

*Ustroń, 8 September 2008*

**Electron pair current  
through the correlated  
quantum dots**

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Lublin, Poland**

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## *Outline*

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### **Introduction**

*/ correlation effects in quantum dots /*

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### **Summary**

# 1. Introduction

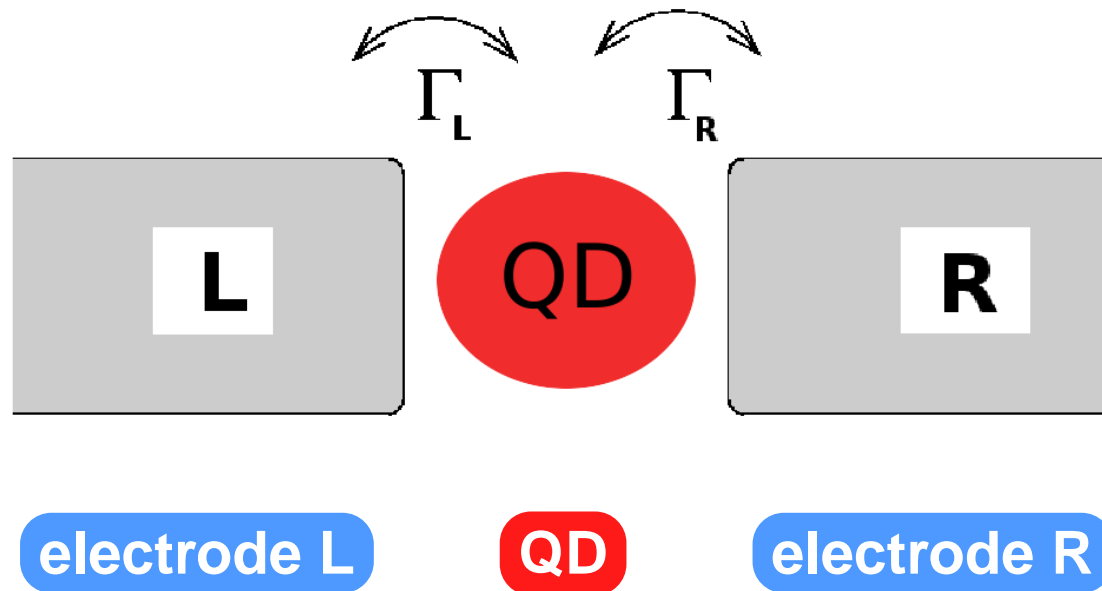
## Physical situation

Let us consider the quantum dot (QD)



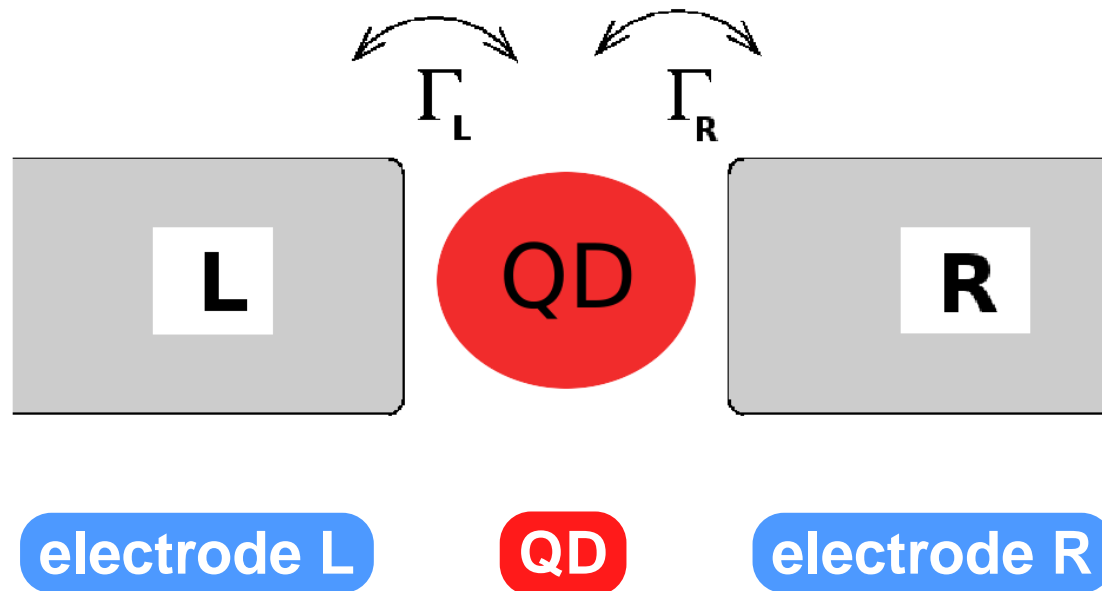
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induced by the external voltage  $eV = \mu_L - \mu_R$ .

