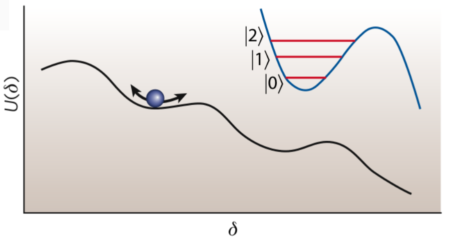
Macroscopic quantum tunneling and quantization: Nobel Prize in Physics 2025

Tadeusz Domański

M. Curie-Skłodowska University
L U B L I N



Institute of Physics

Lublin, 11 December 2025

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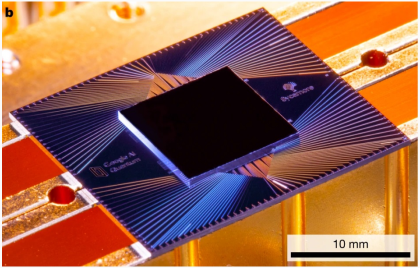

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OUTLINE

- **Josephson effect**

  [Nobel Prize in Physics 1973]

- **Macroscopic tunneling & quantization**

  [Nobel Prize in Physics 2025]

- **Technological applications**

  [superconducting qubits & processors]

- **Current challenges**

  [topological states in Josephson junctions]

Basic properties:

1. **perfect conductor**

2. **perfect diamagnet**

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realized: $\Longrightarrow$ **below critical temperature** T_c

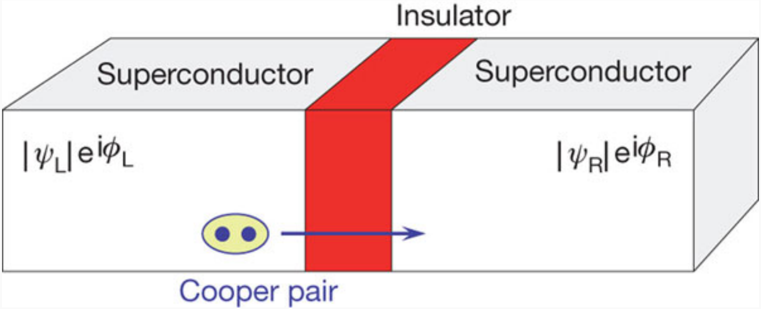
$\Longrightarrow$ **below critical current** I_c

This is Bose-Einstein condensate of the Cooper pairs which are described by a macroscopic wave function:

$$\Psi(\vec{r},t) \equiv |\Psi(\vec{r},t)| \; e^{i\phi(\vec{r},t)}$$

JOSEPHSON EFFECT

Two superconductors, differing in phase $\phi_L \neq \phi_R$, contacted through a narrow insulating region induce a *super*flow of the Cooper pairs.



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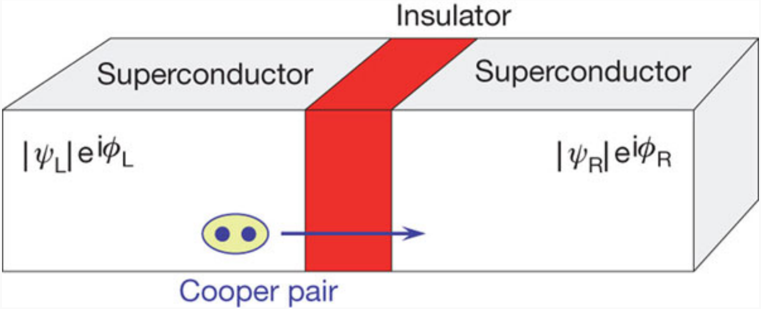

This effect has been predicted by B.D. Josephson in 1962.

/ 22-year-old PhD student at Cambridge, England /

HYDRODYNAMIC REASONING

In quantum mechanics the probability current is defined by

$$\vec{j}(\vec{r},t) = -\frac{i\hbar}{2m}\left[\Psi^\star(\vec{r},t)\nabla\Psi(\vec{r},t) - \Psi(\vec{r},t)\nabla\Psi^\star(\vec{r},t)\right]$$

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Applying this formalism to the wave-function Φ_0 of Cooper pairs

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we can express the Josephson current density

$$\vec{j}_J(\vec{r},t) = -\, q\, \frac{i\hbar}{2m}\left[\Psi_0^\star\nabla\Psi_0 - \Psi_0\nabla\Psi_0^\star\right]$$

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where $q = 2e$ is charge and $\vec{v}(\vec{r},t)$ is velocity of Cooper pairs.

EXPERIMENTAL EVIDENCE

VOLUME 10, NUMBER 6 PHYSICAL REVIEW LETTERS 15 MARCH 1963

PROBABLE OBSERVATION OF THE JOSEPHSON SUPERCONDUCTING TUNNELING EFFECT

P. W. Anderson and J. M. Rowell
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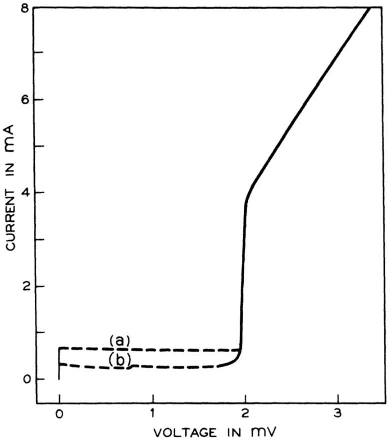
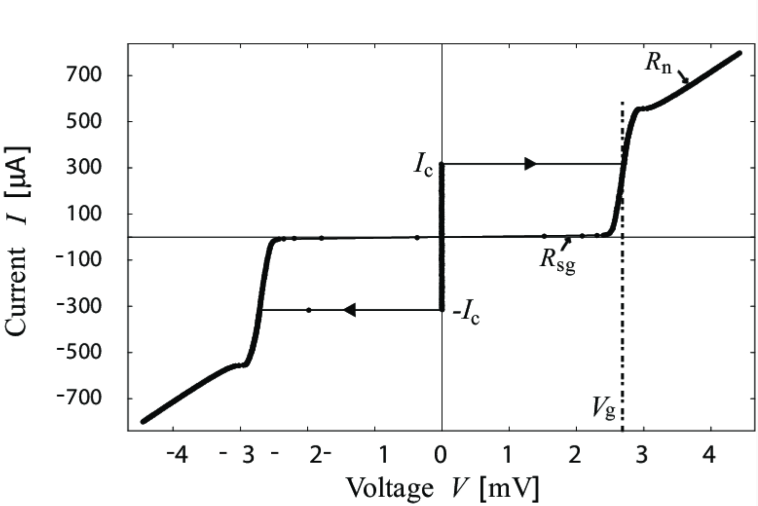


Authors reported:

„dc tunneling current at or near zero voltage in very thin tin oxide barriers between superconducting Sn and Pb"

Typical current-voltage plot, where $V_g = 2\Delta$



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2025

J. Clarke, M.H. Devoret, J.M. Martinis

RECIPIENTS OF NOBEL PRIZE 2025

„for the discovery of macroscopic quantum mechanical tunnelling and energy quantisation in an electric circuit"

Ill. Niklas Elmehed © Nobel Prize Outreach

John Clarke

Prize share: 1/3

Ill. Niklas Elmehed © Nobel Prize Outreach

Michel H. Devoret

Prize share: 1/3

Ill. Niklas Elmehed © Nobel Prize Outreach

John M. Martinis

Prize share: 1/3

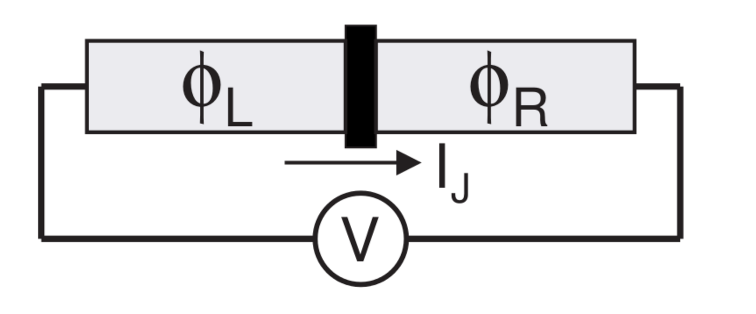
John Clarke - emeritus at the University of California (Berkeley)

