

PLANAR JOSEPHSON JUNCTIONS

Diagram of the trivial and topological superconducting state with respect to (1) phase difference ϕ and (2) in-plane magnetic field



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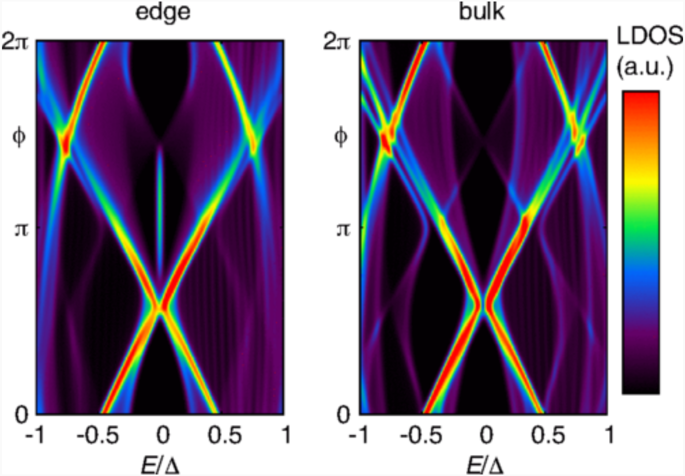
Spectrum averaged near the edge (left) and center (right) of Rashba strip.



F. Pientka, A. Keselman, E. Berg, A. Yacoby, A. Stern & B.I. Halperin,

Phys. Rev. X 7, 021032 (2017).

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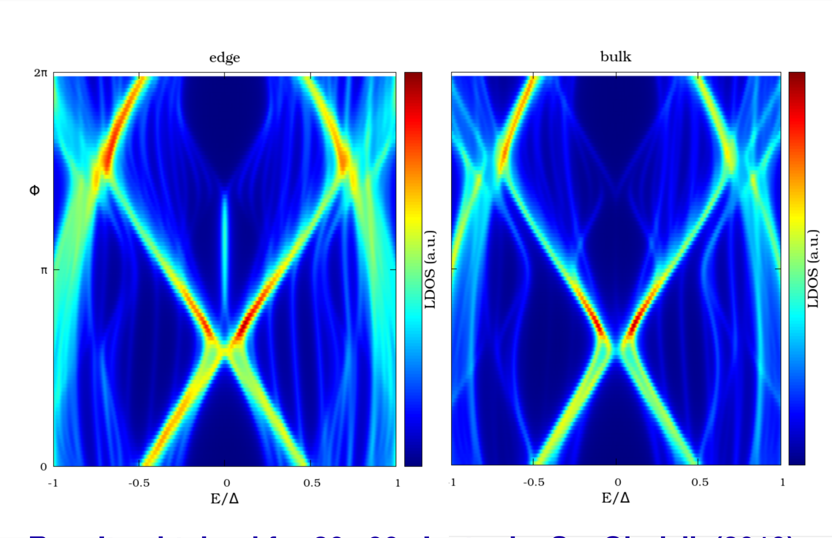

Results obtained for 30×90 cluster by Sz. Głodzik (2019).

SCIENTIFIC HYPOTHESIS

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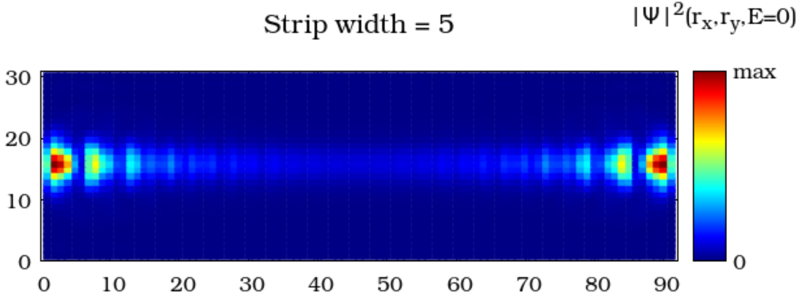
- **Relevant issues:**

⟹ **emergence of any transverse features ?**
⟹ **(e.g. Majorana polarization)**

⟹ **are point-like defects detrimental ?**

JOSEPHSON JUNCTION: INFLUENCE OF WIDTH

Majorana modes induced in the proximitized Rashba strip.