**The underlying physics:  $U > 0$  case**

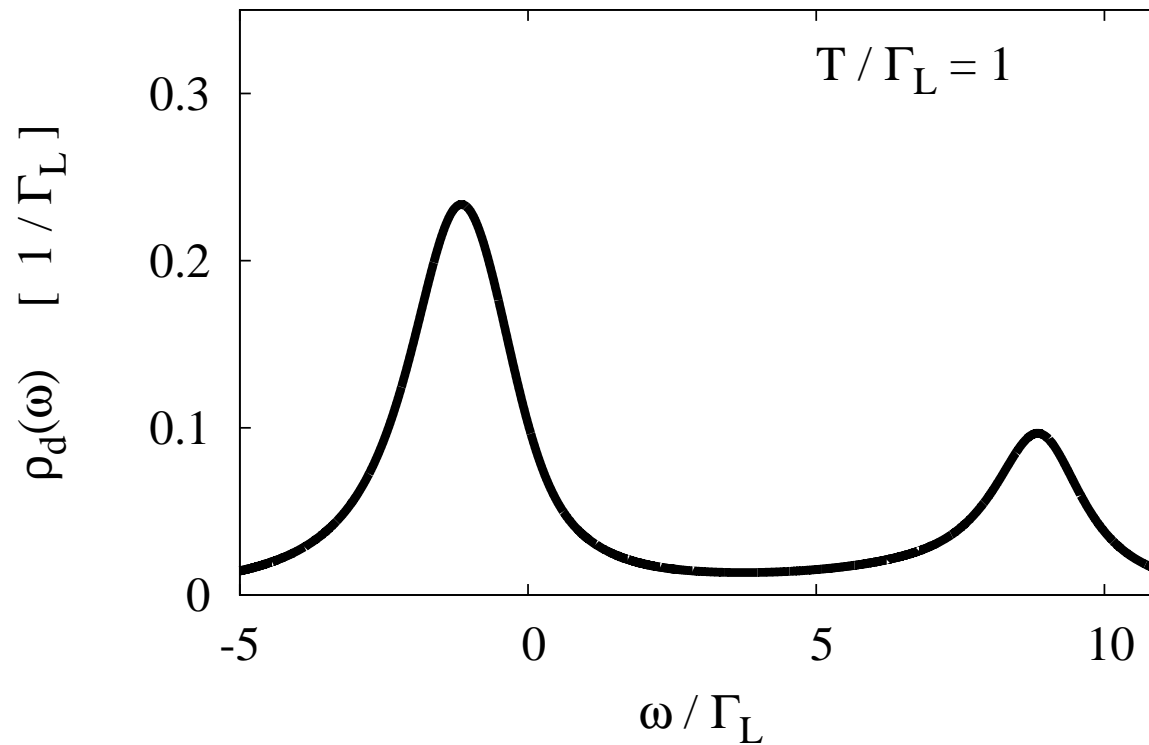


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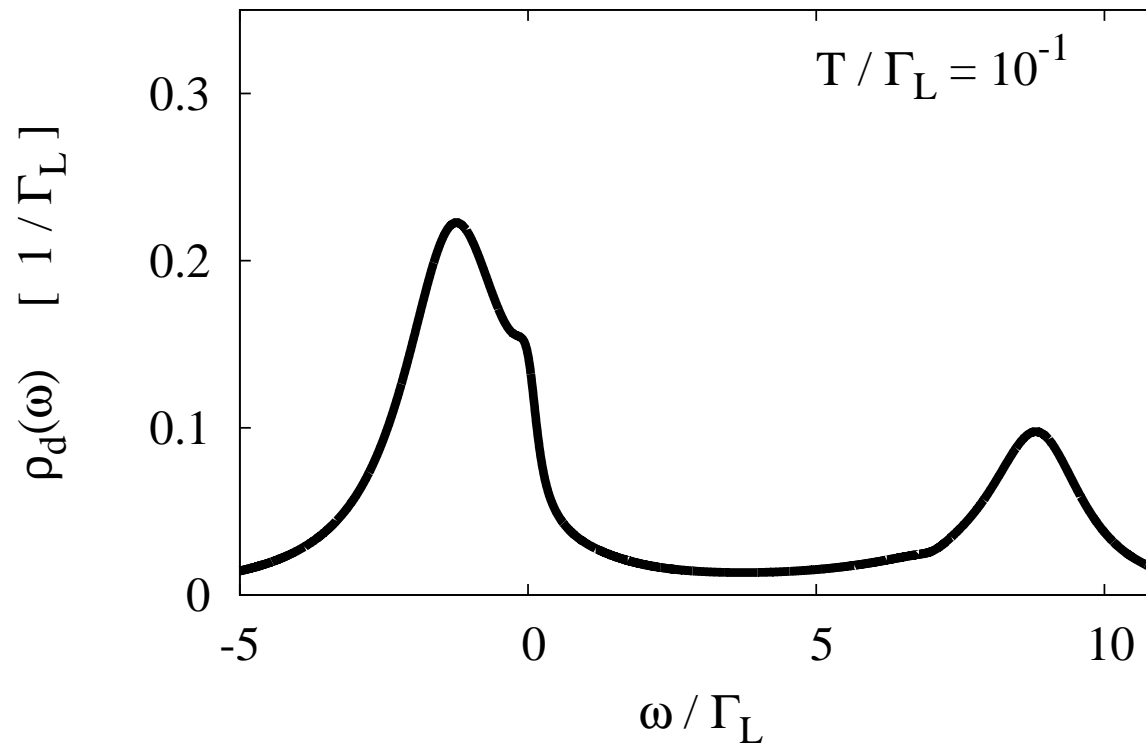
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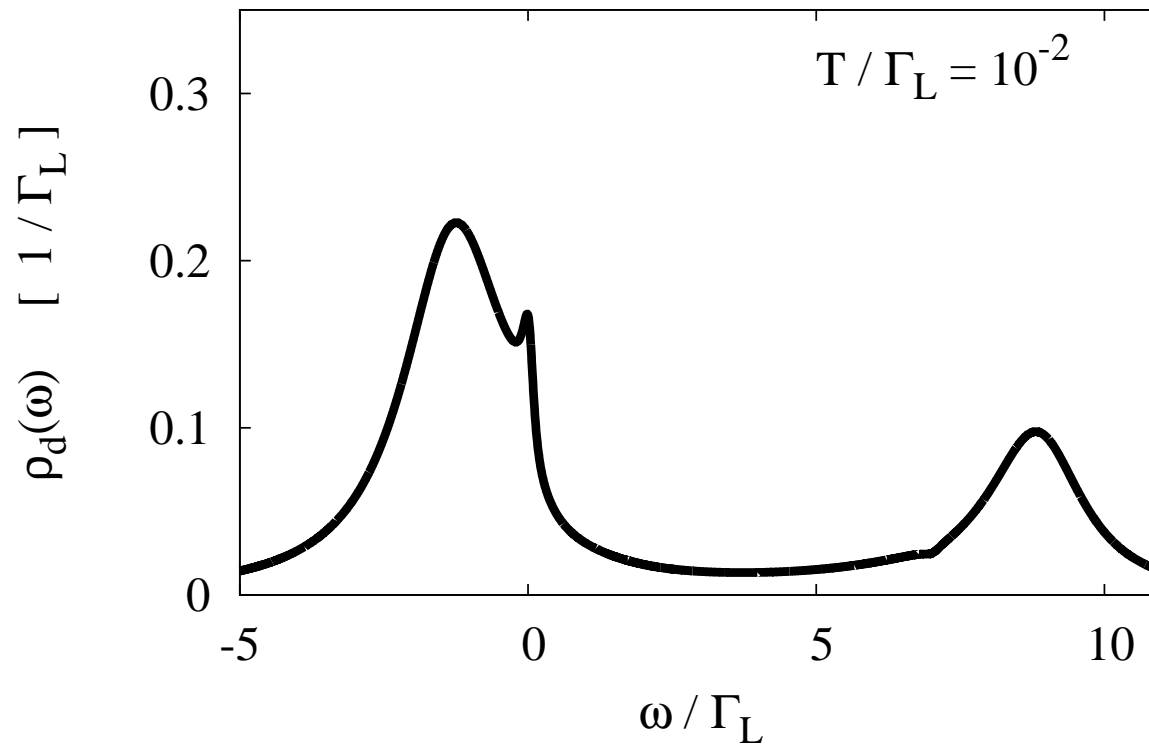
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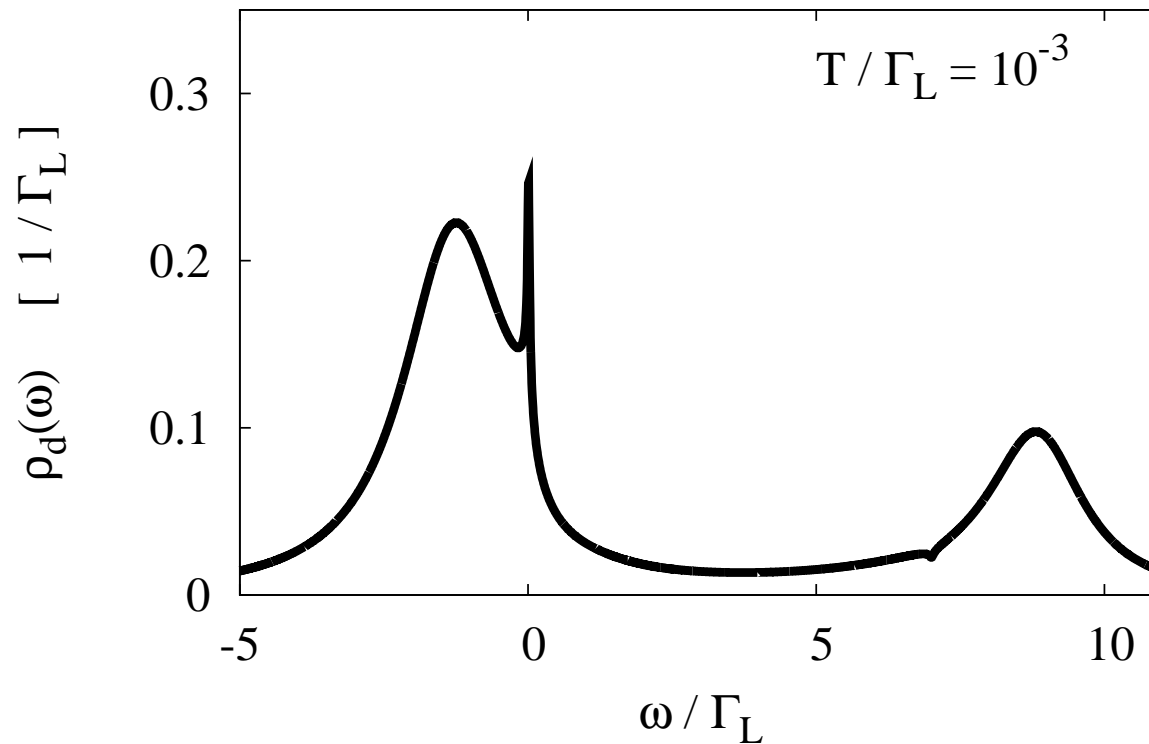
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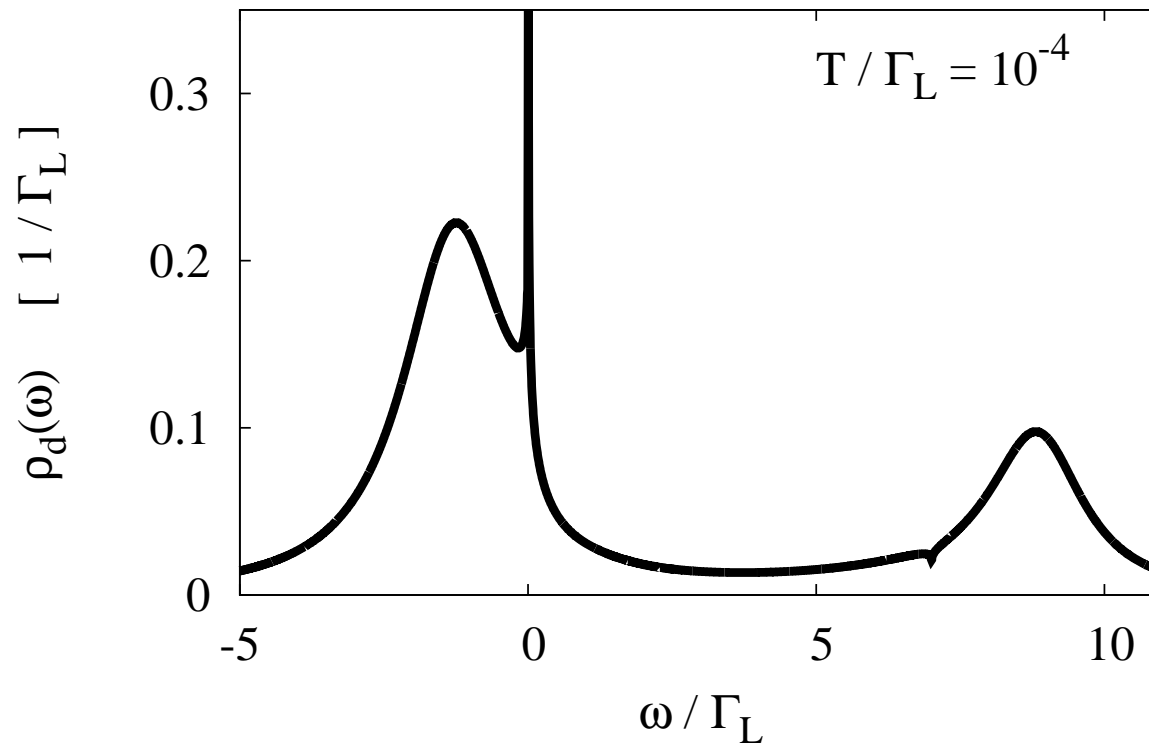