Michel D. Devoret - University of California (Santa Barbara) & Yale University

John M. Martinis - University of California (Santa Barbara)

JOSEPHSON JUNCTION IN ELECTRIC CIRCUIT

Let's consider the Josephson junction enclosed in electric circuit



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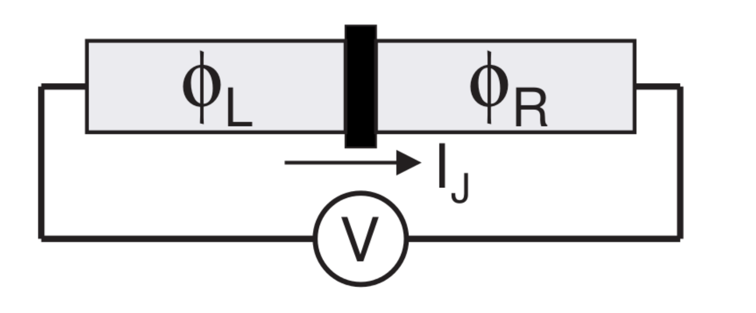

with the macroscopic wave-functions $\Psi_{L/R}$ of the Cooper pairs

$$\Psi_{L/R} \equiv \left|\Psi_{L/R}\right| \; e^{i\phi_{L/R}}$$

where $\left|\Psi_{L/R}\right| = \sqrt{n_{L/R}}$ denote concentrations and $\phi_{L/R}$ phases.

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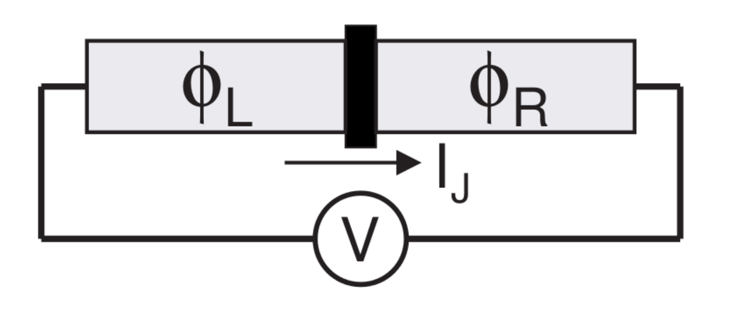

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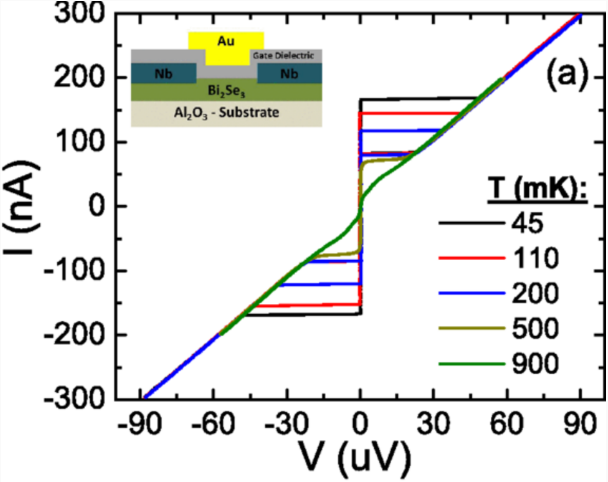
$$I_J = I_{crit} \sin\left(\phi_R - \phi_L\right)$$

RESISTIVE TRANSITION

The critical current I_{crit} diminishes upon increasing temperature.



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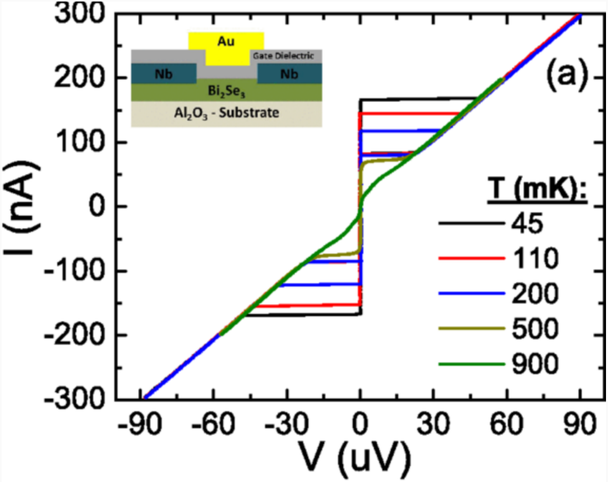
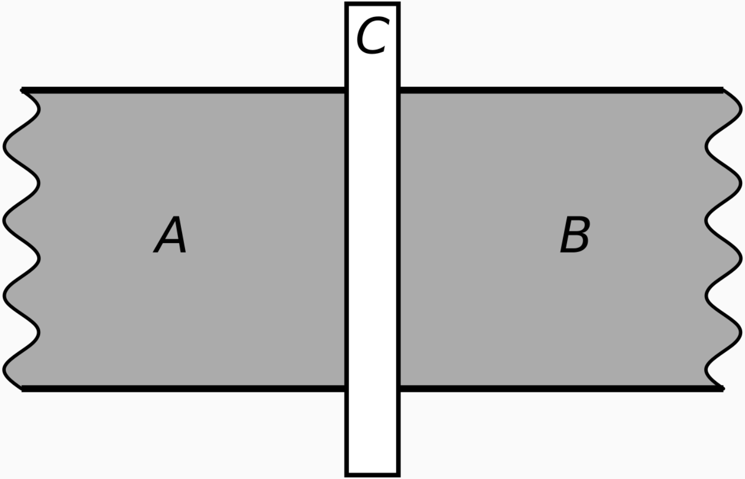


Josephson-type superflow changes to resistive behaviour at $I \to I_{crit}$.

BIASED JOSEPHSON JUNCTION

Let’s denote two sides of this junction by A and B, correspondingly



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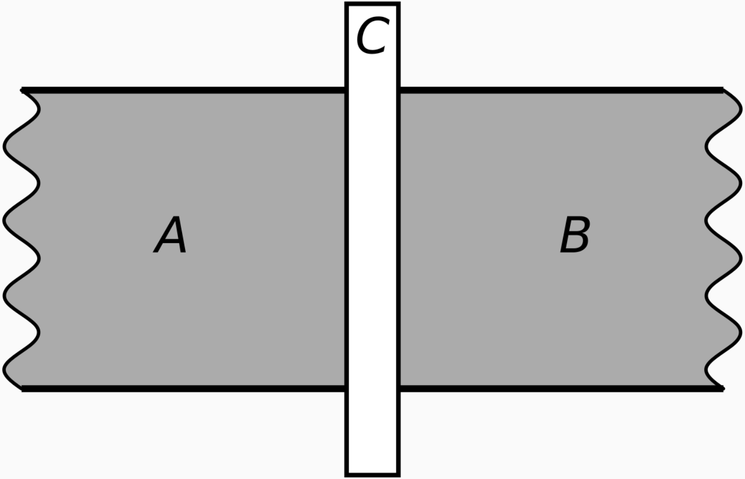

and assume that voltage V is applied across the junction.

DC/AC JOSEPHSON EFFECT .... [WIKIPEDIA]

Schrödinger eqn $i\hbar\frac{\partial\Psi_{A/B}}{\partial t} = \hat{H}\Psi_{A/B}$ for the Josephson junction:

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\sqrt{n_A}e^{i\phi_A}\\ \sqrt{n_B}e^{i\phi_B}\end{pmatrix} = \begin{pmatrix}eV & K\\ K & -eV\end{pmatrix}\begin{pmatrix}\sqrt{n_A}e^{i\phi_A}\\ \sqrt{n_B}e^{i\phi_B}\end{pmatrix} \tag{1}$$

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To solve this equation (1), let us first calculate the time derivative for the wave function of superconductor A:

$$\frac{\partial}{\partial t}(\sqrt{n_A}e^{i\phi_A}) \;=\; \dot{\sqrt{n_A}}e^{i\phi_A} + \sqrt{n_A}(i\dot{\phi}_A e^{i\phi_A})$$

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Schrödinger eqn gives:

$$(\dot{\sqrt{n_A}} + i\sqrt{n_A}\dot{\phi}_A)e^{i\phi_A} = \frac{1}{i\hbar}(eV\sqrt{n_A}e^{i\phi_A} + K\sqrt{n_B}e^{i\phi_B})$$

