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Majorana quasiparticles in nanoscopic superconductors

Tadeusz Domański

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J. Bardeen



J. Kondo



E. Majorana

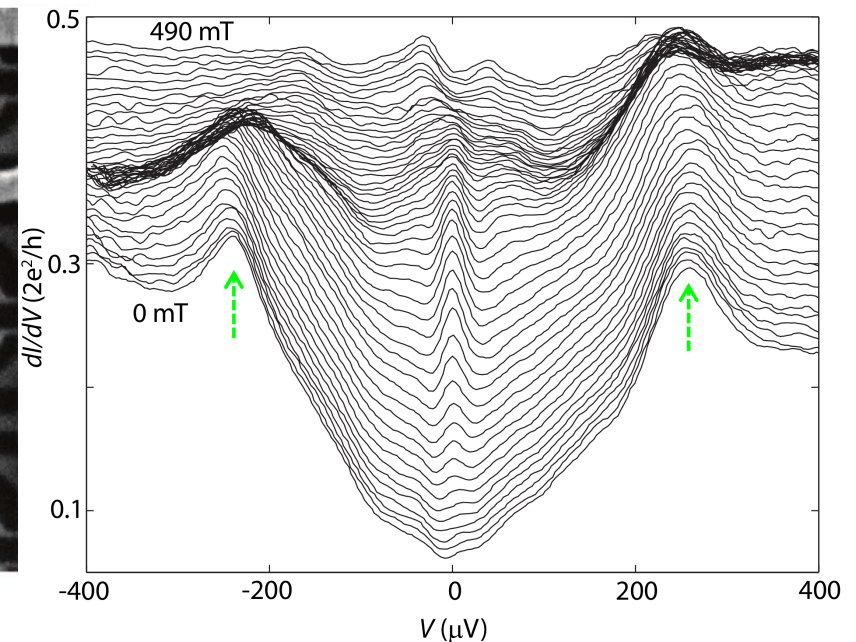
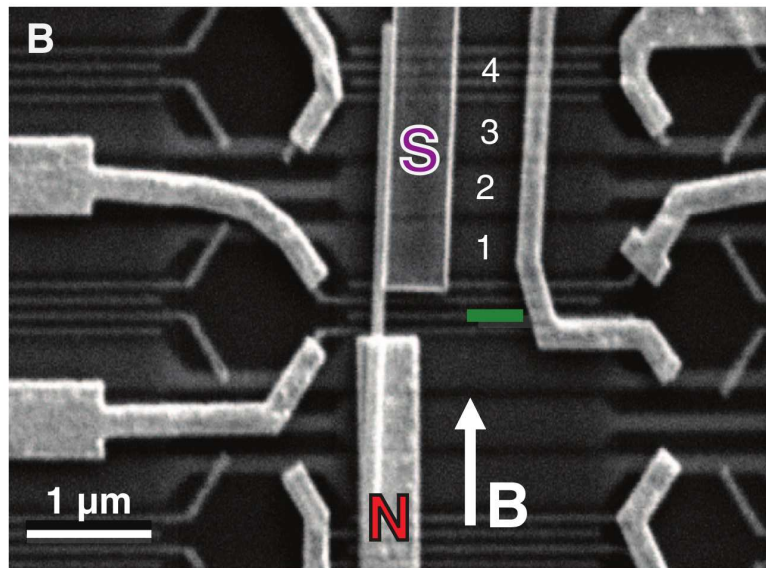
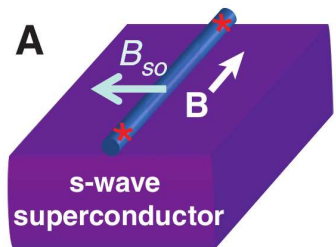
Experimental evidence

– **for Majorana quasiparticles**

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InSb nanowire between a metal (gold) and a superconductor (Nb-Ti-N)



dI/dV measured at 70 mK for varying magnetic field B indicated:

⇒ **a zero-bias enhancement due to Majorana state**

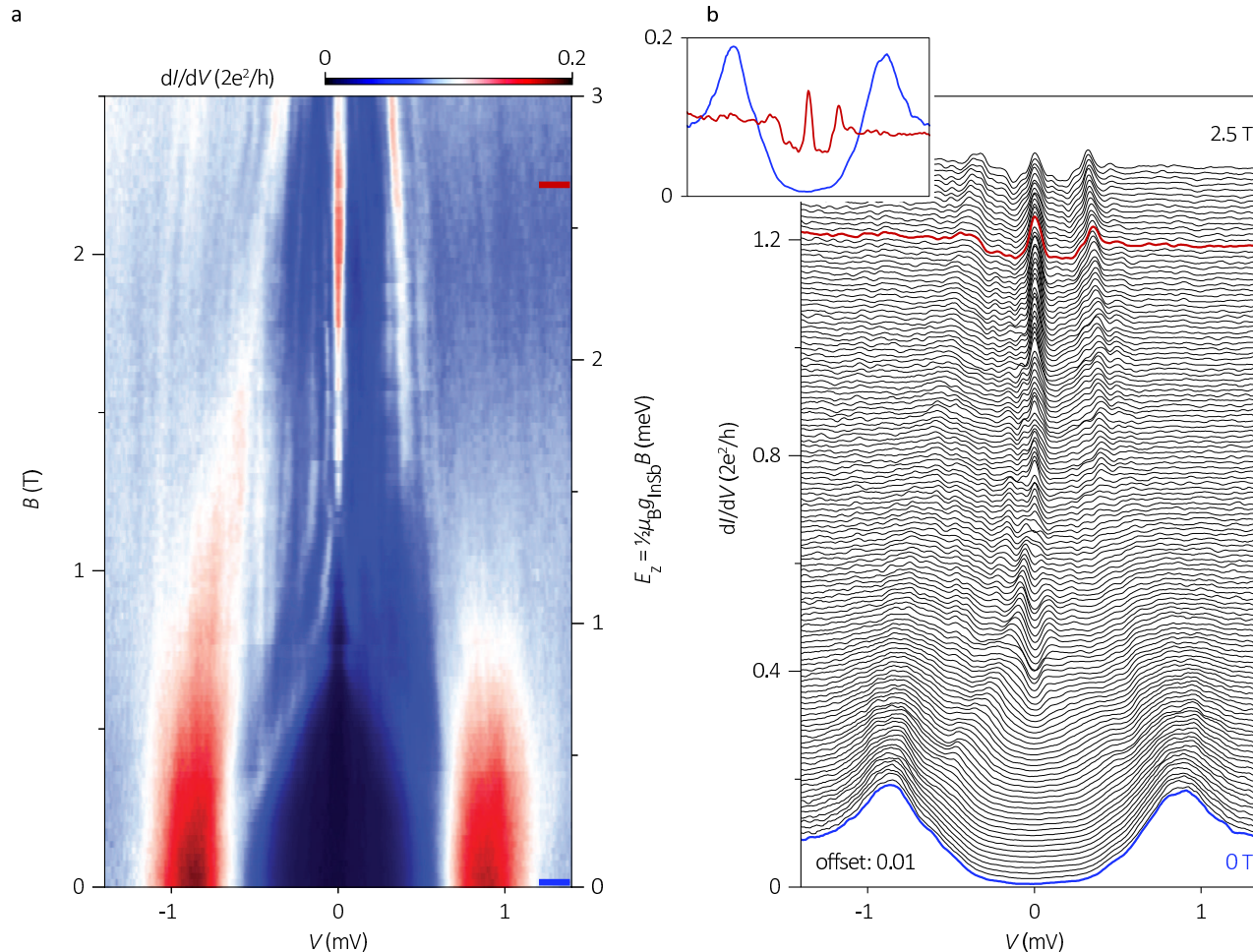
V. Mourik, ..., and L.P. Kouwenhoven, Science **336**, 1003 (2012).

/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /

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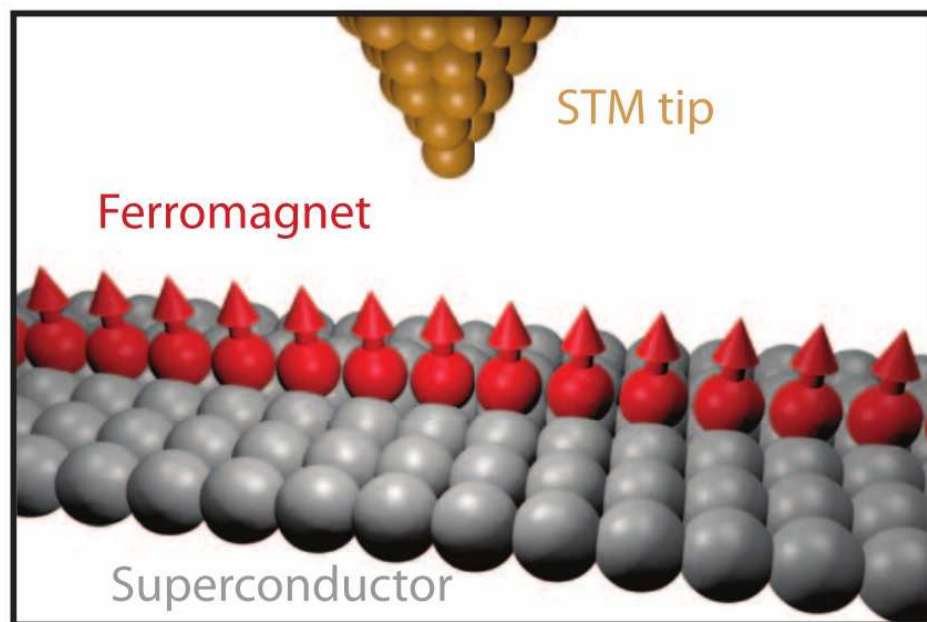
H. Zhang, ..., and L.P. Kouwenhoven, arXiv:1603.04069 (2016).

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A chain of iron atoms deposited on a surface of superconducting lead



STM measurements provided evidence for:

⇒ **Majorana bound states at the edges of a chain.**

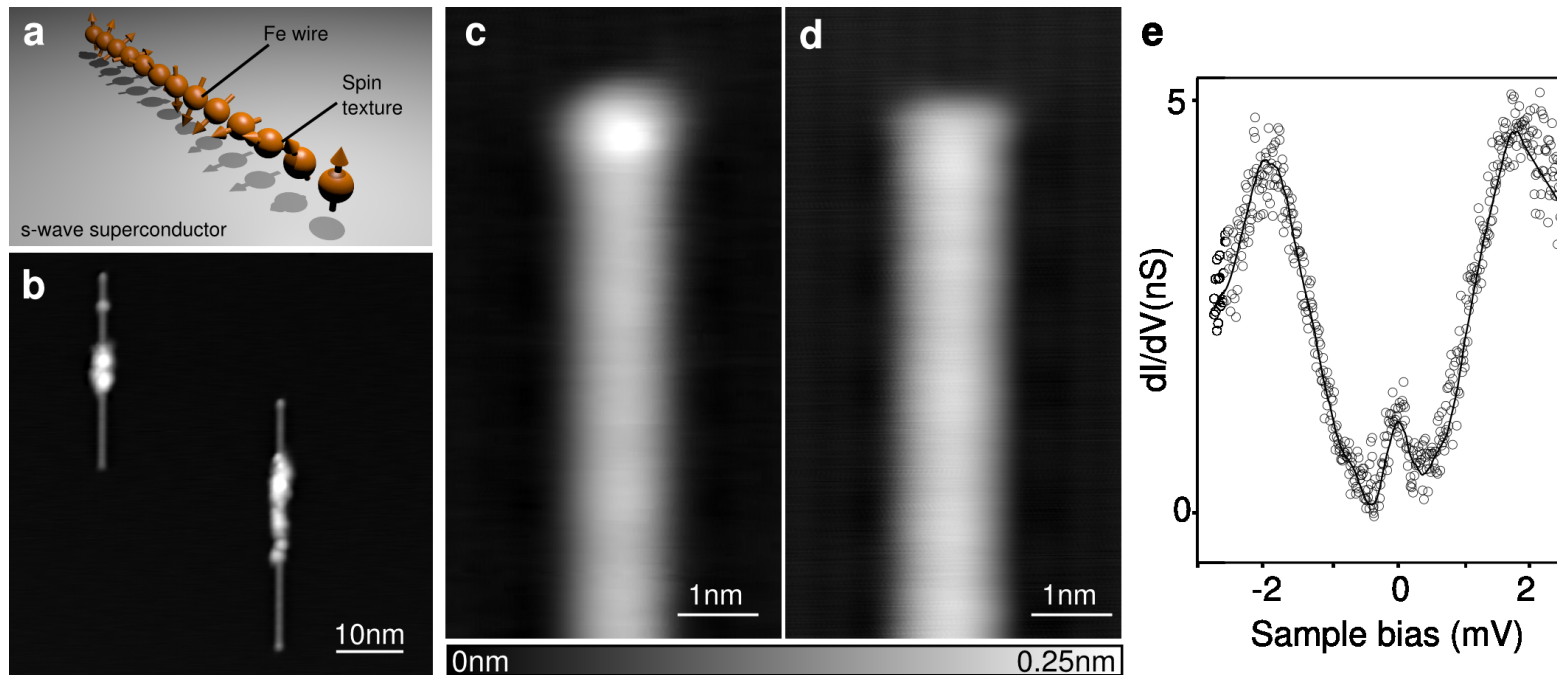
S. Nadj-Perge, ..., and A. Yazdani, Science **346**, 602 (2014).

/ Princeton University, Princeton (NJ), USA /

Experimental evidence

— for Majorana quasiparticles

Self-assembled Fe chain on superconducting Pb(110) surface



AFM combined with STM provided evidence for:

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R. Pawlak, M. Kisiel *et al*, npj Quantum Information 2, 16035 (2016).

/ University of Basel, Switzerland /

Outline:

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- Majorana fermions:

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- Majorana fermions:

⇒ what are they ?

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⇒ what are they ?

⇒ emergent majoranions / in many-body systems /

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⇒ emergent majoranions / in many-body systems /

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- Majorana madness ...

Basic notions

– on Majorana fermions

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- P. Dirac (1928)

$$i\dot{\psi} = (\vec{\alpha} \cdot \vec{p} + \beta m) \psi$$

/ relativistic description of fermions /

particles ($E > 0$),

anti-particles ($E < 0$)

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noticed that particular choice of $\vec{\alpha}$ and β yields a real wave-function !

