Subgap current through the strongly correlated quantum dot hybridized with the normal and superconducting leads

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A short title:

Kondo effect vs superconductivity in quantum dots

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Outline

🌟 Introduction

effects of correlations in the N - QD - S setup
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effects of correlations in the N - QD - S setup

🌟 Procedure

Keldysh formalism & approximations
Outline

🌟 **Introduction**
  
  *effects of correlations in the N - QD - S setup*

🌟 **Procedure**
  
  *Keldysh formalism & approximations*

🌟 **Results**
  
  *a) Kondo effect vs superconductivity,
  b) signatures in the Andreev conductance*
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Introduction

*effects of correlations in the N - QD - S setup*

Procedure

*Keldysh formalism & approximations*

Results

*a) Kondo effect vs superconductivity,*

*b) signatures in the Andreev conductance*

Summary
1. Introduction
Let us consider the quantum dot (QD) in the following setup
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- Metallic lead (N)
- Quantum dot (dot)
- Superconductor (S)
Let us consider the quantum dot (QD) in the following setup:

Physical situation

which is a particular version of SET.
Microscopic model

Since the correlations on the QD are very efficient
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\[
\hat{H}_{QD} = \sum_\sigma \epsilon_d \hat{d}^\dagger \hat{d}_\sigma + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow}
\]
Microscopic model

Since the correlations on the QD are very efficient

\[ \hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}_\sigma^\dagger \, \hat{d}_\sigma + U \, \hat{n}_{d\uparrow} \, \hat{n}_{d\downarrow} \]

they are expected to affect the transport via N-QD-S junction
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\[ \hat{H} = \sum_{\sigma} \epsilon_d \hat{d}_{\sigma}^{\dagger} \hat{d}_{\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} + \hat{H}_N + \hat{H}_S \]

\[ + \sum_{k,\sigma} \sum_{\beta=N,S} \left( V_{k\beta} \hat{d}_{\sigma}^{\dagger} \hat{c}_{k\sigma\beta} + V_{k\beta}^* \hat{c}_{k\sigma,\beta}^{\dagger} \hat{d}_{\sigma} \right) \]
Microscopic model

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$$\hat{H}_{QD} = \sum_\sigma \epsilon_d \hat{d}_\sigma^\dagger \hat{d}_\sigma + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow}$$

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$$+ \sum_{k,\sigma} \sum_{\beta=N,S} \left( V_{k\beta} \hat{d}_\sigma^\dagger \hat{c}_{k\sigma\beta} + V_{k\beta}^* \hat{c}_{k\sigma,\beta}^\dagger \hat{d}_\sigma \right)$$

induced by the external voltage $eV = \mu_N - \mu_S$. 
The underlying physics – part 1
Hybridization of the QD to the metallic lead shows up:
The underlying physics – part 1

Hybridization of the QD to the metallic lead shows up:

\[
\rho_d(\omega) \left[ \frac{1}{\Gamma_N} \right]
\]

\[
\frac{\omega}{\Gamma_N} = 0 \quad \text{Γ}_S = 0 \quad \frac{T}{\Gamma_N} = 1
\]

the charging effect
Hybridization of the QD to the metallic lead shows up:

\[
\rho_d(\omega) \quad \left[ \frac{1}{\Gamma_N} \right]
\]

\[
\Gamma_S = 0 \quad \text{and} \quad \frac{T}{\Gamma_N} = 10^{-1}
\]

the charging effect and ...
Hybridization of the QD to the metallic lead shows up:

\[ \rho_d(\omega) \left[ \frac{1}{\Gamma_N} \right] \]

\[ \frac{\omega}{\Gamma_N} = 0 \]

\[ \frac{T}{\Gamma_N} = 10^{-2} \]

\[ \Gamma_S = 0 \]

The charging effect and ...
The underlying physics – part 1

Hybridization of the QD to the metallic lead shows up:

\[ \rho_d(\omega) \propto \frac{1}{\Gamma_N} \]

\[ \frac{\omega}{\Gamma_N} = \frac{\Gamma_S}{0} = 10^{-3} \]