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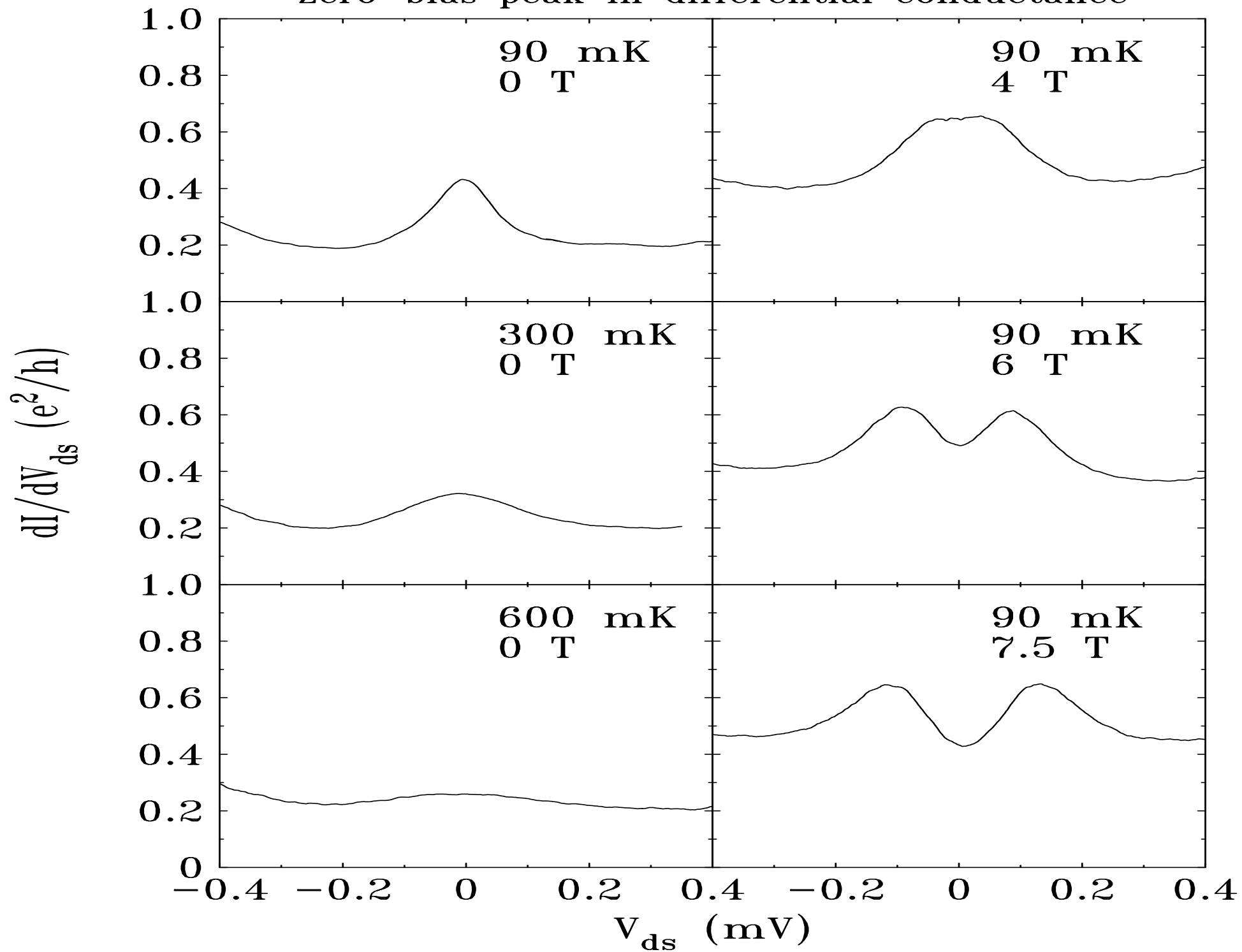
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depends on the correlations through the QD spectral function

$$\rho_{d,\sigma}(\omega) = -\frac{1}{\pi} \text{Imag} \{ G_{d,\sigma}(\omega + i0^+) \}$$

## Zero-bias peak in differential conductance



## **2. The spin vs Kondo effect**

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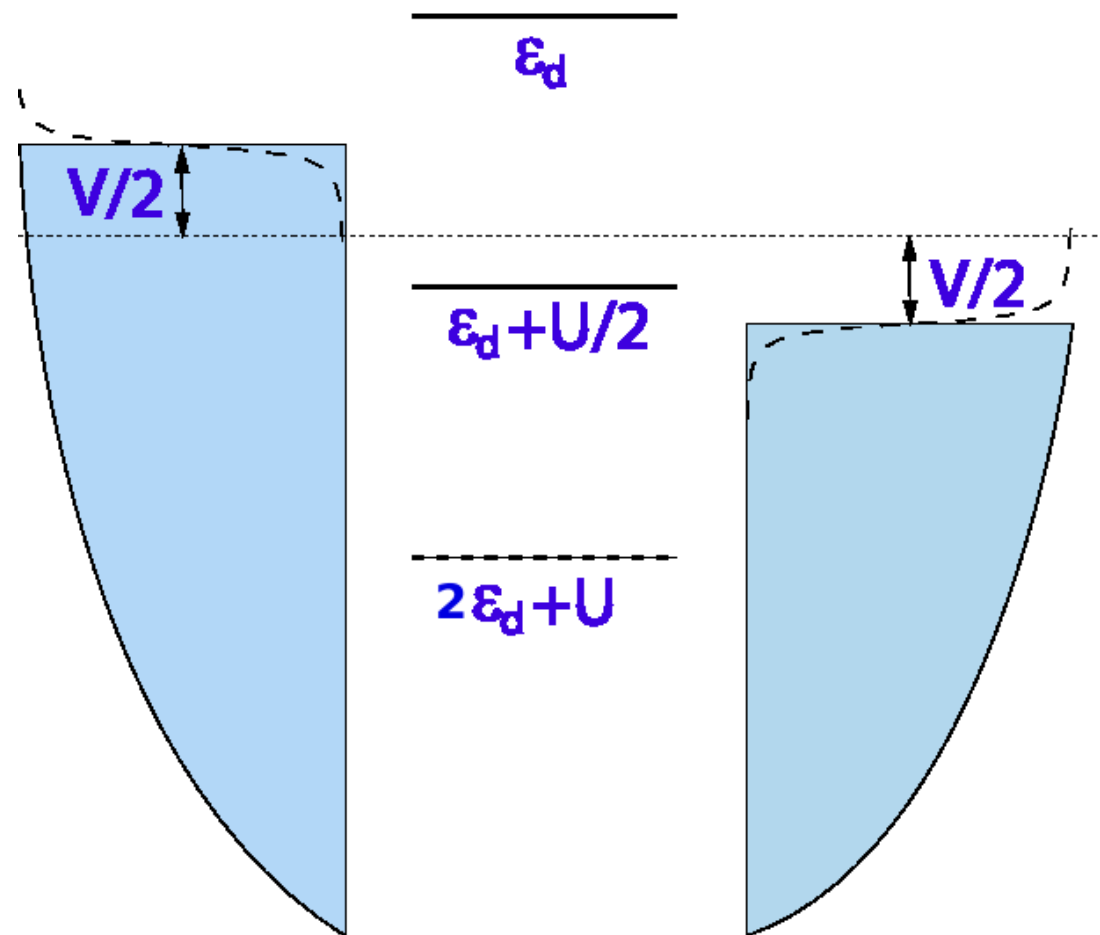
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The spin Kondo effect comes from the antiferromagnetic coupling

$$\mathbf{J}_{\mathbf{k}_F, \mathbf{k}_F}^{\beta, \beta'} = \frac{U}{\epsilon_d(\epsilon_d + U)} V_{\mathbf{k}_F\beta} V_{\mathbf{k}_F\beta'}^*$$

## Situation with the negative $U$ quantum dot



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The low energy physics is effectively described by

$$\hat{H}_{charge}^{Kondo} = \sum_{\mathbf{k}, \beta, \sigma} \xi_{\mathbf{k}\beta} \hat{c}_{\mathbf{k}\beta\sigma}^{\dagger} \hat{c}_{\mathbf{k}\beta\sigma} + 2 \sum_{\mathbf{k}, \mathbf{q}, \beta, \beta'} J_{\mathbf{k}, \mathbf{q}}^{\beta, \beta'} \hat{\mathbf{T}}_d \cdot \hat{\mathbf{T}}_{\mathbf{k}\beta, \mathbf{q}\beta'}$$

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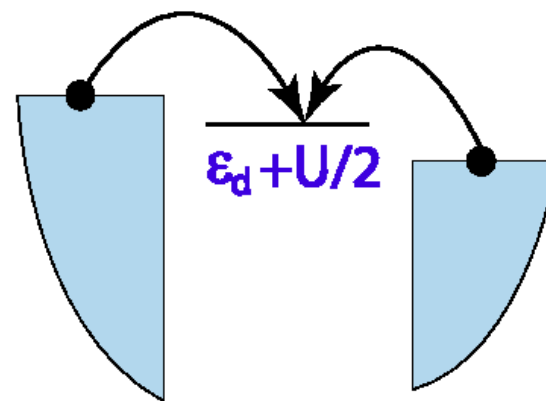
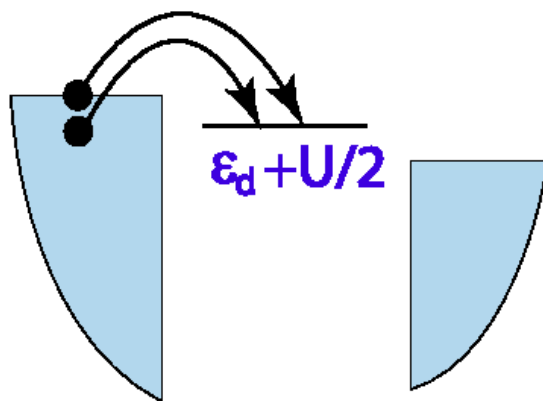
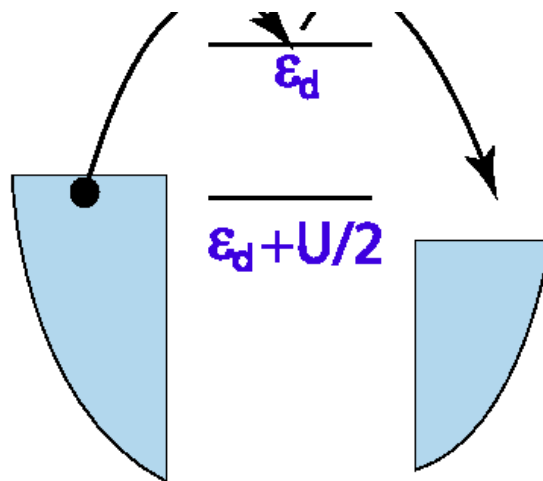
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thus the Kondo effect can be formed in the pseudospin channel.

