Ustroń, 18 Sept. 2012

Andreev scattering :

from the nano- to macroscale

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http://kft.umcs.lublin.pl/doman/lectures

Outline

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/ underlying idea /

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1. Introduction

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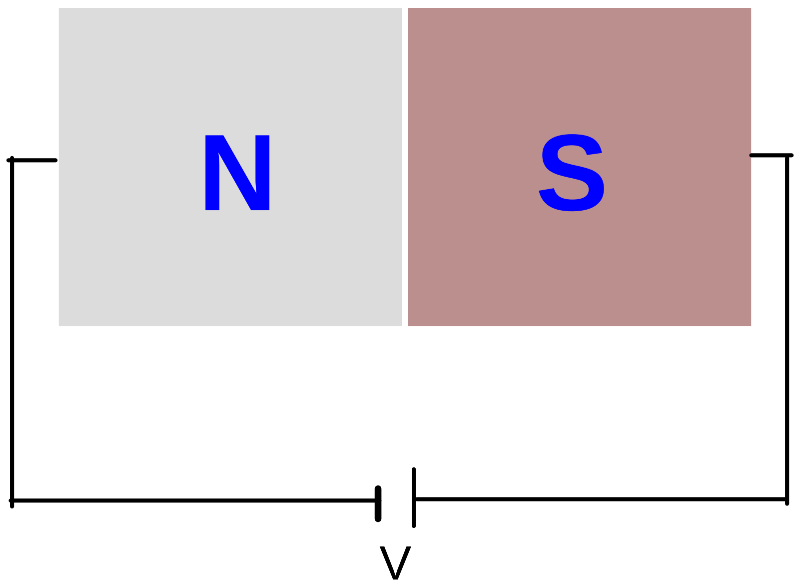

Let us restrict to the subgap regime $|eV| \ll \Delta$ of an applied bias V.

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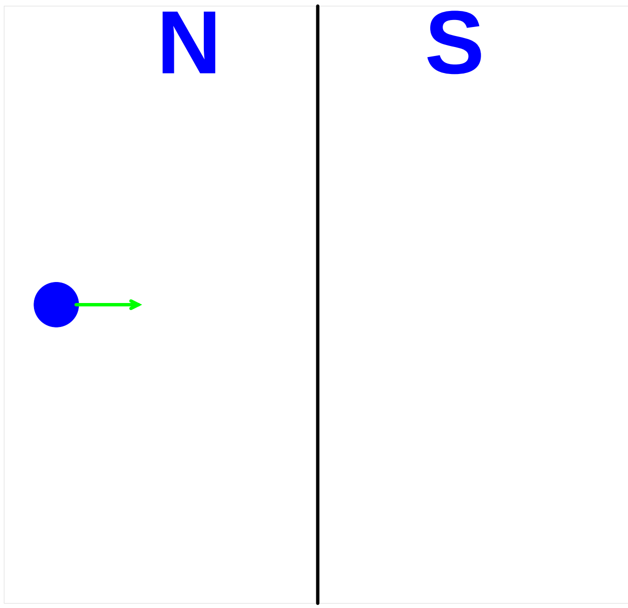

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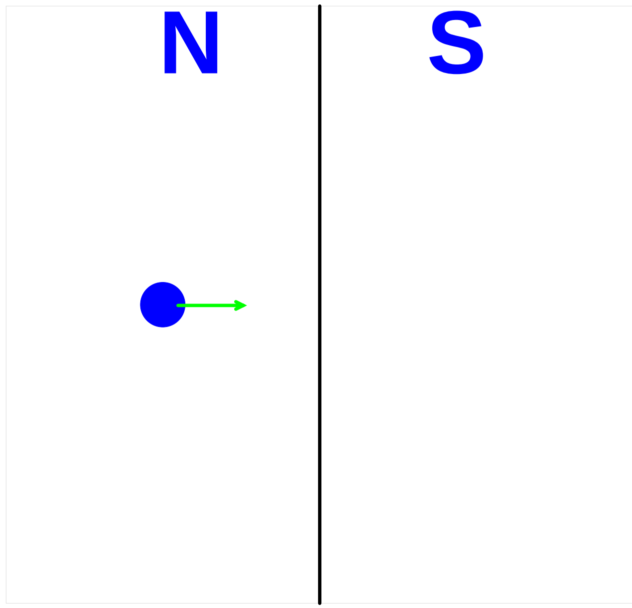

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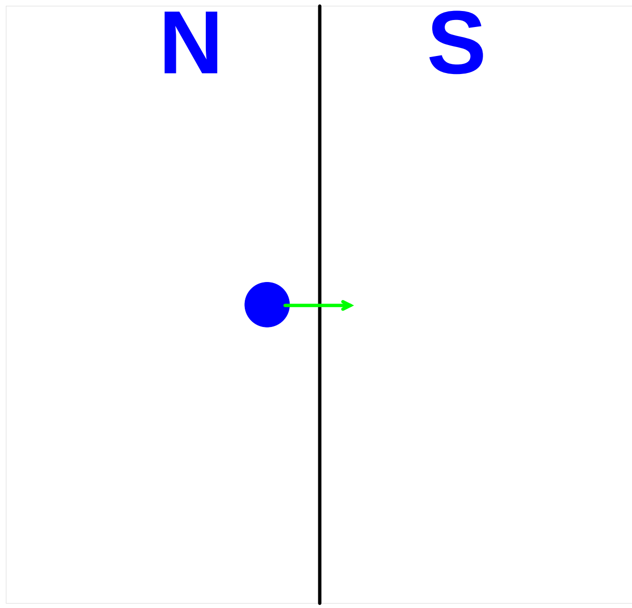

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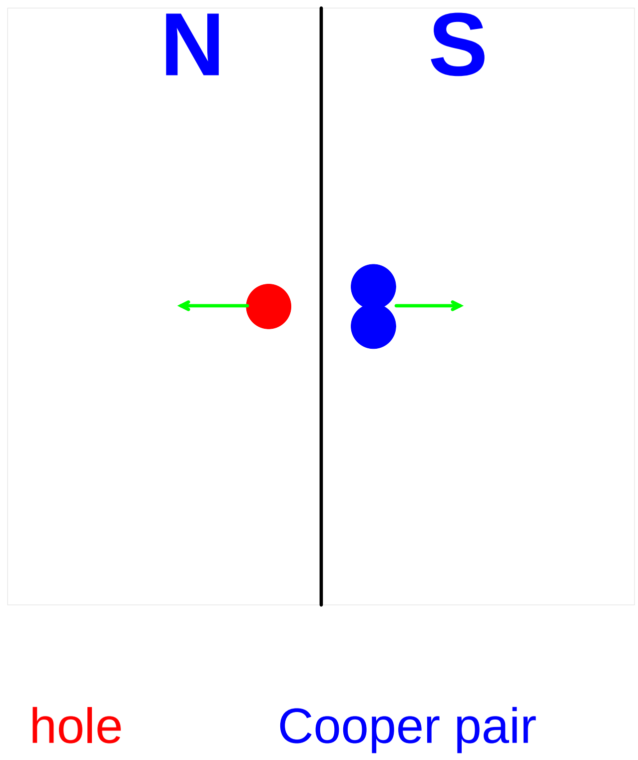

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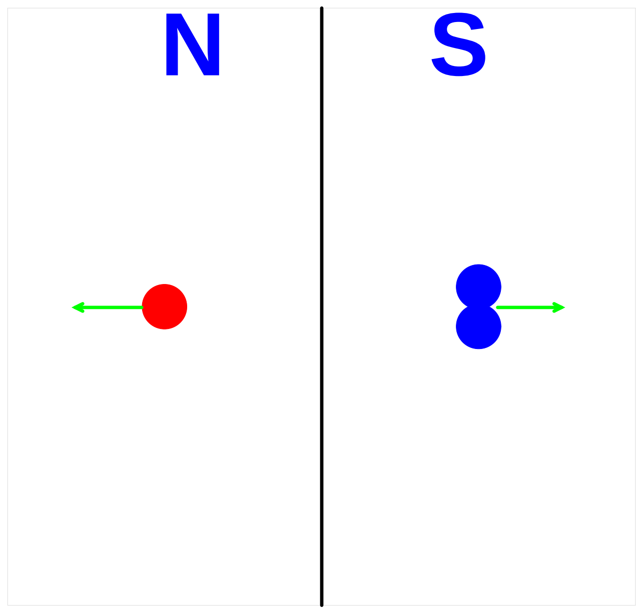

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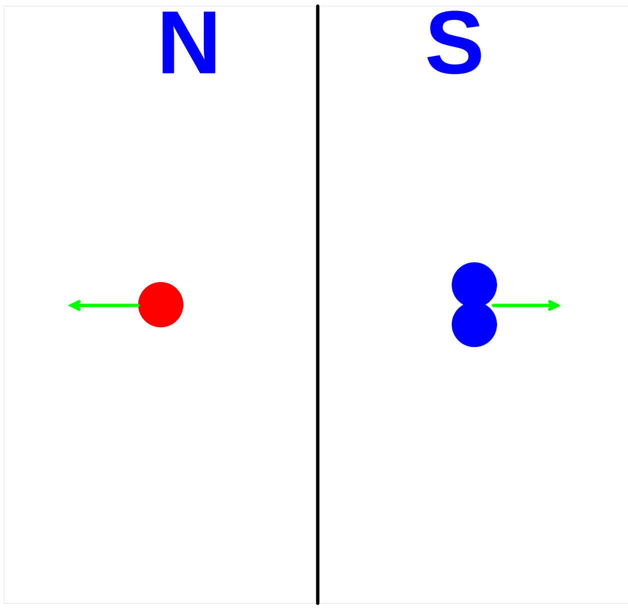

hole Cooper pair

Such double-charge exchange is named the <u>Andreev reflection</u> (scattering).

Andreev reflections – historical remark

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A.F. Andreev

/ P. Kapitza Institute, Moscow (Russia) /

A.F. Andreev, Sov. Phys. JETP **19**, *1228 (1964).*

2. Andreev transport via quantum dot

Physical situation

N-QD-S scheme

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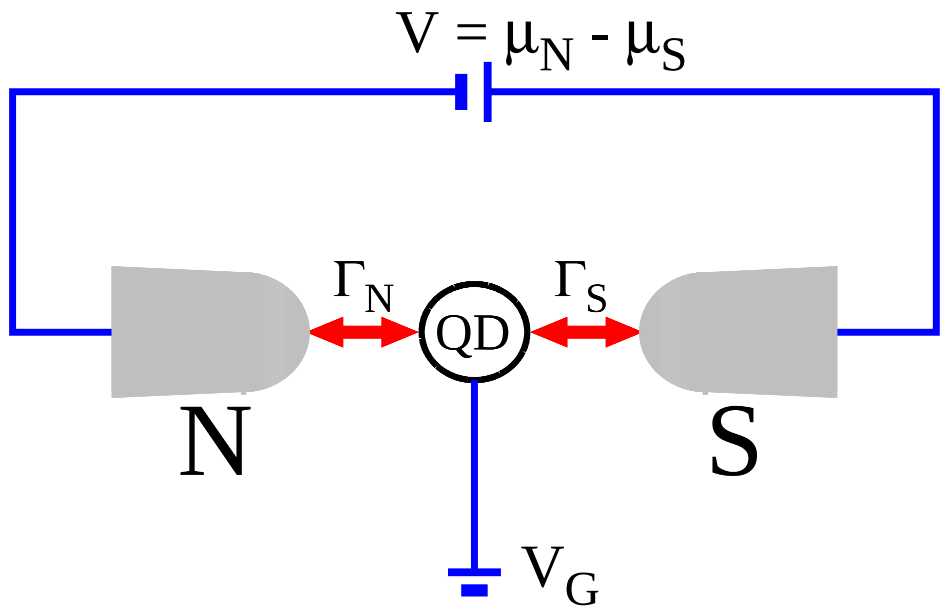

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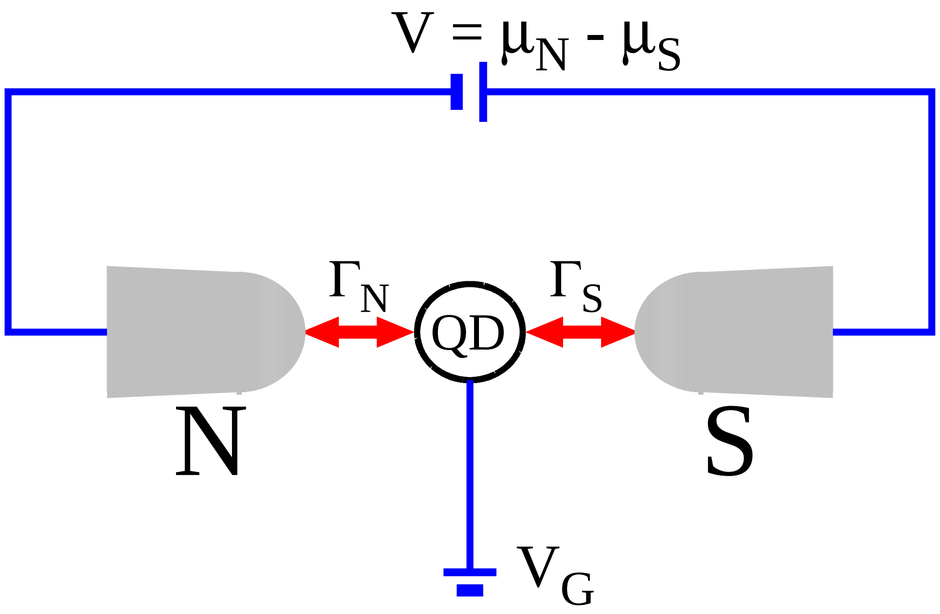

metallic lead QD superconductor

This setup can be thought of as a particular version of the SET.

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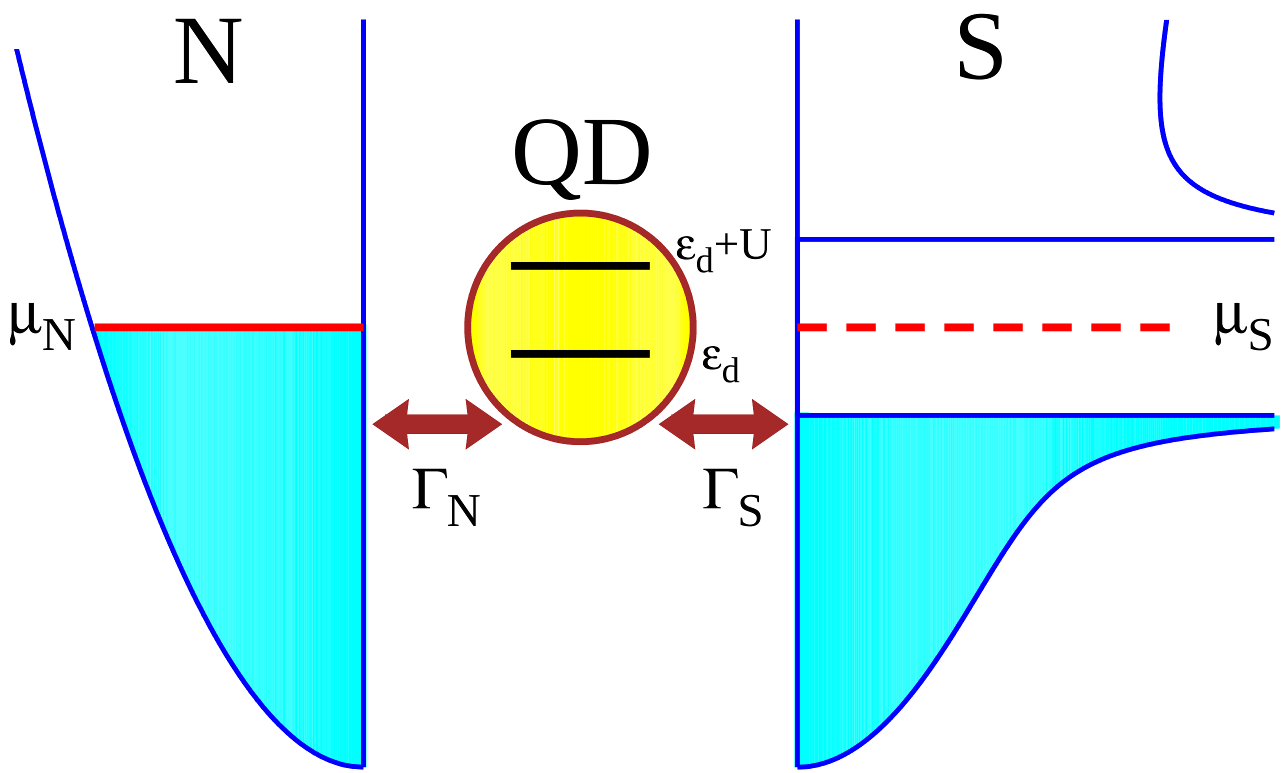

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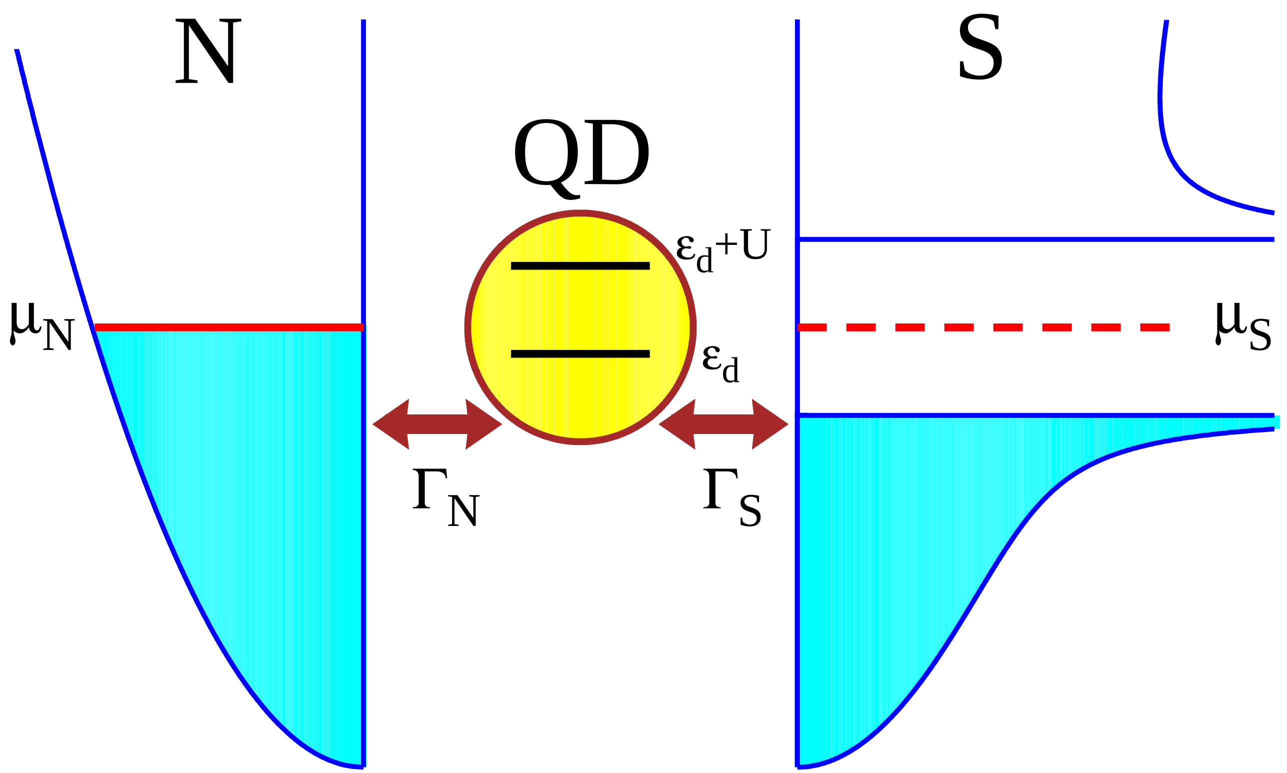

External bias $eV = \mu_N - \mu_S$ induces the current(s) through QD.

Microscopic model

The correlation effects

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$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U \, \hat{n}_{d\uparrow} \, \hat{n}_{d\downarrow}$$

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where

$$\hat{H}_N = \sum_{k,\sigma} \left(\varepsilon_{k,N} - \mu_N \right) \hat{c}^{\dagger}_{k\sigma N} \hat{c}_{k\sigma N}$$

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where

$$\hat{H}_S = \sum_{k,\sigma} \left(\varepsilon_{k,S} - \mu_S \right) \hat{c}^{\dagger}_{k\sigma S} \hat{c}_{k\sigma S} - \sum_{k} \left(\Delta \hat{c}^{\dagger}_{k\uparrow S} \hat{c}^{\dagger}_{k\downarrow S} + \mathrm{h.c.} \right)$$

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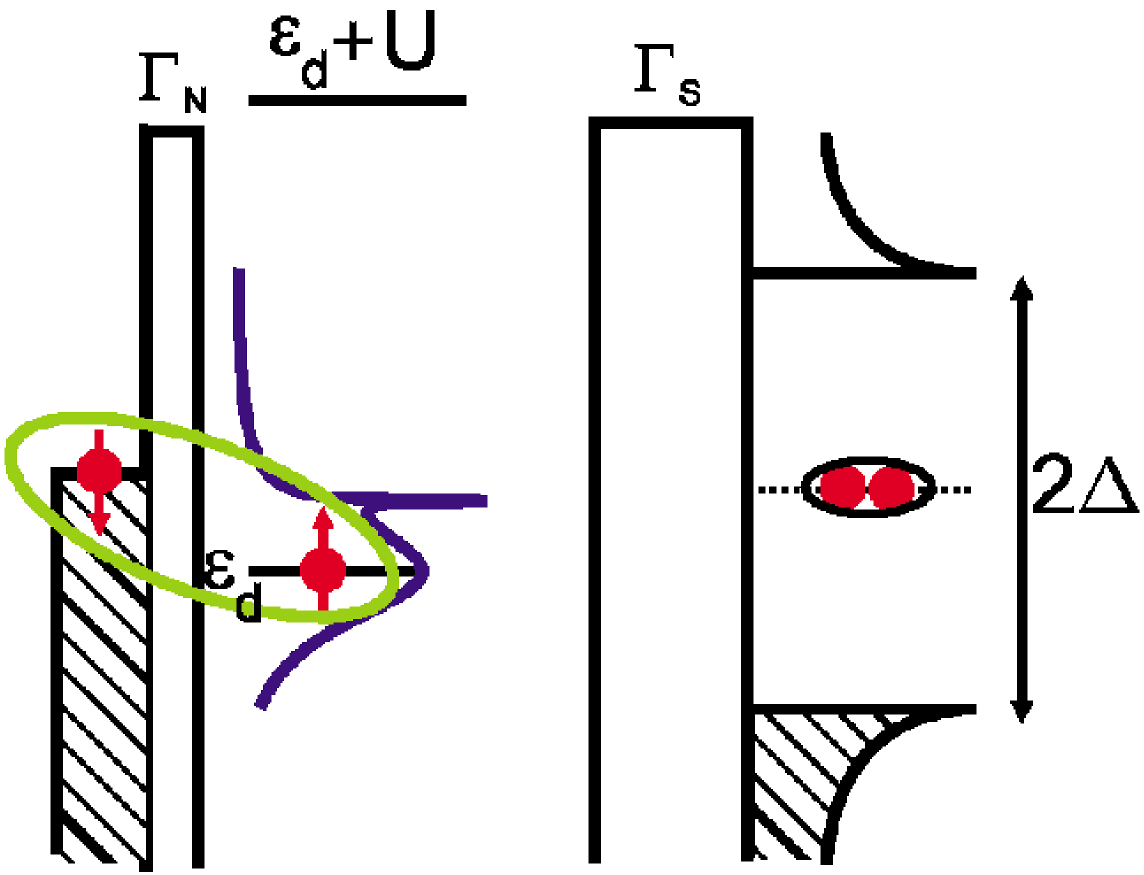

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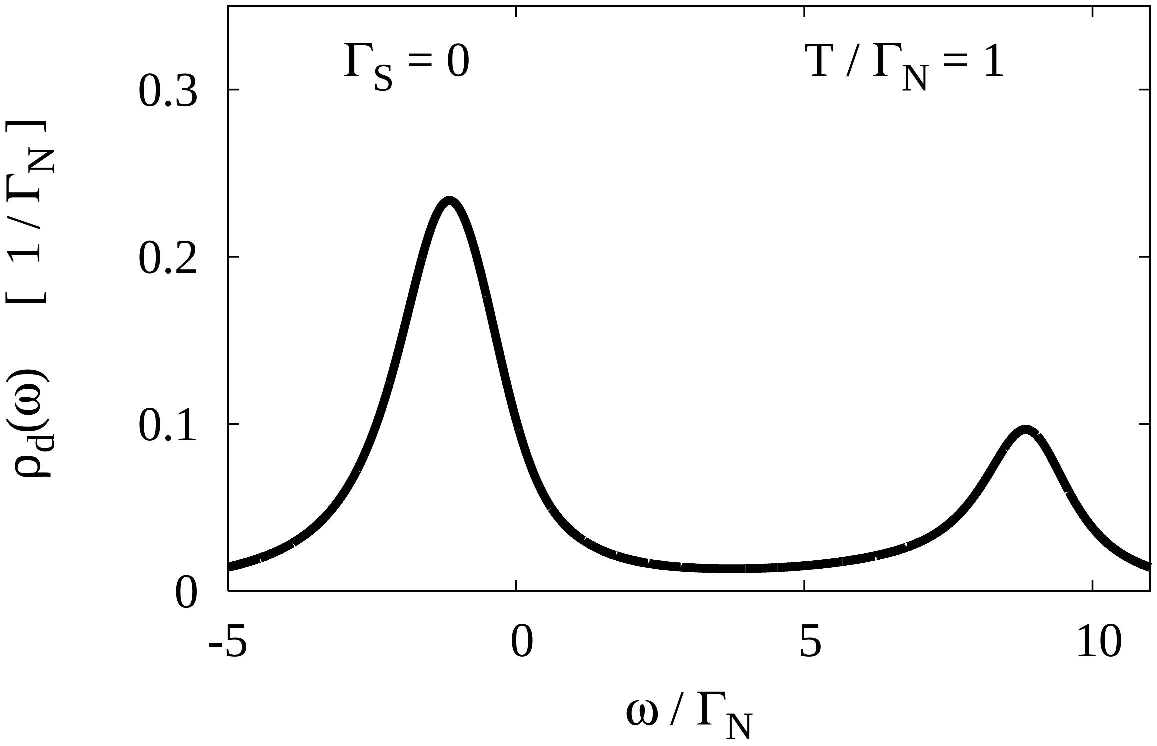

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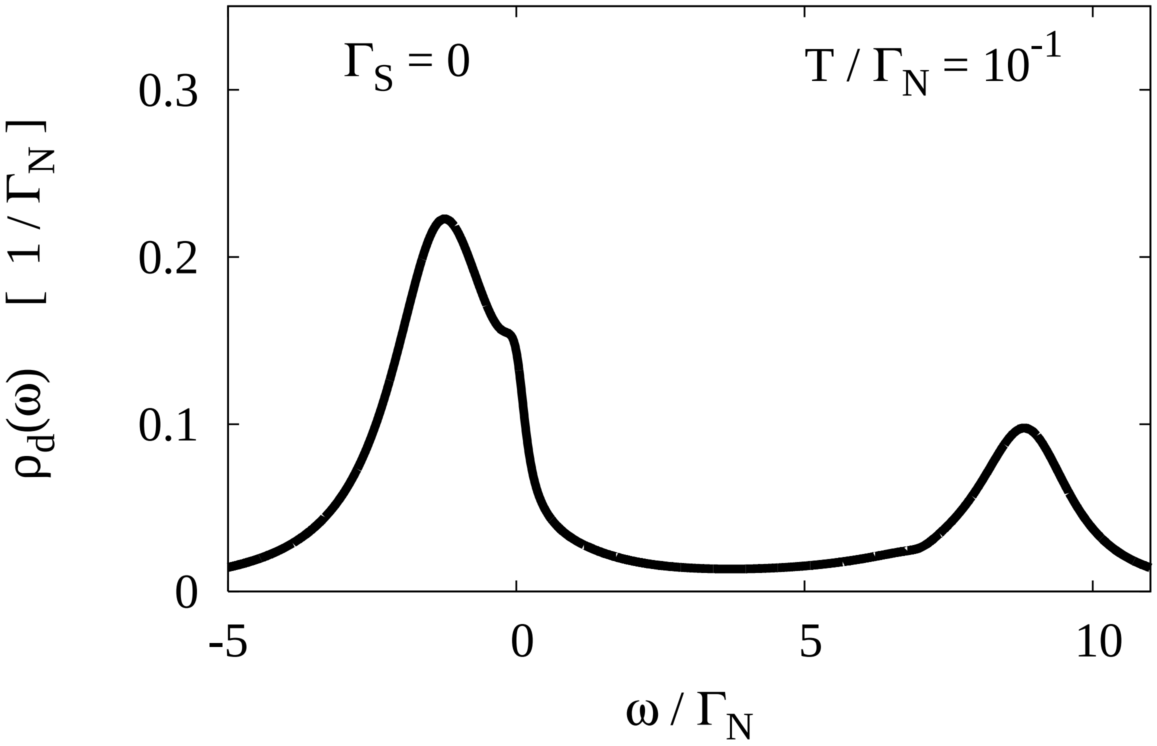

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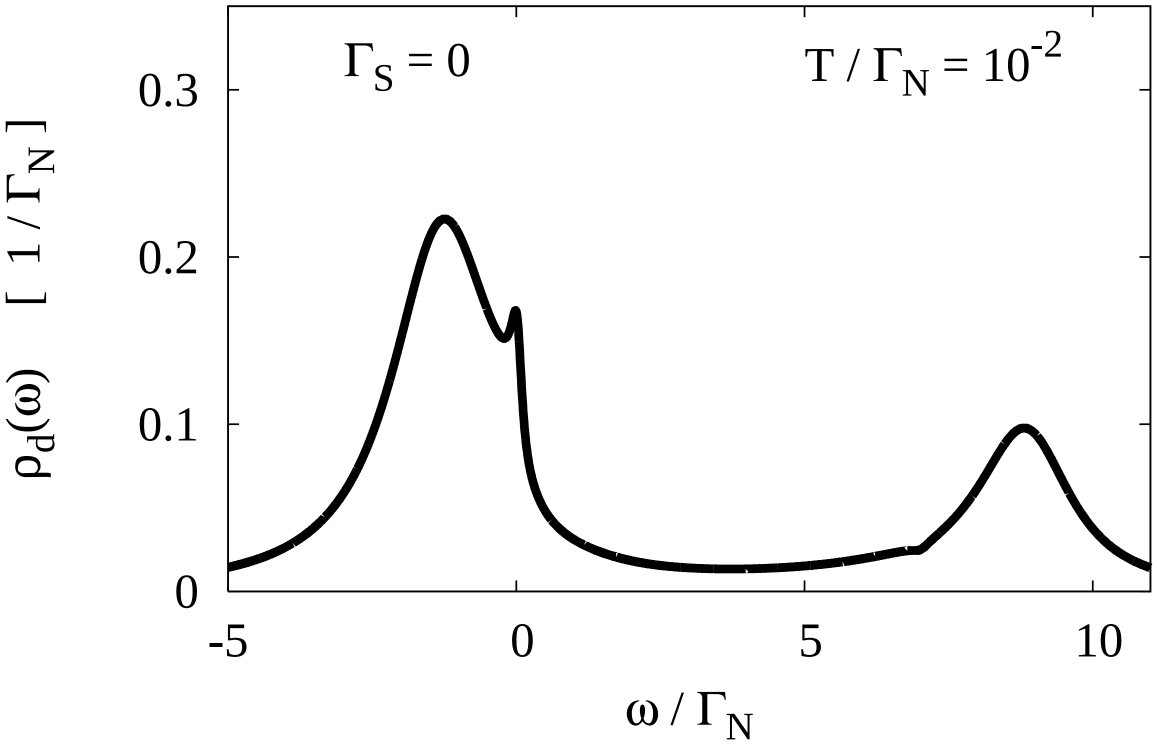

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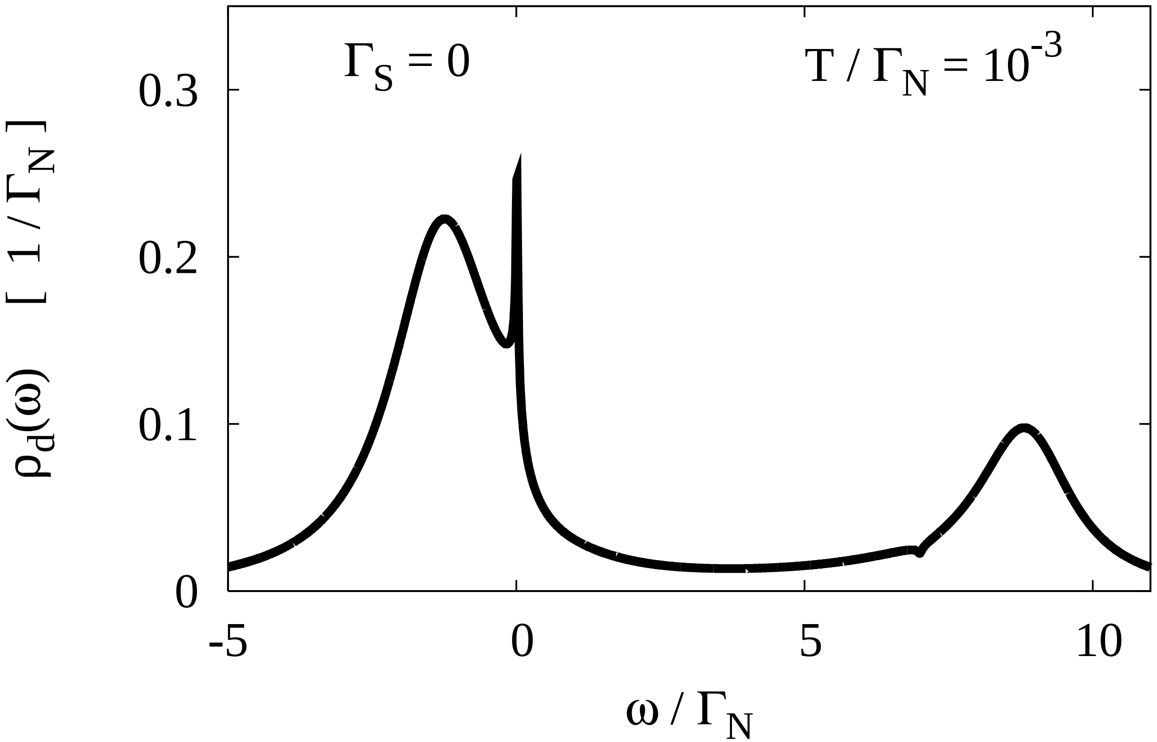

- a broadening of QD levels and
- appearance of the Kondo resonance below T_K.

