Phase transitions in time-domain:

application to superconducting systems

Tadeusz DOMAŃSKI

M. Curie-Skłodowska Univ.

Warsaw University **15 October 2021**

OUTLINE

I. Dynamical phase transitions

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$\Rightarrow$ main concepts

$\Rightarrow$ a few examples

$\Rightarrow$ finite-size effects

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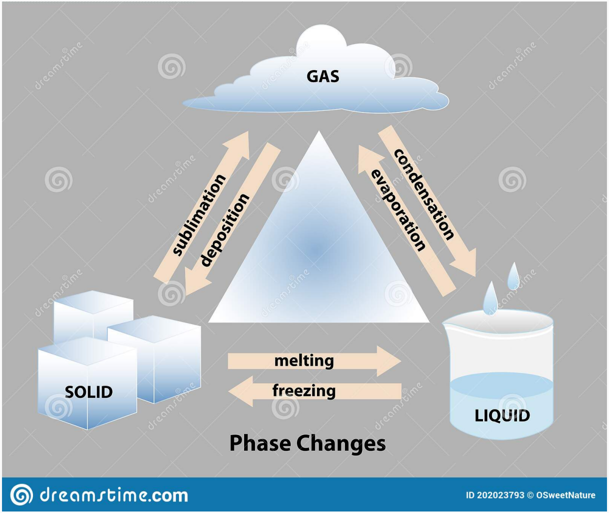
- ⇒ main concepts
- ⇒ a few examples
- ⇒ finite-size effects

II. Application to superconducting systems

- ⇒ bulk systems
- ⇒ nanostructures

Phase transitions in every-day life



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$\Rightarrow$ **transitions in time-domain .... (2013)**

I. Dynamical quantum phase transition

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Idea: **M. Heyl, A. Polkovnikov, S. Kehrein, Phys. Rev. Lett. 110, 135704 (2013).**

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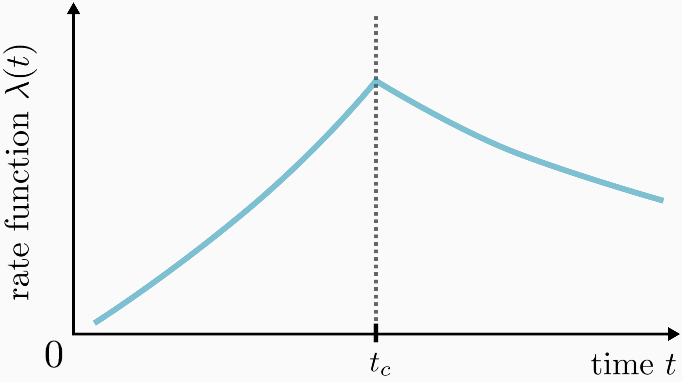
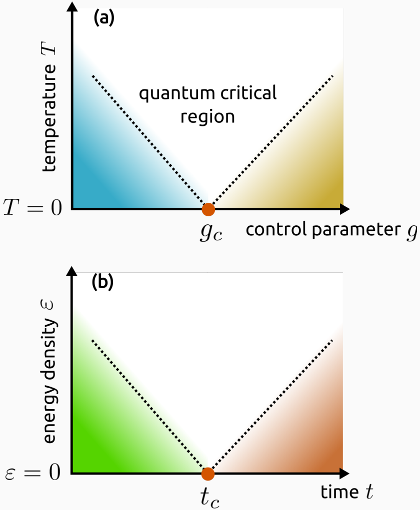
CRITICAL TIME



At critical time t_c the rate function $\lambda(t)$ of the Loschmidt echo $L(t) \equiv e^{-N\lambda(t)}$ exhibits a nonanalytic kink.

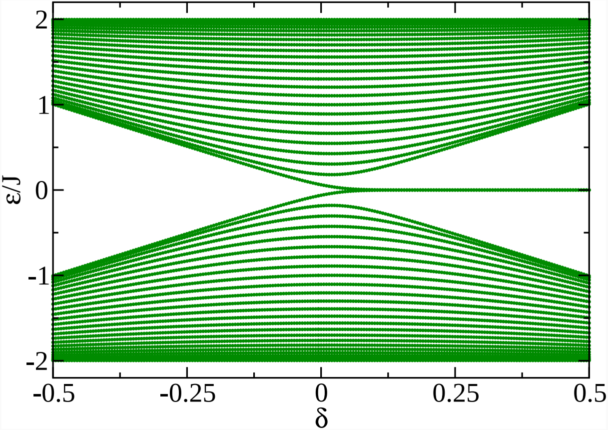
ANALOGY TO QUANTUM-PHASE-TRANSITION



Loschmidt amplitude probes the ground state manifold of the initial Hamiltonian (energy density at $\varepsilon = 0$).

A few examples ...

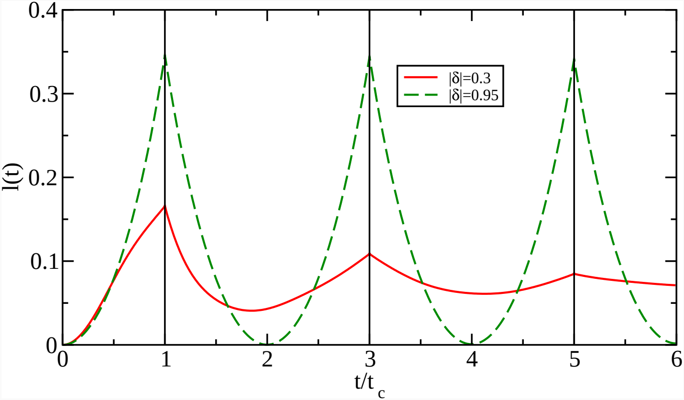
Quasiparticle spectrum of the SSH model under stationary conditions.



$$\hat{H} = -J \sum_j \left[\left(1 + \delta e^{i\pi j} \right) \hat{c}_j^\dagger \, \hat{c}_{j+1} \; + \; \text{h.c.} \right]$$

[N. Sedlmayr, Acta Phys. Polon. A 135, 1191 (2019)]