A. Taraphder and P. Coleman, Phys. Rev. Lett. **66**, 2814 (1991).

## Charge tunneling through $U < 0$ QD

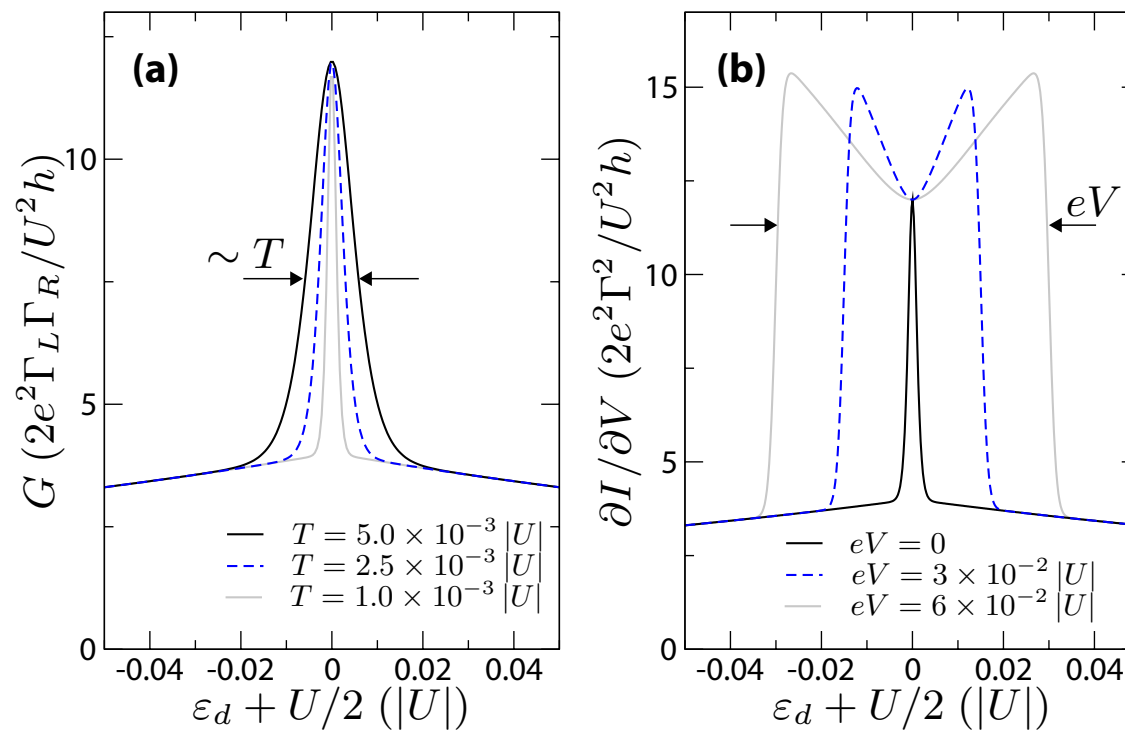


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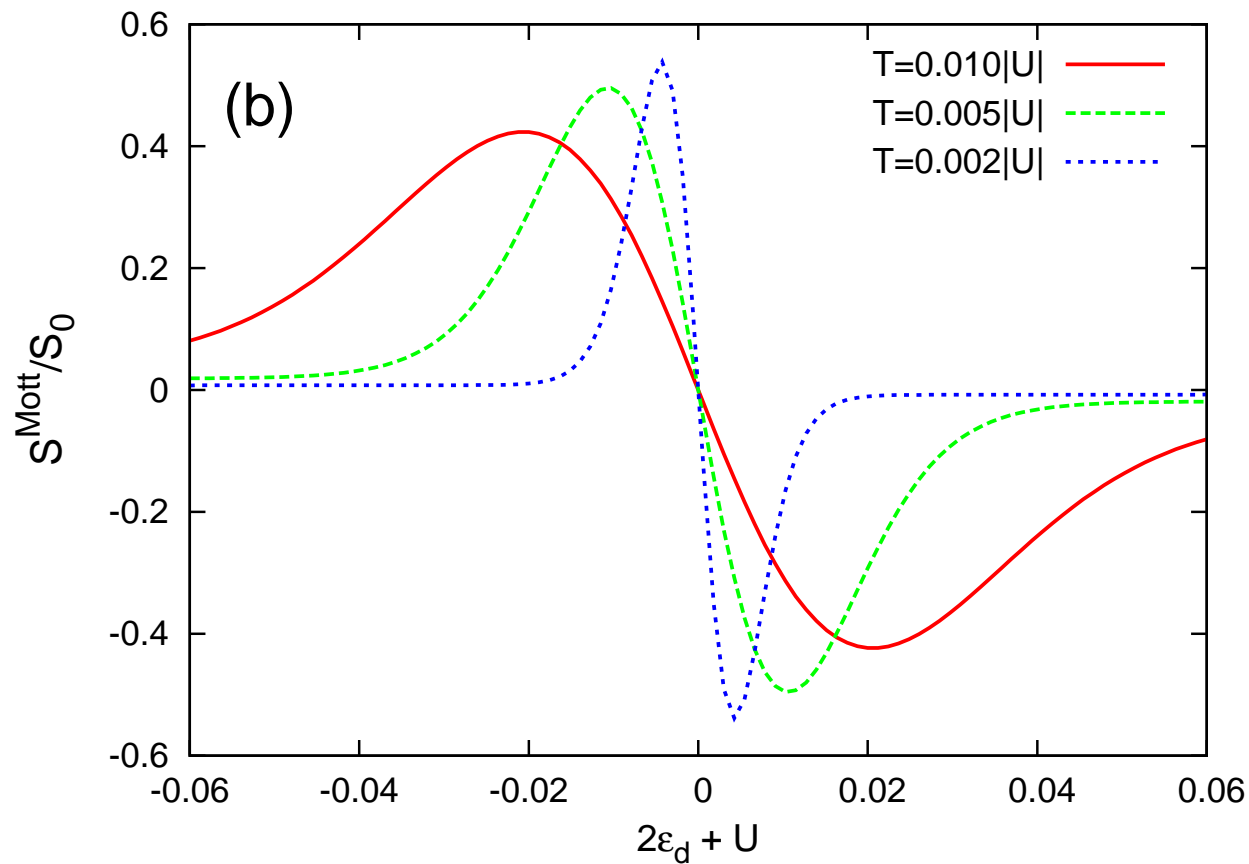
★ differential conductance:



J. Koch, M.E. Raikh, and F. von Oppen, PRL**96**, 056803 (2006).

## Pair tunneling

★ thermoelectric power:



M. Gierczak, K.I. Wysokiński, J.Phys.Conf.Ser. **104**, 012005 (2008).

### **3. The two-channel model**

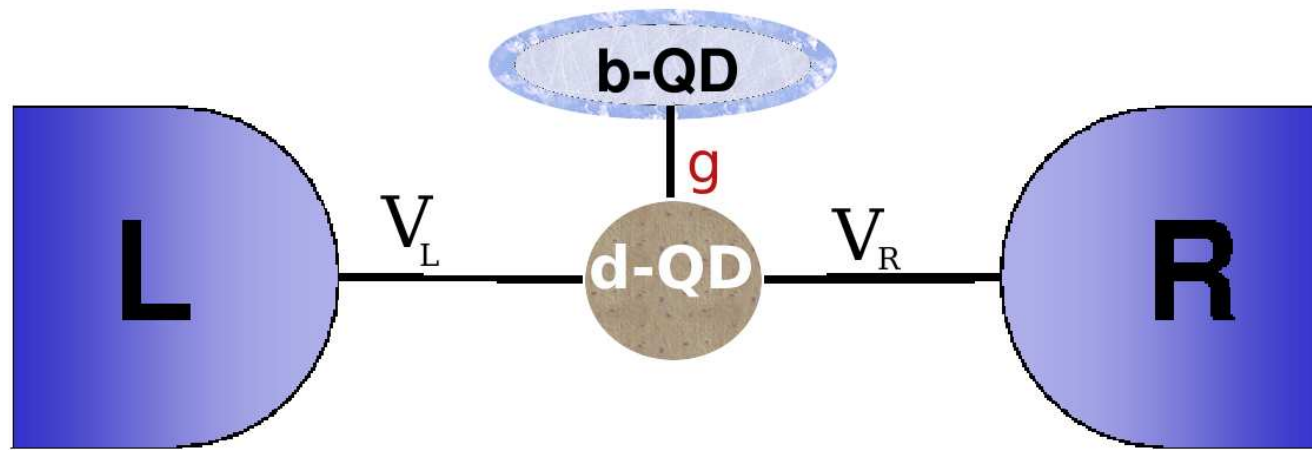


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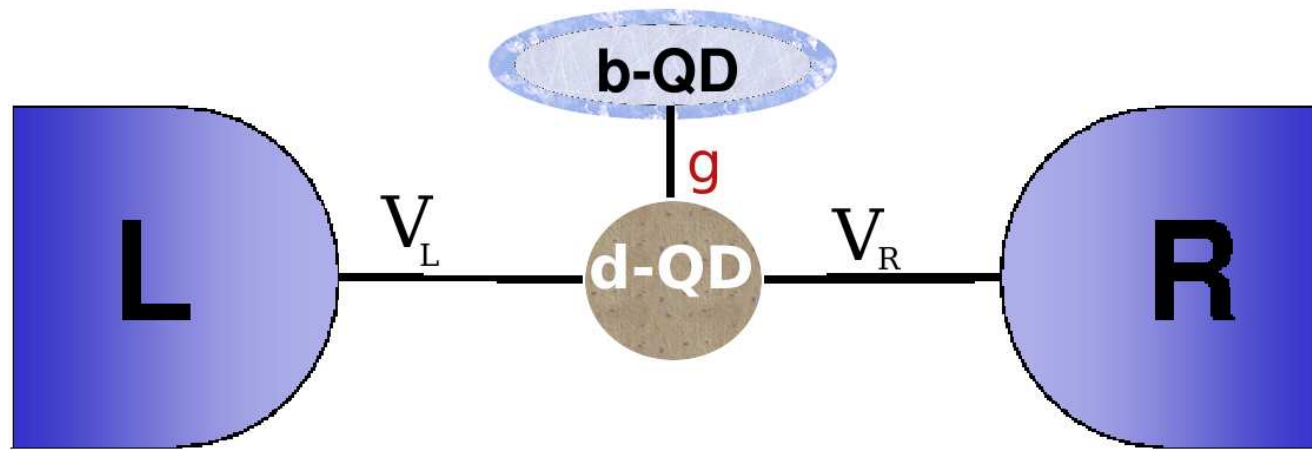
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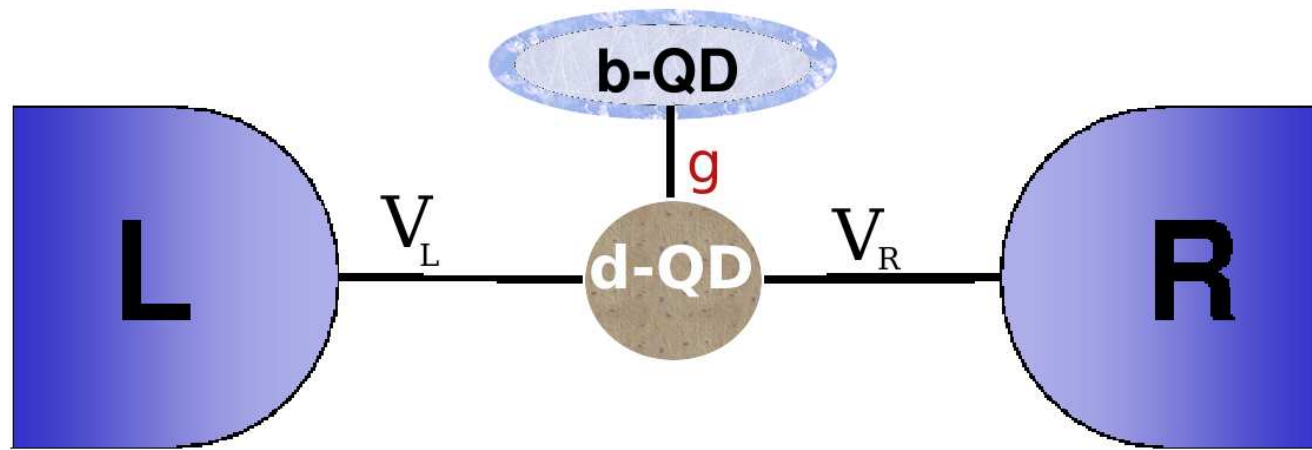
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*T. Domański, Eur. Phys. J. B 33, 41 (2003).*

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To account for a finite hybridization we employ the **Ansatz**

## Quantum fluctuations (c.d.)

where

$$\begin{aligned}E_A, E_B &= \frac{1}{2} [E_{pair} \mp \gamma(2E_d - E_{pair})] \\v^2, u^2 &= \frac{1}{2} \left(1 \mp \frac{1}{\gamma}\right) \\\gamma^2 &= 1 + \left(\frac{2g}{2E_d - E_{pair}}\right)^2\end{aligned}$$

and  $\mathcal{Z}$  is a strongly temperature-dependent coefficient.

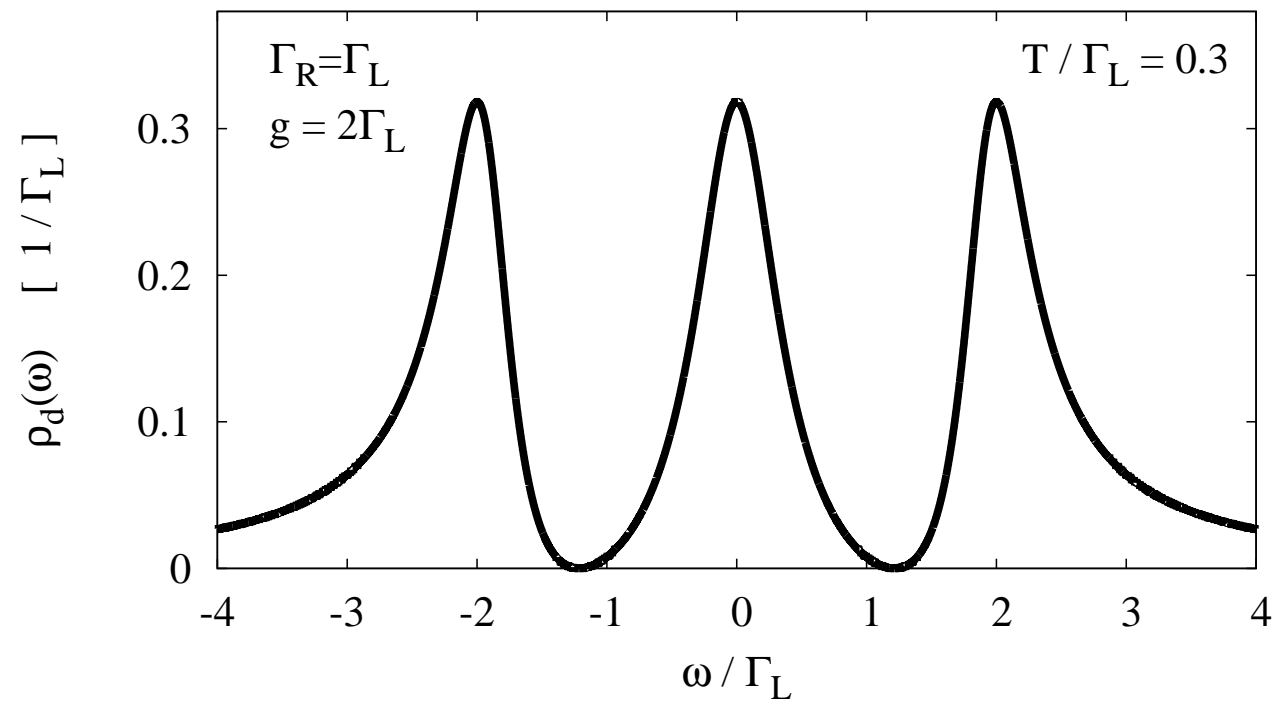
To account for a finite hybridization we employ the **Ansatz**

$$\mathcal{G}_d(\omega)^{-1} = \mathcal{G}_d^0(\omega)^{-1} - \sum_{\mathbf{k}, \beta} \frac{|V_{\mathbf{k}\beta}|^2}{\omega - \xi_{\mathbf{k}\beta}}$$

