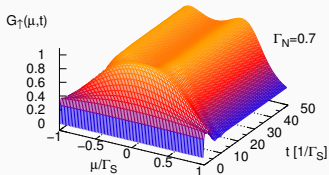
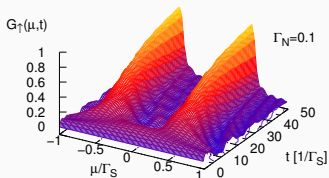


# Dynamical effects of correlated superconducting nanostructures

Tadeusz DOMAŃSKI

M. Curie-Skłodowska University, Lublin



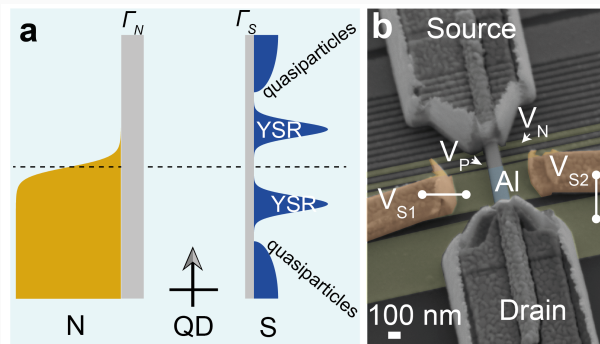
# Superconducting nanostructures

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**a few examples ...**

# HETEROSTRUCTURES WITH SUPERCONDUCTOR(S)

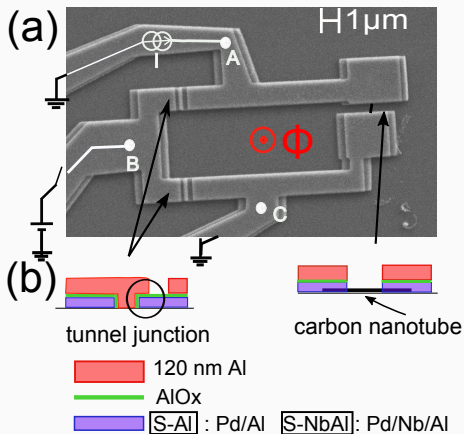
Normal metal (N) – Quantum Dot (QD) – Superconductor (S)



J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, *Commun. Phys.* **3**, 125 (2020).

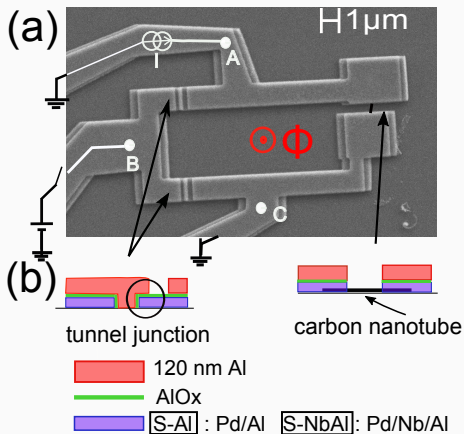
# HETEROSTRUCTURES WITH SUPERCONDUCTOR(S)

Superconductor – Quantum Dot – Superconductor



# HETEROSTRUCTURES WITH SUPERCONDUCTOR(S)

Superconductor – Quantum Dot – Superconductor



Josephson junction

R. Delagrance et al, Phys. Rev. B 93, 195437 (2016).

# SUPERCONDUCTING PROXIMITY EFFECT

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⇒ **in-gap bound states**

- originating from:

⇒ **leakage of Cooper pairs on QD** (Andreev)

⇒ **exchange int. of QD with SC** (Yu-Shiba-Rusinov)

**Why are we interested in this ?**

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**recent headlines ...**

## A perspective on semiconductor-based superconducting qubits

Cite as: Appl. Phys. Lett. **117**, 240501 (2020); doi: [10.1063/5.0024124](https://doi.org/10.1063/5.0024124)

Submitted: 4 August 2020 · Accepted: 9 November 2020 ·

Published Online: 14 December 2020



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Export Citation



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Ramón Aguado<sup>a1</sup> 

### AFFILIATIONS

Instituto de Ciencia de Materiales de Madrid (ICMM), Consejo Superior de Investigaciones Científicas (CSIC), Sor Juana Inés de la Cruz 3, 28049 Madrid, Spain

**Quantum bits (qubits) can be constructed out of in-gap bound states, using either the Josephson junctions (transmons) or the semiconducting-superconducting hybrids (gatemons).**

