PTF Kraków, 23 marca 2017 r.

Majoranyzacja elektronów w nadprzewodnikach topologicznych

Tadeusz Domański

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Współautorzy:

Maciek M. Maśka (Uniwersytet Śląski, Katowice)

Janek Barański (Instytut Fizyki PAN, Warszawa)

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J. Bardeen

E. Majorana

Outline:

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- **Majorana madness ...**

Basic notions – on Majorana fermions

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- **P. Dirac (1928)**

$$i\dot{\psi} = (\vec{\alpha}\cdot\vec{p} + \beta m)\,\psi$$

/ relativistic description of fermions /

particles ($E > 0$), anti-particles ($E < 0$)

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$\Rightarrow$ **Basic features:**

chargeless

zero-energy

Searching for majoranas – in particle and nuclear physics

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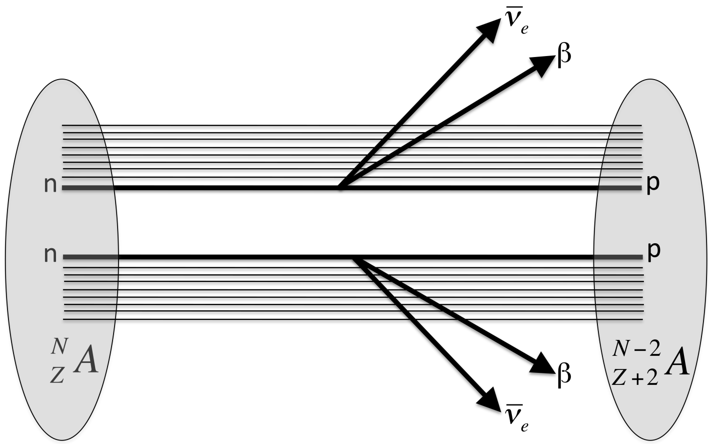
DOUBLE BETA DECAY

Searching for majoranas – **in particle and nuclear physics**

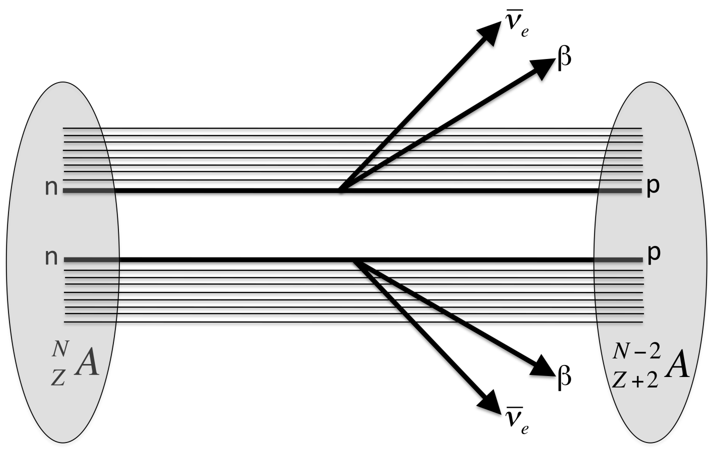
DOUBLE BETA DECAY



$$^N_Z A \longrightarrow {}^{N-2}_{Z+2} A + 2\beta^- + 2\bar{\nu}_e$$

Searching for majoranas – in particle and nuclear physics

DOUBLE BETA DECAY



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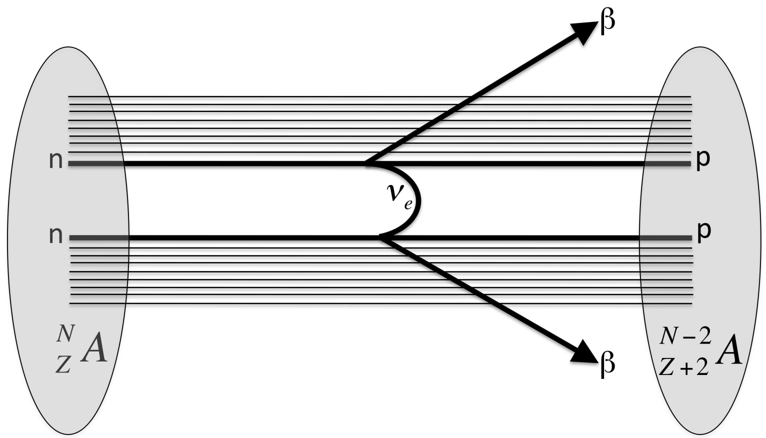

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Searching for majoranas – in particle and nuclear physics

