Ustroń, 5 Sept. 2016

Enhancement of the Kondo effect caused by electron pairing in nanostructures

Tadeusz Domański

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Issues to be addressed:

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- **Superconductivity in nanostructures:**

⇒ **electron pairing** .... **/ due to proximity effect /**

⇒ **subgap quasiparticles** .... **/ Andreev (Shiba) states /**

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⇒ **quantum phase transition** .... **/ spinful $\leftrightarrow$ spinless states /**

⇒ **spin exchange interaction** .... **/ pairing vs Kondo effect /**

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- **Role of Coulomb repulsion:**

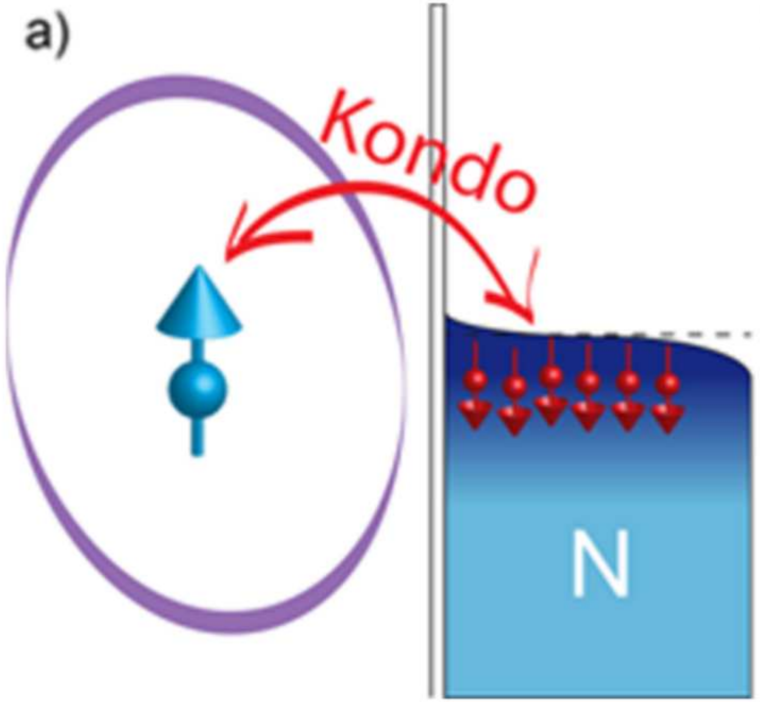
⇒ **quantum phase transition** .... **/ spinful $\leftrightarrow$ spinless states /**

⇒ **spin exchange interaction** .... **/ pairing vs Kondo effect /**

- **What happens when Cooper meets Kondo ?**

Kondo effect – reminder

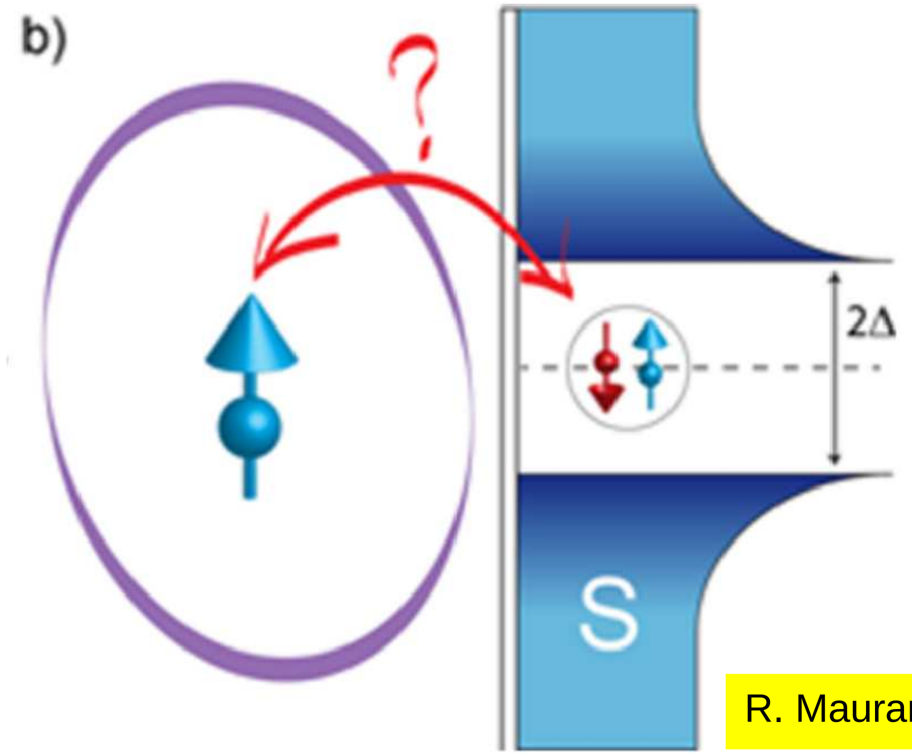
Quantum impurity (dot) embedded in a metallic bath



forms the many-body Kondo state with itinerant electrons thanks to the effective screening (exchange) interactions.

Kondo vs pairing – ′ to screen or not to screen ?′

Quantum impurity (dot) coupled to a superconductor



R. Maurand, Ch. Schönenberger, Physics **6**, 75 (2013).

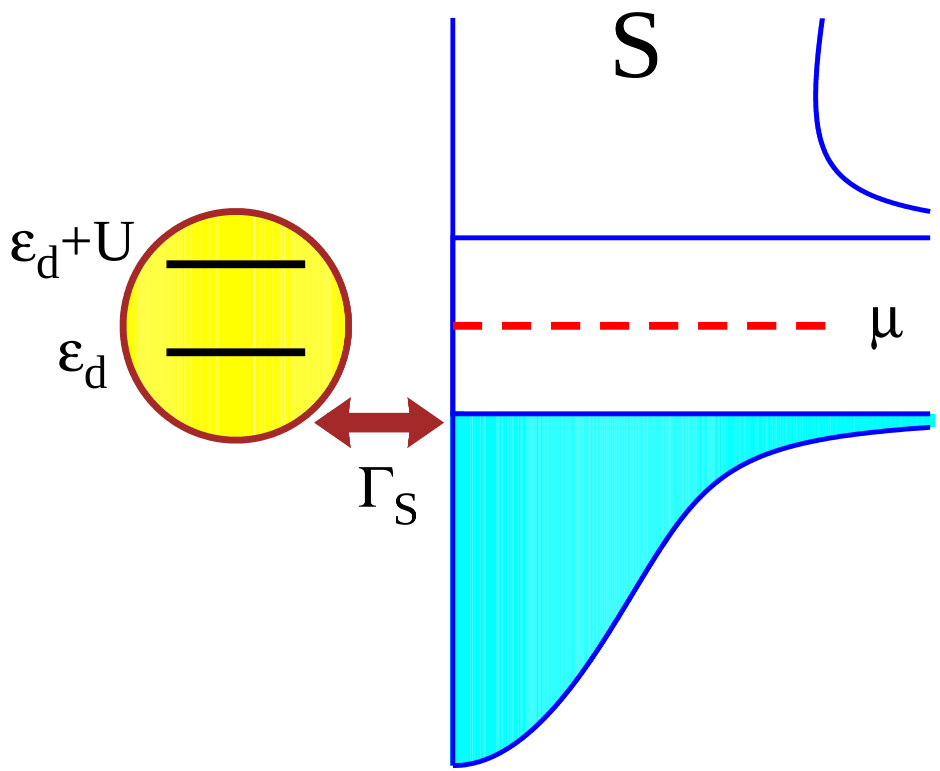
behaves differently, because :

$\Rightarrow$ **electronic states near the Fermi level are missing,**

$\Rightarrow$ **pairing has <u>nontrivial</u> interplay with Kondo state.**

Quantum impurity (dot) in a superconducting host

Electronic spectrum



Microscopic model

Anderson-type Hamiltonian

Quantum impurity (dot)

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^\dagger_\sigma \, \hat{d}_\sigma \; + \; U \; \hat{n}_{d\uparrow} \; \hat{n}_{d\downarrow}$$

coupled with a superconductor

$$\begin{aligned} \hat{H} \; &= \; \sum_{\sigma} \epsilon_d \hat{d}^\dagger_\sigma \hat{d}_\sigma + U \; \hat{n}_{d\uparrow} \; \hat{n}_{d\downarrow} + \hat{H}_S \\ &+ \; \sum_{\mathrm{k},\sigma} \left(V_{\mathrm{k}} \; \hat{d}^\dagger_\sigma \hat{c}_{\mathrm{k}\sigma} + V^*_{\mathrm{k}} \; \hat{c}^\dagger_{\mathrm{k}\sigma} \hat{d}_\sigma \right) \end{aligned}$$

where

$$\hat{H}_S = \sum_{k,\sigma} (\varepsilon_k - \mu) \, \hat{c}^\dagger_{k\sigma} \hat{c}_{k\sigma} - \sum_k \left(\Delta \hat{c}^\dagger_{k\uparrow} \; \hat{c}^\dagger_{k\downarrow} + \mathrm{h.c.} \right)$$

Uncorrelated QD

$U_d = 0$ **(exactly solvable case)**

Uncorrelated QD

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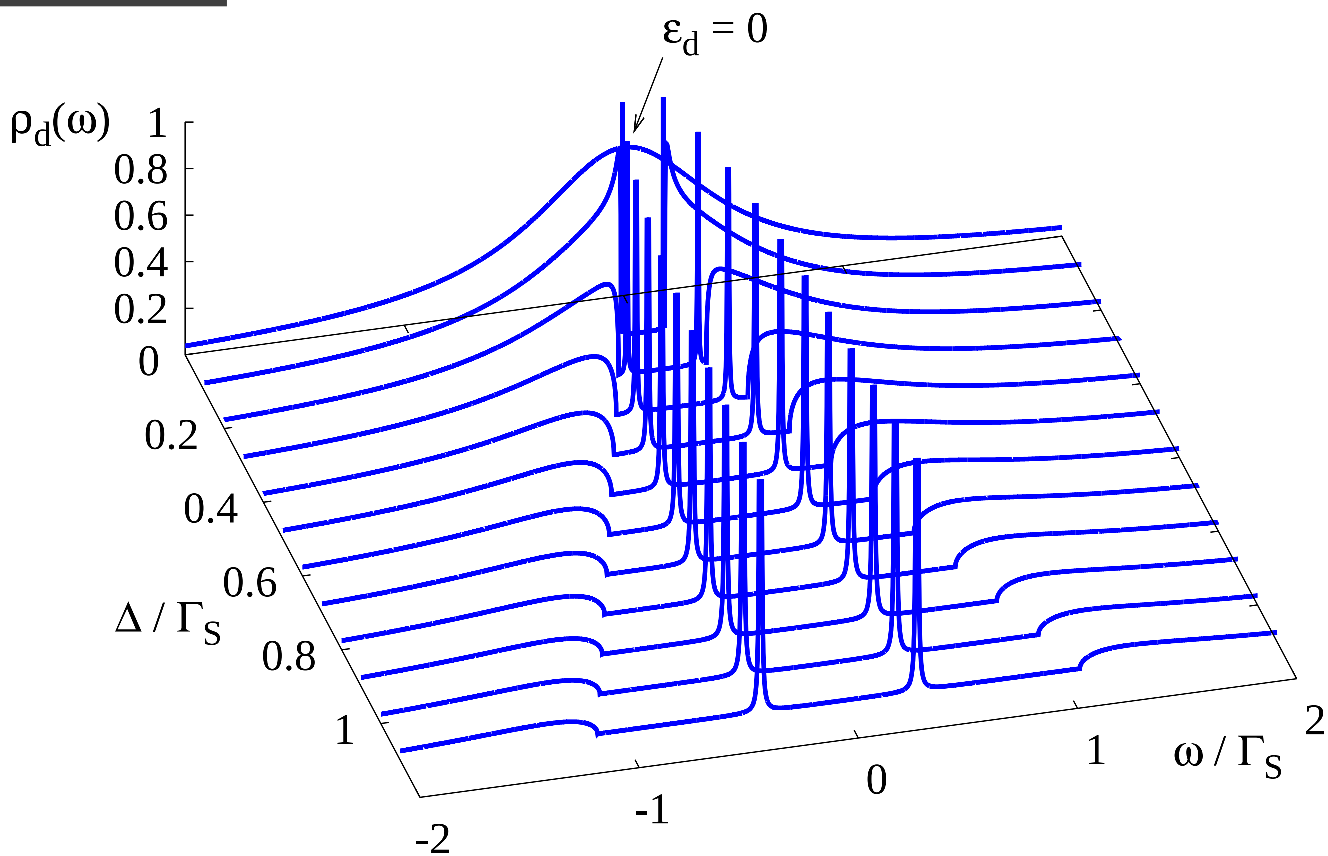


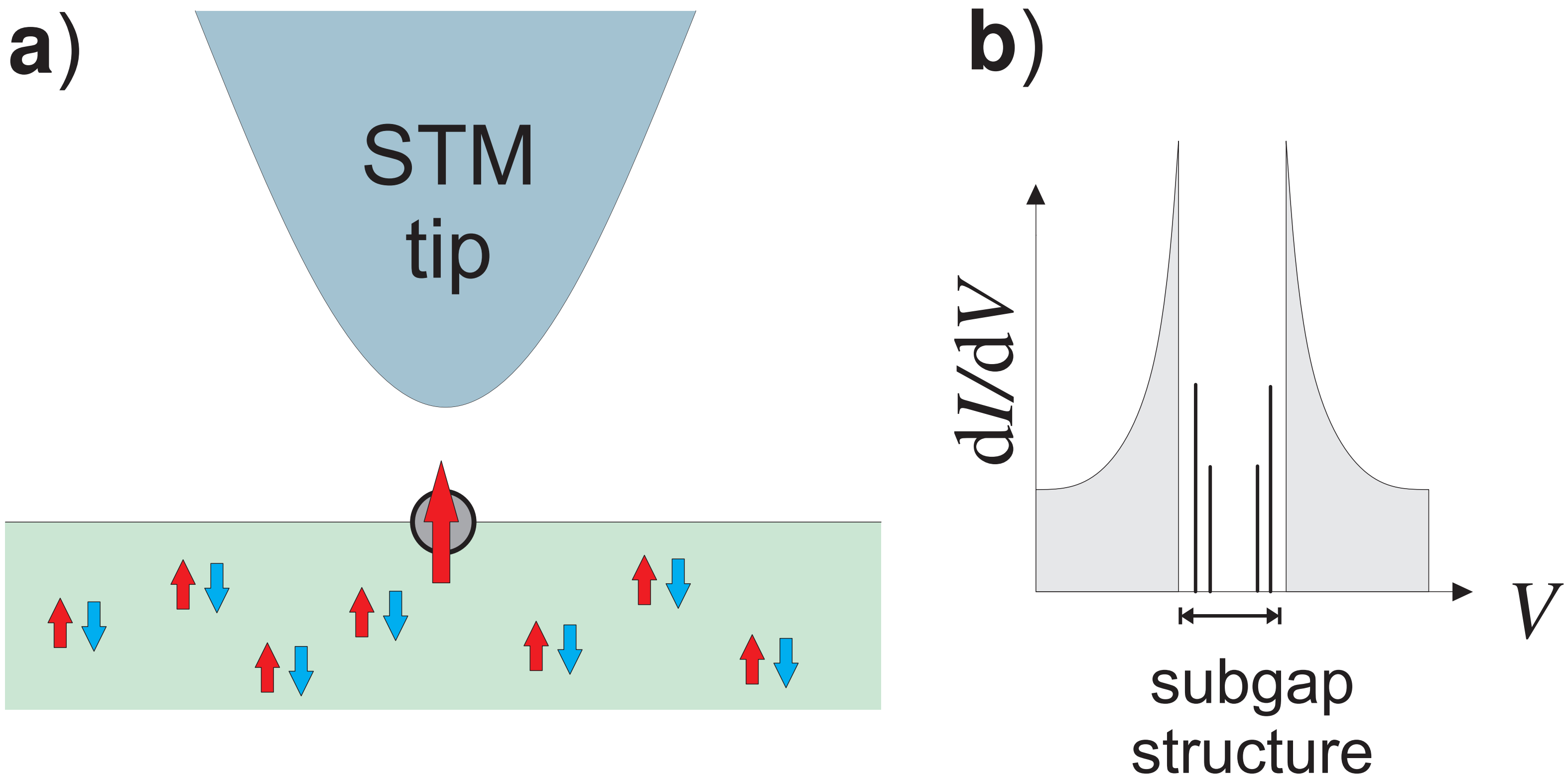
In-gap (Andreev/Shiba) bound states :

⇒ always appear in pairs,

⇒ appear symmetrically at finite energies.