## REPORT

### QUANTUM DEVICES

## Coherent manipulation of an Andreev spin qubit

M. Hays<sup>1\*</sup>, V. Fatemi<sup>1\*</sup>, D. Bouman<sup>2,3</sup>, J. Cerrillo<sup>4,5</sup>, S. Diamond<sup>1</sup>, K. Serniak<sup>1†</sup>, T. Connolly<sup>1</sup>, P. Krogstrup<sup>6</sup>, J. Nygård<sup>6</sup>, A. Levy Yeyati<sup>5,7</sup>, A. Geresdi<sup>2,3,8</sup>, M. H. Devoret<sup>1\*</sup>

Two promising architectures for solid-state quantum information processing are based on electron spins electrostatically confined in semiconductor quantum dots and the collective electrodynamic modes of superconducting circuits. Superconducting electrodynamic qubits involve macroscopic numbers of electrons and offer the advantage of larger coupling, whereas semiconductor spin qubits involve individual electrons trapped in microscopic volumes but are more difficult to link. We combined beneficial aspects of both platforms in the Andreev spin qubit: the spin degree of freedom of an electronic quasiparticle trapped in the supercurrent-carrying Andreev levels of a Josephson semiconductor nanowire. We performed coherent spin manipulation by combining single-shot circuit-quantum-electrodynamics readout and spin-flipping Raman transitions and found a spin-flip time  $T_S = 17$  microseconds and a spin coherence time  $T_{2E} = 52$  nanoseconds. These results herald a regime of supercurrent-mediated coherent spin-photon coupling at the single-quantum level.

Hays *et al.*, *Science* **373**, 430–433 (2021) 23 July 2021

**Recent evidence for experimental realization**

## Yu-Shiba-Rusinov Qubit

Archana Mishra,<sup>1,\*</sup> Pascal Simon,<sup>2,†</sup> Timo Hyart,<sup>1,3,‡</sup> and Mircea Trif<sup>1,§</sup>

<sup>1</sup>*International Research Centre MagTop, Institute of Physics, Polish Academy of Sciences, Aleja Lotnikow 32/46, Warsaw PL-02668, Poland*

<sup>2</sup>*Université Paris-Saclay, CNRS, Laboratoire de Physiques des Solides, Orsay 91405, France*

<sup>3</sup>*Department of Applied Physics, Aalto University, Aalto, Espoo 00076, Finland*



(Received 15 June 2021; accepted 2 November 2021; published 7 December 2021)

Magnetic impurities in  $s$ -wave superconductors lead to spin-polarized Yu-Shiba-Rusinov (YSR) in-gap states. Chains of magnetic impurities offer one of the most viable routes for the realization of Majorana bound states, which hold promise for topological quantum computing. However, this ambitious goal looks distant, since no quantum coherent degrees of freedom have yet been identified in these systems. To fill this gap, we propose an effective two-level system, a YSR qubit, stemming from two nearby impurities. Using a time-dependent wave-function approach, we derive an effective Hamiltonian describing the YSR-qubit evolution as a function of the distance between the impurity spins, their relative orientations, and their dynamics. We show that the YSR qubit can be controlled and read out using state-of-the-art experimental techniques for manipulation of the spins. Finally, we address the effect of spin noise on the coherence properties of the YSR qubit and show robust behavior for a wide range of experimentally relevant parameters. Looking forward, the YSR qubit could facilitate the implementation of a universal set of quantum gates in hybrid systems where they are coupled to topological Majorana qubits.

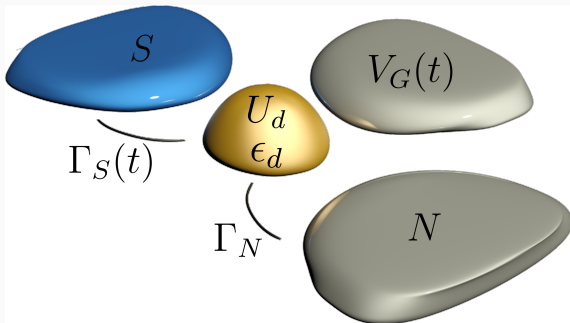
**Are there any characteristic time-scales ?**

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**(of importance for operations on qubits)**

