Electron pair current through the correlated quantum dots

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★ Summary
1. Introduction
Physical situation

Let us consider the quantum dot (QD)
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with the metallic (conducting) external leads.
Microscopic model

On-dot correlations
Microscopic model

On-dot correlations

\[ \hat{H}_{QD} = \sum_{\sigma} \epsilon_d \hat{d}^\dagger_{\sigma} \hat{d}_{\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} \]
On-dot correlations

\[ \hat{H}_{QD} = \sum_\sigma \epsilon_d \, \hat{d}_\sigma^\dagger \, \hat{d}_\sigma + U \, \hat{n}_{d\uparrow} \, \hat{n}_{d\downarrow} \]

efficiently aeffect the transport via L-QD-R junction
Microscopic model

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efficiently affect the transport via L-QD-R junction

\[ \hat{H} = \sum_{\sigma} \epsilon_d \hat{d}_{\sigma}^\dagger \hat{d}_{\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} + \hat{H}_L + \hat{H}_R 
+ \sum_{k,\sigma} \sum_{\beta=L,R} \left( V_{k\beta} \hat{d}_{\sigma}^\dagger \hat{c}_{k\sigma\beta} + V_{k\beta}^* \hat{c}_{k\sigma,\beta}^\dagger \hat{d}_{\sigma} \right) \]
Microscopic model

On-dot correlations

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\[ + \sum_{k,\sigma} \sum_{\beta=L,R} \left( V_{k\beta} \, \hat{d}_\sigma^\dagger \hat{c}_{k\sigma\beta} + V_{k\beta}^* \, \hat{c}_{k\sigma,\beta}^\dagger \hat{d}_\sigma \right) \]

induced by the external voltage \( eV = \mu_L - \mu_R \).
The underlying physics: \( U > 0 \) case
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Correlations manifest themselves by:
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Correlations manifest themselves by:

$\frac{\rho_d(\omega)}{1/\Gamma_L} = 1$

$T/\Gamma_L = 1$

the charging effect
The underlying physics: \( U > 0 \) case

Correlations manifest themselves by:

\[ \rho_d(\omega) = \frac{1}{\Gamma_L} \]

\[ \frac{T}{\Gamma_L} = 10^{-1} \]

\[ \omega / \Gamma_L \]

the charging effect and ...
The underlying physics: $U > 0$ case

Correlations manifest themselves by:

$\rho_d(\omega) \propto \frac{1}{\Gamma_L}$

$\frac{T}{\Gamma_L} = 10^{-2}$

$\star$ the charging effect and ...
The underlying physics: \( U > 0 \) case

Correlations manifest themselves by:

\[ \rho_d(\omega) = \frac{1}{\Gamma_L} \]

\( \frac{T}{\Gamma_L} = 10^{-3} \)

- the charging effect
- the Kondo effect

at temperatures \( T < T_K \).
Correlations manifest themselves by:

\[ \rho_d(\omega) \left[ \frac{1}{\Gamma_L} \right] \]

at temperatures \( T < T_K \).
Non-equilibrium phenomena
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Application of the external voltage induces the current
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\[ J_L = -e \langle \hat{N}_L \rangle \]
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Non-equilibrium phenomena

Application of the external voltage induces the current

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which can be expressed by

\[ J_L = \frac{ie}{\hbar} \sum_{k, \sigma} V_{k, L} \left( \langle \hat{c}_{k, L, \sigma}^\dagger \hat{d}_\sigma \rangle - \langle \hat{d}_\sigma^\dagger \hat{c}_{k, L, \sigma} \rangle \right). \]
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Using the Keldysh equation

\[ G^< = (1 + G^r \Sigma^r) \left( 1 + \Sigma^a G^a \right) + G^r \Sigma^< G^a. \]
Non-equilibrium phenomena

