

# Quantum transition in time-domain of Anderson impurity coupled to superconductor

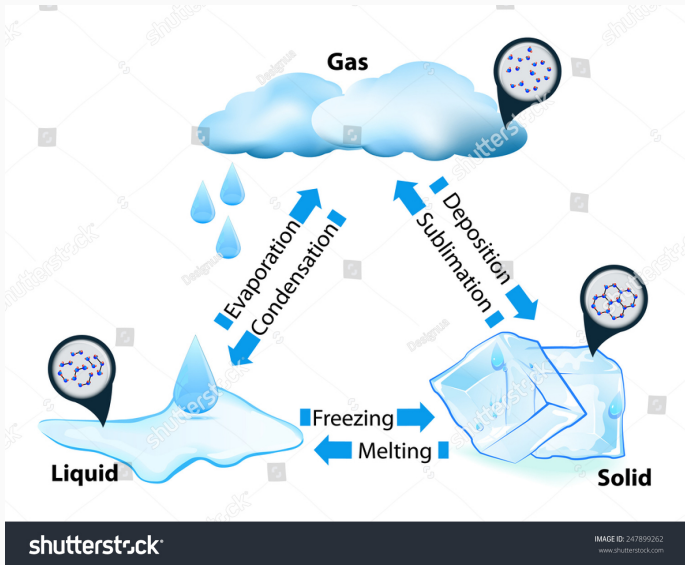
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Tadeusz DOMAŃSKI

M. Curie-Skłodowska Univ., Lublin



# Every-day examples of phase transitions



## VARIETY OF PHASE TRANSITIONS

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at some critical points exhibit:**

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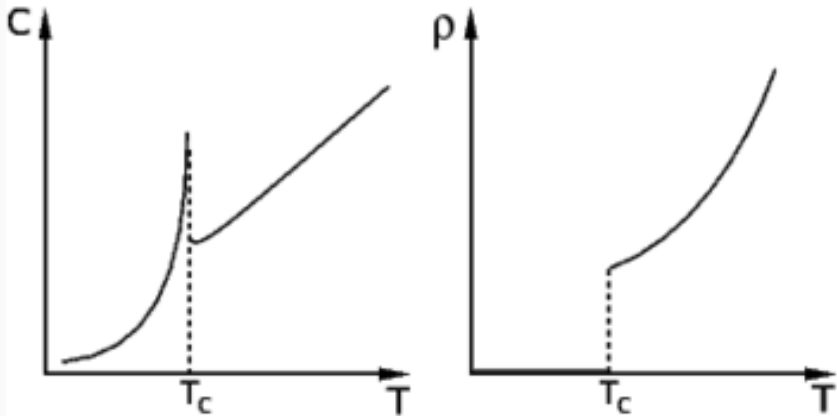
**This list has to be updated by:**

- ⇒ phase transitions in time-domain .....(2013)**



# TRANSITION TO SUPERCONDUCTING STATE

Phase transition is manifested by non-analytic behaviour appearing at critical point in the specific heat



# **I. Dynamical quantum phase transition**

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**(general outline of the idea)**

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**Loschmidt amplitude**



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**nonanalytical**  $\lim_{T \rightarrow T_c} F(T)$

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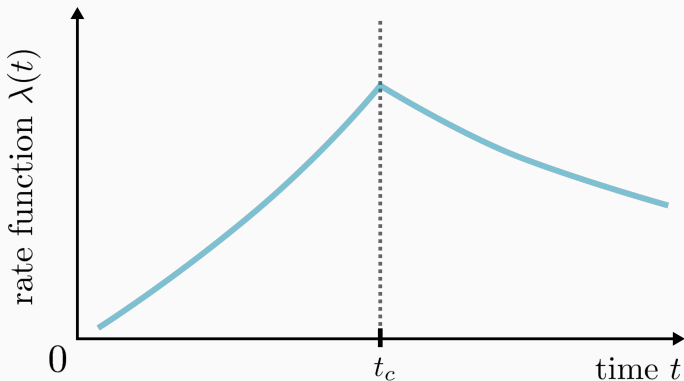
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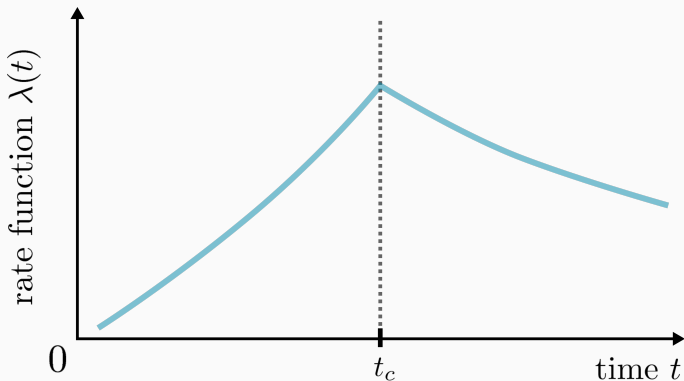
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At critical time  $t_c$  the rate function  $\lambda(t)$  of  $L(t) \equiv e^{-N\lambda(t)}$  has a kink (or other types of nonanalytic behaviour).