QUENCH DRIVEN TRANSITION



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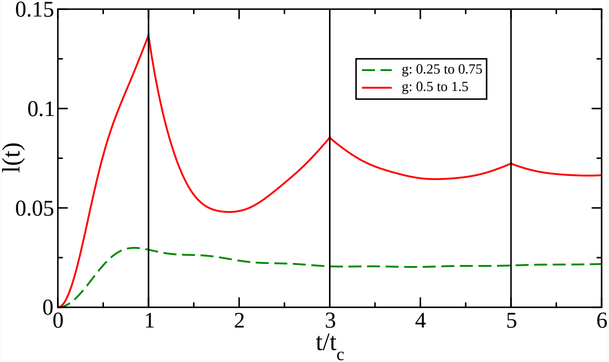
solid red line: $\delta = -0.3 \quad \longrightarrow \quad \delta = +0.3$

dashed green line: $\delta = 0.95 \quad \longrightarrow \quad \delta = -0.95$

[N. Sedlmayr, Acta Phys. Polon. A 135, 1191 (2019)]

QUENCH OF TRANSVERSE FIELD h

Post-quench return rate of the Ising model ($g \equiv h/J$)



$$\hat{H} = -\frac{J}{2}\sum_{j=1}^{N-1} \hat{\sigma}_j^z \, \hat{\sigma}_{j+1}^z + \frac{h}{2}\sum_{j=1}^{N} \hat{\sigma}_j^x$$

solid red line - **across a phase transition ($g_c = 1$)**

dashed green line - **inside the same phase**

SOME REMARKS

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(in most cases, though not always)

$\Rightarrow$ **at finite temperatures**

(however, they are not sharp)

EXAMPLE OF EXPERIMENTAL REALIZATION

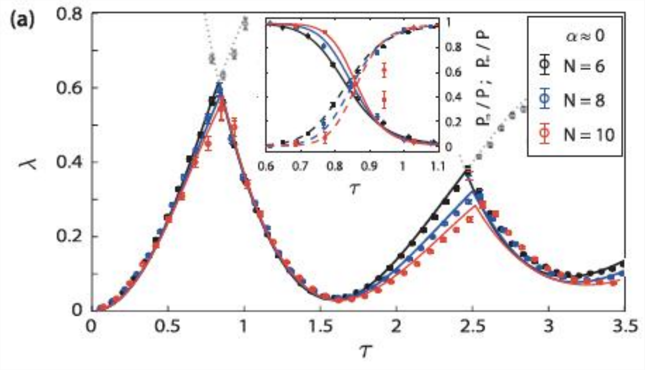
Selected for a Viewpoint in *Physics*
PRL **119**, 080501 (2017) PHYSICAL REVIEW LETTERS week ending 25 AUGUST 2017

Direct Observation of Dynamical Quantum Phase Transitions in an Interacting Many-Body System

P. Jurcevic,[1,2] H. Shen,[1] P. Hauke,[1,3] C. Maier,[1,2] T. Brydges,[1,2] C. Hempel,[1,*] B. P. Lanyon,[1,2] M. Heyl,[4,5] R. Blatt,[1,2] and C. F. Roos[1,2]

[1]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Technikerstr. 21A, 6020 Innsbruck, Austria*
[2]*Institut für Experimentalphysik, Universität Innsbruck, Technikerstrasse 25, 6020 Innsbruck, Austria*
[3]*Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, 6020 Innsbruck, Austria*
[4]*Max-Planck-Institut für Physik komplexer Systeme, 01187 Dresden, Germany*
[5]*Physik Department, Technische Universität München, 85747 Garching, Germany*

(Received 10 March 2017; published 21 August 2017)

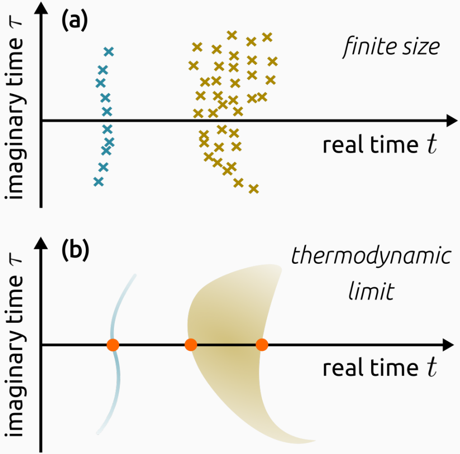


5

Dynamical ferromagnet to paramagnet transition of trapped ions.

Observability in finite-size systems

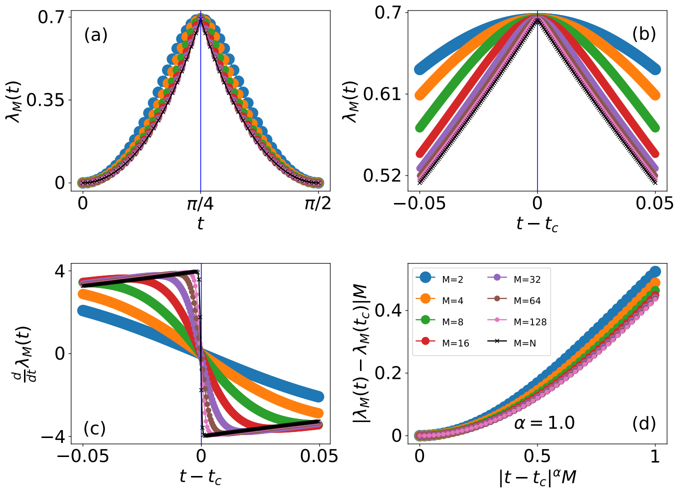
TRANSITIONS OF FINITE-SIZE SYSTEMS



Schematic view of "Fisher zeros" obtained for the Loschmidt amplitude $\left\langle \Psi_0 | e^{-iz\hat{H}} | \Psi_0 \right\rangle$ in the complex plane $z = t + i\tau$.

Marcus Heyl, Rep. Prog. Phys. 81, 054001 (2018).

ISING MODEL: DQPT OF FINITE-SIZE SYSTEM



"Local measures of dynamical quantum phase transitions"

J.C. Halimeh, D. Trapin, M. Damme & M. Heyl, Phys. Rev. B 104, 075130 (2021).