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and (its complex conjugate):

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$$2\dot{\sqrt{n_A}} = \frac{1}{i\hbar}(K\sqrt{n_B}e^{i\varphi} - K\sqrt{n_B}e^{-i\varphi}) = \frac{K\sqrt{n_B}}{\hbar} \cdot 2\sin\varphi$$

and using $\dot{\sqrt{n_A}} = \frac{\dot{n}_A}{2\sqrt{n_A}}$, we finally obtain:

$$\dot{n}_A = \frac{2K}{\hbar}\sqrt{n_A n_B}\ \sin\varphi.$$

Now, by subtracting eqns (2,3) we eliminate $\dot{\sqrt{n_A}}$:

$$2i\sqrt{n_A}\dot{\phi}_A = \frac{1}{i\hbar}(2eV\sqrt{n_A} + K\sqrt{n_B}e^{i\varphi} + K\sqrt{n_B}e^{-i\varphi})$$

which gives:

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Similar equations can be derived for superconductor B:

$$\begin{aligned} \dot{n}_B &= -\frac{2K\sqrt{n_A n_B}}{\hbar}\sin\varphi \\ \dot{\phi}_B &= \frac{1}{\hbar}(eV - K\sqrt{\frac{n_A}{n_B}}\cos\varphi) \end{aligned}$$

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Tunneling current is thus: $I_A = 2e\dot{n}_A = -I_B \propto \sin(\varphi)$

In particular, for $n_A \approx n_B$ this treatment yields:

$$I = I_c \sin(\varphi) \tag{4}$$

$$\frac{\partial \varphi(t)}{\partial t} = \frac{2eV}{\hbar} \tag{5}$$

where $\varphi = \phi_B - \phi_A$.

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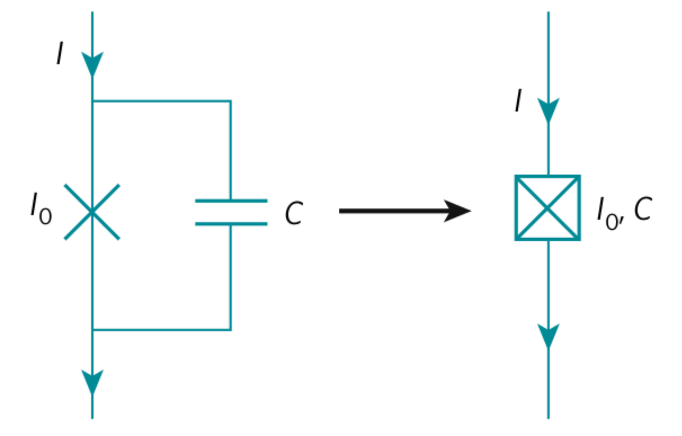
These Josephson equations:

(4) relate the tunneling current I flowing through a junction to the macroscopic phase difference φ (it can be static)

(5) express the time evolution φ in terms of the voltage V developed across a junction (ac Josephson effect)

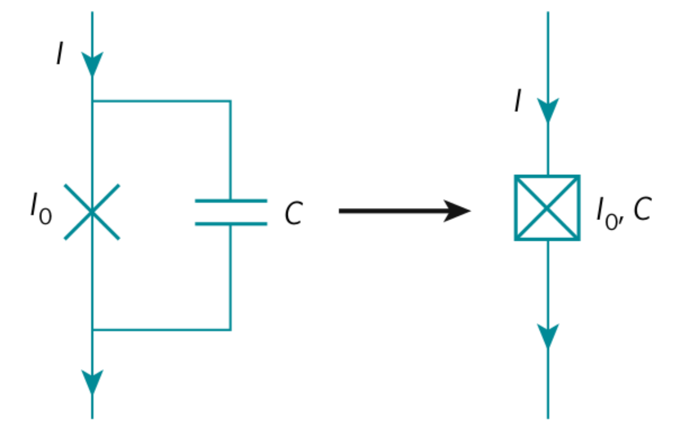
JOSEPHSON JUNCTION + CAPACITOR



Incorporating the Josephson junction into a ciruit with capacitor:

$$I(t) \;=\; I_c \sin(\varphi(t)) + C\frac{\partial V}{\partial t}$$

JOSEPHSON JUNCTION + CAPACITOR



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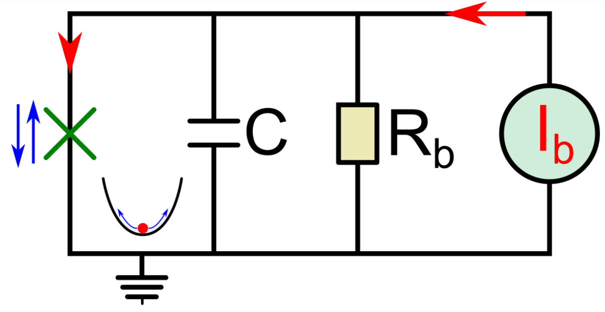
$$I(t) \quad = \quad I_c \sin(\varphi(t)) + \frac{\hbar}{2e} C \frac{\partial^2 \varphi}{\partial t^2}$$

QUANTUM-NESS OF JOSEPHSON CIRCUIT

RCSJ setup:



Phase difference φ of the Josephson junction and charge Q on the capacitor represent canonically conjugated quantities.

They obey the Heisenberg's uncertainty principle, and:

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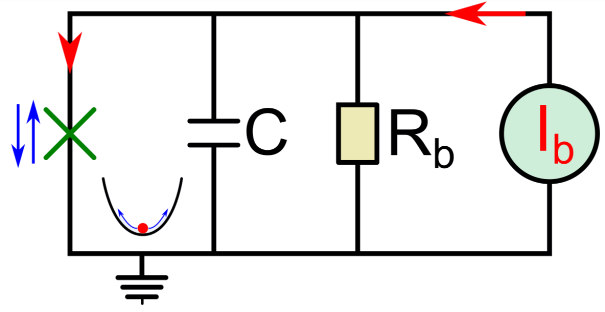

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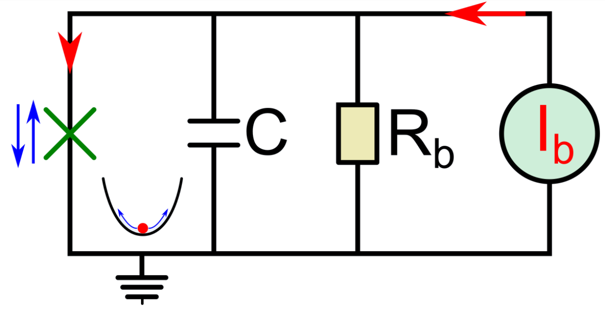

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- **for large C the charge Q is well defined**

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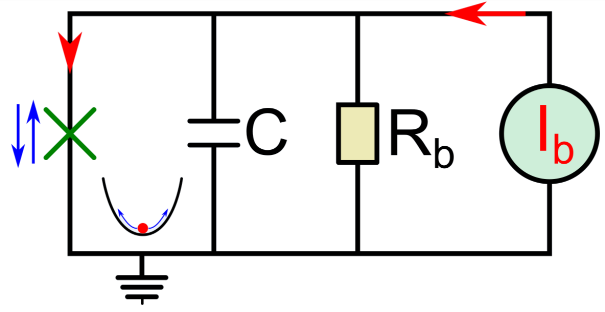

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The charge current consists of:

$$I(t) = I_c \sin(\varphi) + \frac{\hbar}{2e} C \frac{\partial^2 \varphi}{\partial t^2} + \frac{V}{R_b}$$

QUANTUM-NESS OF JOSEPHSON CIRCUIT

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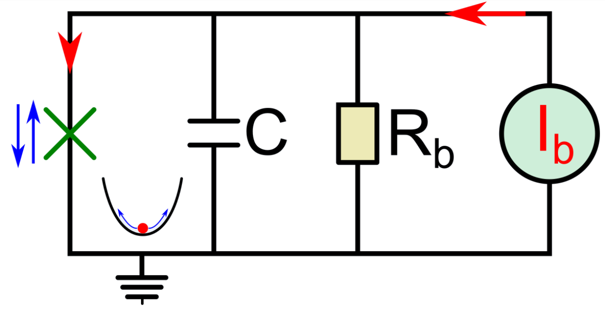