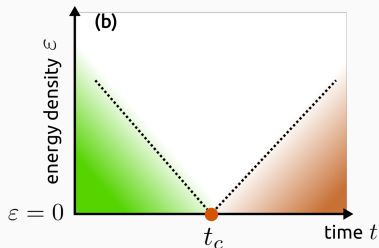
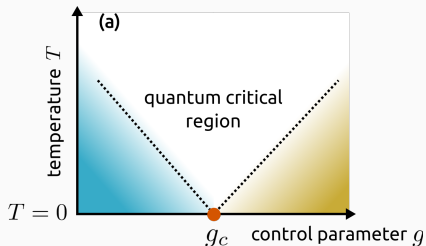
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At this instant the state  $\psi(t_c)$  is orthogonal to initial  $\psi(t_0)$ .

# ANALOGY TO QUANTUM-PHASE-TRANSITION



M. Heyl,

Rep. Prog. Phys. 81, 054001 (2018).

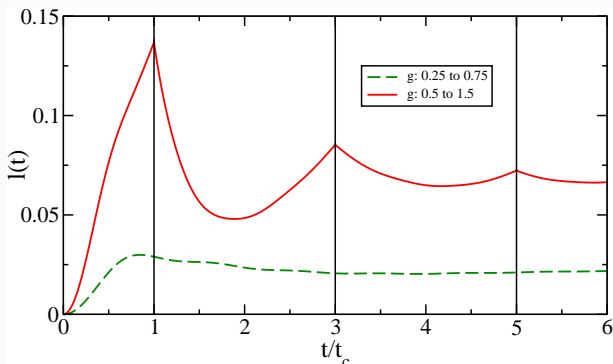
**Loschmidt amplitude probes the ground state manifold of the initial Hamiltonian (energy density at  $\varepsilon = 0$ ).**



**A few examples ...**

# QUENCH OF TRANSVERSE FIELD $h$

Post-quench return rate of the Ising model ( $g \equiv h/J$ )



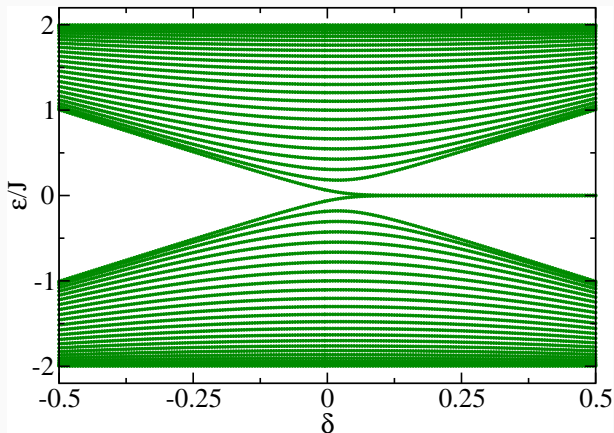
$$\hat{H} = -\frac{J}{2} \sum_{j=1}^{N-1} \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + \frac{h}{2} \sum_{j=1}^N \hat{\sigma}_j^x$$

**solid red line** - across a phase transition ( $g_c = 1$ )

**dashed green line** - inside the same phase

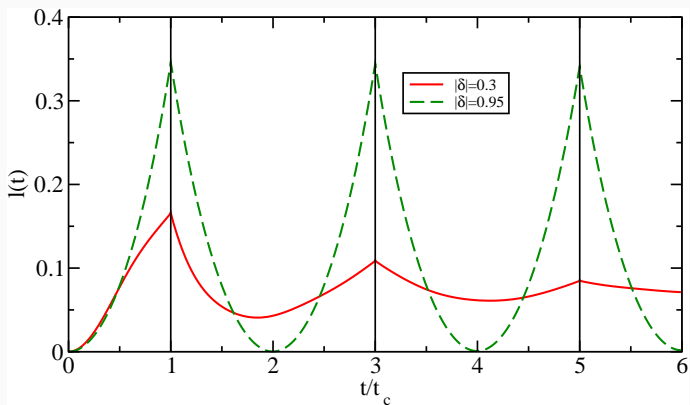
# SU-SCHRIEFFER-HEEGER MODEL

Quasiparticle spectrum of the SSH model under stationary conditions.



$$\hat{H} = -J \sum_j \left[ (1 + \delta e^{i\pi j}) \hat{c}_j^\dagger \hat{c}_{j+1} + \text{h.c.} \right]$$

# QUENCH DRIVEN TRANSITION



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**solid red line:**  $\delta = -0.3 \rightarrow \delta = +0.3$

**dashed green line:**  $\delta = 0.95 \rightarrow \delta = -0.95$

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⇒ **at equidistant critical times**  
(in most cases, but not always)

⇒ **at finite temperatures**  
(where they are no longer sharp)