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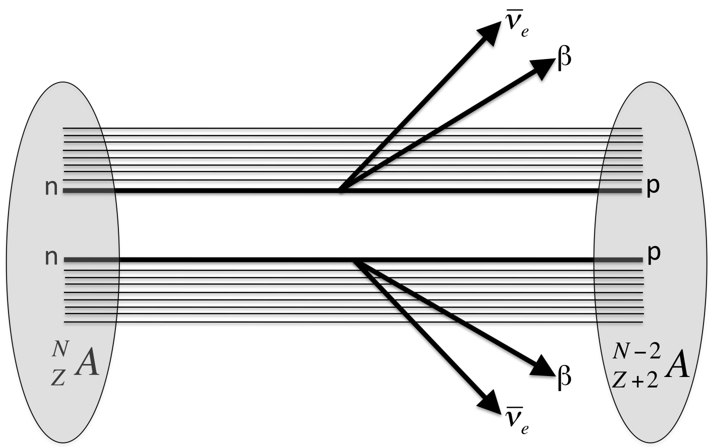
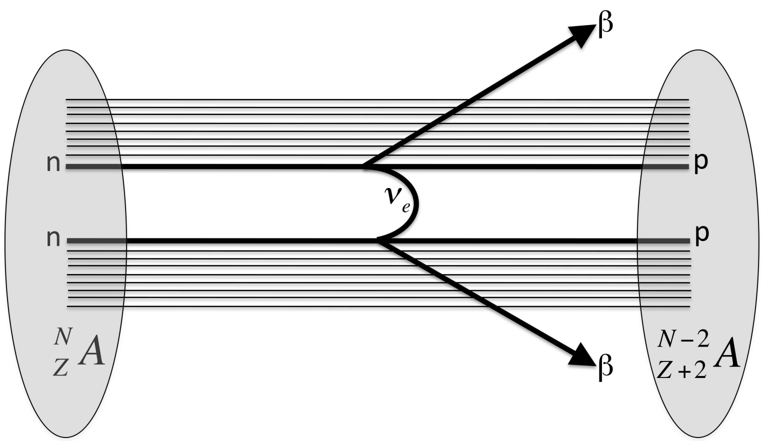


$$ {}^{N}_{Z}A \rightarrow {}^{N-2}_{Z+2}A + 2\beta^- + 2\bar{\nu}_e $$



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This issue is still under dispute.

Quasiparticles – in solids

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★ So, how about <u>majoranions</u> ?

★ Formaly, any usual fermion can be <u>majoranized</u> ...

Majoranization – of normal fermions

- **Normal fermions (e.g. electrons) obey the anticommutation relations**

$$\left\{\hat{c}_i, \hat{c}_j^\dagger\right\} = \delta_{i,j}$$

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- $c_j^{(\dagger)}$ **can be recast in terms of Majorana operators**

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'real' and 'imaginary' parts

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$\hat{\gamma}_{i,n}$ correspond to <u>neutral</u> objects

- **Exotic properties**

$$\hat{\gamma}_{i,n}^\dagger = \hat{\gamma}_{i,n}$$

$$\left\{\hat{\gamma}_{i,n}, \hat{\gamma}_{j,m}^\dagger\right\} = \delta_{i,j}\delta_{n,m}$$

creation = annihilation !

fermionic antisymmetry

Majoranization – of normal fermions

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- Exotic properties (cd)

$$\hat{\gamma}_{i,n}\,\hat{\gamma}_{i,n} = 1/2$$

$$\hat{\gamma}_{i,n}^\dagger\,\hat{\gamma}_{i,n} = 1/2$$

no Pauli principle !

half 'occupied' & half 'empty'

particle = antiparticle – does it make sense ?

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- At the Fermi level $u_{k_F} = v_{k_F} = 1/\sqrt{2}$, and then

$$\hat{\beta}_{k_F\uparrow} \longleftrightarrow \hat{\gamma}_{k_F,1}$$
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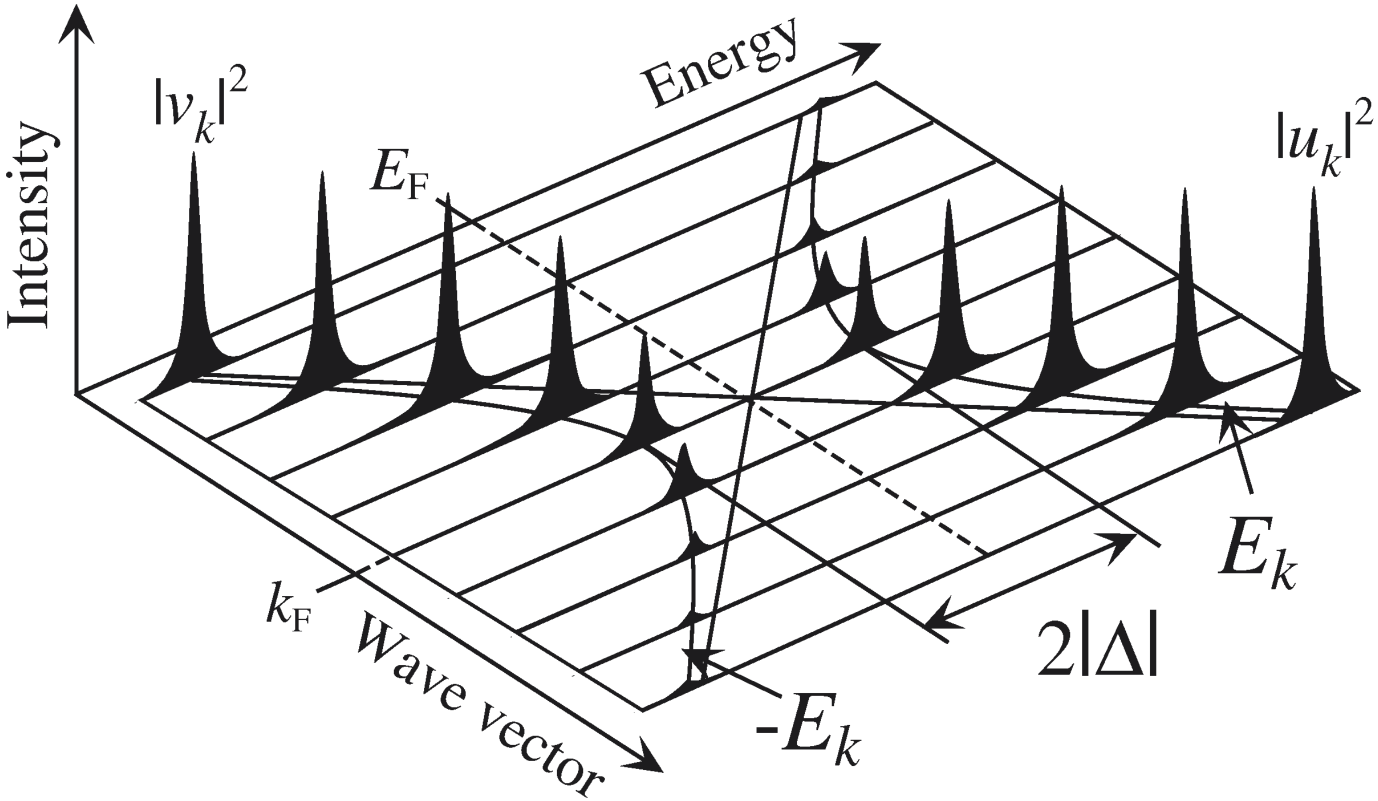
OK, but what about their energy ?