J. Barański and T. Domański, J. Phys.: Condens. Matter **25**, 435305 (2013).

Subgap states of multilevel quantum impurities



a) STM scheme and b) differential conductance for a multilevel quantum impurity adsorbed on a superconductor surface.

R. Žitko, O. Bodensiek, and T. Pruschke, Phys. Rev. B **83**, 054512 (2011).

Correlated QD – spinful and/or spinless states

In a subgap regime $|\omega| \ll \Delta$ the proximized impurity can described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left(\Delta_d \, \hat{d}^{\dagger}_{\uparrow} \hat{d}^{\dagger}_{\downarrow} + \mathrm{h.c.} \right)$$

with the induced pairing potential $\Delta_d = \Gamma_S / 2$.

The true eigen-states of this problem are:

$\vert\uparrow\rangle$ and $\vert\downarrow\rangle$	$\Leftarrow$	doublet (spin $\frac{1}{2}$)
$u\,\vert 0\rangle - v\,\vert\uparrow\downarrow\rangle$, $v\,\vert 0\rangle + u\,\vert\uparrow\downarrow\rangle$	$\Leftarrow$	singlets (spin 0)

Possible quantum phase transition occurs upon varying ε_d, U_d or Γ_S.

Correlated QD – quantum phase transition

An example: ground state of the half-filled QD

$\lvert\uparrow\rangle$ or $\lvert\downarrow\rangle$	when $\Gamma_S < U$	(spinful state)
$u\,\lvert 0\rangle - v\,\lvert\uparrow\downarrow\rangle$	when $\Gamma_S > U$	(spinless state)

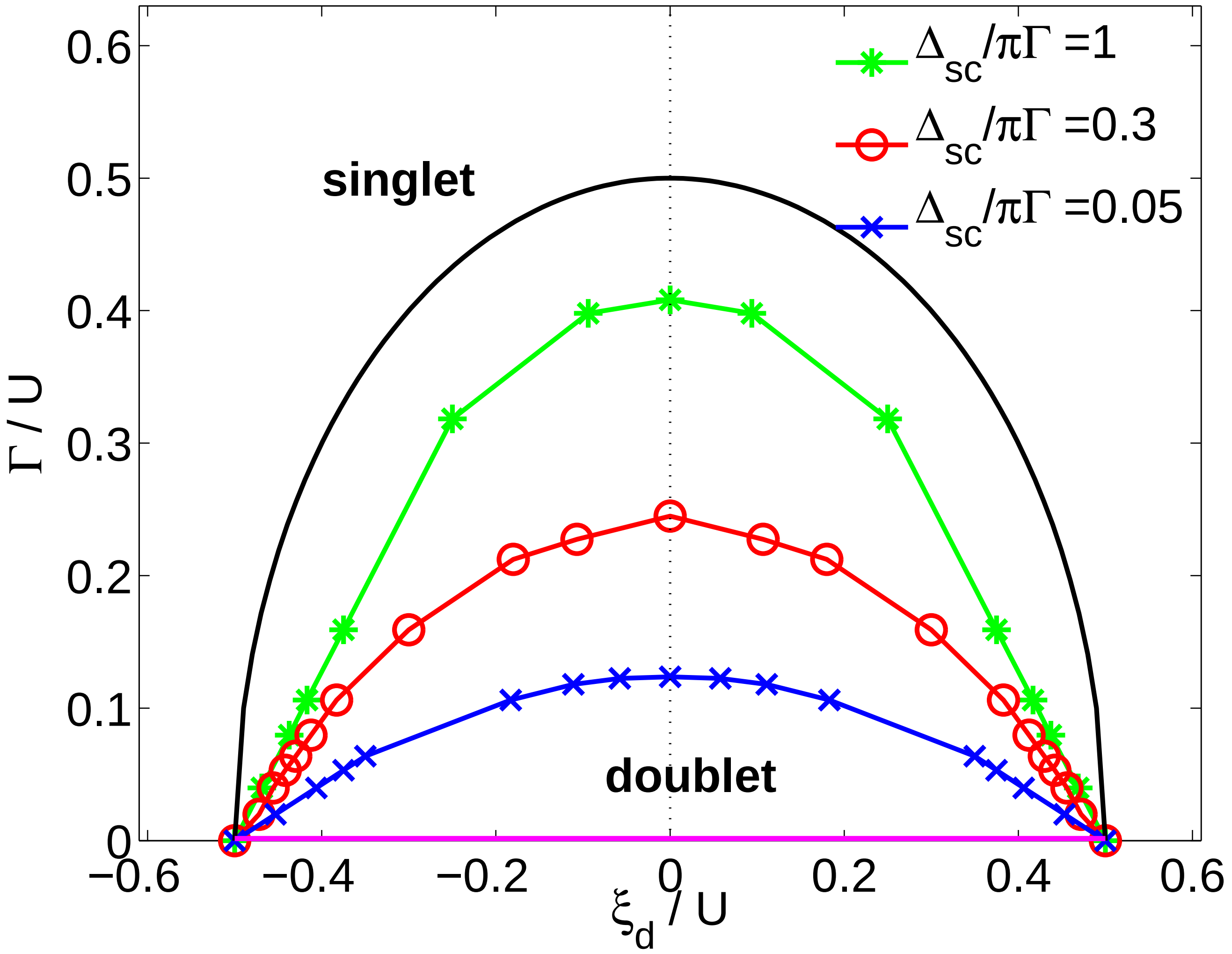
Correlated QD – quantum phase transition

An example: ground state of the half-filled QD

$\|\uparrow\rangle$ or $\|\downarrow\rangle$	when $\Gamma_S < U$	(spinful state)
$u\,\|0\rangle - v\,\|\uparrow\downarrow\rangle$	when $\Gamma_S > U$	(spinless state)

Important remark : the spinless state cannot be screened !

Singlet-doublet transition – NRG results for arbitrary Δ



$\xi_d \equiv \varepsilon_d + \frac{1}{2}U$

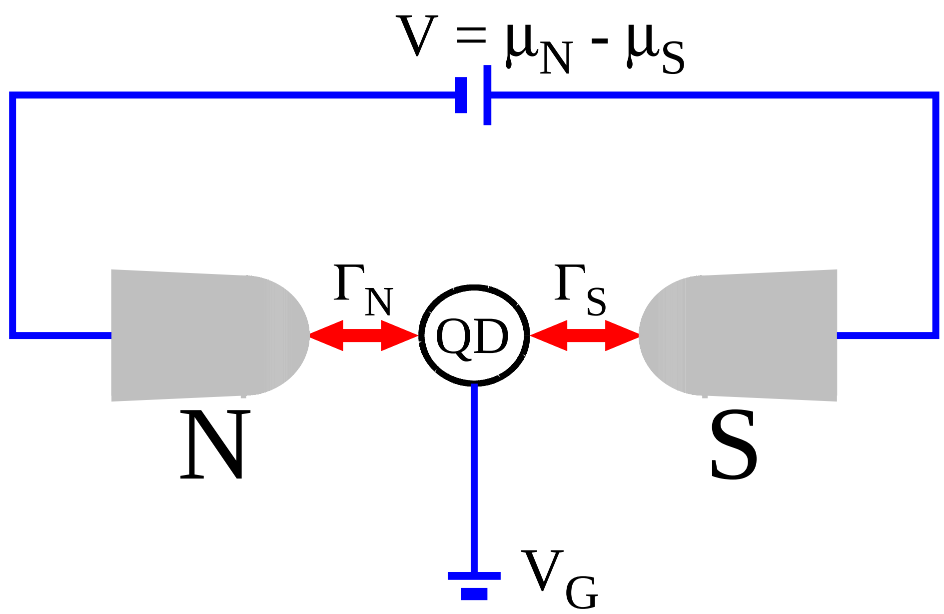
J. Bauer, A. Oguri, and A.C. Hewson, J. Phys.: Condens. Matter **19**, *486211 (2007).*

N–QD–S heterojunction

/Confrontation: Cooper vs Kondo /

Physical situation

To probe the subgap states one can study the electron transport through a quantum dot (QD) coupled between the normal (N) and superconducting (S) electrodes



metallic lead

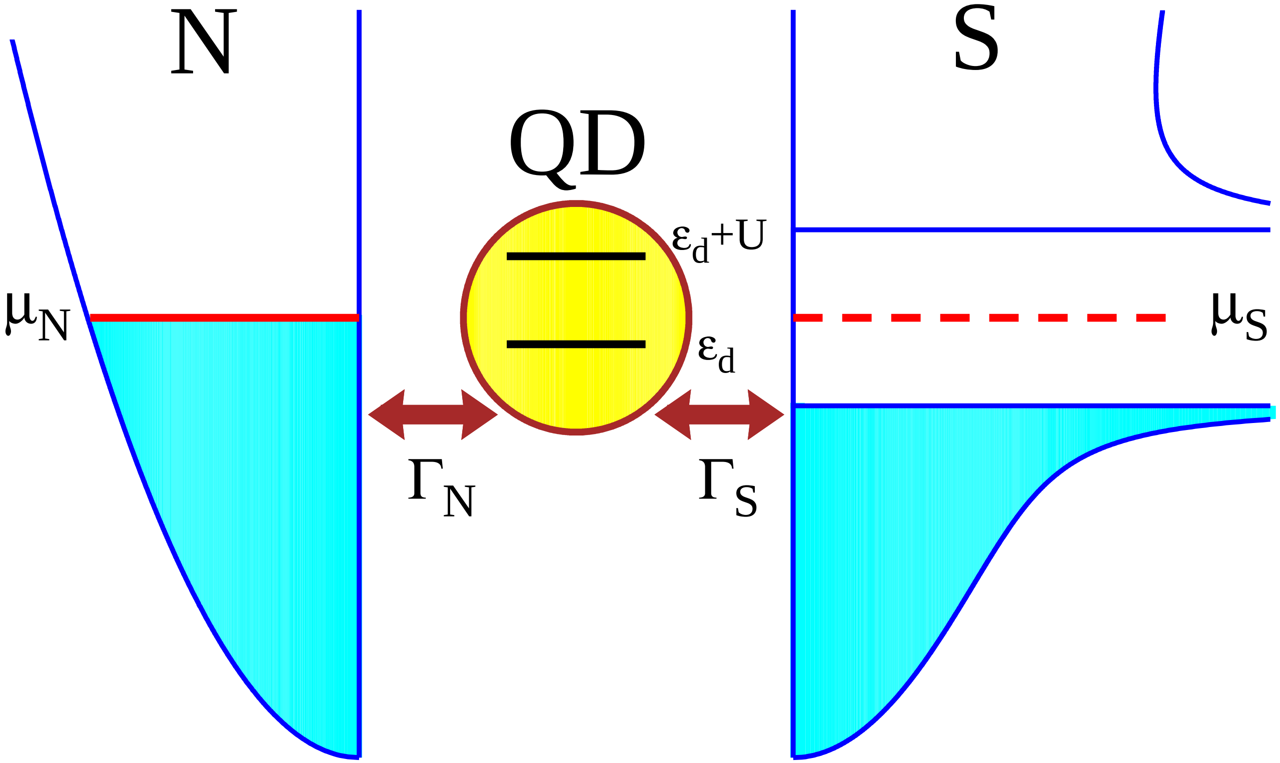
QD

superconductor

This N–QD–S setup has been practically studied in several recent experiments.

Electronic spectrum

Spectrum of the N-QD-S heterostructure

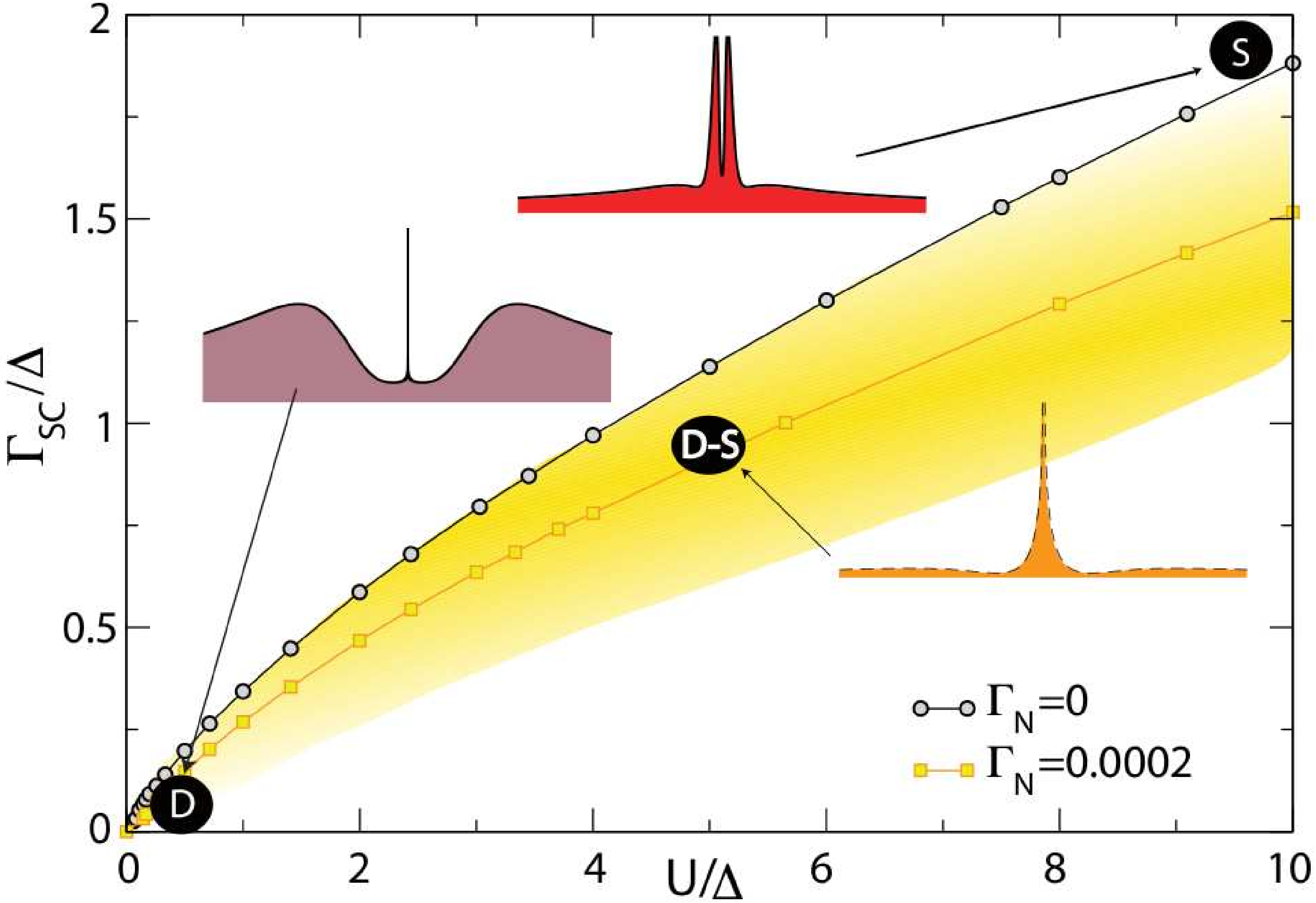


Kondo state in a subgap regime

singlet $\longleftrightarrow$ doublet crossover

Kondo state in a subgap regime

singlet ↔ doublet crossover



Phase diagram obtained by NRG Ljubljana code.