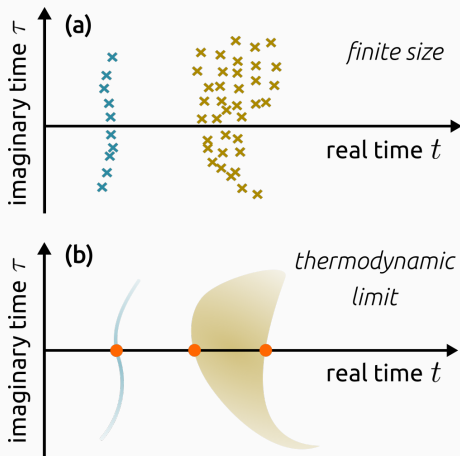


# Finite-size systems

# **Finite-size systems**

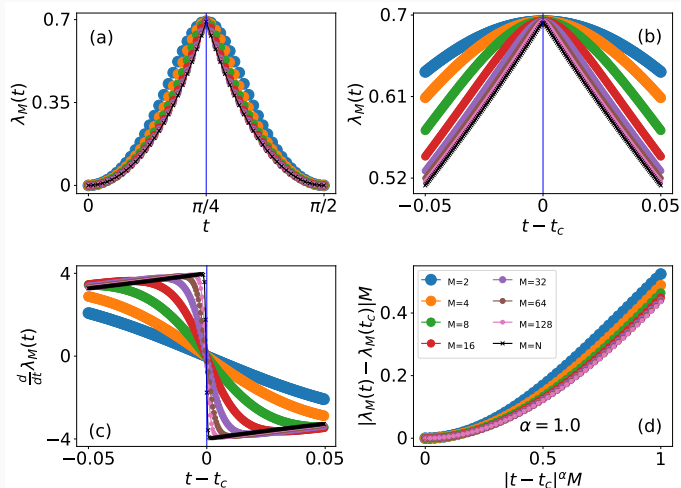
**(possible observability)**

# TRANSITIONS OF FINITE-SIZE SYSTEMS



Schematic view of "Fisher zeros" for the Loschmidt amplitude  $\langle \Psi_0 | e^{-iz\hat{H}} | \Psi_0 \rangle$  obtained in the complex plane  $z = t + i\tau$ .

# ISING MODEL: DQPT OF FINITE-SIZE SYSTEM



**"Local measures of dynamical quantum phase transitions"**

J.C. Halimeh, D. Trapin, M. Damme & M. Heyl, Phys. Rev. B 104, 075130 (2021).

## SIMILAR IDEAS

**"Exact zeros of the Loschmidt echo and quantum speed limit time for the dynamical quantum phase transitions in finite-size systems"**

B. Zhou, Y. Zeng & S. Chen, Phys. Rev. B 104, 094311 (2021).

**"Finite-component dynamical quantum phase transitions"**

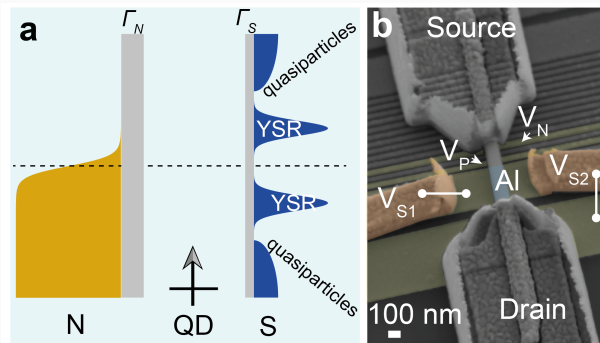
R. Puebla, Phys. Rev. B 102, 220302(R) (2020).

## **II. Application to superconducting nanostructures**

# Examples of superconducting nanostructures

# HETEROSTRUCTURES WITH SUPERCONDUCTOR(S)

normal metal (N) - quantum dot (QD) - superconductor (S)

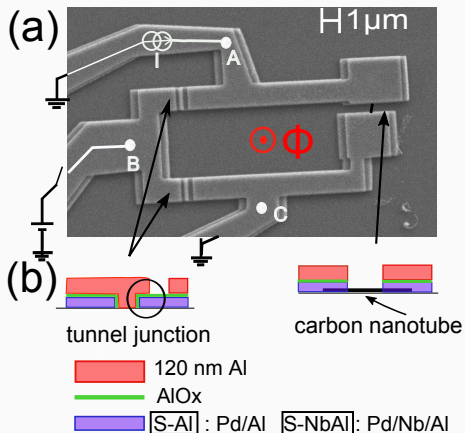


J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, *Commun. Phys.* **3**, 125 (2020).



# HETEROSTRUCTURES WITH SUPERCONDUCTOR(S)

superconductor (S) - quantum dot (QD) - superconductor (S)



R. Delagrangé, R. Weil, A. Kasumov, M. Ferrier, H. Bouchiat, R. Deblock,  
Phys. Rev. B **93**, 195437 (2016).

# SUPERCONDUCTING PROXIMITY EFFECT

- Coupling of the localized (QD) to itinerant (SC) electrons induces:

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⇒ **in-gap bound states**

- originating from:

⇒ **leakage of Cooper pairs on QD** (Andreev)

⇒ **exchange int. of QD with SC** (Yu-Shiba-Rusinov)

**Why are we interested in this ?**

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**A few headlines ...**

## A perspective on semiconductor-based superconducting qubits

Cite as: Appl. Phys. Lett. **117**, 240501 (2020); doi: [10.1063/5.0024124](https://doi.org/10.1063/5.0024124)