Physical implication: **particle = antiparticle**

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⇒ Basic features:

- chargeless**
- zero-energy**

Searching for majoranas

– **in particle and nuclear physics**

Searching for majoranas

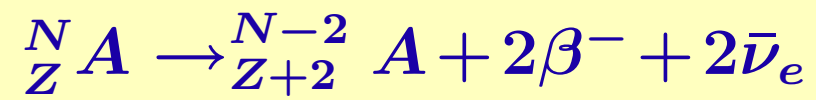
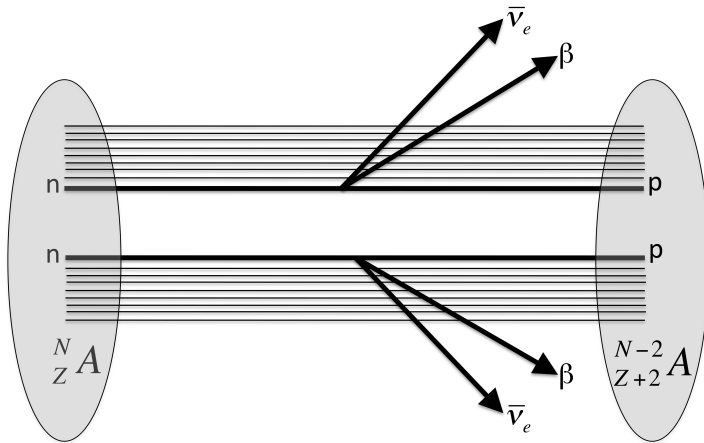
– **in particle and nuclear physics**

DOUBLE BETA DECAY

Searching for majoranas

– in particle and nuclear physics

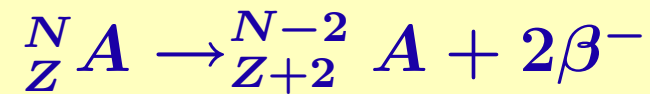
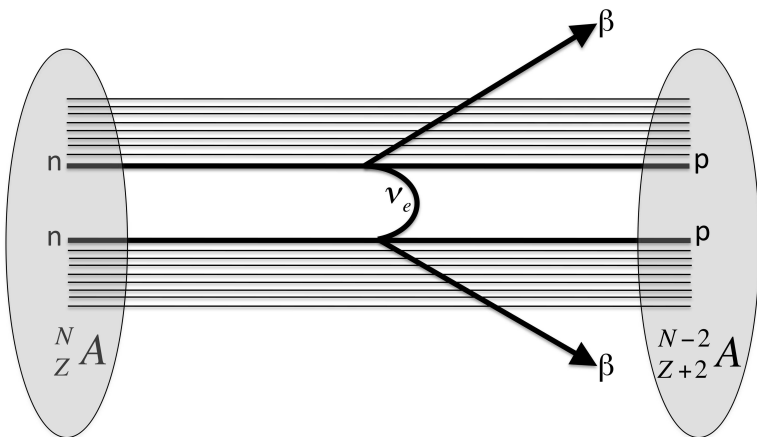
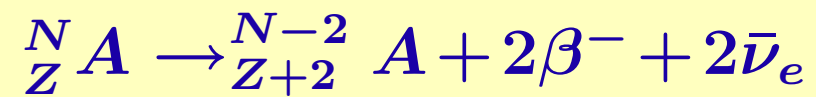
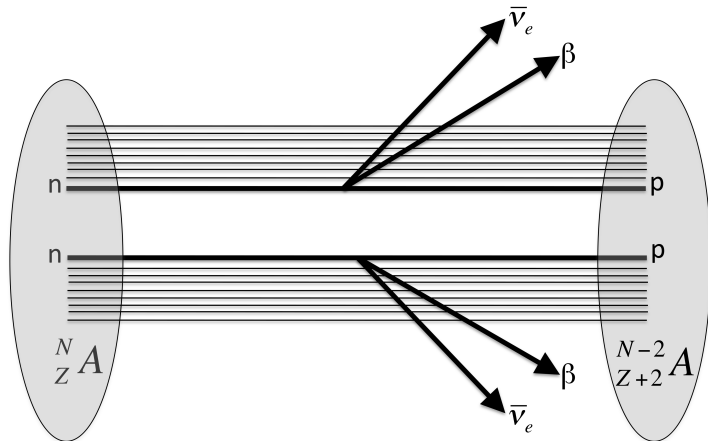
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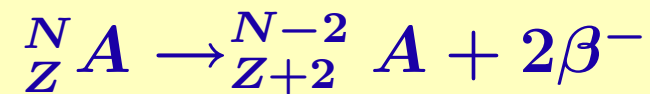
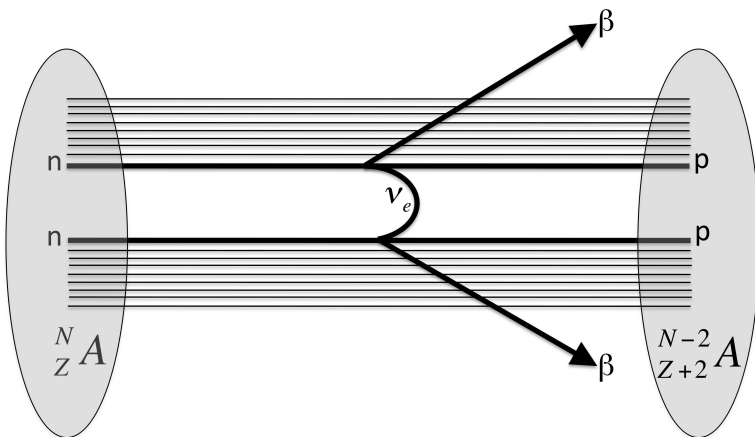
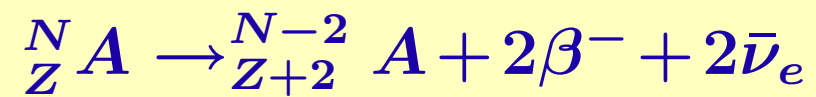
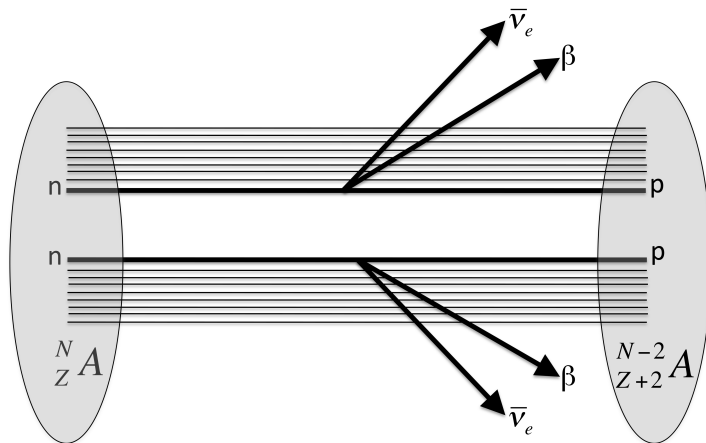


Neutrinoless decay would imply neutrinos to be majoranas.

Searching for majoranas

– in particle and nuclear physics

DOUBLE BETA DECAY



Neutrinoless decay would imply neutrinos to be majoranas.

This issue is still under dispute.

Quasiparticles – in solids

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- ★ Many-body effects, however, can induce a plethora of **emergent** objects like **phonons, magnons, polaritons, holons, bogoliubons** etc.
/ **'More is different'** P.W. Anderson (1972) /

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Quasiparticles – in solids

- ★ Properties of solids are (predominantly) due to conduction **electrons**, that represent the usual **Dirac fermions**
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/ 'More is different' P.W. Anderson (1972) /
- ★ So, how about **majoranions** ?
- ★ Formally, any Dirac fermion can be **majoranized** ...

Majoranization – of normal fermions

- Dirac fermions (e.g. electrons) obey the anticommutation relations

$$\{\hat{c}_i, \hat{c}_j^\dagger\} = \delta_{i,j}$$

$$\{\hat{c}_i, \hat{c}_j\} = 0 = \{\hat{c}_i^\dagger, \hat{c}_j^\dagger\}$$

i,j – any quantum numbers

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i,j – any quantum numbers

- $c_j^{(\dagger)}$ can be recast in terms of Majorana operators

$$\hat{c}_j \equiv (\hat{\gamma}_{j,1} + i\hat{\gamma}_{j,2}) / \sqrt{2}$$

$$\hat{c}_j^\dagger \equiv (\hat{\gamma}_{j,1} - i\hat{\gamma}_{j,2}) / \sqrt{2}$$

'real' and 'imaginary' parts

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- Exotic properties

$$\begin{aligned}\hat{\gamma}_{i,n}^\dagger &= \hat{\gamma}_{i,n} \\ \{\hat{\gamma}_{i,n}, \hat{\gamma}_{j,m}^\dagger\} &= \delta_{i,j} \delta_{n,m}\end{aligned}$$

creation = annihilation !

fermionic antisymmetry

Majoranization – of normal fermions

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- Exotic properties (cd)

$$\begin{aligned}\hat{\gamma}_{i,n} \hat{\gamma}_{i,n} &= 1/2 \\ \hat{\gamma}_{i,n}^\dagger \hat{\gamma}_{i,n} &= 1/2\end{aligned}$$

no Pauli principle !

half 'occupied' & half 'empty'

Majoranization – does it make sense ?

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- resemble the Bogoliubov qps of BCS theory

$$\begin{aligned}\hat{\beta}_{k\uparrow} &\equiv u_k \hat{c}_{k\uparrow} + v_k \hat{c}_{-k\downarrow}^\dagger \\ \hat{\beta}_{-k\downarrow}^\dagger &\equiv -v_k \hat{c}_{k\uparrow} + u_k \hat{c}_{-k\downarrow}^\dagger\end{aligned}$$

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- At Fermi level $u_{k_F} = v_{k_F} = 1/\sqrt{2}$, thus

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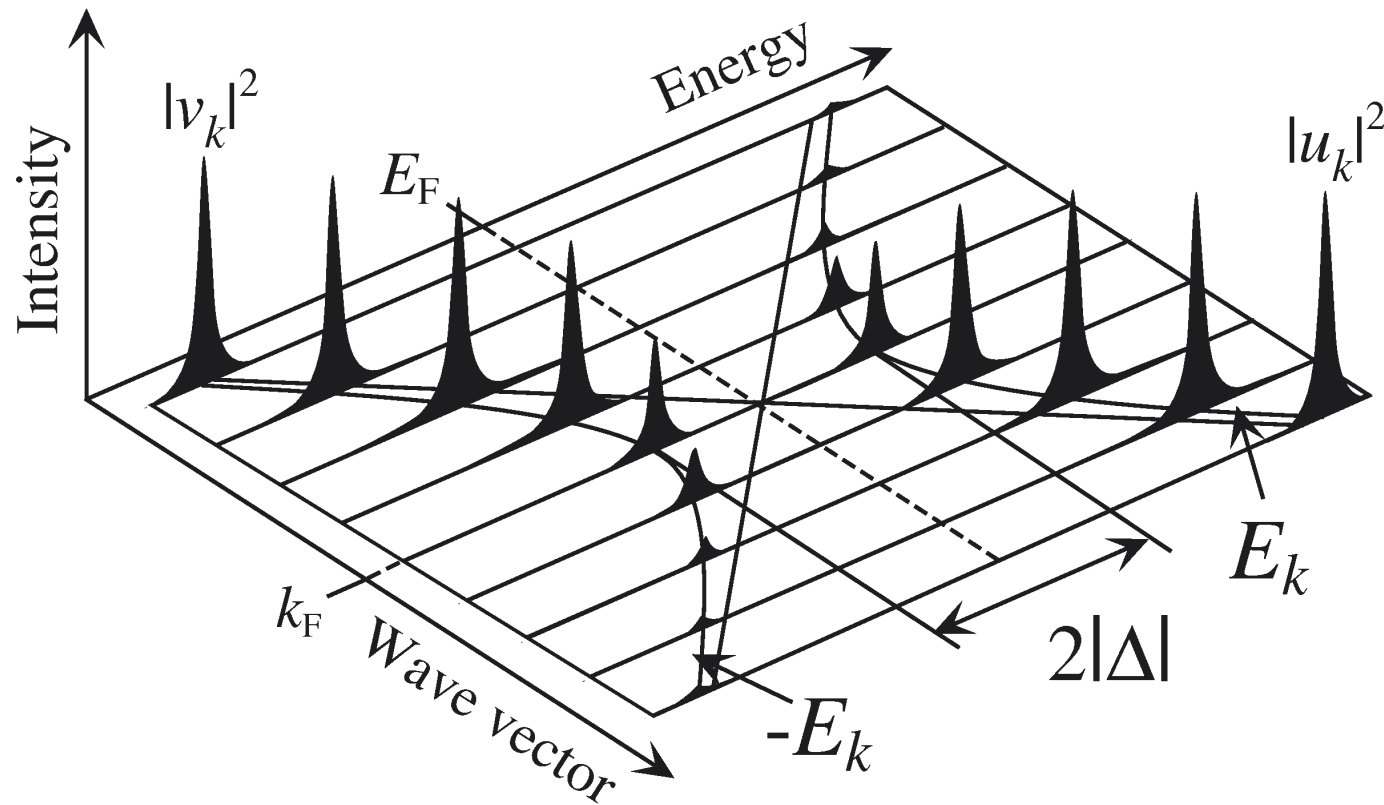
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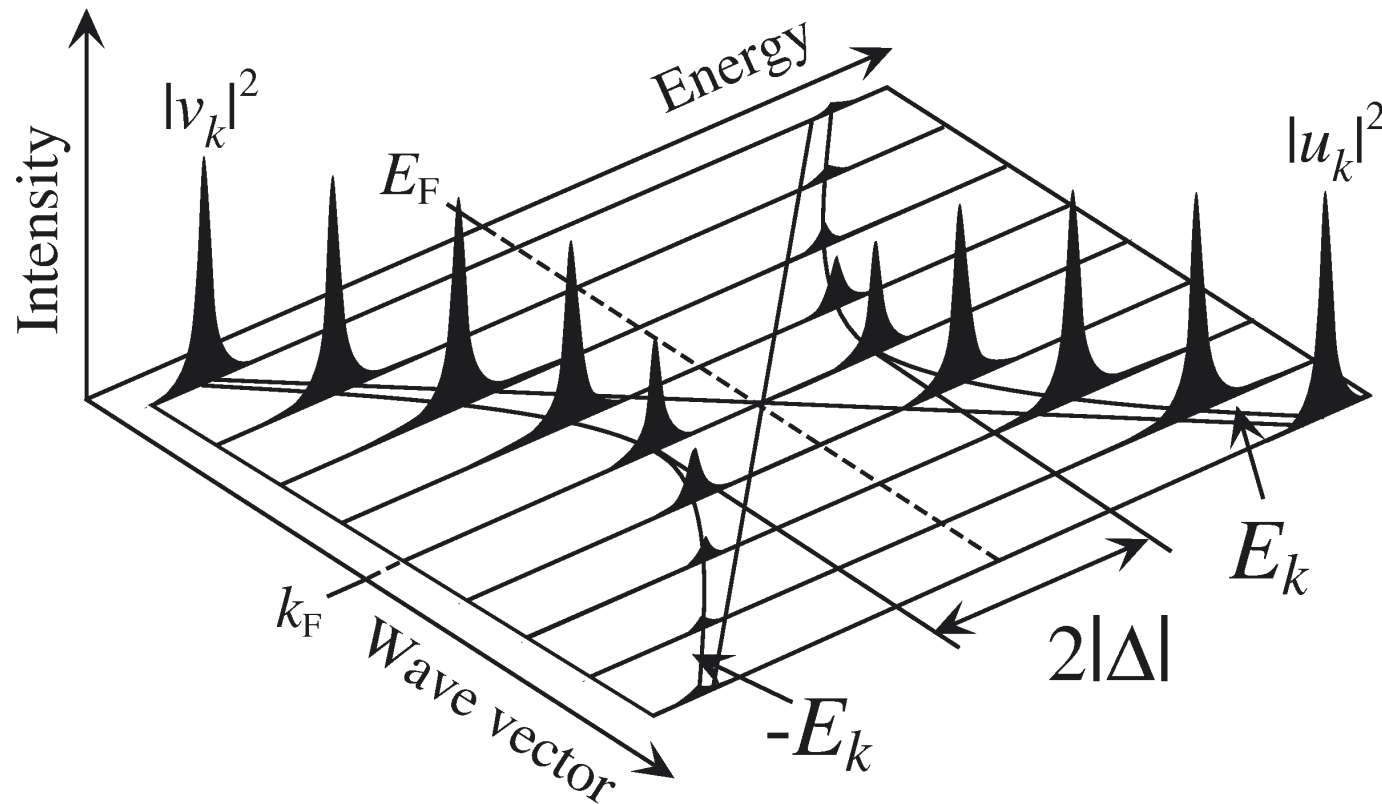
OK, what is their energy ?

Bogoliubov quasiparticles – in superconductors



Bogoliubov qps of s-wave superconductors are gapped $\pm \sqrt{\epsilon_k^2 + \Delta^2}$.