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Using the Keldysh equation

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one obtains ...
Non-equilibrium phenomena
Non-equilibrium phenomena

steady current
Non-equilibrium phenomena

steady current

\[ J_L = -J_R \]
Non-equilibrium phenomena

steady current

\[ J_L = -J_R \]

given by the Landaer-type formula

\[ J(V) = \frac{2e}{\hbar} \int d\omega \ T(\omega) \ [f(\omega - \mu_L, T) - f(\omega - \mu_R, T)] \]
Non-equilibrium phenomena

steady current

\[ J_L = -J_R \]

given by the Landaer-type formula

\[ J(V) = \frac{2e}{h} \int d\omega \ T(\omega) \left[ f(\omega - \mu_L, T) - f(\omega - \mu_R, T) \right] \]

where the transmittance

\[ T_\sigma(\omega) = \sum_\sigma \frac{\Gamma_L(\omega)\Gamma_R(\omega)}{\Gamma_L(\omega) + \Gamma_R(\omega)} \rho_{d,\sigma}(\omega) \]
Non-equilibrium phenomena

steady current

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where the transmittance

\[ T_{\sigma}(\omega) = \sum_{\sigma} \frac{\Gamma_L(\omega)\Gamma_R(\omega)}{\Gamma_L(\omega) + \Gamma_R(\omega)} \rho_{d,\sigma}(\omega) \]

depends on the correlations through the QD spectral function

\[ \rho_{d,\sigma}(\omega) = -\frac{1}{\pi} \text{Imag} \{ G_{d,\sigma}(\omega + i0^+) \} \]
Experimental data

Zero-bias peak in differential conductance

\[ \frac{dI}{dV_{ds}} \ (e^2/h) \]

\[
\begin{array}{c|c}
90 \text{ mK} & 90 \text{ mK} \\ 0 \text{ T} & 4 \text{ T} \\
\hline
300 \text{ mK} & 90 \text{ mK} \\ 0 \text{ T} & 6 \text{ T} \\
\hline
600 \text{ mK} & 90 \text{ mK} \\ 0 \text{ T} & 7.5 \text{ T} \\
\end{array}
\]

\[ V_{ds} \ (\text{mV}) \]

0 - 0.4 - 0.2 0 0.2 0.4 -0.2 0 0.2 0.4
2. The spin vs Kondo effect
Effective interactions: $U > 0$
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Perturbative treatment of the hybridization terms
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yields for a subspace of the relevant (singly occupied) states

$$\hat{H}^{Kondo}_{spin} = \sum_{k,\beta,\sigma} \xi_{k\beta} \hat{c}^{+}_{k\beta\sigma} \hat{c}_{k\beta\sigma} - \sum_{k,q,\beta,\beta'} J^{\beta,\beta'}_{k,q} \hat{S}_{d} \cdot \hat{S}_{k\beta,q\beta'}$$
Effective interactions: \( U > 0 \)

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\[
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\hat{H}_{\text{spin}}^{Kondo} = \sum_{k,\beta,\sigma} \xi_{k\beta} \hat{c}_{k\beta\sigma}^+ \hat{c}_{k\beta\sigma} - \sum_{k,q,\beta,\beta'} J_{k,q}^{\beta,\beta'} \hat{S}_d \cdot \hat{S}_{k\beta,q\beta'}
\]

\[
\hat{S}_d^+ = \hat{d}_\uparrow \hat{d}_\downarrow, \quad \hat{S}_d^- = \hat{d}_\downarrow \hat{d}_\uparrow, \quad \hat{S}_d^z = \frac{1}{2}(\hat{d}_\uparrow \hat{d}_\uparrow - \hat{d}_\downarrow \hat{d}_\downarrow)
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$$\hat{H}_{\text{spin}}^{Kondo} = \sum_{k,\beta,\sigma} \xi_{k\beta} \hat{c}_{k\beta\sigma}^{+} \hat{c}_{k\beta\sigma} - \sum_{k,q,\beta,\beta'} J_{k,q}^{\beta,\beta'} \hat{S}_d \cdot \hat{S}_{k\beta,q\beta'}$$

$$\hat{S}_d^+ = \hat{d}_\uparrow \hat{d}_\downarrow, \quad \hat{S}_d^- = \hat{d}_\downarrow \hat{d}_\uparrow, \quad \hat{S}_d^z = \frac{1}{2} (\hat{d}_\uparrow \hat{d}_\uparrow - \hat{d}_\downarrow \hat{d}_\downarrow)$$