Submitted: 4 August 2020 · Accepted: 9 November 2020 ·

Published Online: 14 December 2020



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Ramón Aguado<sup>a1</sup> 

### AFFILIATIONS

Instituto de Ciencia de Materiales de Madrid (ICMM), Consejo Superior de Investigaciones Científicas (CSIC), Sor Juana Inés de la Cruz 3, 28049 Madrid, Spain

**Quantum bits (qubits) can be constructed out of in-gap bound states, using either the Josephson junctions (transmons) or the semiconducting-superconducting hybrids (gatemons).**

## REPORT

### QUANTUM DEVICES

## Coherent manipulation of an Andreev spin qubit

M. Hays<sup>1\*</sup>, V. Fatemi<sup>1\*</sup>, D. Bouman<sup>2,3</sup>, J. Cerrillo<sup>4,5</sup>, S. Diamond<sup>1</sup>, K. Serniak<sup>1†</sup>, T. Connolly<sup>1</sup>, P. Krogstrup<sup>6</sup>, J. Nygård<sup>6</sup>, A. Levy Yeyati<sup>5,7</sup>, A. Geresdi<sup>2,3,8</sup>, M. H. Devoret<sup>1\*</sup>

Two promising architectures for solid-state quantum information processing are based on electron spins electrostatically confined in semiconductor quantum dots and the collective electrodynamic modes of superconducting circuits. Superconducting electrodynamic qubits involve macroscopic numbers of electrons and offer the advantage of larger coupling, whereas semiconductor spin qubits involve individual electrons trapped in microscopic volumes but are more difficult to link. We combined beneficial aspects of both platforms in the Andreev spin qubit: the spin degree of freedom of an electronic quasiparticle trapped in the supercurrent-carrying Andreev levels of a Josephson semiconductor nanowire. We performed coherent spin manipulation by combining single-shot circuit-quantum-electrodynamics readout and spin-flipping Raman transitions and found a spin-flip time  $T_S = 17$  microseconds and a spin coherence time  $T_{2E} = 52$  nanoseconds. These results herald a regime of supercurrent-mediated coherent spin-photon coupling at the single-quantum level.

Hays *et al.*, *Science* **373**, 430–433 (2021) 23 July 2021

**Recent evidence for experimental realization**



## Yu-Shiba-Rusinov Qubit

Archana Mishra,<sup>1,\*</sup> Pascal Simon,<sup>2,†</sup> Timo Hyart,<sup>1,3,‡</sup> and Mircea Trif<sup>1,§</sup>

<sup>1</sup>*International Research Centre MagTop, Institute of Physics, Polish Academy of Sciences, Aleja Lotnikow 32/46, Warsaw PL-02668, Poland*

<sup>2</sup>*Université Paris-Saclay, CNRS, Laboratoire de Physiques des Solides, Orsay 91405, France*

<sup>3</sup>*Department of Applied Physics, Aalto University, Aalto, Espoo 00076, Finland*



(Received 15 June 2021; accepted 2 November 2021; published 7 December 2021)

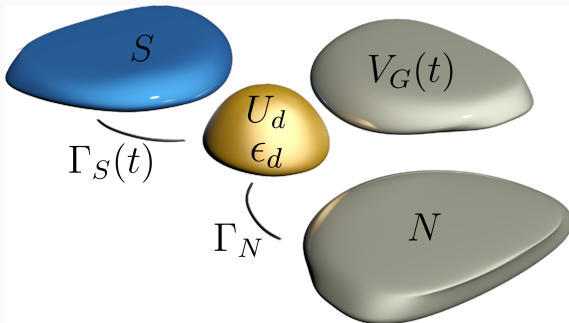
Magnetic impurities in  $s$ -wave superconductors lead to spin-polarized Yu-Shiba-Rusinov (YSR) in-gap states. Chains of magnetic impurities offer one of the most viable routes for the realization of Majorana bound states, which hold promise for topological quantum computing. However, this ambitious goal looks distant, since no quantum coherent degrees of freedom have yet been identified in these systems. To fill this gap, we propose an effective two-level system, a YSR qubit, stemming from two nearby impurities. Using a time-dependent wave-function approach, we derive an effective Hamiltonian describing the YSR-qubit evolution as a function of the distance between the impurity spins, their relative orientations, and their dynamics. We show that the YSR qubit can be controlled and read out using state-of-the-art experimental techniques for manipulation of the spins. Finally, we address the effect of spin noise on the coherence properties of the YSR qubit and show robust behavior for a wide range of experimentally relevant parameters. Looking forward, the YSR qubit could facilitate the implementation of a universal set of quantum gates in hybrid systems where they are coupled to topological Majorana qubits.