Sz. Głodzik, N. Sedlmayr & T. Domański, (2020) [to be submitted].

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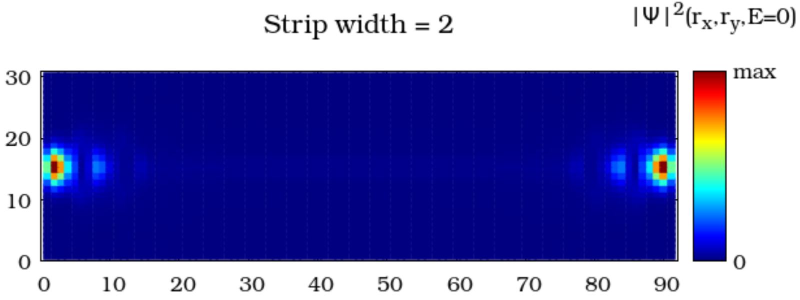

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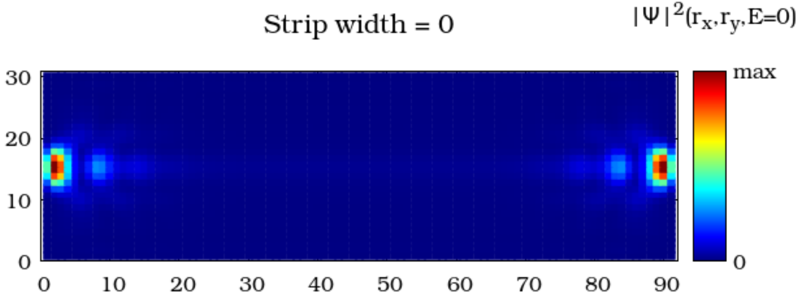

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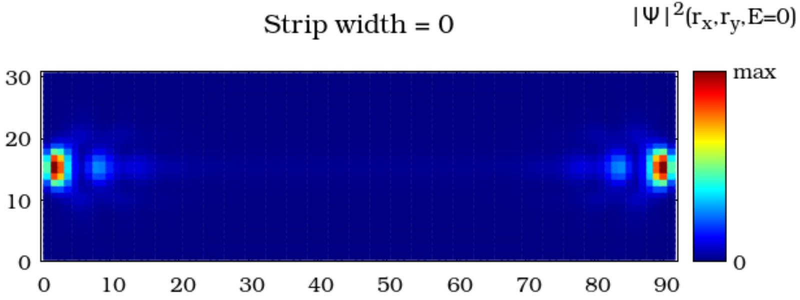
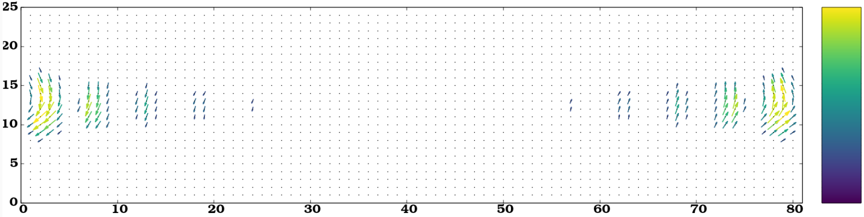


No strip needed ! Could such interface be feasible ?

Sz. Głodzik, N. Sedlmayr & T. Domański, (2020) [to be submitted].

JOSEPHSON JUNCTION: POLARIZATION

Majorana polarization $u_{\uparrow,n}v_{\uparrow,n} - u_{\downarrow,n}v_{\downarrow,n}$ (where $E_n = 0$).