SIMILAR IDEAS

"Exact zeros of the Loschmidt echo and quantum speed limit time for the dynamical quantum phase transitions in finite-size systems"

B. Zhou, Y. Zeng & S. Chen, Phys. Rev. B 104, 094311 (2021).

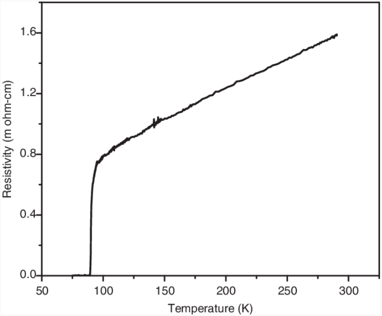
"Finite-component dynamical quantum phase transitions"

R. Puebla, Phys. Rev. B 102, 220302(R) (2020).

II. Application to superconducting systems

BULK SUPERCONDUCTOR: PROPERTIES

Perfect conductor



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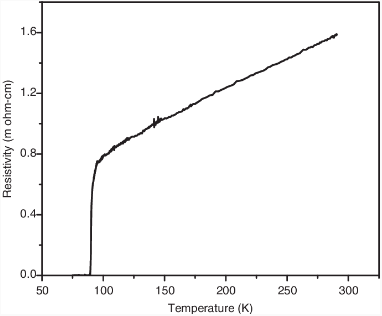

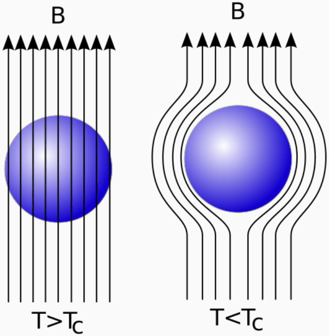

Perfect diamagnet

ELECTRON PAIRING

BCS (non-Fermi liquid) ground state :

$$|\mathbf{BCS}\rangle = \prod_k \left(u_k + v_k \, \hat{c}^\dagger_{k\uparrow} \, \hat{c}^\dagger_{-k\downarrow} \right) \; |\mathbf{vacuum}\rangle$$

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Charge is conserved modulo-2e due to Bose-Einstein condensation of Cooper pairs

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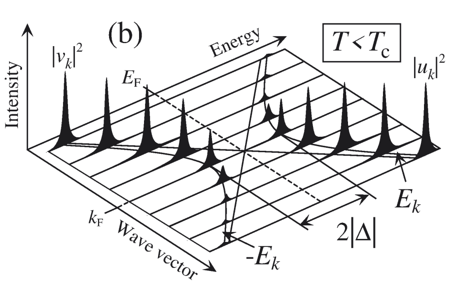
BOGOLIUBOV QUASIPARTICLES

Quasiparticle spectrum of conventional superconductors consists of two Bogoliubov (p/h) branches, gaped around E_F



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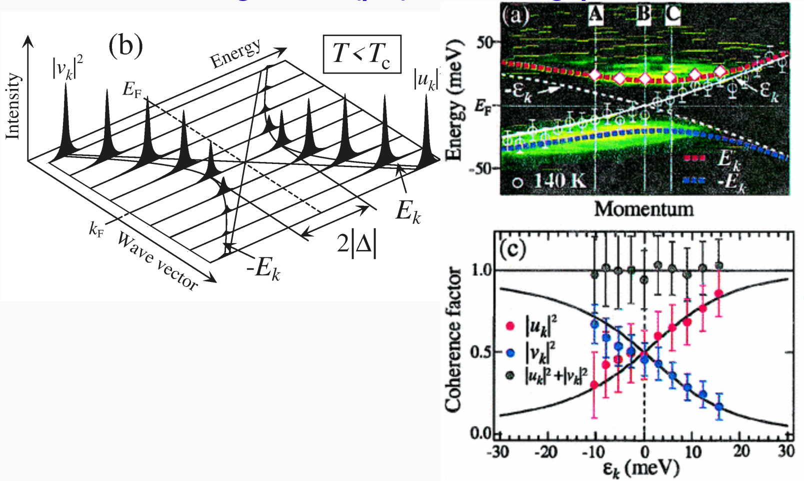

H. Matsui et al, Phys. Rev. Lett. 90, 217002 (2003).

PARTICLE VS HOLE

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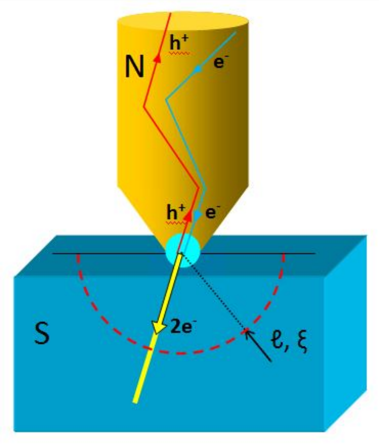
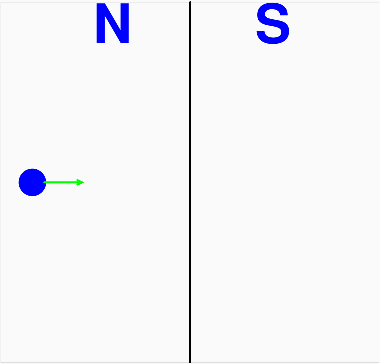


PARTICLE VS HOLE

Let us consider the interface of metal N and superconductor S



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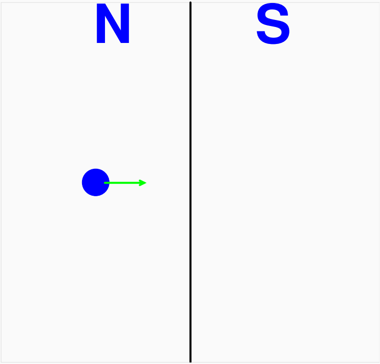