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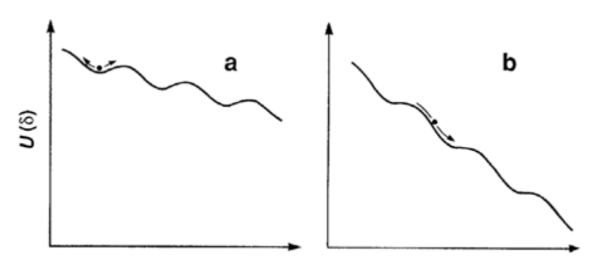
The effective Hamiltonian:

$$\hat{H} = \frac{1}{2C}\hat{Q}^2 - I_c\frac{\hbar}{2e}\cos(\hat{\varphi}) - I_c\frac{\hbar}{2e}\hat{\varphi}$$

SUPERCONDUCTING CIRCUIT

The effective „washboard potential" $U(\delta) = -I_0 \frac{\hbar}{2e}\delta - E_J \cos\delta$



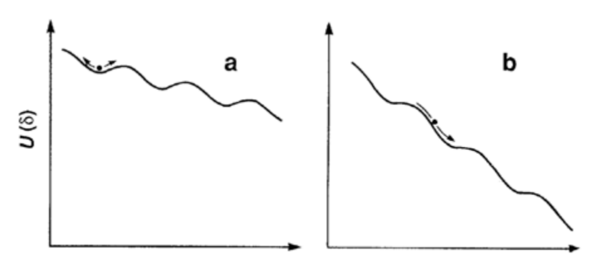
Josephson phase [δ]

In absence of fluctuations & ac external fields

$\Longrightarrow$ **for $I < I_{crit}$ the system „stays" in a local minimum, oscillating with the plasma frequency ω_p (in zero-voltage state)**

SUPERCONDUCTING CIRCUIT

The effective „washboard potential” $U(\delta) = -I_0 \frac{\hbar}{2e}\delta - E_J \cos\delta$



Josephson phase [δ]

In absence of fluctuations & ac external fields

$\Longrightarrow$ for $I < I_{crit}$ the system „stays” in a local minimum, oscillating with the plasma frequency ω_p (in zero-voltage state)

$\Longrightarrow$ for $I > I_{crit}$ the system „runs” down the washboard $\langle \frac{d}{dt}\delta \rangle > 0$ (switching to the resistive state)

QUANTUM TUNNELING: THEORETICAL STUDIES

VOLUME 46 | 26 JANUARY 1981 | NUMBER 4

Influence of Dissipation on Quantum Tunneling in Macroscopic Systems

A. O. Caldeira and A. J. Leggett
School of Mathematical and Physical Sciences, University of Sussex, Brighton BN1 9QH, Sussex, United Kingdom

(Received 28 July 1980)

A quantum system which can tunnel, at $T = 0$, out of a metastable state and whose interaction with its environment is adequately described in the classically accessible region by a phenomenological friction coefficient η, is considered. By only assuming that the environment response is linear, it is found that dissipation multiplies the tunneling probability by the factor $\exp[-A\eta(\Delta q)^2/\hbar]$, where Δq is the "distance under the barrier" and A is a numerical factor which is generally of order unity.

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PHYSICAL REVIEW B | VOLUME 30, NUMBER 11 | 1 DECEMBER 1984

Quantum dynamics of a superconducting tunnel junction

Ulrich Eckern
Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, D-7500 Karlsruhe 1, Federal Republic of Germany

Gerd Schön* and Vinay Ambegaokar†
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 18 June 1984)

Basing our model and method on the microscopic theory we formulate a quantum-mechanical description for the relevant variable in a superconducting tunnel junction, i.e., the phase difference across the junction. The quasiparticle degrees of freedom are responsible for dissipation and noise in the system. Because of the discreteness of the charge-transfer process, the noise is shot noise. The energy gaps in the superconductors lead to further interesting features. We discuss the consequences of these physical effects on macroscopic quantum phenomena.

MACROSCOPIC TUNNELING & QUANTIZATION

VOLUME 55, NUMBER 15 PHYSICAL REVIEW LETTERS 7 OCTOBER 1985

Energy-Level Quantization in the Zero-Voltage State of a Current-Biased Josephson Junction

John M. Martinis, Michel H. Devoret,[a] and John Clarke
Department of Physics, University of California, Berkeley, California 94720, and Materials and Molecular Research Division, Lawrence Berkeley Laboratory, Berkeley, California 94720
(Received 14 June 1985)

We report the first observation of quantized energy levels for a macroscopic variable, namely the phase difference across a current-biased Josephson junction in its zero-voltage state. The position of these energy levels is in quantitative agreement with a quantum mechanical calculation based on parameters of the junction that are measured in the classical regime.

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VOLUME 55, NUMBER 18 PHYSICAL REVIEW LETTERS 28 OCTOBER 1985

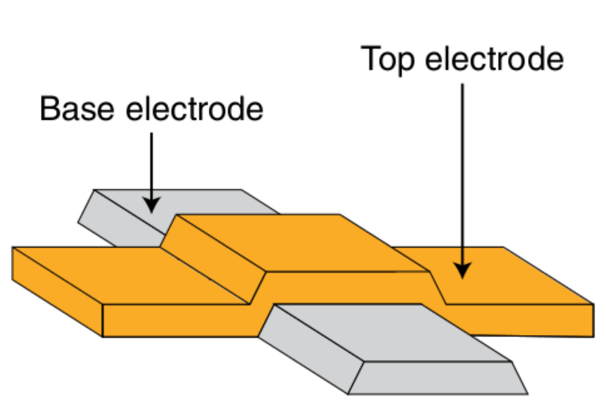
Measurements of Macroscopic Quantum Tunneling out of the Zero-Voltage State of a Current-Biased Josephson Junction

Michel H. Devoret,[a] John M. Martinis, and John Clarke
Department of Physics, University of California, Berkeley, California 94720, and Materials and Molecular Research Division, Lawrence Berkeley Laboratory, Berkeley, California 94720
(Received 26 July 1985)

The escape rate of an underdamped ($Q \approx 30$), current-biased Josephson junction from the zero-voltage state has been measured. The relevant parameters of the junction were determined *in situ* in the thermal regime from the dependence of the escape rate on bias current and from resonant activation in the presence of microwaves. At low temperatures, the escape rate became independent of temperature with a value that, with no adjustable parameters, was in excellent agreement with the zero-temperature prediction for macroscopic quantum tunneling.

EXPERIMENTAL SETUP

Josephson junction used by J.M. Martinis, M.H. Devoret and J. Clarke

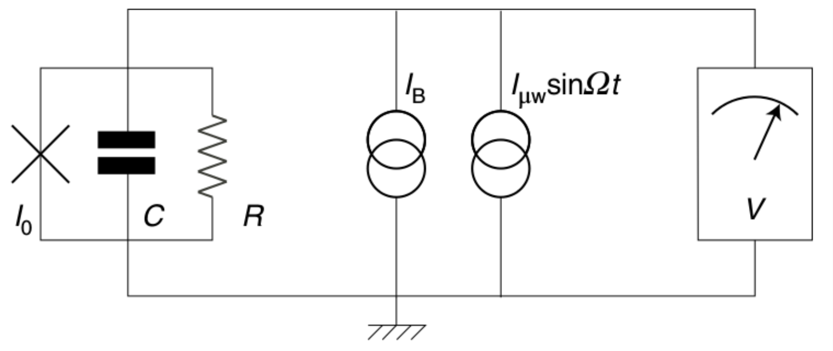


Device size:
10 μm $\times$ 10 μm

Superconducting Nb (base electrode) and PbIn alloy (top electrode) separated by 1-nm thick NbO_x (insulating layer) which was formed by plasma-oxidation.

EXPERIMENTAL SETUP

Electric circuit used by J.M. Martinis, M.H. Devoret and J. Clarke



Josephson junction (cross) shunted by a capacitance (C) and resistance (R) connected to both the static bias I_B and microwave $I_{\mu w}$ current sources.