R. Žitko, J.S. Lim, R. López, and R. Aguado, Phys. Rev. B **91**, 045441 (2015).

Kondo state in a subgap regime

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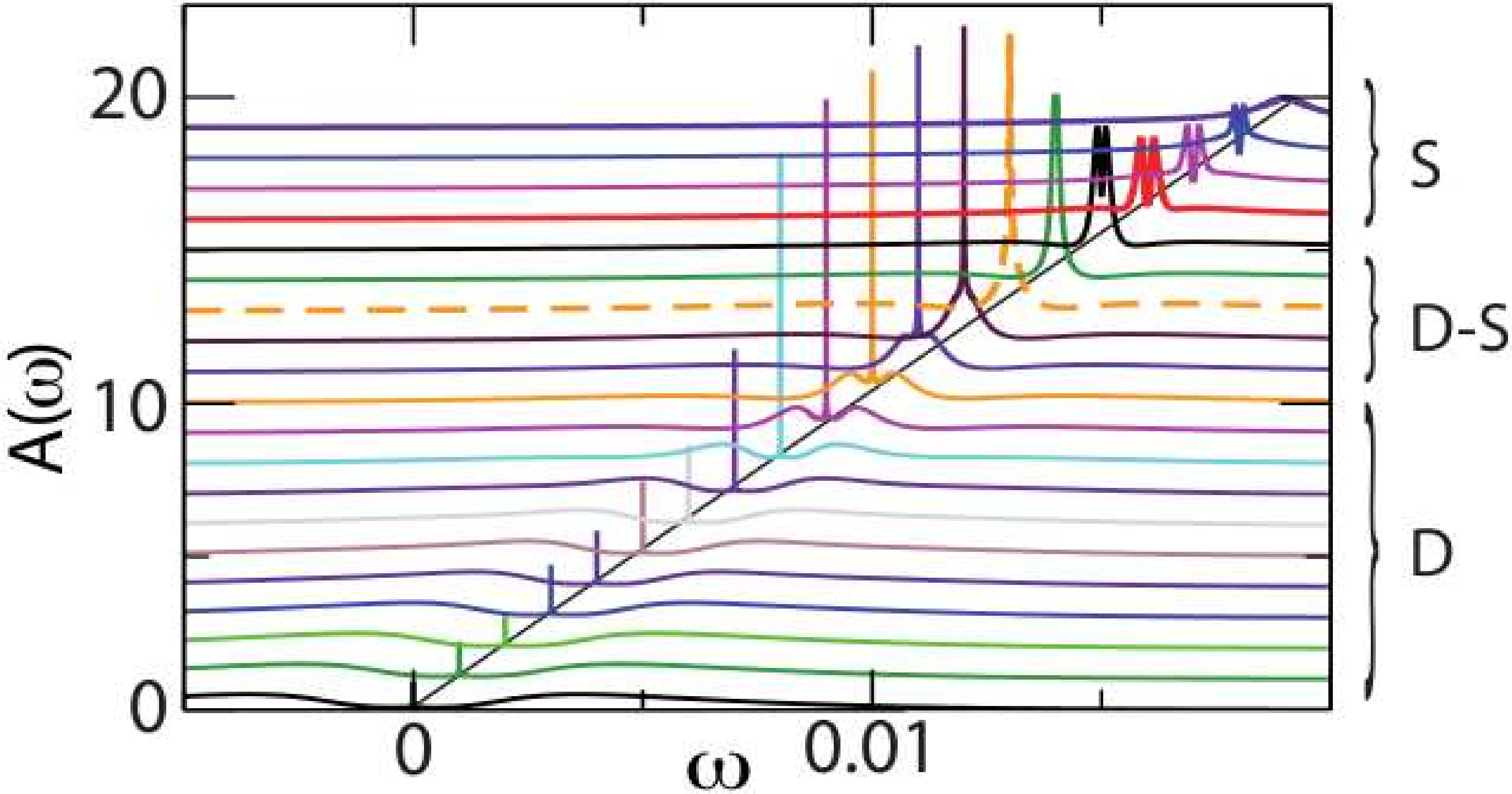

QD spectrum obtained by NRG Ljubljana code.

R. Žitko, J.S. Lim, R. López, and R. Aguado, Phys. Rev. B **91**, 045441 (2015).

Motivation $\longrightarrow$ strategy

/ after Ustroń-2014 conference /

Intrigued by such weird evolution of the Kondo peak we revisited the same problem, using several complementary methods:

$\Rightarrow$ **perturbative treatment of $V_{k,N}$ (a lá Schrieffer and Wolff),**

$\Rightarrow$ **$2^{\rm nd}$ order perturbative treatment of the Coulomb potential,**

$\Rightarrow$ **NRG calculations (Budapest code).**

T. Domański, I. Weymann, M. Barańska & G. Górski, Scientific Reports **6**, 23336 (2016).

Correlated quantum dot – exact solution for $\Gamma_N = 0^+$

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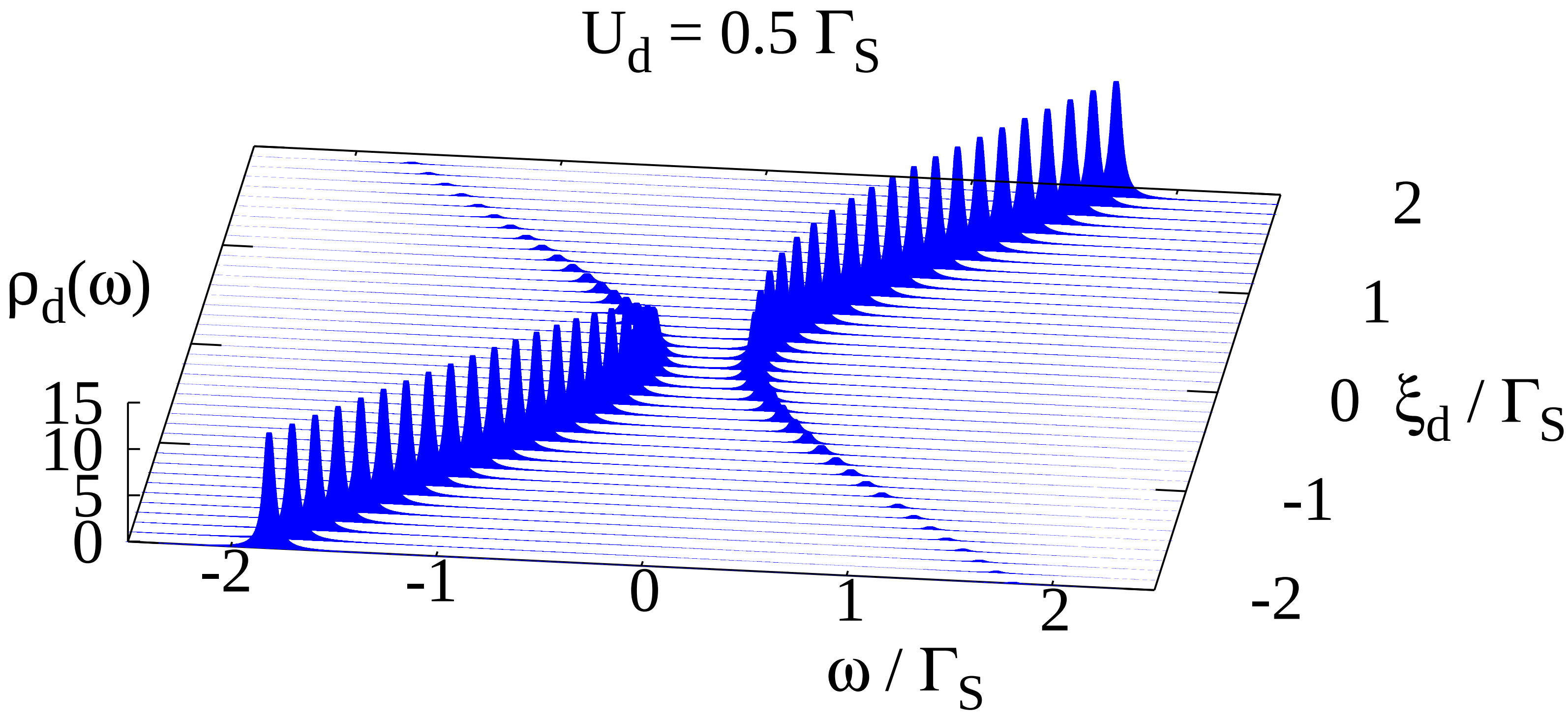
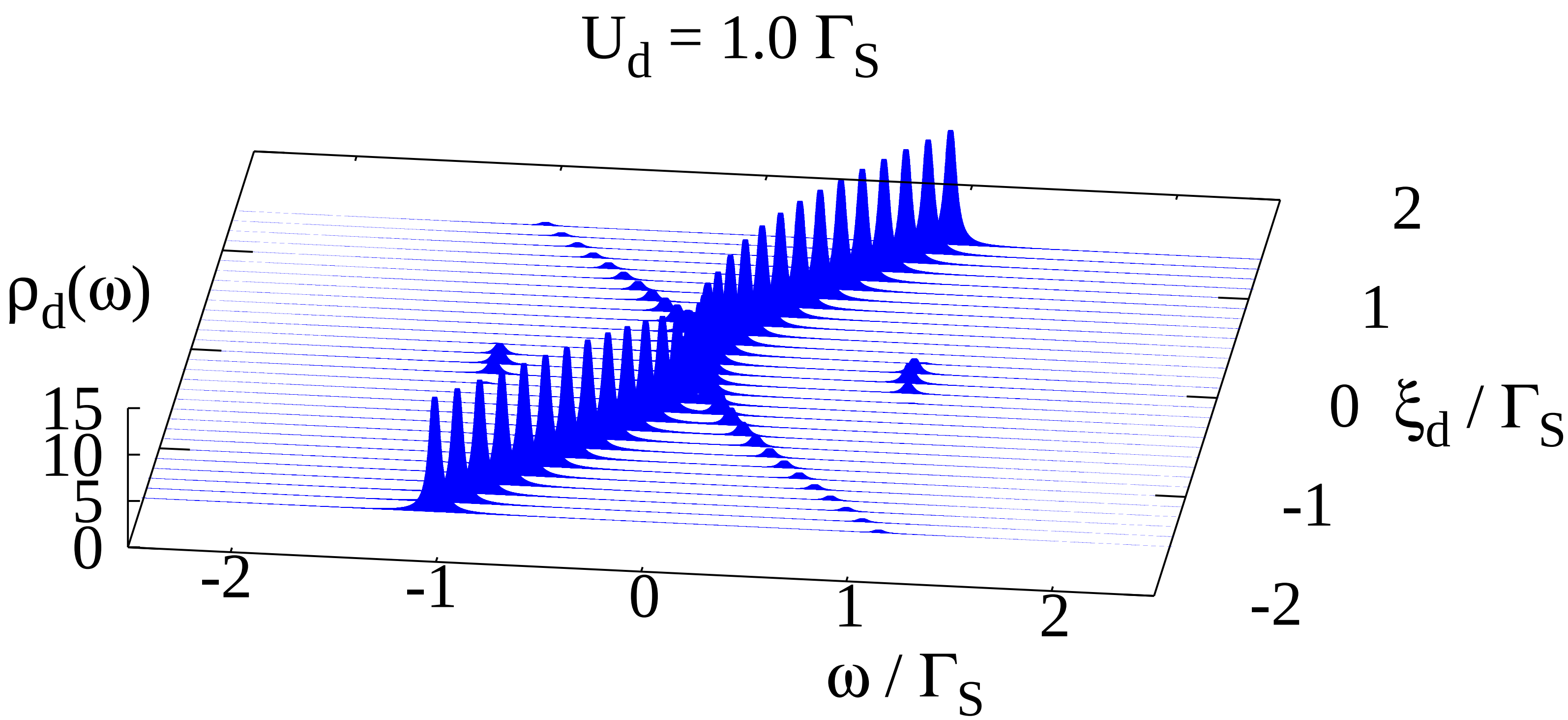
Subgap spectrum $\rho_d(\omega)$ for varying $\xi_d \equiv \varepsilon_d + \frac{1}{2}U_d$