Bogoliubov quasiparticles – in superconductors



Bogoliubov qps of s-wave superconductors are gapped $\pm\sqrt{\varepsilon_k^2 + \Delta^2}$.

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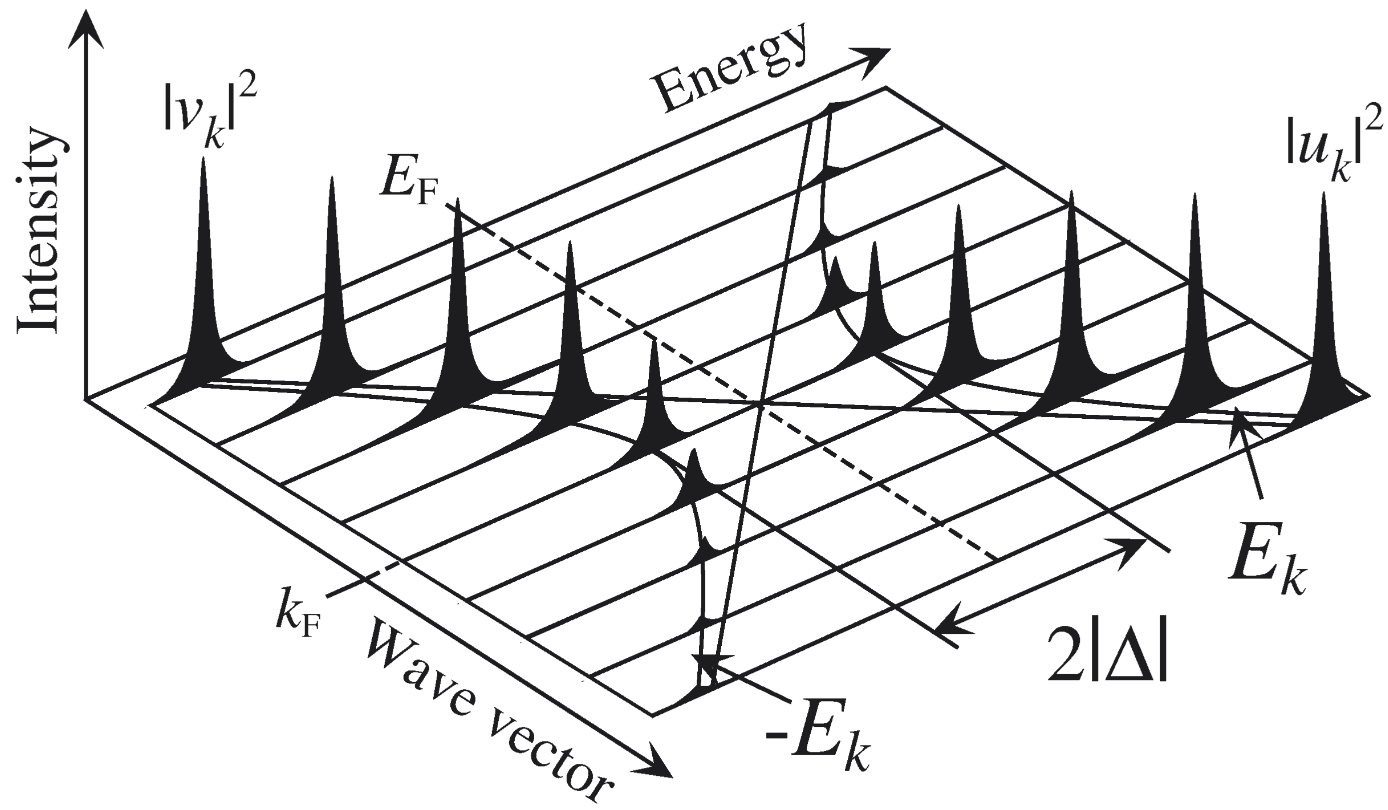