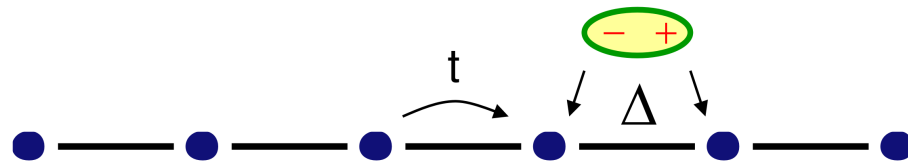
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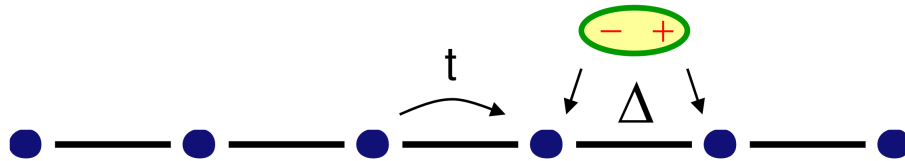
True majoranas have to be the zero energy ($E_k = 0$) quasiparticles !

Kitaev chain – a paradigm for Majorana modes



p-wave pairing of spinless 1D fermions

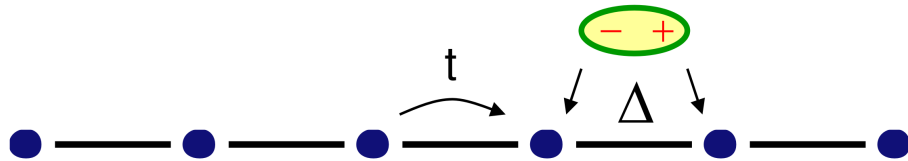
Kitaev chain – a paradigm for Majorana modes



p-wave pairing of spinless 1D fermions

$$\hat{H} = t \sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1} + \text{h.c.} \right) - \mu \sum_i \hat{c}_i^\dagger \hat{c}_i + \Delta \sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1}^\dagger + \text{h.c.} \right)$$

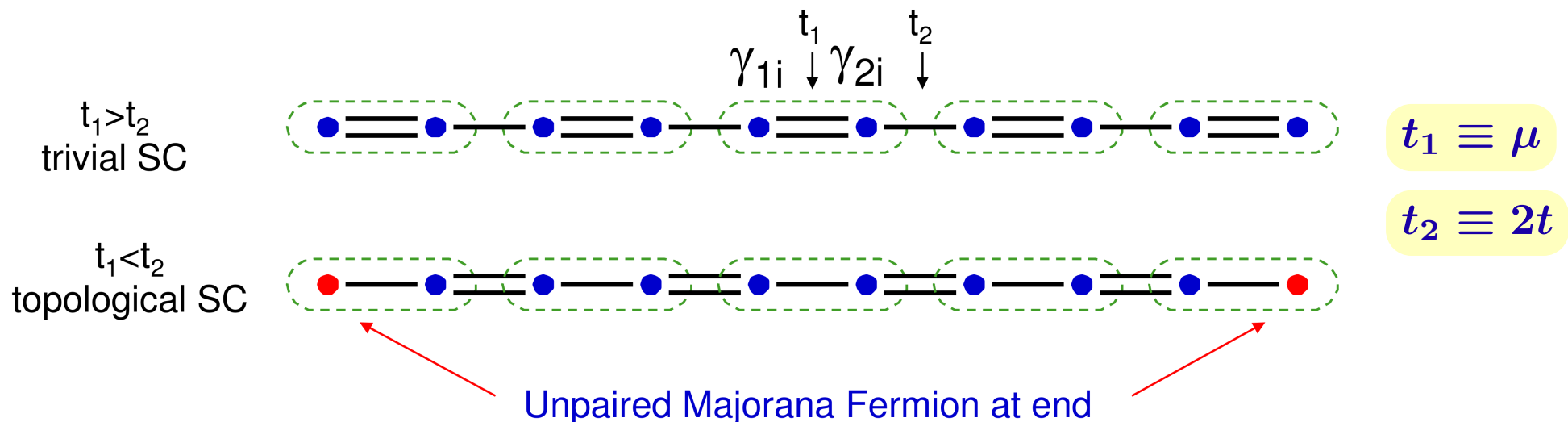
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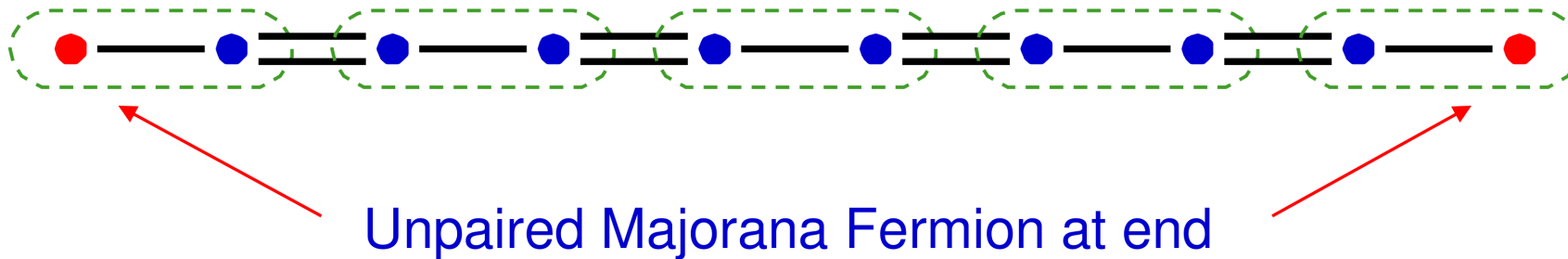
This toy-model can be **exactly solved** in Majorana basis. For $\Delta = t$ one obtains:



Kitaev toy model

/ Phys. Usp. 44, 131 (2001) /

★ In the special case $\Delta = t$ and $|\mu| < 2t$



operators $\hat{\gamma}_{1,1}$ and $\hat{\gamma}_{2,N}$ are decoupled from all the rest, what implies

the zero-energy modes at the chain edges

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Unpaired Majorana Fermion at end

operators $\hat{\gamma}_{1,1}$ and $\hat{\gamma}_{2,N}$ are decoupled from all the rest, what implies

the zero-energy modes at the chain edges

★ Similar ideas have been considered for 1D Heisenberg chain of 1/2 spins



$$\bullet - \bullet = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

F.D.M. Haldane, Phys. Rev. Lett. 50, 1153 (1983)

Nobel Prize, 2016

Various scenarios

– **for Majorana quasiparticles**

Various scenarios – for Majorana quasiparticles

- vortex states in p -wave superconductors

Volovik (1999)

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- edge states of 2D and 3D topological insulators

Hasan & Kane (2010); Qi & Zhang (2011); Franz & Molenkamp (2013)

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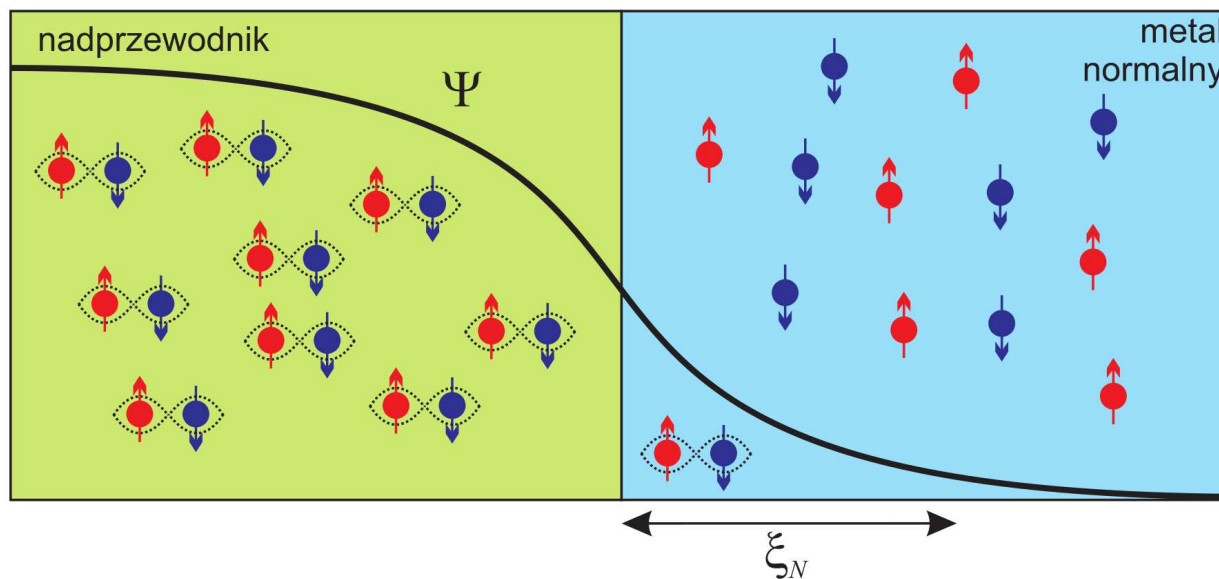
- magnetic atoms chain on superconducting substrate

Choy *et al* (2011); Martin & Morpugo (2012); Nadj-Perge *et al* (2013)

2. Electron pairing in nanosystems

Superconductivity in nanosystems

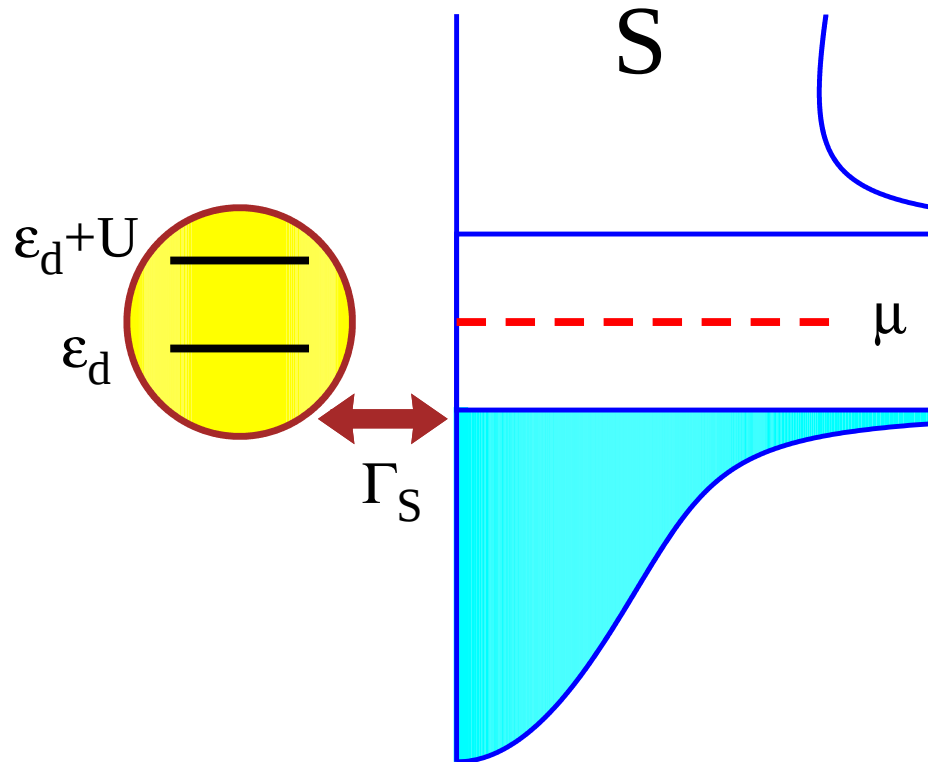
1 Any material brought in contact with a bulk superconductor absorbs the Cooper pairs



Usually the size of quantum dots is smaller than ξ_N

Prototype model – single Anderson impurity

The single quantum impurity (dot) coupled to superconducting reservoir



ϵ_d – energy level, U – Coulomb potential, Γ_S – hybridization

Microscopic model

Anderson-type Hamiltonian

Quantum impurity (dot)

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \hat{d}_{\sigma}^{\dagger} \hat{d}_{\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow}$$

coupled with a superconductor

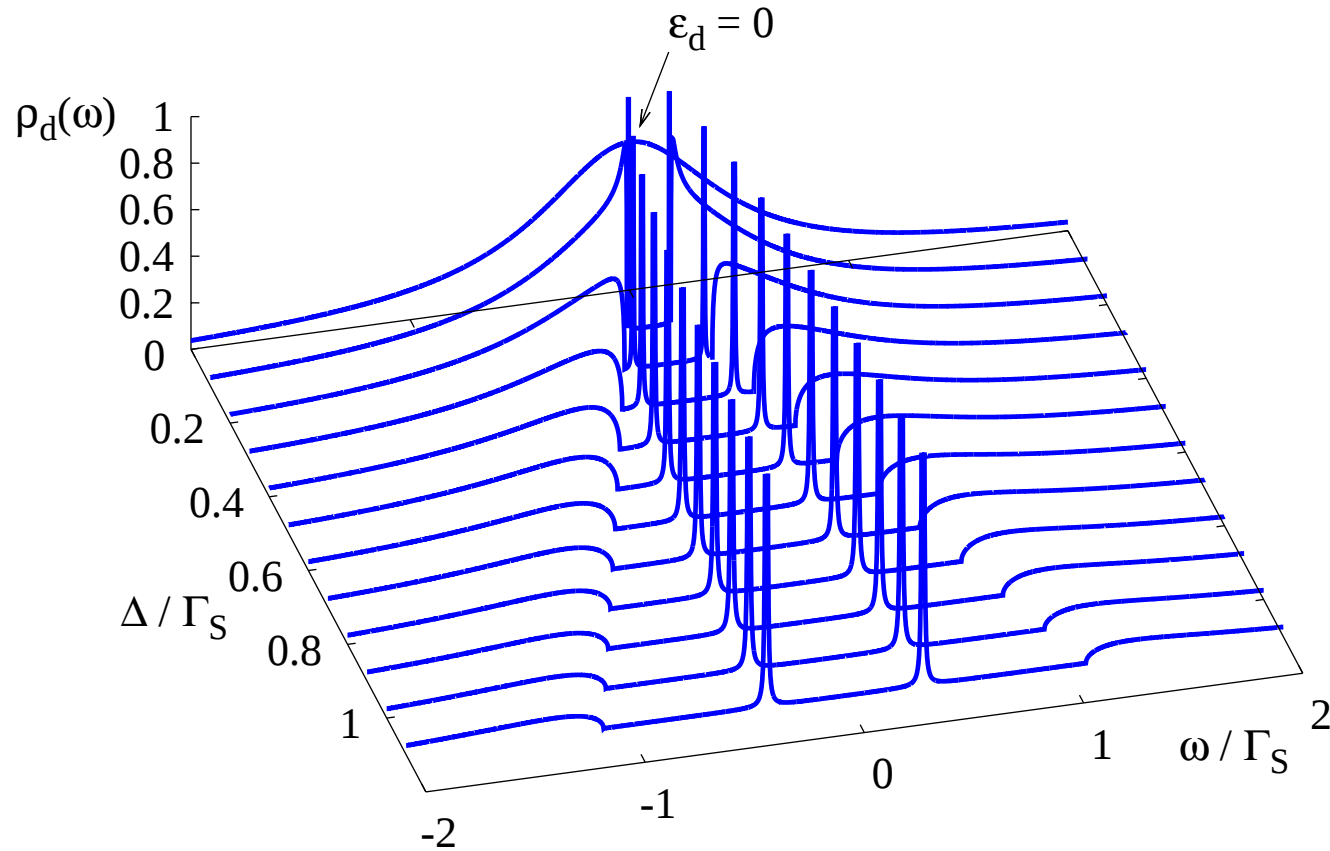
$$\begin{aligned} \hat{H} = & \sum_{\sigma} \epsilon_d \hat{d}_{\sigma}^{\dagger} \hat{d}_{\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} + \hat{H}_S \\ & + \sum_{\mathbf{k}, \sigma} \left(V_{\mathbf{k}} \hat{d}_{\sigma}^{\dagger} \hat{c}_{\mathbf{k}\sigma} + V_{\mathbf{k}}^* \hat{c}_{\mathbf{k}\sigma}^{\dagger} \hat{d}_{\sigma} \right) \end{aligned}$$

where

$$\hat{H}_S = \sum_{\mathbf{k}, \sigma} (\epsilon_{\mathbf{k}} - \mu) \hat{c}_{\mathbf{k}\sigma}^{\dagger} \hat{c}_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} \left(\Delta \hat{c}_{\mathbf{k}\uparrow}^{\dagger} \hat{c}_{\mathbf{k}\downarrow}^{\dagger} + \text{h.c.} \right)$$