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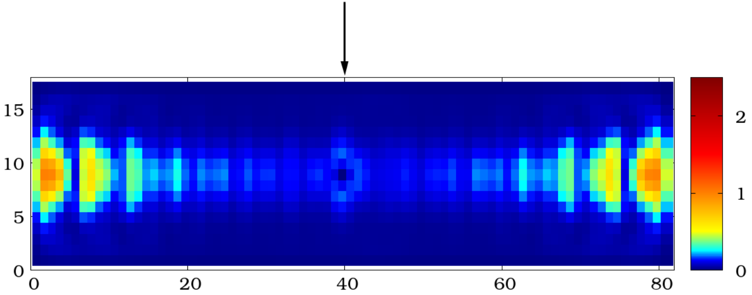
Spatial profile of the Majorana modes:

influence of an electrostatic defect placed in the center.



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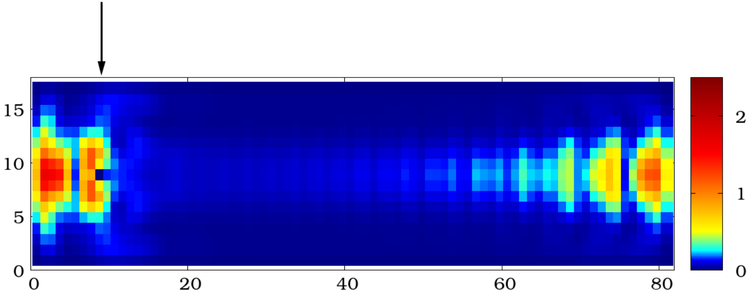

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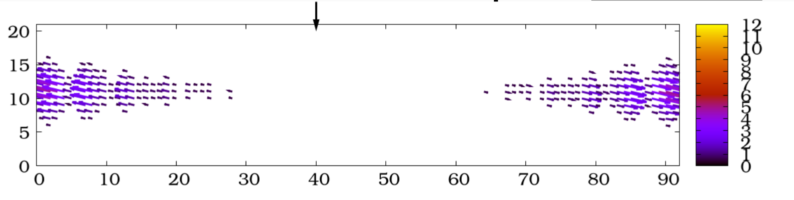

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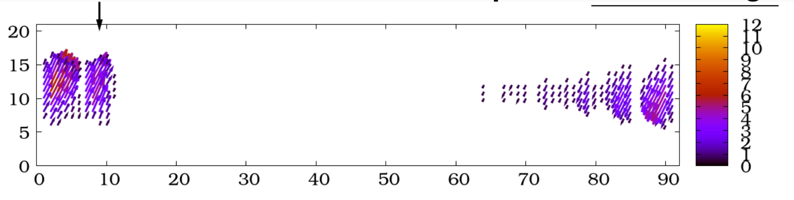

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"Benefits of Weak Disorder in One-Dimensional Topological Superconductors"
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⟹ **finite width affects both a topography**

and polarization of Majorana modes

SUMMARY/CONCLUSIONS

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- taking a form of either localized or dispersive

$\Longrightarrow$ **protected edge modes.**

ACKNOWLEDGEMENTS

- **Majorana quasiparticles**

$\Longrightarrow$ **A. Kobiałka (Lublin), A. Ptok (Kraków),**
M. Maśka & A. Gorczyca-Goraj (Katowice),
J. Tworzydło (Warsaw), N. Sedlmayr (Lublin).

- **Shiba states/bands in topological phases**

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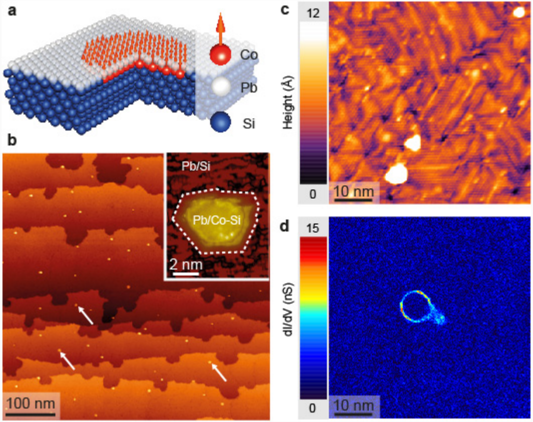
New co-operators are kindly welcome !