BCS-like states

Correlated quantum dot – exact solution for $\Gamma_N = 0^+$

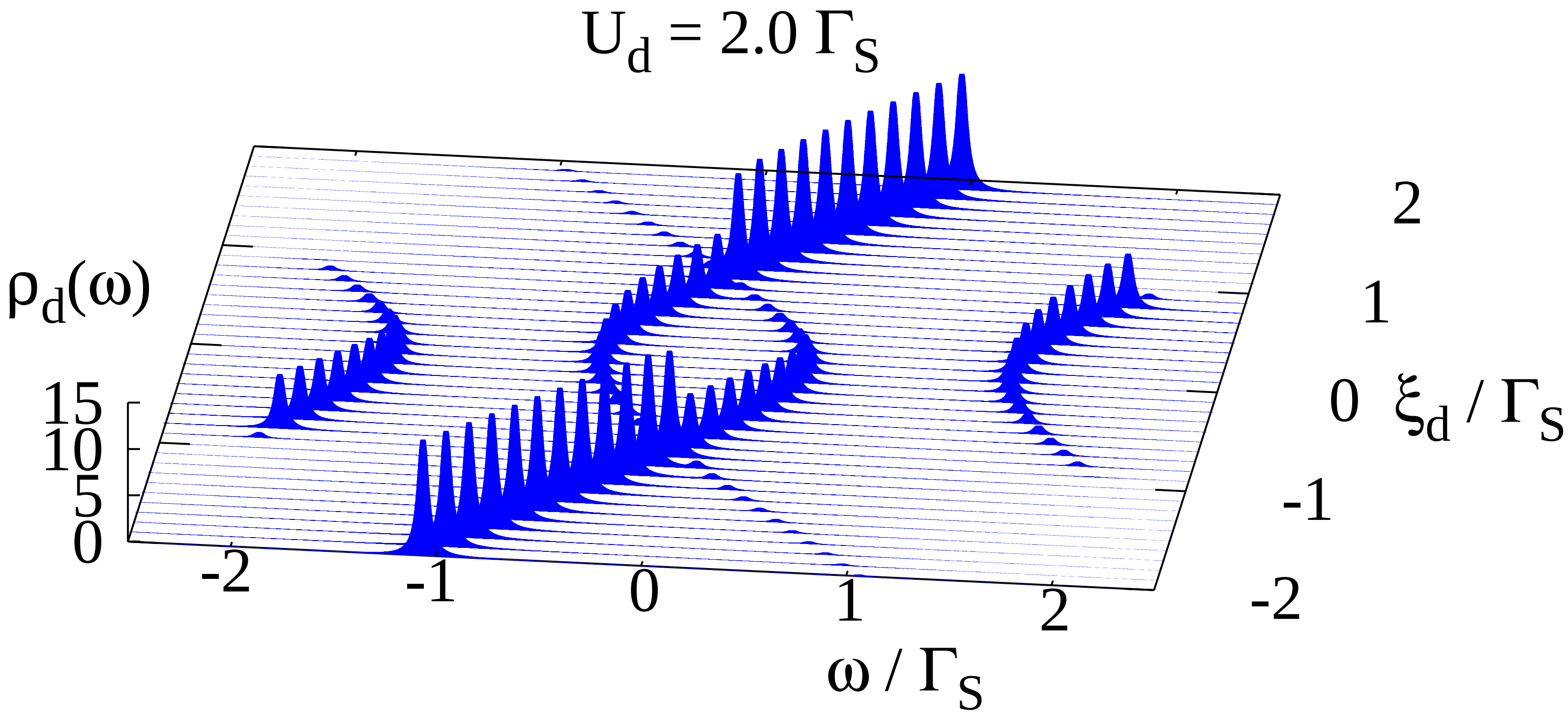
Subgap spectrum $\rho_d(\omega)$ for varying $\xi_d \equiv \varepsilon_d + \frac{1}{2}U_d$



near the quantum phase transition

Correlated quantum dot – exact solution for $\Gamma_N = 0^+$

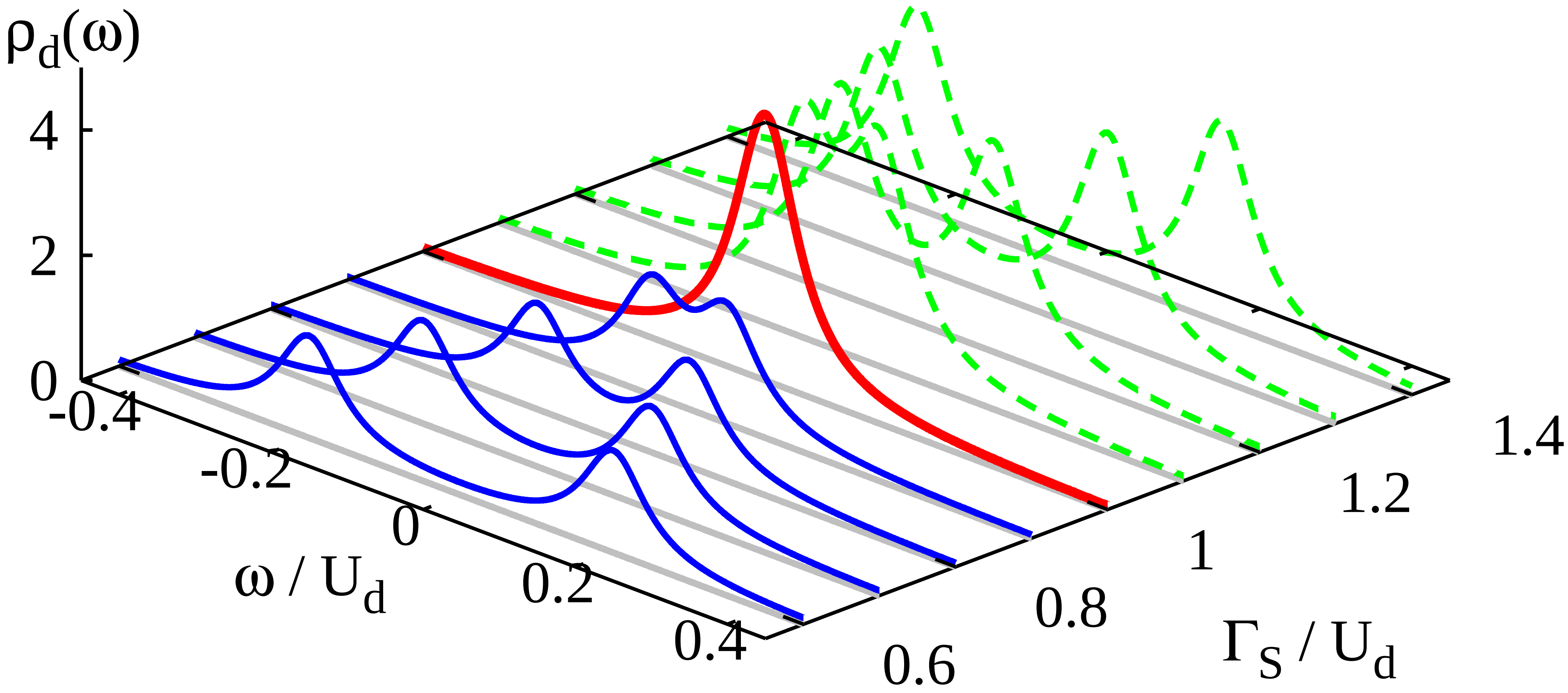
Subgap spectrum $\rho_d(\omega)$ for varying $\xi_d \equiv \varepsilon_d + \frac{1}{2}U_d$



crossings of the in-gap states

Correlated quantum dot – exact solution for $\Gamma_N = 0^+$

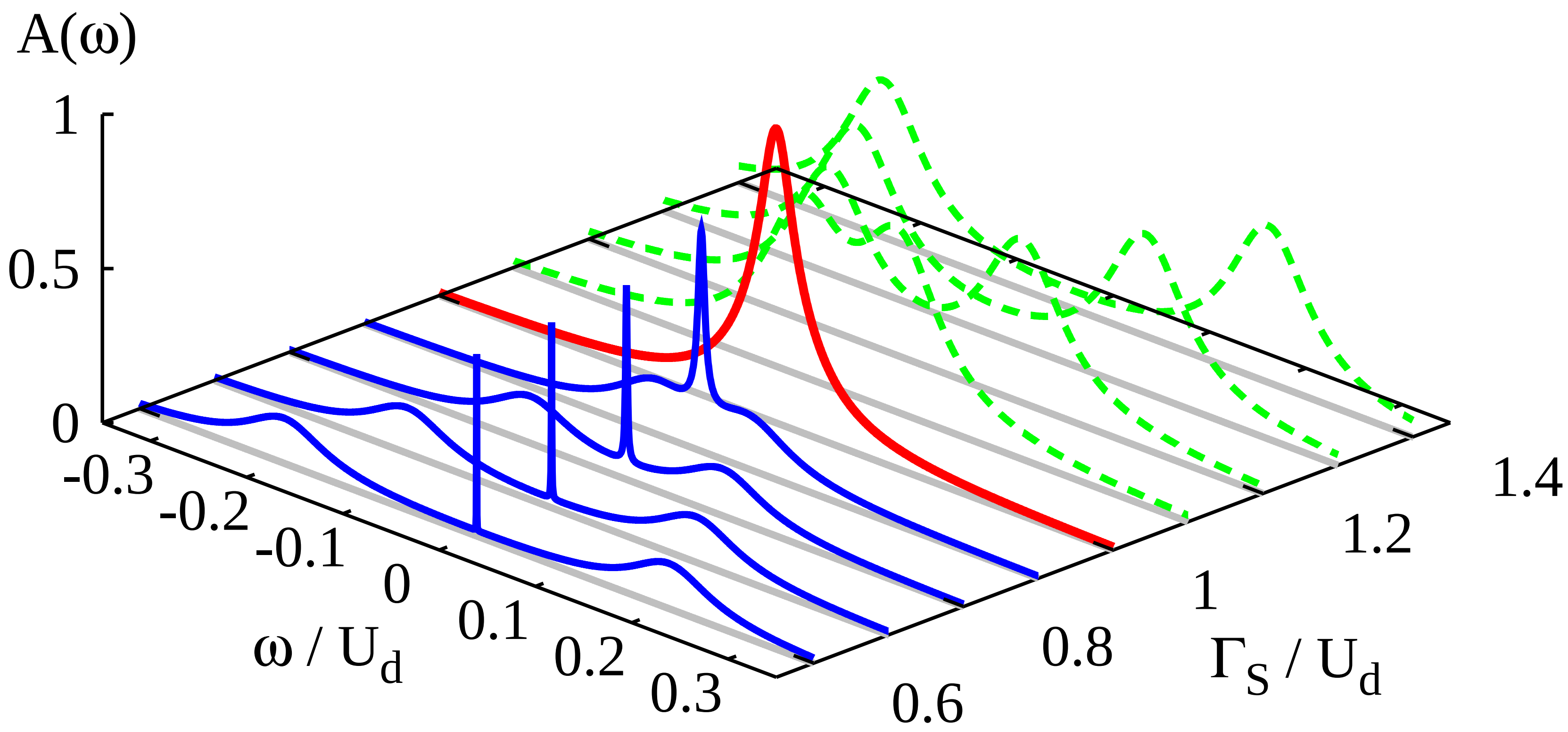
QD spectrum for $\varepsilon_d = -\frac{1}{2}U_d$



Quantum phase transition from the doublet to singlet states

Correlated quantum dot – $\Gamma_N \ll \Gamma_S$

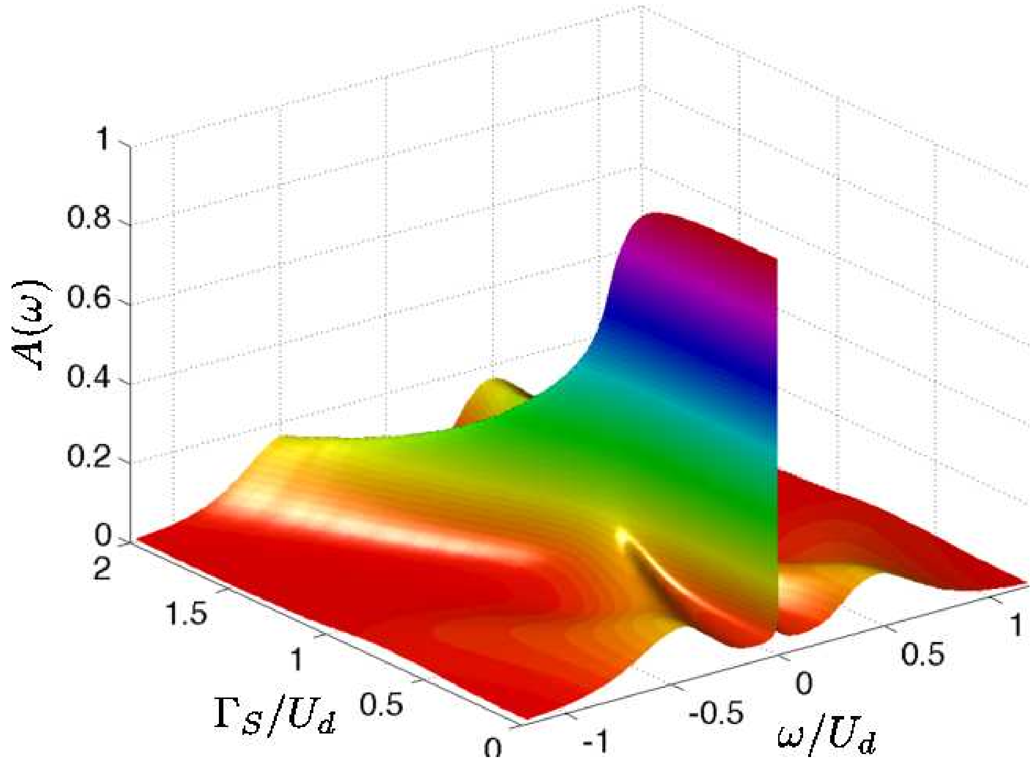
The half-filled quantum dot ($\varepsilon_d = -\frac{1}{2}U_d$)



Kondo peak is present only in the spinful (doublet) state

Results obtained from the generalized Schrieffer-Wolff transformation

Correlated quantum dot – NRG results



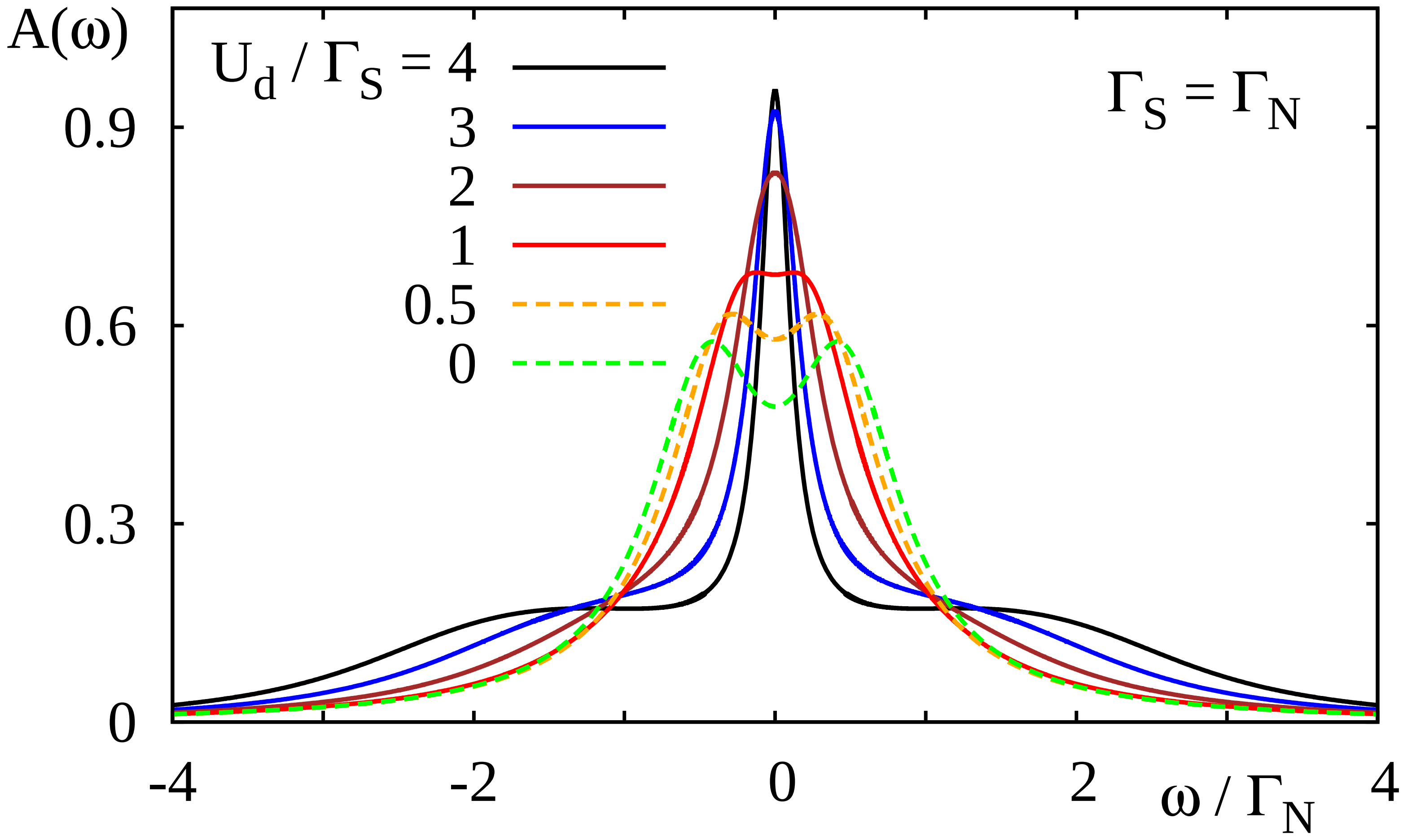
Very intriguing observation: T_K is enhanced with increasing Γ_S

Results obtained from the NRG calculations (Budapest code).