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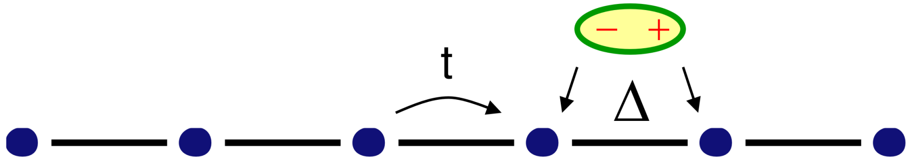
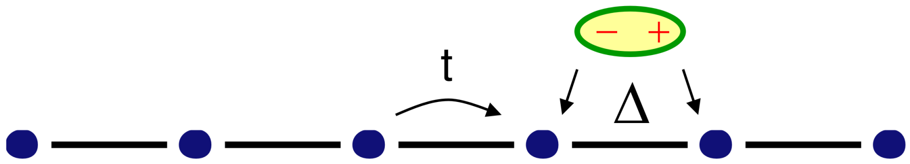
True majoranas have to be the zero energy ($E_k = 0$) quasiparticles !

Kitaev chain – **a paradigm for Majorana modes**



p-wave pairing of spinless 1D fermions

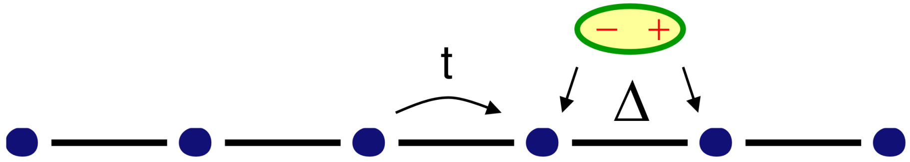
Kitaev chain – a paradigm for Majorana modes



p-wave pairing of spinless 1D fermions

$$\hat{H} = t\sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1} + \text{h.c.}\right) - \mu \sum_i \hat{c}_i^\dagger \hat{c}_i + \Delta \sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1}^\dagger + \text{h.c.}\right)$$

Kitaev chain – a paradigm for Majorana modes



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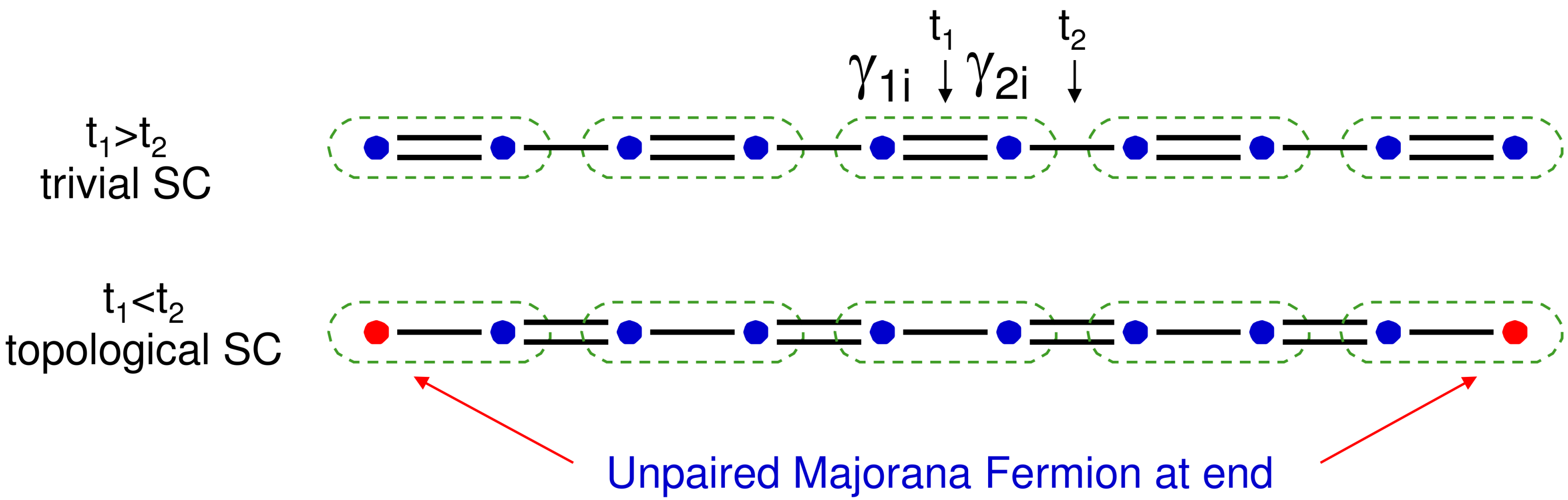
This toy-model can be exactly solved in Majorana basis. For $\Delta = t$ one obtains:



$t_1 \equiv \mu$

$t_2 \equiv 2t$

Kitaev toy model

/ Phys. Usp. 44, 131 (2001) /

★ **In the special case $\Delta = t$ and $|\mu| < 2t$**

operators $\hat{\gamma}_{1,1}$ and $\hat{\gamma}_{2,N}$ are decoupled from all the rest. This implies

zero-energy modes appearing at the chain edges

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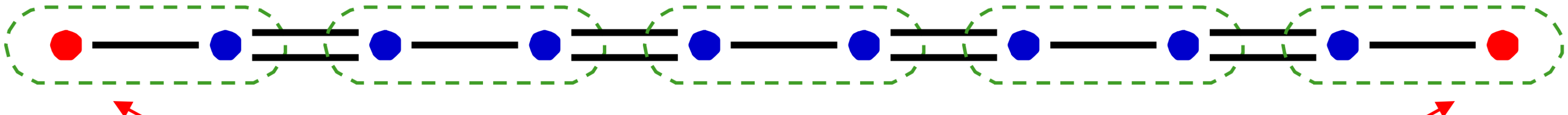