**Are there any characteristic time-scales ?**

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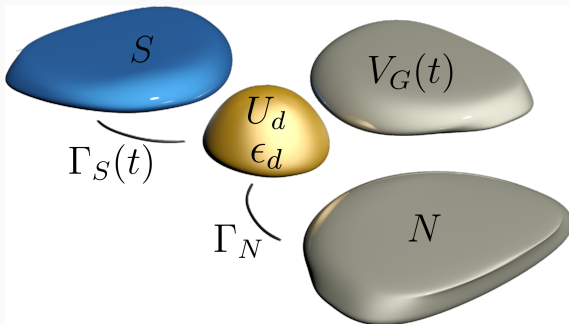
**(of importance for operations on qubits)**

# QUENCH DRIVEN DYNAMICS



**Possible quench protocols:**

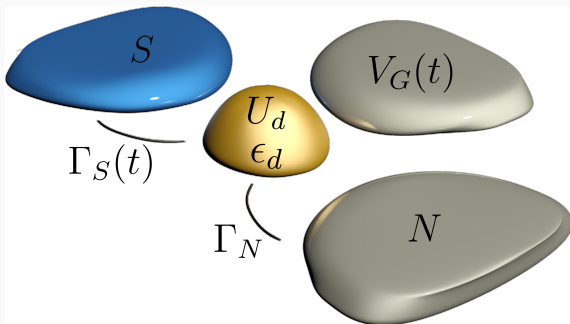
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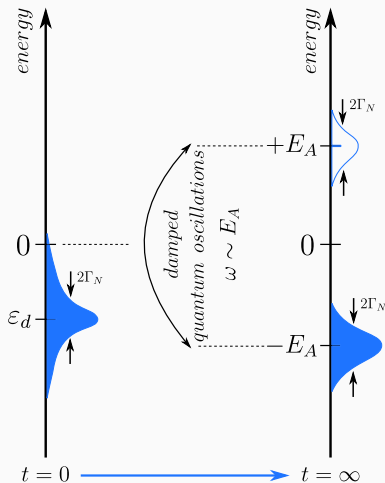
## Possible quench protocols:

$\Rightarrow$  sudden coupling to superconductor  $0 \rightarrow \Gamma_S$

$\Rightarrow$  abrupt application of gate potential  $0 \rightarrow V_G$

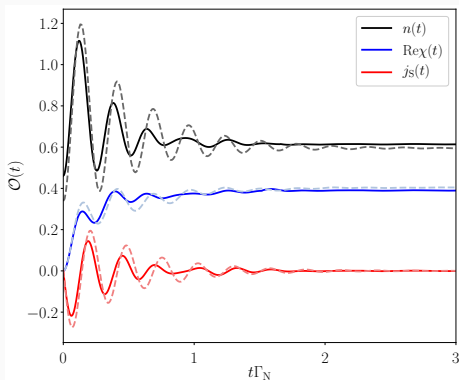
# BUILDUP OF IN-GAP STATES

Schematics of the Andreev states formation induced by quench  $0 \rightarrow \Gamma_5$



# BUILDUP OF IN-GAP STATES

Time-dependent observables driven by the quantum quench  $0 \rightarrow \Gamma_S$



**solid lines** - time dependent NRG

**dashed lines** - Hartree-Fock-Bogolubov



# Singlet-doublet (quantum phase) transition

**Singlet-doublet (quantum phase) transition**

**(static version)**

# SINGLY OCCUPIED VS BCS-TYPE CONFIGURATIONS

Quantum dot proximitized to superconductor can be described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \hat{d}_{\sigma}^{\dagger} \hat{d}_{\sigma} + U_d \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left( \Gamma_s \hat{d}_{\uparrow}^{\dagger} \hat{d}_{\downarrow}^{\dagger} + \text{h.c.} \right)$$

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Eigen-states of this problem are represented by:

$$\begin{array}{ll} |\uparrow\rangle \quad \text{and} \quad |\downarrow\rangle & \Leftarrow \quad \text{doublet states (spin } \frac{1}{2} \text{)} \\ \left. \begin{array}{l} u |0\rangle - v |\uparrow\downarrow\rangle \\ v |0\rangle + u |\uparrow\downarrow\rangle \end{array} \right\} & \Leftarrow \quad \text{singlet states (spin 0)} \end{array}$$

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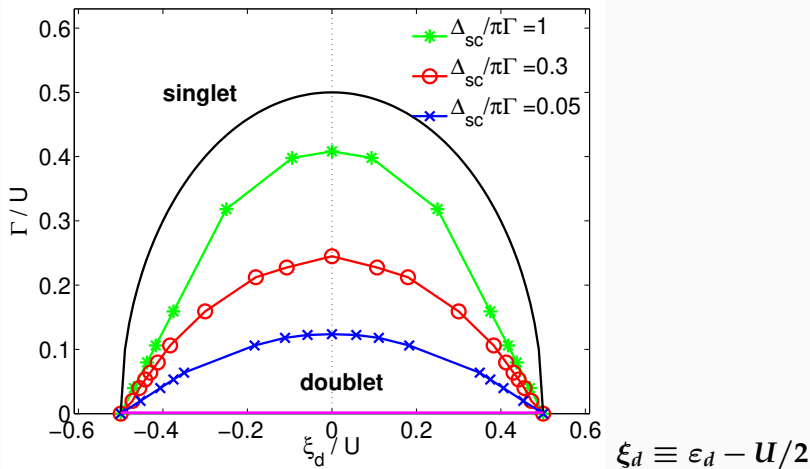
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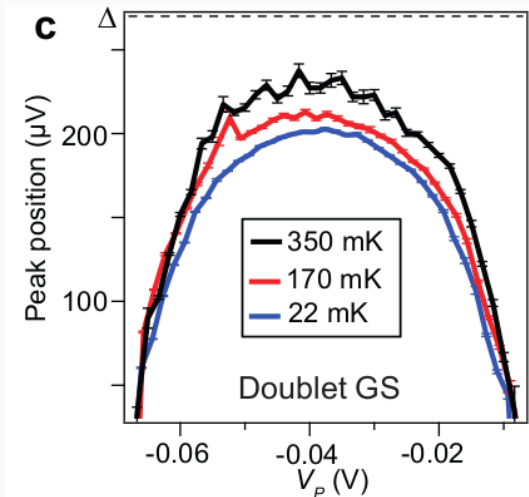
Upon varying the parameters  $\epsilon_d$ ,  $U_d$  or  $\Gamma_S$  there can be induced a **transition** between these doublet/singlet ground states.