Uncorrelated QD

$U_d = 0$ (exactly solvable case)



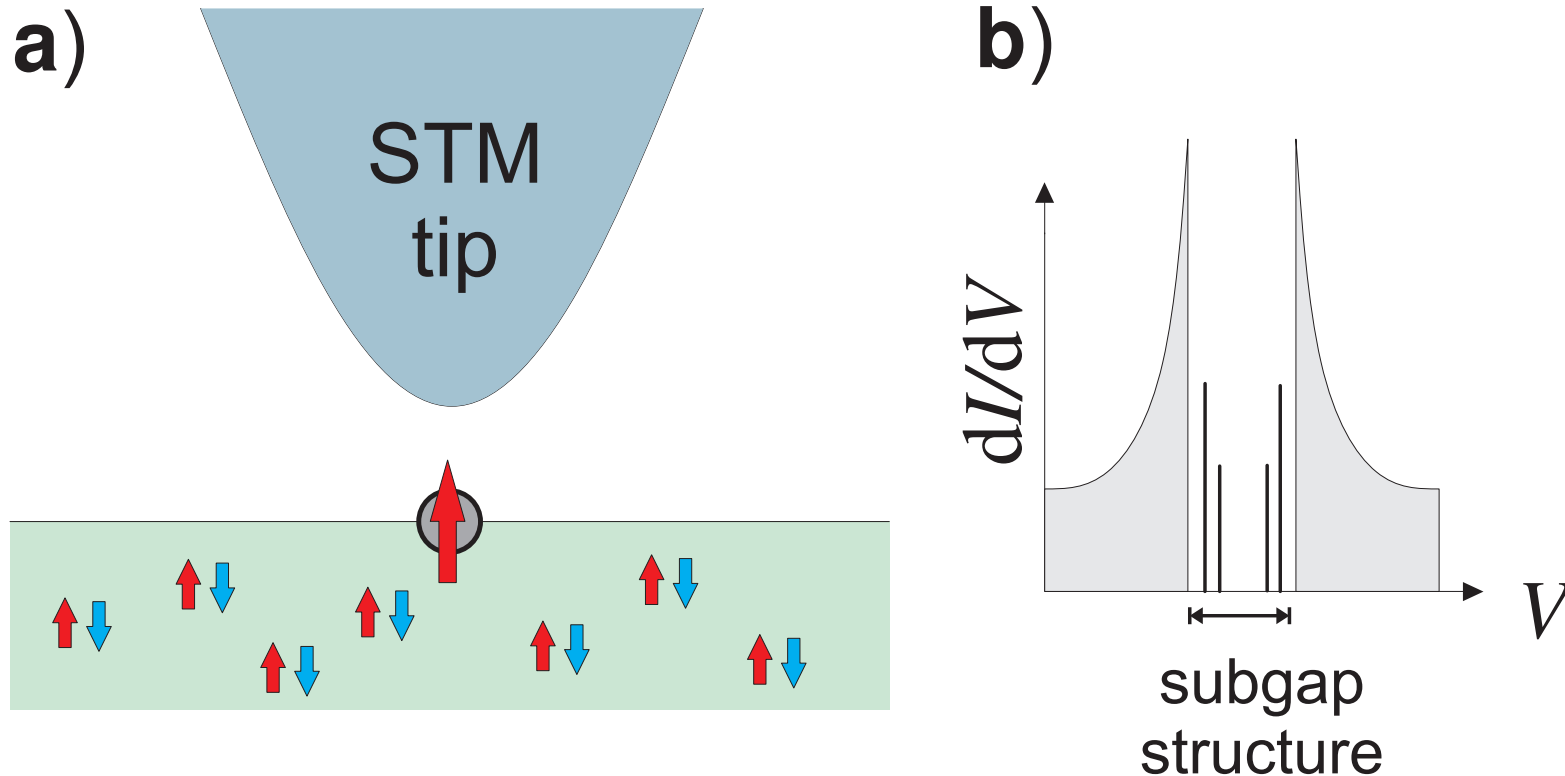
In-gap (**Andreev/Shiba**) bound states :

$\Rightarrow$ always appear in pairs,

$\Rightarrow$ appear symmetrically at finite energies.

Subgap states

of multilevel quantum impurities



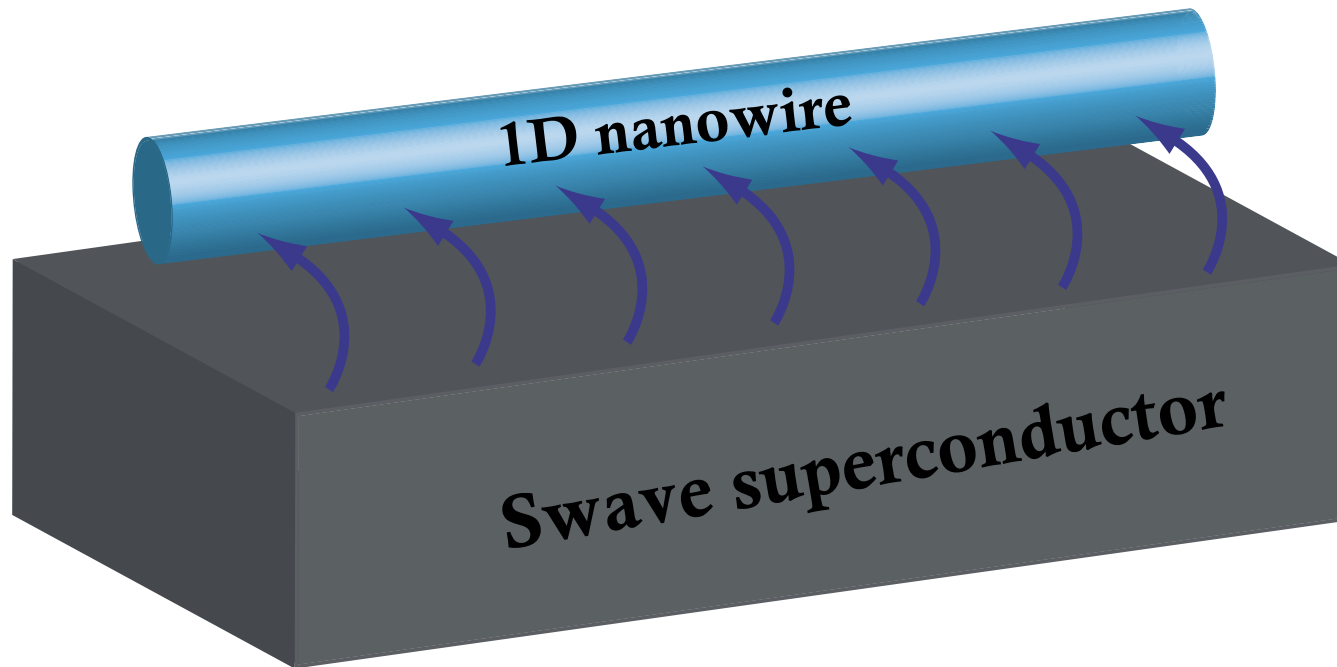
a) STM scheme and b) differential conductance for a multilevel quantum impurity adsorbed on a superconductor surface.

R. Žitko, O. Bodensiek, and T. Pruschke, Phys. Rev. B **83**, 054512 (2011).

Andreev vs Majorana states – a story of mutation

Andreev vs Majorana states – a story of mutation

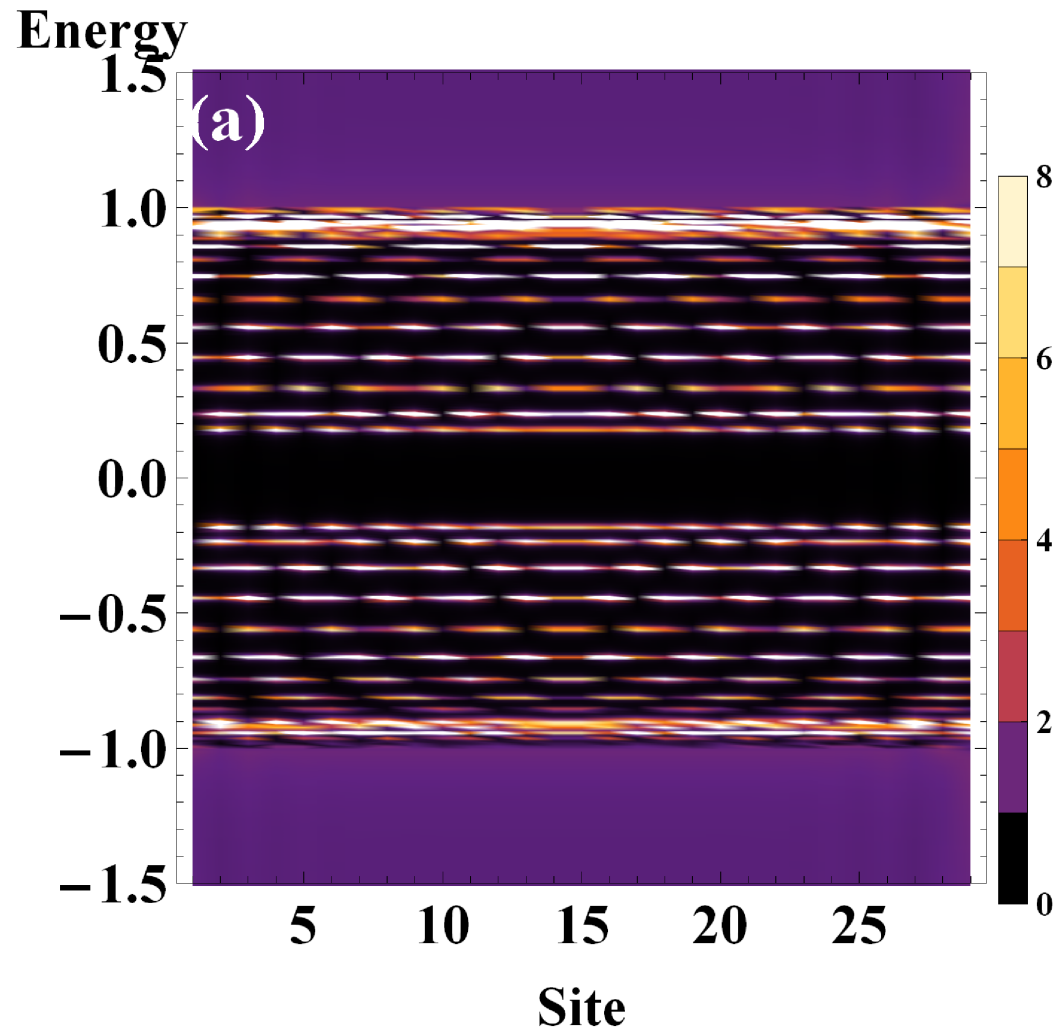
Let us consider:



1D quantum wire deposited on s-wave superconductor

*D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B **88**, 165401 (2013).*

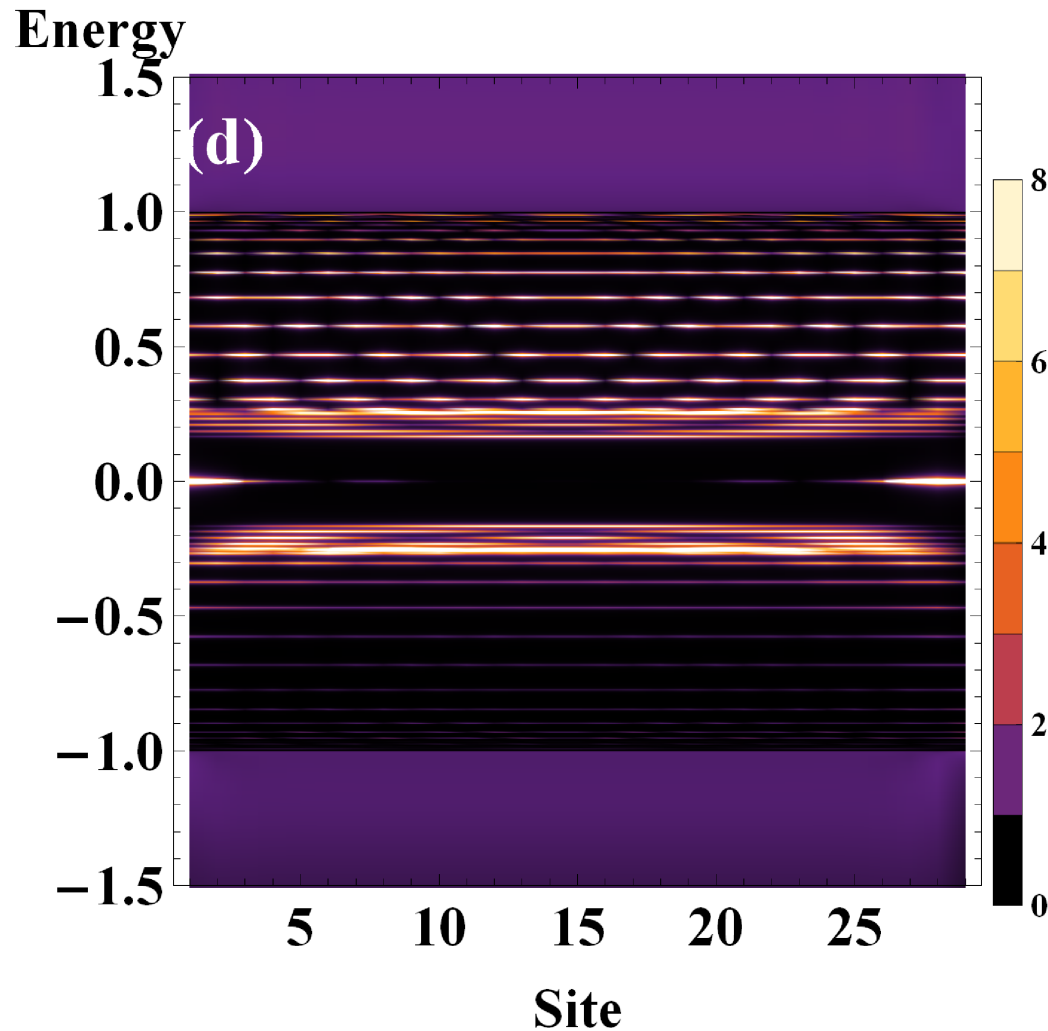
Andreev vs Majorana states – a story of mutation



Spectrum of a quantum wire has a series of Andreev states.

D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B **88**, 165401 (2013).

Andreev vs Majorana states – a story of mutation



Spin-orbit coupling induces the Majorana-type quasiparticles.