Edge modes in dim=2 systems

TWO-DIMENSIONAL MAGNETIC STRUCTURES

Magnetic island of Co atoms deposited on the superconducting Pb surface



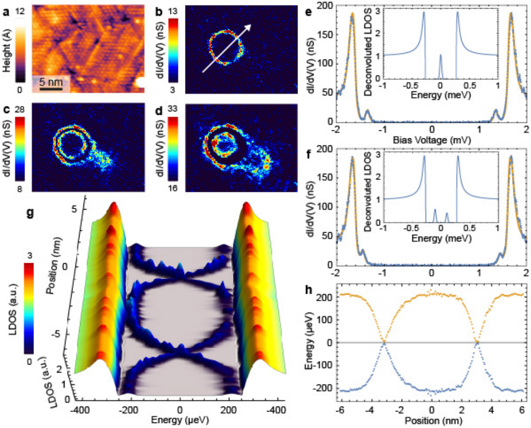
Diameter of island:

5 – 10 nm

G. Ménard, ..., and P. Simon, Nature Commun. 8, 2040 (2017).

Pierre & Marie Curie University (Paris, France)

Majorana modes propagating along magnetic islands



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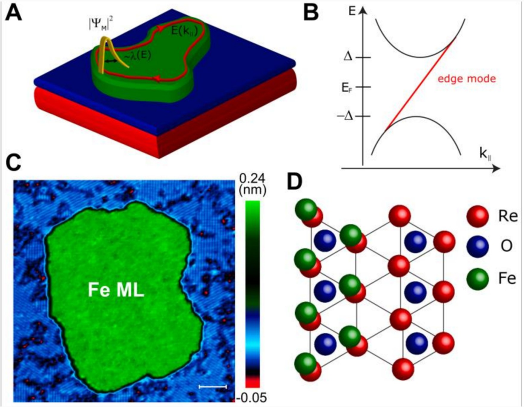
PROPAGATING MAJORANA EDGE MODES

Magnetic island of Fe atoms deposited on the superconducting Re surface



Chern number:

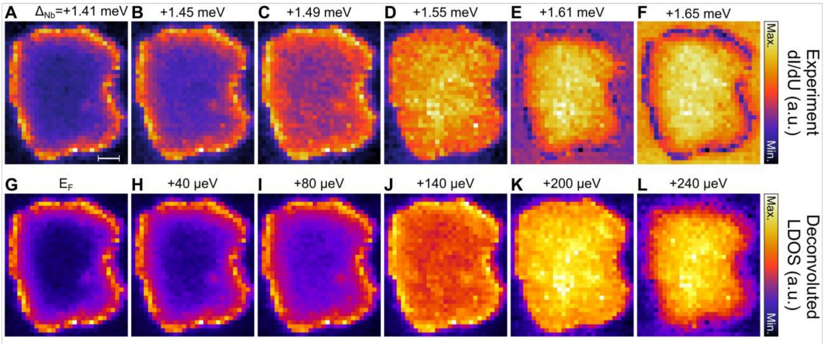
C = 20

A. Palacio-Morales, ... & R. Wiesendanger, Science Adv. 5, eaav6600 (2019).

University of Hamburg (Germany)

PROPAGATING MAJORANA EDGE MODES

Real space maps of the tunneling conductance (top panel) and deconvoluted DOS (bottom panel) obtained for various energies (as indicated) in the subgap regime ($\Delta = 240\mu eV$).