# QUANTUM PHASE TRANSITION (STATIC VERSION)

## Singlet-doublet quantum (phase transition): NRG results



# QUANTUM PHASE TRANSITION: EXPERIMENT



J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, *Commun. Phys.* **3**, 125 (2020).

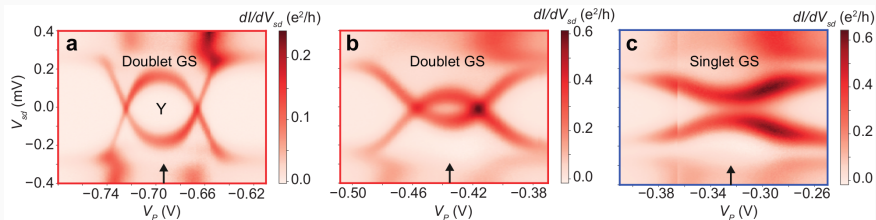
# SINGLET VS DOUBLET: EXPERIMENT

Differential conductance vs source-drain bias  $V_{sd}$  (vertical axis) and gate potential  $V_p$  (horizontal axis) measured for various  $\Gamma_s/U$

$$U \gg \Gamma_s$$

$$U \geq \Gamma_s$$

$$U < \Gamma_s$$



J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup,  
K. Grove-Rasmussen and J. Nygård, *Commun. Phys.* **3**, 125 (2020).



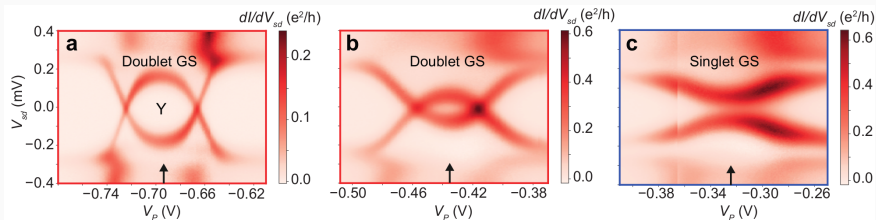
# SINGLET VS DOUBLET: EXPERIMENT

Differential conductance vs source-drain bias  $V_{sd}$  (vertical axis) and gate potential  $V_p$  (horizontal axis) measured for various  $\Gamma_s/U$

$$U \gg \Gamma_s$$

$$U \geq \Gamma_s$$

$$U < \Gamma_s$$

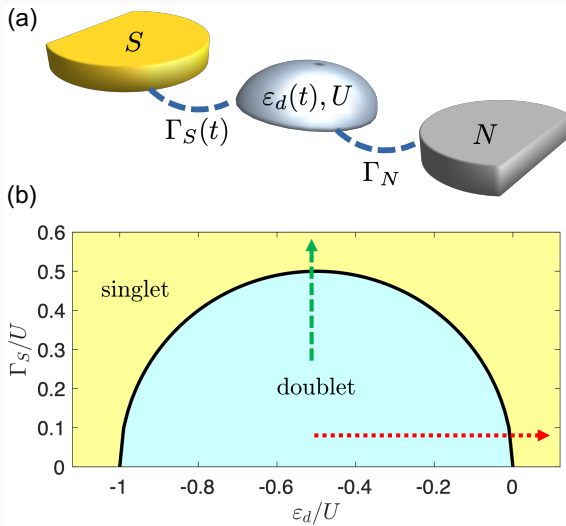


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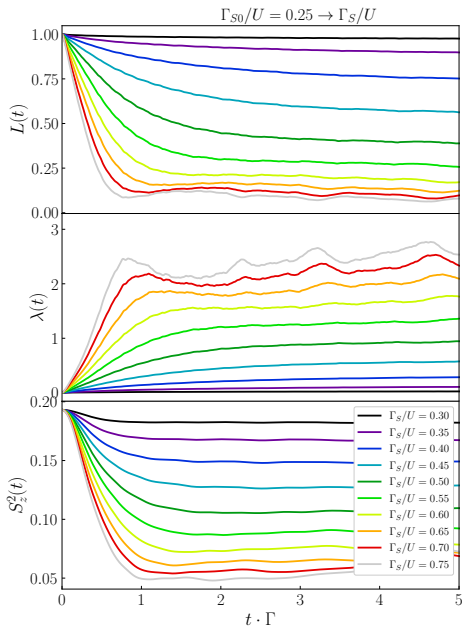
**Crossings of in-gap states correspond to the singlet-doublet QPT.**