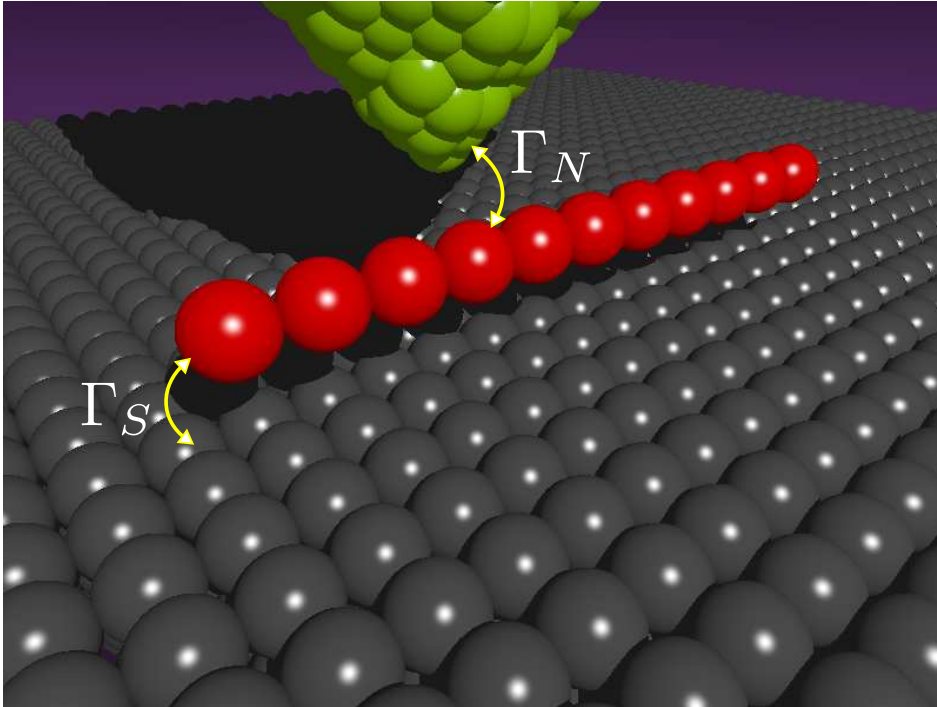
D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B **88**, 165401 (2013).

Towards more realistic situation

/ Rashba chain + pairing /

Towards more realistic situation

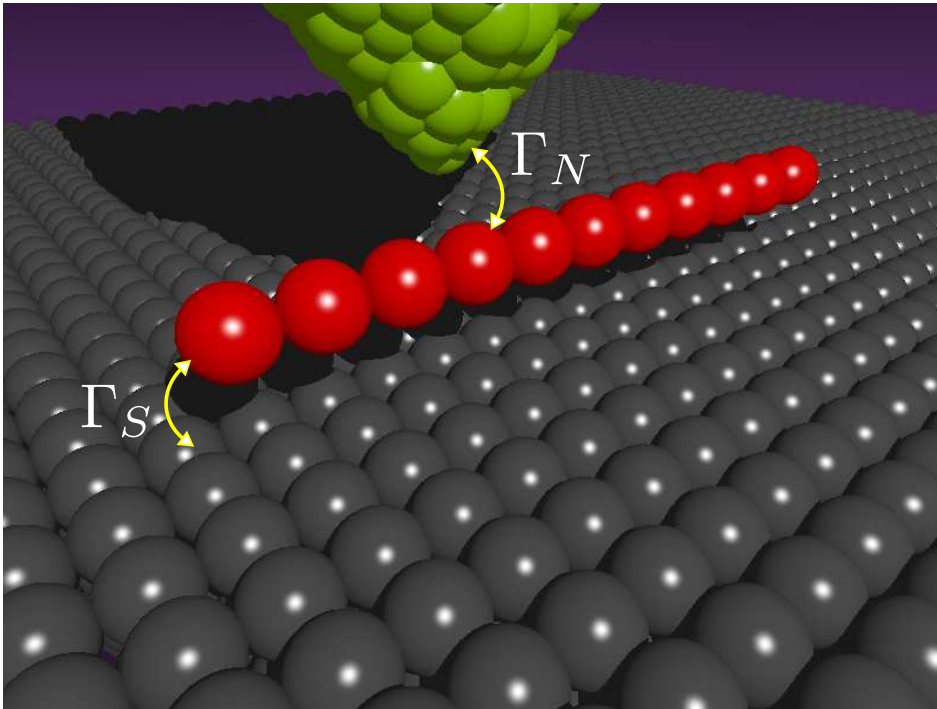
/ Rashba chain + pairing /



Scheme of STM configuration

Towards more realistic situation

/ Rashba chain + pairing /



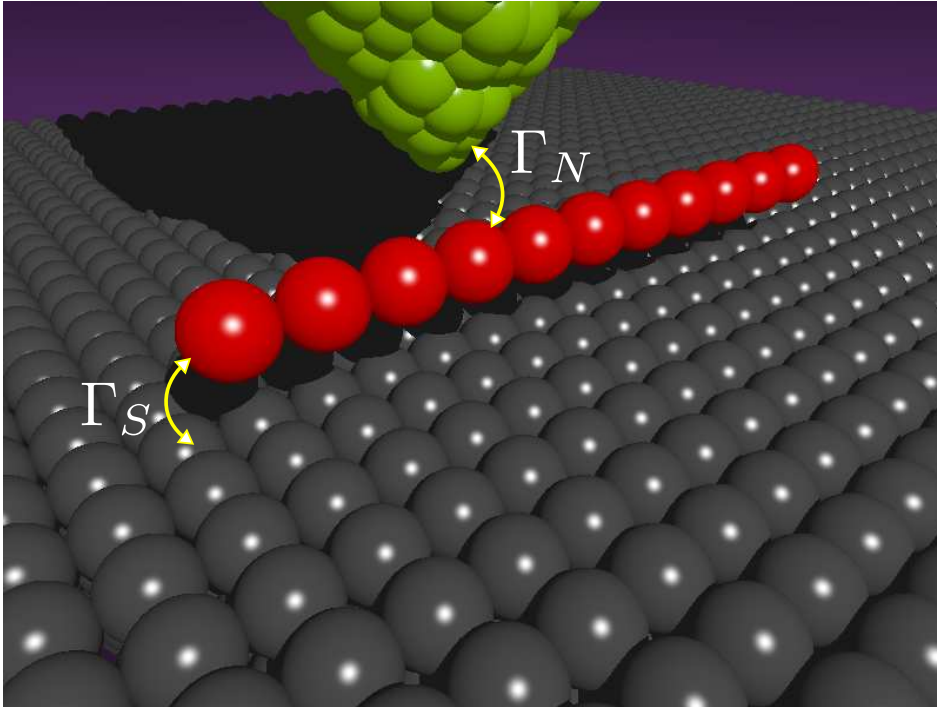
Scheme of STM configuration

$$\hat{H} = \hat{H}_{tip} + \hat{H}_{chain} + \hat{H}_S + \hat{V}_{hybr}$$

We studied this model, focusing on the deep subgap regime $|E| \ll \Delta_{sc}$.

Towards more realistic situation

/ Rashba chain + pairing /



Scheme of STM configuration

where

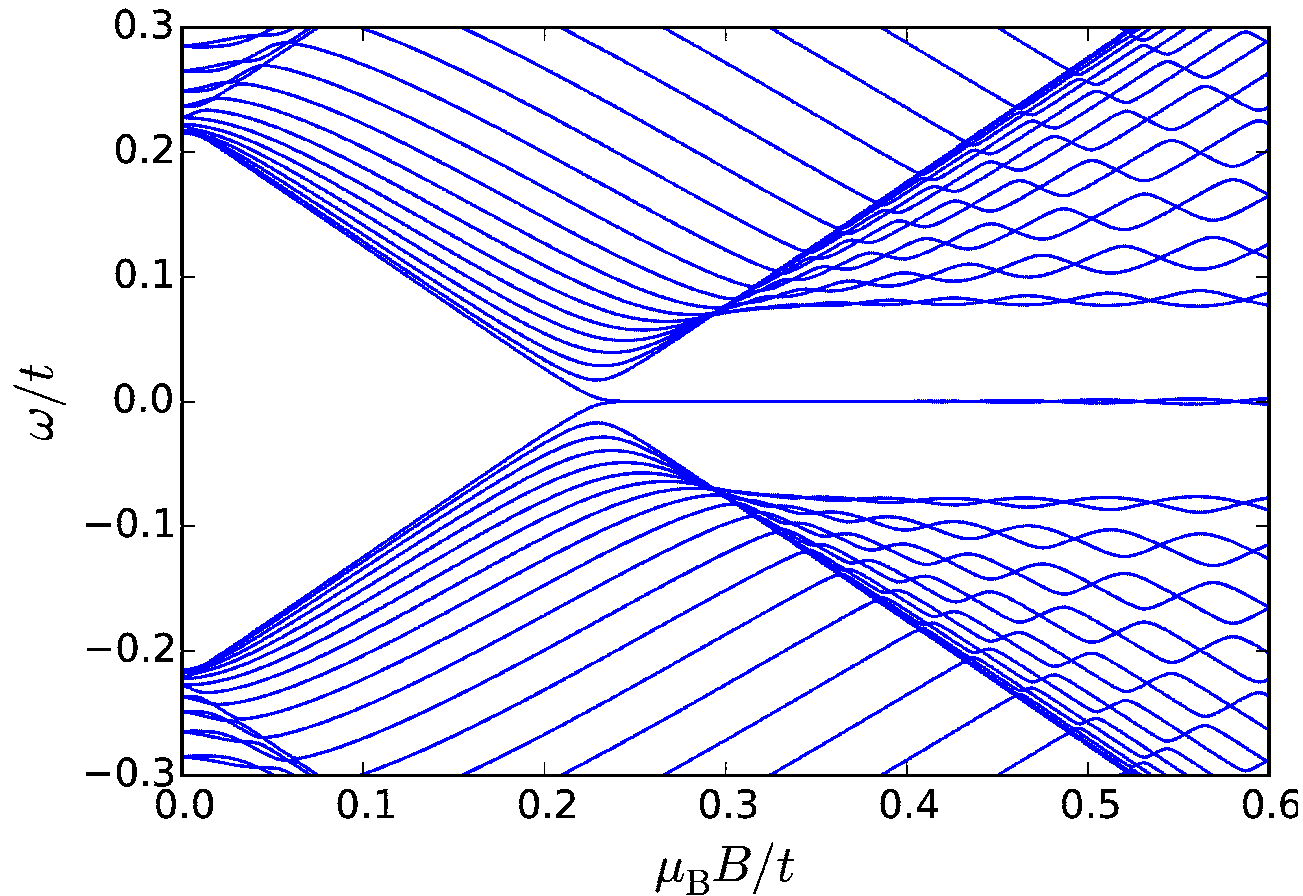
$$\hat{H}_{chain} = \sum_{i,j,\sigma} (t_{ij} - \delta_{ij}\mu) \hat{d}_{i,\sigma}^\dagger \hat{d}_{j,\sigma} + \hat{H}_{Rashba} + \hat{H}_{Zeeman}$$

M. Mańska, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv:1609.00685 (2016).

Majorana states – of the Rashba chain

Majorana states – of the Rashba chain

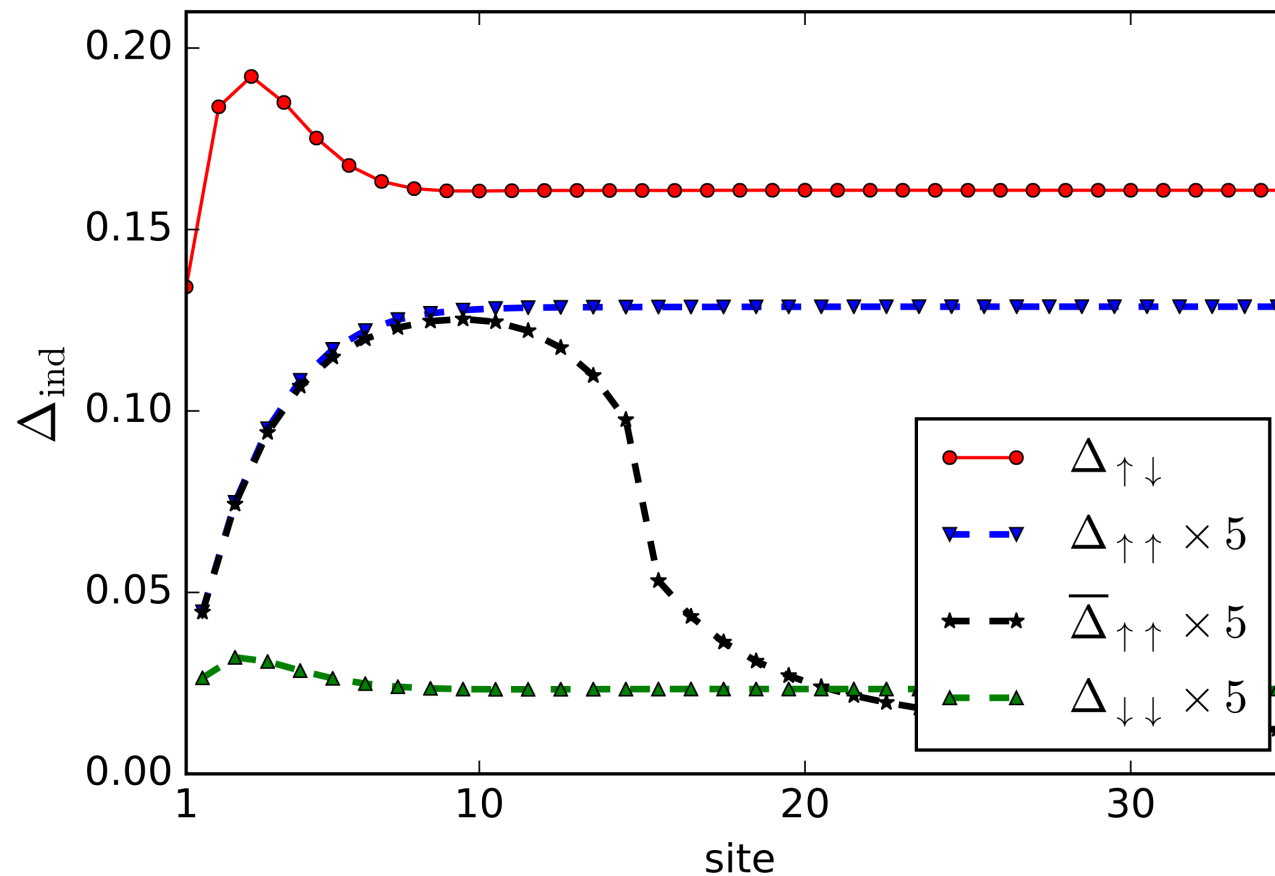
Mutation of Andreev states into zero-energy (Majorana) mode



M. Mańska, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv:1609.00685 (2016).

Majorana states – of the Rashba chain

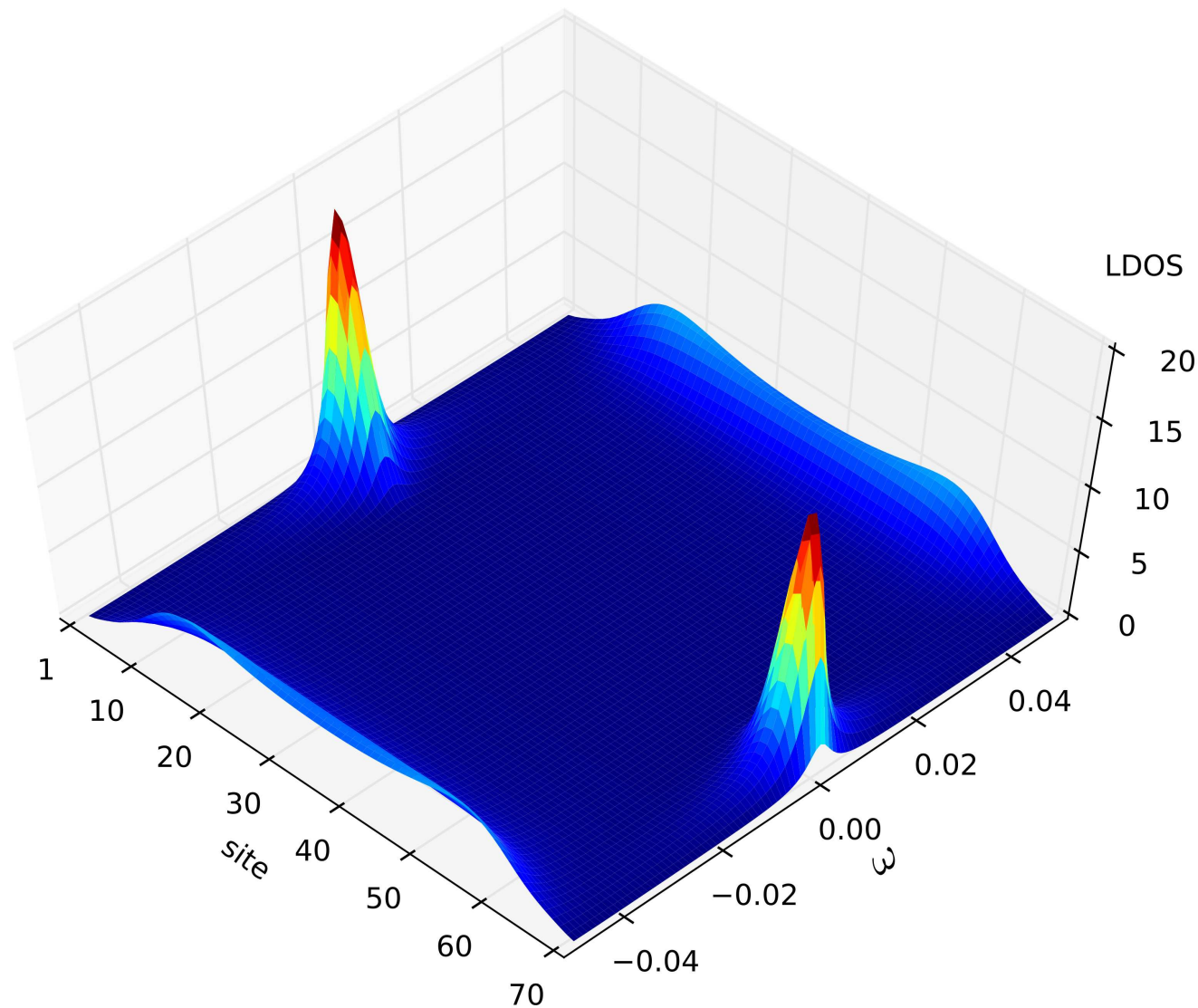
Spatial variation of the induced pairings $\Delta_{\sigma,\sigma'} = \langle \hat{d}_{i,\sigma} \hat{d}_{i+1,\sigma'} \rangle$



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv:1609.00685 (2016).