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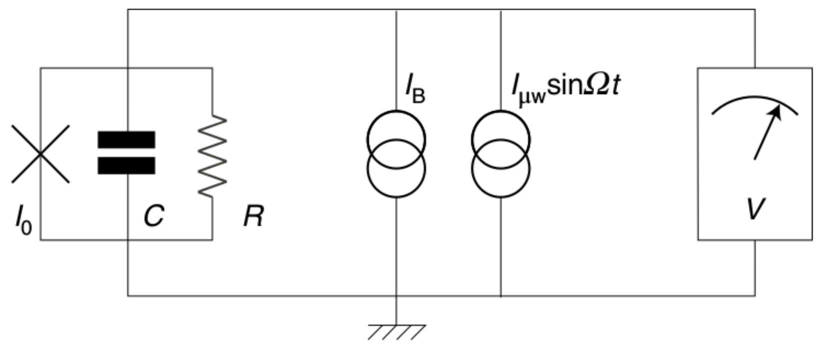

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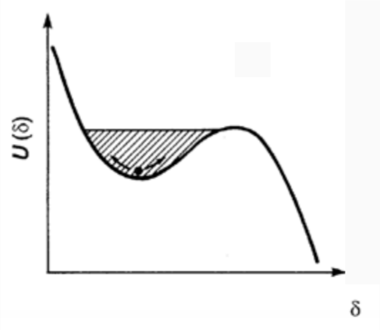
Voltage V across the junction was measured by a low-noise audio-frequency amplifier.

1. MACROSCOPIC TUNNELING: IDEA

Characteristic values: ω_p - plasma frequency, ΔU - potential barrier



Empirical method for probing the escape rate from a local mimimum:

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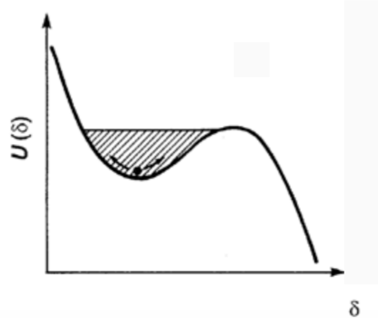

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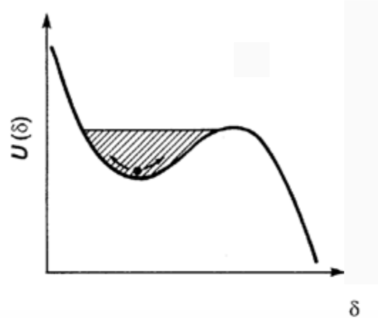

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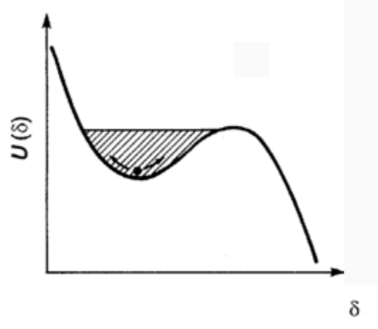

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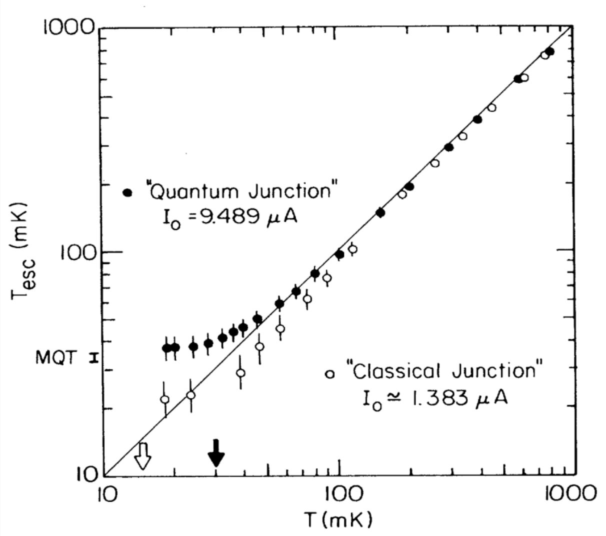
⟹ **A.O. Caldeira and A. Leggett, Ann. Phys. (N.Y.) 149, 374 (1983).**

1. MACROSCOPIC TUNNELING: RESULTS

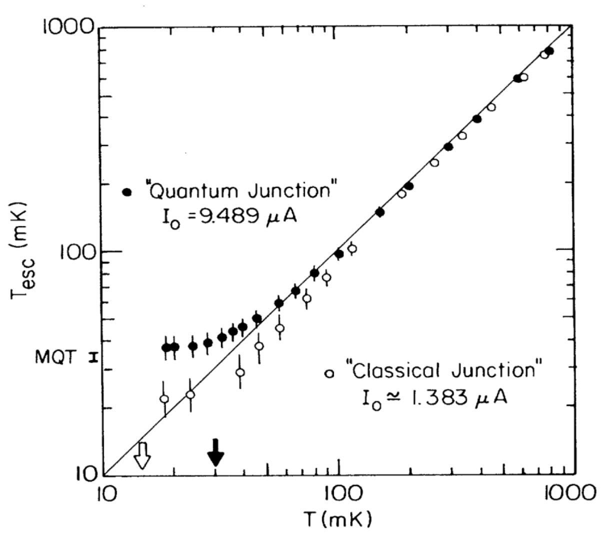
The measured "escape temperature" T_{esc} from the local minimum



Phys. Rev. Lett. 55, 1908 (1985).

1. MACROSCOPIC TUNNELING: RESULTS

The macroscopic quantum tunneling occurs at: $T \leq 30$ mK.



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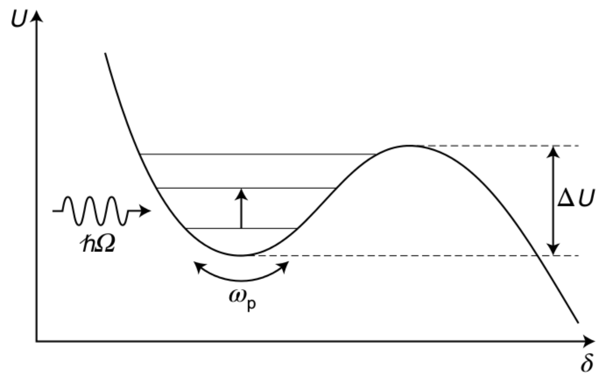
Empirical method for probing (detecting) the quantized energy levels:



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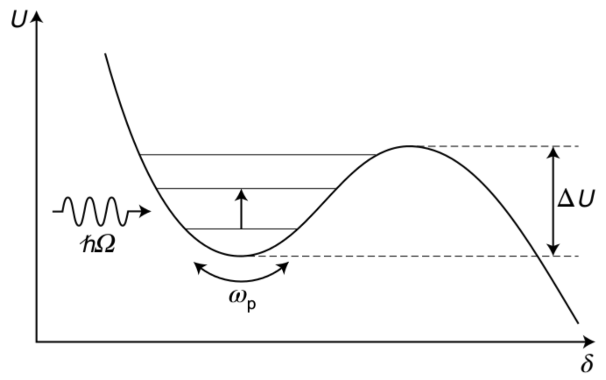

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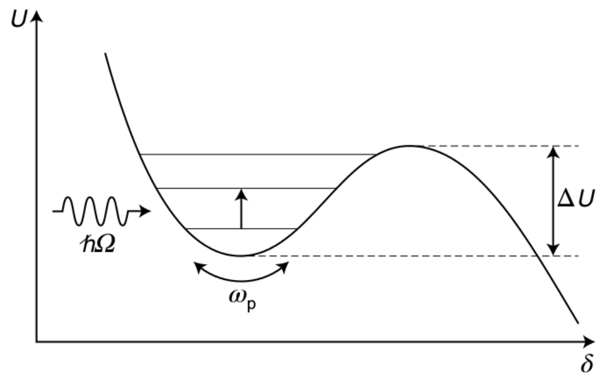

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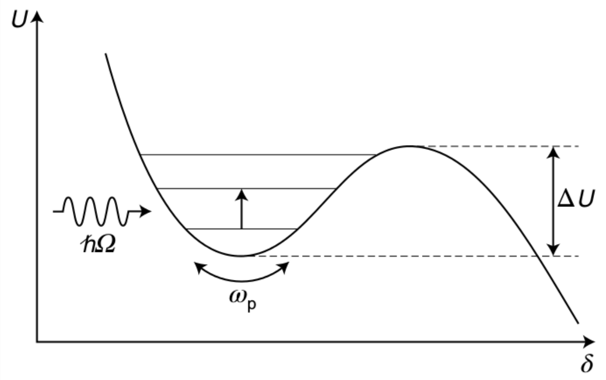