Relevant problems : issue # 2

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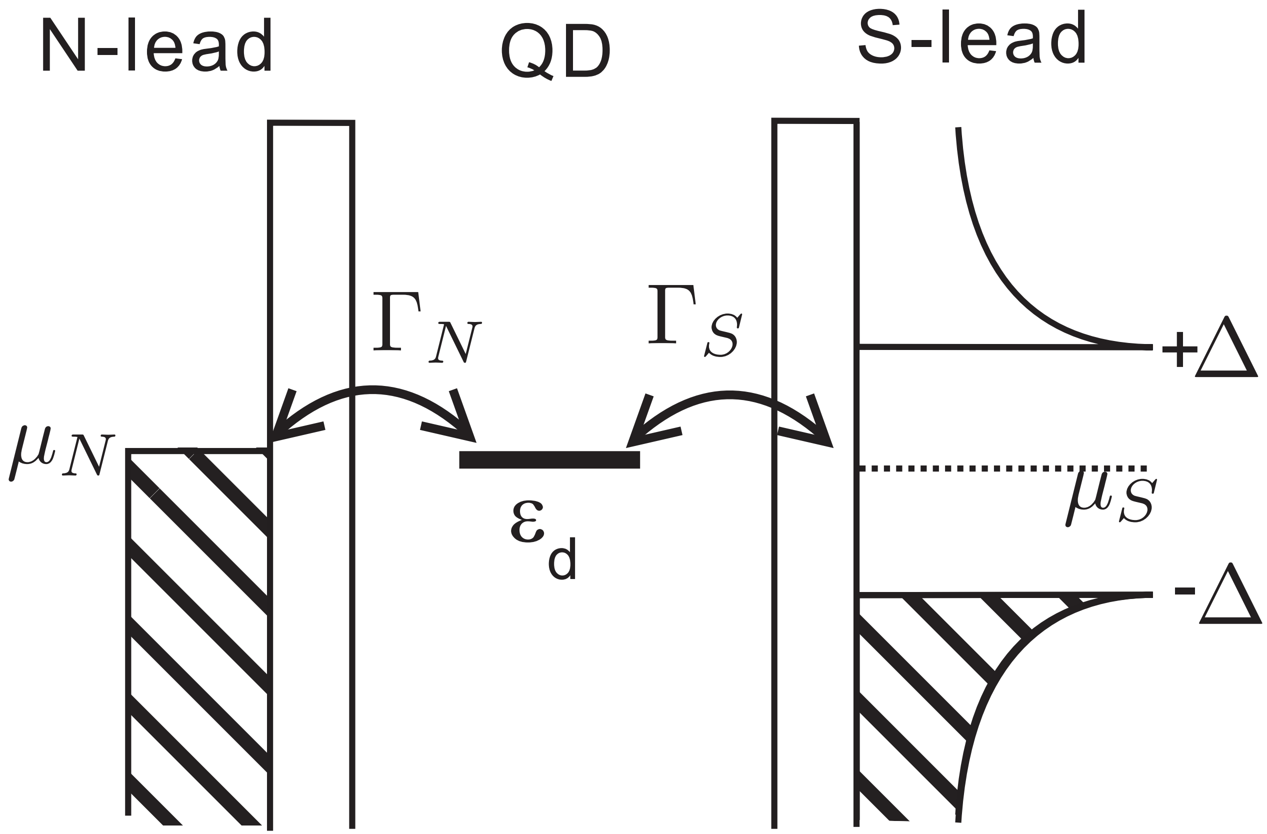
Hybridization of QD to the superconducting lead



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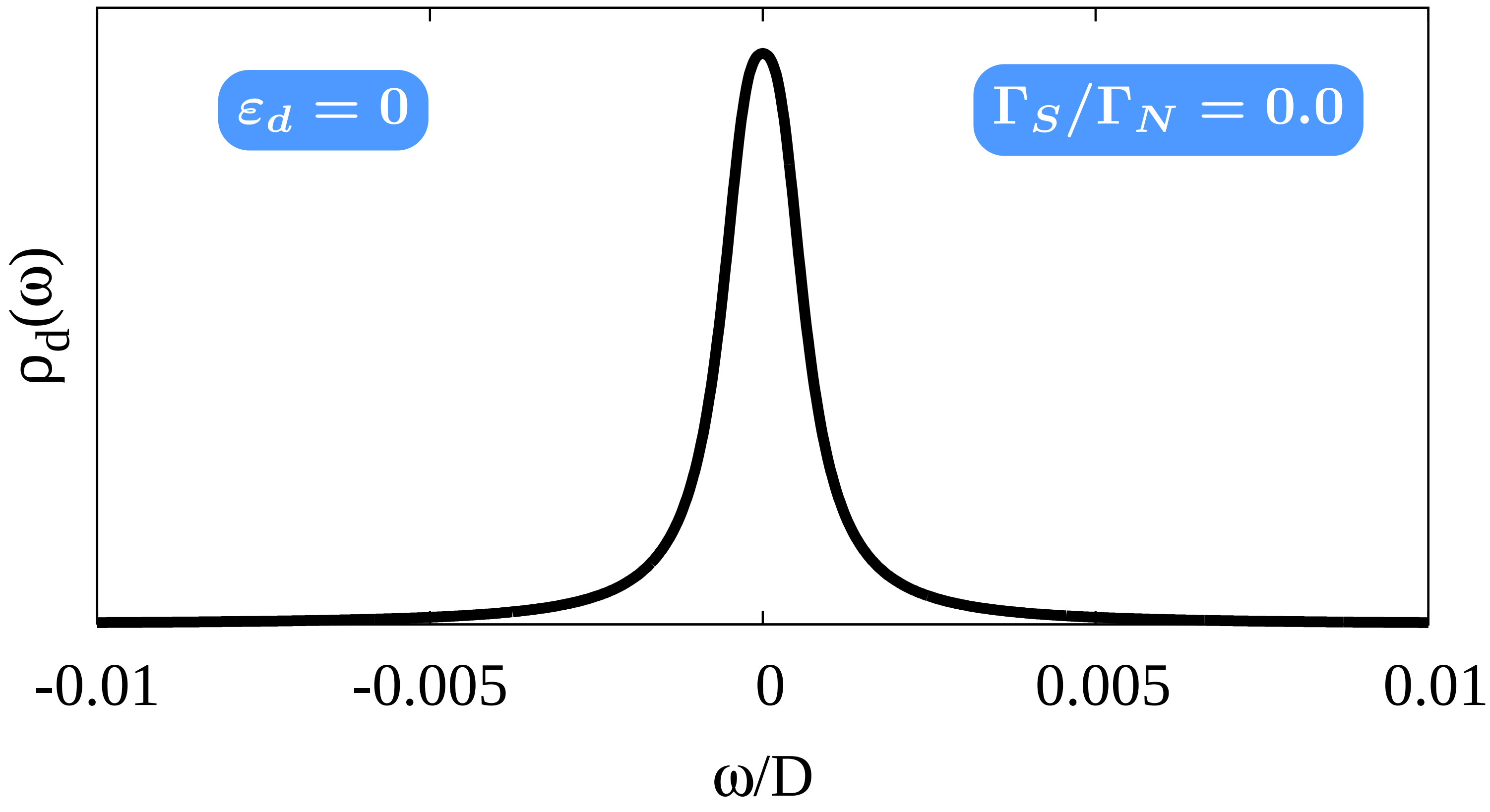

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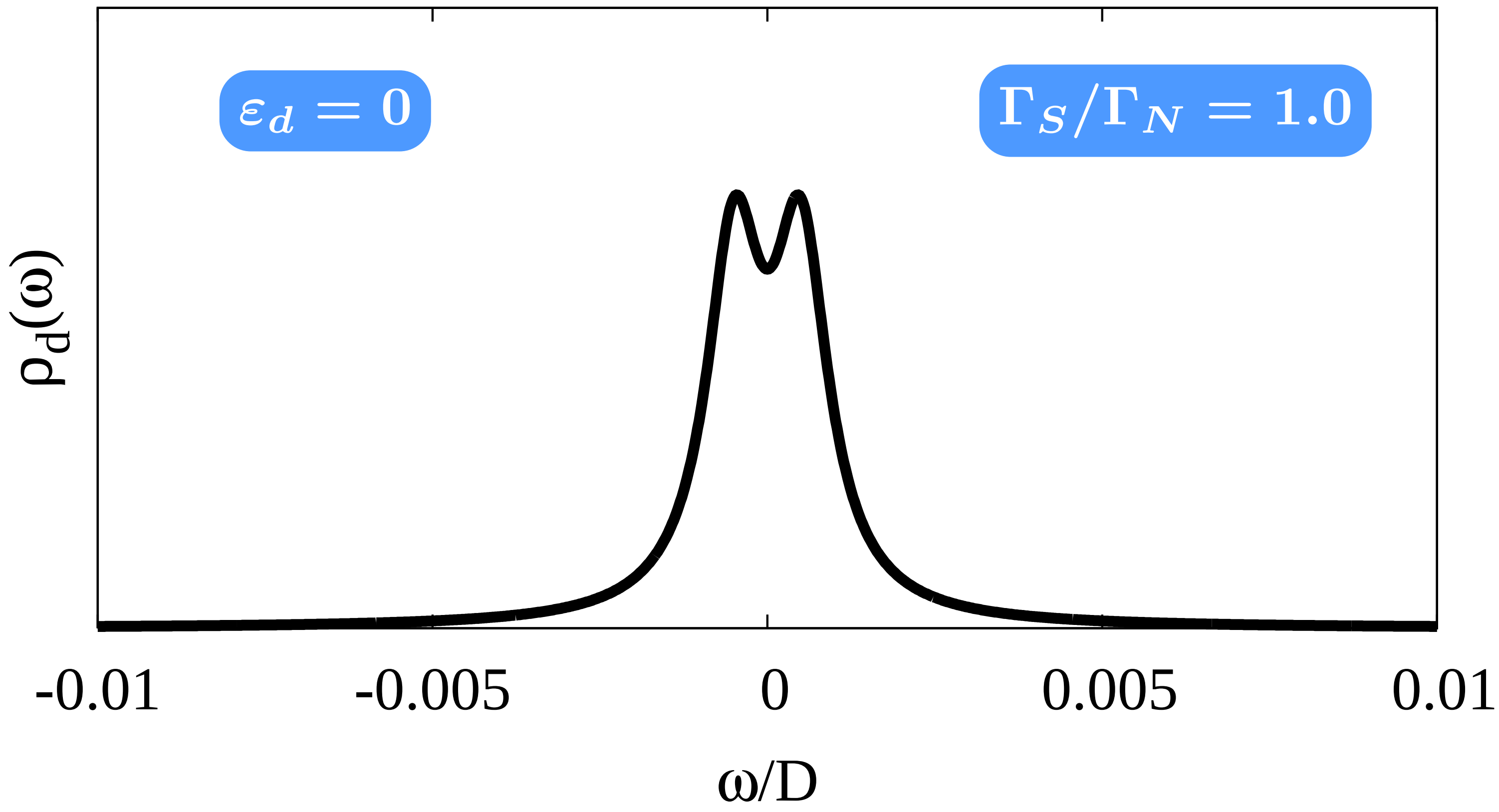

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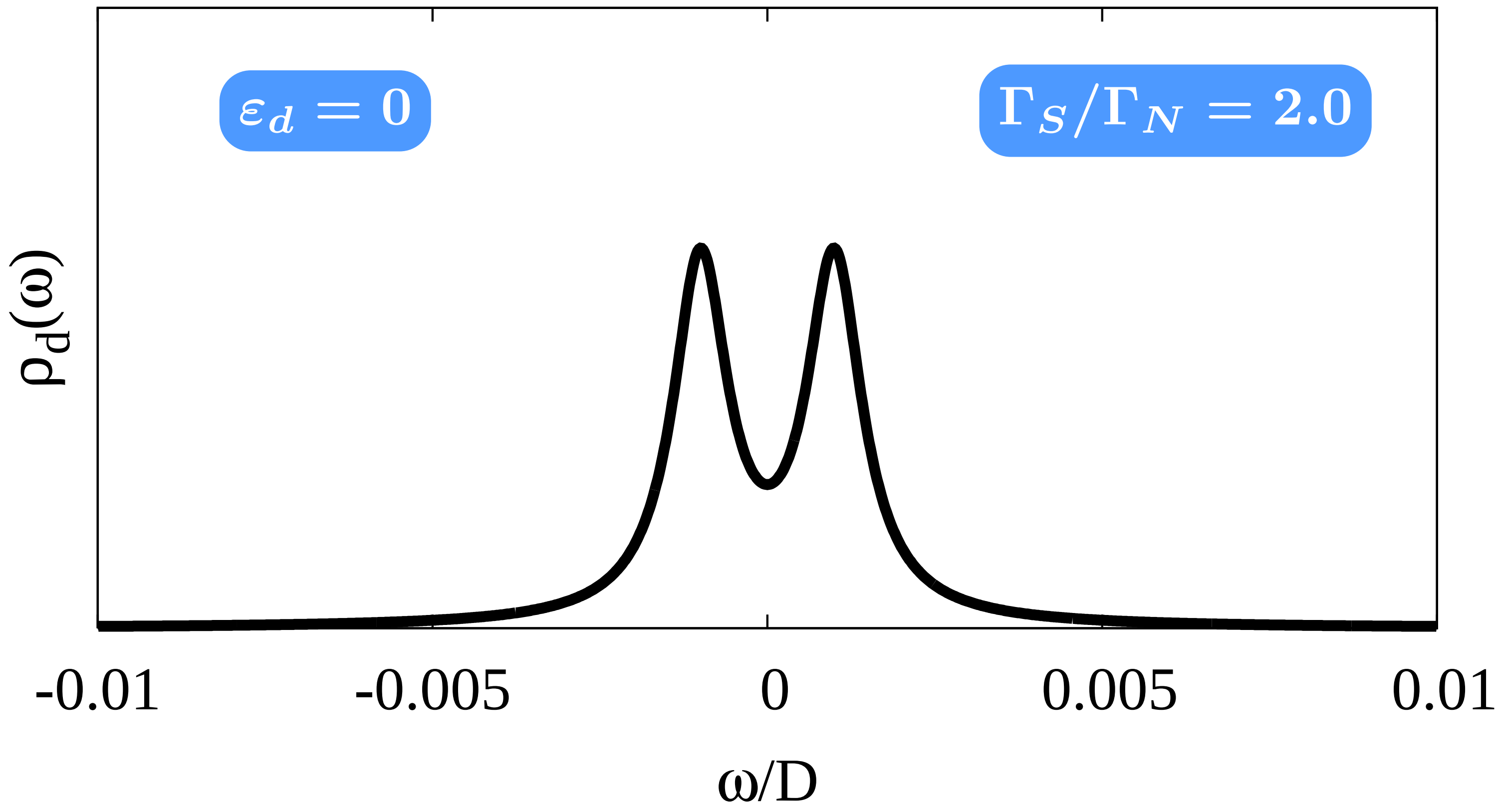

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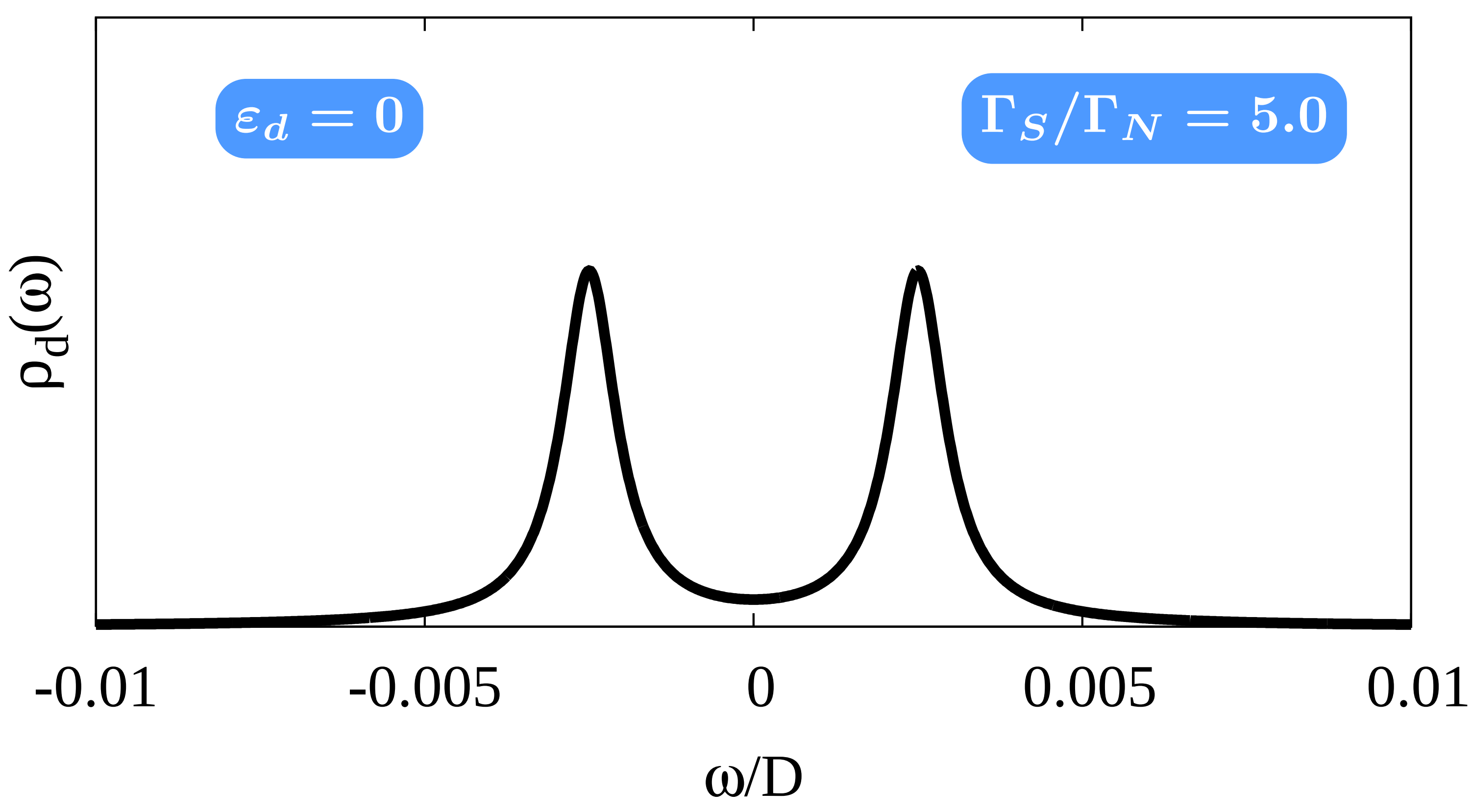

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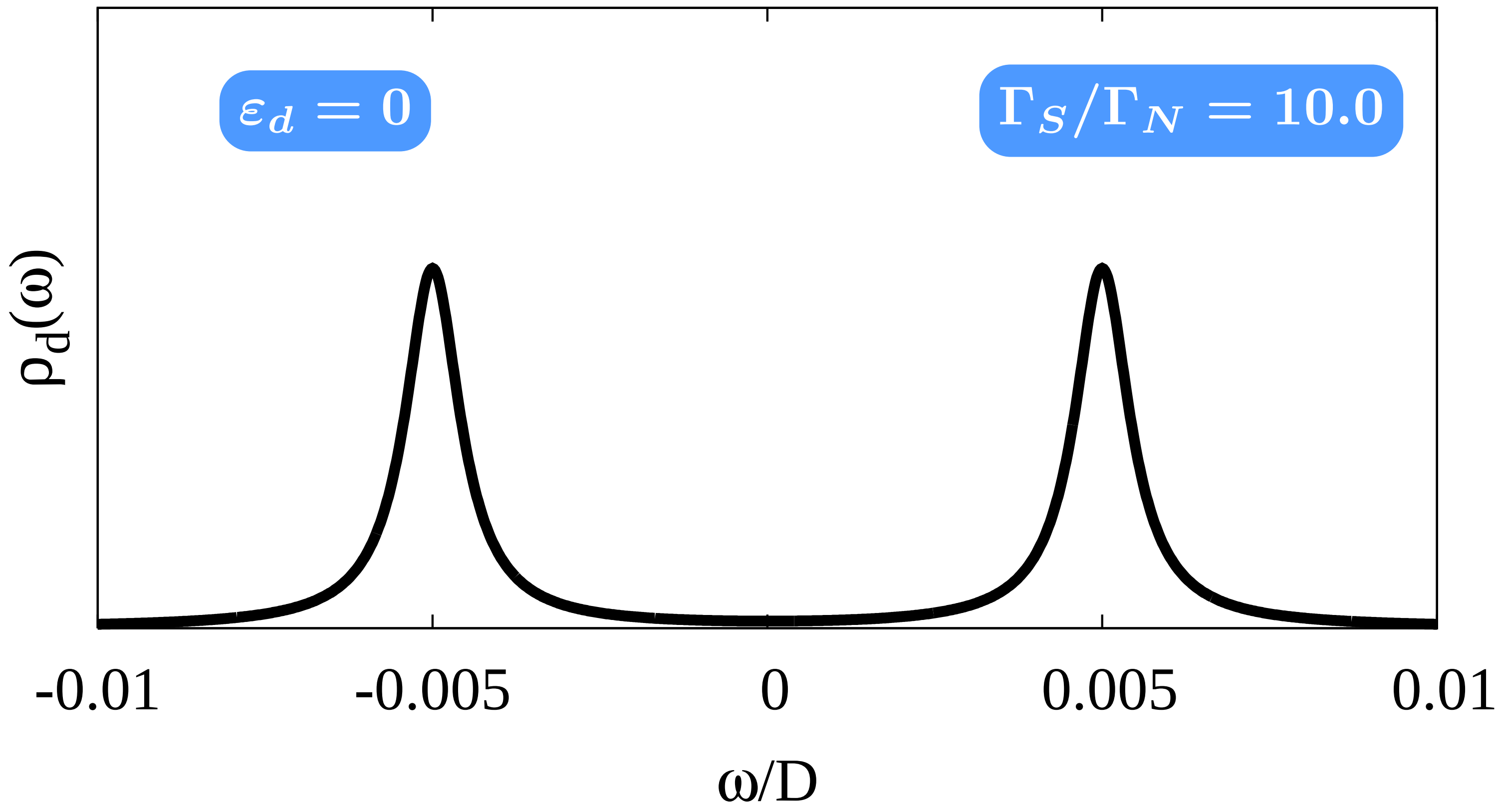

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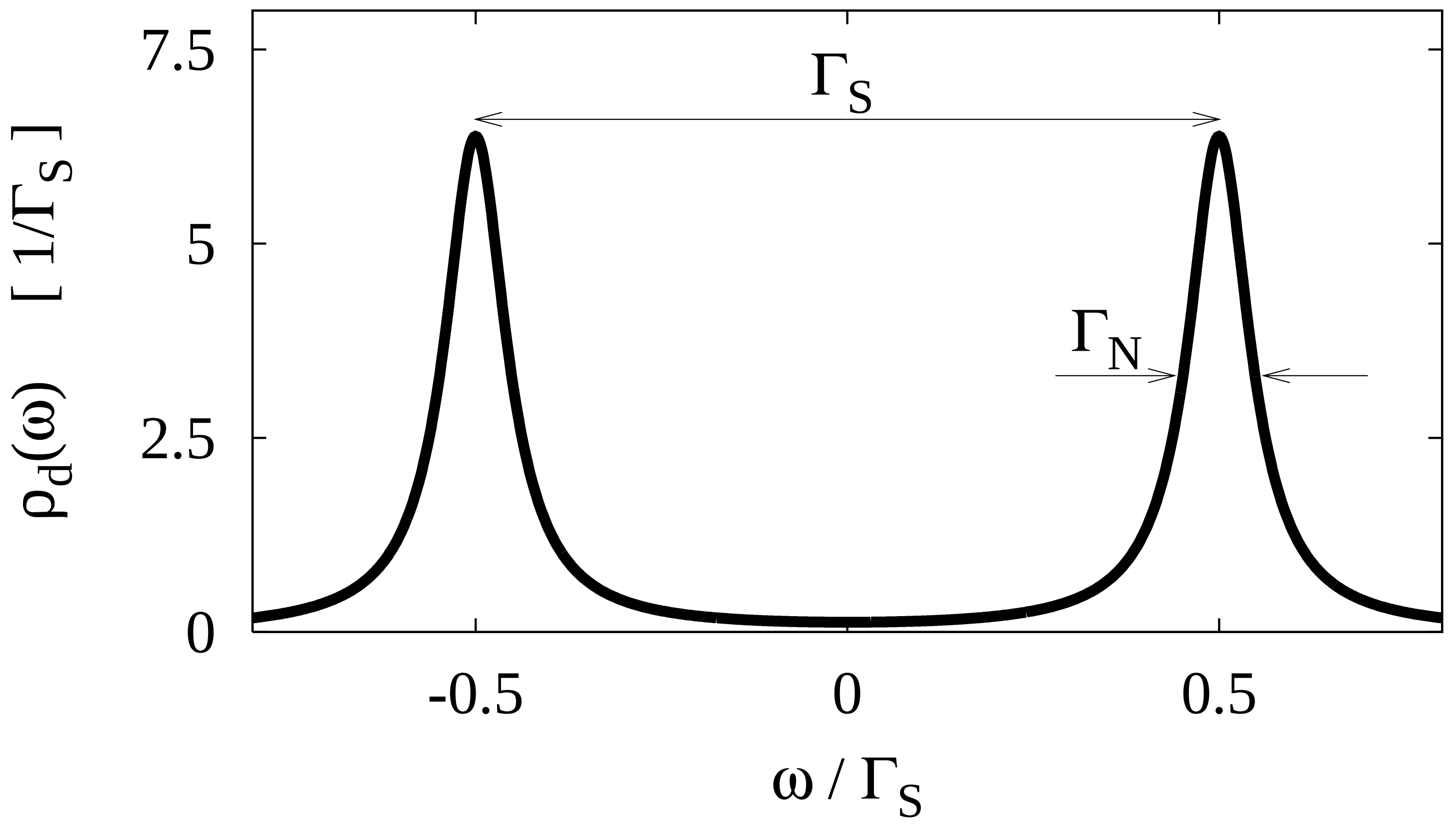

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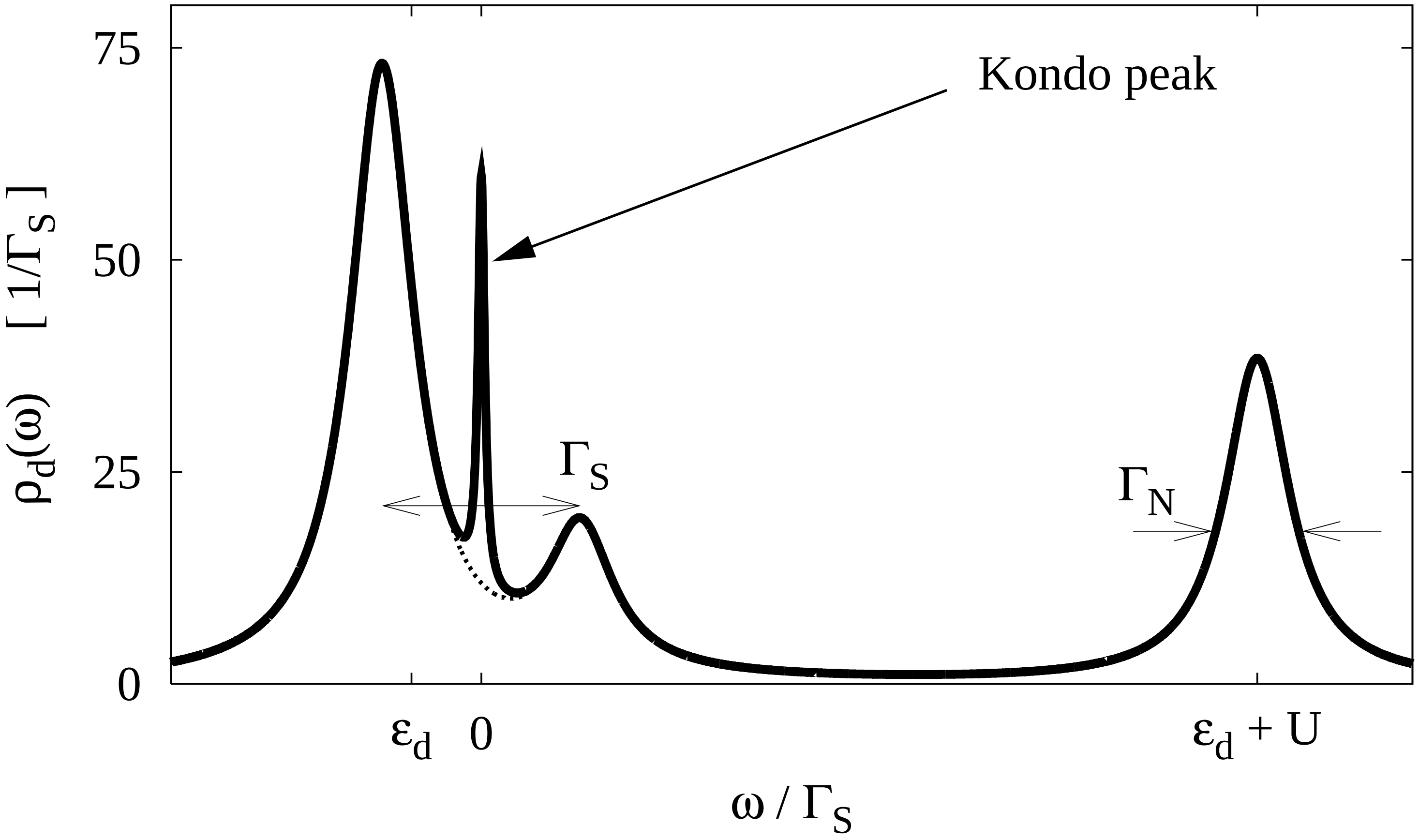

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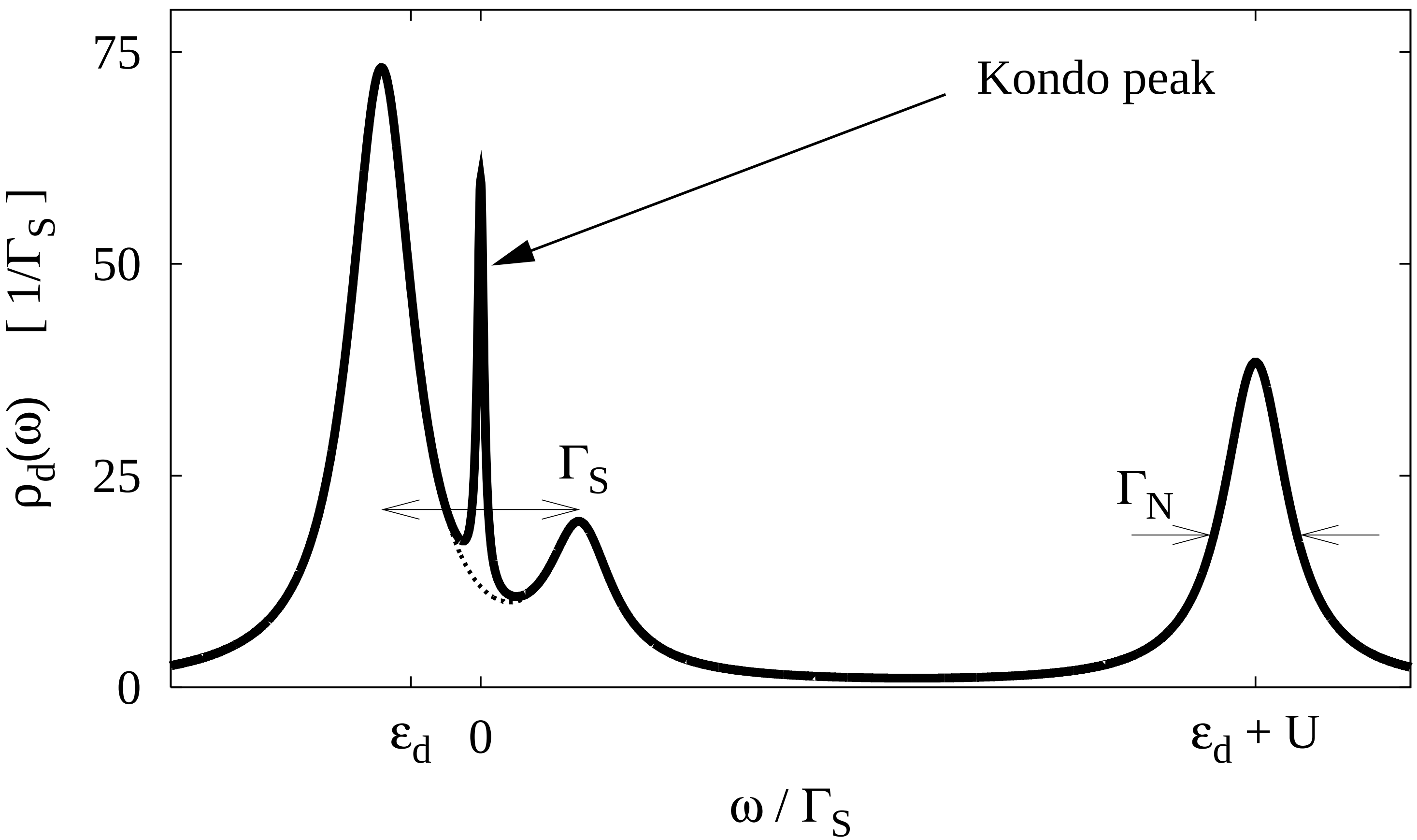

/ interplay between the Kondo effect and superconductivity /

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Are there any particular features ?