Majorana states – of the Rashba chain

Spectrum with the edge Majorana quasiparticles

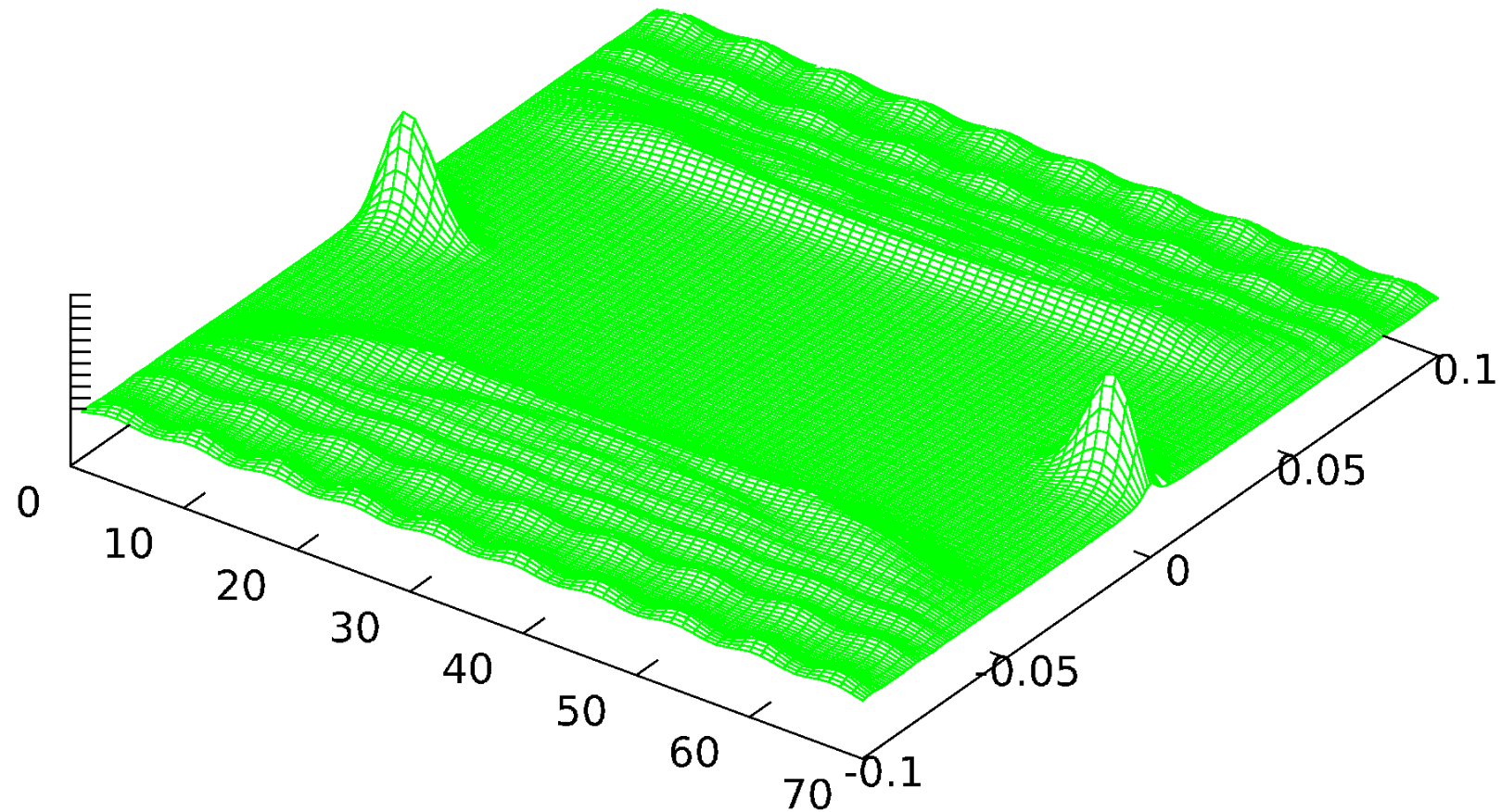


Breaking the Rashba chain

by a reduced hopping at site $i = N/2$

Breaking the Rashba chain

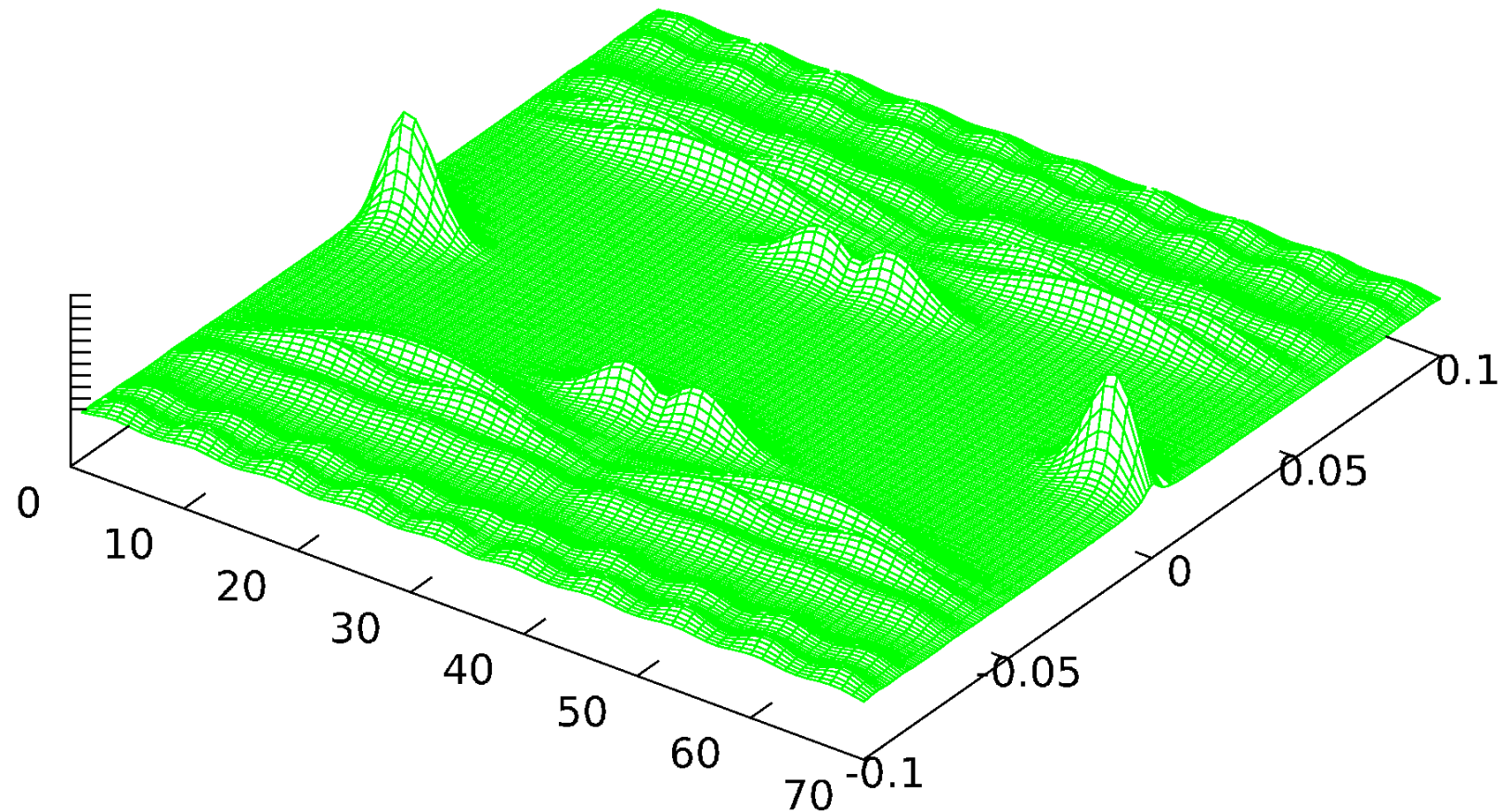
by a reduced hopping at site $i = N/2$



$t_i/t = 1$

Breaking the Rashba chain

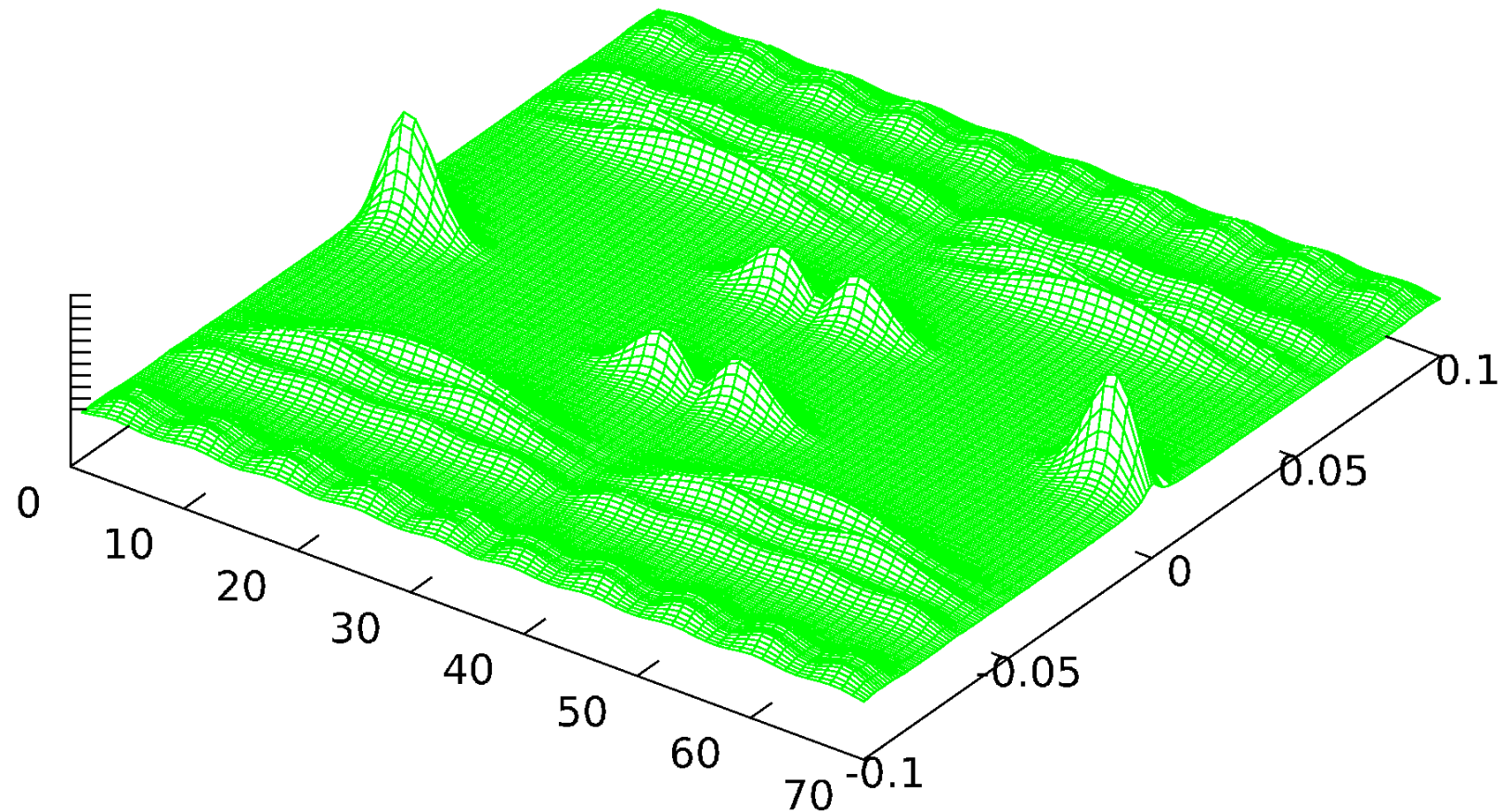
by a reduced hopping at site $i = N/2$



$t_i/t = 0.8$

Breaking the Rashba chain

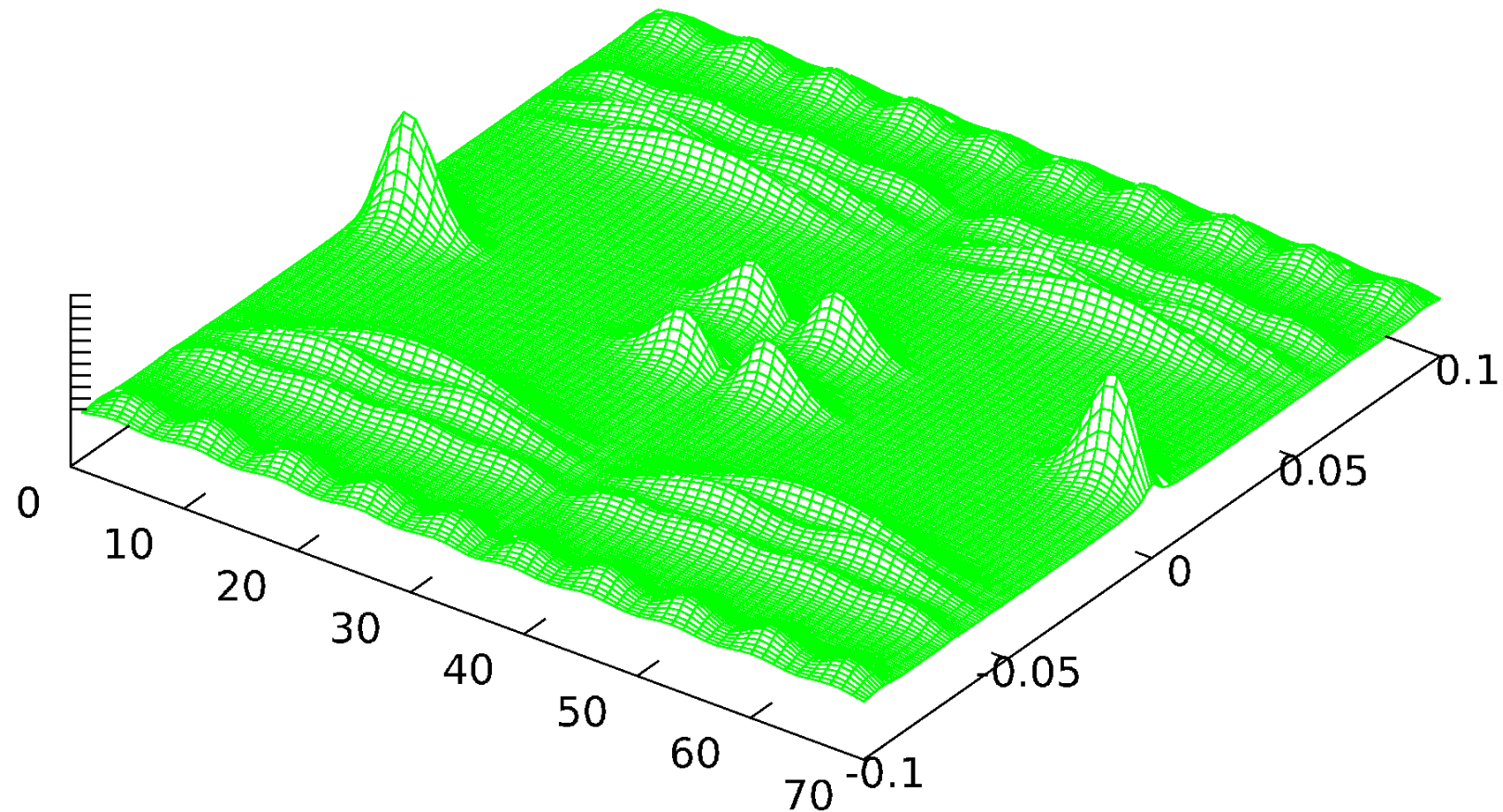
by a reduced hopping at site $i = N/2$