# **Dynamical singlet-doublet transition**

# QUANTUM QUENCHES ACROSS QPT



# $t$ NRG RESULTS: ABRUPT CHANGE OF $\Gamma_S$



**Loschmidt amplitude**

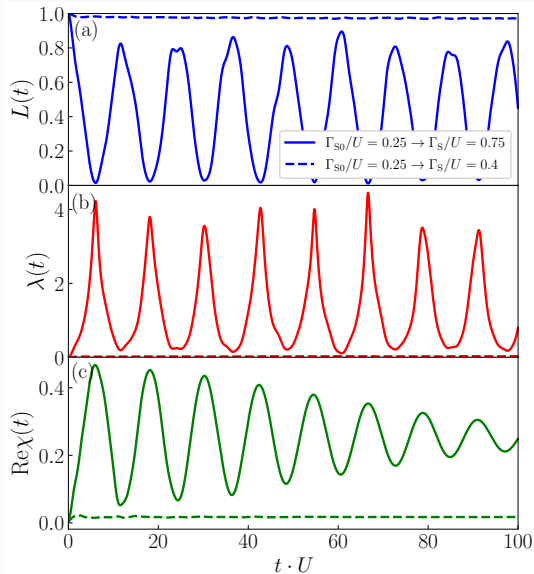
$$L(t) \equiv |\langle \Psi(0) | \Psi(t) \rangle|^2$$

**Return rate**

$$\lambda = -\frac{1}{N} \log [L(t)]$$

**The squared magnetic moment  $\langle S_z^2(t) \rangle$**

# $t$ NRG RESULTS: ABRUPT CHANGE OF $\Gamma_S$

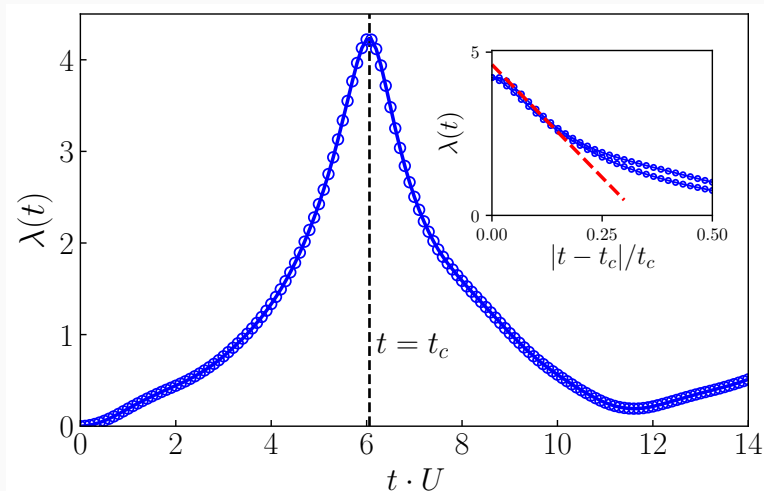


$$\epsilon_d = -U/2$$

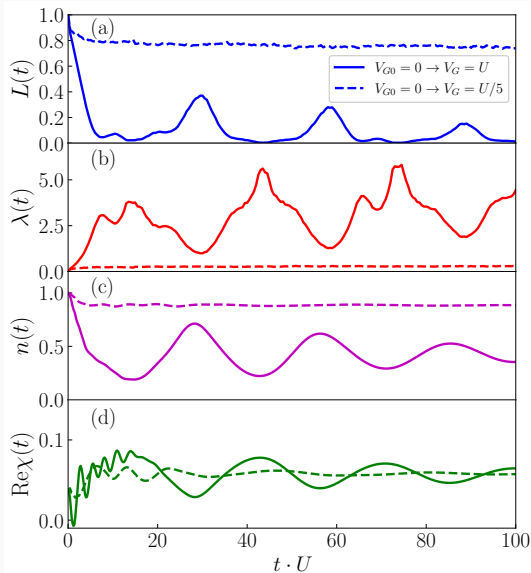
$$\Gamma_N = U/100$$

# $t$ NRG RESULTS: ABRUPT CHANGE OF $\Gamma_S$

Return-rate nearby the first critical time  $t_c$  (rounding due to finite-size).



# $t$ NRG RESULTS: QUANTUM QUENCH $\varepsilon_d \rightarrow \varepsilon_d + V_G$



$$\Gamma_S = U/5$$

$$\Gamma_N = U/100$$

## FURTHER CHALLENGES

**How can we detect such dynamical singlet-transition(s) ?**

- **by enhancements of the time-dependent Andreev current**



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- **by suppressions of the time-dependent magnetic moment**

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**How can we detect such dynamical singlet-transition(s) ?**

- **by enhancements of the time-dependent Andreev current**
- **by suppressions of the time-dependent magnetic moment**

**Specific evaluations will be provided soon .....(on-going project)**

# SIMILAR IDEAS: #1 ULTRACOLD SUPERFLUIDS

Annals of Physics 435 (2021) 168554



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## Loschmidt echo of far-from-equilibrium fermionic superfluids

Colin Rylands<sup>a,\*</sup>, Emil A. Yuzbashyan<sup>b,1</sup>, Victor Gurarie<sup>c,1</sup>,  
Aidan Zabalo<sup>b,1</sup>, Victor Galitski<sup>a,1</sup>