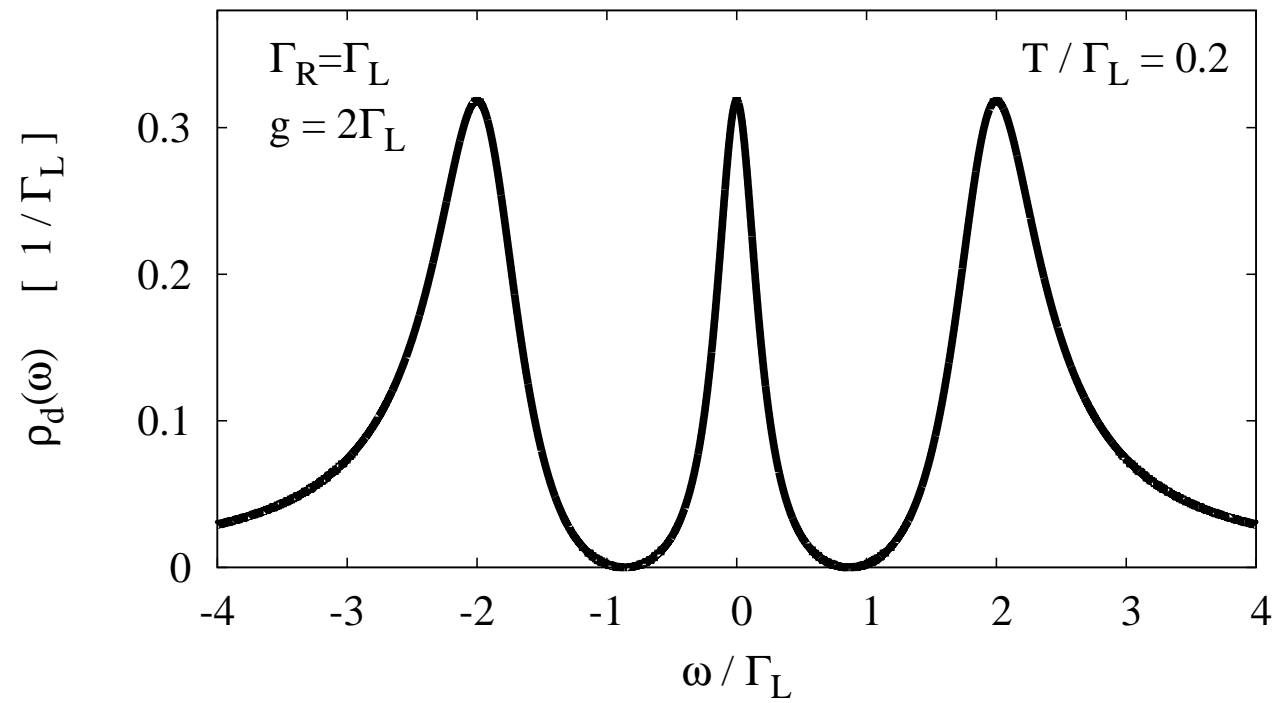


## Spectrum of the d-QD

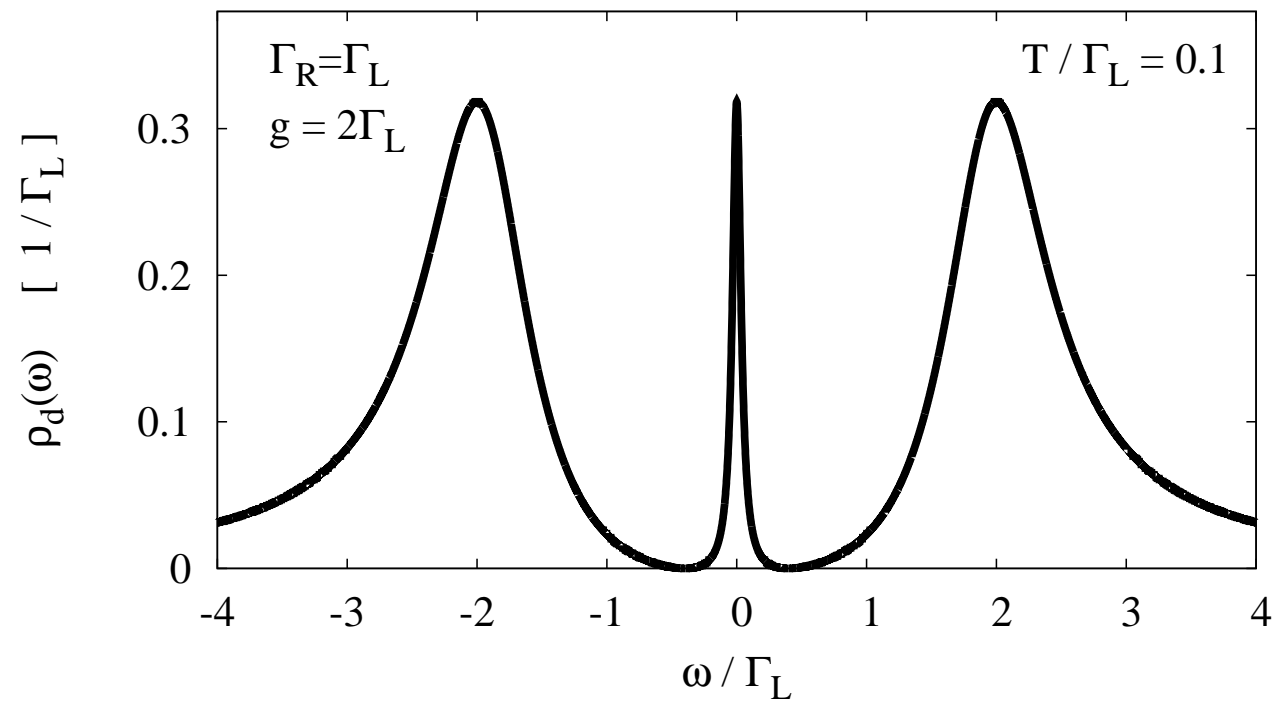
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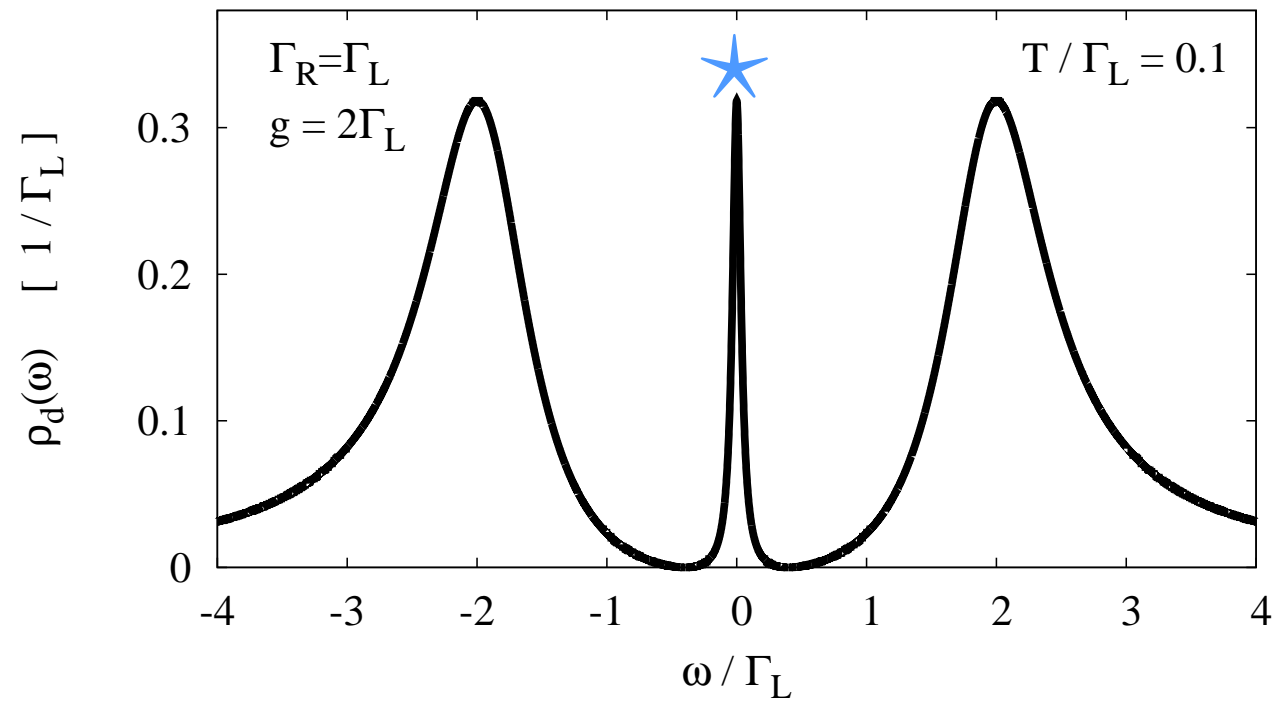
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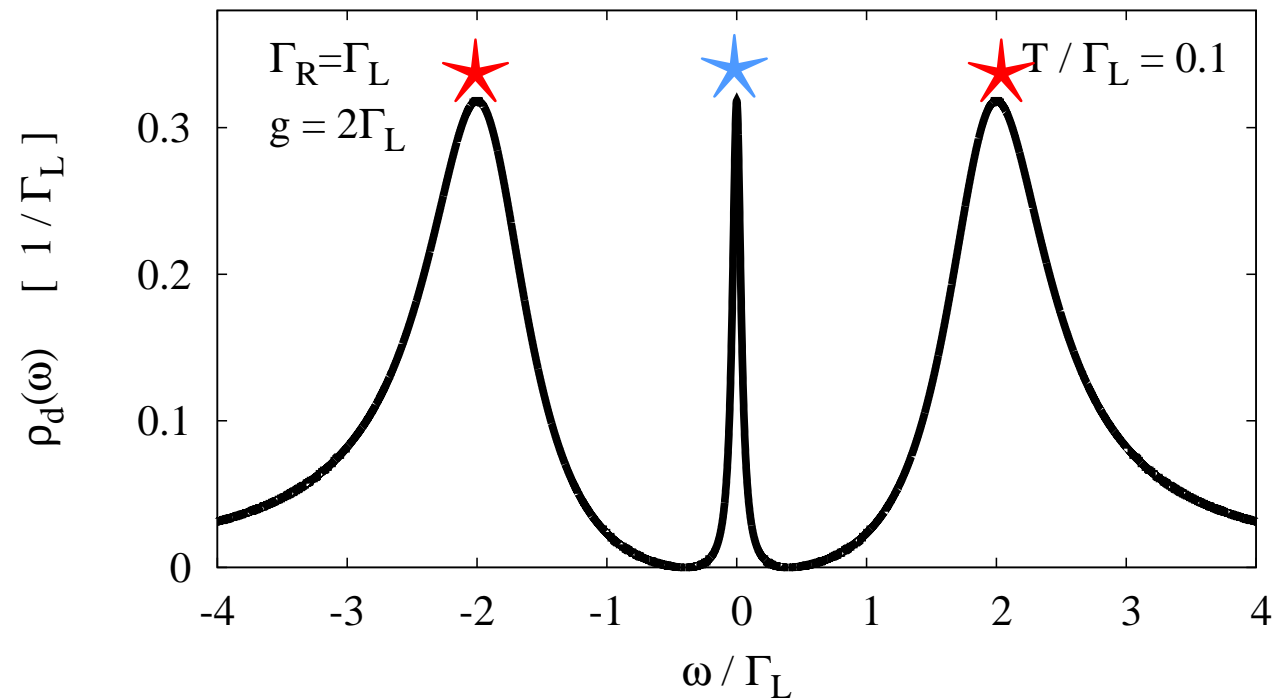


## Spectrum of the d-QD



★ the middle peak: **superradiant state,**

## Spectrum of the d-QD

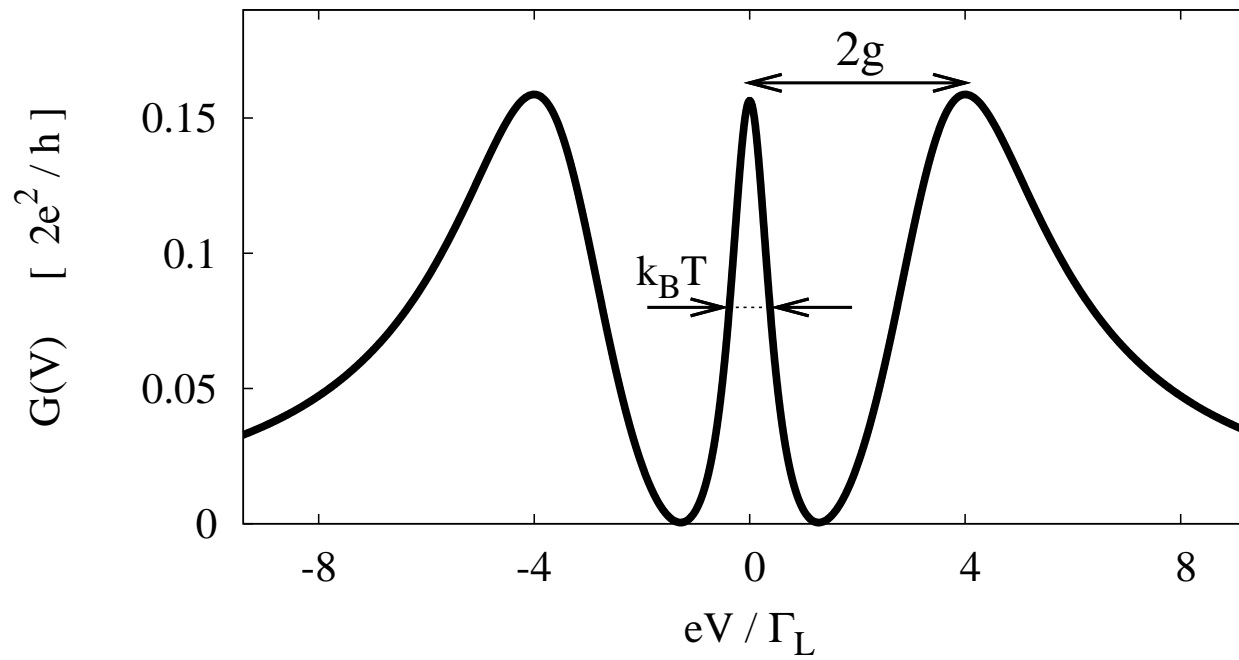


★ the middle peak: superradiant state,

★ the side peaks: subradiant states.