Formal aspects

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$\Sigma^0_d(\omega)$ **the selfenergy for** $U = 0$

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with

$\Sigma_d^U(\omega)$ correction due to $U \neq 0$.

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with the transmittance

$$T_1(\omega) = \Gamma_N\Gamma_S\left(\left|G^r_{11}(\omega)\right|^2+\left|G^r_{12}(\omega)\right|^2-\frac{2\Delta}{|\omega|}\mathrm{Re}G^r_{11}(\omega)G^r_{12}(\omega)\right)$$

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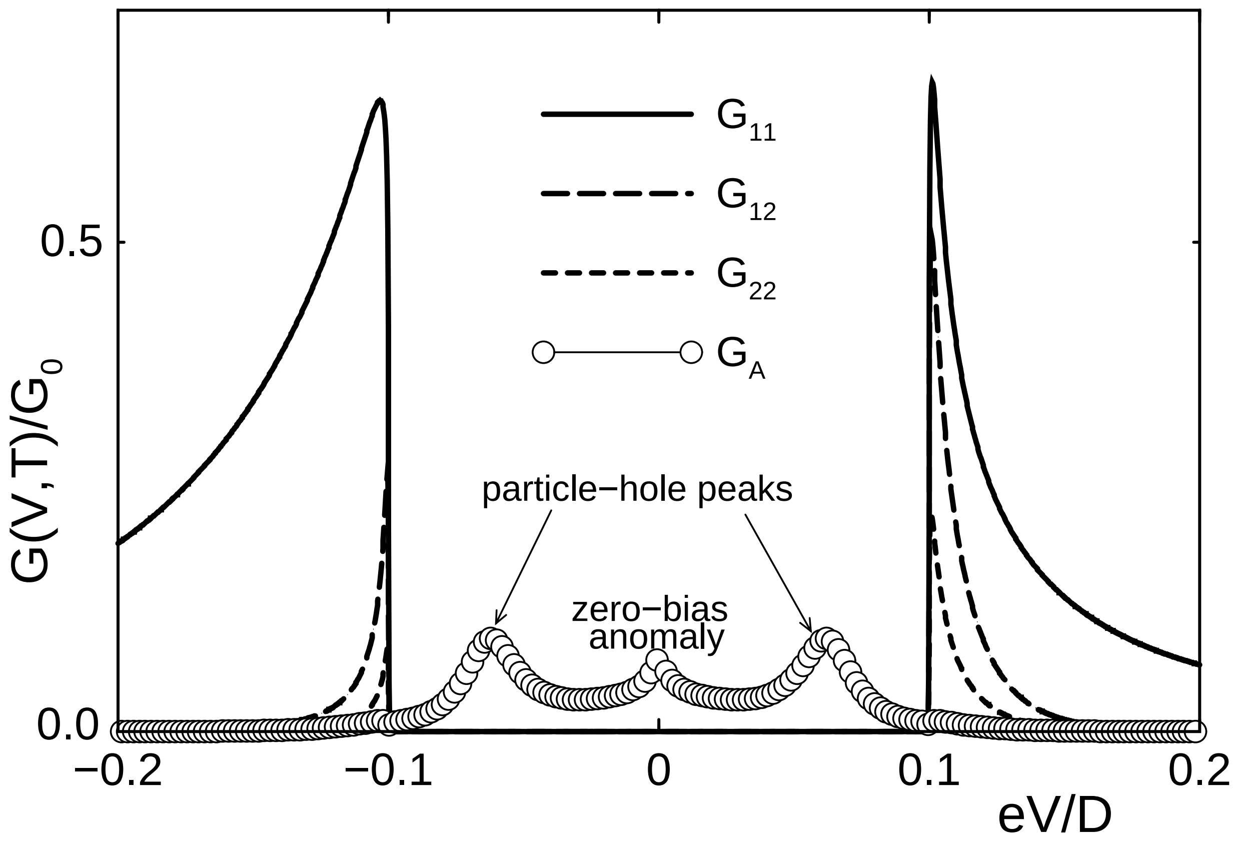

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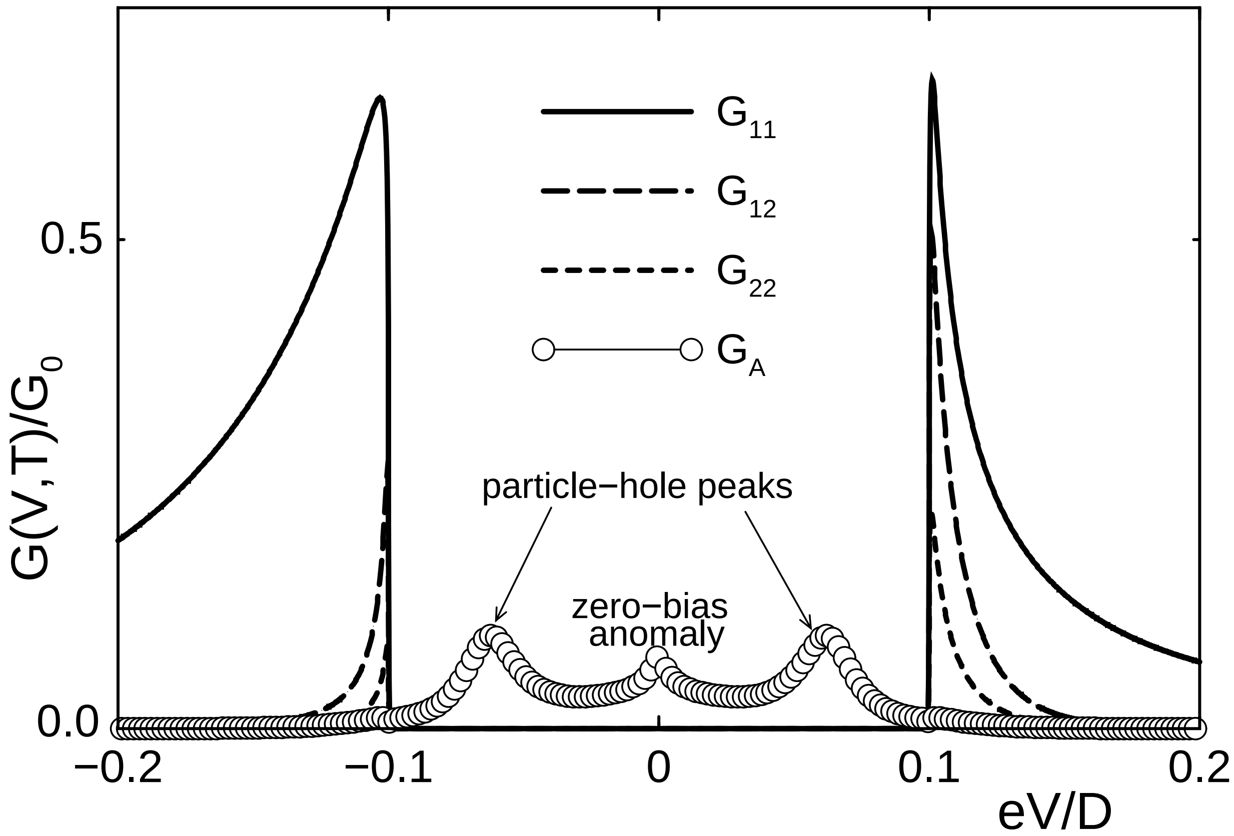

T. Domański, A. Donabidowicz, K.I. Wysokiński, PRB **76**, 104514 (2007).

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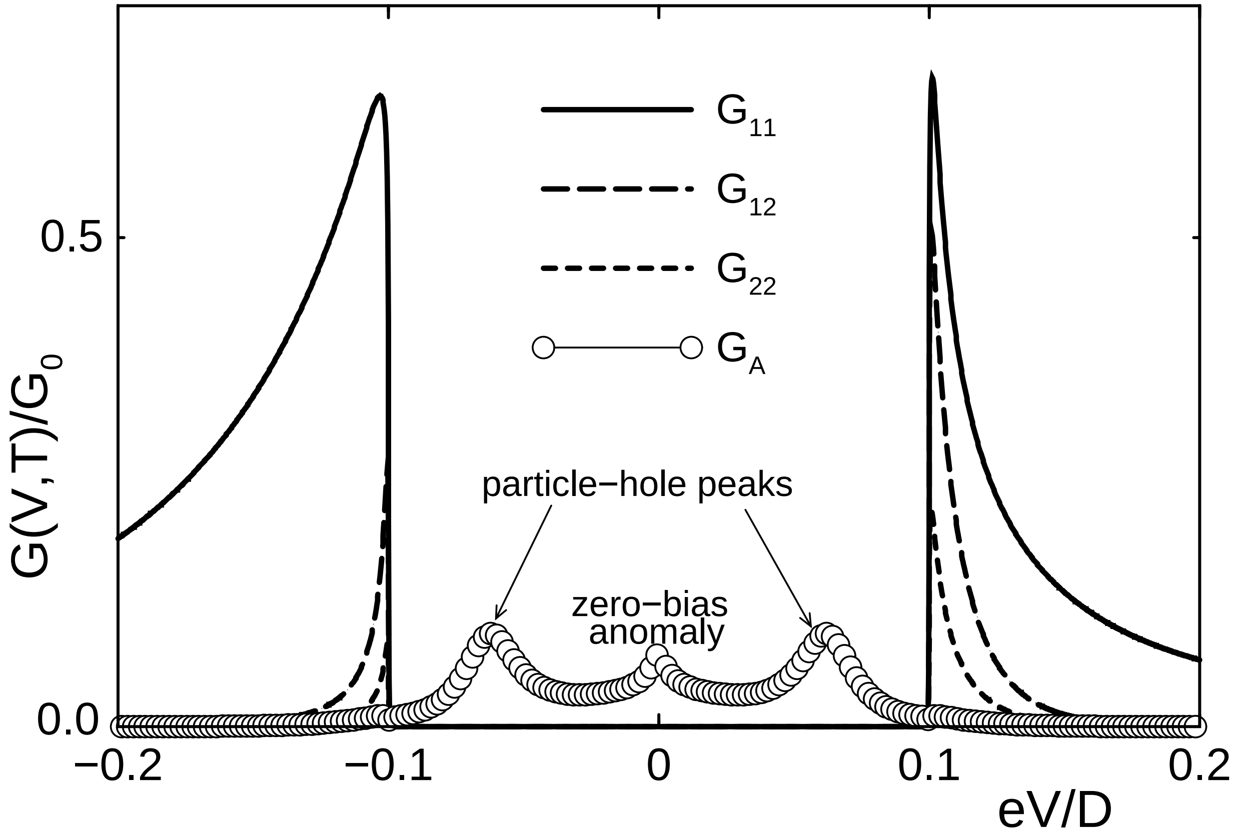

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We shall now focus on the subgap Andreev conductance.

Correlated QD – effect of the asymmetry Γ_S/Γ_N

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Spectral function obtained below T_K for $U = 10\Gamma_N$

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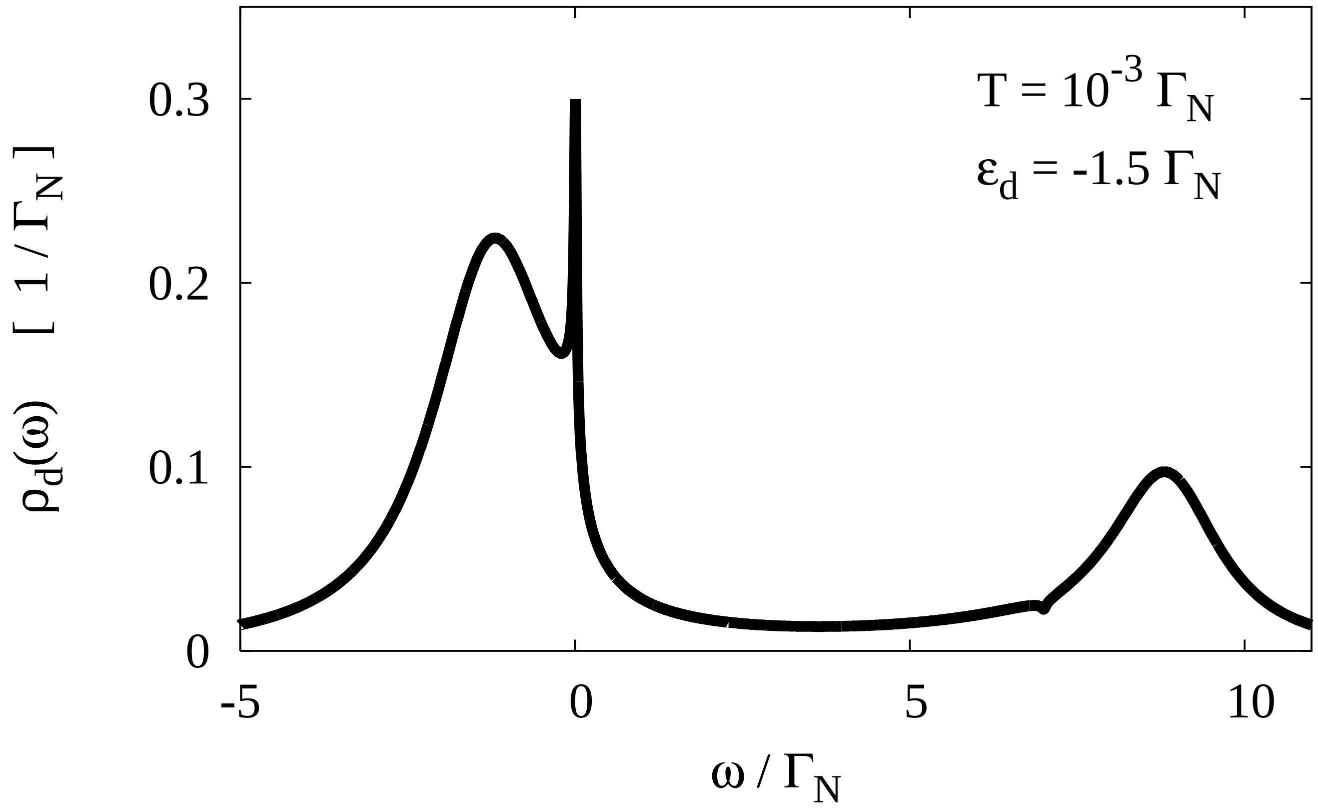
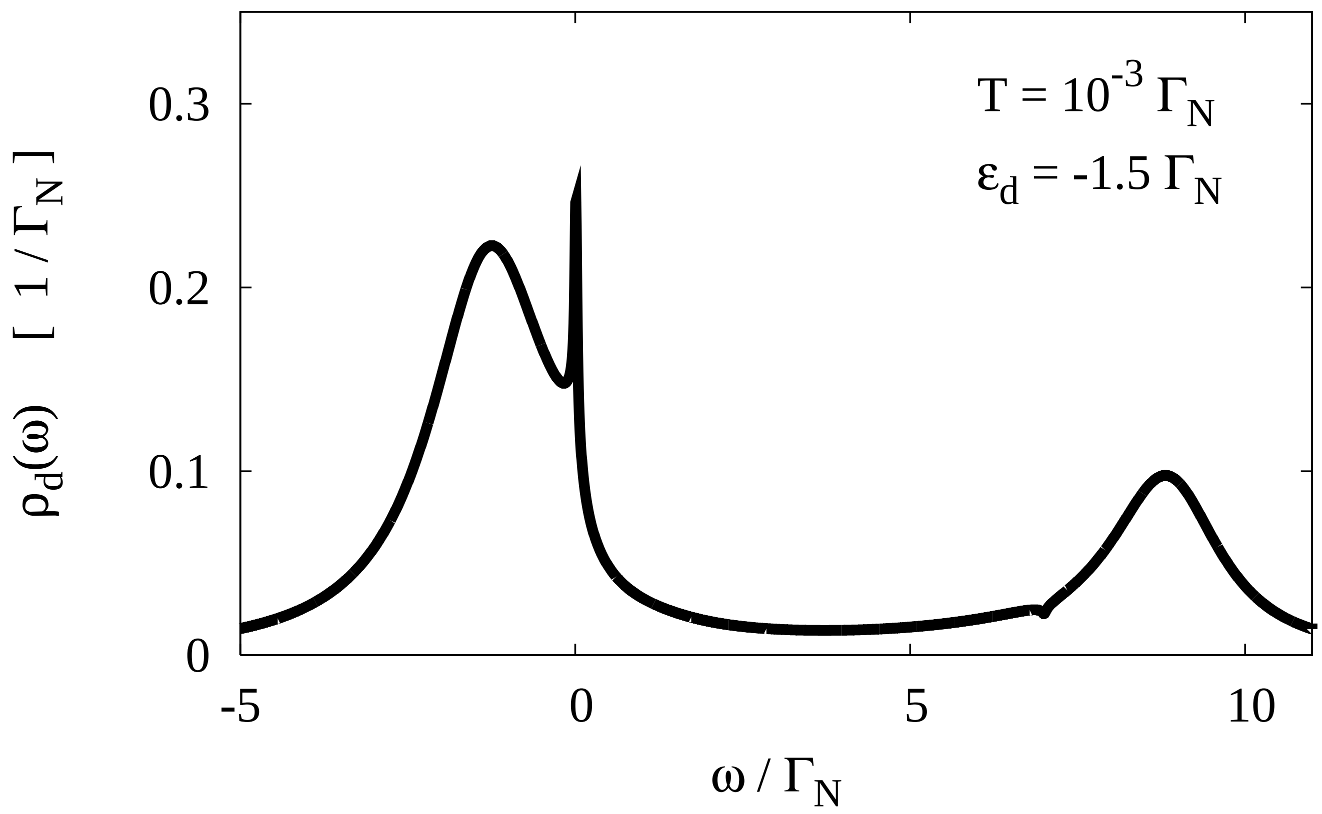
Spectral function obtained below T_K for $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 0$$

Correlated QD – **effect of the asymmetry Γ_S/Γ_N**

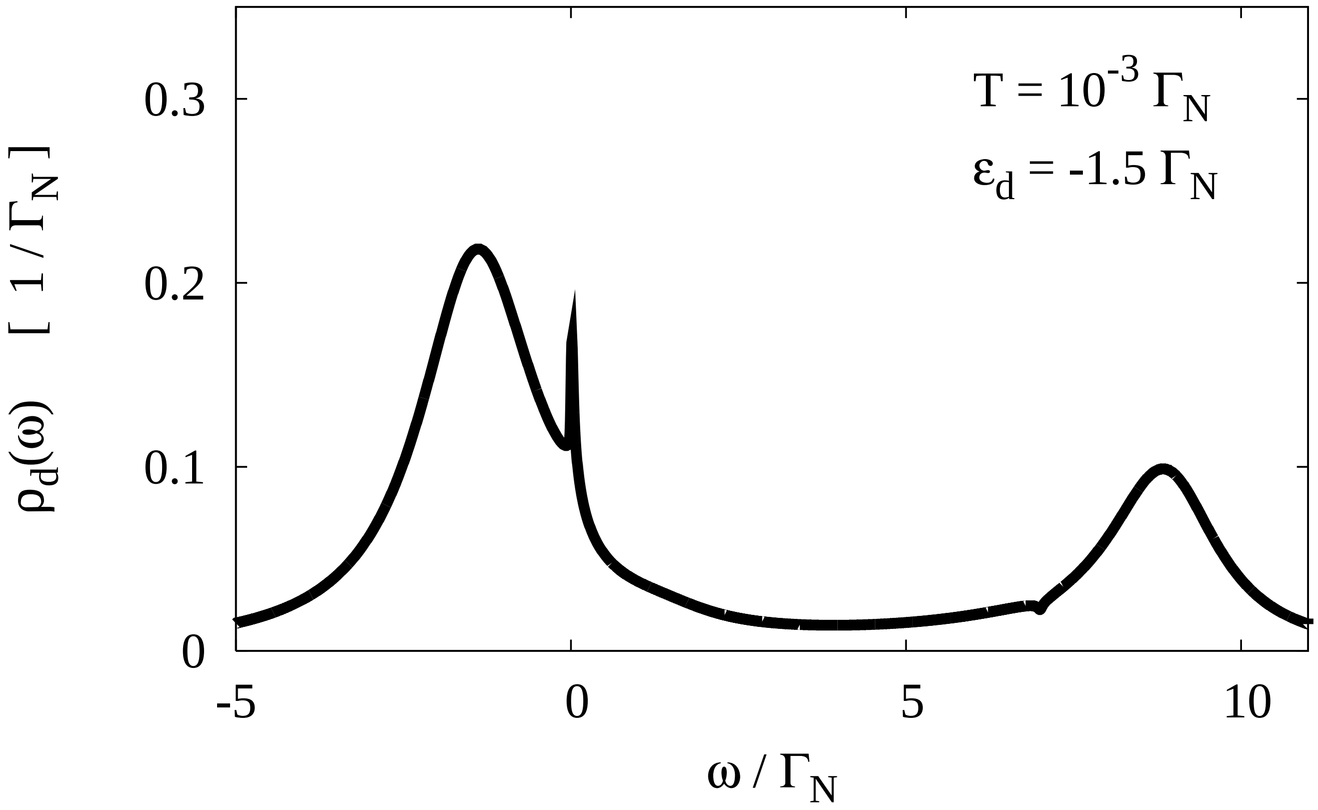
Spectral function obtained below T_K for $U=10\Gamma_N$



$\Gamma_S/\Gamma_N = 1$

Correlated QD – **effect of the asymmetry Γ_S/Γ_N**

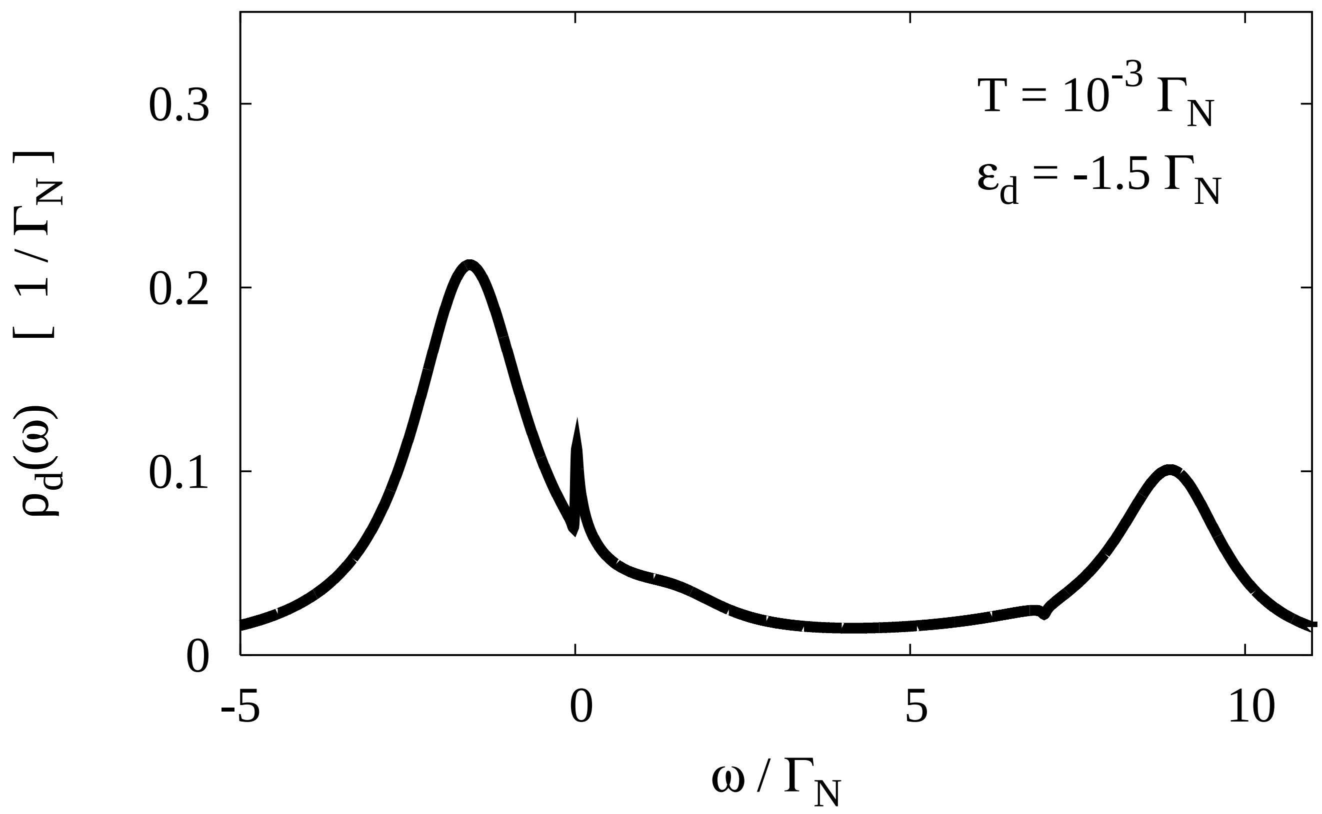
Spectral function obtained below T_K for $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 2$$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

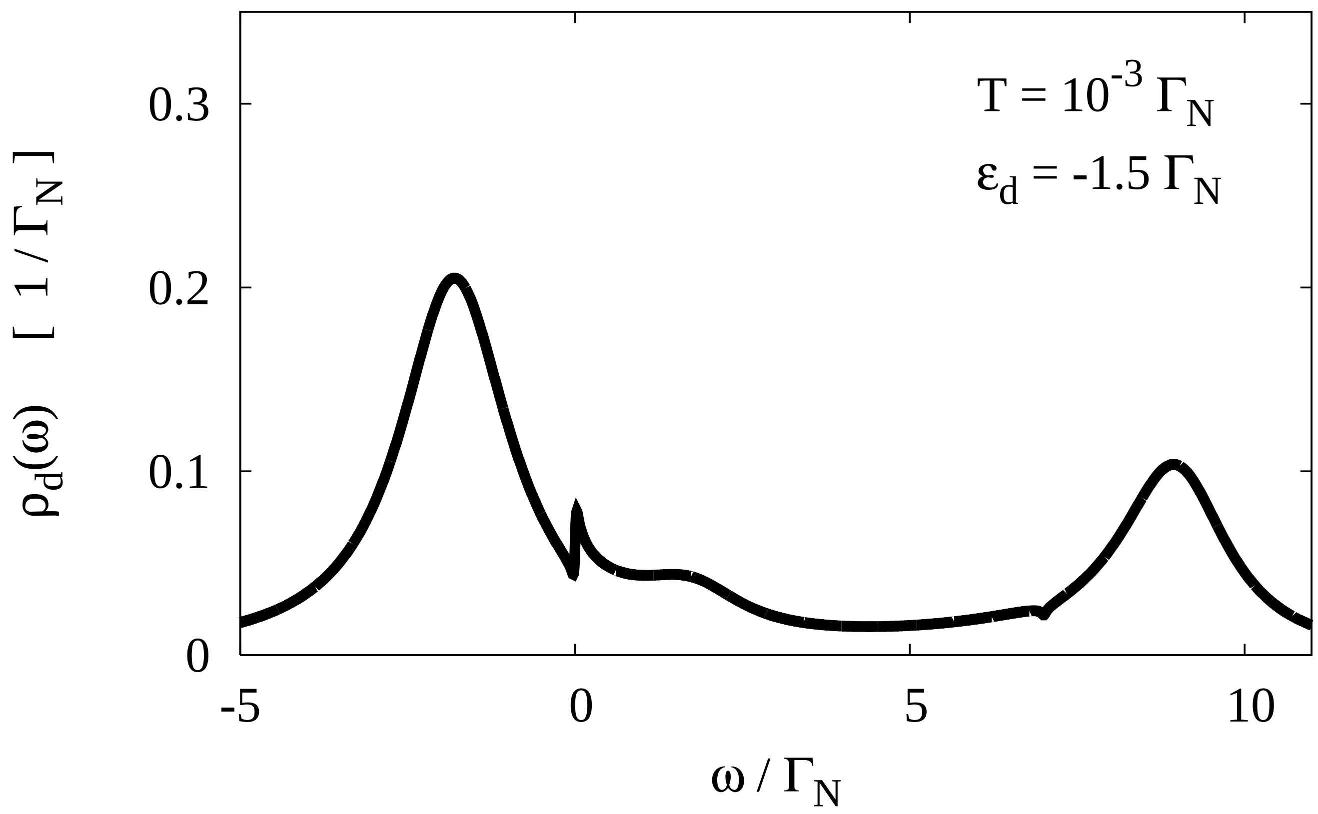
Spectral function obtained below T_K for $U = 10\Gamma_N$



$\Gamma_S/\Gamma_N = 3$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

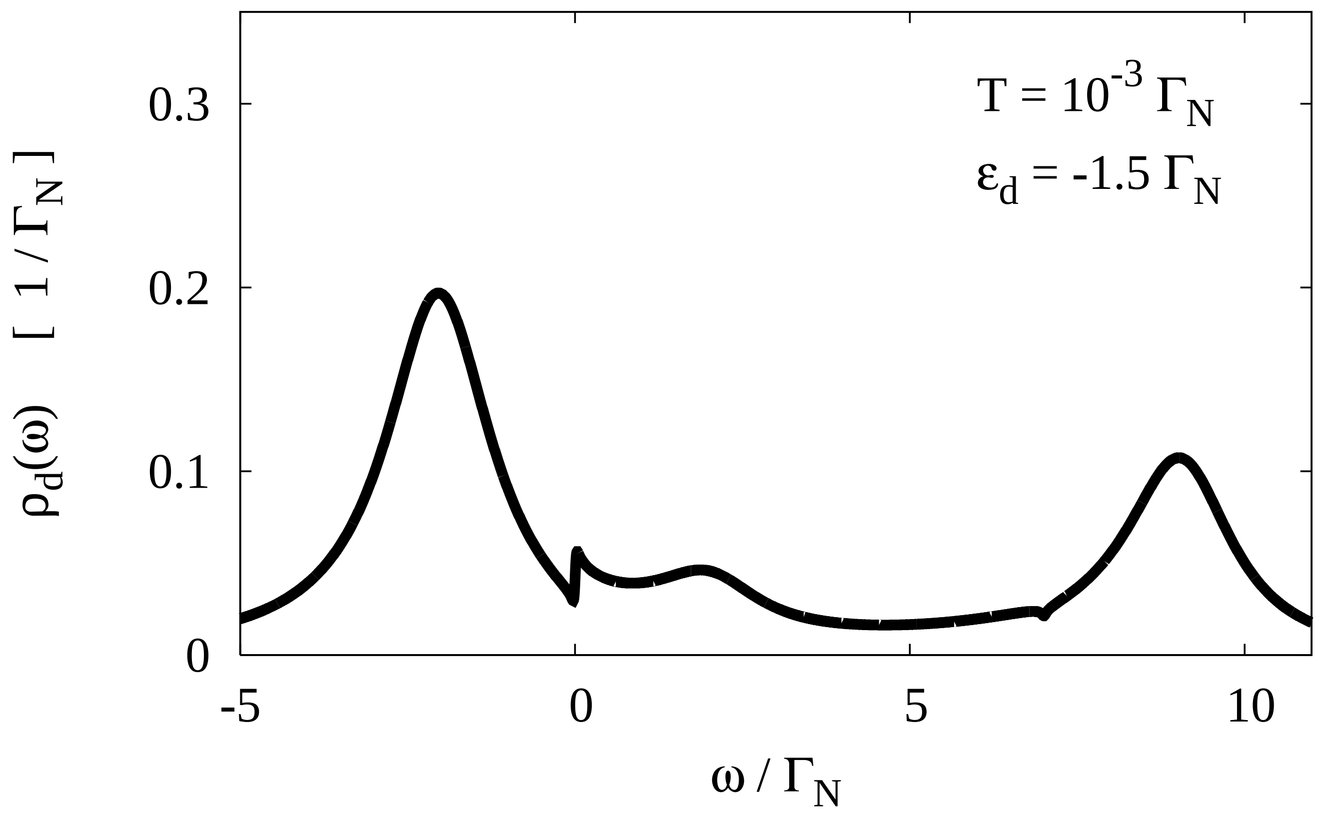
Spectral function obtained below T_K for $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 4$$