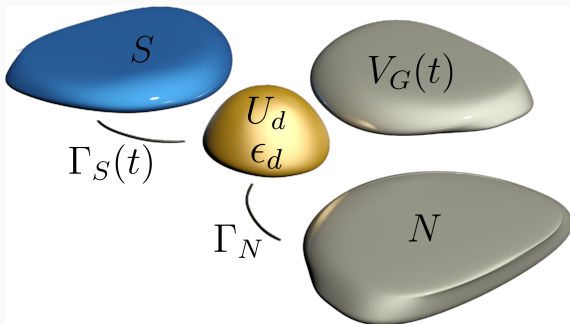


# QUENCH DRIVEN DYNAMICS



**Possible quench protocols:**

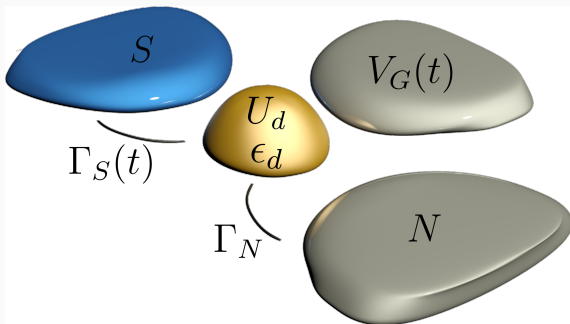
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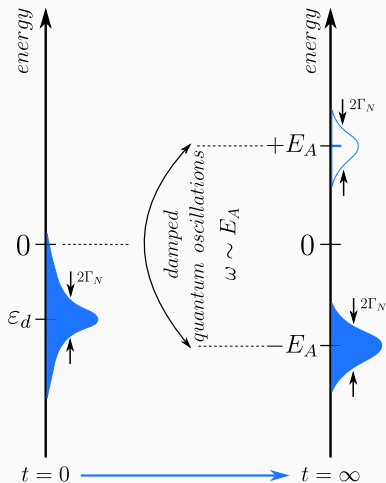
## Possible quench protocols:

$\Rightarrow$  sudden coupling to superconductor  $0 \rightarrow \Gamma_S$

$\Rightarrow$  abrupt application of gate potential  $0 \rightarrow V_G$

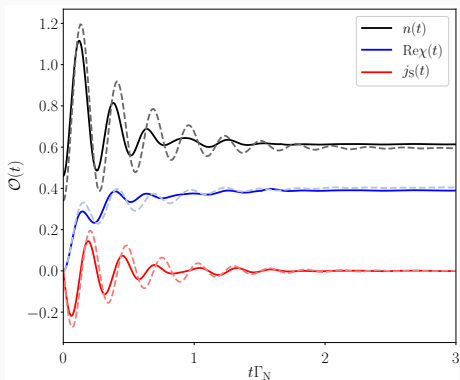
# BUILDUP OF IN-GAP STATES

Schematics of the Andreev states formation induced by quench  $0 \rightarrow \Gamma_5$



# BUILDUP OF IN-GAP STATES

Time-dependent observables driven by the quantum quench  $0 \rightarrow \Gamma_S$



**solid lines** - time dependent NRG

**dashed lines** - Hartree-Fock-Bogolubov

# Singlet-doublet (quantum phase) transition

**Singlet-doublet (quantum phase) transition**

**(static version)**

# SINGLY OCCUPIED VS BCS-TYPE CONFIGURATIONS

Quantum dot proximitized to superconductor can be described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \hat{d}_{\sigma}^{\dagger} \hat{d}_{\sigma} + U_d \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left( \Gamma_s \hat{d}_{\uparrow}^{\dagger} \hat{d}_{\downarrow}^{\dagger} + \text{h.c.} \right)$$



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Eigen-states of this problem are represented by:

$$\begin{array}{ll} |\uparrow\rangle \quad \text{and} \quad |\downarrow\rangle & \Leftarrow \quad \text{doublet states (spin } \frac{1}{2}) \\ \left. \begin{array}{l} u |0\rangle - v |\uparrow\downarrow\rangle \\ v |0\rangle + u |\uparrow\downarrow\rangle \end{array} \right\} & \Leftarrow \quad \text{singlet states (spin 0)} \end{array}$$