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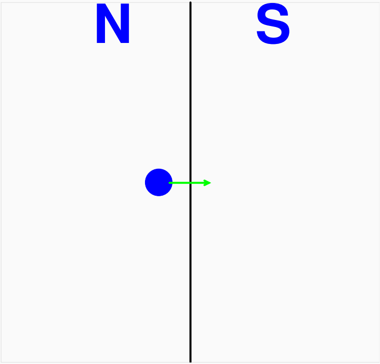

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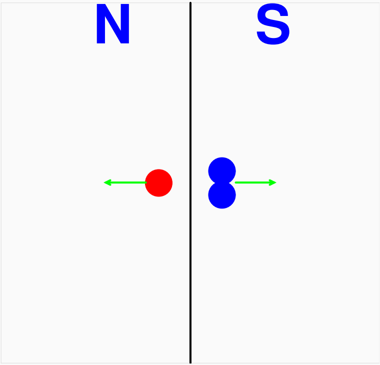

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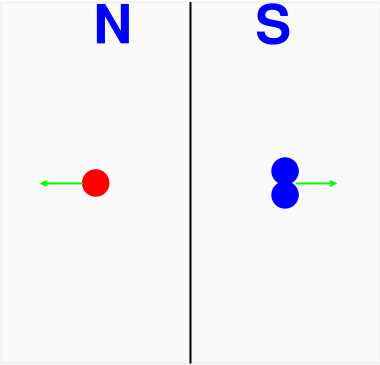

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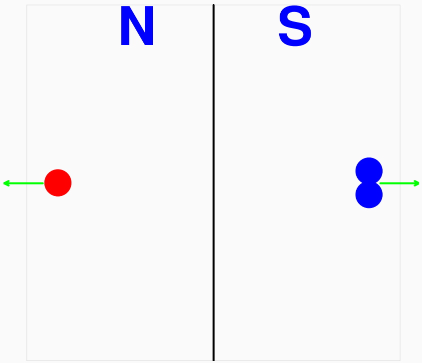

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Dynamical effects in bulk superconductors

Evolution inside the paired state:

$$|\mathbf{BCS}(t)\rangle = \prod_k \left(u_k(t) + v_k(t)\ \hat{c}^\dagger_{k\uparrow}\ \hat{c}^\dagger_{-k\downarrow} \right)\ |\mathbf{vacuum}\rangle$$

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$$L(t) = \sum_k \left| u_k^\star(0) u_k(t) + v_k^\star(0) v_k(t) \right|$$

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For a comprehensive analysis and useful references, see:

"Loschmidt echo of far-from-equilibrium fermionic superfluids"

C. Rylands E.A. Yuzbashyan, V. Gurarie, A. Zabalo, V. Galitski, arXiv:2103.03754 (2021).

POSSIBLE SCENARIOS

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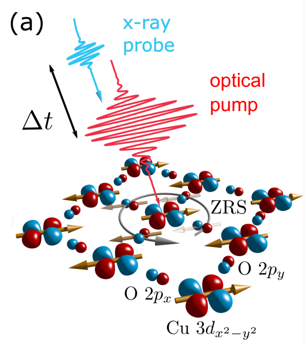
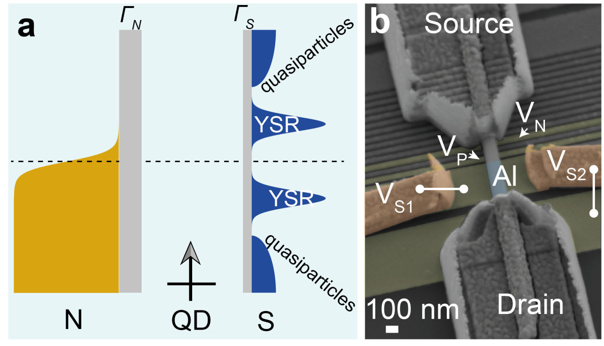
⇒ **by time-resolved X-ray absorption spectroscopy**



D.R. Baykusheva et al, arXiv:2109.13229 (2021)

Superconducting nanostructures

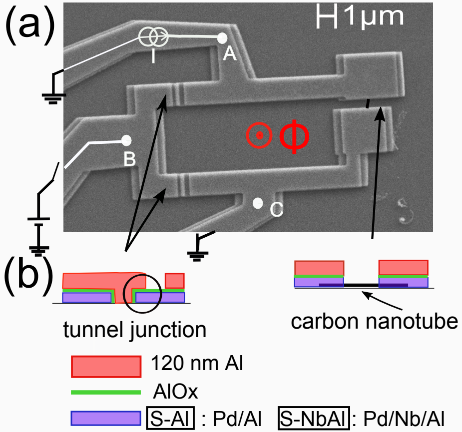
normal metal (N) - quantum dot (QD) - superconductor (S)



J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, Commun. Phys. **3**, 125 (2020).

HETEROSTRUCTURES WITH SUPERCONDUCTOR(S)

superconductor (S) - quantum dot (QD) - superconductor (S)



R. Delagrange, R. Weil, A. Kasumov, M. Ferrier, H. Bouchiat, R. Deblock, Phys. Rev. B **93**, 195437 (2016).

PROXIMITY EFFECT: BASIC ISSUES

- **Coupling of the localized (QD) to itinerant (SC) electrons induces:**

$\Longrightarrow$ **on-dot pairing**

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- Coupling of the localized (QD) to itinerant (SC) electrons induces:

$\Longrightarrow$ **on-dot pairing**

- This is manifested spectroscopically by:

$\Longrightarrow$ **in-gap bound states**

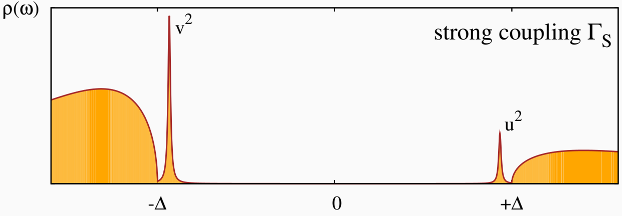
- originating from:

$\Longrightarrow$ **leakage of Cooper pairs on QD** **(Andreev)**

$\Longrightarrow$ **exchange int. of QD with SC** **(Yu-Shiba-Rusinov)**

IN-GAP STATES

Spectrum of a single impurity coupled to bulk superconductor:



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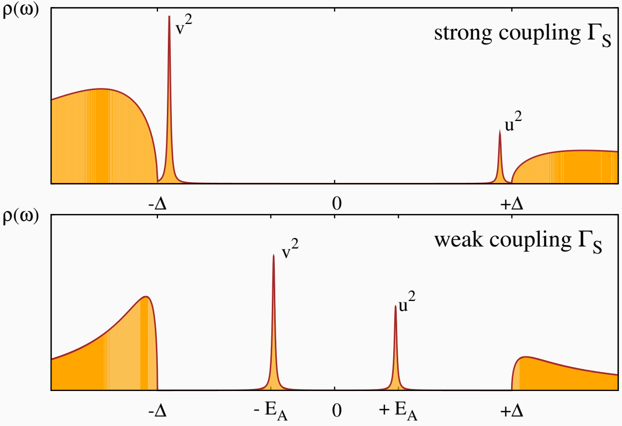

Bound states appearing in the subgap region $-\Delta < \omega < \Delta$.

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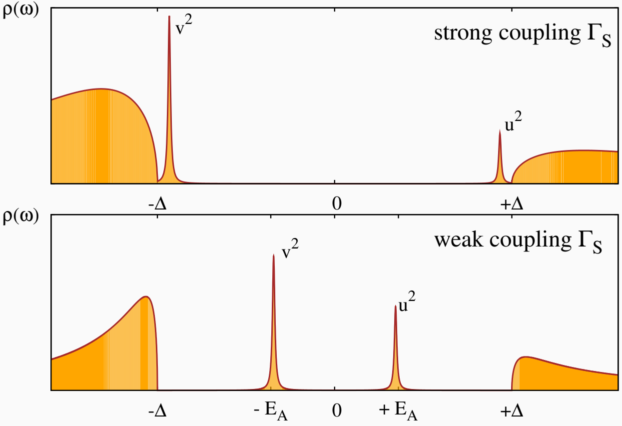

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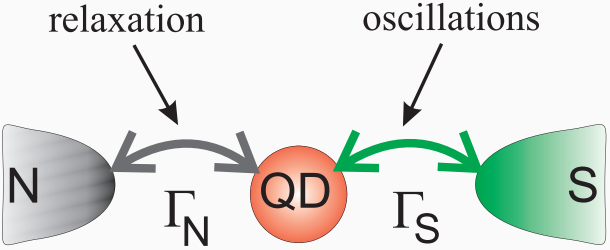
Yu-Shiba-Rusinov (Andreev) bound states

Characteristic temporal scales

Consider a sudden coupling of QD to external leads



R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

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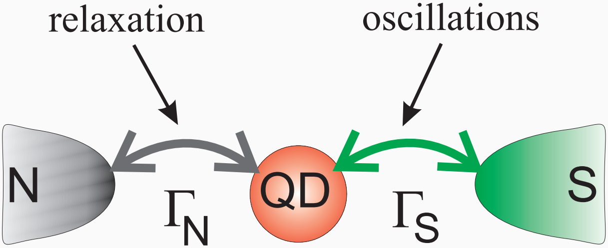

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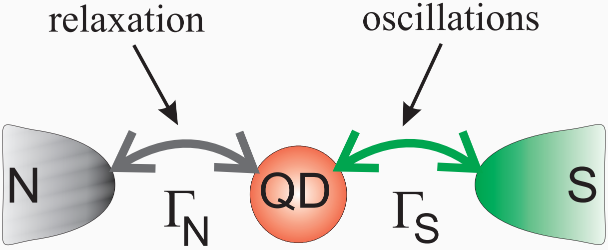
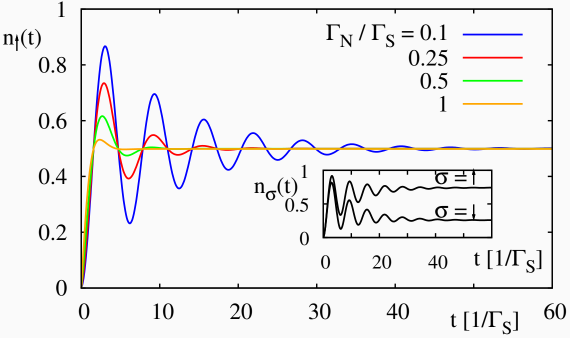


R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

- **how much time is needed to create in-gap states ?**
- **are there any characteristic features ?**

RELAXATION VS QUANTUM OSCILLATIONS

Time-dependent charge of an initially empty QD

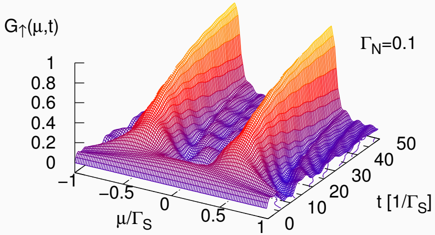


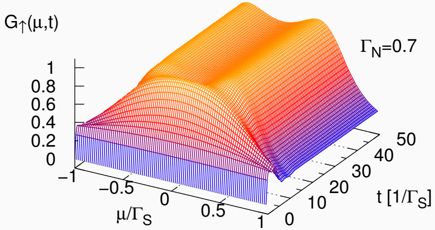
- **relaxation rate is proportional to Γ_N**
- **oscillations depend on energies of in-gap states**