Correlated QD – **effect of the asymmetry Γ_S/Γ_N**

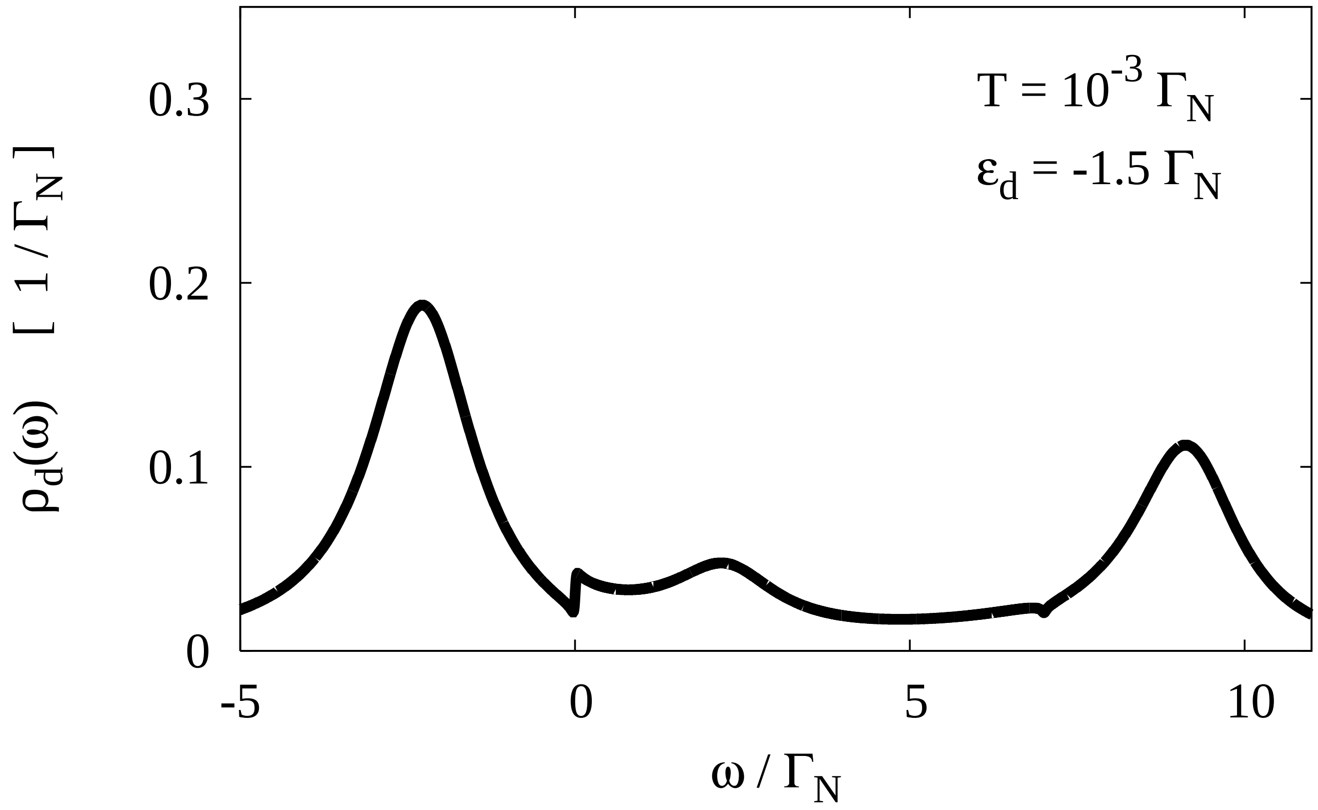
Spectral function obtained below T_K for $U = 10\Gamma_N$



$\Gamma_S/\Gamma_N = 5$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

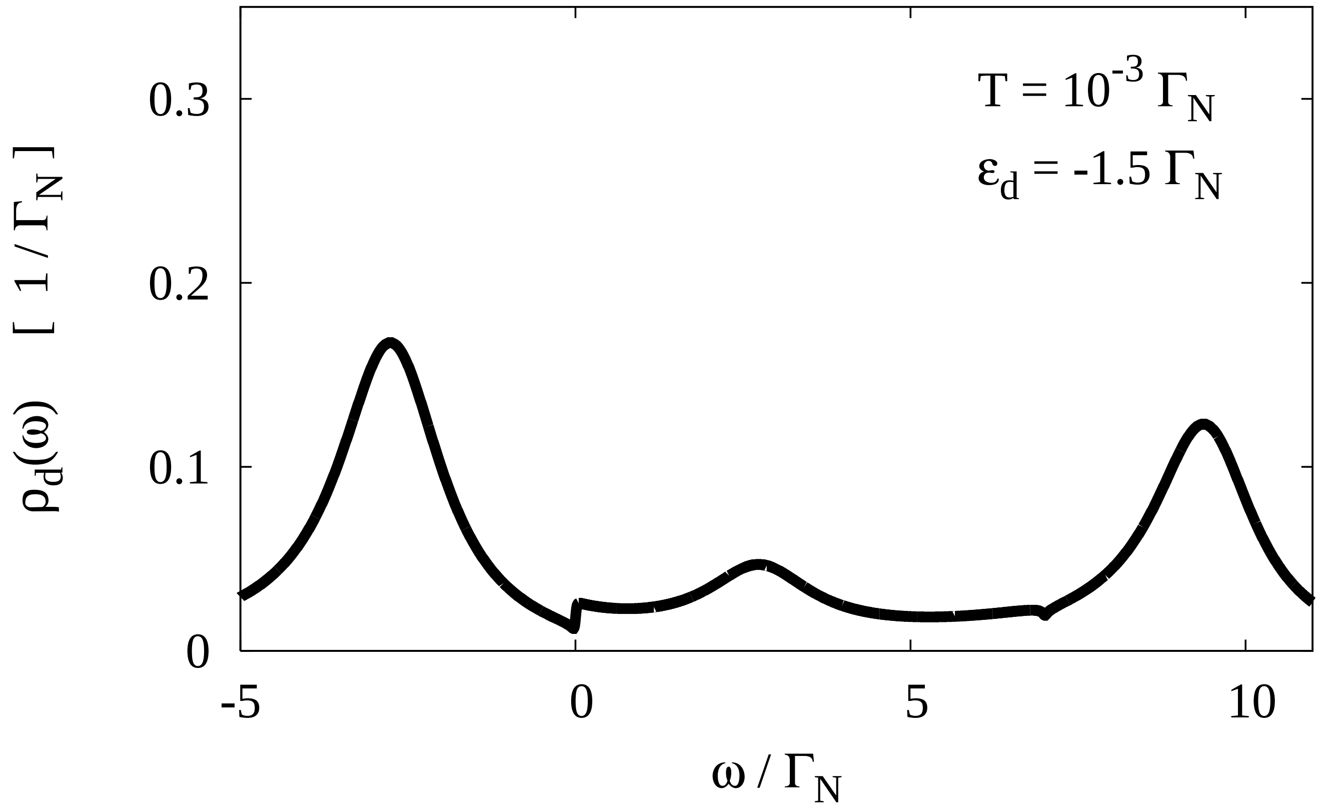
Spectral function obtained below T_K for $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 6$$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

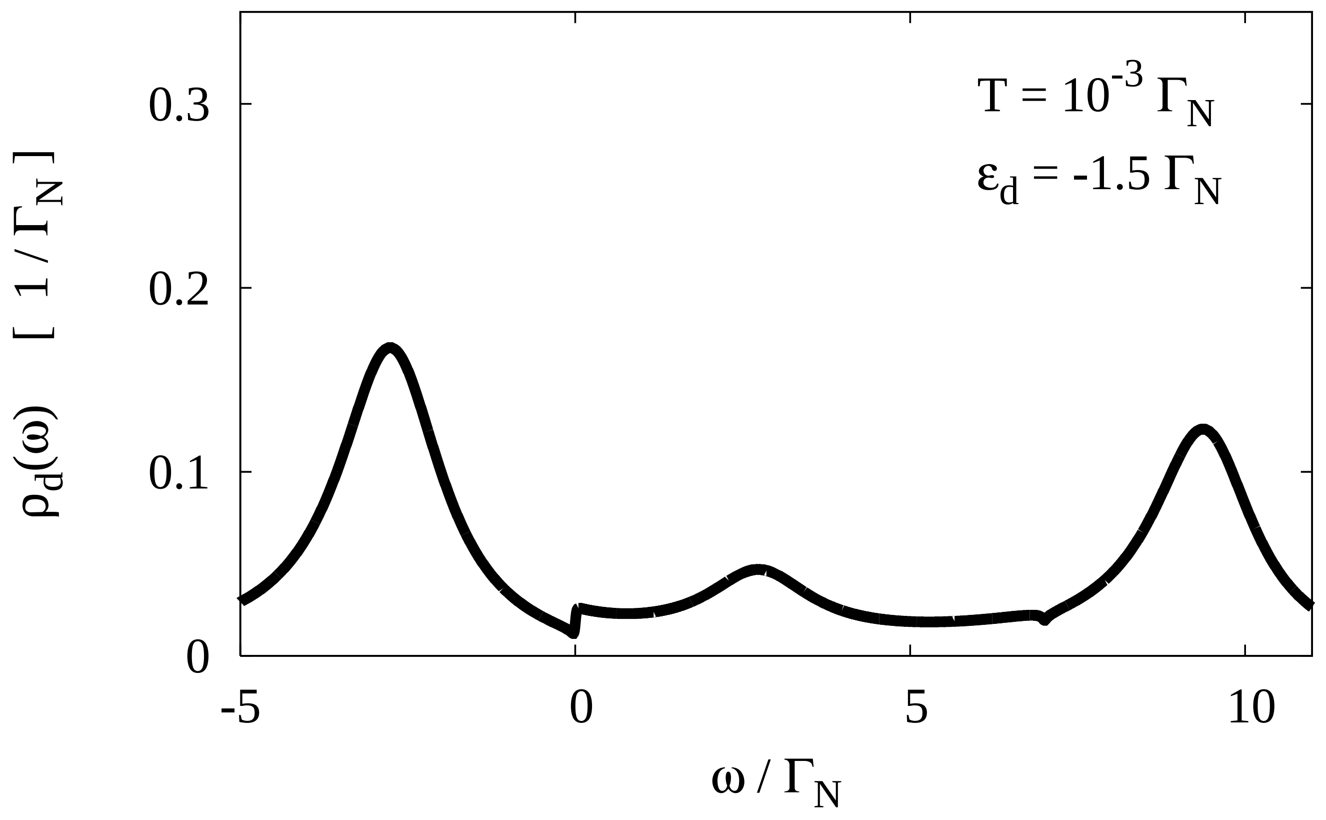
Spectral function obtained below T_K for $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 8$$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

Spectral function obtained below T_K for $U=10\Gamma_N$



Superconductivity suppresses the Kondo resonance

Correlated QD – effect of the asymmetry Γ_S/Γ_N

Andreev conductance $G_A(V)$ for:

$U = 10\Gamma_N$

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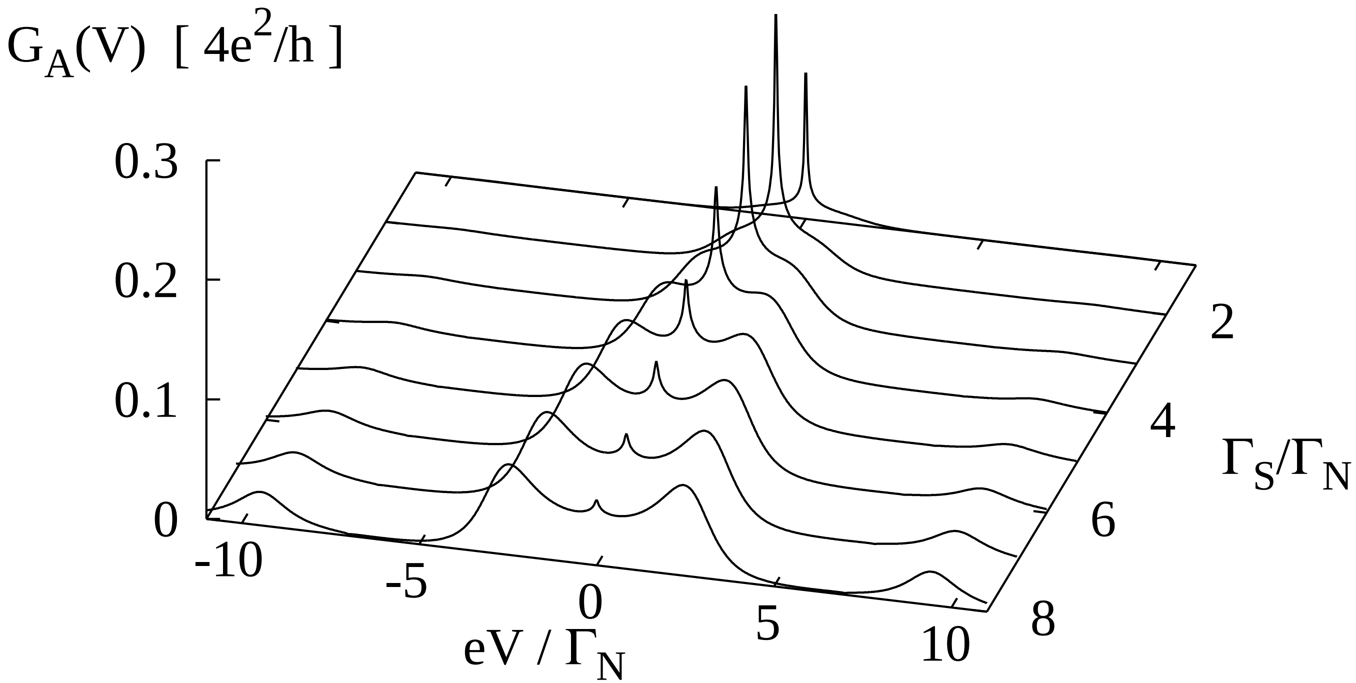

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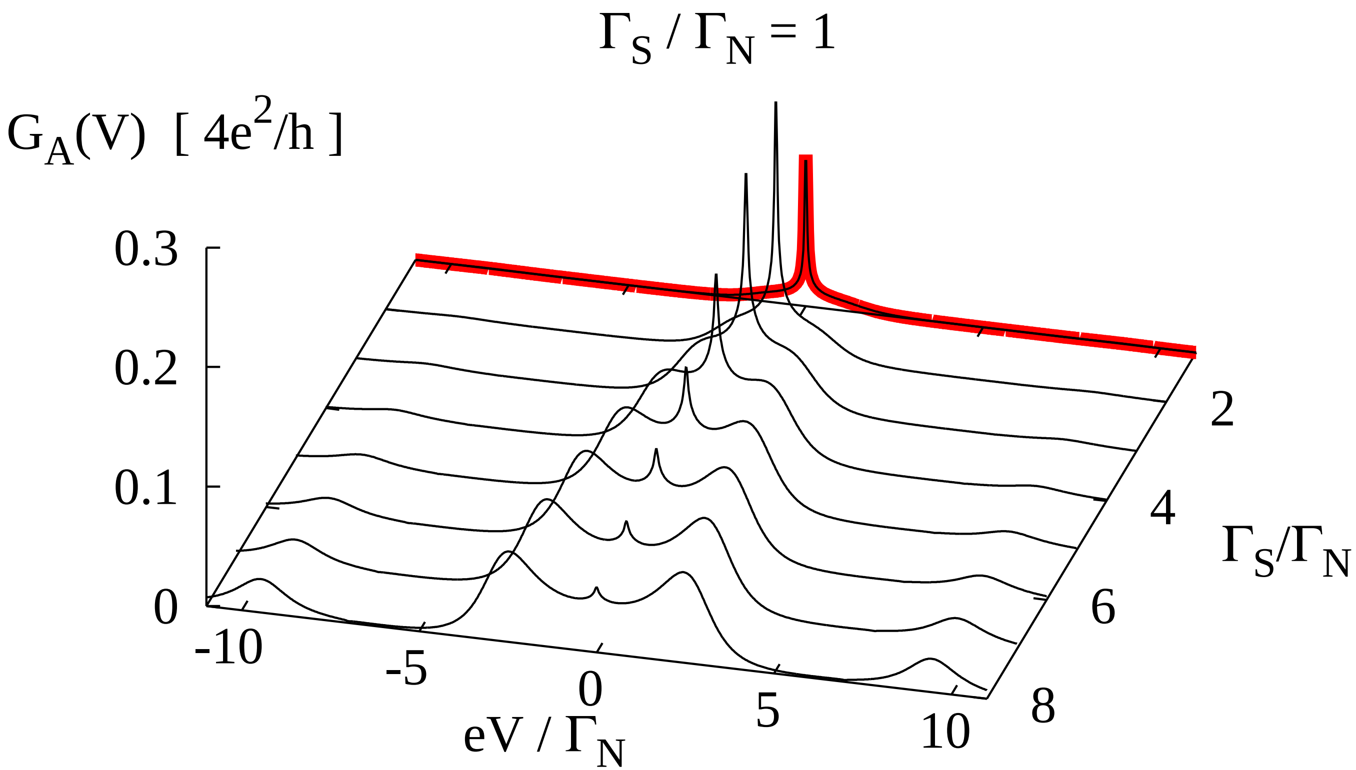

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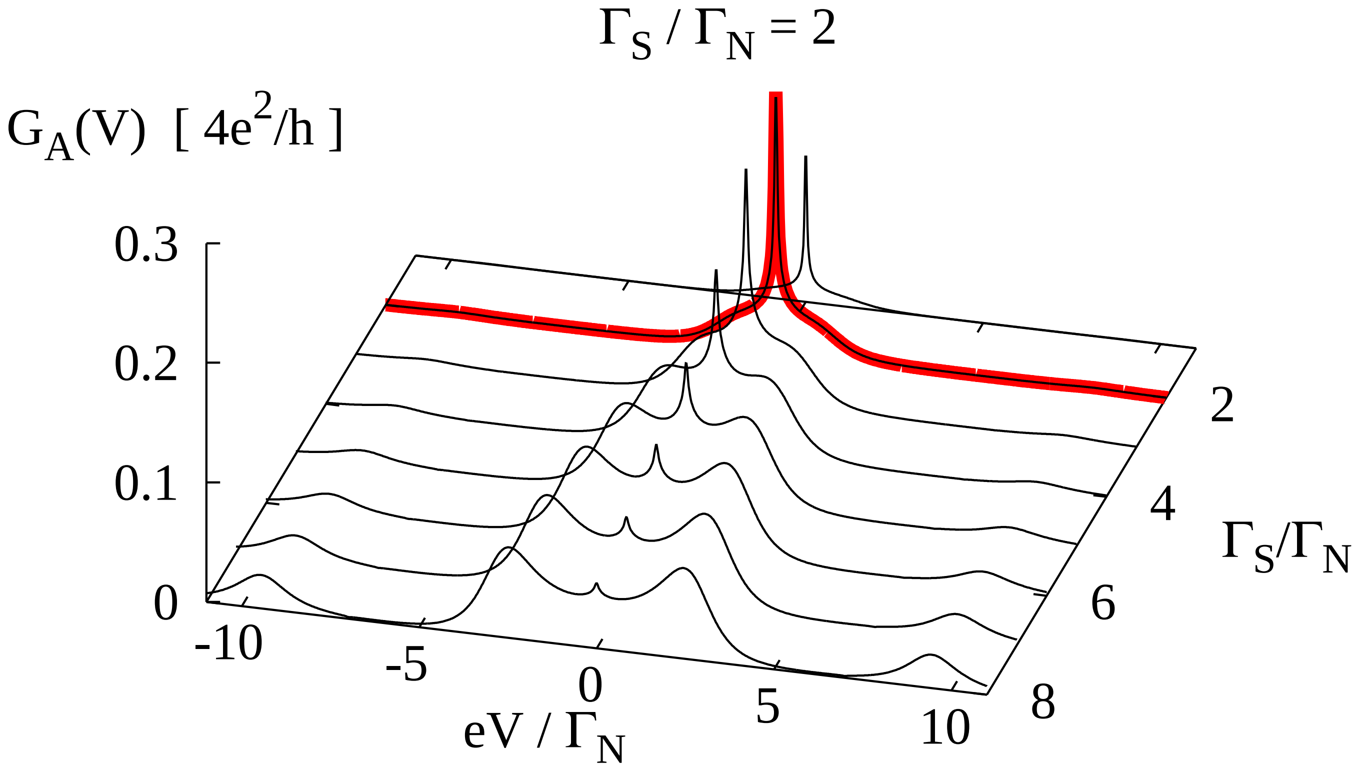

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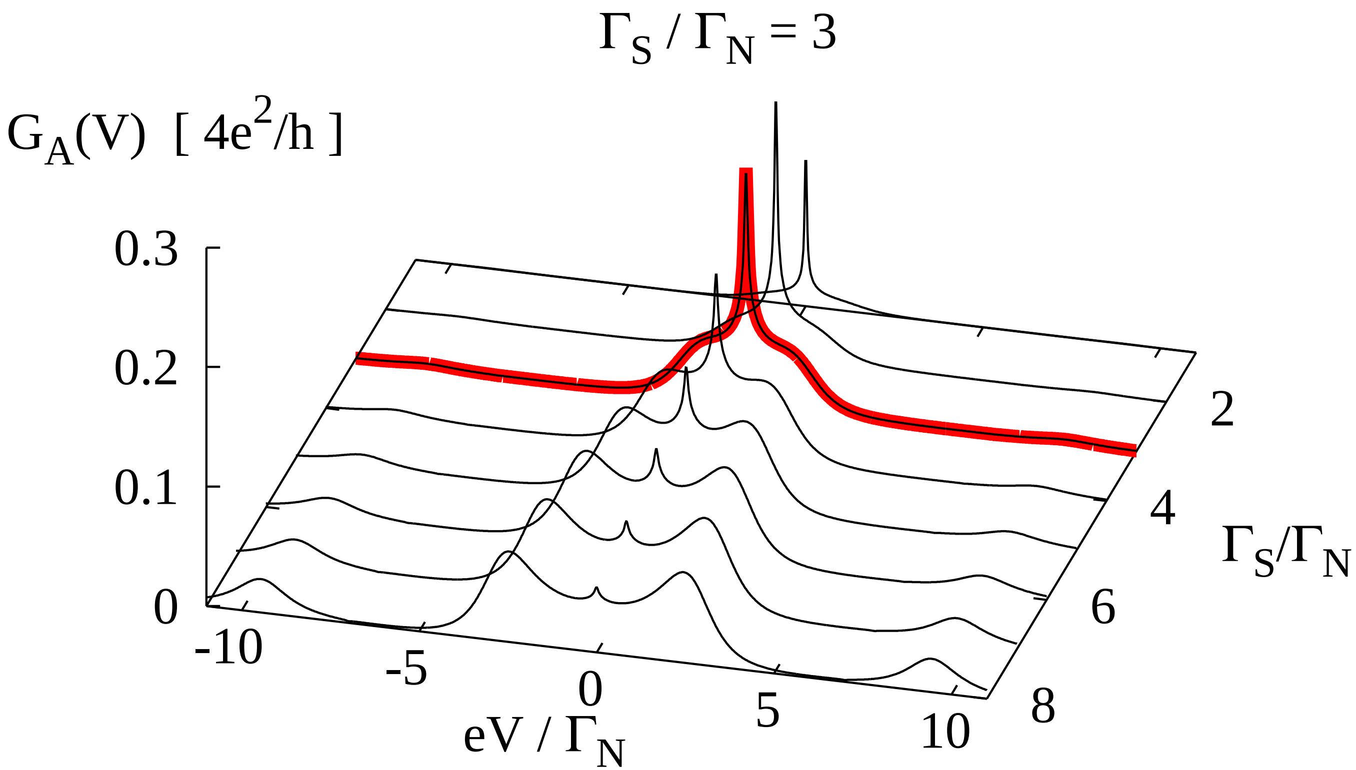

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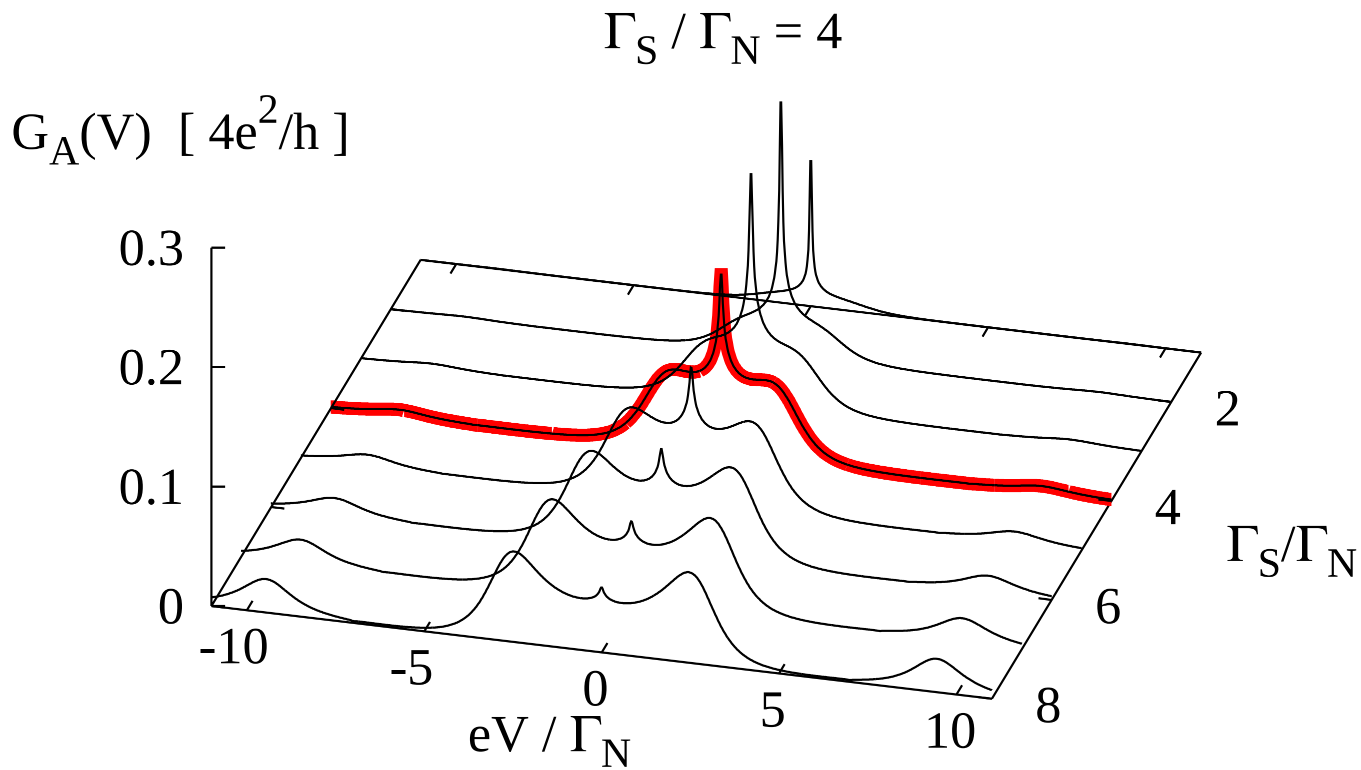

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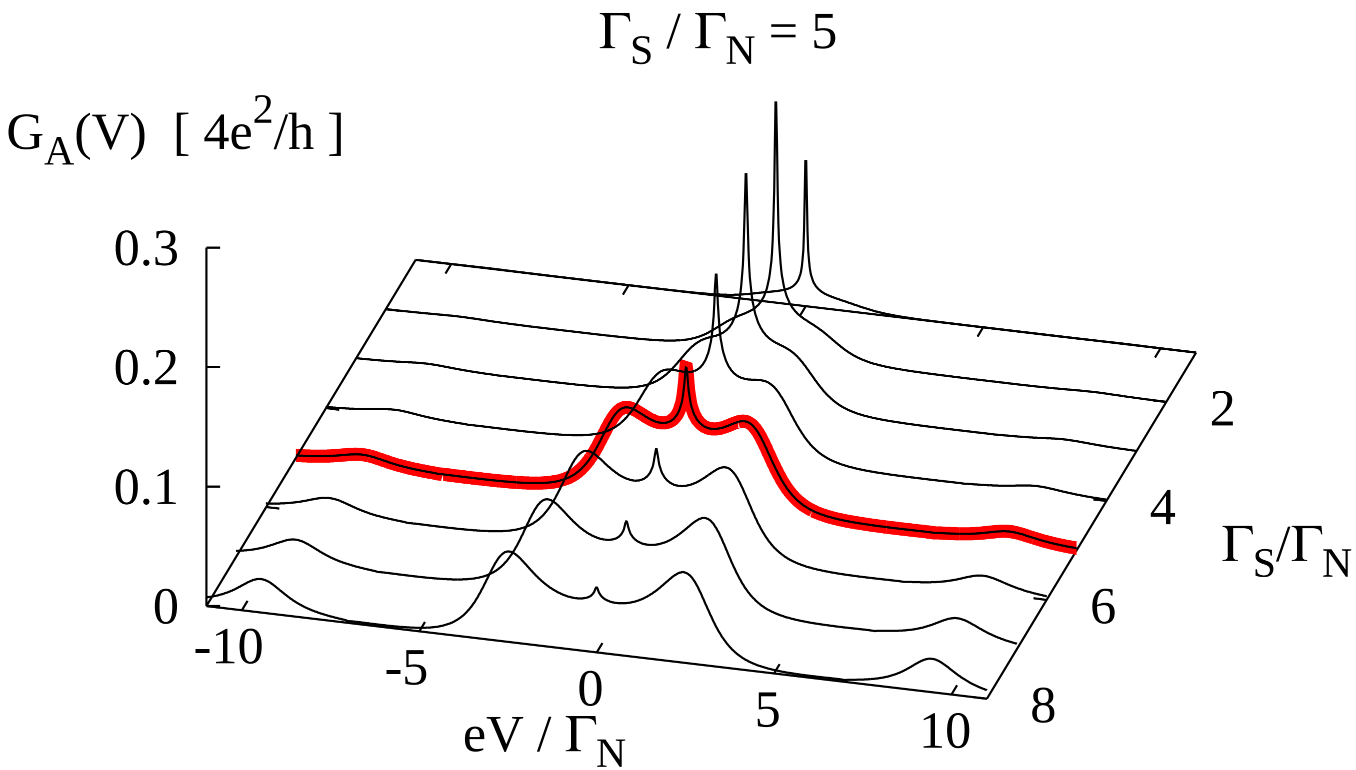