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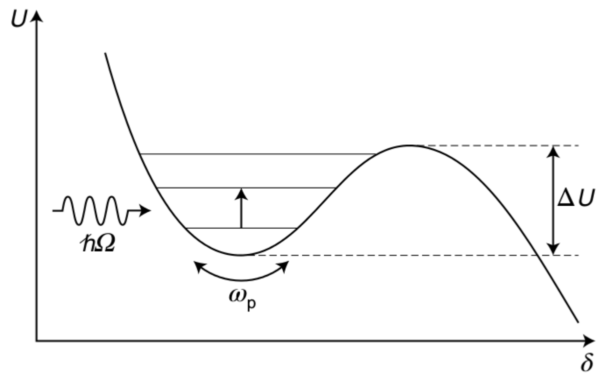

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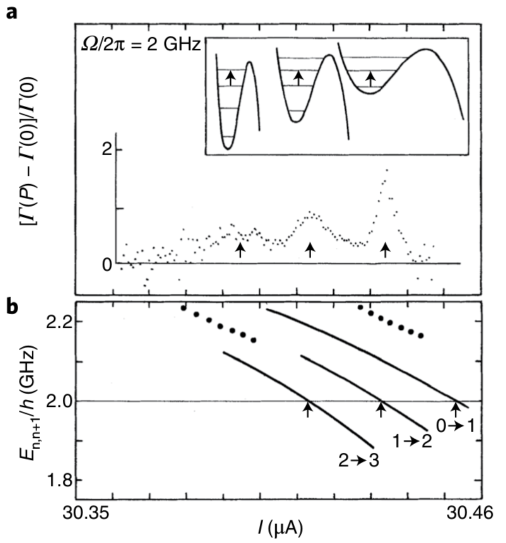
- **observe the thermal escape rate from the excited level E_{n+1}**

2. ENERGY QUANTIZATION: RESULTS

Inter-level transitions induced by the microwaves $\Omega = 2$ GHz at $T = 28$ mK.



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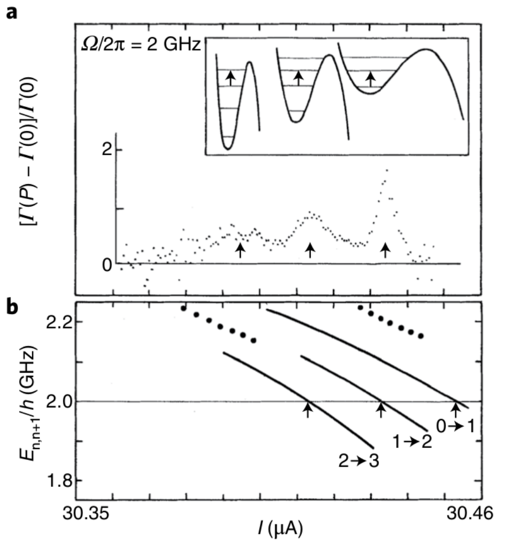
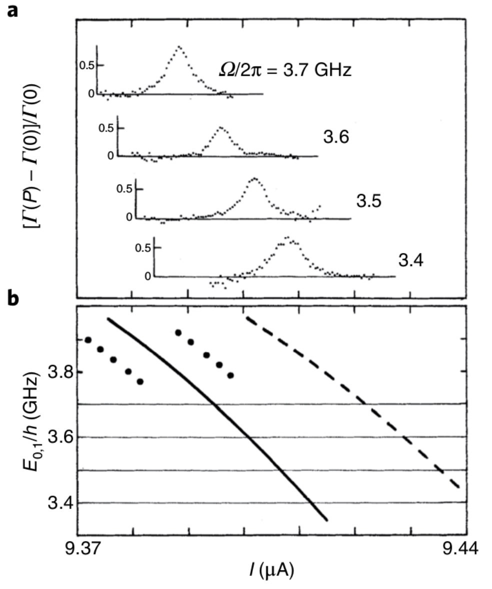


Arrows indicate positions of the resonances (bottom pannel shows the calculated results).

2. ENERGY QUANTIZATION: RESULTS

Transition $E_0 \to E_1$ induced by the microwaves $\Omega = 3.4, 3.5, 3.6, 3.7$ GHz.



Quantum bits

Quantum bits

based on Josephson junctions

JOSEPHSON JUNCTION $\longrightarrow$ QUANTUM BIT

The next step in the demonstration of macroscopic quantum physics was to implement devices, showing a superposition of two quantum states.

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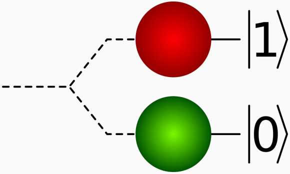
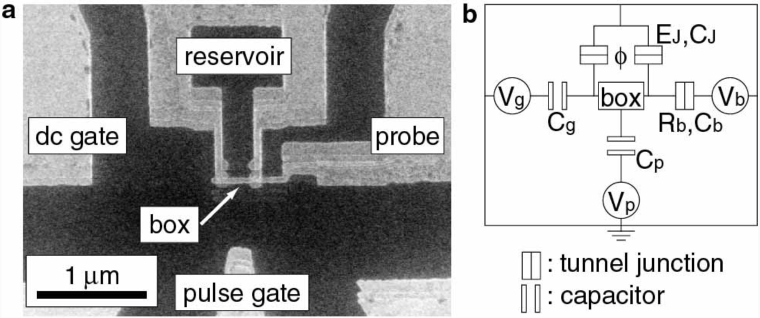
$$\alpha |0\rangle + \beta |1\rangle$$



SUPERCONDUCTING QUBIT: FIRST REALIZATION

Y. Nakamura et al carried out the first experiment on charge qubit,



Y. Nakamura, Y.A. Pashkin, J.S. Tsai, Nature 398, 786 (1999).

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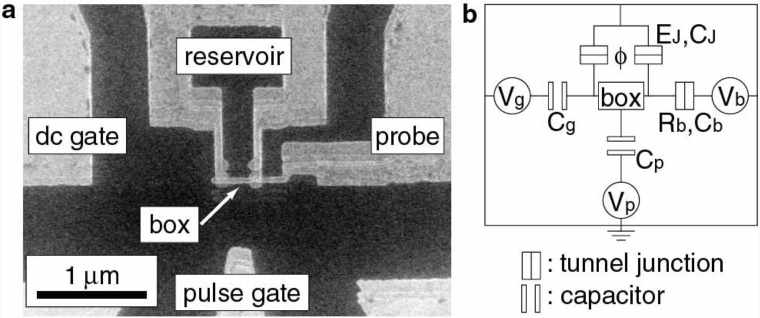

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showing a superposition of the Cooper-pair states $|n\rangle$ and $|n+1\rangle$, where n is the number of electron pairs (in the Cooper-pair-box).

$\Rightarrow$ The quantum state was controlled by gate potential, hence gatemon.

SUBSEQUENT ACTIVITIES

Realizations of superconducting quantum bits:

$\Longrightarrow$ **(2000) flux qubit** **(J. Friedman et al, C. van der Wal et al)**

/superconducting loop interrupted by one or three Josephson junctions/

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$\Rightarrow$ (later) many other ...

Schematic idea of the Josephson phase qubit (phason).

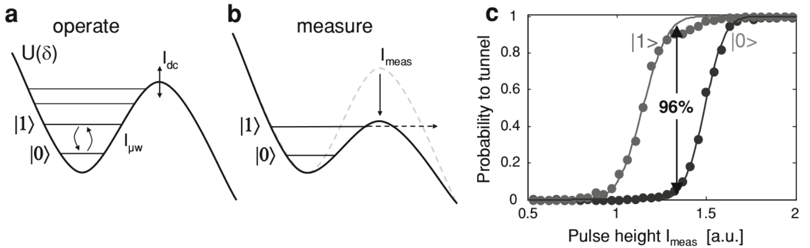


Fig. 1 **a** Plot of non-linear potential $U(\delta)$ for the Josephson phase qubit. The qubit states $|0\rangle$ and $|1\rangle$ are the two lowest eigenstates in the well. The junction bias I_{dc} is typically chosen to give 3–7 states in the well. Microwave current $I_{\mu w}$ produces transitions between the qubit states. **b** Plot of potential during state measurement. The well barrier is lowered with a bias pulse I_{meas} so that the $|1\rangle$ state can rapidly tunnel. **c** Plot of tunneling probability versus I_{meas} for the states $|0\rangle$ and $|1\rangle$. The arrow indicates the optimal height of I_{meas}, which gives a fidelity of measurement close to the maximum theoretical value 96%

J.M. Martinis, Quantum Inf Process 8, 81-103 (2009).