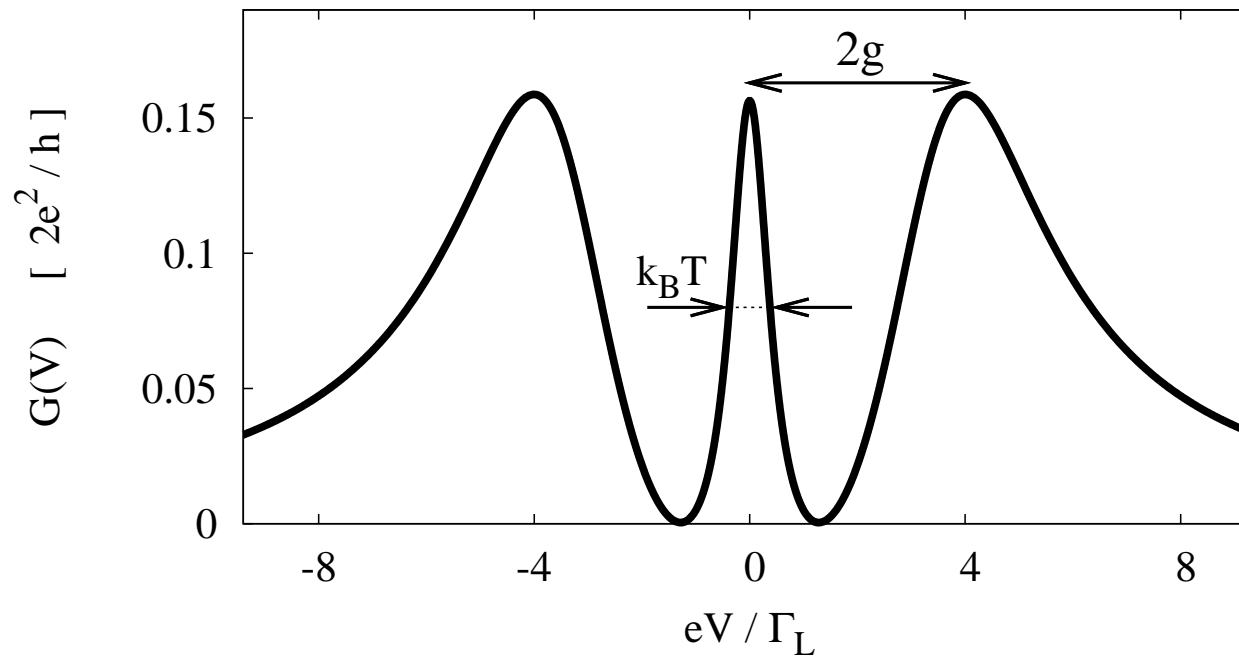
## Differential conductance

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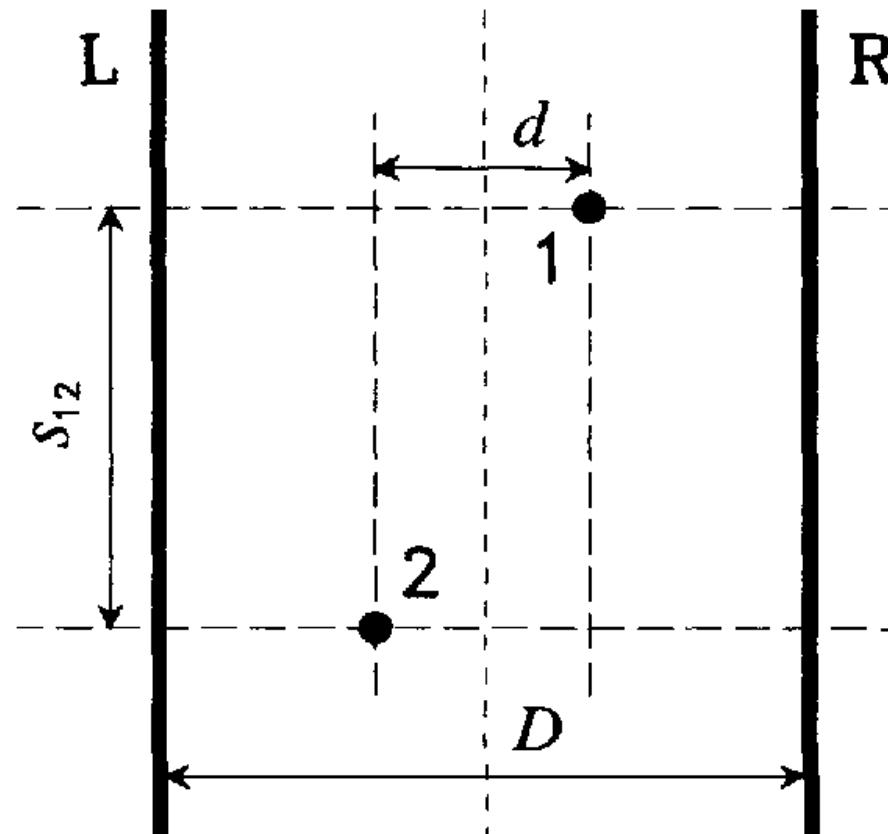