R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

EXPERIMENTALLY ACCESSIBLE QUANTITIES

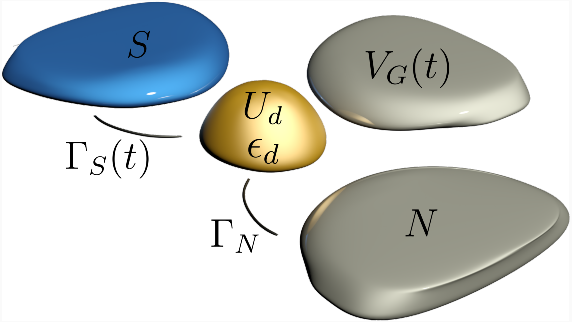
QD level $\varepsilon = 0$



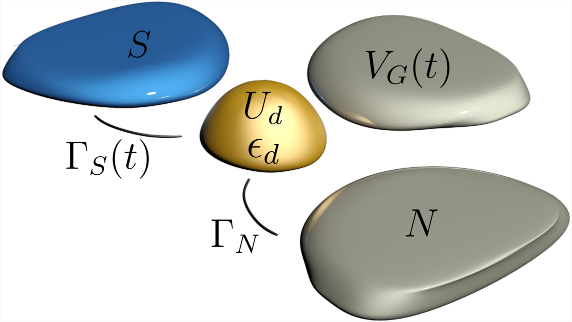


Subgap tunneling conductance $G_\sigma = \frac{\partial I_\sigma}{\partial t}$ vs time (t) and voltage (μ)

QUENCH DRIVEN DYNAMICS



Possible quench protocols:



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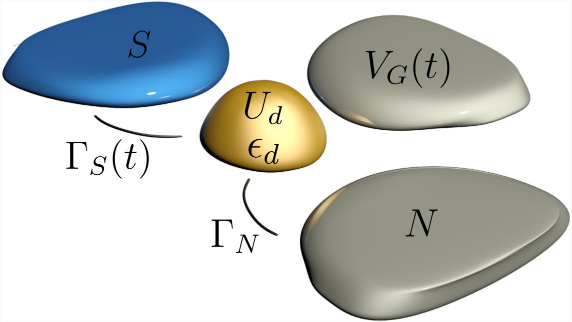


Possible quench protocols:

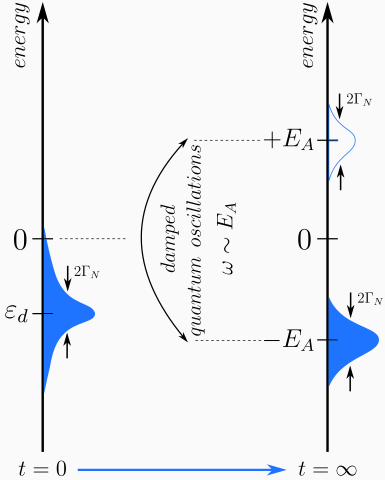
$\Longrightarrow$ **sudden coupling to superconductor** $0 \to \Gamma_S$

$\Longrightarrow$ **abrupt application of gate potential** $0 \to V_G$

K. Wrześniewski, B. Baran, R. Taranko, T. Domański & I. Weymann, PRB 103, 155420 (2021).

BUILDUP OF IN-GAP STATES

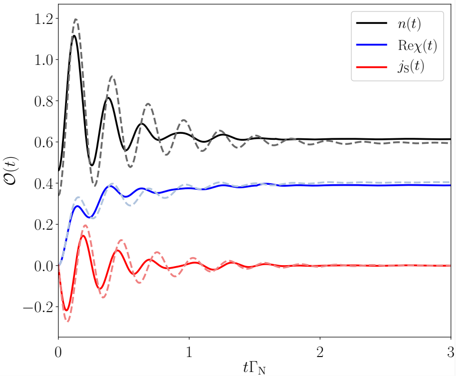
Rabbi-type oscillations observable in development of the in-gap states



K. Wrześniewski, B. Baran, R. Taranko, T. Domański & I. Weymann, PRB 103, 155420 (2021).

BUILDUP OF IN-GAP STATES

Time-dependent observables driven by the quantum quench $0 \rightarrow \Gamma_S$



solid lines - time dependent NRG

dashed lines - Hartree-Fock-Bogolubov

K. Wrześniewski, B. Baran, R. Taranko, T. Domański & I. Weymann, PRB 103, 155420 (2021).

Singlet-doublet transition

SINGLY OCCUPIED VS BCS-TYPE CONFIGURATIONS

The proximitized quantum dot can described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left(\Delta_d \, \hat{d}^{\dagger}_{\uparrow} \hat{d}^{\dagger}_{\downarrow} + \text{h.c.} \right)$$

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Eigen-states of this problem are represented by:

$|\uparrow\rangle$ **and** $|\downarrow\rangle$ $\Leftarrow$ **doublet states (spin $\frac{1}{2}$)**

$$\left. \begin{array}{l} u\,|0\rangle - v\,|\uparrow\downarrow\rangle \\ v\,|0\rangle + u\,|\uparrow\downarrow\rangle \end{array} \right\} \Leftarrow$$

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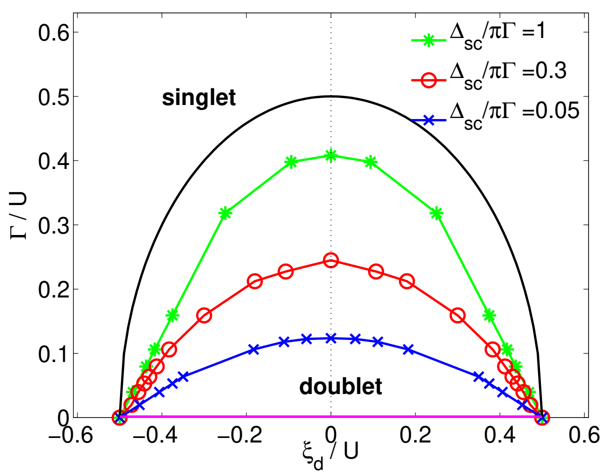
Upon varrying the parameters ε_d, U_d or Γ_S there can be induced quantum phase transition between these doublet/singlet states.