Unpaired Majorana Fermion at end

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zero-energy modes appearing at the chain edges

★ **Similar ideas have been considered for 1D Heisenberg chain of 1/2 spins**

$$\bullet\!-\!\bullet \;=\; \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

F.D.M. Haldane, Phys. Rev. Lett. 50, 1153 (1983)

Nobel Prize, 2016

Various scenarios – for Majorana quasiparticles

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- **vortex states in p-wave superconductors**

  Volovik (1999)

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- **magnetic atoms chain on superconducting substrate**

  Choy *et al* (2011); Martin & Morpugo (2012); Nadj-Perge *et al* (2013)

Experimental evidence – for Majorana quasiparticles

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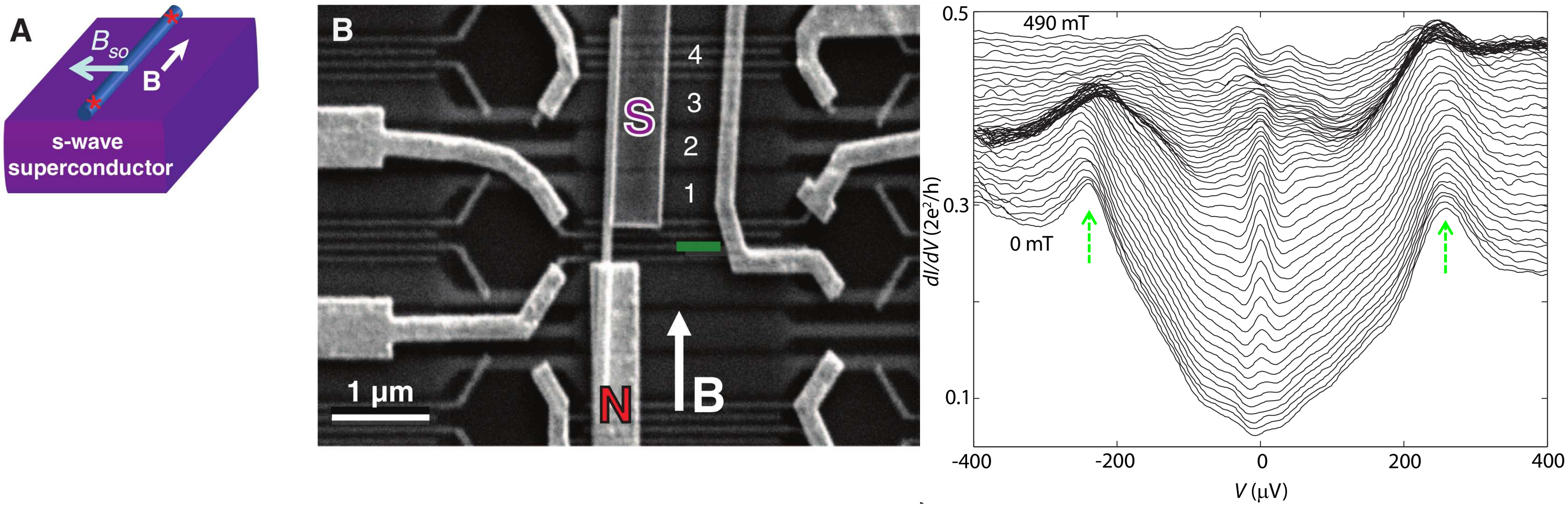
InSb nanowire between a metal (gold) and a superconductor (Nb-Ti-N)