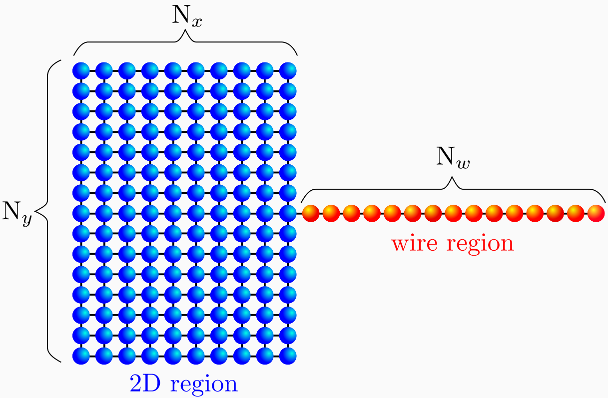
A. Palacio-Morales, ... & R. Wiesendanger, Science Adv. 5, eaav6600 (2019).

University of Hamburg (Germany)

Mixed – dimensionality structures

CAN MAJORANA QPS BE DECONFINED ?

Main idea: **Majorana qps in 1D–2D hybrid structure**



A. Kobiałka, T. Domański & A. Ptok, Scientific Reports 9, 12933 (2019).

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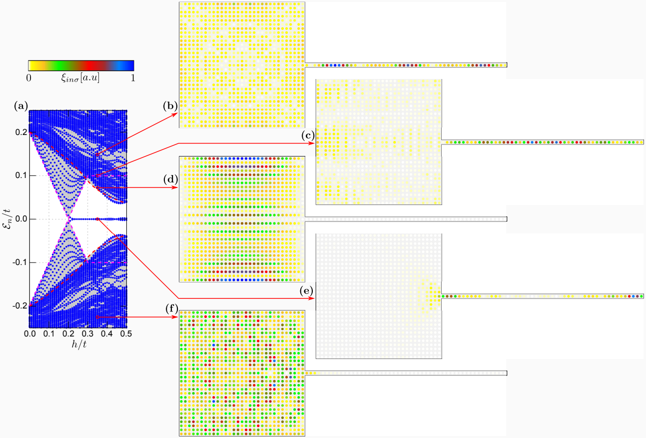
For details, concerning the topological criteria see:

- **A. Kitaev, AIP Conf. Proc. 1134, 22 (2009);**
- **M.Z. Hasan & C.L. Kane, Rev. Mod. Phys. 82, 3045 (2010);**
- **X.-L. Qi & S.-C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).**

DELOCALIZATION OF MAJORANA MODES

Majorana/Andreev quasiparticles of a wire-plaquette hybrid



plaquette: nontopological ***nanowire: topological***

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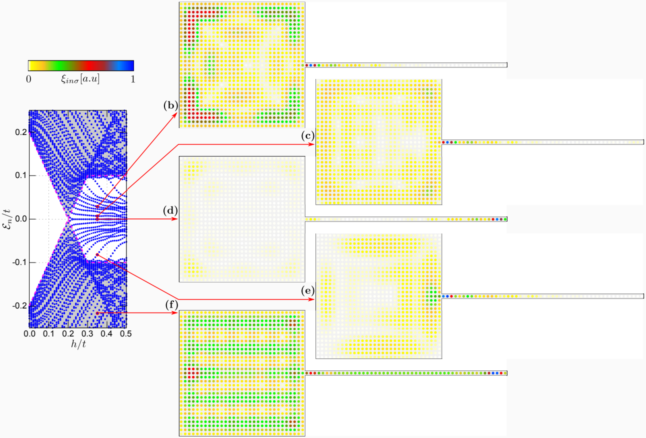
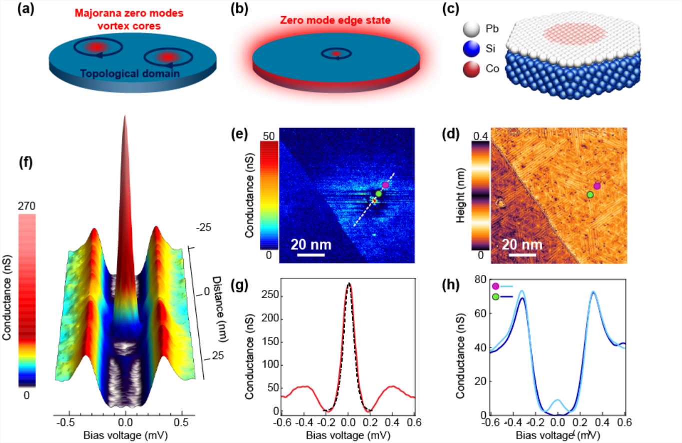