- the charging effect
- the Kondo effect

at temperatures \( T < T_K \).
Hybridization of the QD to the metallic lead shows up:

\[
\frac{\rho_d(\omega)}{\Gamma_N} = \frac{1}{\Gamma_N} \frac{\Gamma_S = 0}{\Gamma_N = 10^{-4}}
\]

\[\frac{\omega}{\Gamma_N} = \frac{10^{-4}}{10^{-4}}\]

\[\Gamma_S = 0\]

\[T < T_K\]

\[T / \Gamma_N = 10^{-4}\]

\[\Gamma_N / \Gamma_N = 10^{-4}\]

\[\Gamma_S = 0\]

\[\omega / \Gamma_N\]

\[\rho_d(\omega)\]

\[1 / \Gamma_N\]

\[\omega / \Gamma_N\]

\[\Gamma_N / \Gamma_N\]

\[\Gamma_S = 0\]

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\[\rho_d(\omega)\]

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\[\rho_d(\omega)\]

\[1 / \Gamma_N\]
The underlying physics – part 2
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Such induced paring is responsible for the particle-hole splitting.

Effective QD spectrum obtained for $U = 0$, $\varepsilon_d = 0$. 
The relevant questions
a) What kind of interplay occurs between the induced superconductivity and the Kondo effects?
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b) What is their influence on measurable charge current through the N-QD-S junction?
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a) What kind of interplay occurs between the induced superconductivity and the Kondo effects?

Do they cooperate or rather compete?

b) What is their influence on measurable charge current through the N-QD-S junction?

Are there any particular features?
2. Procedure

/ Keldysh formalism & approximations /
Correlations on the QD
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To account for the proximity effect and for correlations we use the matrix Green’s function
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\[
G_d(\tau) = - \begin{pmatrix}
\hat{T}_\tau \langle \hat{d}_\uparrow(\tau)\hat{d}_\uparrow \rangle & \hat{T}_\tau \langle \hat{d}_\uparrow(\tau)\hat{d}_\downarrow \rangle \\
\hat{T}_\tau \langle \hat{d}_\downarrow(\tau)\hat{d}_\uparrow \rangle & \hat{T}_\tau \langle \hat{d}_\downarrow(\tau)\hat{d}_\downarrow \rangle
\end{pmatrix}
\]
Correlations on the QD

To account for the proximity effect and for correlations, we use the matrix Green’s function

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G_d(\tau) = -\left( \begin{array}{ll}
\hat{T}_\tau \langle \hat{d}^\uparrow(\tau)\hat{d}^\dagger \rangle & \hat{T}_\tau \langle \hat{d}^\uparrow(\tau)\hat{d} \rangle \\
\hat{T}_\tau \langle \hat{d}^\dagger(\tau)\hat{d}^\dagger \rangle & \hat{T}_\tau \langle \hat{d}^\dagger(\tau)\hat{d} \rangle
\end{array} \right)
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whose Fourier transform obeys the following Dyson equation
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\hat{T}_\tau \langle \hat{d} \downarrow(\tau) \hat{d}^\uparrow \rangle & \hat{T}_\tau \langle \hat{d} \downarrow(\tau) \hat{d} \downarrow \rangle
\end{pmatrix}
\]

whose Fourier transform obeys the following Dyson equation

\[
G_d(\omega)^{-1} = \begin{pmatrix}
\omega - \varepsilon_d & 0 \\
0 & \omega + \varepsilon_d
\end{pmatrix} - \Sigma_d^0(\omega) - \Sigma_d^U(\omega)
\]
Correlations on the QD

To account for the proximity effect and for correlations we use the matrix Green’s function

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\hat{T}_\tau \langle \hat{d}^\dagger \downarrow(\tau) \hat{d}^\dagger \uparrow \rangle & \hat{T}_\tau \langle \hat{d}^\dagger \downarrow(\tau) \hat{d}^\dagger \downarrow \rangle
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\]

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\end{pmatrix} - \Sigma_0^d(\omega) - \Sigma_U^d(\omega)
\]

where

\[
\Sigma_0^d(\omega) - \text{the selfenergy for } U = 0
\]
Correlations on the QD

To account for the proximity effect and for correlations we use the matrix Green’s function

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G_d(\tau) = - \left( \begin{array}{cc}
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\end{array} \right)
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0 & \omega + \varepsilon_d 
\end{array} \right) - \Sigma_d^0(\omega) - \Sigma_d^U(\omega)
\]

where

\[
\Sigma_d^U(\omega) — \text{correction due to } U \neq 0.
\]
Non-equilibrium phenomena
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Application of external voltage induces the charge current
Application of external voltage induces the charge current

\[ J_N = -e \langle \hat{N}_N \rangle \]
Non-equilibrium phenomena

Application of external voltage induces the charge current

\[ J_{N(S)} = -e\langle \hat{N}_{N(S)} \rangle = - \frac{e}{i\hbar} \langle [\hat{N}_{N(S)}, \hat{H}] \rangle \]
Non-equilibrium phenomena