QUANTUM PHASE TRANSITION (STATIC VERSION)

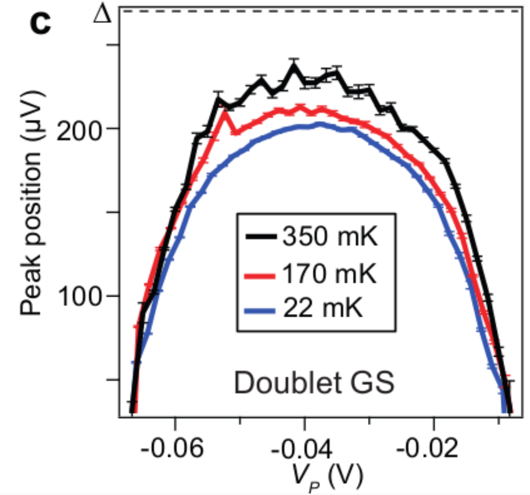
Singlet-doublet quantum phase transition: NRG results



$\xi_d \equiv \varepsilon_d - U/2$

J. Bauer, A. Oguri & A.C. Hewson, J. Phys.: Condens. Matter 19, 486211 (2007).

QUANTUM PHASE TRANSITION: EXPERIMENT

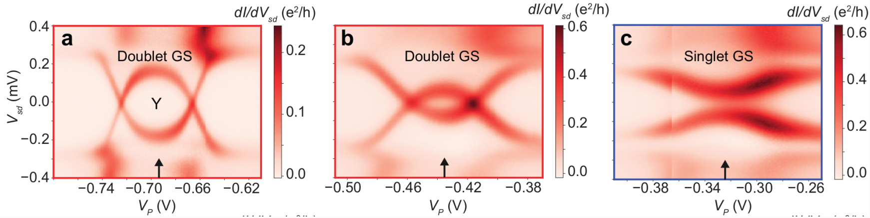


J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, Commun. Phys. 3, 125 (2020).

SINGLET VS DOUBLET: EXPERIMENT

Differential conductance vs source-drain bias V_{sd} (vertical axis) and gate potential V_p (horizontal axis) measured for various Γ_S/U

$U \gg \Gamma_s$ $\quad$ $U \geq \Gamma_s$ $\quad$ $U < \Gamma_s$



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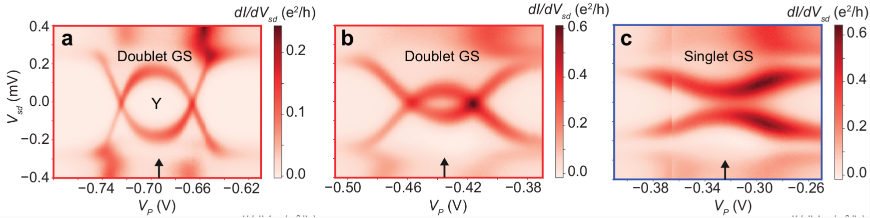

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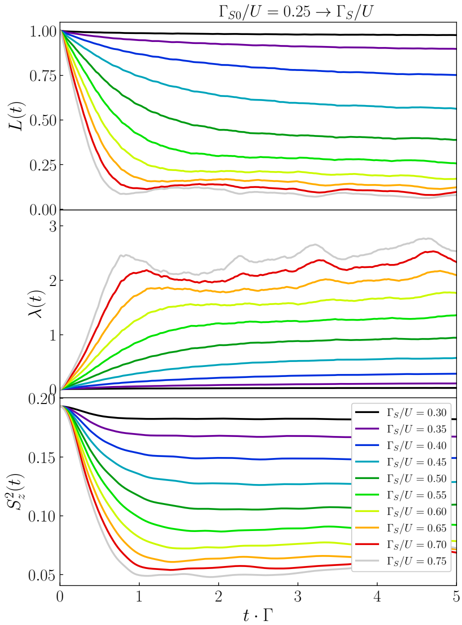
Crossings of in-gap states correspond to the singlet-doublet QPT.

Dynamical singlet-doublet transition

tNRG RESULTS: ABRUPT CHANGE OF Γ_S



Loschmidt echo

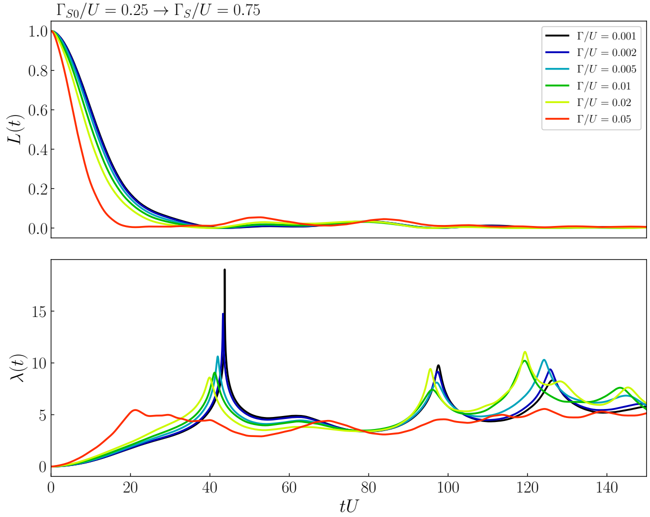
$L(t) \equiv |\langle \Psi(0)|\Psi(t)\rangle|^2$

Return rate

$\lambda(t) \equiv -\frac{1}{N} \ln\{L(t)\}$

The squared magnetic moment $\langle S_z^2(t) \rangle$

tNRG RESULTS: $\Gamma_S = U/4 \longrightarrow \Gamma_S = 3U/4$



Loschmidt echo $L(t)$ and return rate $\lambda(t)$ obtained for various $\Gamma_N \equiv \Gamma$

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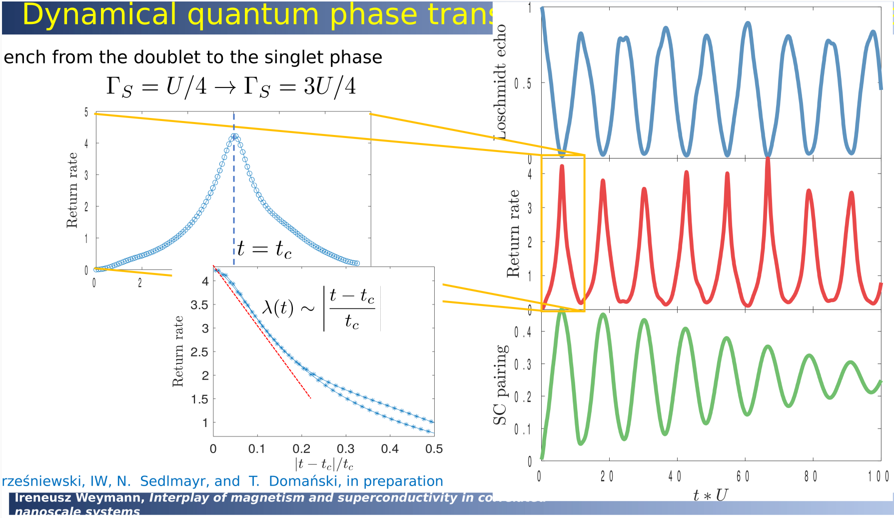

Finite-size scaling analysis near the critical-time point.