The spin Kondo effect comes from the antiferromagnetic coupling

$$J_{k_F,k_F}^{\beta,\beta'} = \frac{U}{\varepsilon_d (\varepsilon_d + U)} V_{k_F\beta} V_{k_F\beta'}^*$$
Situation with the negative $U$ quantum dot
Effective interactions: $U < 0$
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The perturbative treatment of the hybridization terms formally proceeds along the same line as for $U < 0$ case.
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However, in the present case the empty/double occupied sites are more favorable and they give rise to the pair-hopping

$$
\sum_{k,q,\beta,\sigma,\sigma'} J_{k,q}^{\beta,\beta'} \hat{d}_{\sigma}^{\dagger} \hat{d}_{-\sigma} \hat{c}_{k\beta-\sigma'} \hat{c}_{q\beta',\sigma'} + \text{h.c.}
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$$\sum_{k,q,\beta,\sigma,\sigma'} J_{k,q}^{\beta,\beta'} \hat{d}^\dagger_\sigma \hat{d}^\dagger_{-\sigma} \hat{c}_{k\beta-\sigma'} \hat{c}_{q\beta'\sigma'} + \text{h.c.}$$

The low energy physics is effectively described by

$$\hat{H}^{Kondo}_{\text{charge}} = \sum_{k,\beta,\sigma} \xi_{k\beta} \hat{c}^{+}_{k\beta\sigma} \hat{c}_{k\beta\sigma} + 2 \sum_{k,q,\beta,\beta'} J_{k,q}^{\beta,\beta'} \hat{T}_d \cdot \hat{T}_{k\beta,q\beta'}$$
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$$
\hat{H}_{\text{charge}}^{Kondo} = \sum_{k,\beta,\sigma} \xi_{k\beta} \hat{c}^+_k \hat{c}_{k\beta\sigma} + 2 \sum_{k,q,\beta,\beta'} J_{k,q}^{\beta,\beta'} \hat{T}_{d} \cdot \hat{T}_{k\beta,q\beta'}
$$

$$
\hat{T}_d^+ \hat{d}^\dagger_\uparrow \hat{d}^\dagger_\downarrow \hat{T}_d^- = \hat{d}_\downarrow \hat{d}_\uparrow, \quad \hat{T}_d^z = \frac{1}{2} (\hat{d}^\dagger_\uparrow \hat{d}_\uparrow + \hat{d}^\dagger_\downarrow \hat{d}_\downarrow - 1)
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$$\sum_{k,q,\beta,\sigma,\sigma'} J_{k,q}^{\beta,\beta'} \hat{d}_\sigma^\dagger \hat{d}_{-\sigma}^\dagger \hat{c}_{k\beta} \hat{c}_{q\beta'} + \text{h.c.}$$

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thus the Kondo effect can be formed in the pseudospin channel.

Charge tunneling through $U < 0$ QD

Pair tunneling
Pair tunneling

\( \frac{G}{2e^2 \Gamma_L \Gamma_R / U^2 h} \)  
\( \frac{\partial I}{\partial V} (2e^2 / U^2 h) \)

Pair tunneling

thermoelectric power:

\[
\frac{S_{\text{Mott}}}{S_0}
\]

\[2\varepsilon_d + U\]

3. The two-channel model
Electron pair trapping

We propose the following molecular quantum dot (mQD)
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\[ \hat{H}_L + \hat{V}_L + \hat{H}_{mQD} + \hat{V}_R + \hat{H}_R \]
Electron pair trapping

We propose the following molecular quantum dot (mQD)

\[
\hat{H}_L + \hat{V}_L + \hat{H}_{mQD} + \hat{V}_R + \hat{H}_R
\]

\[
\hat{H}_{mQD} = \sum_{\sigma} E_d \ \hat{d}^{\dagger}_{\sigma} \hat{d}_{\sigma} + E_{pair} \ \hat{b}^{\dagger} \hat{b} + g \left( \hat{b}^{\dagger} \hat{d}_{\downarrow} \hat{d}_{\uparrow} + \hat{d}_{\uparrow}^{\dagger} \hat{d}_{\downarrow}^{\dagger} \hat{b} \right)
\]
Quantum fluctuations
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The charge Kondo effect requires a degeneracy of the states
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Quantum fluctuations