Application of external voltage induces the charge current

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\]

The current is then expressed by

\[
J_{N(S)} = \frac{ie}{\hbar} \sum_{k,\sigma} V_{k,N} \left( \langle \hat{c}^\dagger_{k,\sigma} \hat{d}_\sigma \rangle - \langle \hat{d}^\dagger_\sigma \hat{c}_{k,\sigma} \rangle \right)
\]
Non-equilibrium phenomena

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Using the Keldysh equation

\[ G^{<} = (1 + G^{r} \Sigma^{r}) (1 + \Sigma^{a} G^{a}) + G^{r} \Sigma^{<} G^{a} \]
Non-equilibrium phenomena

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Using the Keldysh equation

\[ G^< = (1 + G^r \Sigma^r) (1 + \Sigma^a G^a) + G^r \Sigma^< G^a \]

we obtain ...
Non-equilibrium phenomena
Non-equilibrium phenomena

... we obtain that for a small voltage
Non-equilibrium phenomena

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\[ |eV| \ll \Delta \]
Non-equilibrium phenomena

... we obtain that for a small voltage

\[ |eV| \ll \Delta \]

(subgap) current can be expressed by the Landaer-type formula

\[ J(V) = \frac{2e}{\hbar} \int d\omega \ T(\omega) \ [f(\omega + eV, T) - f(\omega - eV, T)] \]
Non-equilibrium phenomena

... we obtain that for a small voltage

$$|eV| \ll \Delta$$

(subgap) current can be expressed by the Landaer-type formula

$$J(V) = \frac{2e}{h} \int d\omega \ T(\omega) \ [f(\omega+eV, T) - f(\omega-eV, T)]$$

where the transmittance

$$T(\omega) = \frac{\Gamma_N^2}{|G_{12}(\omega)|^2}$$
Non-equilibrium phenomena

... we obtain that for a small voltage

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\[
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\]

where the transmittance

\[
T(\omega) = \Gamma^2_N \ |G_{12}(\omega)|^2
\]

depends on the off-diagonal part of \( G(\omega) \).
Schematic illustration

Charge current between the N and S electrodes
We assume the external bias $V$ to be small $|eV| \ll \Delta$. 
Schematic illustration

Charge current between the N and S electrodes

electron
Schematic illustration

Charge current between the N and S electrodes

electron
Schematic illustration

Charge current between the N and S electrodes

electron
Schematic illustration

Charge current between the N and S electrodes

N

S

hole

Cooper pair
Schematic illustration

Charge current between the N and S electrodes

N

S

hole

Cooper pair
Schematic illustration

Charge current between the N and S electrodes

This process is called Andreev reflection.
3. Results
Uncorrelated QD
Uncorrelated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 0$
Uncorrelated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 0$

$\varepsilon_d = 0 \quad \Gamma_S/\Gamma_N = 0.5$
Uncorrelated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 0$

$\varepsilon_d = 0$

$\Gamma_S/\Gamma_N = 1.0$
Uncorrelated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 0$

- $\varepsilon_d = 0$
- $\Gamma_S/\Gamma_N = 2.0$
Uncorrelated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 0$

$\epsilon_d = 0$  $\Gamma_S/\Gamma_N = 5.0$
Uncorrelated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 0$

$\varepsilon_d = 0$  $\Gamma_S/\Gamma_N = 10.0$
The particle-hole splitting is due to superconductivity!
Uncorrelated QD

Andreev conductance $G_A(V)$
Uncorrelated QD

Andreev conductance $G_A(V)$

The case $U = 0$ with $\epsilon_d = 0$
Uncorrelated QD

Andreev conductance $G_A(V)$

The case $U = 0$ with $\epsilon_d = 0$
Uncorrelated QD

Andreev conductance $G_A(V)$

The case $U = 0$ with $\varepsilon_d = 0$

$\Gamma_S/\Gamma_N = 2$
Uncorrelated QD

Andreev conductance $G_A(V)$

The case $U = 0$ with $\epsilon_d = 0$
Uncorrelated QD

Andreev conductance $G_A(V)$

The case $U = 0$ with $\epsilon_d = 0$
The zero-bias conductance is optimal near $\Gamma_S \sim \Gamma_N$!
Strongly correlated QD
Spectral function $\rho_d(\omega)$ obtained for $U = 10\Gamma_N$
Strongly correlated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 10\Gamma_N$