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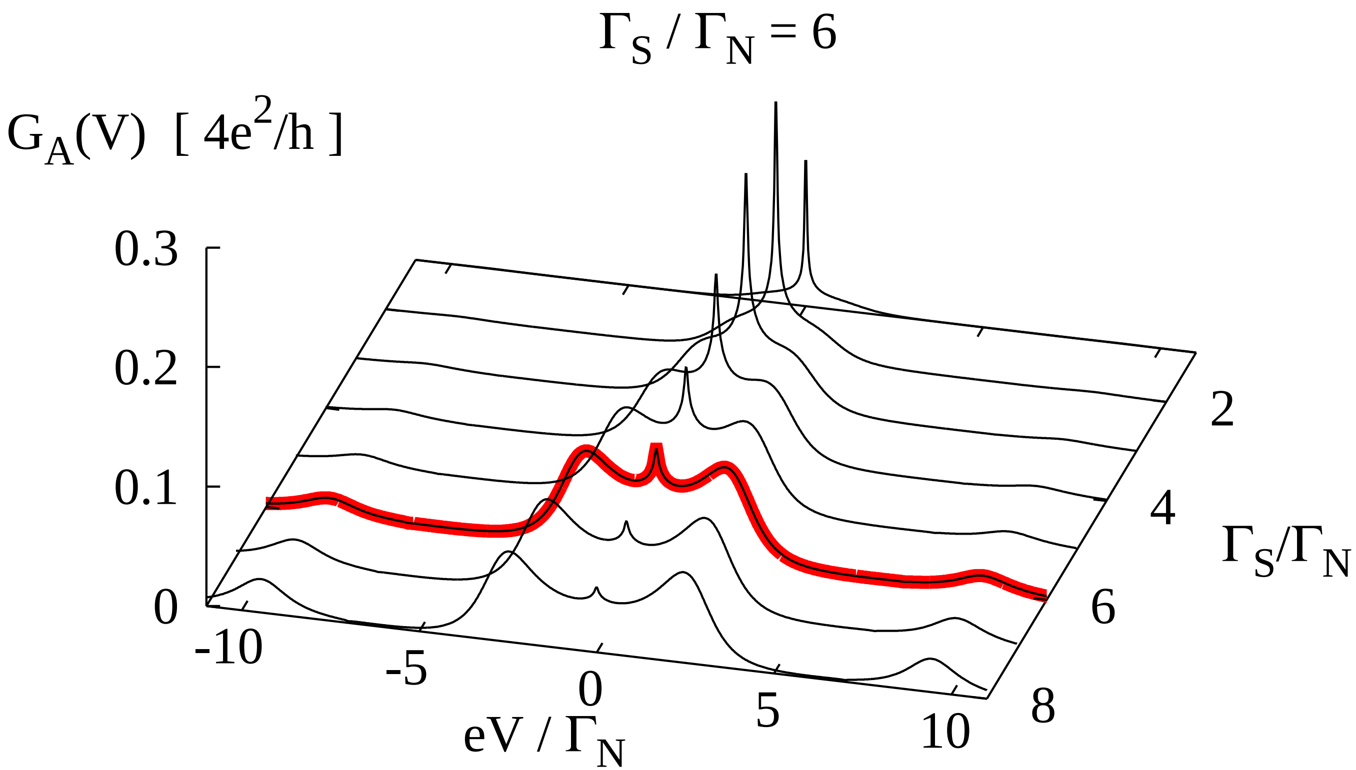

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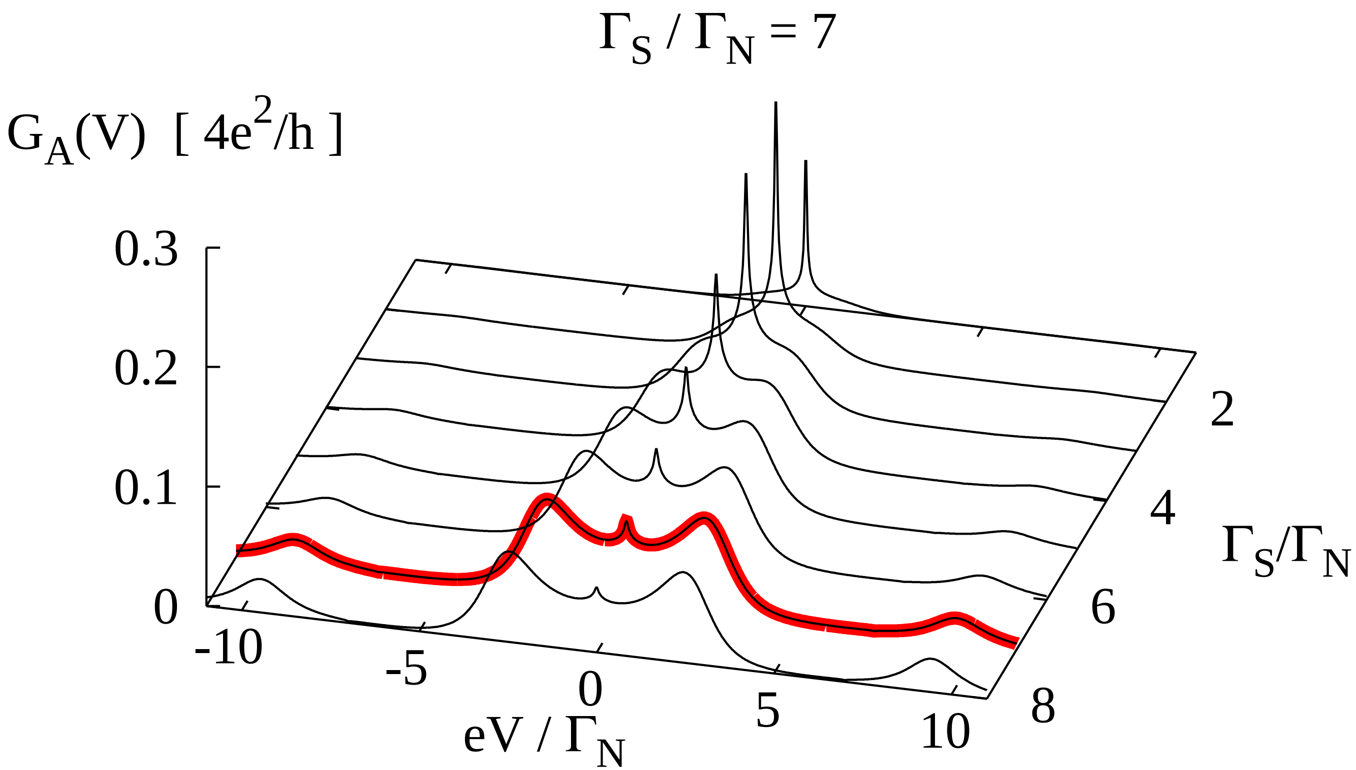

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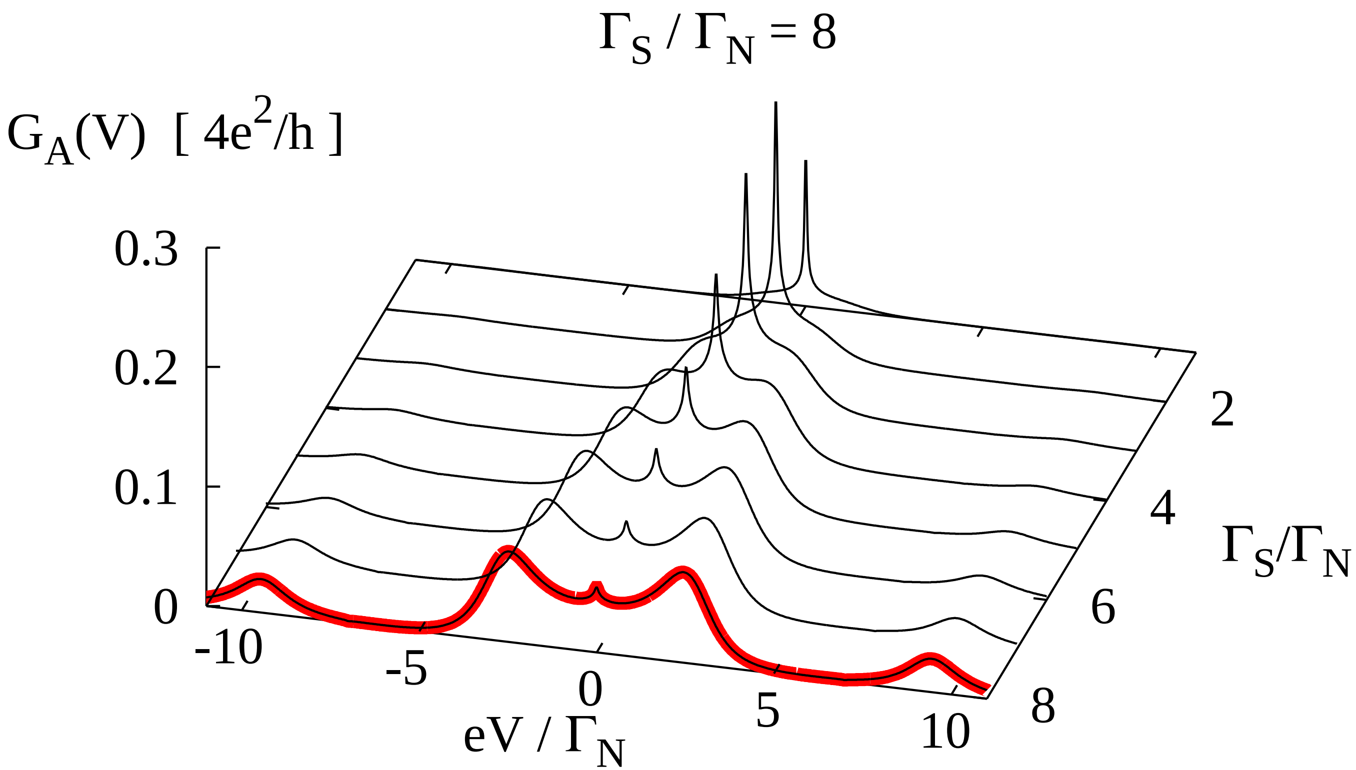

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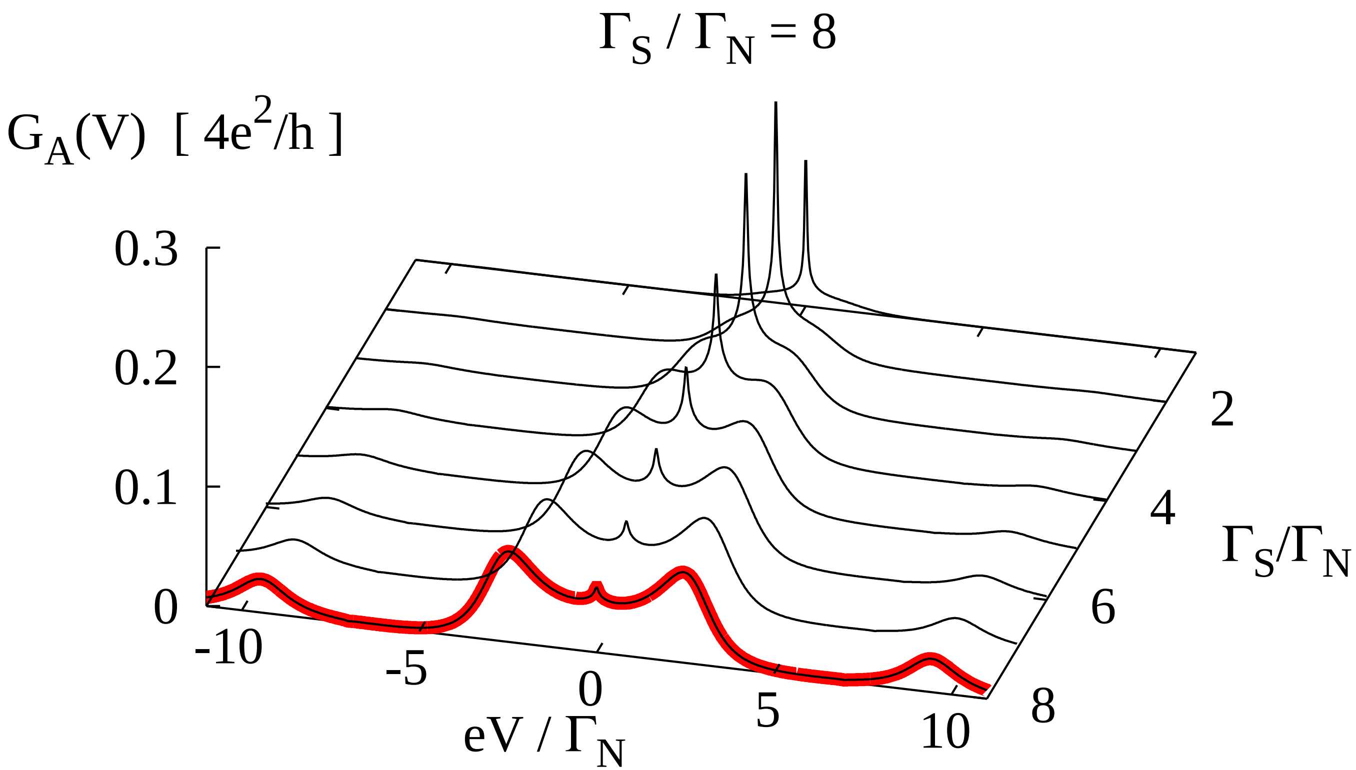

T. Domański and A. Donabidowicz, PRB **78**, 073105 (2008).

Kondo resonance slightly enhances the zero-bias Andreev conductance, especially for $\Gamma_S \sim \Gamma_N$!

Strongly correlated QD – influence of temperature

Temperature dependence of $G_A(V)$ for: $U = 10\Gamma_N$

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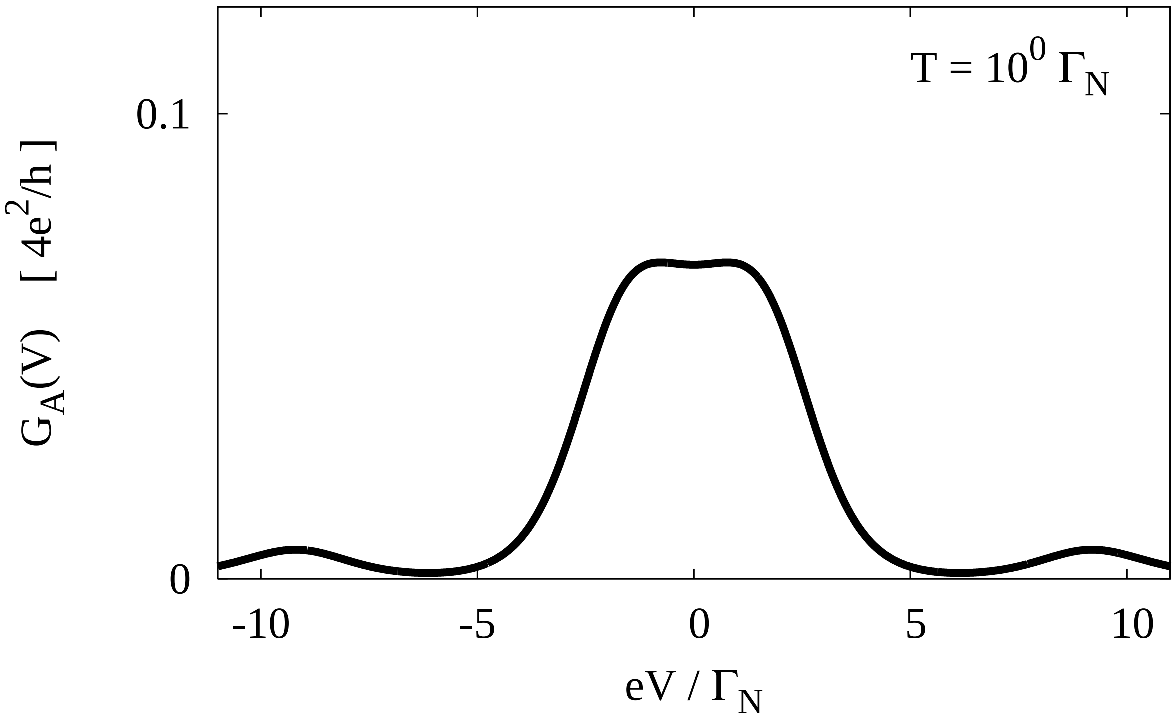

$$k_B T = \Gamma_N$$

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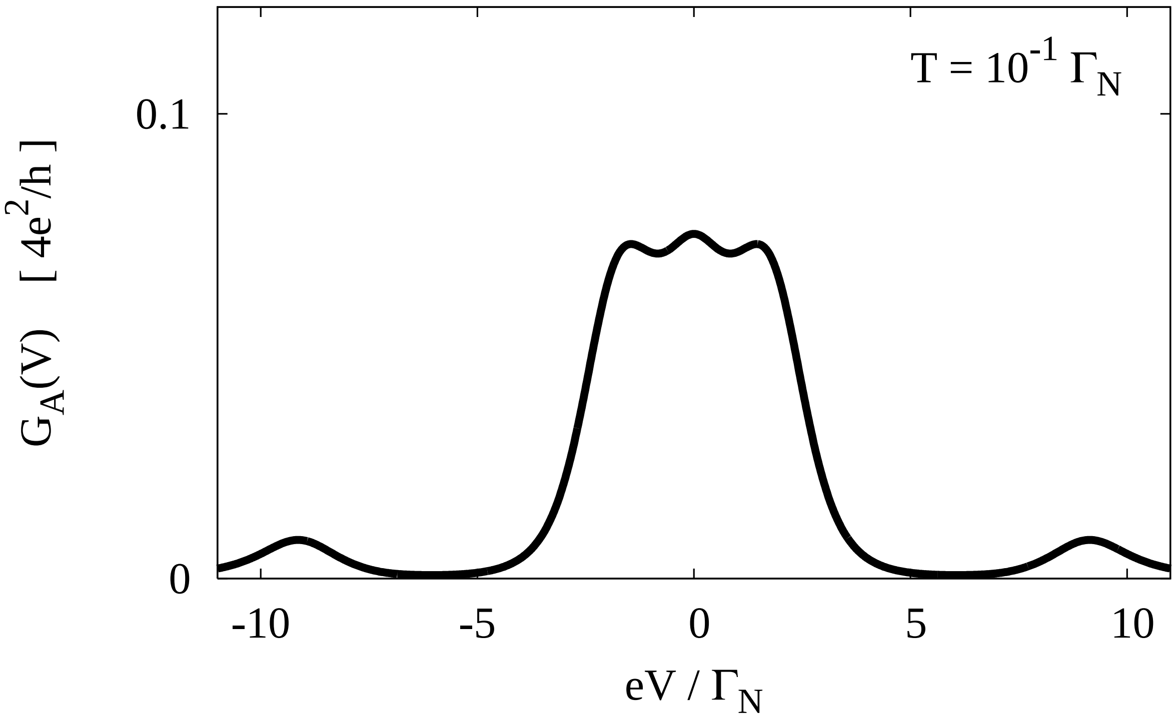

$$k_B T = \Gamma_N / 10$$

Strongly correlated QD – influence of temperature

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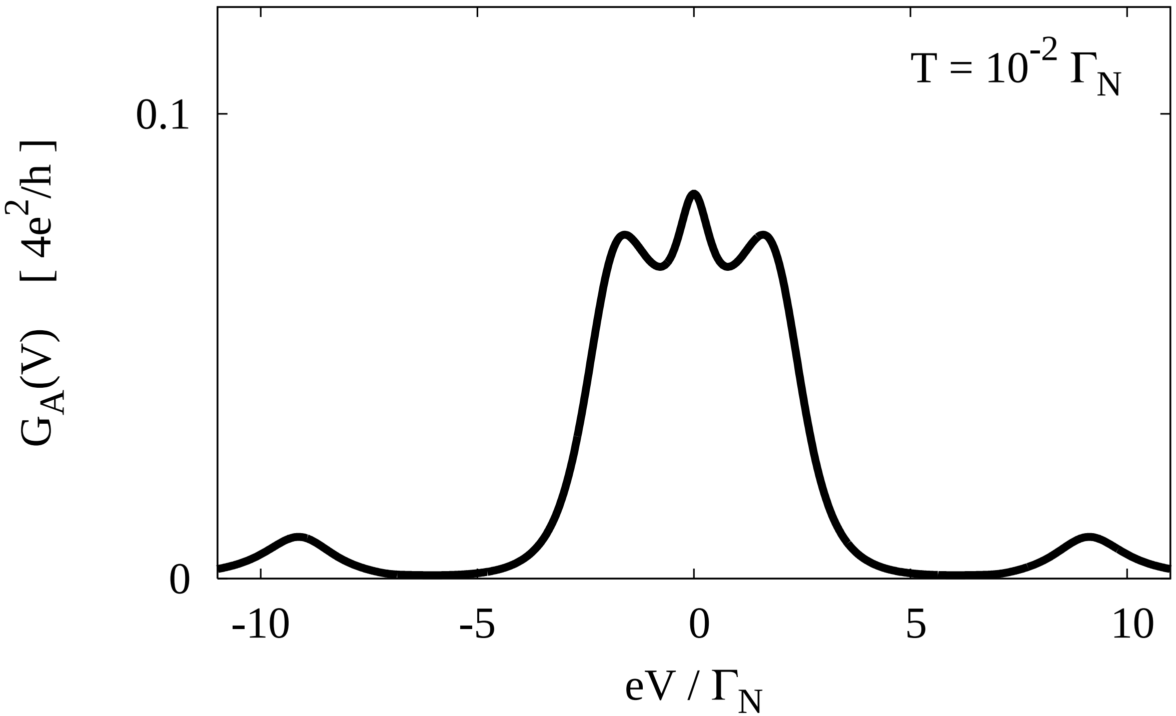

$$k_BT = \Gamma_N/100$$

Strongly correlated QD – influence of temperature

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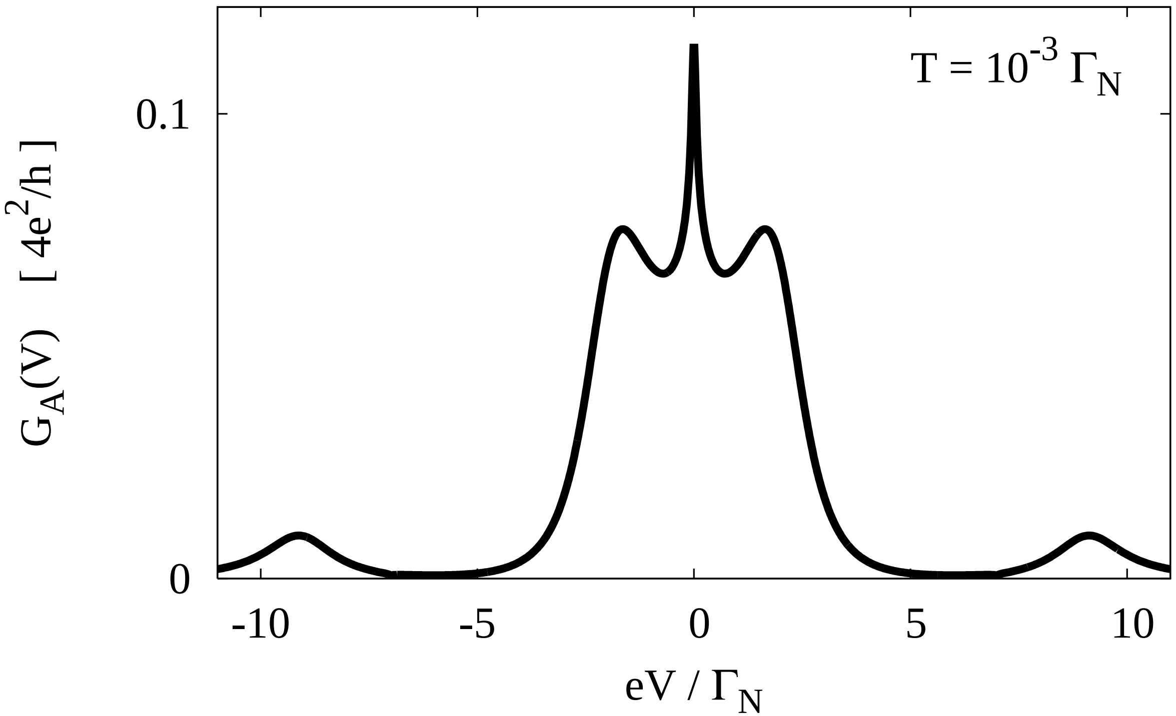


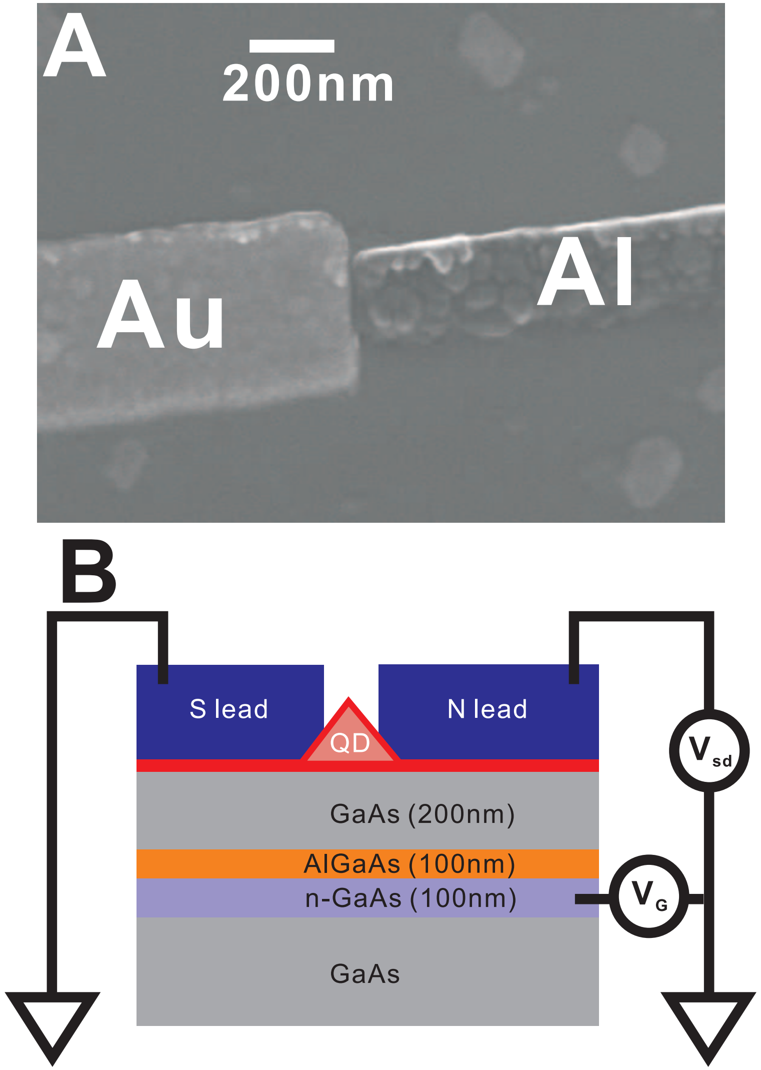
$$k_B T = \Gamma_N / 1000$$

T. Domański and A. Donabidowicz, PRB **78**, 073105 (2008).

Experimental setup

Experimental setup

/ University of Tokyo /



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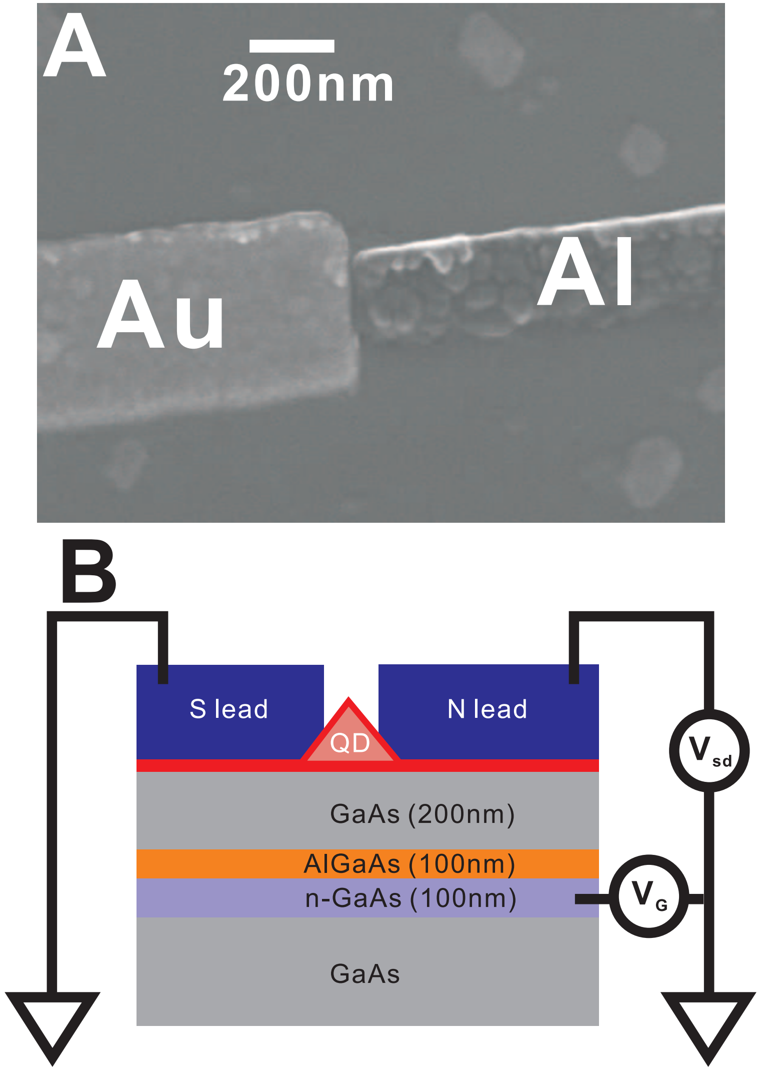


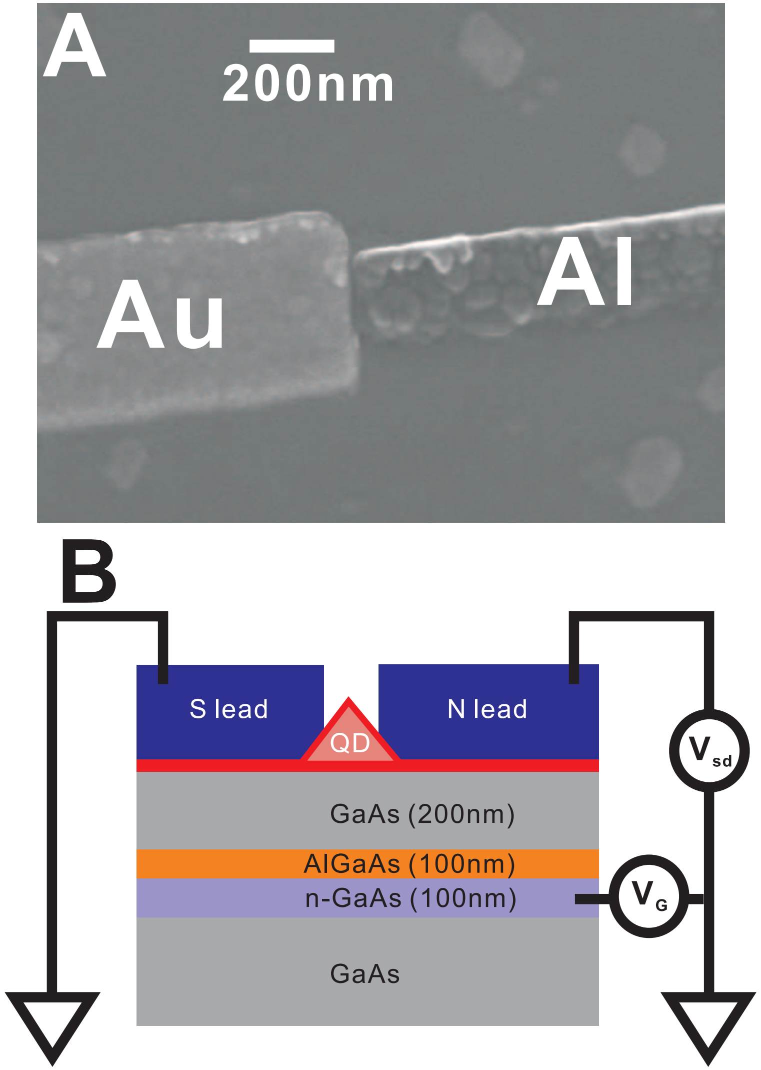
QD : self-assembled InAs

diameter ~ 100 nm

backgate : Si-doped GaAs

Experimental setup

/ University of Tokyo /



$T_c \simeq 1\text{K}$

$\Delta \simeq 152\mu\text{eV}$

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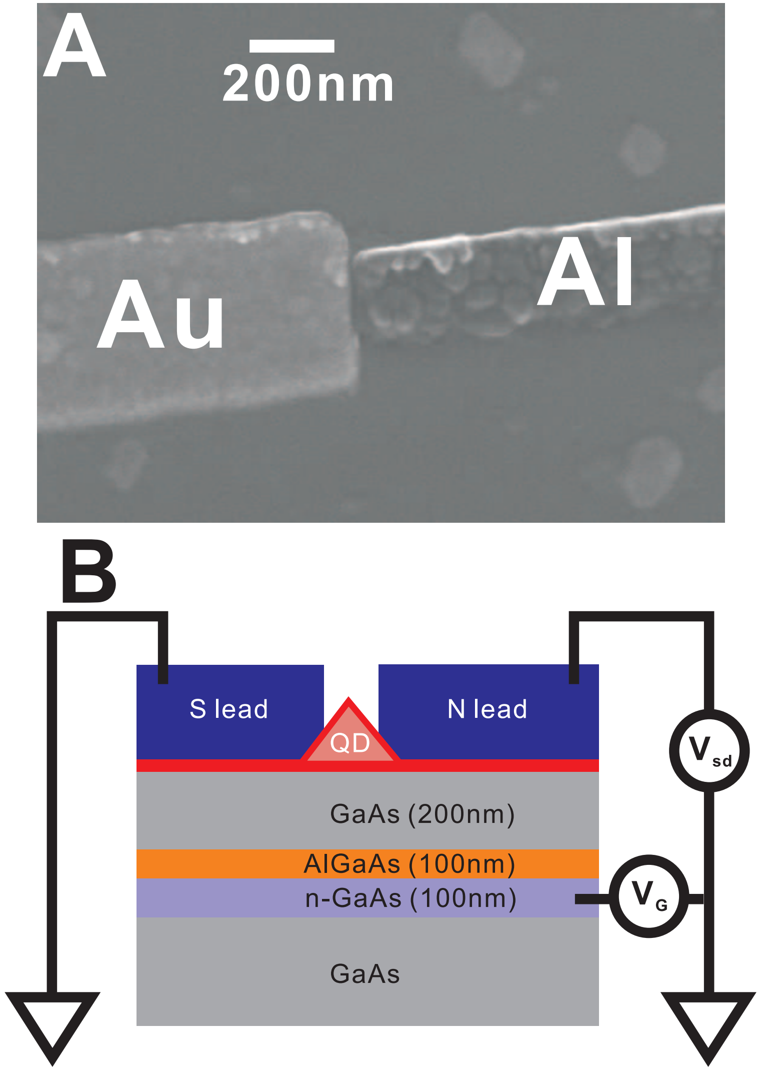