$t_i/t = 0.6$

Breaking the Rashba chain

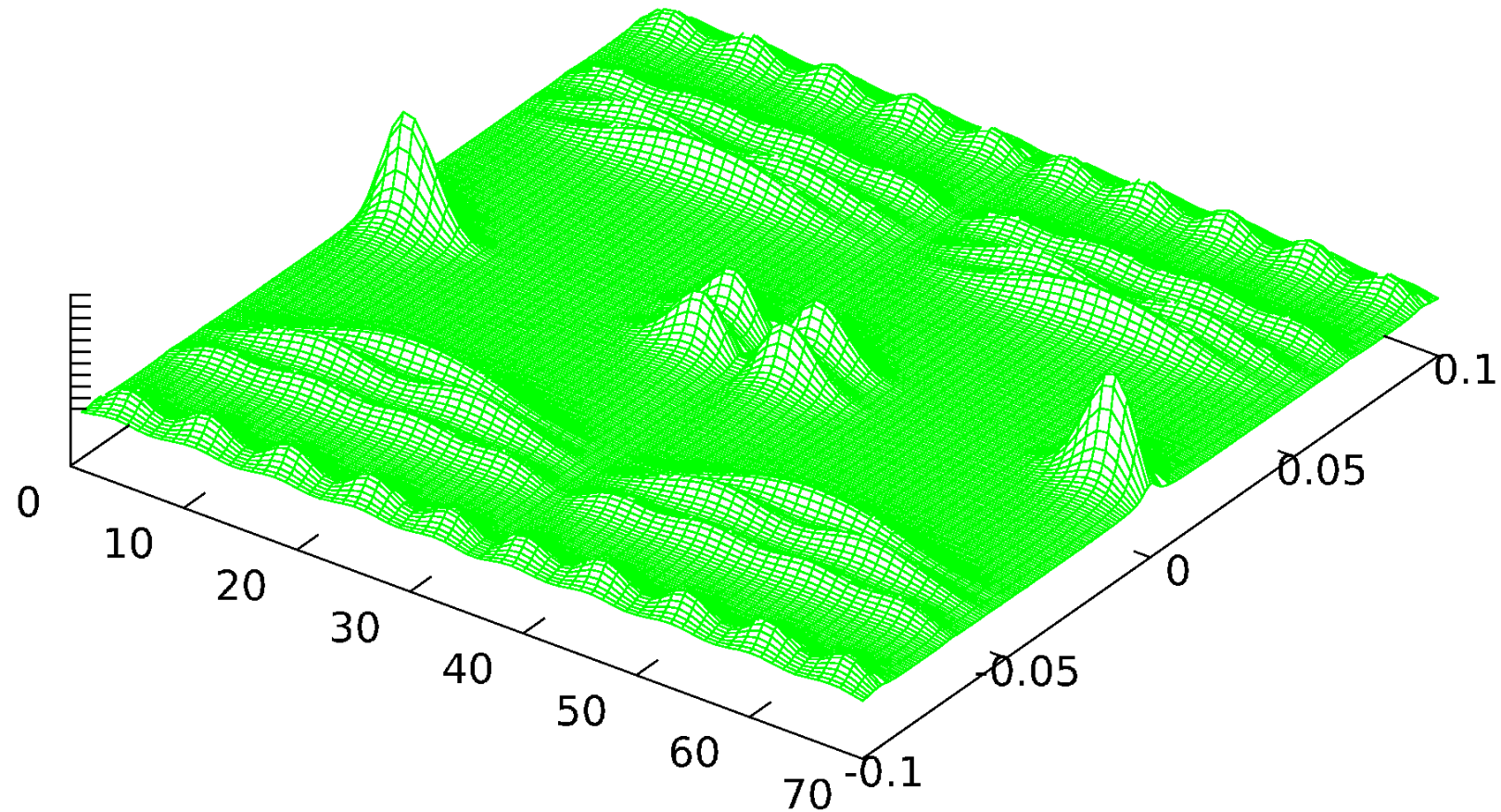
by a reduced hopping at site $i = N/2$



$t_i/t = 0.4$

Breaking the Rashba chain

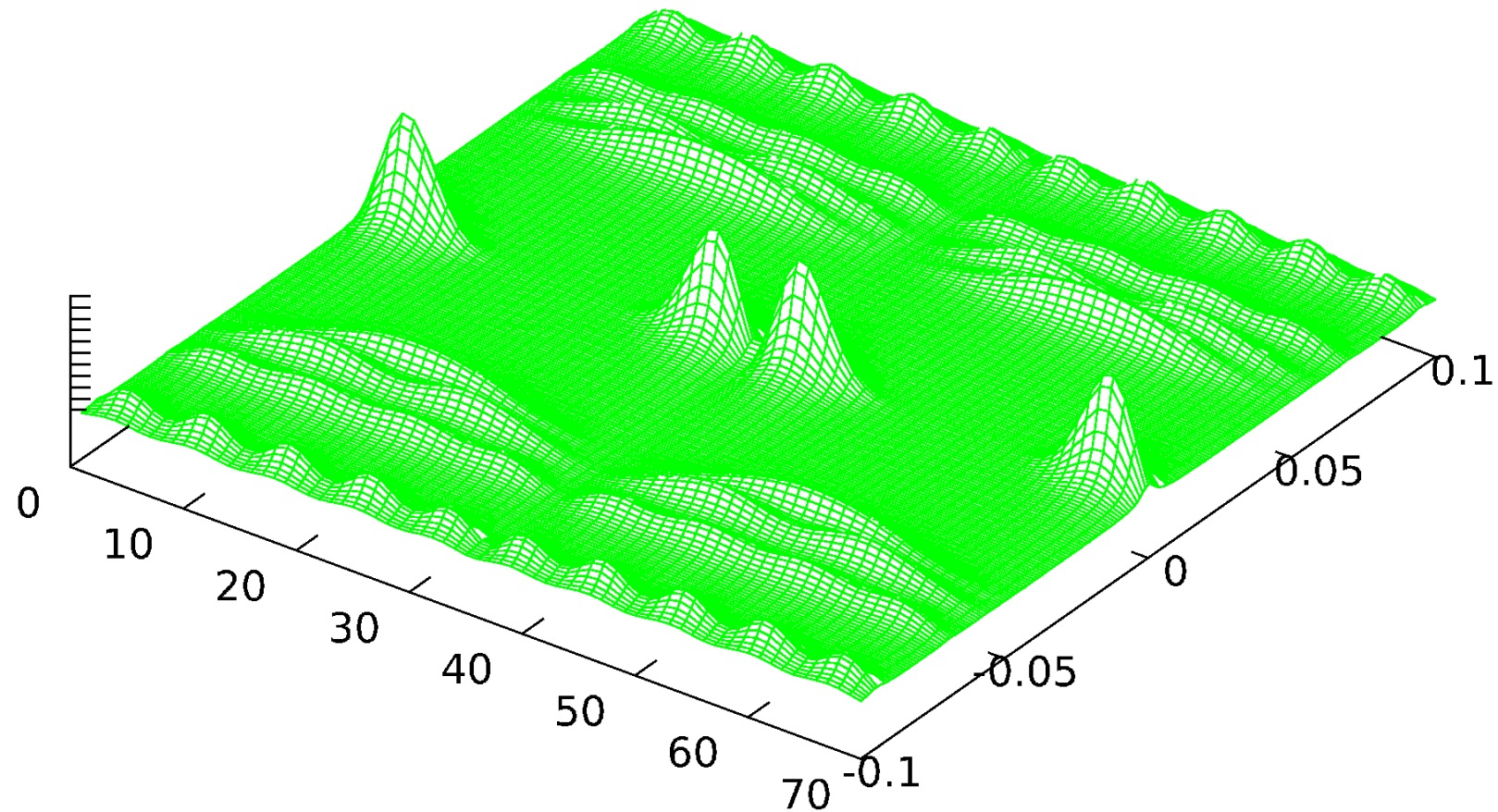
by a reduced hopping at site $i = N/2$



$t_i/t = 0.2$

Breaking the Rashba chain

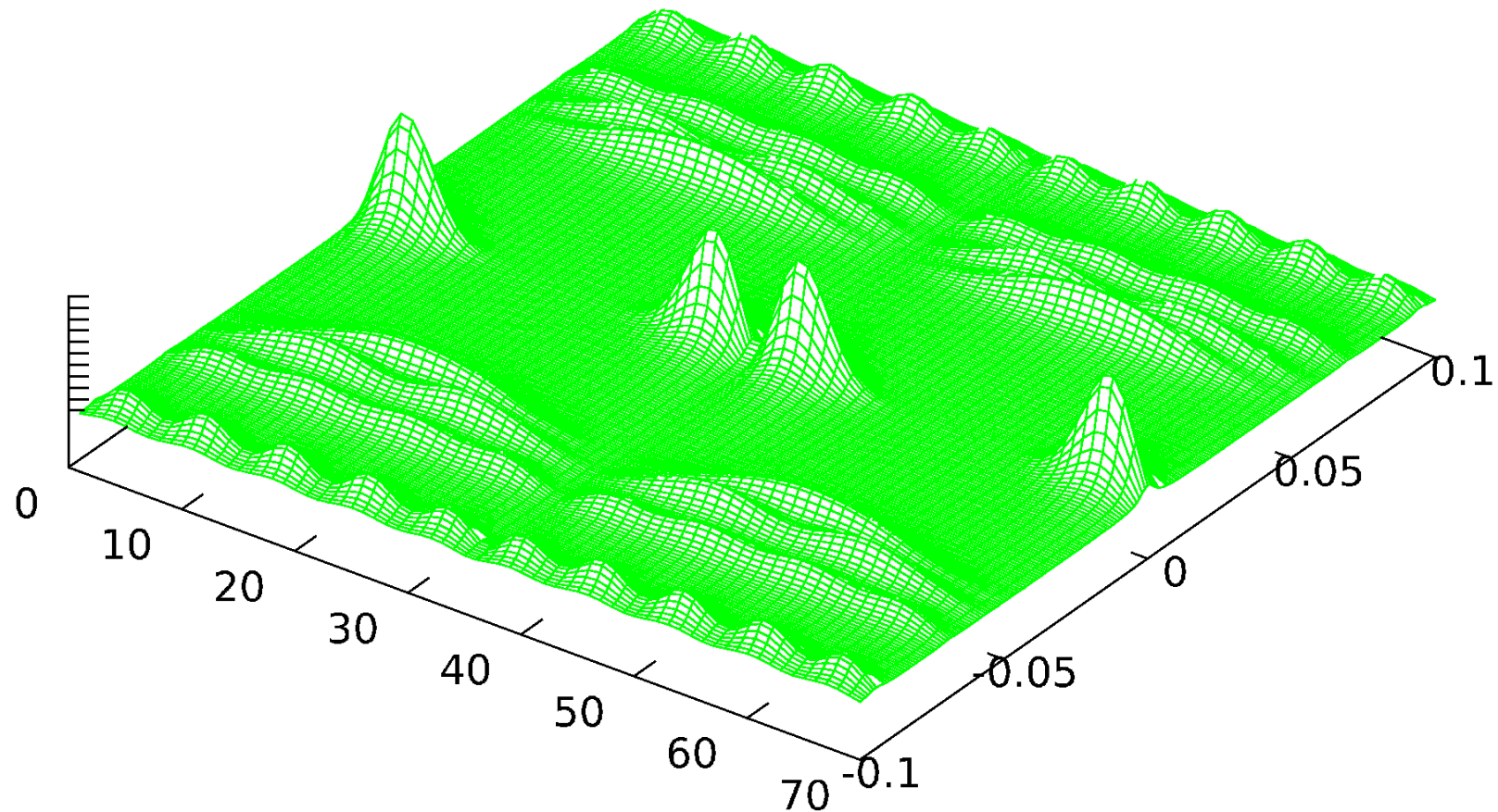
by a reduced hopping at site $i = N/2$



$t_i/t = 0$

Breaking the Rashba chain

by a reduced hopping at site $i = N/2$

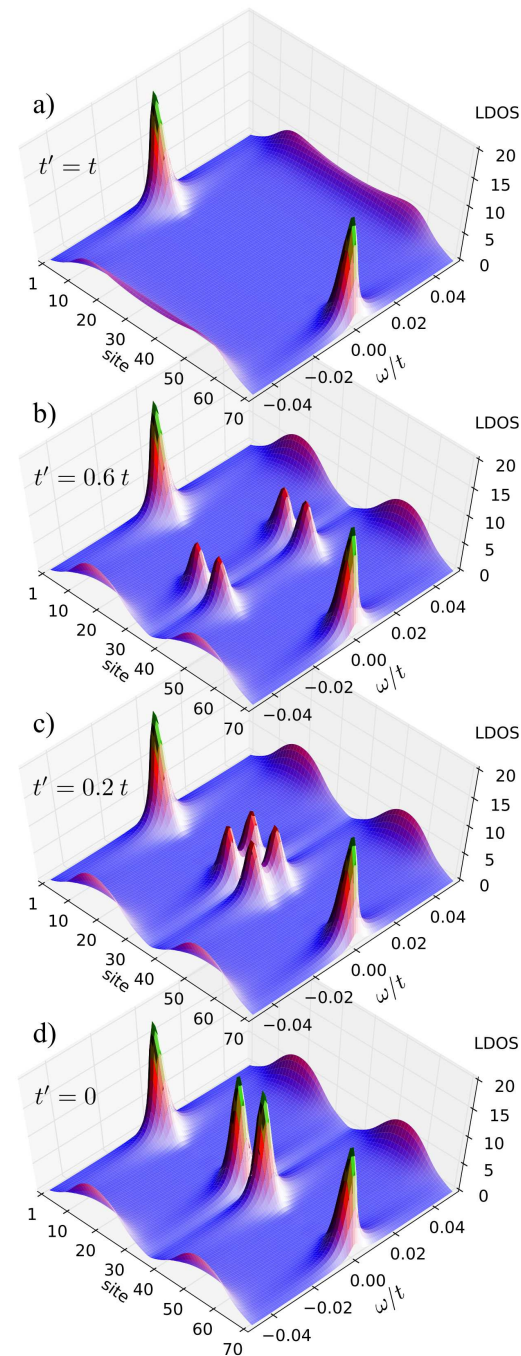


Number of Majorana quasiparticles doubled !