$\Gamma_S/\Gamma_N = 0$

$T = 10^{-3}\Gamma_N$

$\epsilon_d = -1.5\Gamma_N$
Strongly correlated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 10\Gamma_N$

\[ \frac{\rho_d(\omega)}{\Gamma_N} = \frac{10^{-3} \Gamma_N}{\Gamma_N} \quad \text{for} \quad \frac{\epsilon_d}{\Gamma_N} = -1.5 \Gamma_N \]

$\Gamma_S/\Gamma_N = 1$
Strongly correlated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 10\Gamma_N$

\[
\frac{\Gamma}{\Gamma_N} = \frac{\Gamma_S}{\Gamma_N} = 2
\]

$T = 10^{-3} \Gamma_N$

$\varepsilon_d = -1.5 \Gamma_N$
Strongly correlated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 10\Gamma_N$

$\Gamma_S/\Gamma_N = 3$

$T = 10^{-3} \Gamma_N$
$\varepsilon_d = -1.5 \Gamma_N$
Strongly correlated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 10\Gamma_N$

$\rho_d(\omega) \left[ \frac{1}{\Gamma_N} \right]$  

$\omega / \Gamma_N$

$\Gamma_S / \Gamma_N = 4$

$T = 10^{-3} \Gamma_N$

$\epsilon_d = -1.5 \Gamma_N$
Strongly correlated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 10\Gamma_N$

\begin{align*}
\Gamma_S / \Gamma_N &= 5 \\
T &= 10^{-3} \Gamma_N \\
\epsilon_d &= -1.5 \Gamma_N
\end{align*}
Strongly correlated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 10\Gamma_N$

\[ \frac{\omega}{\Gamma_N} \]

$T = 10^{-3} \Gamma_N$

$\varepsilon_d = -1.5 \Gamma_N$

$\Gamma_S/\Gamma_N = 6$
Strongly correlated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 10 \Gamma_N$

$\rho_d(\omega)$ [1/$\Gamma_N$]

$\omega / \Gamma_N$

$\Gamma_S / \Gamma_N = 8$

$T = 10^{-3} \Gamma_N$

$\varepsilon_d = -1.5 \Gamma_N$
Strongly correlated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 10\Gamma_N$

\[\frac{\Gamma_S}{\Gamma_N} = 10\]
Strongly correlated QD

Spectral function $\rho_d(\omega)$ obtained for $U = 10\Gamma_N$

Superconductivity competes with the Kondo effect
Strongly correlated QD

Andreev conductance $G_A(V)$ for: $U = 10\Gamma_N$
Strongly correlated QD

Andreev conductance $G_A(V)$ for: $U = 10\Gamma_N$
Strongly correlated QD

Andreev conductance $G_A(V)$ for: $U = 10 \Gamma_N$

Notice enhancement of the zero-bias Andreev conductance for $\Gamma_S \sim \Gamma_N$!
Strongly correlated QD

Andreev conductance $G_A(V)$ for: $U = 10 \Gamma_N$. 
Andreev conductance $G_A(V)$ for: $U = 10\Gamma_N$.

$T = 10^0 \Gamma_N$
Andreev conductance $G_A(V)$ for: $U = 10\Gamma_N$.

$T = 10^{-1}\Gamma_N$
Strongly correlated QD

Andreev conductance $G_A(V)$ for: $U = 10\Gamma_N$.

$T = 10^{-1}\Gamma_N$
Strongly correlated QD

Andreev conductance $G_A(V)$ for: $U = 10\Gamma_N$.

$G_A(V) \ [4e^2/h]\ eV / \Gamma_N$

$T = 10^{-3} \Gamma_N$
4. Summary
Summary

- QD in a contact with the superconducting lead is converted into the superconducting grain.
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- Coupling of the QD to the metallic leads to formation of the Kondo resonance at \( \omega = 0 \).
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- Subgap current arises for $|eV| < \Delta$ solely from the mechanism of Andreev reflections.
Summary

- QD in a contact with the superconducting lead is converted into the superconducting grain.

- Coupling of the QD to the metallic leads to formation of the Kondo resonance at $\omega = 0$.

- Superconductivity and the Kondo effect compete with one another in quantum dots.

- Subgap current arises for $|eV| < \Delta$ solely from the mechanism of Andreev reflections.

- Kondo effect slightly enhances the zero-bias Andreev conductance when $\Gamma_S \sim \Gamma_N$. 