dI/dV **measured at 70 mK for varying magnetic field B indicated:**

$\Rightarrow$ **a zero-bias enhancement due to Majorana state**

V. Mourik, ..., and L.P. Kouwenhoven, Science **336**, 1003 (2012).

/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /

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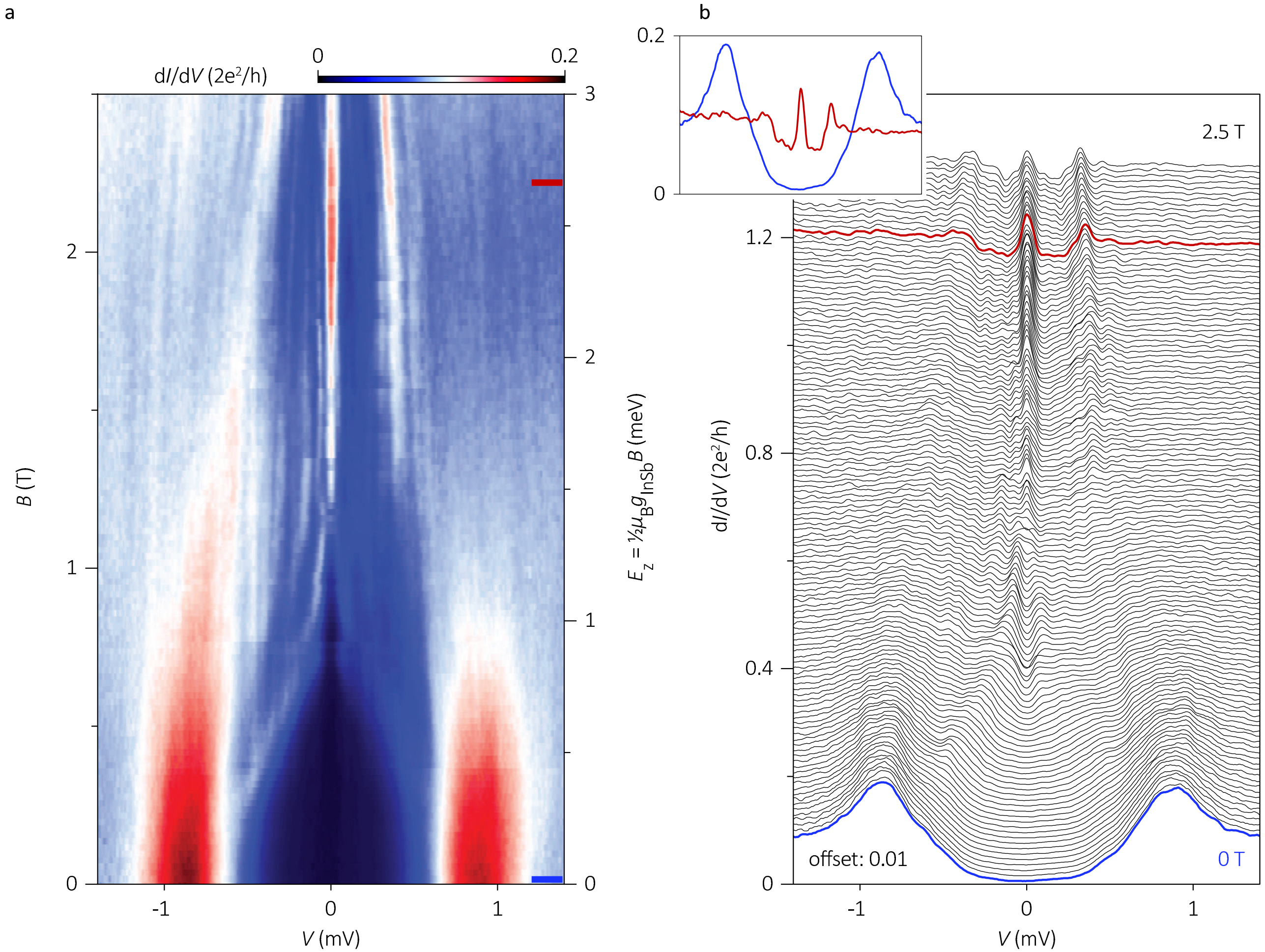
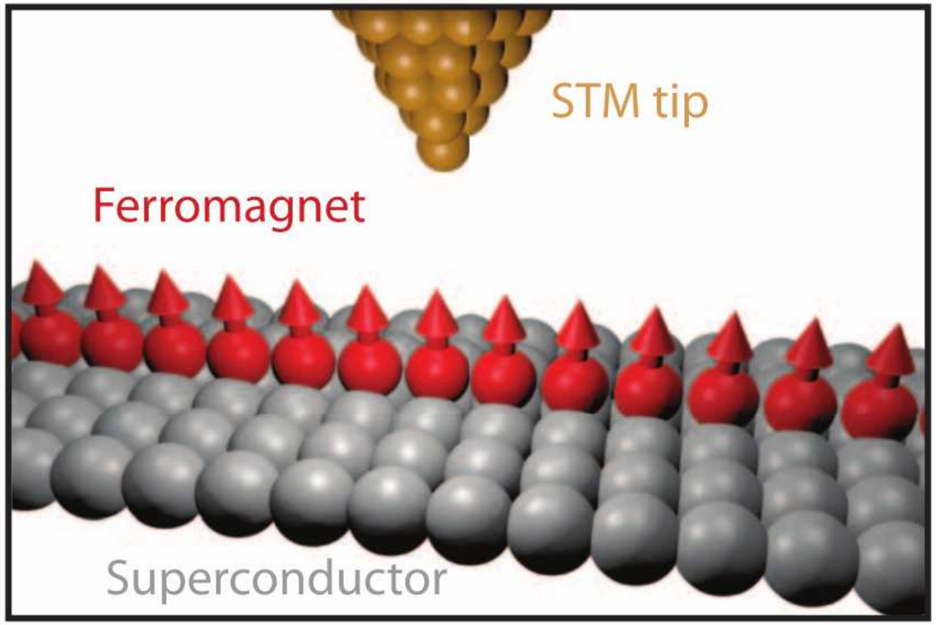


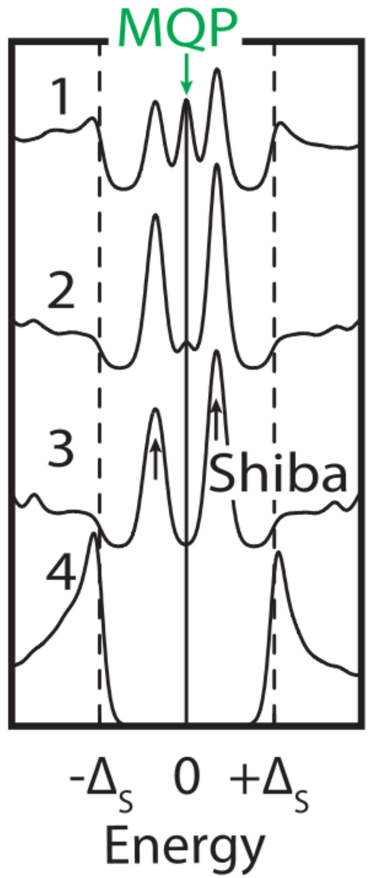
H. Zhang, ..., and L.P. Kouwenhoven, arXiv:1603.04069 (2016).