T. Domański, I. Weymann, M. Barańska & G. Górski, Scientific Reports **6**, 23336 (2016).

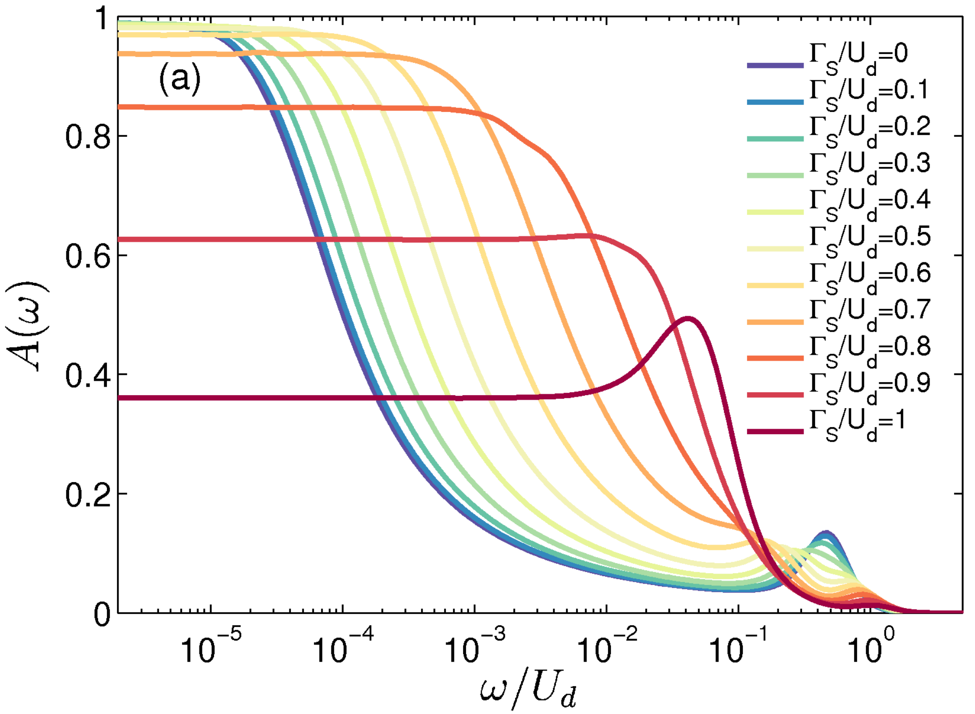
Correlated quantum dot – arbitrary Γ_N

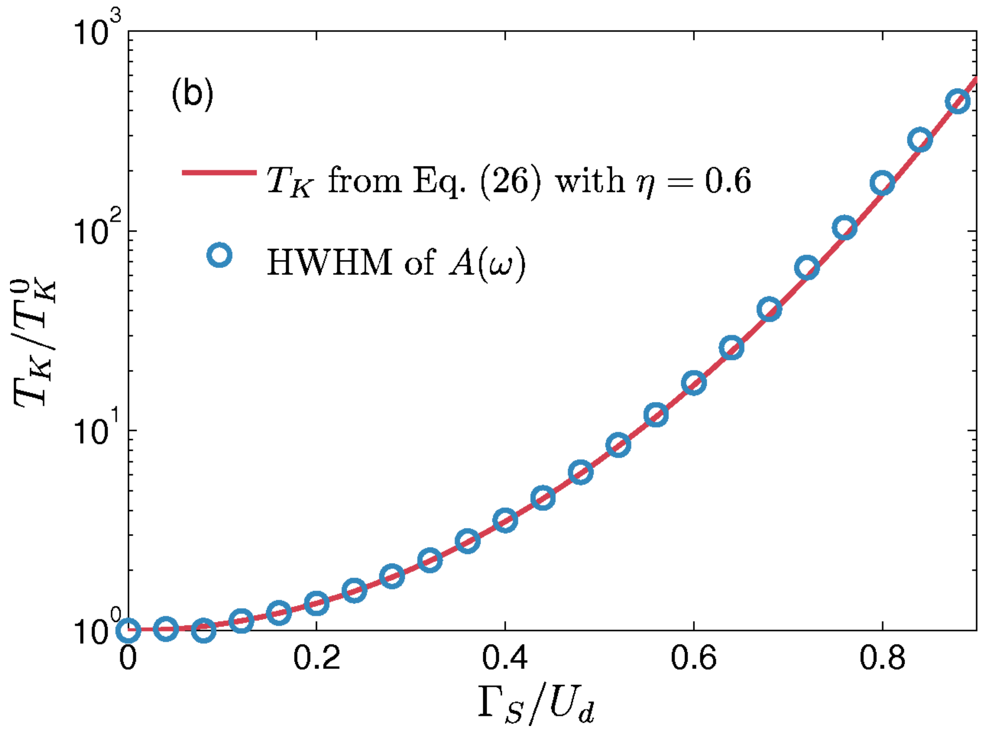
The half-filled quantum dot ($\varepsilon_d = -\frac{1}{2}U_d$)



Results obtained from the 2-nd order perturbative treatment of U_d.

Correlated quantum dot – Kondo temperature T_K





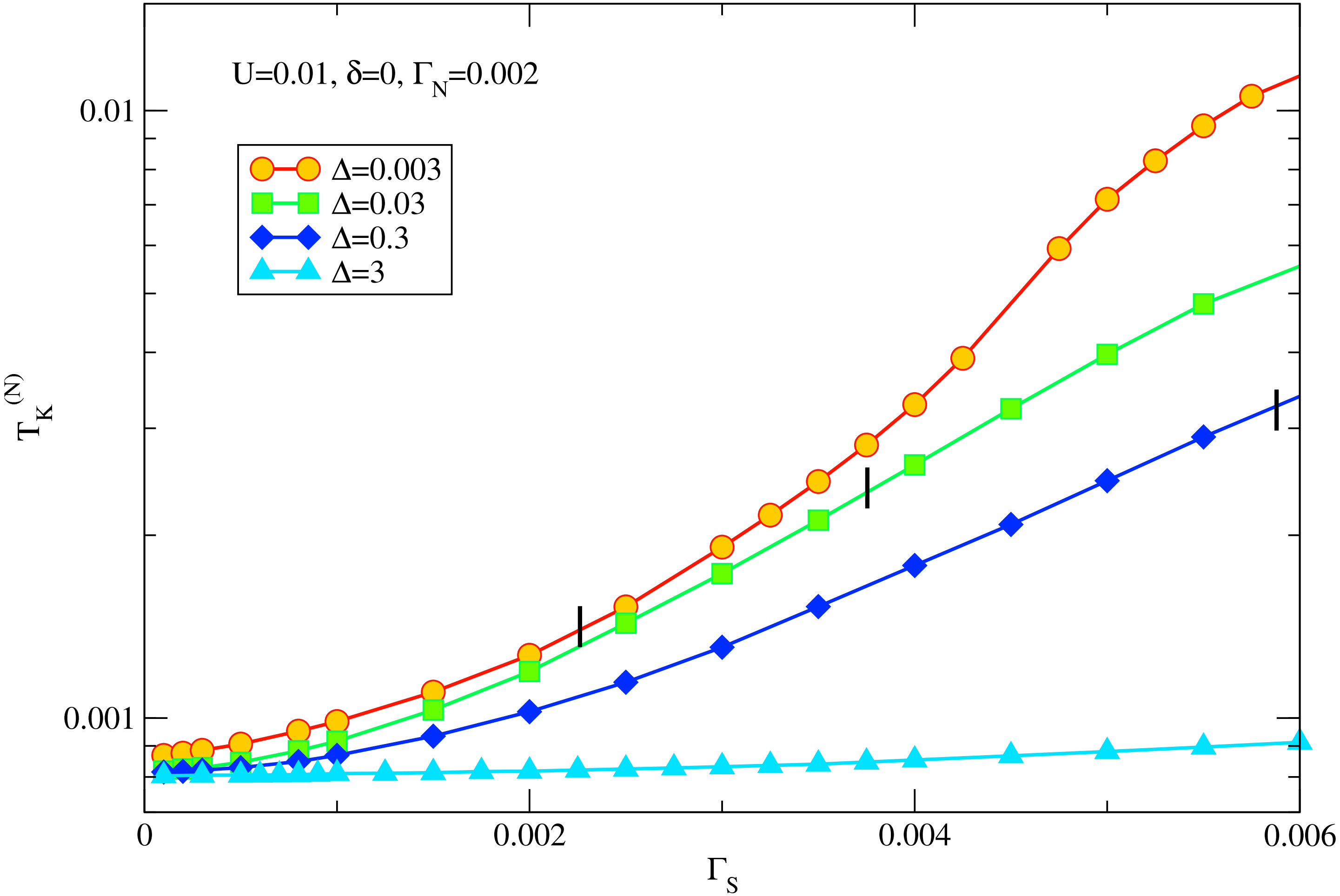
T_K estimated from the NRG and the extended Schrieffer-Wolff transformation

$$T_K \simeq 0.3\,\sqrt{\Gamma_N U_d}\,\exp\left[\frac{\pi\varepsilon_d(\varepsilon_d+U_d)+(\Gamma_S/2)^2}{\Gamma_N U_d}\right]$$

T. Domański et al, Scientific Reports **6**, 23336 (2016).

Correlated quantum dot – **Kondo temp. T_K for finite Δ_{sc}**

R. Žitko (private information)



Results obtained by the NRG calculations (Ljubljana code).

Conclusions

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Electron pairing in nanosystems:

⇒ **induces the subgap (Andreev/Shiba) states**

⇒ **has unusual relationship with correlations**

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Part 2

$\Rightarrow$ **Majorana quasiparticles**

Experimental evidence – for Majorana quasiparticles

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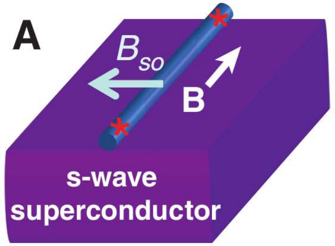
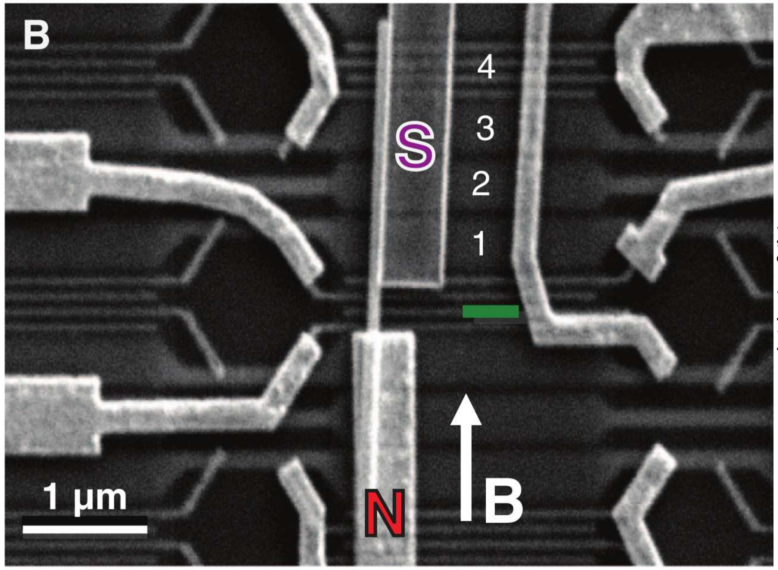
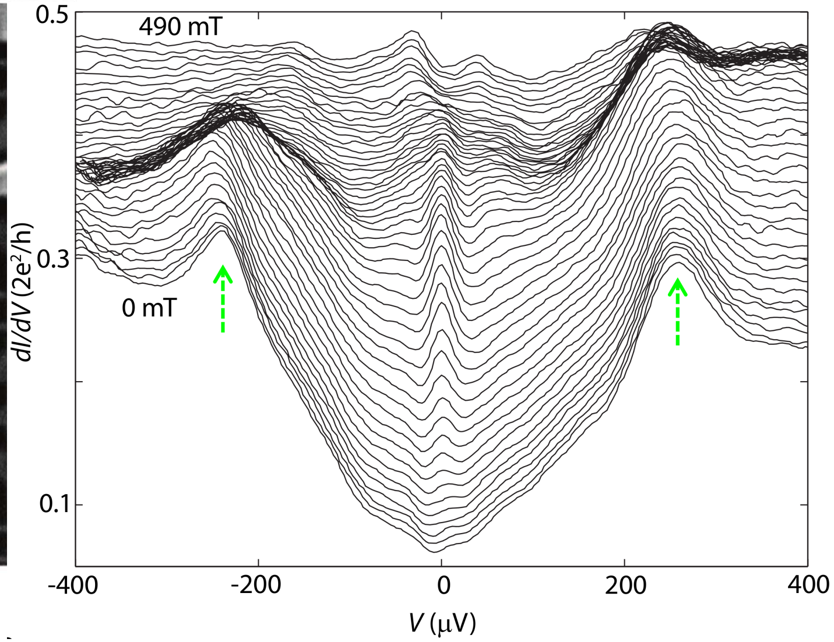
InSb nanowire between a metal (gold) and a superconductor (Nb-Ti-N)







dI/dV measured at 70 mK for varying magnetic field B indicated:

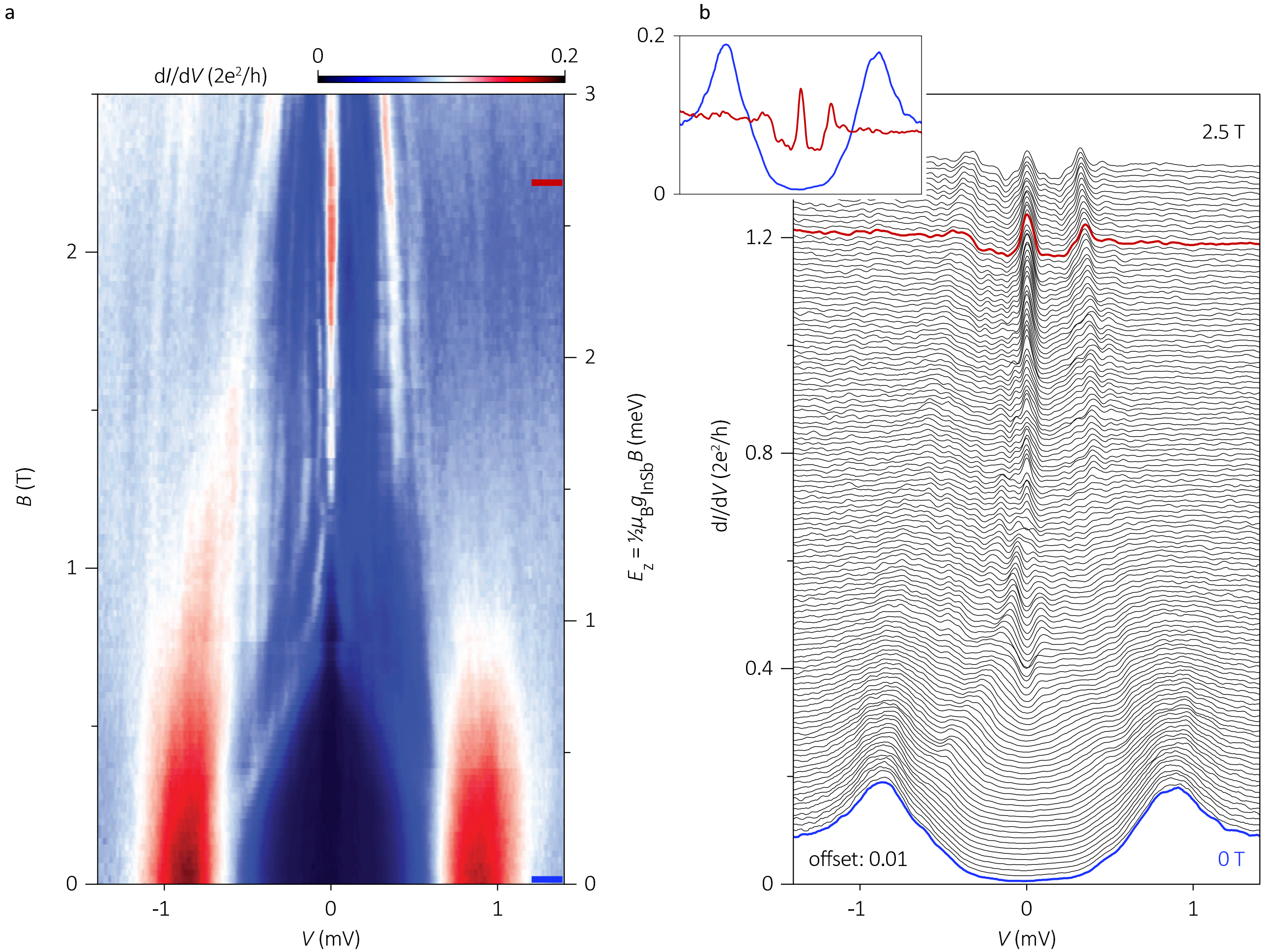
$\Rightarrow$ a zero-bias enhancement due to Majorana state

V. Mourik, ..., and L.P. Kouwenhoven, Science **336**, 1003 (2012).

/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /

Experimental evidence – for Majorana quasiparticles

InSb nanowire between a metal (gold) and a superconductor (Nb-Ti-N)

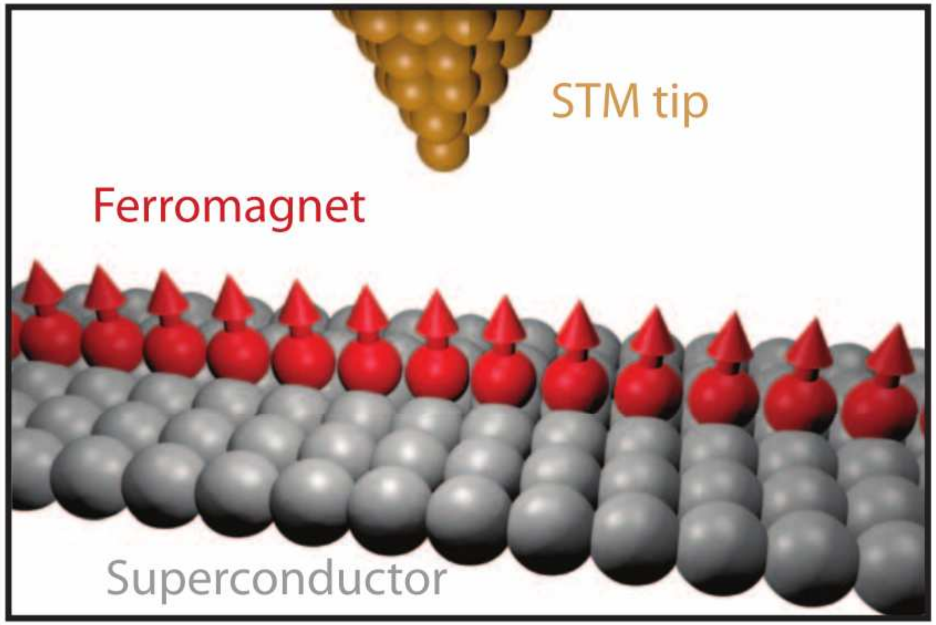


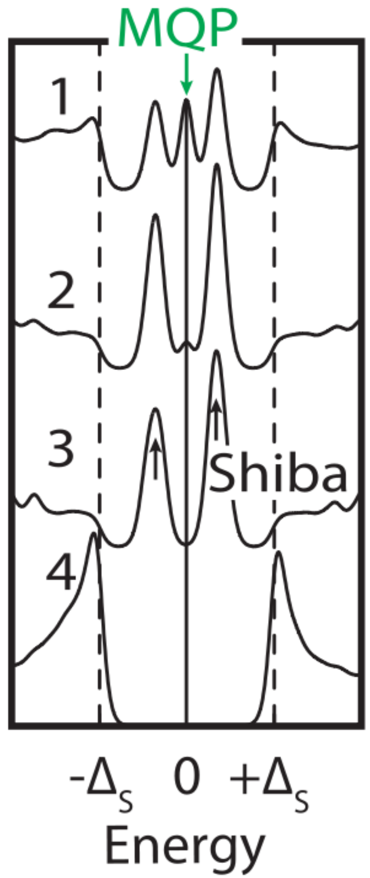
H. Zhang, ..., and L.P. Kouwenhoven, arXiv:1603.04069 (2016).

/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /

Experimental evidence – for Majorana quasiparticles

A chain of iron atoms deposited on a surface of superconducting lead





STM measurements provided evidence for:

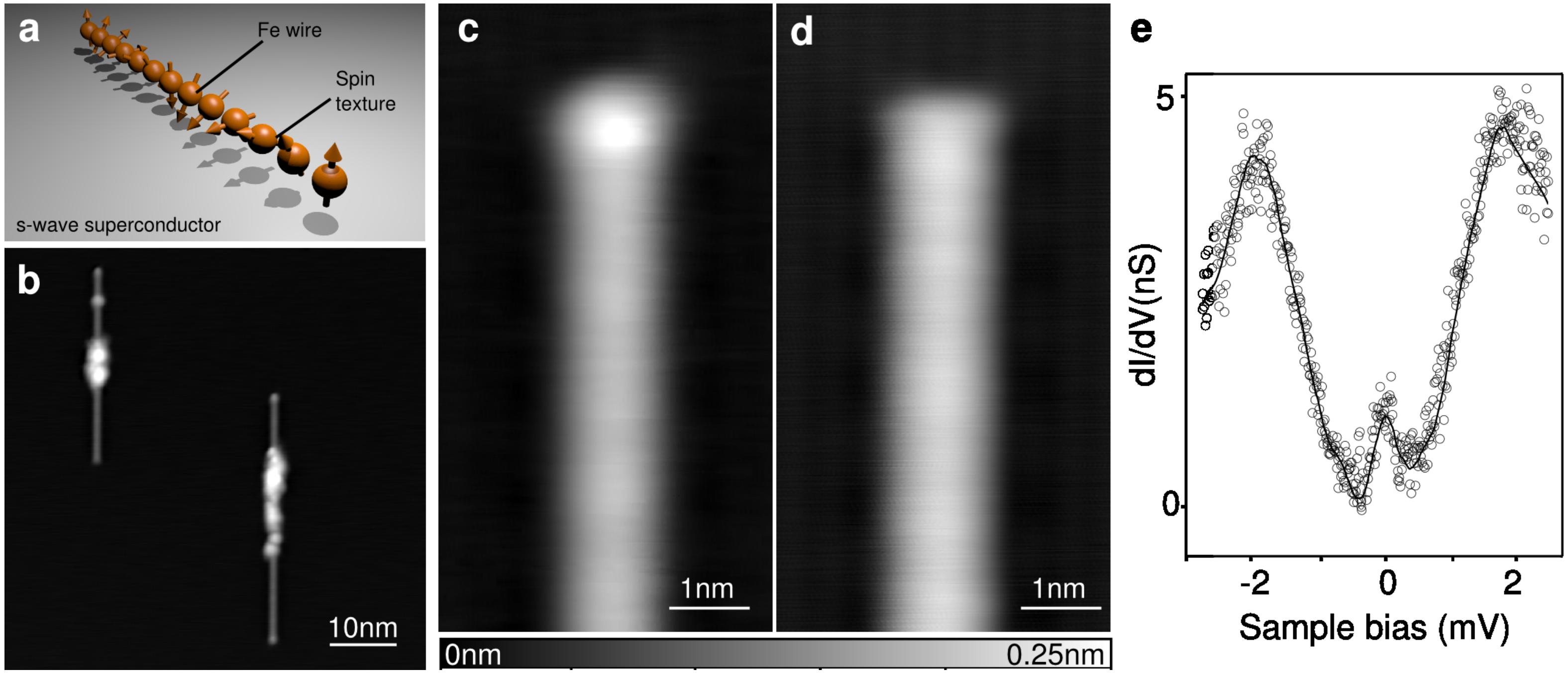
⇒ **Majorana bound states at the edges of a chain.**

S. Nadj-Perge, ..., and A. Yazdani, Science **346**, 602 (2014).

/ Princeton University, Princeton (NJ), USA /

Experimental evidence – for Majorana quasiparticles

Self-assembled Fe chain on superconducting Pb(110) surface



AFM combined with STM provided evidence for:

$\Rightarrow$ **Majorana bound states at the edges of a chain.**

R. Pawlak, M. Kisiel , ..., and E. Meyer, arXiv:1505.06078 (2015).

/ University of Basel, Switzerland /

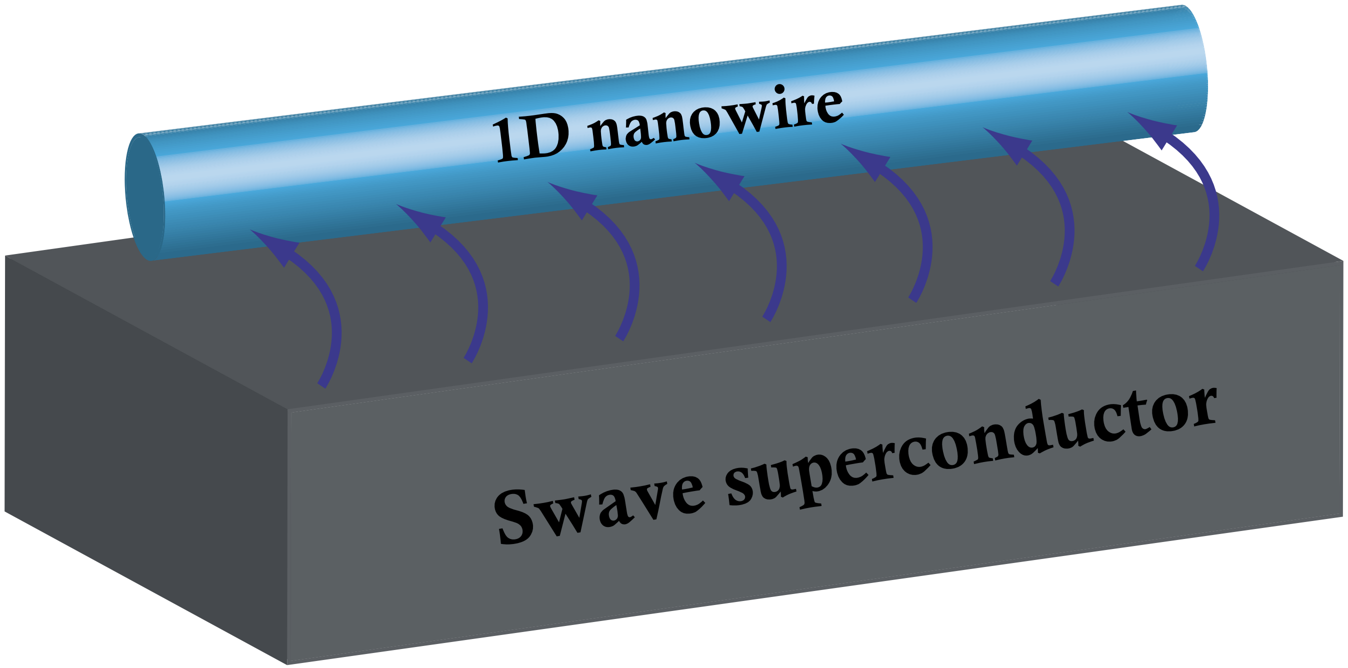
Question:

$\Rightarrow$ **where does Majorana come from ?**

Andreev vs Majorana states – a story of mutation

Andreev vs Majorana states – a story of mutation

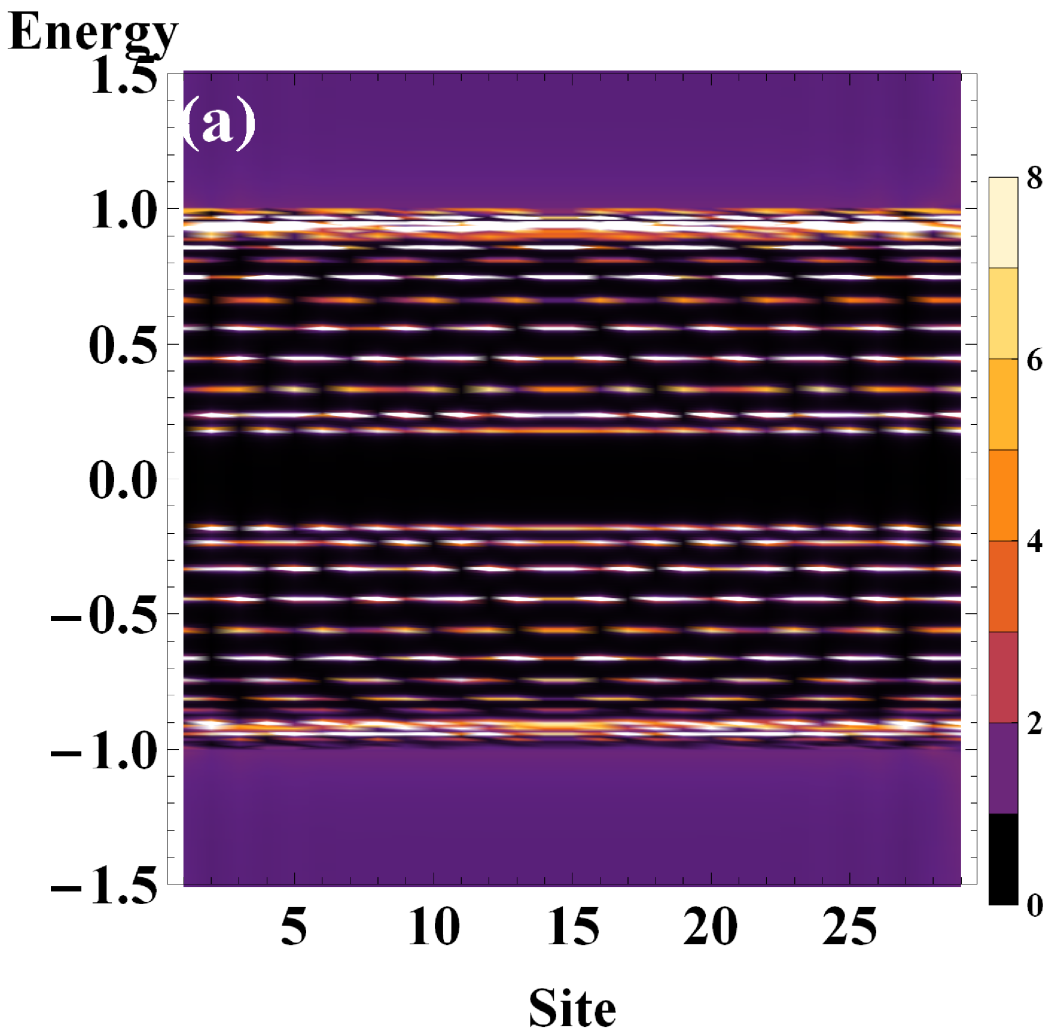
Let us consider:



1D quantum wire deposited on s-wave superconductor

D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B ***88****, 165401 (2013).*

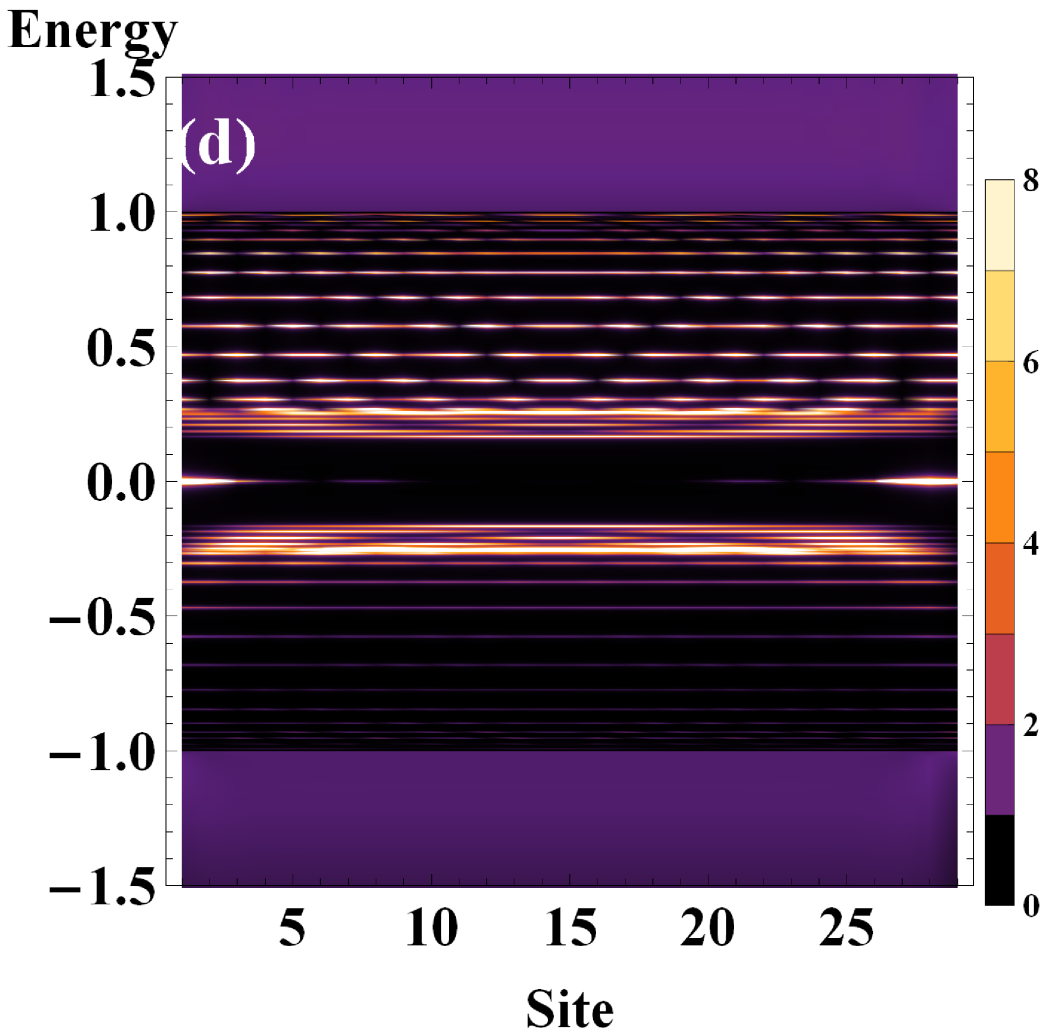
Andreev vs Majorana states – a story of mutation



Spectrum of a quantum wire has a series of Andreev states.

D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B ***88****, 165401 (2013).*

Andreev vs Majorana states – a story of mutation



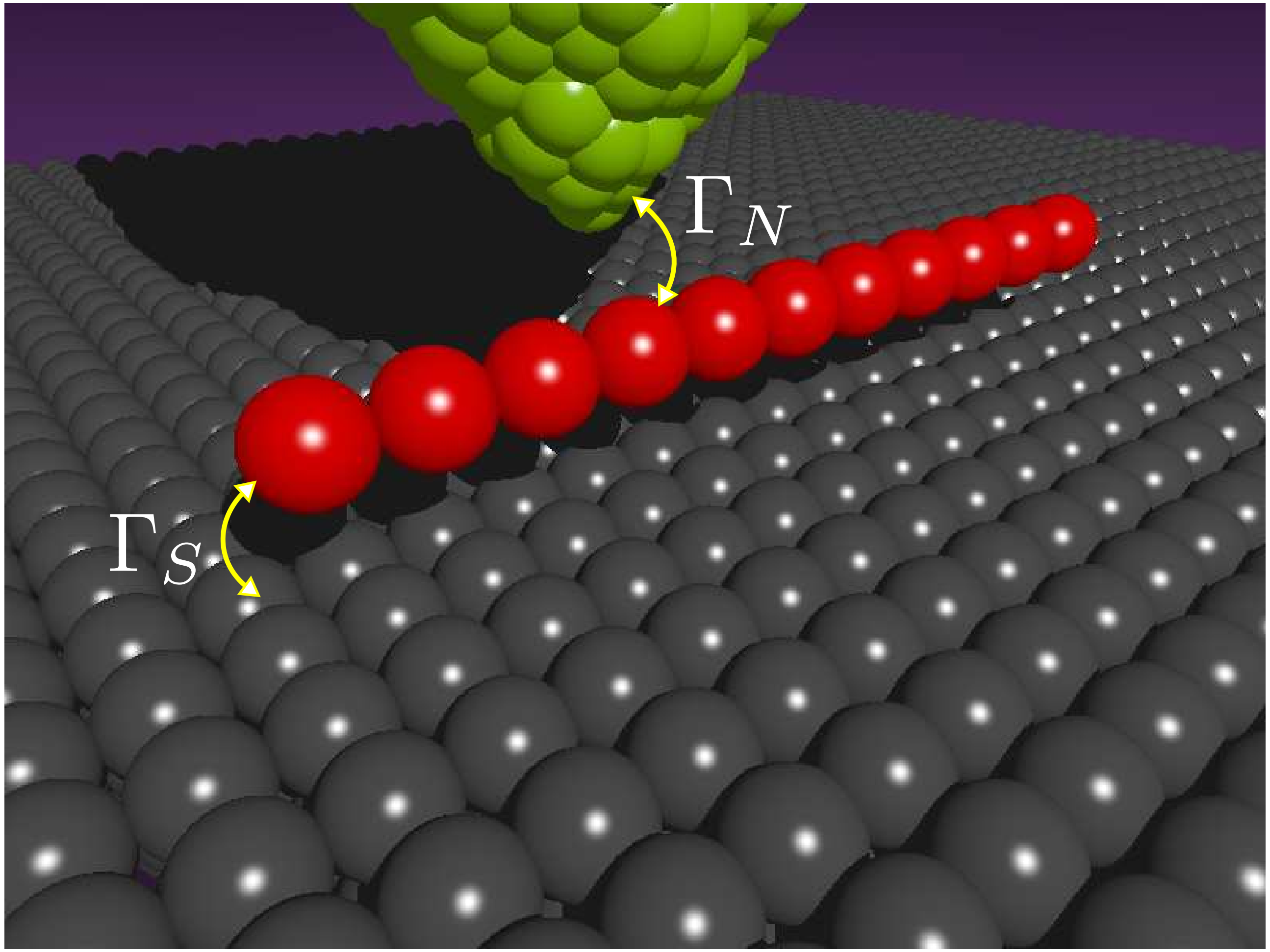
Spin-orbit coupling induces the Majorana-type quasiparticles.

D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B ***88****, 165401 (2013).*

Majorana states – of the Rashba chain

Majorana states – of the Rashba chain

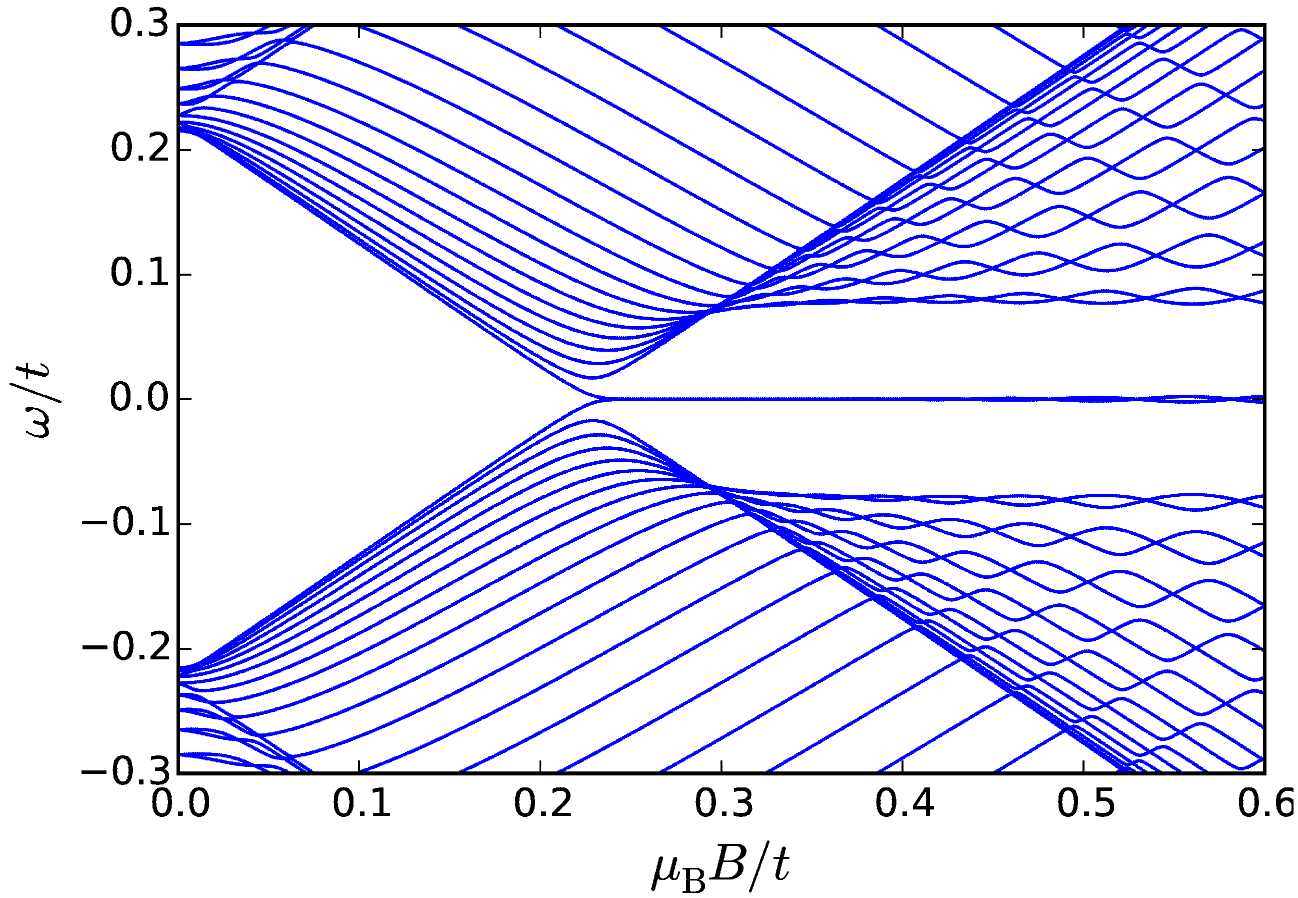
STM setup:



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv-preprint (5 September 2016).

Majorana states – of the Rashba chain

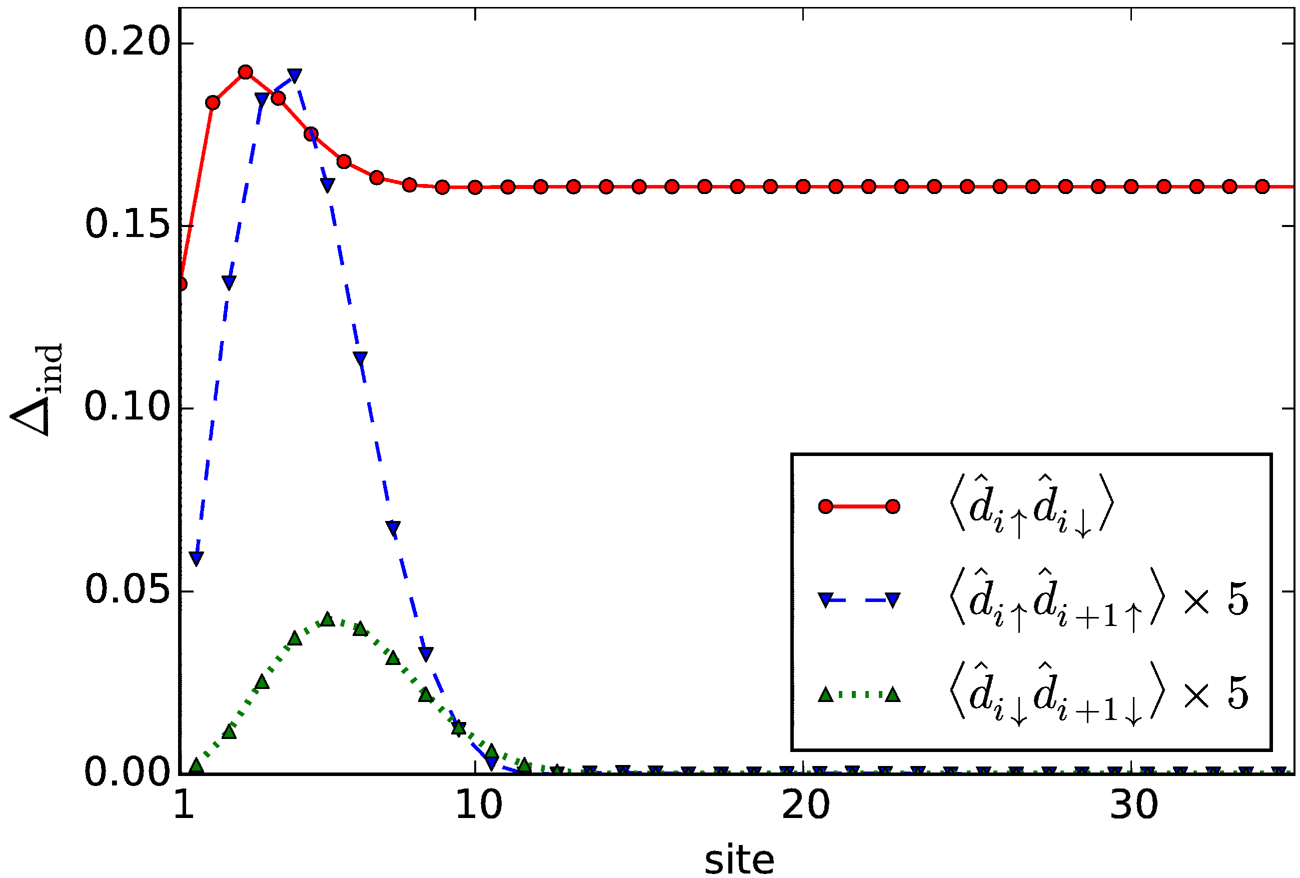
Mutation of Andreev states into zero-energy (Majorana) mode



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv-preprint (5 September 2016).