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R.S. Deacon et al, *Phys. Rev. Lett.* **104**, *076805 (2010).*

Interplay with the Kondo effect

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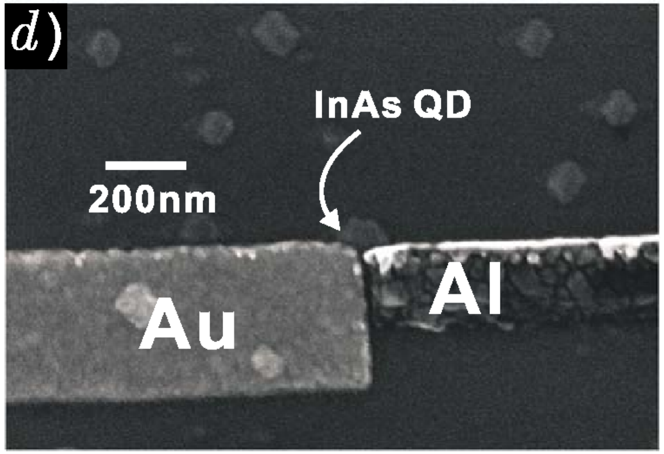

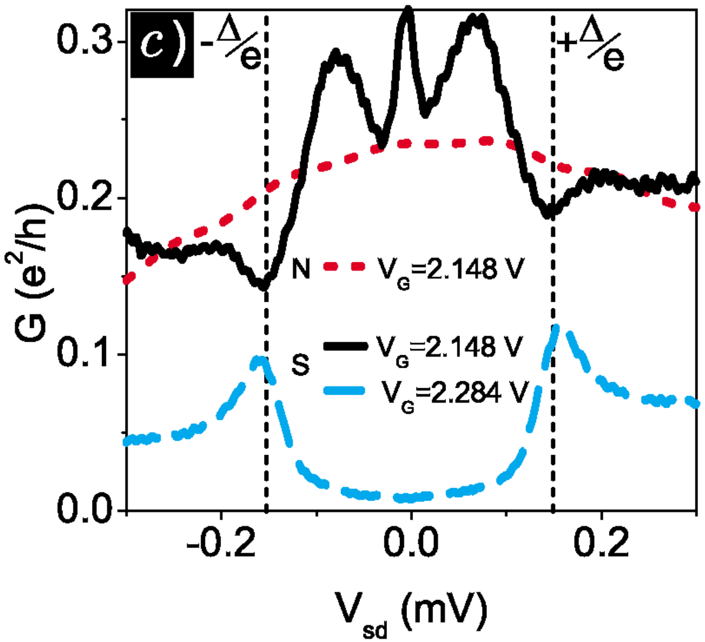

R.S. Deacon et al, *Phys. Rev. B* **81**, *121308(R) (2010).*

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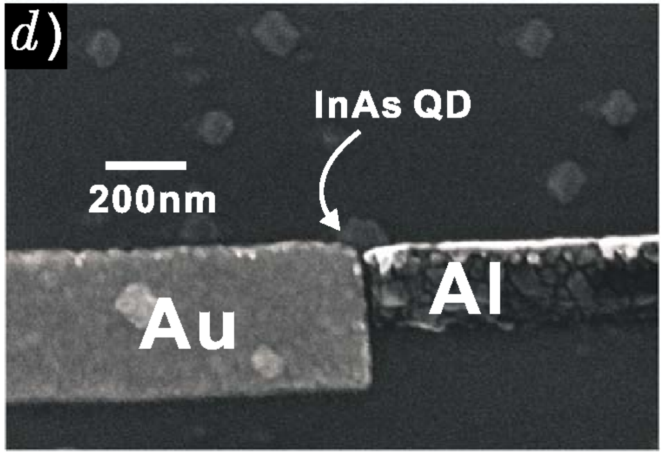
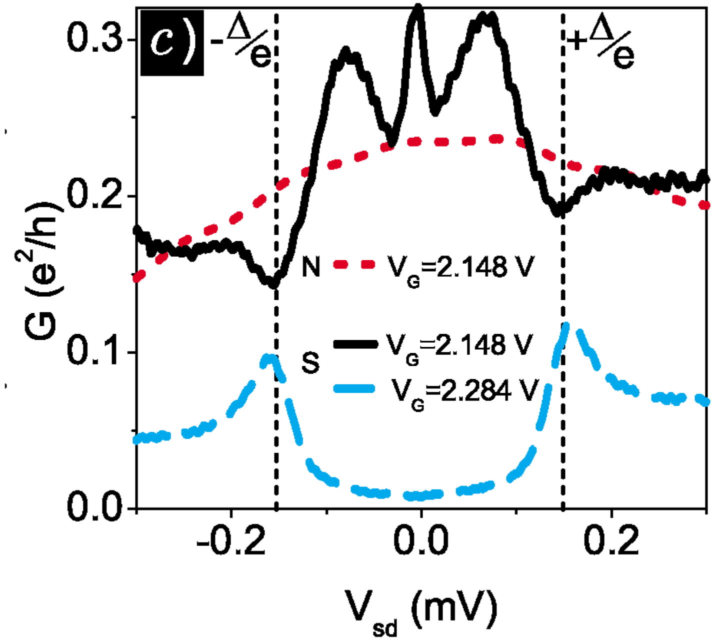


"The zero-bias conductance peak is consistent with Andreev transport enhanced by the Kondo singlet state"



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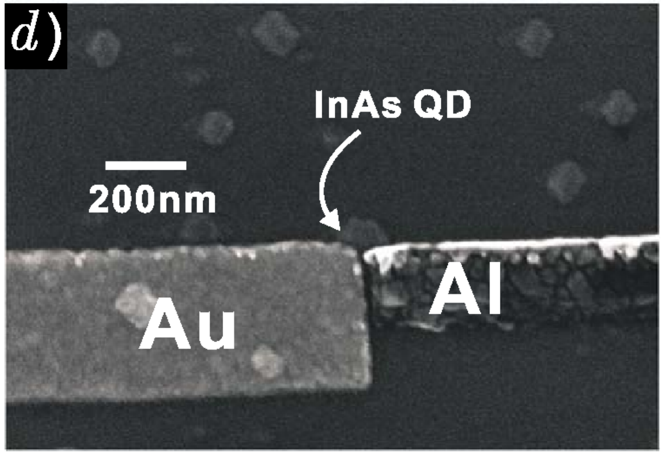

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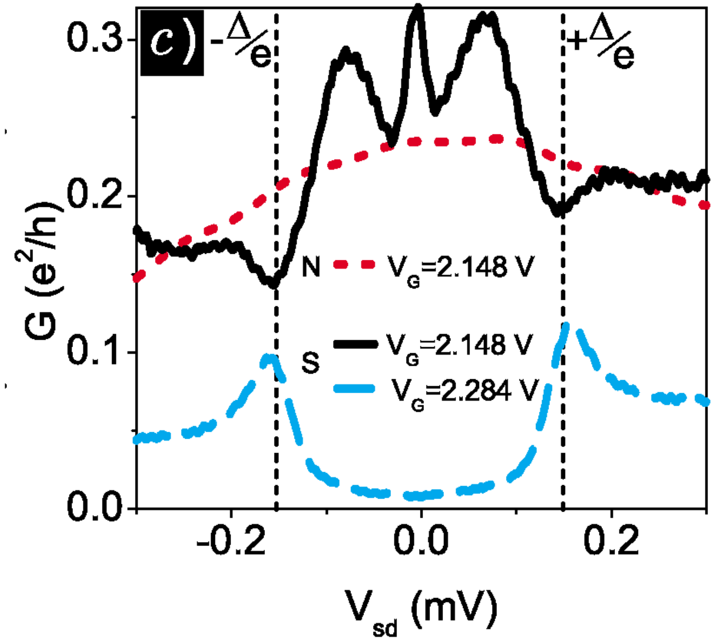

"We note that the feature exhibits excellent qualitative agreement with a recent theoretical treatment by Domanski et al"

R.S. Deacon et al, *Phys. Rev. B* **81**, *121308(R) (2010).*

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/ for the part 2 /

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QD coupled between N and S electrodes:

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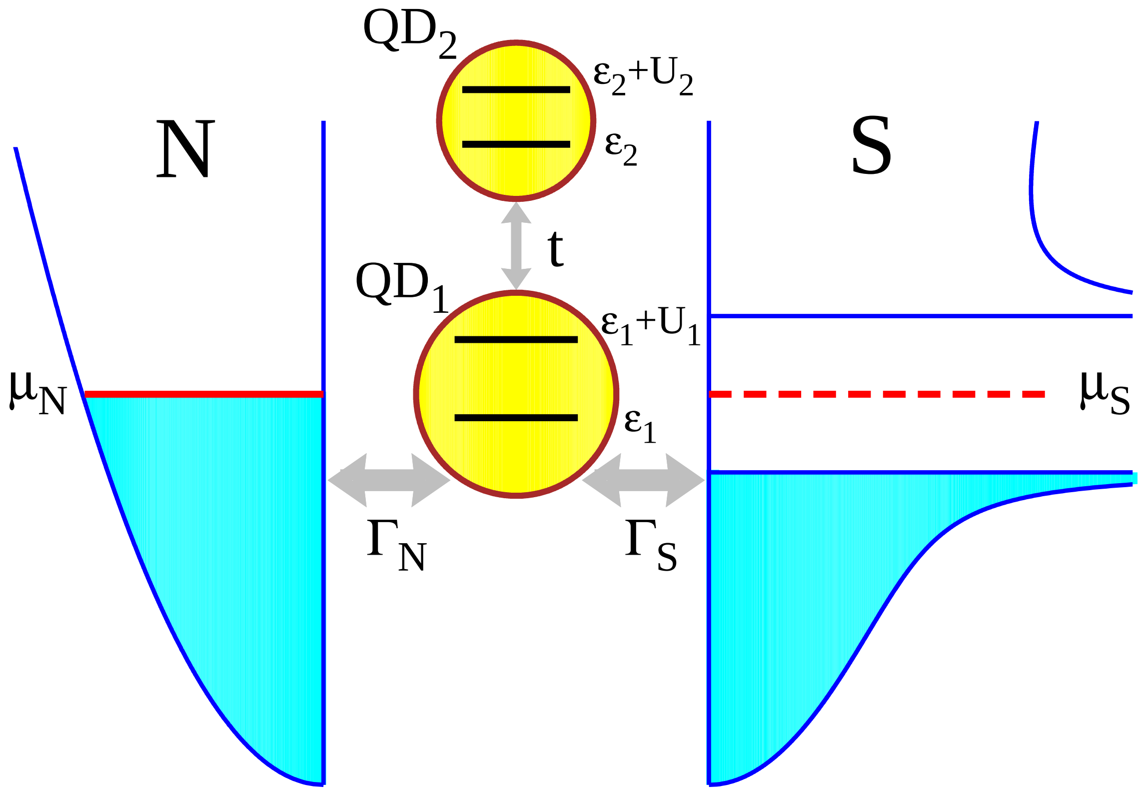
$\Rightarrow$ the zero-bias enhancement / below T_K /

3. Further extensions

Double QD – **between a metal and superconductor**



(T-shape configuration)

Relevant issues:

⇒ **induced on-dot pairing** . . . . (due to $\boldsymbol{\Gamma_S}$)

⇒ **Coulomb blockade & Kondo effect** . . . . (via $\boldsymbol{U_1}$ and $\boldsymbol{\Gamma_N}$)

⇒ **quantum interference** . . . . (because of $\boldsymbol{t}$)

Quantum interference – in the particle and hole channels

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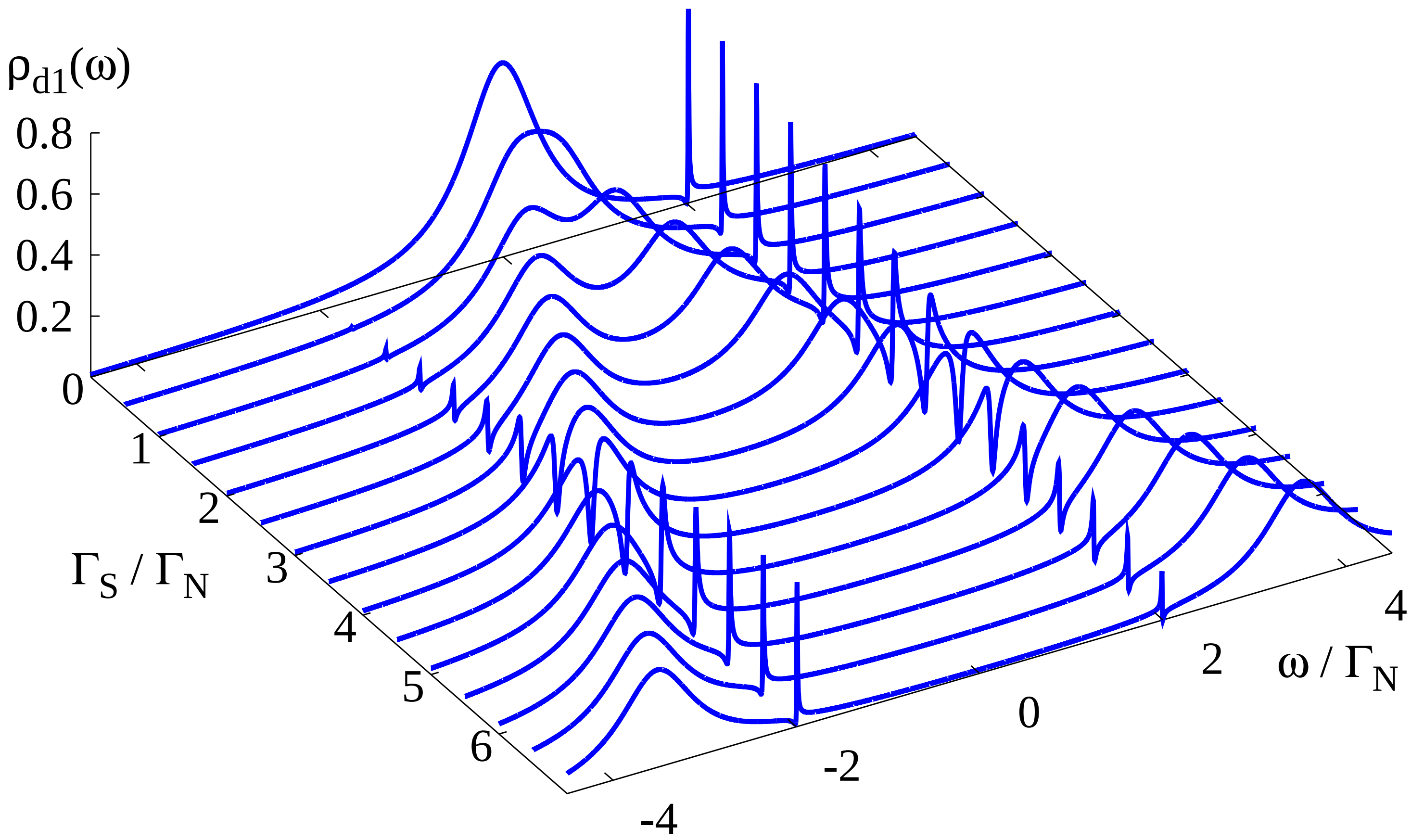
Fano-type lineshapes appear simultaneously at $\pm\varepsilon_2$



/ The case: $U_1 = 0$ and $U_2 = 0$ /

J. Barański and T. Domański, Phys. Rev. B (2012).

Quantum interference – **in the particle and hole channels**

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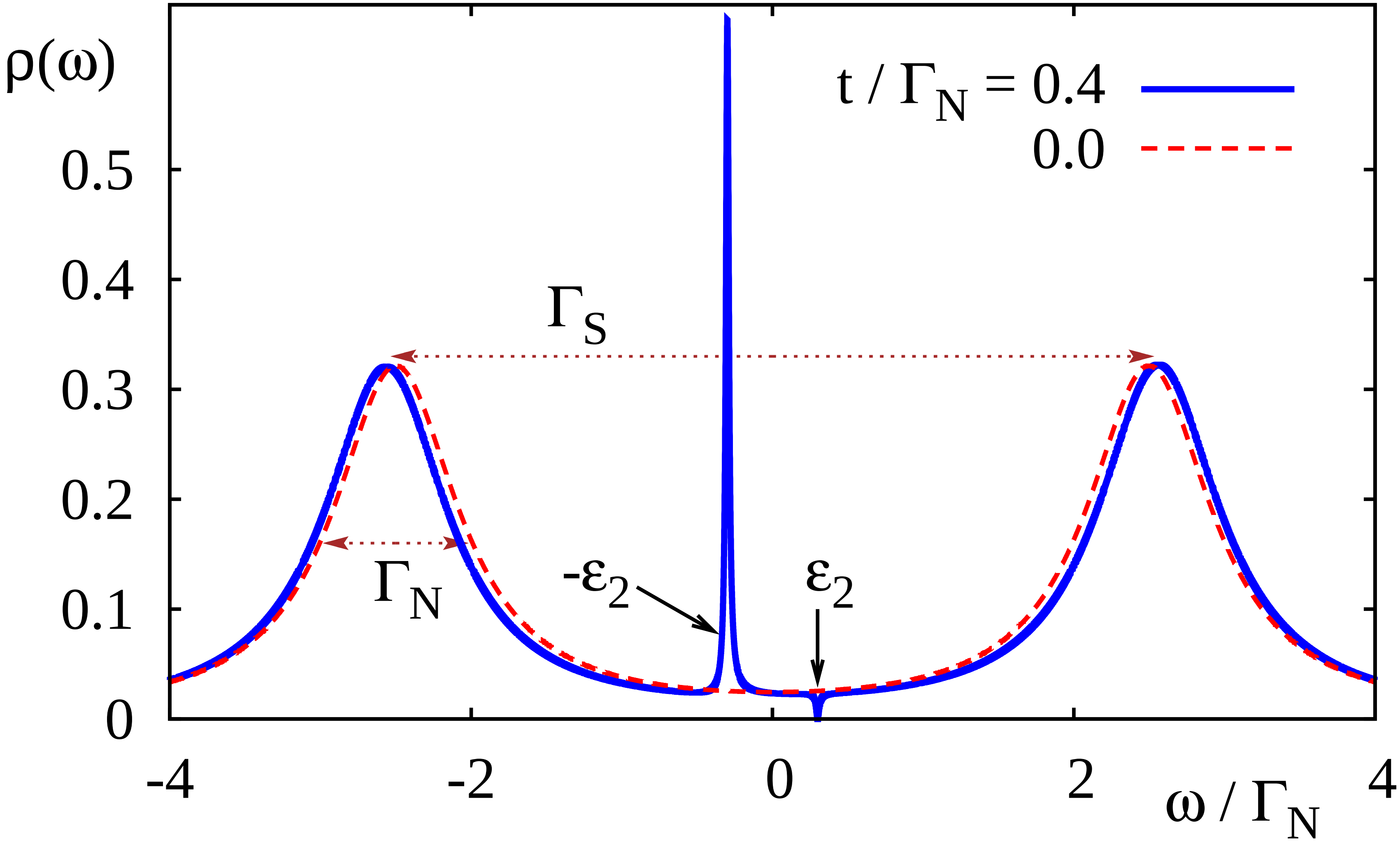

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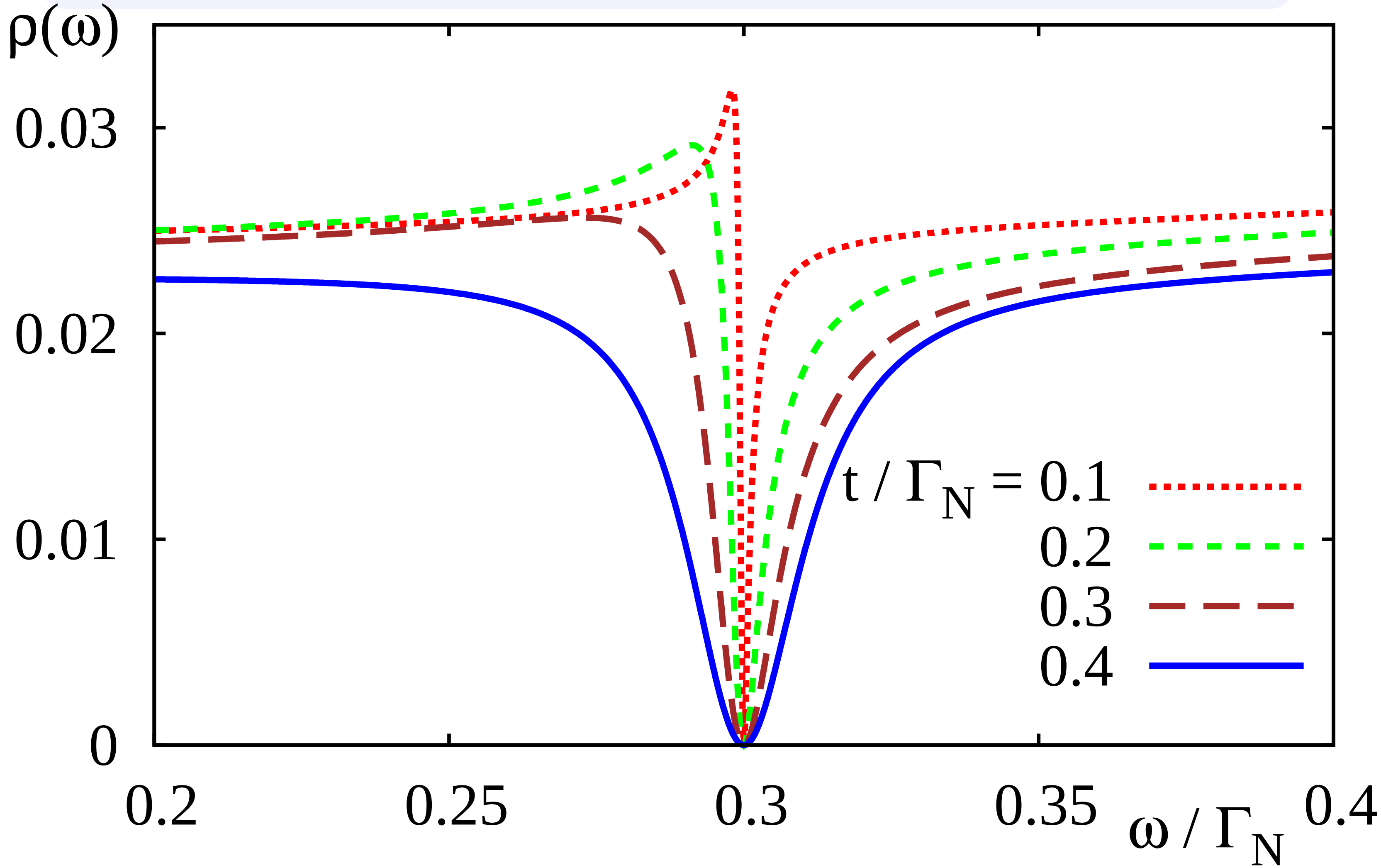

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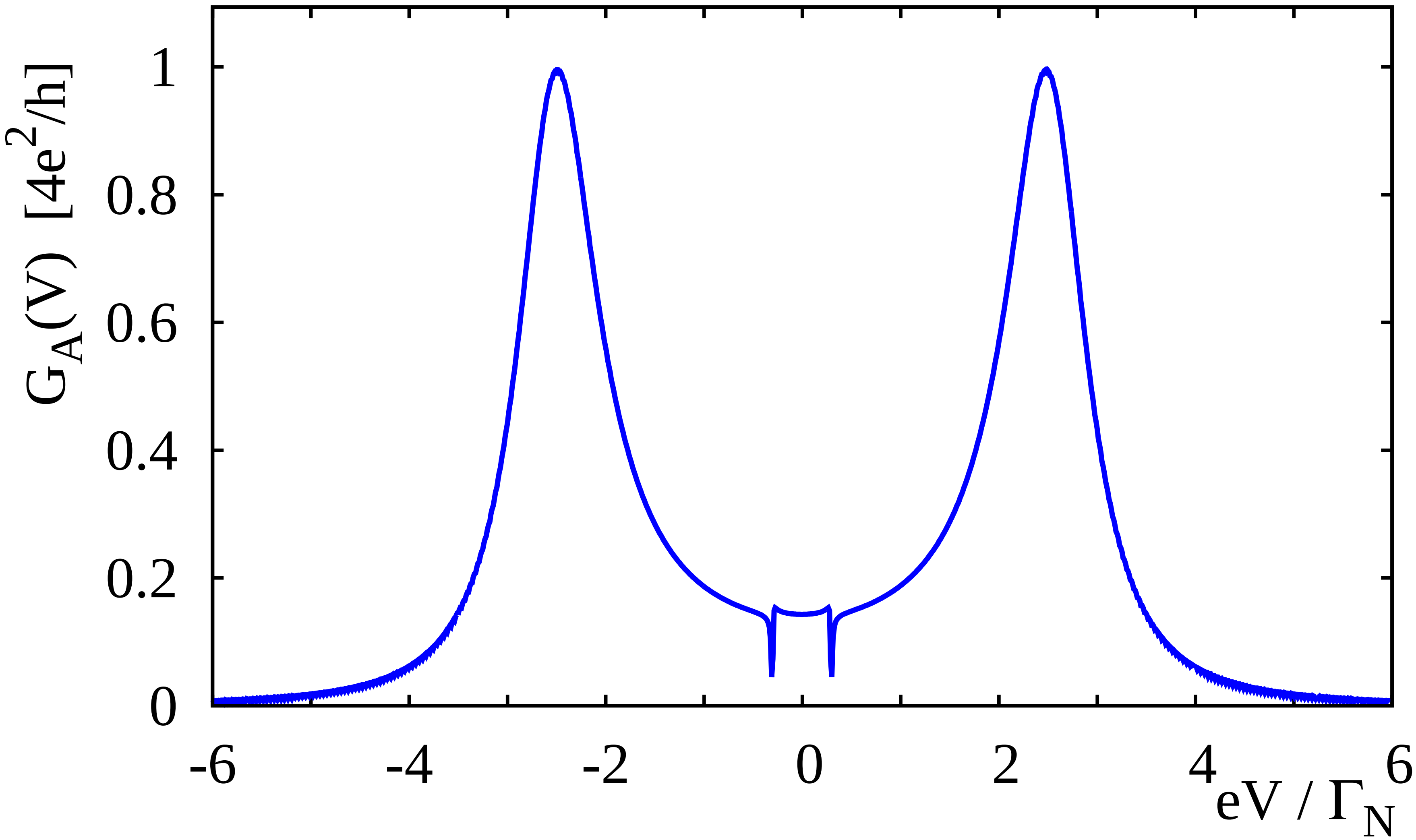
Quantum interference – in the particle and hole channels

Differential Andreev conductance $G_A = \frac{\partial J_A(V)}{\partial V}$



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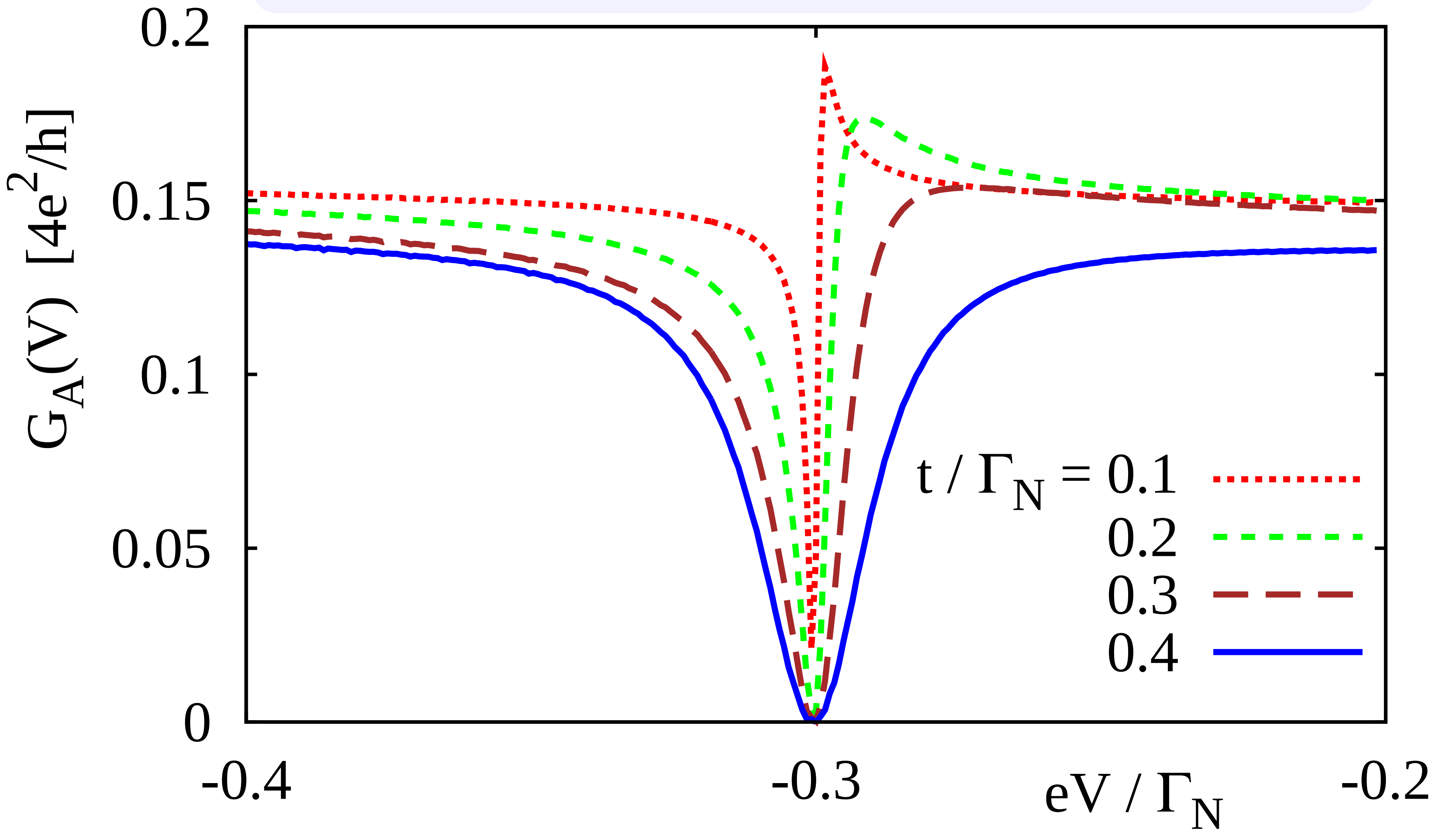