/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /

Experimental evidence – **for Majorana quasiparticles**

A chain of iron atoms deposited on a surface of superconducting lead





STM measurements provided evidence for:

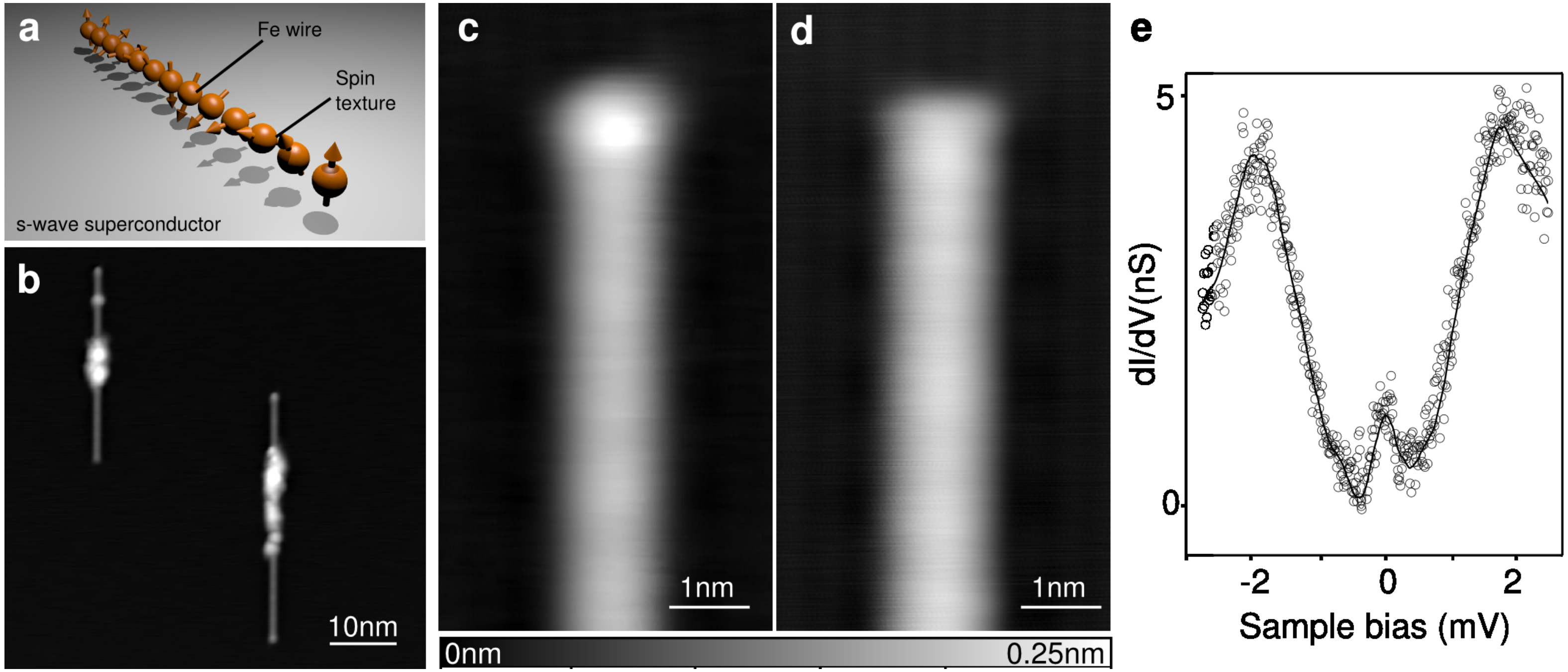
$\Rightarrow$ **Majorana bound states at the edges of a chain.**

S. Nadj-Perge, ..., and A. Yazdani, Science **346**, 602 (2014).

/ Princeton University, Princeton (NJ), USA /

Experimental evidence – **for Majorana quasiparticles**

Self-assembled Fe chain on superconducting Pb(110) surface



AFM combined with STM provided evidence for:

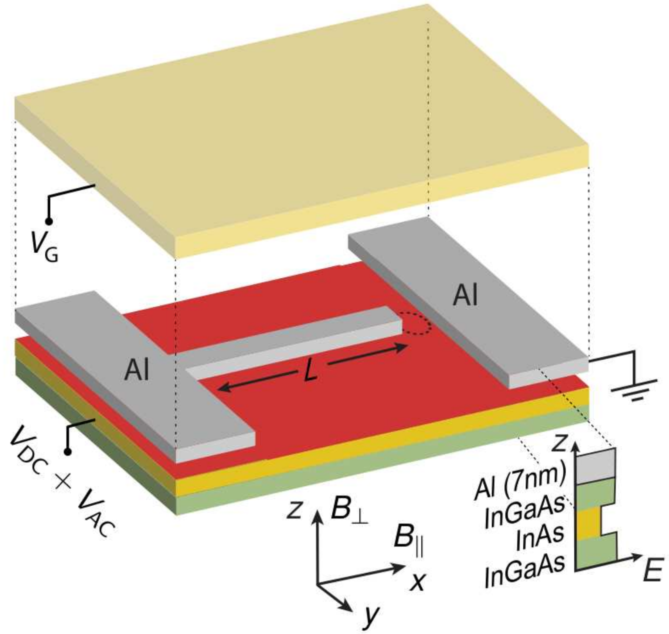
⇒ **Majorana bound states at the edges of a chain.**