**Superradiant** line broadening is proportional to  **$T$**  !

## Dicke effect in mesoscopic physics

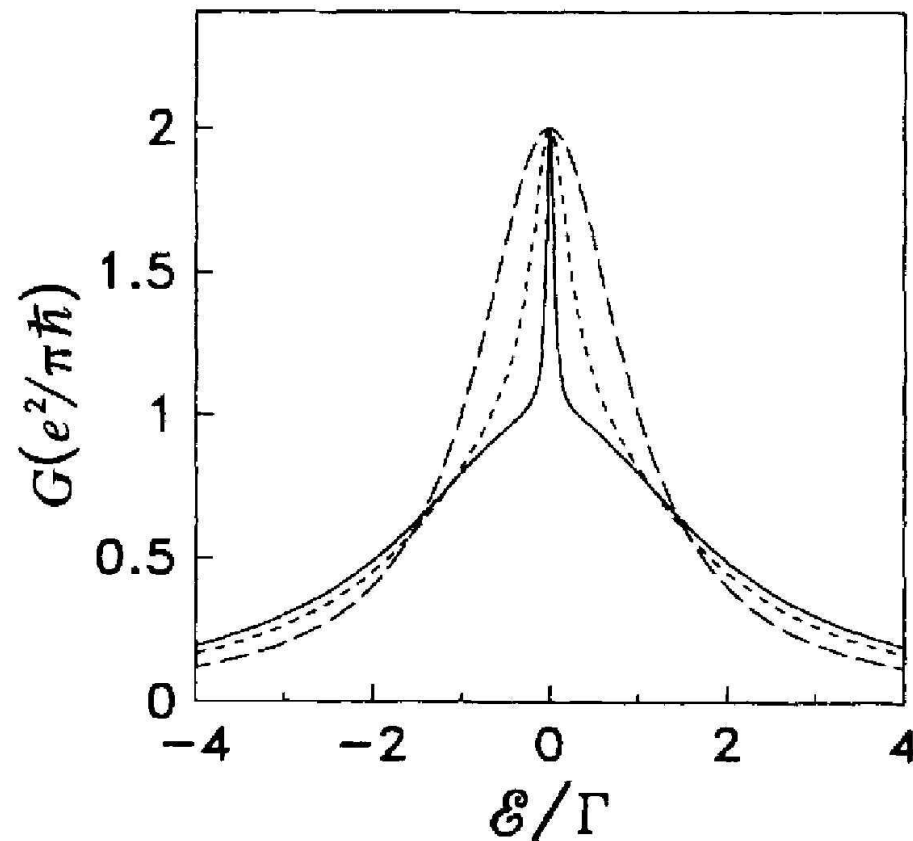
## Dicke effect in mesoscopic physics

# 1 tunneling via two quantum dots



## Dicke effect in mesoscopic physics

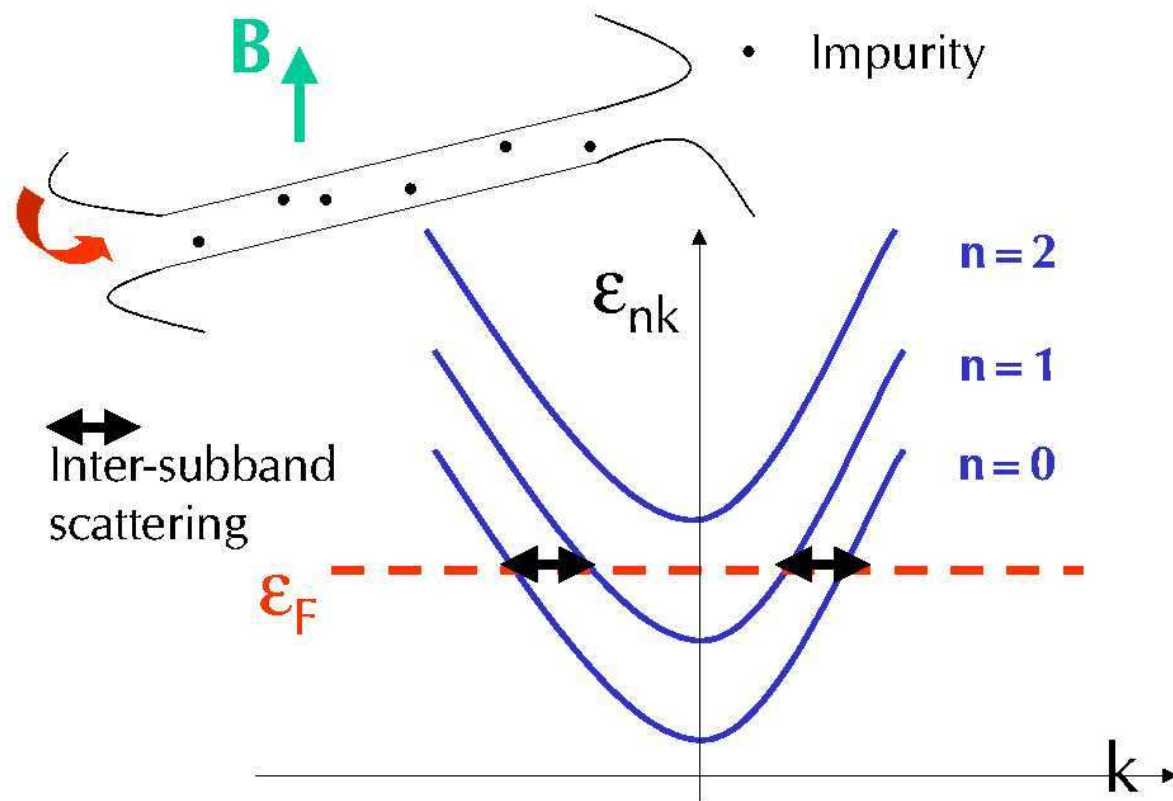
# 1 tunneling via two quantum dots



T.V. Shahbazyan and M.E. Raikh, PRB **49**, 17123 (1994).

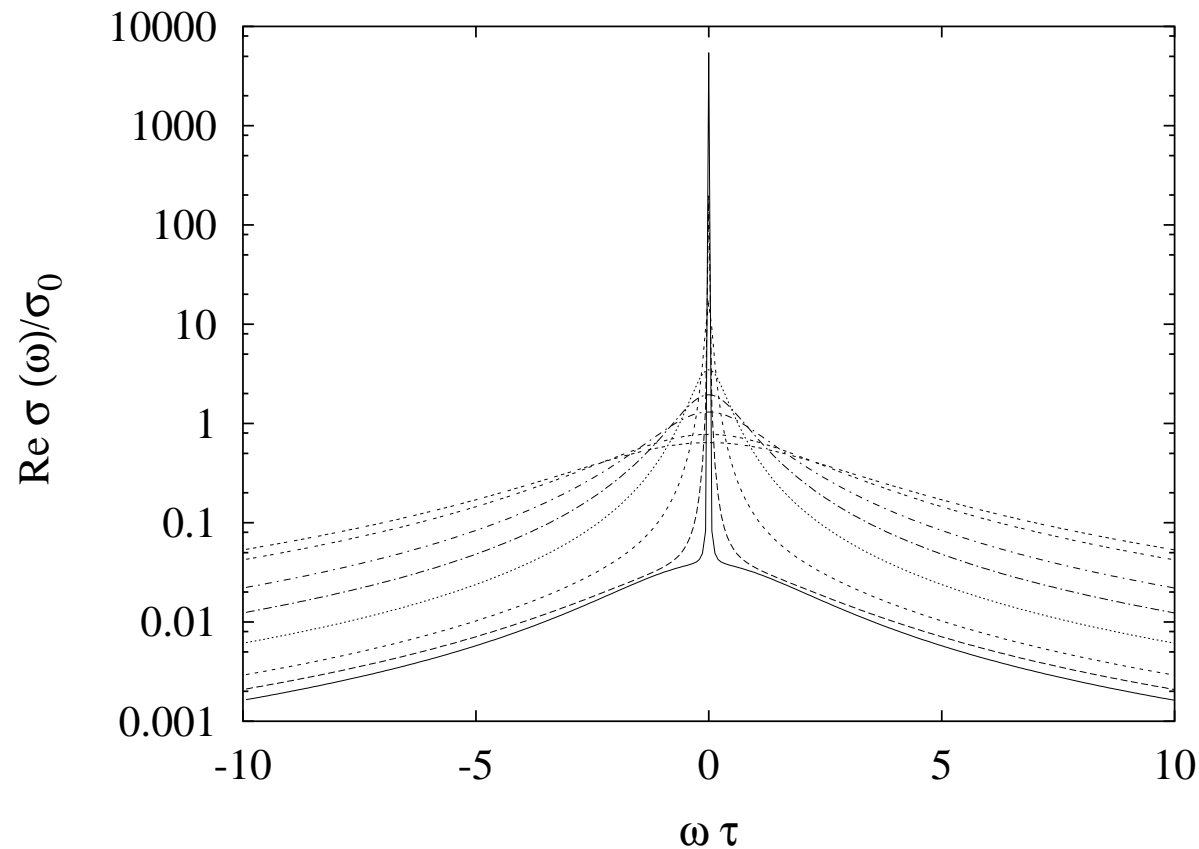
## Dicke effect in mesoscopic physics

# 2 tunneling via the quantum wire + magnetic field



## Dicke effect in mesoscopic physics

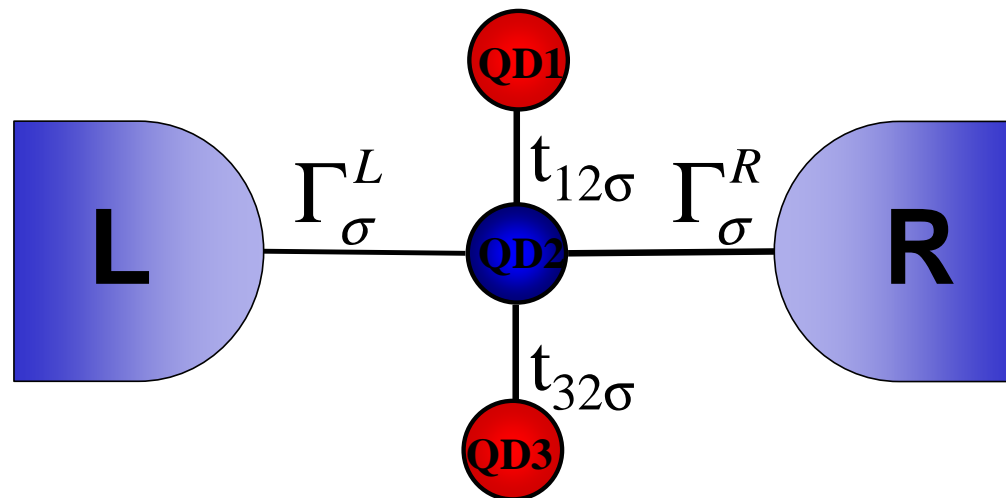
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T. Brandes, Phys. Rep. **408**, 315 (2005).

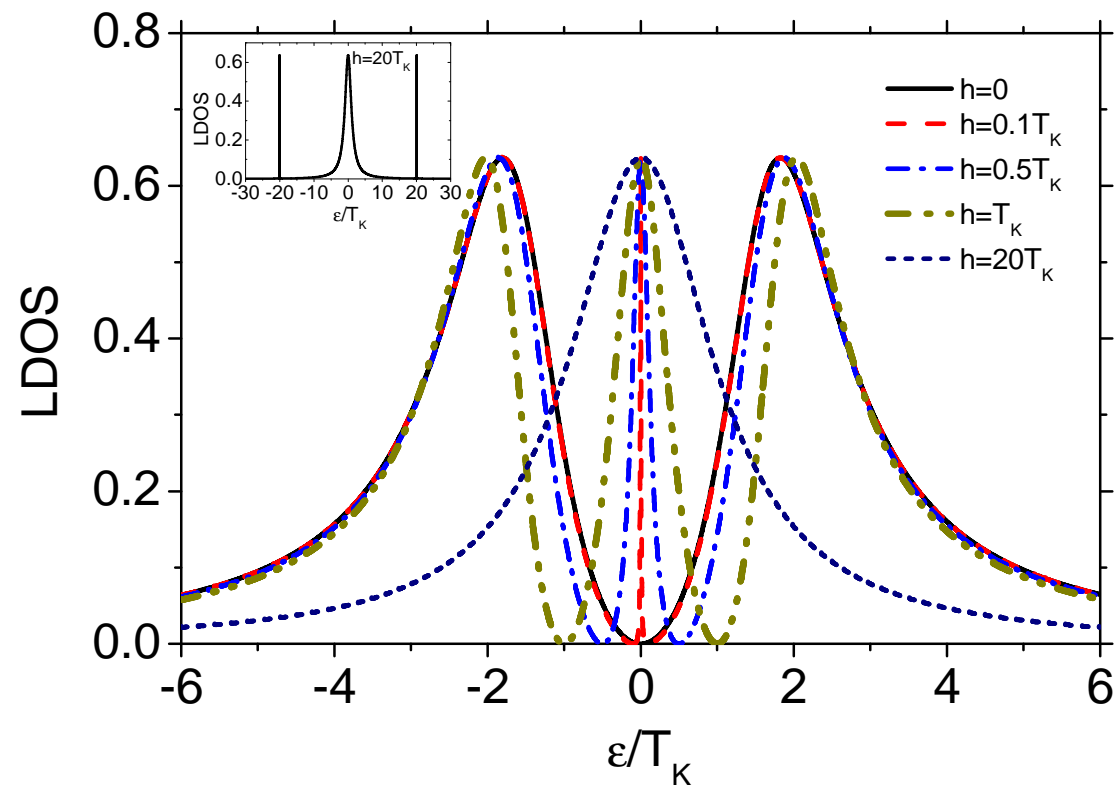
## Dicke effect in mesoscopic physics

# 3 tunneling via three quantum dots



# Dicke effect in mesoscopic physics

# 3 tunneling via three quantum dots



P. Trocha and J. Barnaś, PRB **78**, 075242 (2008).



## 4. Summary

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<http://kft.umcs.lublin.pl/doman>