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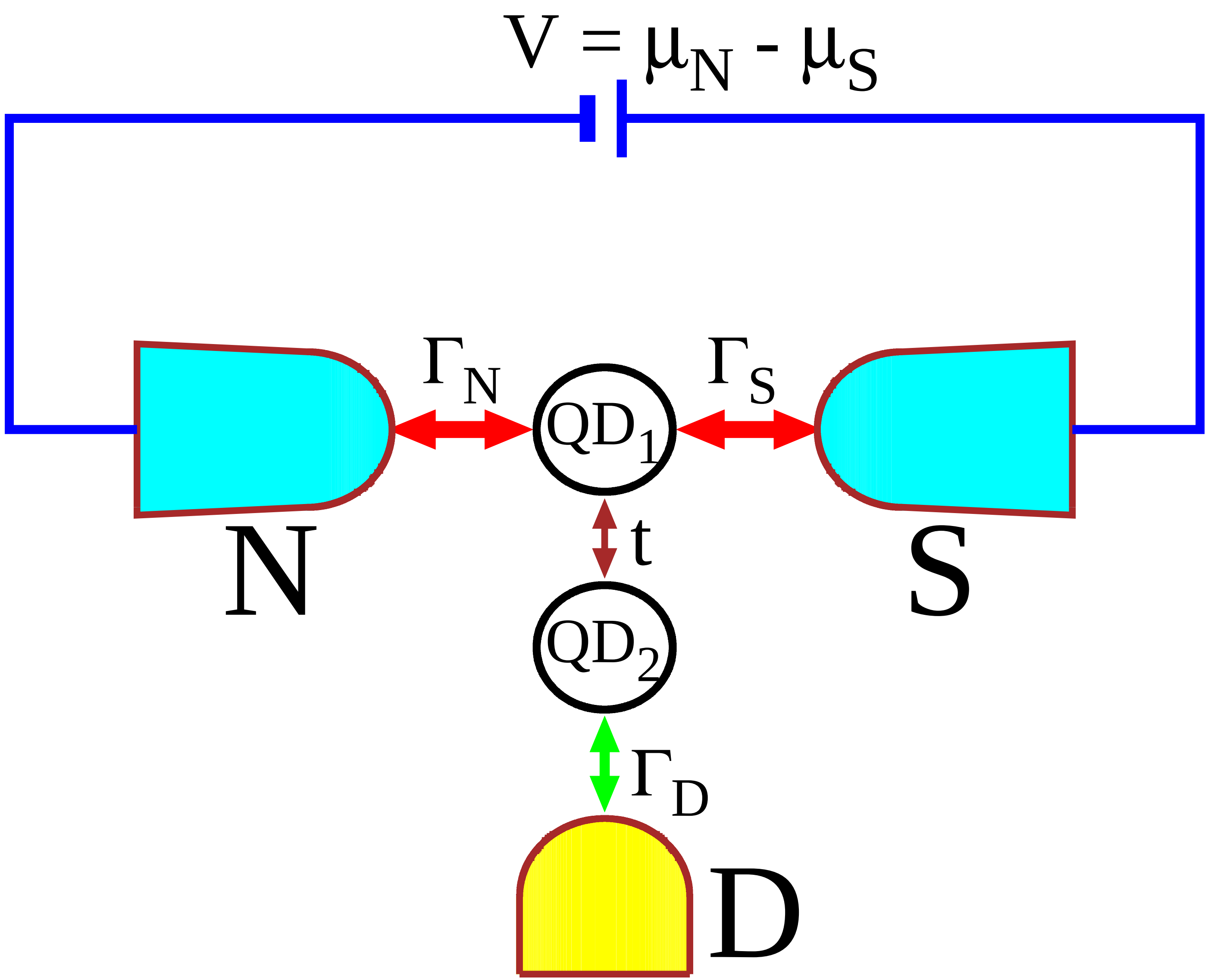
Double QD – **decoherence effects**

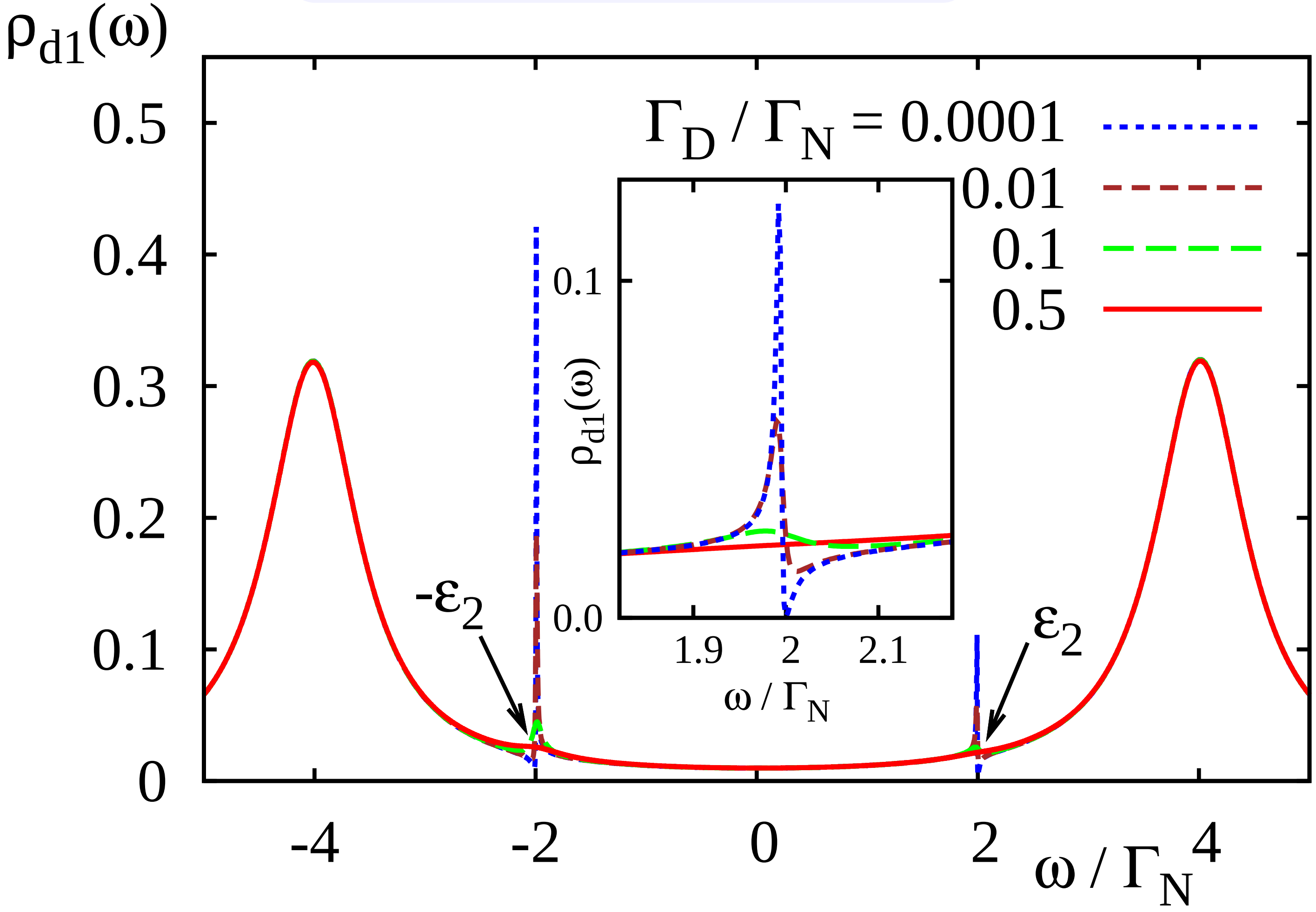


Floating lead (D) does not contribute any current but it serves as a source of decoherence.

Quantum interference – influence of the decoherence

Quantum interference – influence of the decoherence

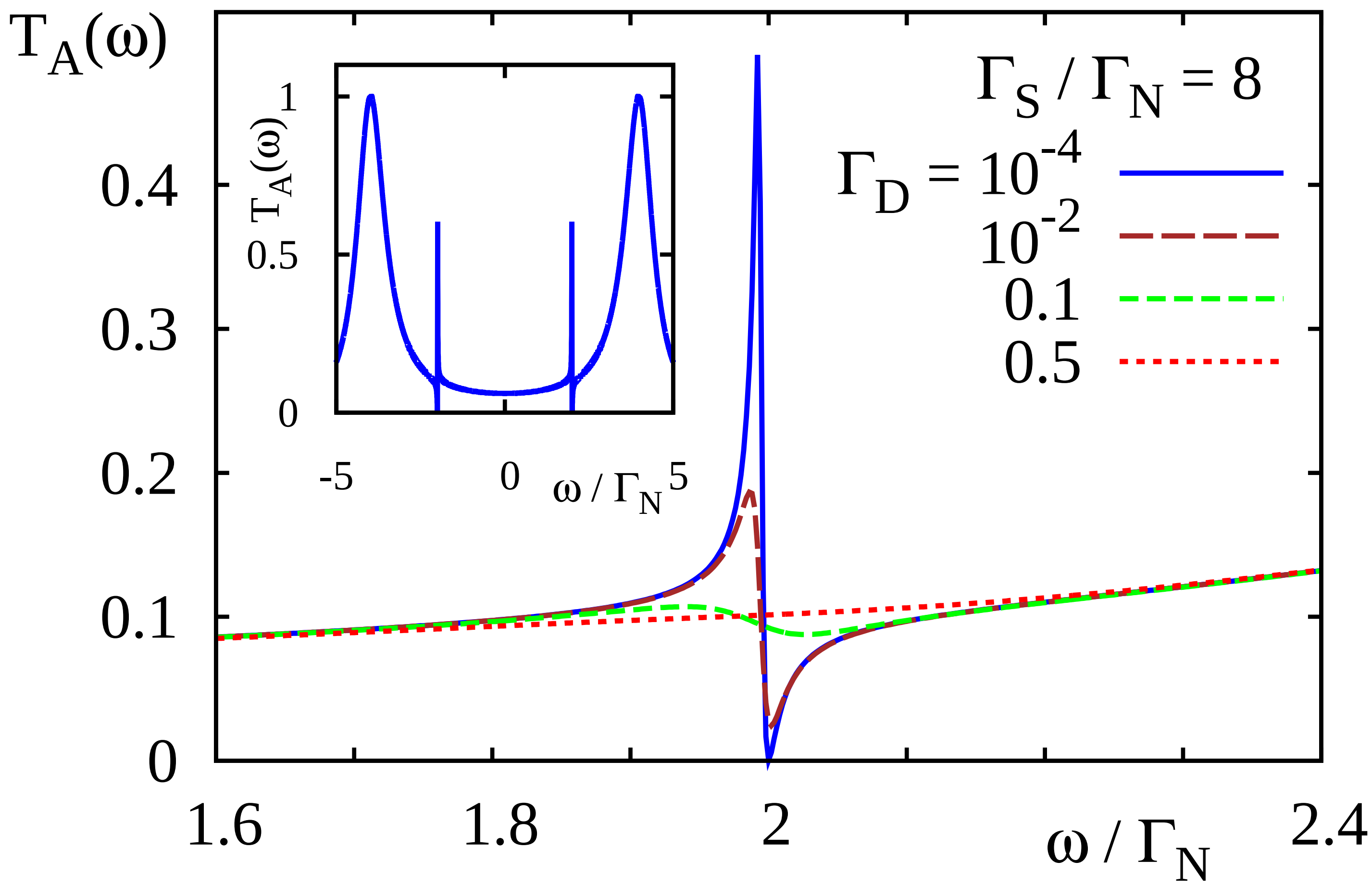
Density of states $\rho_{d1}(\omega)$



J. Barański and T. Domański, Phys. Rev. B (2012).

Quantum interference – influence of the decoherence

Andreev conductance $T_A(\omega)$



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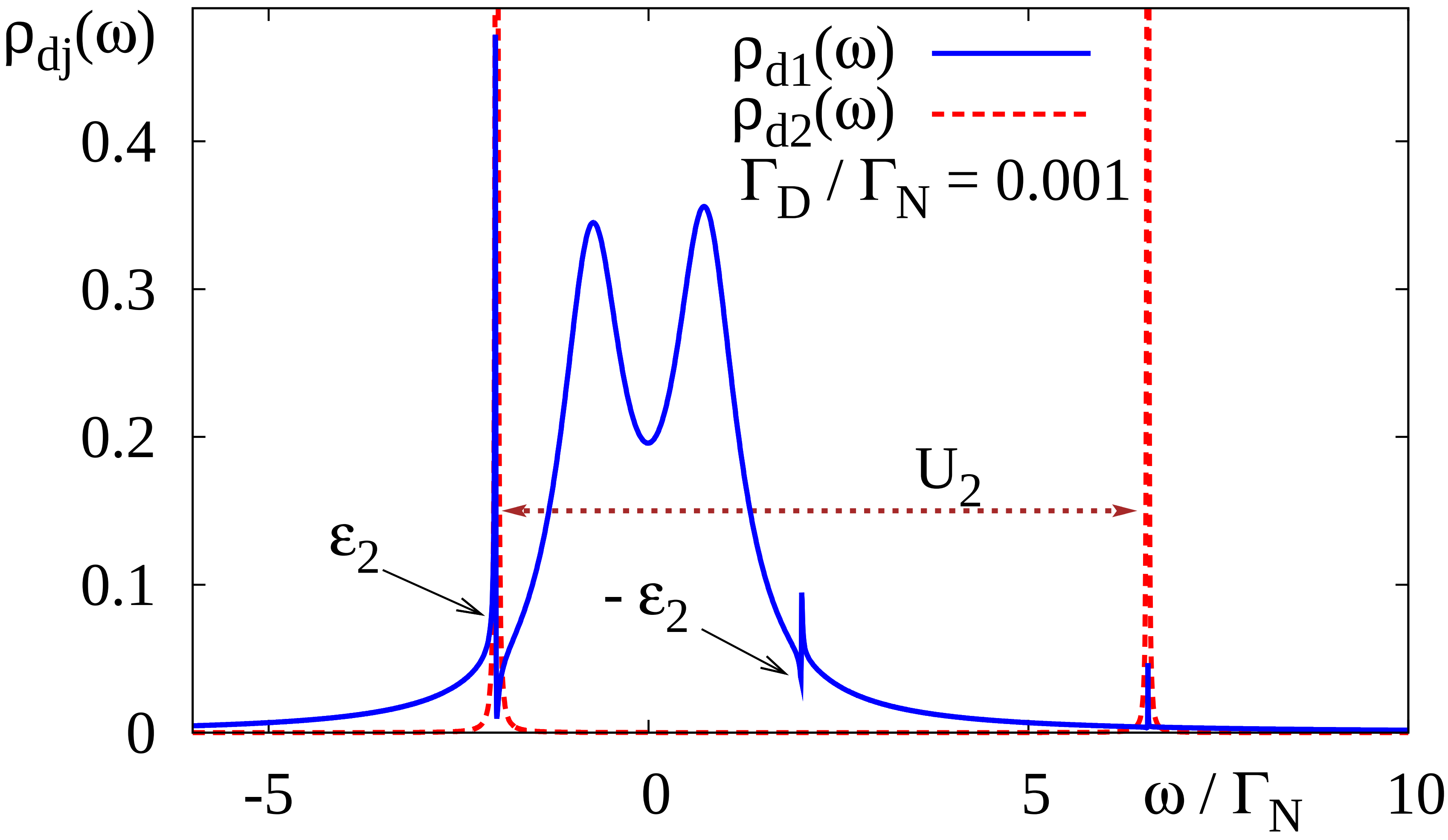
Quantum interference – **influence of the decoherence**

Effect of U_2 and the coupling Γ_D



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Quantum interference – influence of the decoherence

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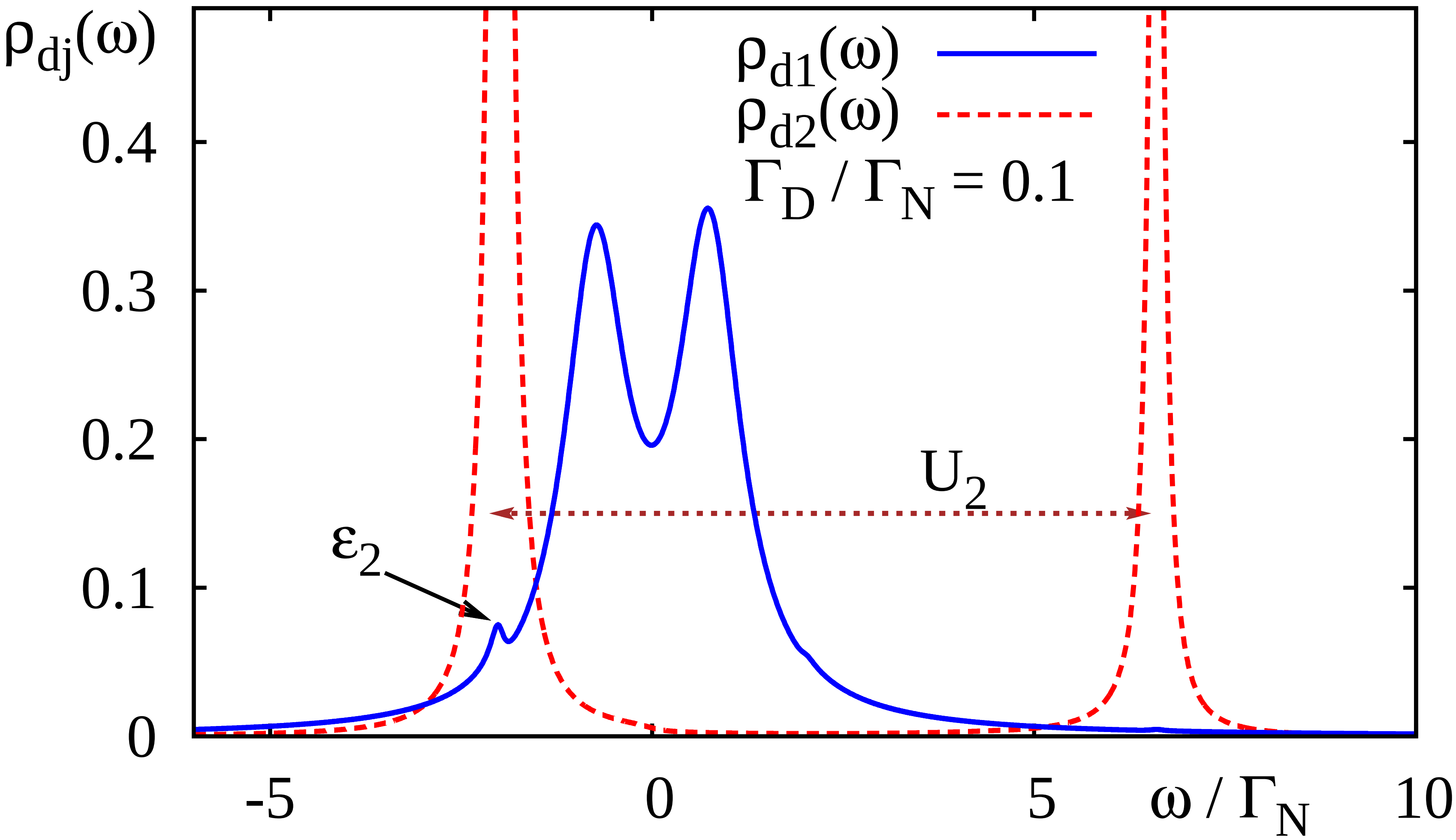

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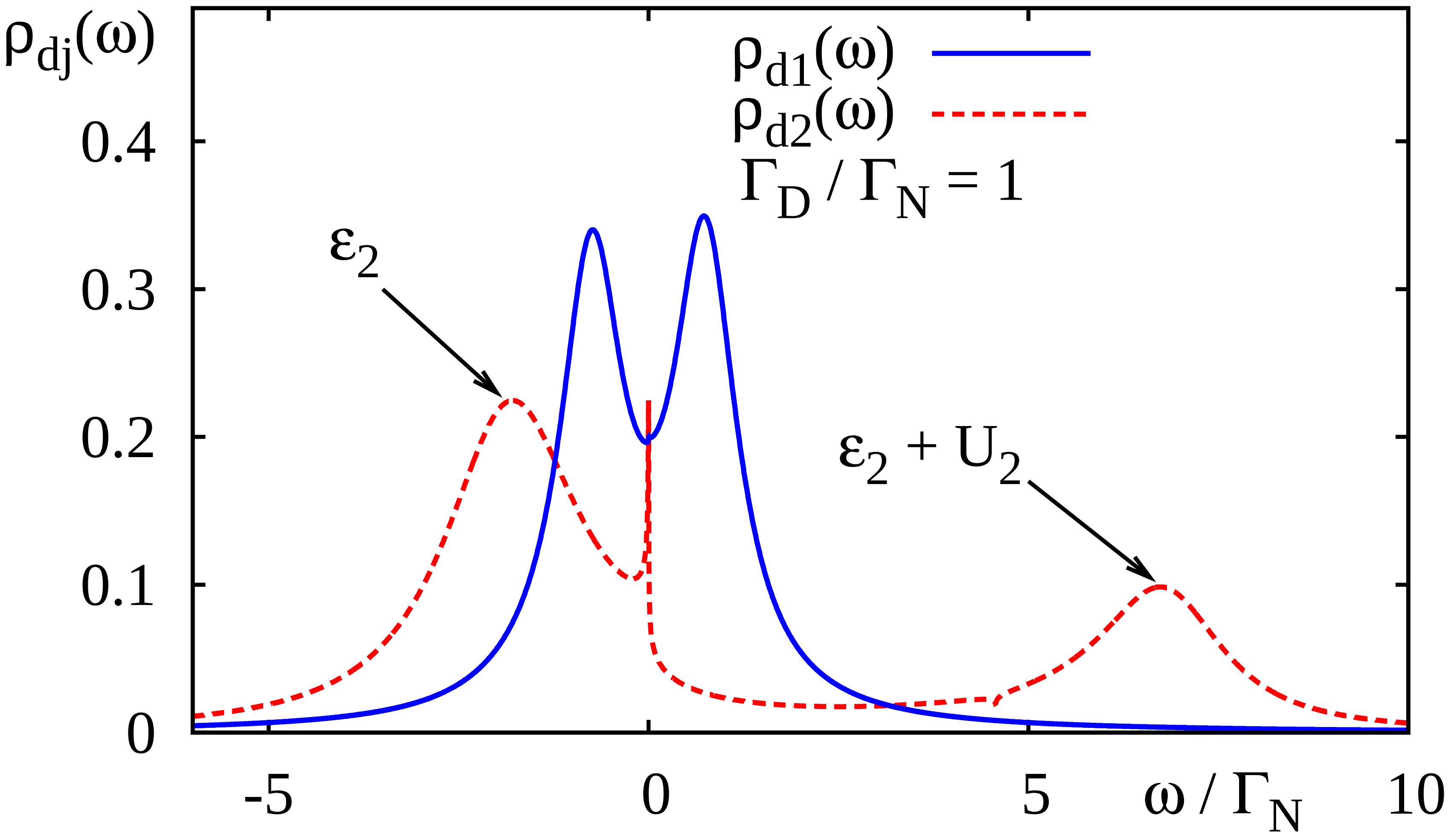

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Furthermore:

$\Rightarrow$ **Fano structure can suppress the Kondo resonance** / below T_K /

Conclusions

/ for the 3^{rd} part /

Double QD between the N and S electrodes:

$\Rightarrow$ **is affected by the quantum interference**
/ Fano-type lineshapes /

$\Rightarrow$ **simultaneously in the particle and hole channels**
/ particle-hole Fano structures /

Furthermore:

$\Rightarrow$ **Fano structure can suppress the Kondo resonance** / below T_K /

$\Rightarrow$ **decoherence has a detrimental effect on the Fano lineshapes**
/ already for a weak coupling /

4. Bulk superconductors

Andreev spectroscopy – for bulk superconductors

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The subgap Andreev spectroscopy is also a valuable tool for studying various superconducting compounds.

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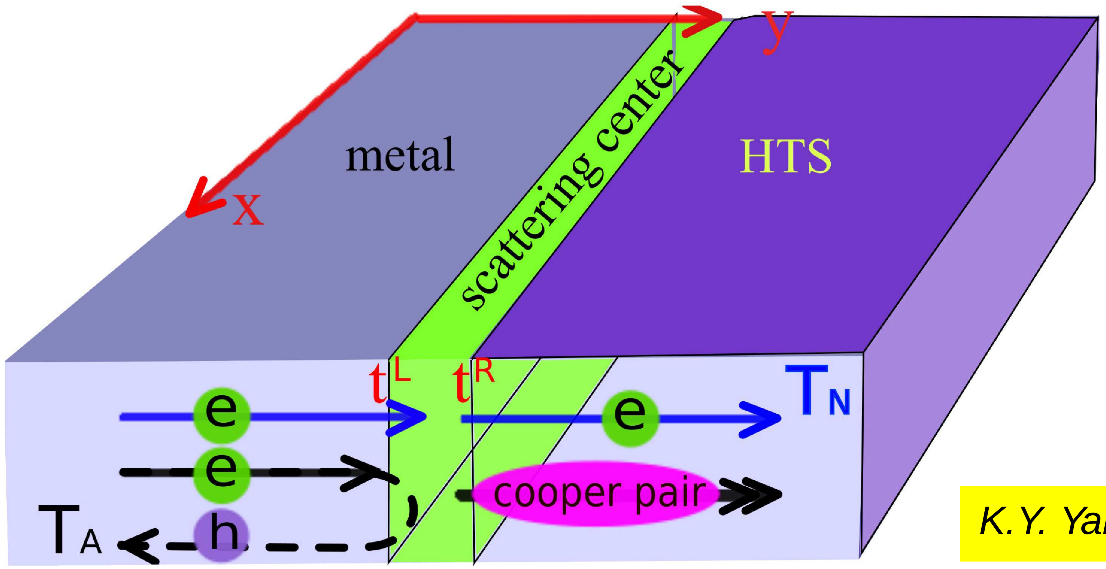
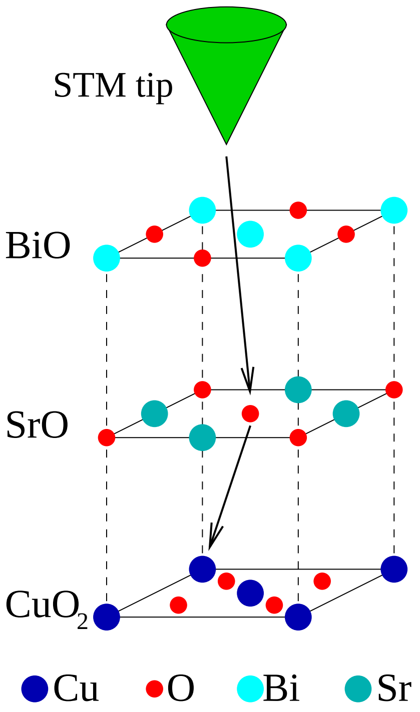


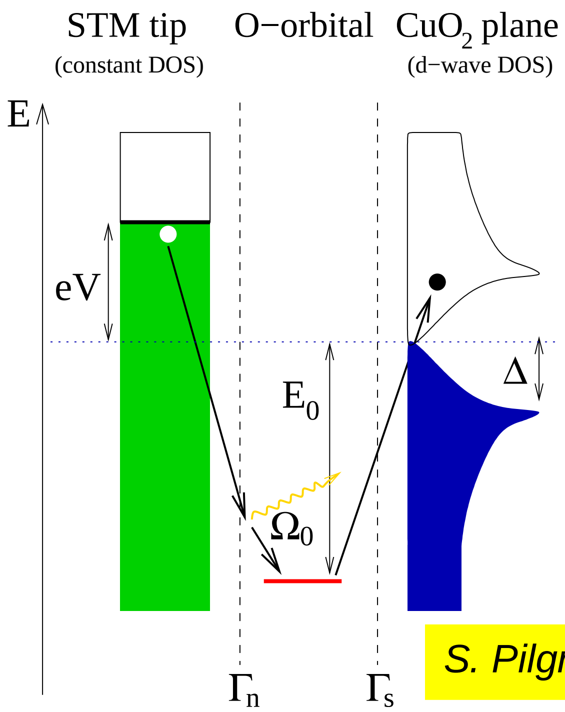
K.Y. Yang et al, *Phys. Rev. Lett.* **105**, *167004 (2010).*

For practical experimental realizations one can e.g. use an insulating barrier sandwiched between the conducting (N) and the probed superconductor (S).

Andreev spectroscopy – for bulk superconductors

The subgap Andreev spectroscopy is also a valuable tool for studying various superconducting compounds.



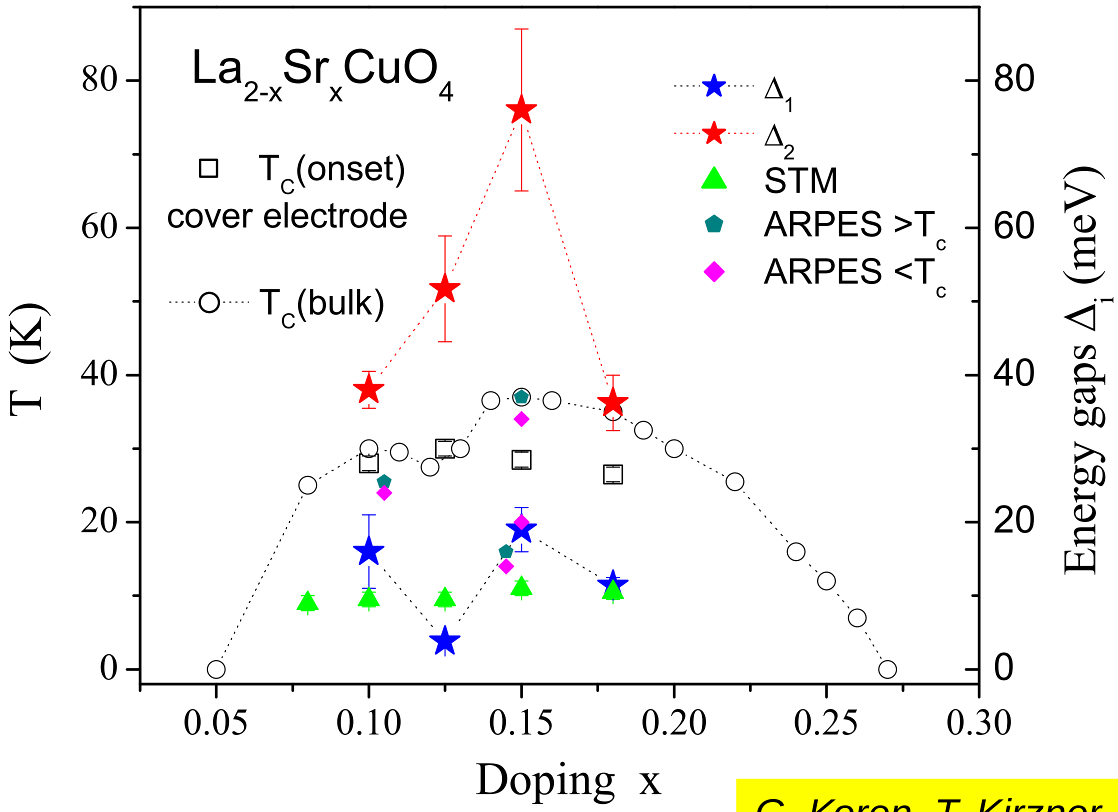


S. Pilgram et al, *Phys. Rev. Lett.* **97**, *117003 (2006).*

Other experimental realizations are also possible in the STM configuration, where the apex oxygen atoms play a role similar to QD in the N-QD-S setup.

Andreev spectroscopy – **for bulk superconductors**

The subgap Andreev spectroscopy is also a valuable tool for studying various superconducting compounds.



G. Koren, T. Kirzner, Phys. Rev. Lett. **106**, *017002 (2011).*

Such Andreev spectroscopy has revealed the intriguing two-gap feature.

Andreev scattering – on a microscopic level

Andreev scattering – on a microscopic level

Besides the specific Andreev-type spectroscopy we can, however, think of the Andreev scattering in a much broader perspective.

We consider the strongly correlated fermion system

$$\hat{H} = \hat{T}_{kin} + U \int d\vec{r}\ \ \hat{c}^{\dagger}_{\uparrow}(\vec{r})\ \hat{c}^{\dagger}_{\downarrow}(\vec{r})\ \hat{c}_{\downarrow}(\vec{r})\ \hat{c}_{\uparrow}(\vec{r})$$

Strongly correlated systems / Hubbard-Stratonovich transf. /

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In a basis of the coherent states and using the Grassmann fields

$$\hat{c}\,|\psi\rangle = \psi\,|\psi\rangle \quad \text{and} \quad \langle\psi|\,\hat{c}^{\dagger} = \langle\psi|\,\bar{\psi}$$

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we can express the partition function by the path integral

$$Z = \int D \left[\bar{\psi}, \psi\right] e^{-S[\bar{\psi},\psi]}$$

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$$Z = \int D\left[\bar{\psi},\psi\right] e^{-S[\bar{\psi},\psi]}$$

where the imaginary-time fermionic action

$$S[\bar{\psi},\psi] = \int_0^{\beta} d\tau \int d\vec{r} \Big[\sum_{\sigma} \bar{\psi}_{\sigma}(\vec{r},\tau) \left(\partial_{\tau} + \hat{\xi}\right) \psi_{\sigma}(\vec{r},\tau) - g\; \bar{\psi}_{\uparrow}(\vec{r},\tau)\; \bar{\psi}_{\downarrow}(\vec{r},\tau)\; \psi_{\downarrow}(\vec{r},\tau) \psi_{\uparrow}(\vec{r},\tau)\Big]$$

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and $\hat{\xi} \equiv -\hbar^2\nabla^2/2m - \mu, \qquad g = -U$.