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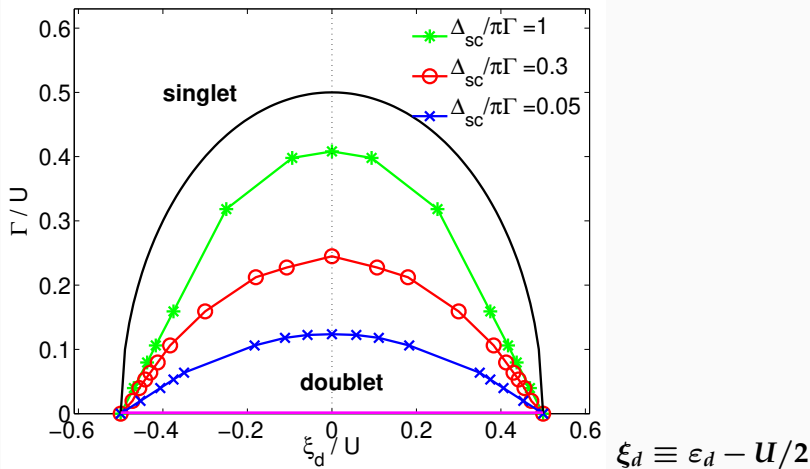
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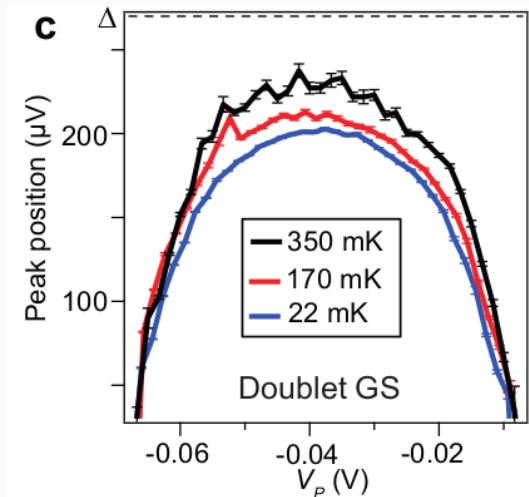
Upon varying the parameters  $\epsilon_d$ ,  $U_d$  or  $\Gamma_S$  there can be induced a **transition** between these doublet/singlet ground states.

# QUANTUM PHASE TRANSITION (STATIC VERSION)

## Singlet-doublet quantum (phase transition): NRG results



# QUANTUM PHASE TRANSITION: EXPERIMENT



J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, *Commun. Phys.* **3**, 125 (2020).

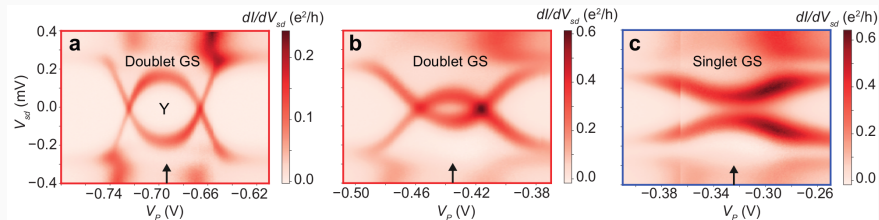
# SINGLET VS DOUBLET: EXPERIMENT

Differential conductance vs source-drain bias  $V_{sd}$  (vertical axis) and gate potential  $V_p$  (horizontal axis) measured for various  $\Gamma_s/U$

$$U \gg \Gamma_s$$

$$U \geq \Gamma_s$$

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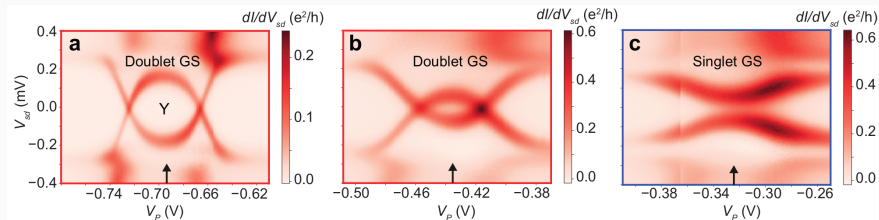
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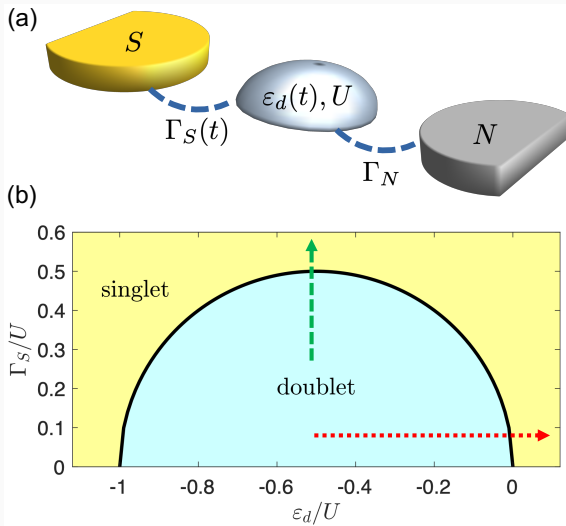