Both regions are assumed to be in topological sc phase.

A. Kobiałka, T. Domański & A. Ptok, Scientific Reports 9, 12933 (2019).

DELOCALIZED MAJORANAS: EXPERIMENTAL FACTS

Majorana localized at point-like defect coexists with another itinerant edge mode observed in Co-Si island deposited on disordered Pb.

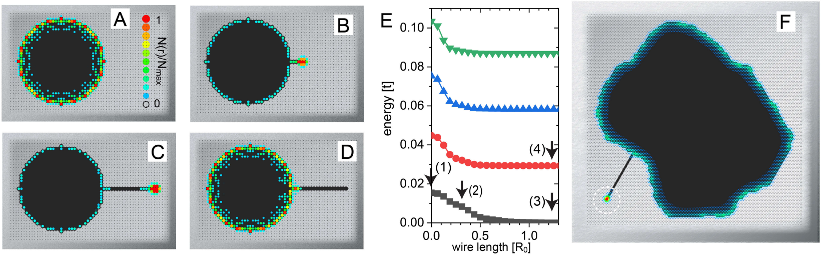


G.C. Ménard, ..., P. Simon and T. Cren, Nature Comm. 10, 2587 (2019).

Paris (France)

TOOL TO DETECT THE CHERN NUMBER

Itinerant Majorana mode leaking into the side-attached nanowire.



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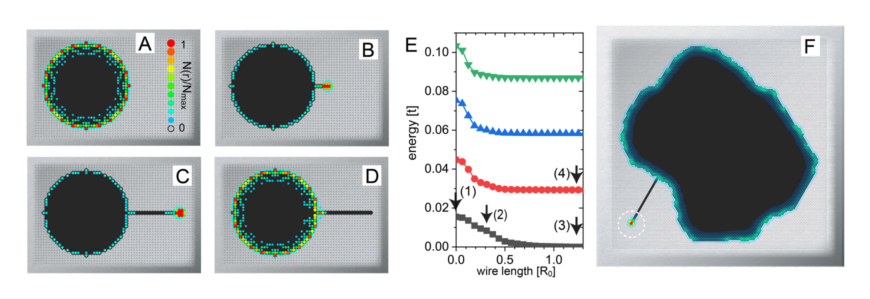

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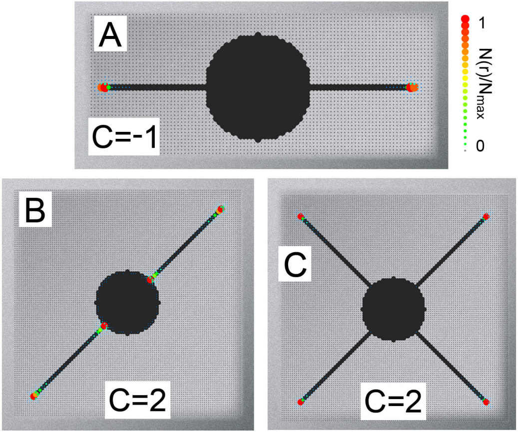
"A similar spatial structure of the zero-energy LDOS was also found in plaquette-nanowire hybrid structure [A. Kobiałka et al, 2019]."

TOOL TO DETECT THE CHERN NUMBER

Leakage of the Majorana modes between nanowire and island can help to detect the Chern number, characterizing topological phase of 2D-systems.



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