Hubbard-Stratonovich – continued

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To eliminate the quartic term we can introduce the auxiliary pairing fields

$$Z = \int D\left[\bar{\boldsymbol{\Delta}}, \boldsymbol{\Delta}, \bar{\psi}, \psi\right] e^{-S[\bar{\boldsymbol{\Delta}}, \boldsymbol{\Delta}, \bar{\psi}, \psi]}$$

Hubbard-Stratonovich – continued

To eliminate the quartic term we can introduce the auxiliary pairing fields

$$Z = \int D\left[\bar{\Delta}, \Delta, \bar{\psi}, \psi\right] e^{-S[\bar{\Delta}, \Delta, \bar{\psi}, \psi]}$$

simplifying the action to a bi-linear form

$$S = \int_0^\beta d\tau \int d\vec{r} \left[\sum_\sigma \bar{\psi}_\sigma(\vec{r},\tau)\left(\partial_\tau + \hat{\xi}\right)\psi_\sigma(\vec{r},\tau) + \frac{|\Delta(\vec{r},\tau)|^2}{g}\right.$$
$$\left. - \bar{\Delta}(\vec{r},\tau)\ \psi_\downarrow(\vec{r},\tau)\psi_\uparrow(\vec{r},\tau) - \Delta(\vec{r},\tau)\ \bar{\psi}_\uparrow(\vec{r},\tau)\bar{\psi}_\downarrow(\vec{r},\tau)\right]$$

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The mean field (*saddle point*) solution usually relies on the assumption of a static and uniform pairing field

$\Delta(\vec{r},\tau) = \Delta$, $\bar{\Delta}(\vec{r},\tau) = \bar{\Delta}$.

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We tried to go beyond this scheme treating the fermionic and bosonic degrees of freedom on an equal footing !

Boson-Fermion scenario

[in the lattice representation]

$$\hat{H} = \sum_{i,j,\sigma} (t_{ij} - \mu\, \delta_{i,j})\, \hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} + \sum_{l} \left(E^{(B)}_{l} - 2\mu \right) \hat{b}^{\dagger}_{l} \hat{b}_{l}$$

$$+ \sum_{i,j} g_{ij} \left[\hat{b}^{\dagger}_{l} \hat{c}_{i,\downarrow} \hat{c}_{j,\uparrow} + \text{h.c.} \right]$$

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For a more specific recent derivation see for instance:

E. Altman and A. Auerbach, Phys. Rev. B **65**, *104508 (2002).*

or *Y. Yildirim and Wei Ku, Phys. Rev. X* **1**, *011011 (2011).*

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T. Domański and J. Ranninger, Phys. Rev. **B 63**, *134505 (2001).*

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\hat{c}_{\mathbf{k}\uparrow}(l) &= u_{\mathbf{k}}(l)\,\hat{c}_{\mathbf{k}\uparrow} + v_{\mathbf{k}}(l)\,\hat{c}^{\dagger}_{-\mathbf{k}\downarrow} + \\
&\quad \frac{1}{\sqrt{N}}\sum_{\mathbf{q}\neq 0}\left[u_{\mathbf{k},\mathbf{q}}(l)\,\hat{b}^{\dagger}_{\mathbf{q}}\hat{c}_{\mathbf{q}+\mathbf{k}\uparrow} + v_{\mathbf{k},\mathbf{q}}(l)\,\hat{b}_{\mathbf{q}}\hat{c}^{\dagger}_{\mathbf{q}-\mathbf{k}\downarrow}\right], \\
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\end{aligned}$$

with the boundary conditions

$u_{\mathbf{k}}(0)=1$ **and** $v_{\mathbf{k}}(0)=v_{\mathbf{k},\mathbf{q}}(0)=u_{\mathbf{k},\mathbf{q}}(0)=0.$

The corresponding fixed point values $\lim_{l\to\infty} u_{\mathbf{k}}(l)$ (and other parameters) have to be determined from the set of coupled flow equations

$$\frac{\partial}{\partial l}u_{\mathbf{k}}(l)\,,\quad \frac{\partial}{\partial l}v_{\mathbf{k}}(l)\,,\quad \frac{\partial}{\partial l}u_{\mathbf{k},\mathbf{q}}(l)\,,\quad \frac{\partial}{\partial l}v_{\mathbf{k},\mathbf{q}}(l)\,.$$

Single particle spectrum above T_c

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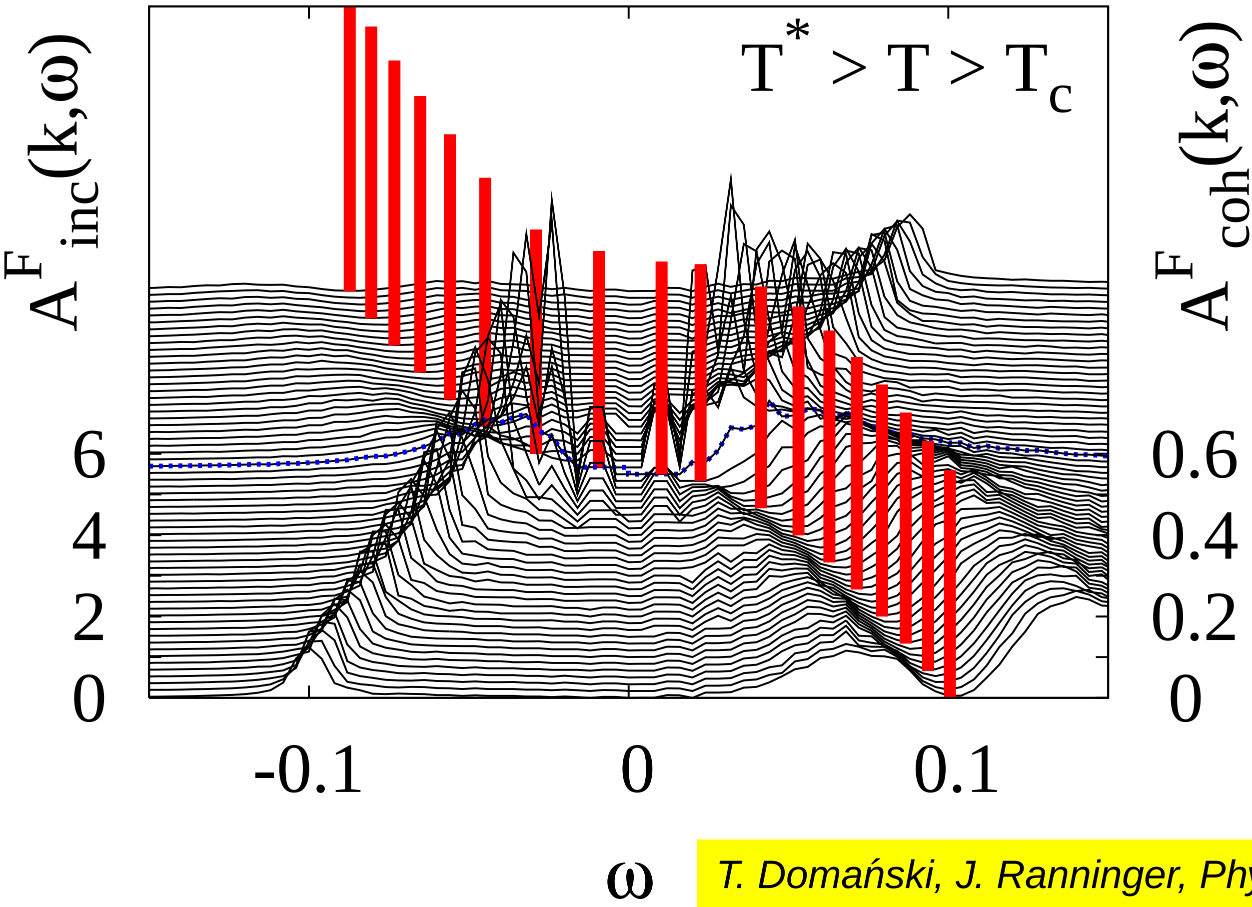

T. Domański, J. Ranninger, Phys. Rev. Lett. **91**, *255301 (2003).*

The Bogoliubov-type quasiparticles survive above T_c, being responsible for a partial destruction of the Fermi surface.

Evidence for Bogoliubov QPs above T_c

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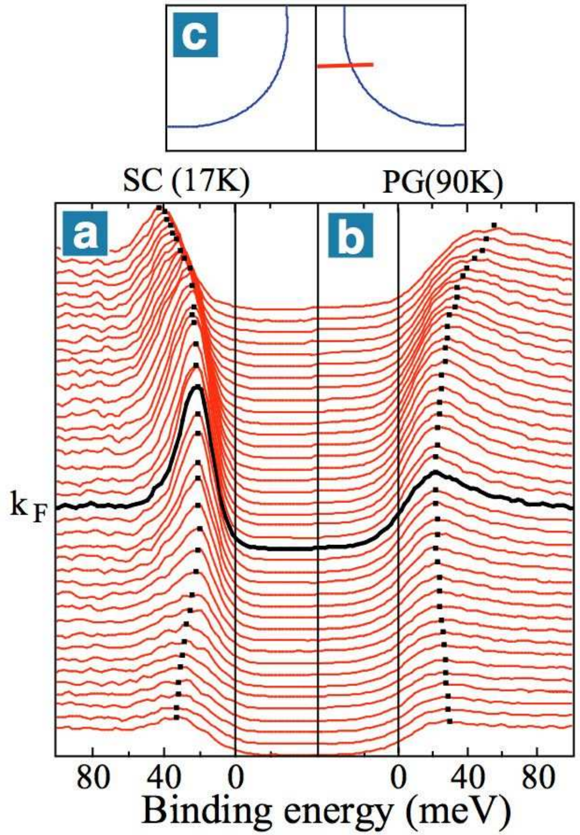
J. Campuzano group (Chicago, USA)



Results for: $Bi_2Sr_2CaCu_2O_8$

A. Kanigel et al, Phys. Rev. Lett. **101**, *137002 (2008).*

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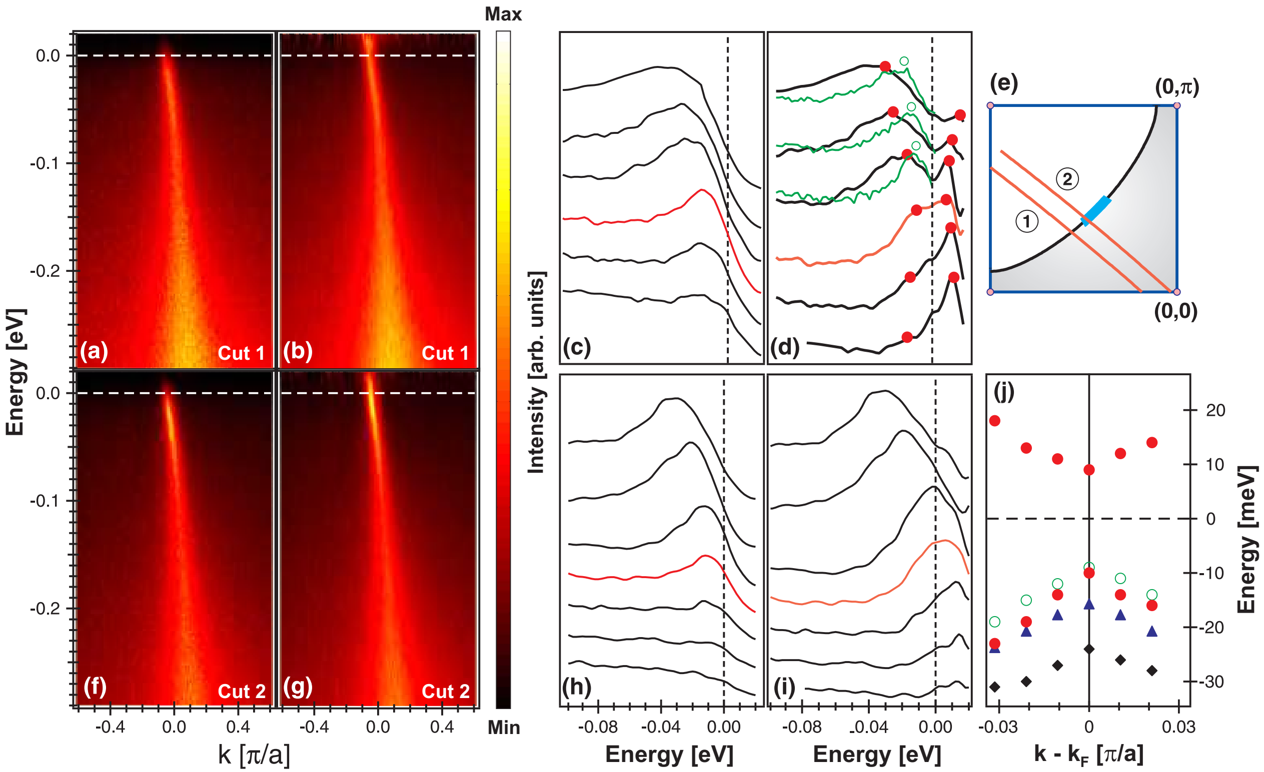

Results for: $La_{1.895}Sr_{0.105}CuO_4$

M. Shi et al, Eur. Phys. Lett. **88***, 27008 (2009).*

5. Ultracold gasses

Andreev spectroscopy – for ultracold atoms

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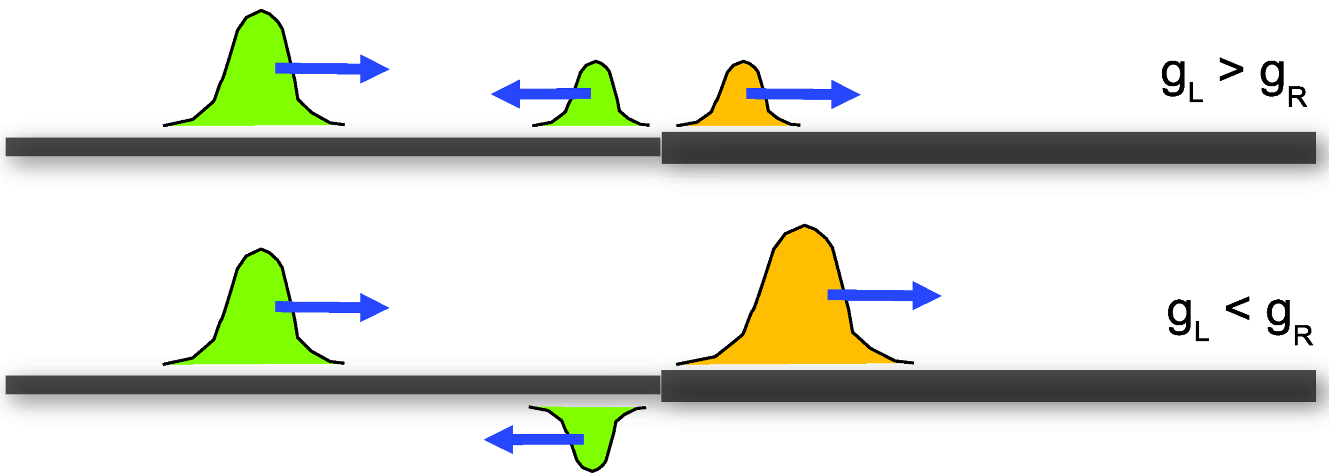
Proposal for the Andreev-type spectroscopy has been discussed also in a context of the superfluid ultracold fermion atom systems.

Andreev spectroscopy – for ultracold atoms

Proposal for the Andreev-type spectroscopy has been discussed also in a context of the superfluid ultracold fermion atom systems.



A.J. Daley, P. Zoller, and B. Trauzettel, Phys. Rev. Lett. **100**, *110404 (2008).*

The wave packet propagating along the 1-dimensional optical lattice can be scattered at an interaction boundary in the Andreev-type fashion.

Feshbach resonance

[local problem]

$$\hat{H}_{loc}(\mathbf{r}) = \sum_{\sigma} \varepsilon(\mathbf{r})\, \hat{c}^{\dagger}_{\sigma}(\mathbf{r})\hat{c}_{\sigma}(\mathbf{r}) + E(\mathbf{r})\, \hat{b}^{\dagger}(\mathbf{r})\hat{b}(\mathbf{r})$$
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$\hat{c}^{(\dagger)}_{\sigma}(\mathbf{r})$ **fermion atoms** .... *(open channel)*

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M.L. Chiofalo, S.J.J.M.F. Kokkelmans, J.N. Milstein, and M.J. Holland, Phys. Rev. Lett. **88***, 090402 (2002).*

Local solution

[exact]

$$\mathcal{G}_{loc}(i\omega_n) = [1-Z(T)]\left(\frac{u^2}{i\omega_n-\varepsilon_+}+\frac{v^2}{i\omega_n-\varepsilon_-}\right)+\frac{Z(T)}{i\omega_n-\varepsilon}$$

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T. Domański, Eur. Phys. J. B **33**, *41 (2003);* *T. Domański* et al, *Sol. State Commun.* **105**, *473 (1998).*

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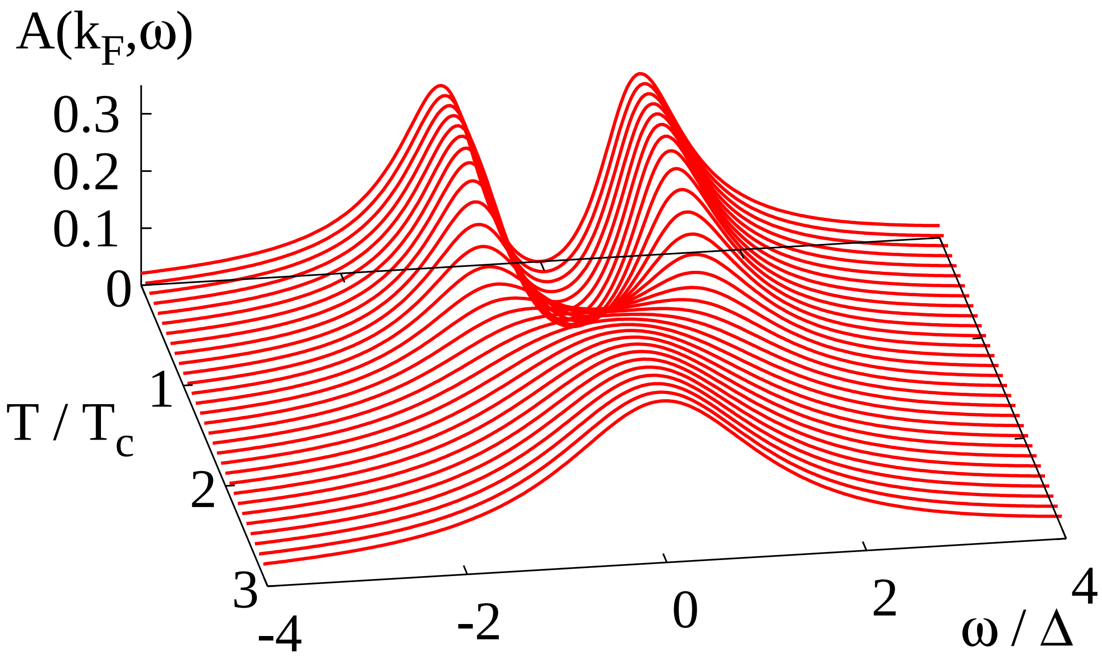

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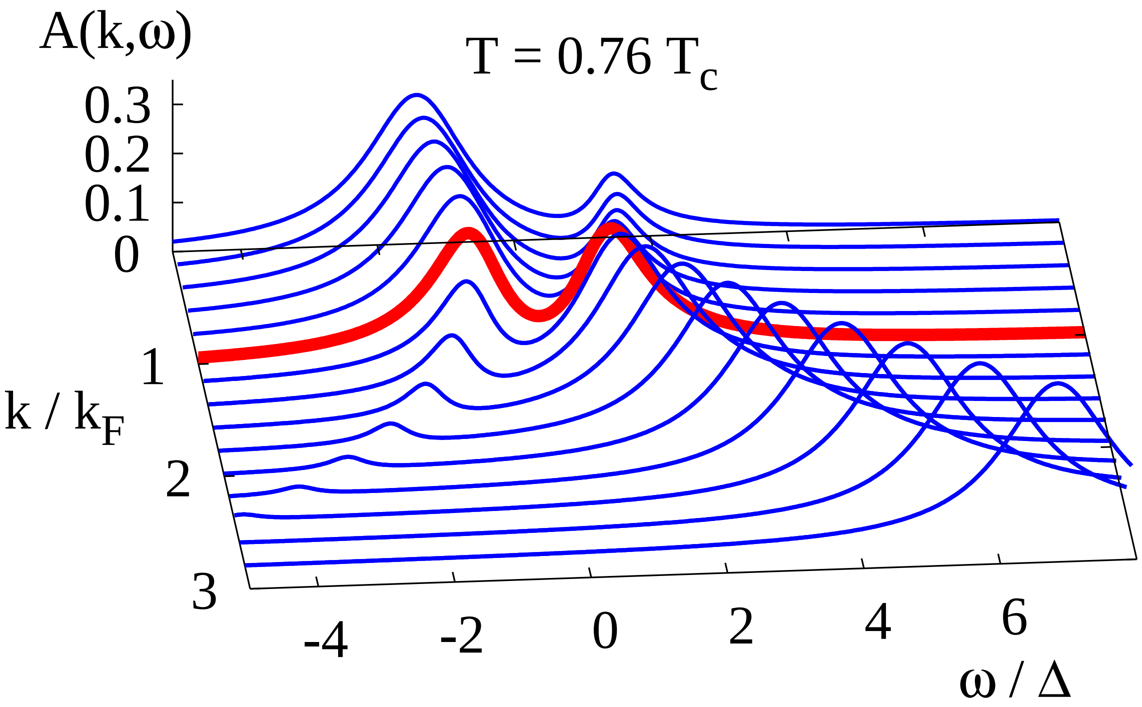

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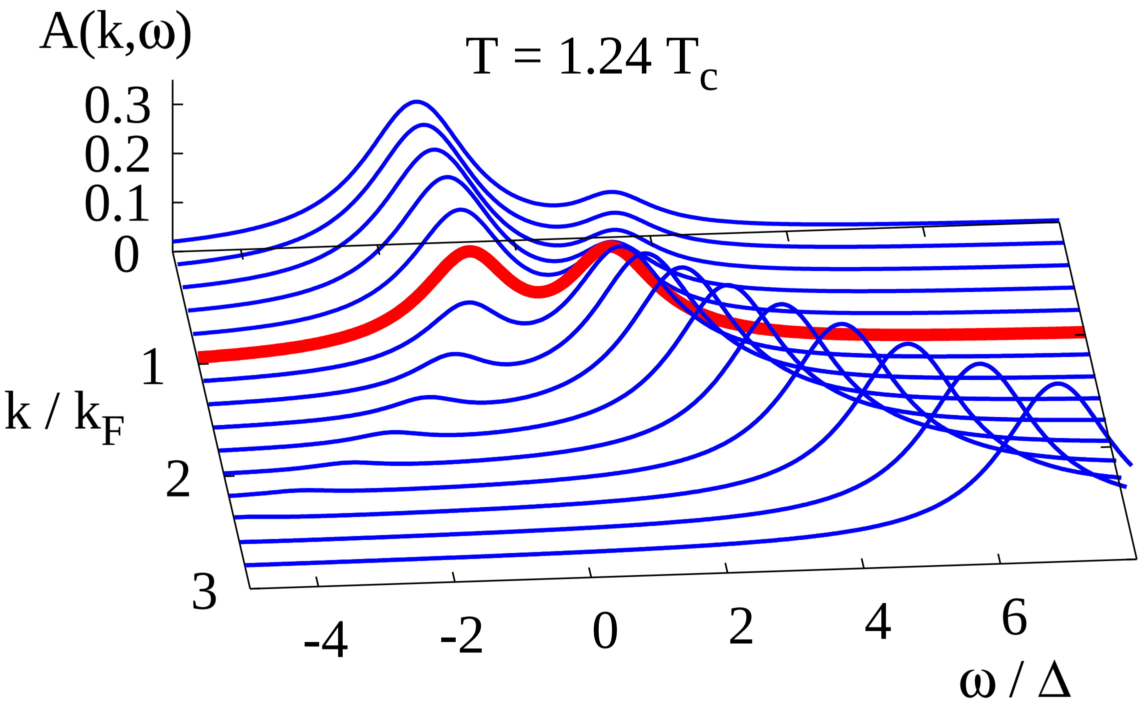

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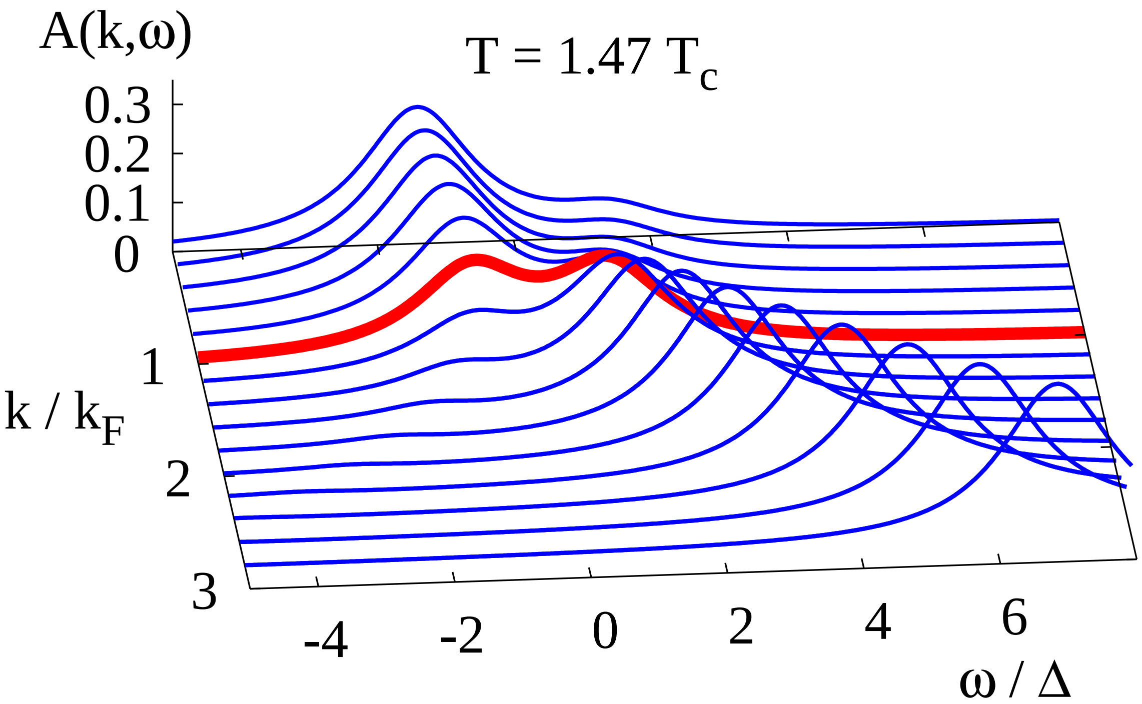

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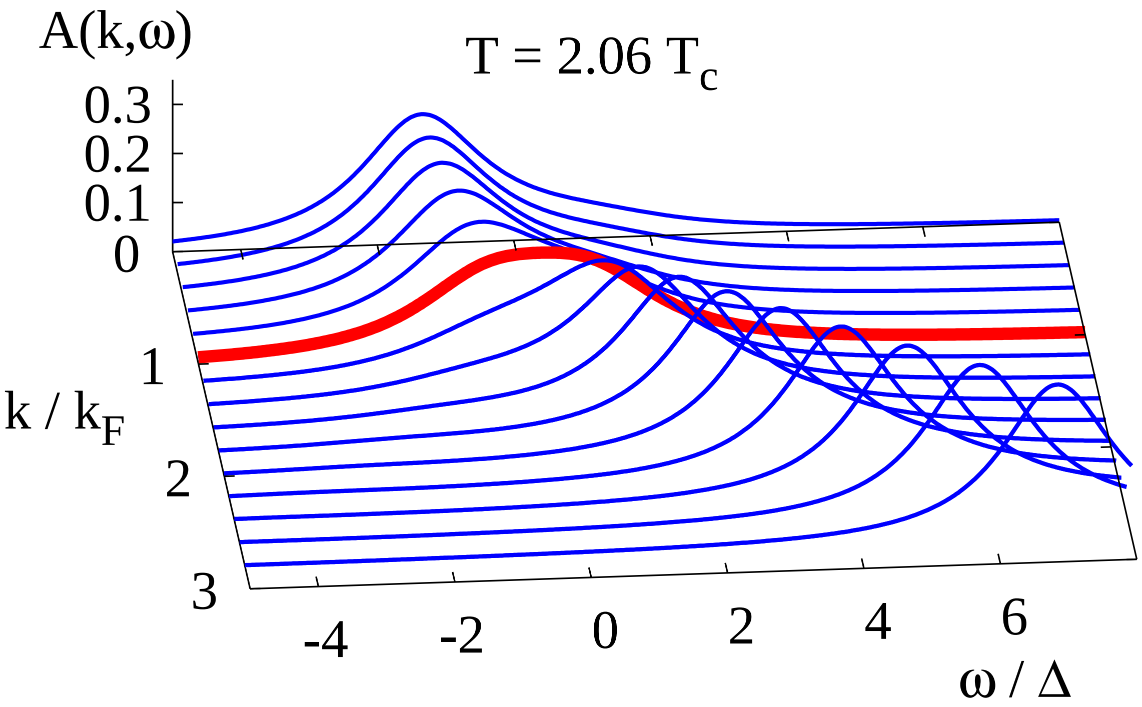

T. Domański, Phys. Rev. A **84**, *023634 (2011).*

Evidence for Bogoliubov QPs above T_c

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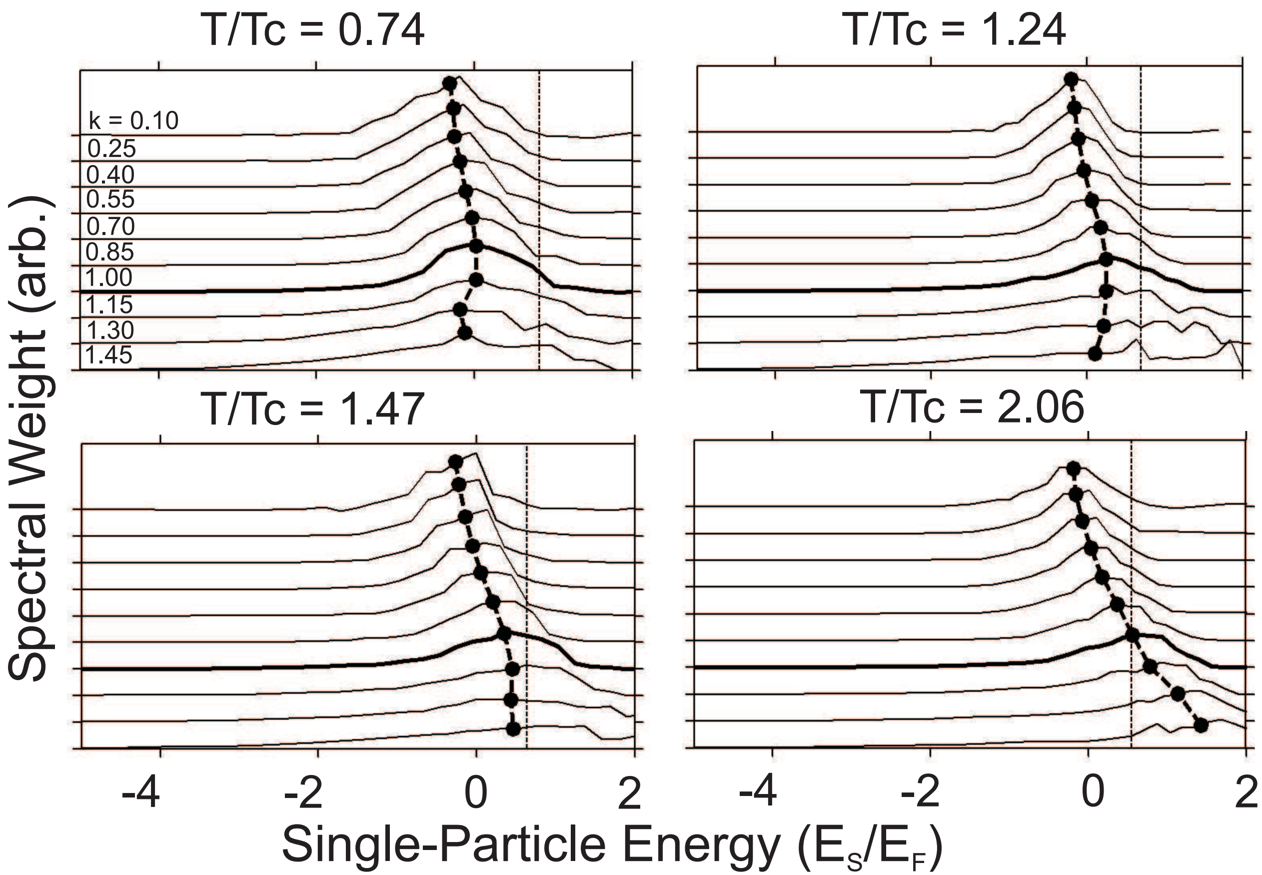
D. Jin group (Boulder, USA)



Results for the ultracold ^{40}K atoms

J.P. Gaebler et al, Nature Phys. **6**, *569 (2010).*

Conclusions

/ for parts 4 & 5 /

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http://kft.umcs.lublin.pl/doman/lectures