Fussion/splitting of Majorana states

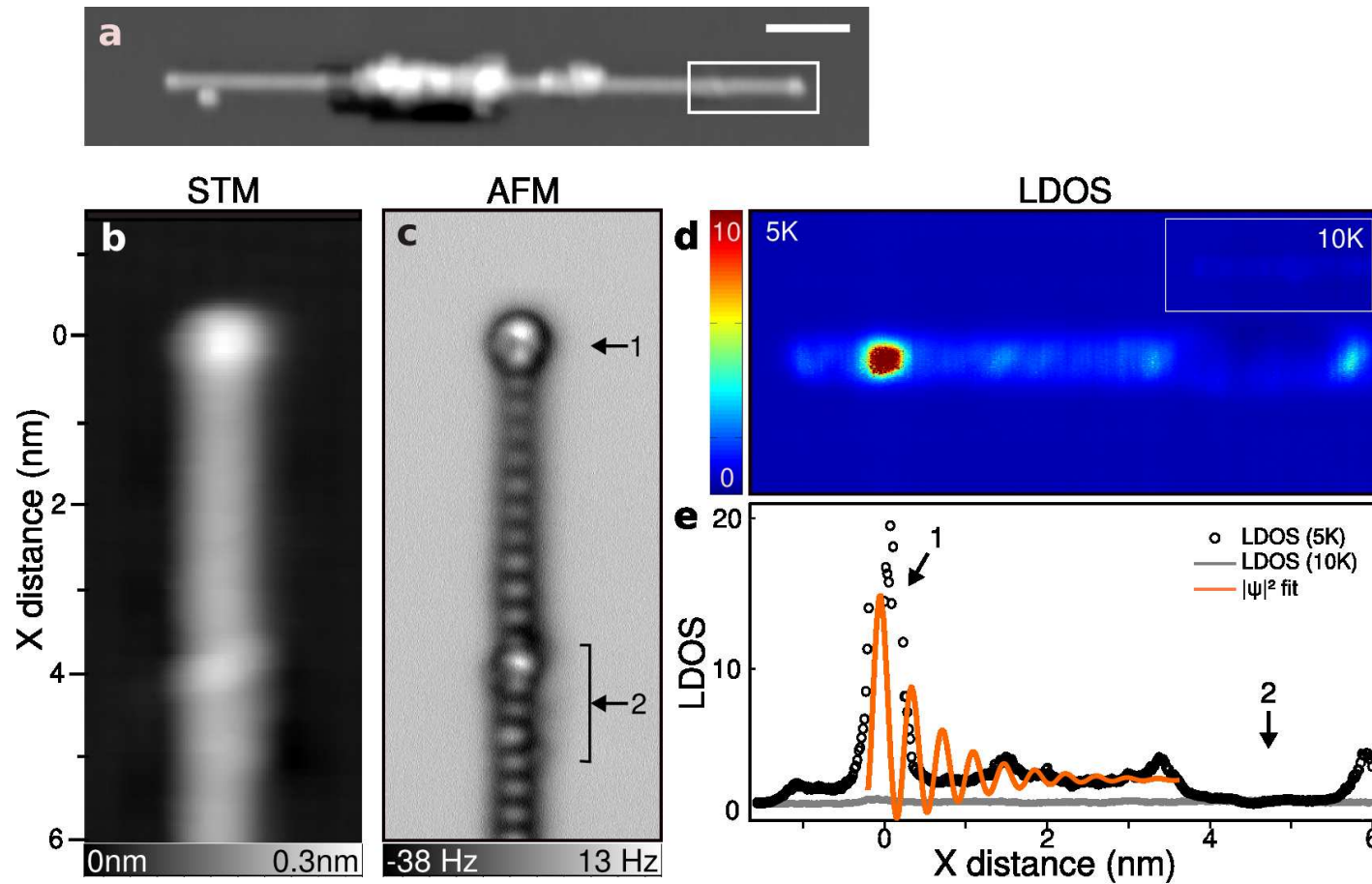
Partitioning
the Rashba chain
into two pieces
via reduced hopping
 t_j at site $j = N/2$

M. Maška et al, arXiv:1609.00685 (2016).



Majoranas at quantum defects

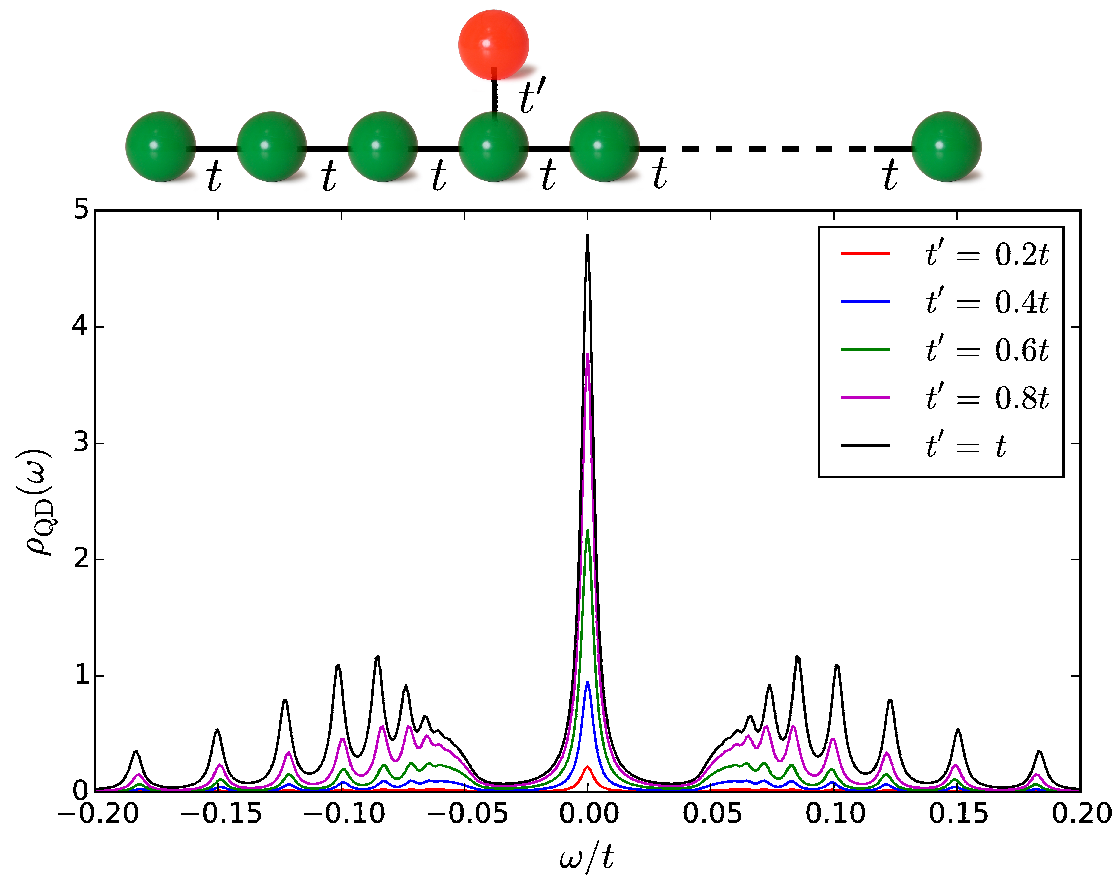
– experimental relevance



R. Pawlak, **M. Kisiel**, ..., and E. Meyer, npj Quantum Information **2**, 16035 (2016).

Majorana states – proximity effect

Majorana state can 'leak' into a normal quantum impurity



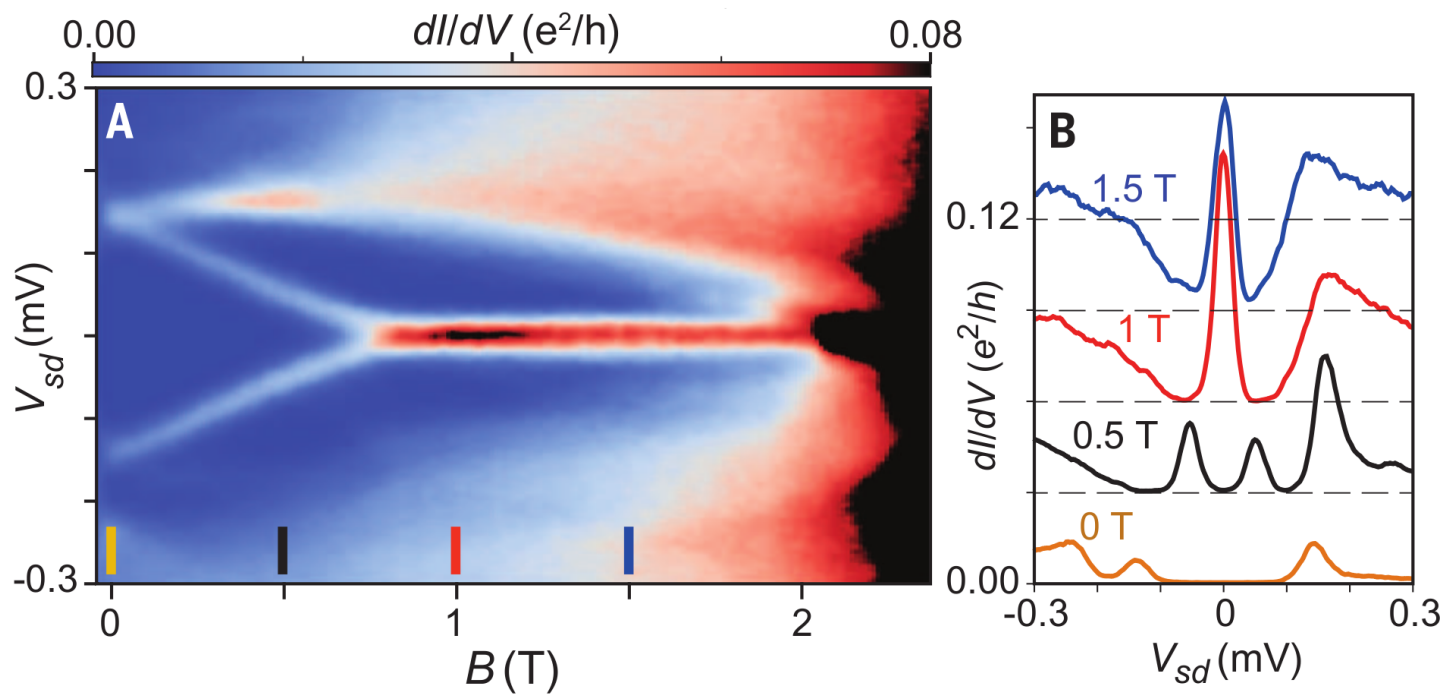
M. Mańska, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv:1609.00685 (2016).

- **experimental realization (27 Dec 2016)**



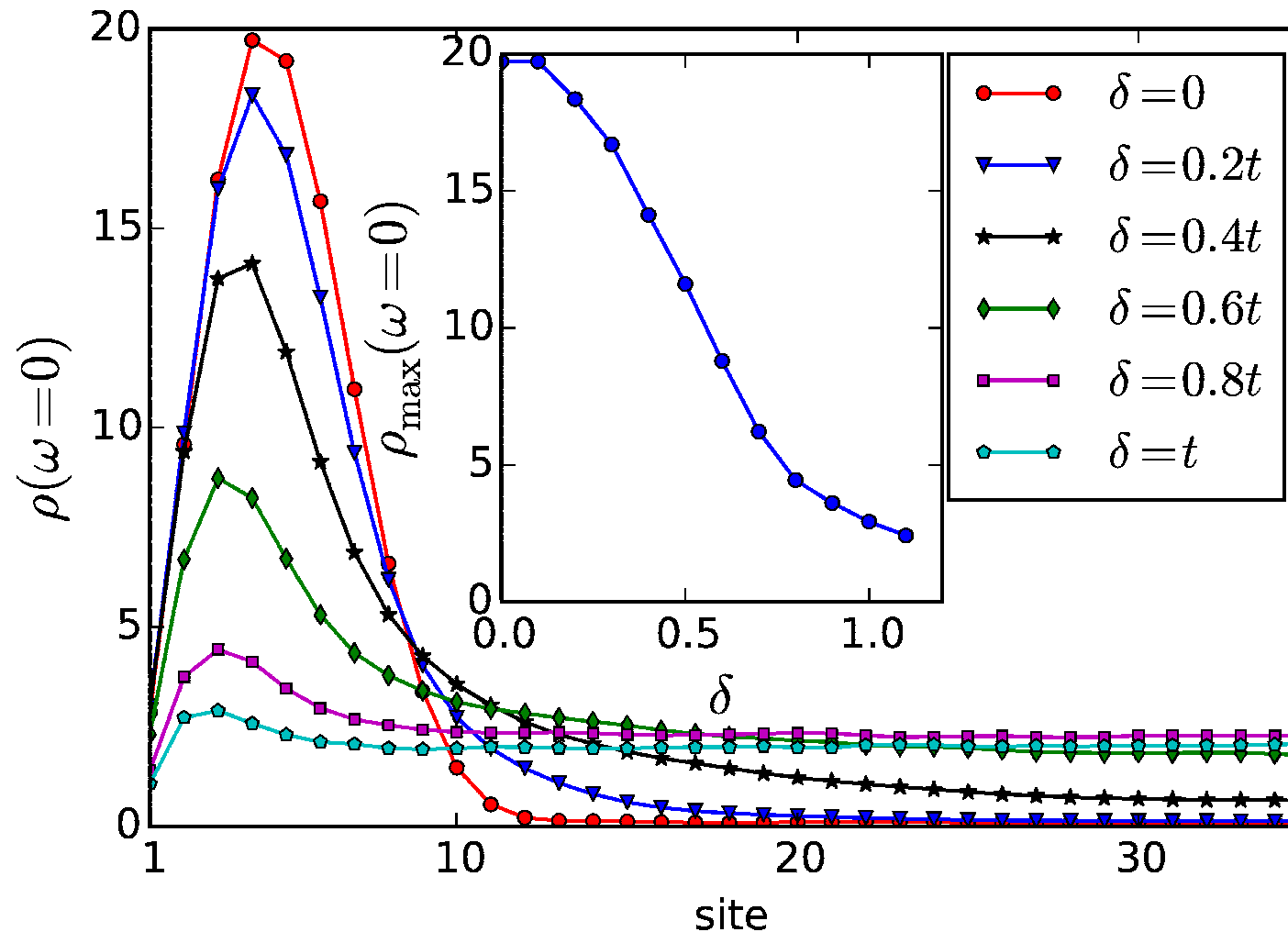
M. T. Deng,^{1,2} S. Vaitiekėnas,^{1,3} E. B. Hansen,¹ J. Danon,^{1,4} M. Leijnse,^{1,5} K. Flensberg,¹
J. Nygård,¹ P. Krogstrup,¹ C. M. Marcus^{1*}

Science **354**, 1557 (2016).



Majorana states – effect of disorder

Majorana quasiparticles are not truly immune to disorder !



Conclusions

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Majorana quasiparticles:

- ⇒ evolve out of the Andreev/Shiba states
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Majorana quasiparticles:

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<http://kft.umcs.lublin.pl/doman/lectures>