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In the limit $V_{k,\beta} = 0$ the true eigenstates are given by

$$|B\rangle = \sin(\varphi) \; |0\rangle_d \otimes |1\rangle_b + \cos(\varphi) \; |↑↓\rangle_d \otimes |0\rangle_b$$

$$|A\rangle = \cos(\varphi) \; |0\rangle_d \otimes |1\rangle_b - \sin(\varphi) \; |↑↓\rangle_d \otimes |0\rangle_b$$
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\[
|A\rangle = \cos(\varphi) |0\rangle_d \otimes |1\rangle_b - \sin(\varphi) |\uparrow\downarrow\rangle_d \otimes |0\rangle_b
\]

and the d-QD Green's function has a three-pole structure

\[
G^V_{k\beta=0}_d(\omega) = \frac{Z}{\omega-E_d} + (1-Z) \left[ \frac{u^2}{\omega-E_B} + \frac{v^2}{\omega-E_A} \right]
\]
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Quantum fluctuations (c.d.)
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where

\[
E_A, E_B = \frac{1}{2} [E_{pair} \mp \gamma (2E_d - E_{pair})]
\]

\[
v^2, u^2 = \frac{1}{2} \left( 1 \mp \frac{1}{\gamma} \right)
\]

\[
\gamma^2 = 1 + \left( \frac{2g}{2E_d - E_{pair}} \right)^2
\]
Quantum fluctuations (c.d.)

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and \( Z \) is a strongly temperature-dependent coefficient.
Quantum fluctuations (c.d.)

where

\[
\begin{align*}
E_A, E_B &= \frac{1}{2} \left[ E_{\text{pair}} \mp \gamma (2E_d - E_{\text{pair}}) \right] \\
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\end{align*}
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To account for a finite hybridization we employ the Ansatz
Quantum fluctuations (c.d.)

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\]

and $Z$ is a strongly temperature-dependent coefficient.

To account for a finite hybridization we employ the Ansatz

\[
G_d(\omega)^{-1} = G_d^0(\omega)^{-1} - \sum_{k,\beta} \frac{|V_{k\beta}|^2}{\omega - \xi_{k\beta}}
\]
Spectrum of the d-QD
Spectrum of the d-QD

\[ \rho_d(\omega) = \frac{1}{\Gamma_L} \]

\[ \frac{\Gamma_R}{\Gamma_L} = 0.3 \]

\[ g = 2\Gamma_L \]

\[ \frac{T}{\Gamma_L} = 0.3 \]
Spectrum of the d-QD

\[ \rho_d(\omega) = \frac{1}{\Gamma_L} \]

\[ \Gamma_R = \Gamma_L \]

\[ g = 2\Gamma_L \]

\[ T / \Gamma_L = 0.2 \]
Spectrum of the d-QD

\[ \rho_d(\omega) \left[ \frac{1}{\Gamma_L} \right] \]

\[ \frac{\Gamma_R}{\Gamma_L} = \frac{g}{2\Gamma_L} \]

\[ T / \Gamma_L = 0.1 \]
Spectrum of the d-QD

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\[ \frac{\Gamma_R}{\Gamma_L} = g = 2\Gamma_L \]

\[ T / \Gamma_L = 0.1 \]

The middle peak: superradiant state,
Spectrum of the d-QD

\[ \rho_d(\omega) \equiv \frac{1}{\Gamma_L} \]

\[ \frac{\omega}{\Gamma_L} = \frac{T}{\Gamma_L} = 0.1 \]

- the middle peak: superradiant state,
- the side peaks: subradiant states.
Differential conductance
Differential conductance

\[ G(V) \sim \frac{2e^2}{h} \]

\[ eV / \Gamma_L \]

\[ 2g \]

\[ k_B T \]
Superradiant line broadening is proportional to $T$!
Dicke effect in mesoscopic physics
Dicke effect in mesoscopic physics

# 1 tunneling via two quantum dots
Dicke effect in mesoscopic physics

1 tunneling via two quantum dots

Dicke effect in mesoscopic physics

# 2 tunneling via the quantum wire + magnetic field

Diagram showing inter-subband scattering, energy levels $\varepsilon_{nk}$, Fermi level $\varepsilon_F$, and impurity with $n = 0, 1, 2$.
Dicke effect in mesoscopic physics

2 tunneling via the quantum wire + magnetic field

\[ \text{Re} \sigma(\omega)/\sigma_0 \]

Dicke effect in mesoscopic physics

3 tunneling via three quantum dots

\[ \Gamma^L_\sigma \quad t_{12\sigma} \quad \Gamma^R_\sigma \]

QD1  QD2  QD3  L  R
Dicke effect in mesoscopic physics

3 tunneling via three quantum dots

4. Summary
Summary:

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- Low energy physics is reminiscent of the Dicke effect with supperadiant line broadening $\sim k_B T$. 
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