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**Crossings of in-gap states correspond to the singlet-doublet QPT.**

# **Dynamical singlet-doublet transition**

# QUANTUM QUENCHES ACROSS QPT





# DYNAMICS IMPOSED BY QUANTUM QUENCH

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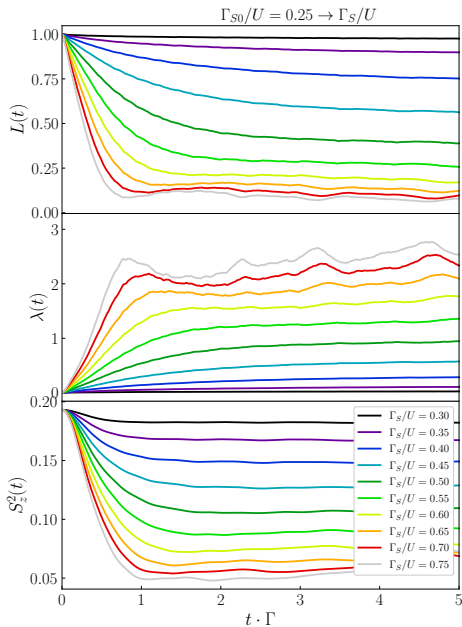
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# $t$ NRG RESULTS: ABRUPT CHANGE OF $\Gamma_S$



**Loschmidt amplitude**

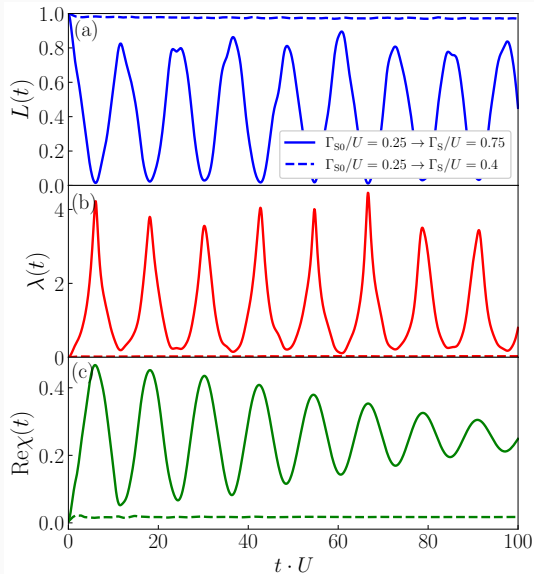
$$L(t) \equiv |\langle \Psi(0) | \Psi(t) \rangle|^2$$

**Return rate**

$$\lambda(t) = -\frac{1}{N} \log [L(t)]$$

**The squared magnetic moment  $\langle S_z^2(t) \rangle$**

# $t$ NRG RESULTS: ABRUPT CHANGE OF $\Gamma_S$

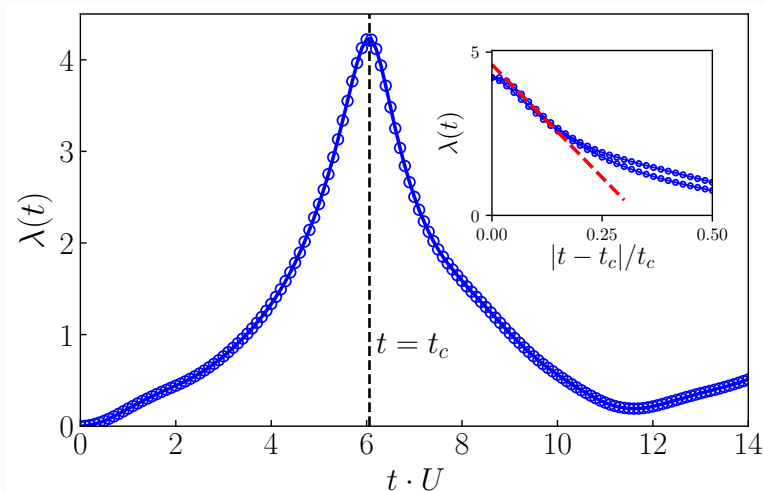


$$\epsilon_d = -U/2$$

$$\Gamma_N = U/100$$

# $t$ NRG RESULTS: ABRUPT CHANGE OF $\Gamma_S$

Return-rate nearby the first critical time  $t_c$  (rounding due to finite-size).