Majorana states – **of the Rashba chain**

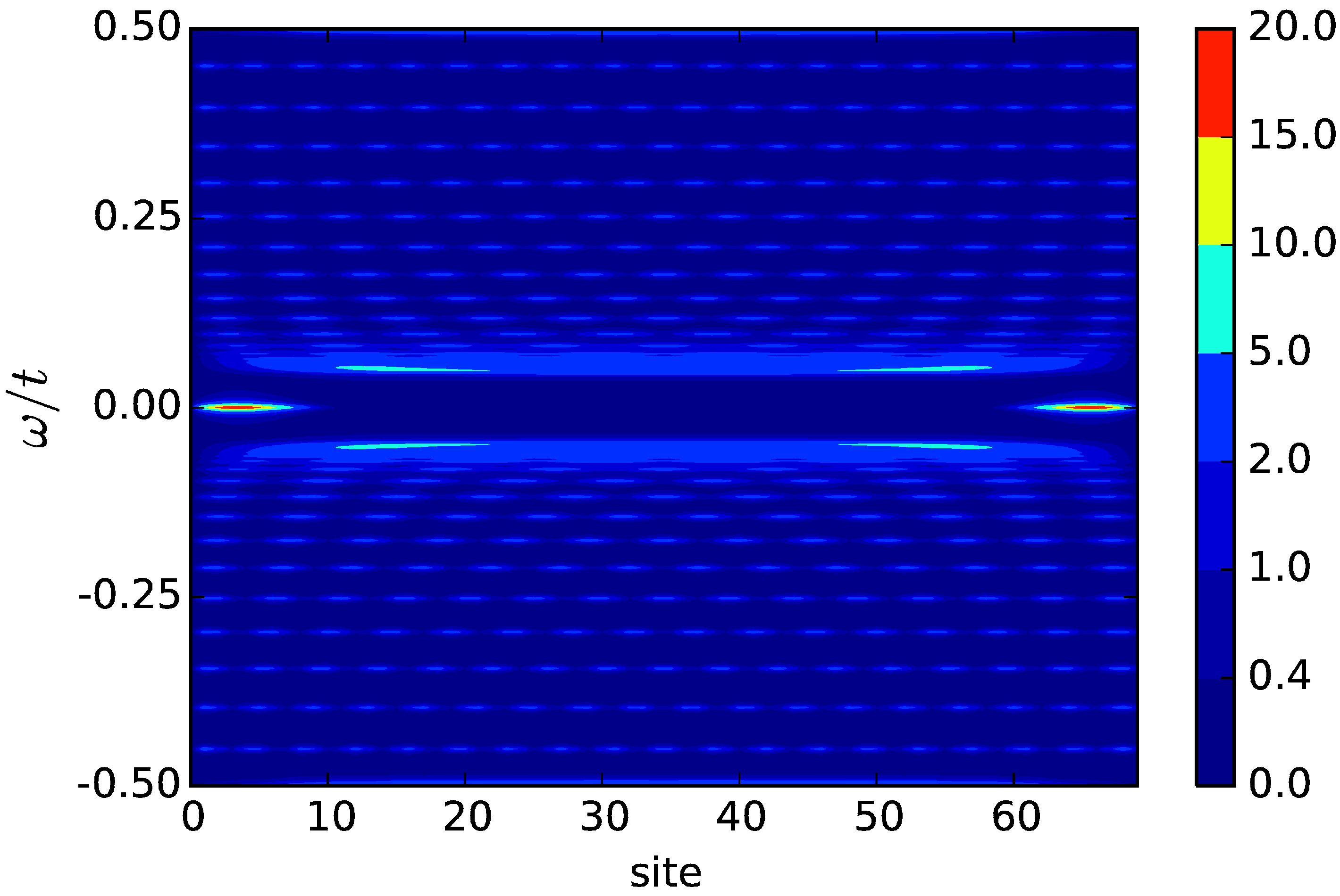
Spatial variation of the effective (trivial and nontrivial) pairing



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv-preprint (5 September 2016).

Majorana states – of the Rashba chain

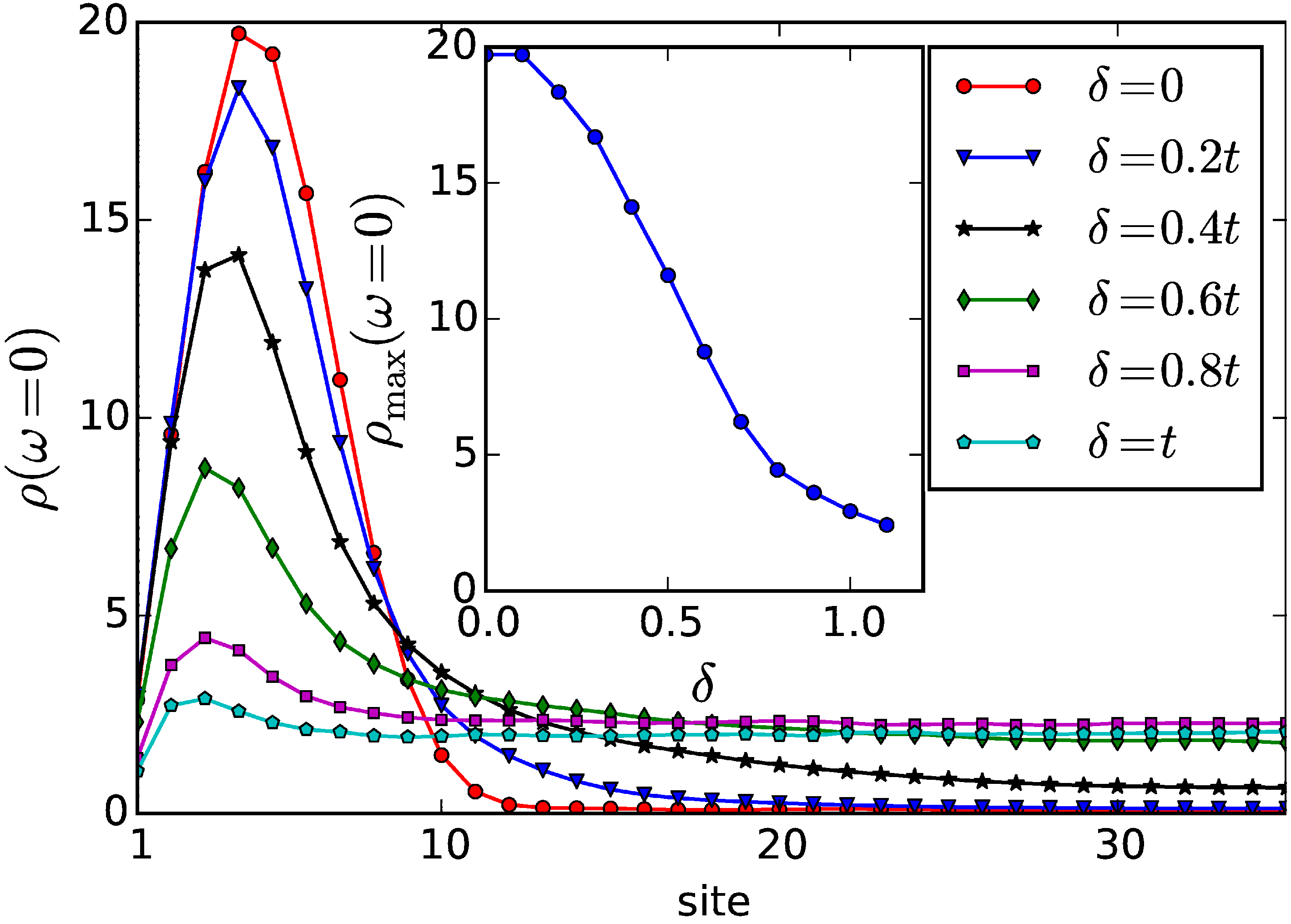
In-gap spectrum with the edge Majorana quasiparticles



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv-preprint (5 September 2016).

Majorana states – of the Rashba chain

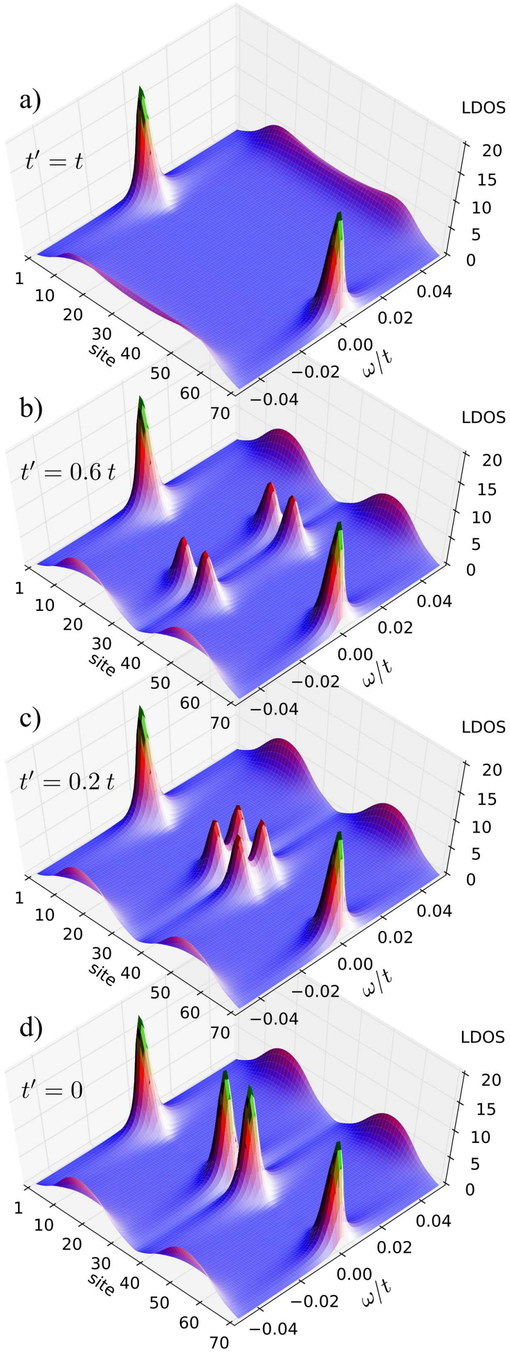
Are Majorana quasiparticles really immune to disorder ?



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv-preprint (5 September 2016).

Majorana states – of the Rashba chain

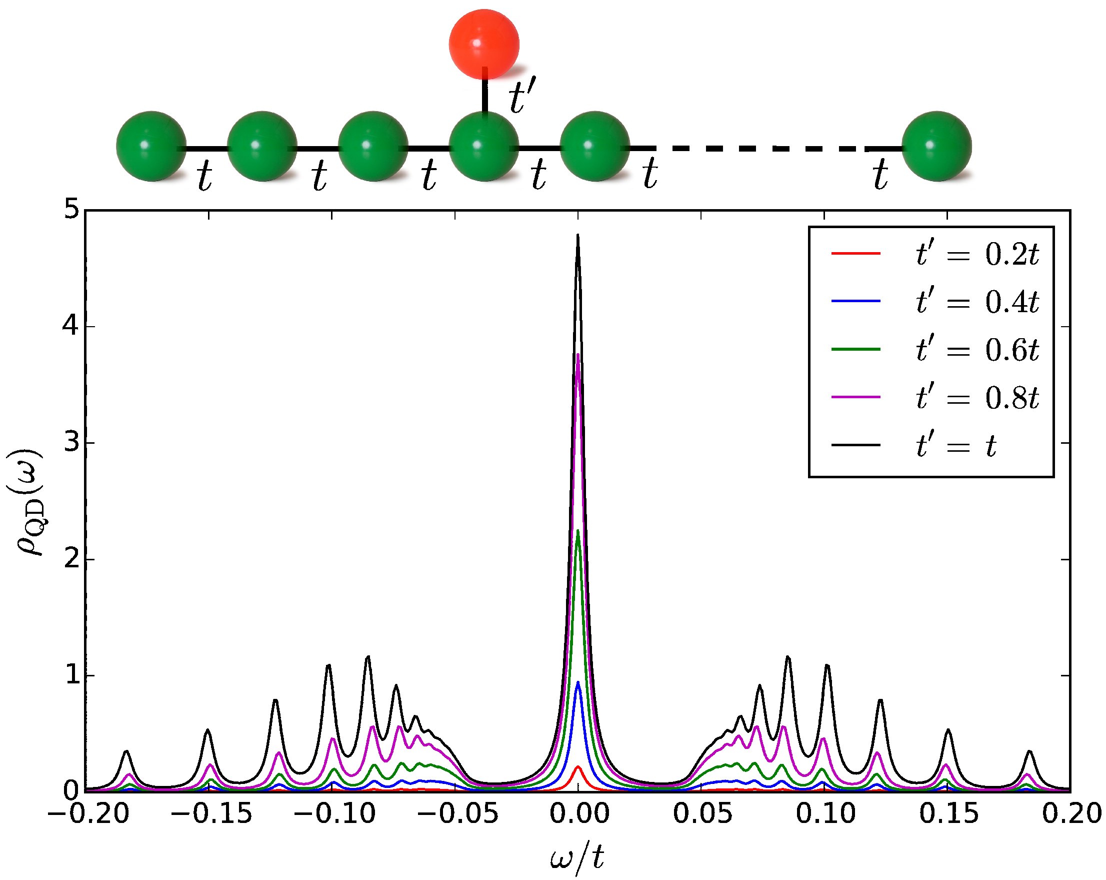
Gradual partitioning of the Rashba chain into separate pieces by reducing hopping t_j at site $j = N/2$



M. Maśka et al, arXiv-preprint (5 September 2016).

Majorana states – of the Rashba chain

Majorana quasiparticle 'leaking' into a normal quantum impurity



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv-preprint (5 September 2016).

Conclusions (part 2)

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$\Longrightarrow$ **evolve out of the Andreev/Shiba states**

$\Longrightarrow$ **are not completely immunue to disorder**

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