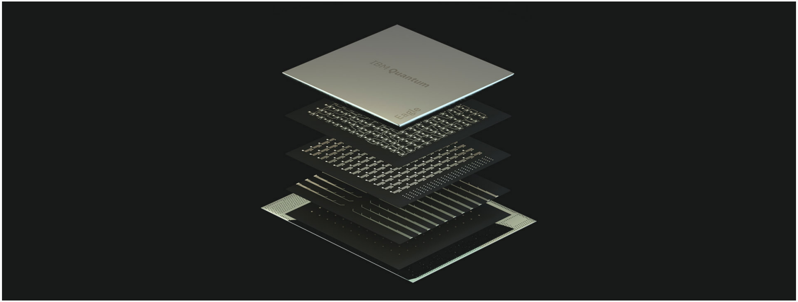
Quantum processors

Quantum processors

based on superconducting qubits

SUPERCONDUCTING PROCESSOR: EAGLE

In November 2021 IBM informed about construction of 127-qubit superconducting processor Eagle.



https://postquantum.com/industry-news/ibm-eagle/

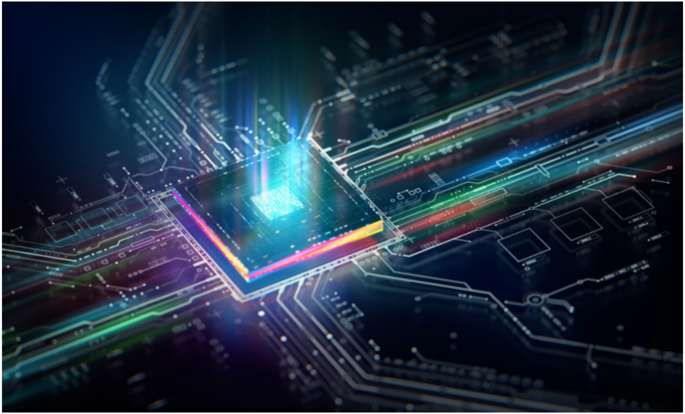
SUPERCONDUCTING PROCESSOR: WILLOW

In December 2024 Google demonstrated 105-qubit processor based on superconducting qubits (transmons).

Google Quantum AI and collaborators, Nature 638, 920 (2024).

SUPERCONDUCTING PROCESSOR: WILLOW

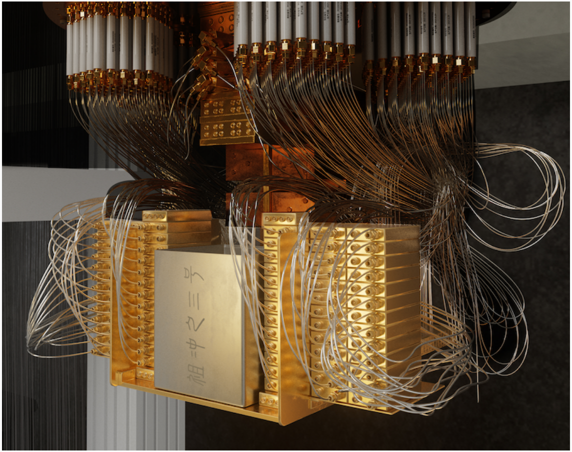
Simulation of the probability distribution obtained in 5 minutes by processor Willow would take about 10^{25} years by the fastest classical computer.



H. Neven (Google blog, 9 December 2024).

SUPERCONDUCTING PROCESSOR: ZUCHONGZHI 3.0

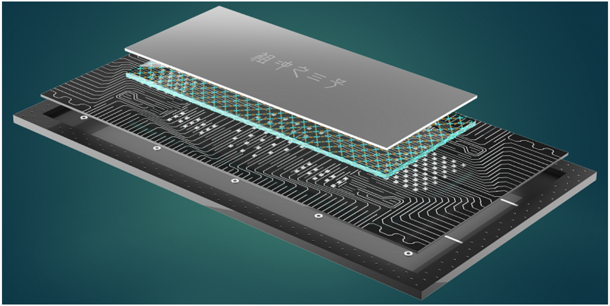
105-qubit processor constructed by the group of prof. Jian-Wei Pan (University of Science and Technology, China)



D. Gao et al, Phys. Rev. Lett. 134, 090601 (2025).

SUPERCONDUCTING PROCESSOR: ZUCHONGZHI 3.0

Simulation of the probability distribution obtained in 100 seconds by processor Zuchongzhi 3.0 would take at least several 10^6 years by the fastest classical computer.



Zuchongzhi 3.0 processor consists of 105 qubits: 15 qubits in 7 arrays.

D. Gao et al, Phys. Rev. Lett. 134, 090601 (2025).

PRESENT & FUTURE APPLICATIONS

Superconducting quantum circuits can address the challenges associated with:

⋆ **sensing spins, phonons, and exotic particles**

⋆ **quantum communication between different chips or subsystems**

⋆ **transduction between microwave and optical photons**

⋆ **simulations of many-body systems**

⋆ **and computations of test algorithms**

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$\Longrightarrow$ **For details see:**

J.M. Marinis, M.H. Devoret, J. Clarke, Nature Phys. 16, 234 (2020).

SUMMARY

⇒ **Unquestionable facts:**

1. **superconducting qubits based on Josephson junctions**

   (gatemon, transmon, fluxonium, Xmon, Unimon, ...)

2. **superconducting quantum processors**

   (Google, IBM, Intel, IMEC, BBN Technology, Rigetti)

SUMMARY

⇒ **Unquestionable facts:**

1. superconducting qubits based on Josephson junctions

(gatemon, transmon, fluxonium, Xmon, Unimon, ...)

2. superconducting quantum processors

(Google, IBM, Intel, IMEC, BBN Technology, Rigetti)

⇒ **Challenges:**

⋆ **topological qubits & processors**

(protection, braiding of Majorana quasiparticles, ...)

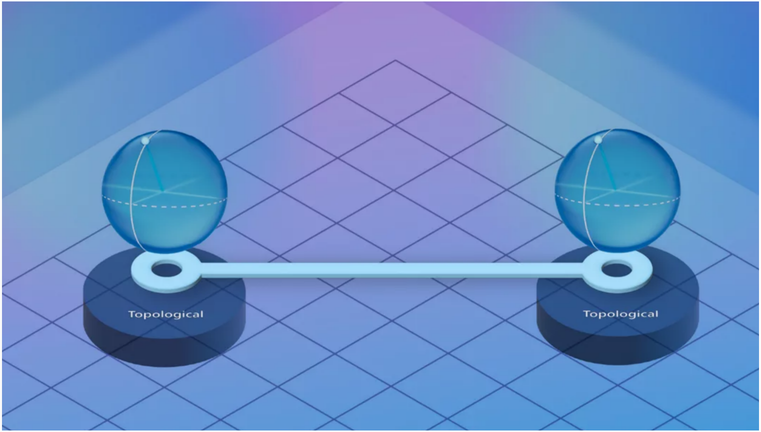
Topological quantum computer

(superconducting qubits based on parity)

TOPOLOGICAL QUBIT

Topological superconducting qubit based on:

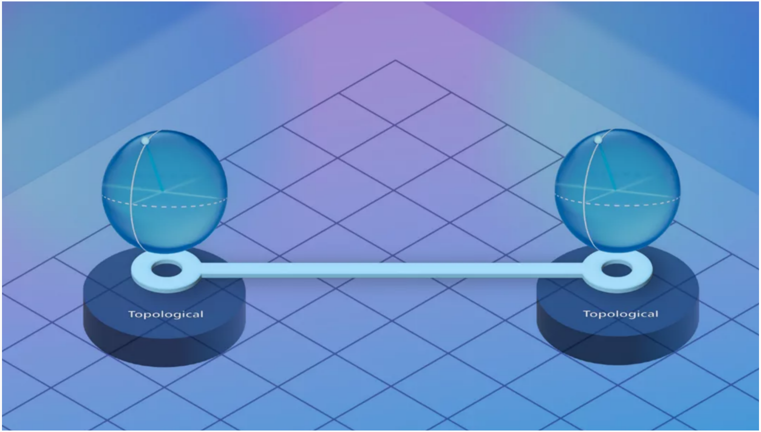
$\Rightarrow$ **Majorana quasiparticles**