# SUMMARY (PART 1)

**Quench imposed onto the nanostructure consisting of normal metal – quantum dot – superconductor:**

- **activates Rabi-type oscillations** (due to particle-hole mixing)

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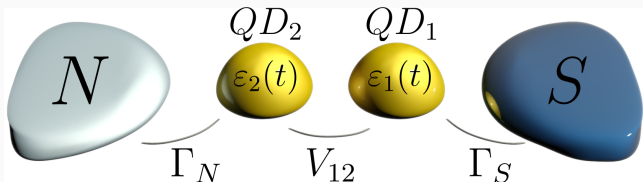
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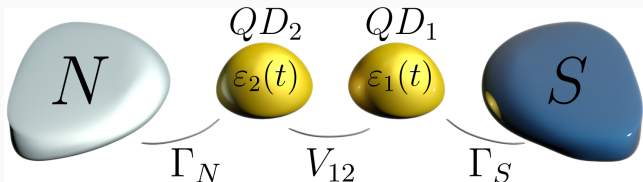
**These phenomena are detectable in charge transport properties.**

# QUENCH IN DOUBLE QUANTUM DOTS



**Physical issues:**

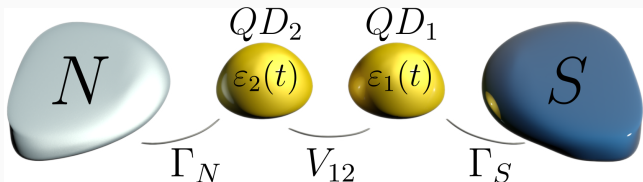
# QUENCH IN DOUBLE QUANTUM DOTS



**Physical issues:**

$\Rightarrow$  **role of initial conditions**

# QUENCH IN DOUBLE QUANTUM DOTS



## Physical issues:

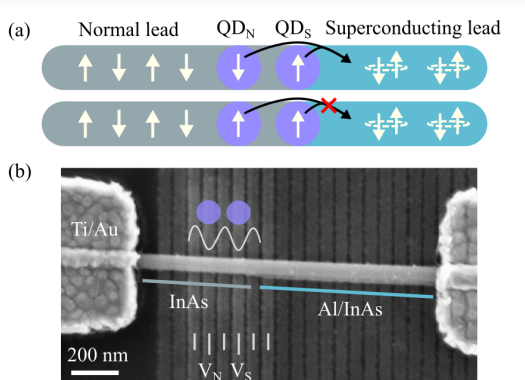
⇒ role of initial conditions

⇒ blocking of superconducting proximity effect

R. Taranko, K. Wrześniewski, B. Baran, I. Weymann, T. Domański,  
Phys. Rev. B 103, 165430 (2021).

B. Baran, R. Taranko, T. Domański, Scientific Reports 11, 11148 (2021).

# ANDREEV BLOCKADE

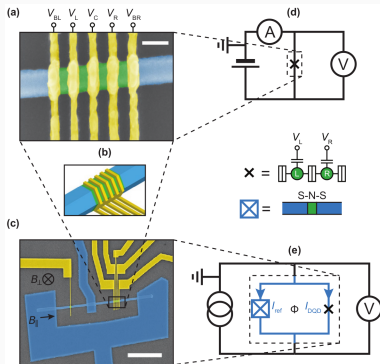


**N = Au, DQD=InAs, S=Al**

**Suppression of Andreev reflection by spin-triplet configuration of DQD**

P. Zhang, H. Wu, J. Chen, S.A. Khan, P. Krogstrup, D. Pekker, S.M. Frolov,  
Phys. Rev. Lett. 128, 046801 (2022).

# TRIPLET BLOCKADE IN JOSEPHSON JUNCTION



**DQD=InAs, S=Al**

**Triplet-blockaded Josephson supercurrent in double quantum dots**

D. Bouman, R.J.J. van Gulik, G. Steffensen, D. Pataki, P. Boross, P. Krogstrup, J. Nygård, J. Paaske, A. Pályi, A. Geresdi, Phys. Rev. B 102, 220505(R) (2020).

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- dynamical phenomena reveal **quantum beating** effects

# ACKNOWLEDGEMENTS

- **dynamical singlet-doublet phase transition**

⇒ K. Wrześniewski (Poznań), I. Weymann (Poznań),  
N. Sedlmayr (Lublin),

- **dynamics of in-gap states (transients effects, etc.)**

⇒ R. Taranko (Lublin), B. Baran (Lublin).