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<sup>b</sup> *Department of Physics and Astronomy, Center for Materials Theory, Rutgers University, Piscataway, NJ 08854, USA*

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**Rapid change across the BCS-BEC limits in the ultracold atom superfluids.**

# SIMILAR IDEAS: #2 DYNAMICS OF SHIBA STATES

## Emergence and Manipulation of non-equilibrium Yu-Shiba-Rusinov states

Jasmin Bedow, Eric Mascot and Dirk K. Morr  
*University of Illinois at Chicago, Chicago, IL 60607, USA*  
(Dated: December 16, 2021)

The experimental advances in the study of time-dependent phenomena has opened a new path to investigating the complex electronic structure of strongly correlated and topological materials. Yu-Shiba-Rusinov (YSR) states induced by magnetic impurities in  $s$ -wave superconductors provide an ideal candidate system to study the response of a system to time-dependent manipulations of the magnetic environment. Here, we show that by imposing a time-dependent change in the magnetic exchange coupling, by changing the relative alignment of magnetic moments in an impurity dimer, or through a periodic drive of the impurity moment, one can tune the system through a time-dependent quantum phase transition, in which the system undergoes a transition from a singlet to a doublet ground state. We show that the electronic response of the system to external perturbations can be imaged through the time-dependent differential conductance,  $dI(t)/dV$ , which, in analogy to the equilibrium case, is proportional to a non-equilibrium local density of states. Our results open the path to visualizing the response of complex quantum systems to time-dependent external perturbations.

arXiv:2112.07733v1

## Signatures of the Higgs mode in transport through a normal-metal–superconductor junction

Gaomin Tang<sup>1</sup>, Wolfgang Belzig<sup>2</sup>, Ulrich Zülicke<sup>3</sup>, and Christoph Bruder<sup>1</sup>

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<sup>2</sup>*Fachbereich Physik, Universität Konstanz, D-78457 Konstanz, Germany*

<sup>3</sup>*School of Chemical and Physical Sciences and MacDiarmid Institute for Advanced Materials and Nanotechnology, Victoria University of Wellington, P.O. Box 600, Wellington 6140, New Zealand*



(Received 21 February 2020; revised manuscript received 5 June 2020; accepted 8 June 2020; published 29 June 2020)

A superconductor subject to electromagnetic irradiation in the terahertz range can show amplitude oscillations of its order parameter. However, coupling this so-called Higgs mode to the charge current is notoriously difficult. We propose to achieve such a coupling in a particle-hole-asymmetric configuration using a DC-voltage-biased normal-metal–superconductor tunnel junction. Using the quasiclassical Green's function formalism, we demonstrate three characteristic signatures of the Higgs mode: (i) The AC charge current exhibits a pronounced resonant behavior and is maximal when the radiation frequency coincides with the order parameter. (ii) The AC charge current amplitude exhibits a characteristic nonmonotonic behavior with increasing voltage bias. (iii) At resonance for large voltage bias, the AC current vanishes inversely proportional to the bias. These signatures provide an electric detection scheme for the Higgs mode.

**Possibility to observe the collective amplitude (Higgs-type) mode of the order parameter in presence of ultrafast ac field.**

# SIMILAR IDEAS: #4 HIGGS AND GOLDSTONE MODES

PHYSICAL REVIEW B **103**, 045414 (2021)

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## Higgs-like pair amplitude dynamics in superconductor–quantum-dot hybrids

Mathias Kamp and Björn Sothmann

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(Received 2 October 2020; revised 11 December 2020; accepted 11 December 2020; published 14 January 2021)

We consider a quantum dot weakly tunnel coupled to superconducting reservoirs. A finite superconducting pair amplitude can be induced on the dot via the proximity effect. We investigate the dynamics of the induced pair amplitude after a quench and under periodic driving of the system by means of a real-time diagrammatic approach. We find that the quench dynamics is dominated by an exponential decay towards equilibrium. In contrast, the periodically driven system can sustain coherent oscillations of both the amplitude and the phase of the induced pair amplitude in analogy to Higgs and Nambu-Goldstone modes in driven bulk superconductors.

**Possibility to observe the collective amplitude (Higgs-type) and phasal (Goldstone-type) modes of the order parameter.**

# CONCLUSIONS

**Quench imposed onto the nanostructure consisting of metal – quantum dot – superconductor:**

- **activates Rabi-type oscillations** (due to particle-hole mixing)

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Quench imposed onto the nanostructure consisting of metal – quantum dot – superconductor:

- **activates Rabi-type oscillations** (due to particle-hole mixing)
- **rescales energies of in-gap quasiparticles**
- **can exhibit dynamical transitions** (upon varying ground states)

**These phenomena are detectable in charge transport properties.**

# ACKNOWLEDGEMENTS

- **dynamical singlet-doublet phase transition**

⇒ K. Wrześniewski (Poznań), I. Weymann (Poznań),

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- **transients effects of superconducting hybrids**

⇒ R. Taranko (Lublin),

- **in-gap states of periodically driven system**

⇒ B. Baran (Lublin),

- **time-resolved leakage of Majorana quasiparticles**

⇒ J. Barański (Dęblin)