TOPOLOGICAL QUBIT

Topological superconducting qubit based on:

⇒ **Majorana quasiparticles**

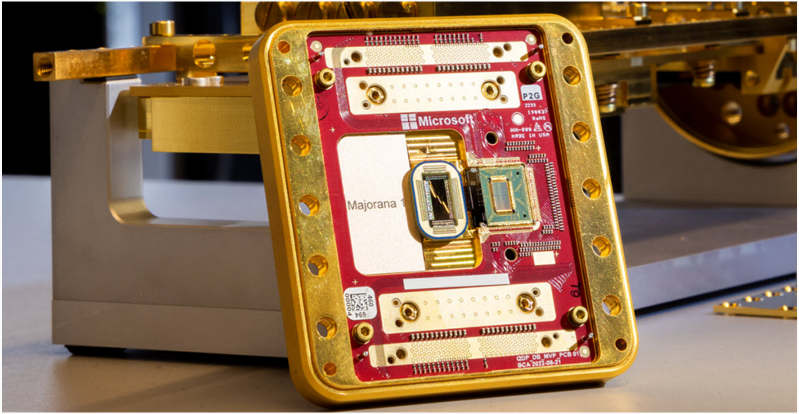


Such qubits would have:

⇒ **topological protection against environmental influence.**

RECENT NEWS

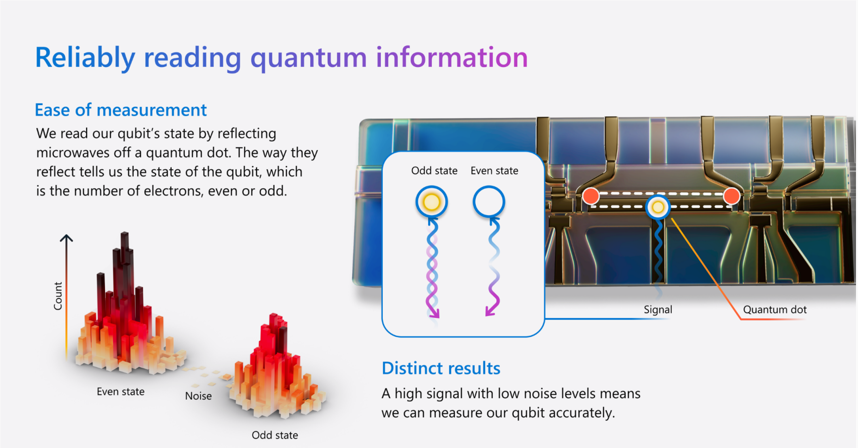
In February 2025 Microsoft informed about construction of the first processor based on topological superconducting qubits



Microsof Azure Quantum, Nature 638, 651 (2025).

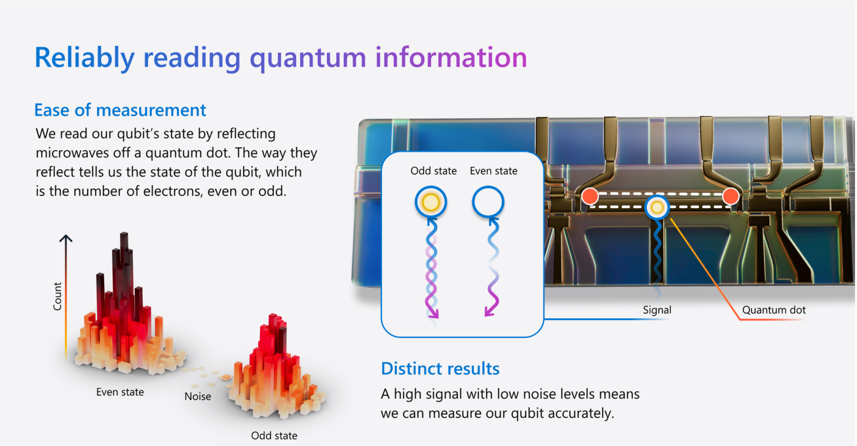
https://www.youtube.com/shorts/jPrI2wO1GfM

CONTROVERSIES



"Tetron" device in the shape of H-letter, consisting of four Majorana quasiparticles

CONTROVERSIES



"Tetron" device in the shape of H-letter, consisting of four Majorana quasiparticles

Parity measurement was announced during APS March Meeting (14 000 participants) but the scientific community expressed high scepticism.

M. Rini, Physics 18, 68 (2025).

Thank You

Thank You

https://sites.google.com/view/domanskit/lectures

TOPOLOGICAL QUBIT

Topological qubit can be constructed of four Majorana qps, which consist of two nonlocal fermions (electrons)

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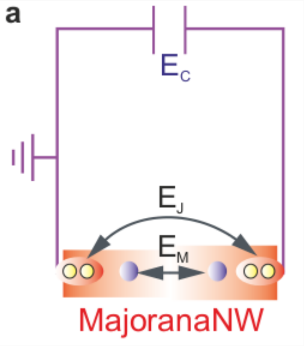
Topological qubit can be constructed of four Majorana qps, which consist of two nonlocal fermions (electrons)

$$\hat{f}_1 = \frac{1}{\sqrt{2}}(\hat{\gamma}_1 + i\hat{\gamma}_2) \qquad \hat{f}_2 = \frac{1}{\sqrt{2}}(\hat{\gamma}_3 + i\hat{\gamma}_4)$$

$$\hat{f}_1^\dagger = \frac{1}{\sqrt{2}}(\hat{\gamma}_1 - i\hat{\gamma}_2) \qquad \hat{f}_2^\dagger = \frac{1}{\sqrt{2}}(\hat{\gamma}_3 - i\hat{\gamma}_4)$$



Number of the fermions

$$\hat{n}_1 = \hat{f}_1^\dagger \hat{f}_1 = \frac{1}{2}\left(1 + i\hat{\gamma}_1\hat{\gamma}_2\right)$$

$$\hat{n}_2 = \hat{f}_2^\dagger \hat{f}_2 = \frac{1}{2}\left(1 + i\hat{\gamma}_3\hat{\gamma}_4\right)$$

is related to the parity:

$|0,0\rangle \quad ; \quad |1,1\rangle$ **even number of fermions**

$|1,0\rangle \quad ; \quad |0,1\rangle$ **odd number of fermions**

TOPOLOGICAL QUBIT

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Within a given parity the setup is topologically protected, i.e. it is immune to decoherence etc.

TOPOLOGICAL QUBIT: AN EXAMPLE

A possible choice for the topological qubit can be:

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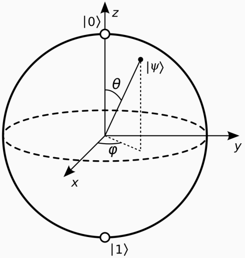

where θ and ϕ are the angles in Bloch sphere.

TOPOLOGICAL COMPUTATION: CONCEPTS

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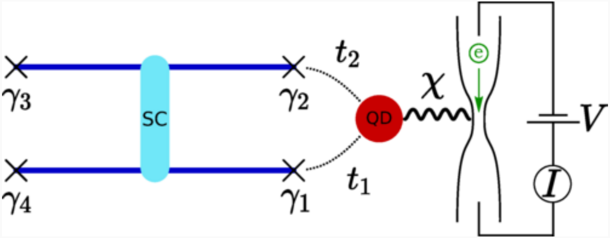

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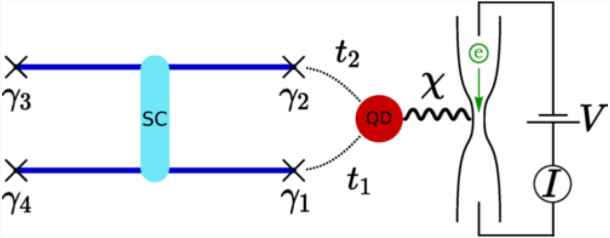

⟹ **Detection:** **measurement of charge on the side-attached QD.**