CONCLUSIONS

Quench imposed on N – QD – S junction:

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These phenomena are detectable in transport properties !

ACKNOWLEDGEMENTS

- **dynamical singlet-doublet transition**

$\Longrightarrow$ **I. Weymann (Poznań), K. Wrześniewski (Poznań), N. Sedlmayr (Lublin),**

- **transients phenomena, Floquet formalism**

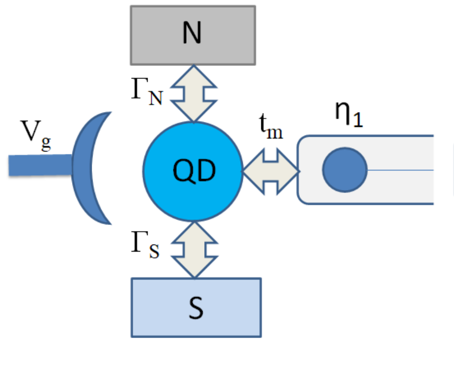
$\Longrightarrow$ **R. Taranko (Lublin), B. Baran (Lublin),**

- **time-resolved leakage of Majorana qps**

$\Longrightarrow$ **J. Barański (Dęblin)**

Other related topics

Abrupt coupling (t_m) of quantum dot to topological SC nanowire



J. Barański, ... & T. Domański,

Phys. Rev. B 103, 235416 (2021).

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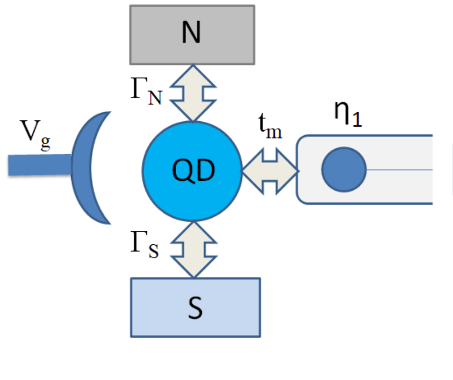

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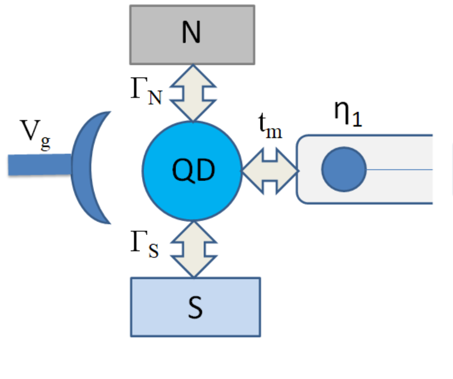

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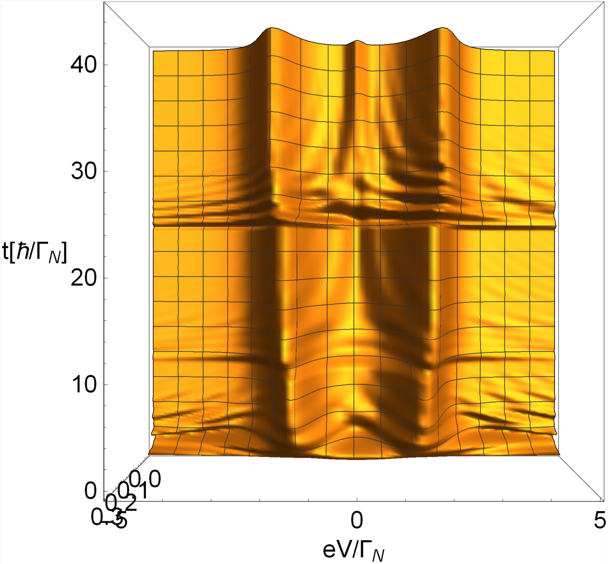
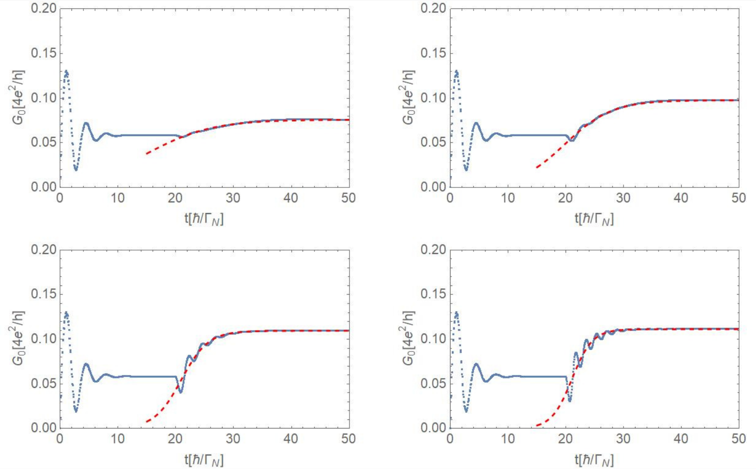
- time needed for Majorana leakage on QD,
- time-resoveld zero bias conductance.

TIME-RESOLVED MAJORANA LEAKAGE



The differential Andreev conductance vs bias voltage V and time

TIME-RESOLVED ZERO BIAS CONDUCTANCE



The zero-bias differential conductivity obtained for $\Gamma_S = 3\Gamma_N$ and $\epsilon_d = \Gamma_N$, assuming: $t_m = 0.25$ (upper left), 0.5 (upper right), 1 (lower left), 1.5 (lower right) Γ_N. QD is abruptly connected to Majorana mode at time $t = 20\hbar/\Gamma_N$.