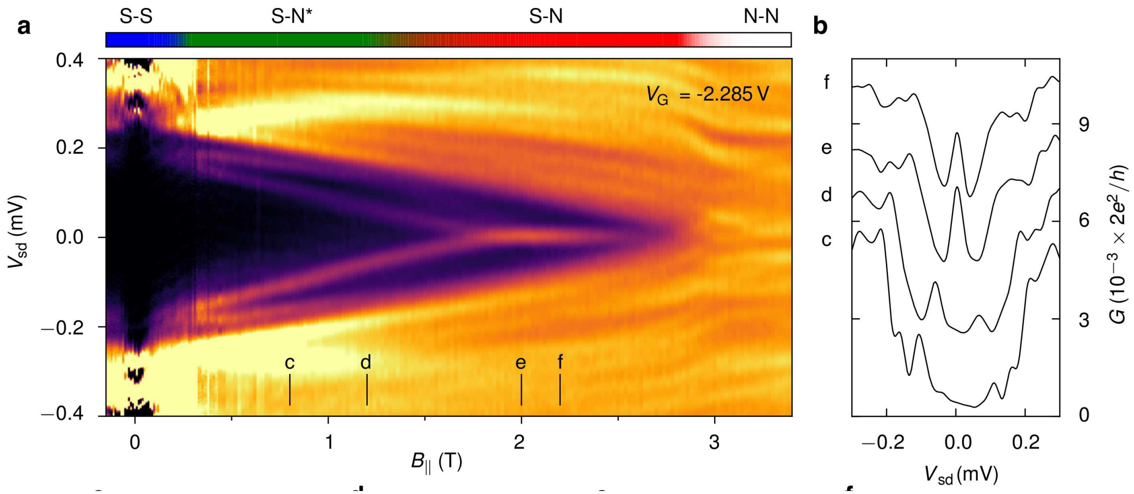
R. Pawlak, M. Kisiel *et al*, npj Quantum Information **2**, 16035 (2016).

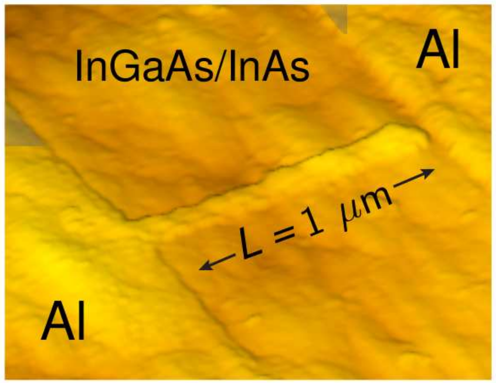
/ University of Basel, Switzerland /

Experimental evidence – for Majorana quasiparticles

Wire-like device constructed lithographically







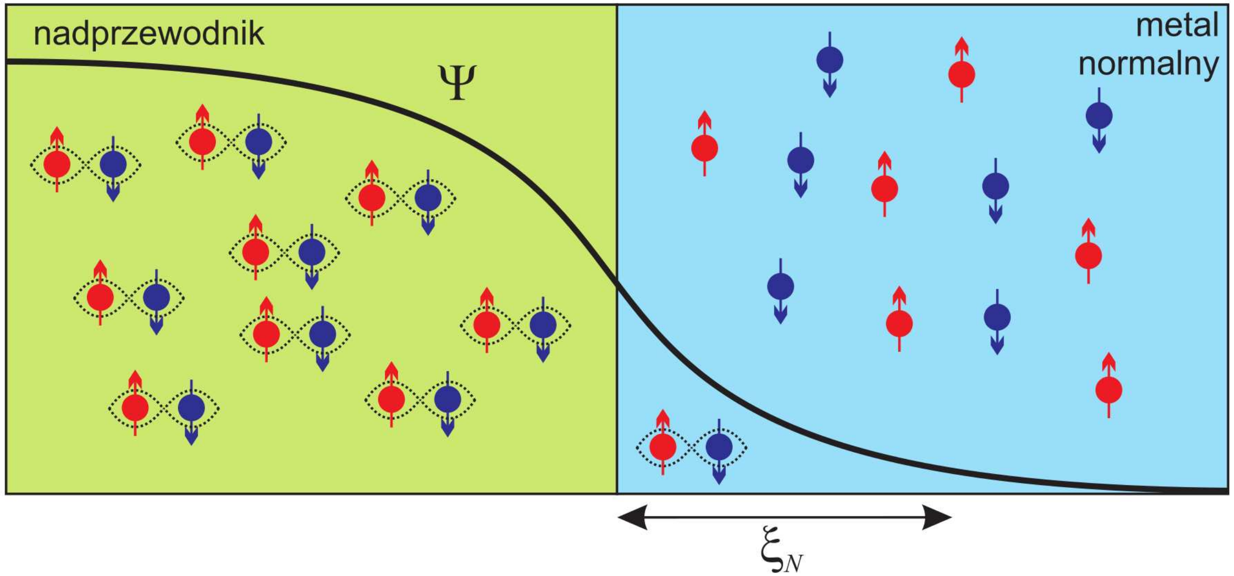
H.J. Suominen *et al*, arXiv:1703.03699 (2017).

/ University of Copenhagen, Denmark /

2. Electron pairing in nanosystems

Superconductivity in nanosystems

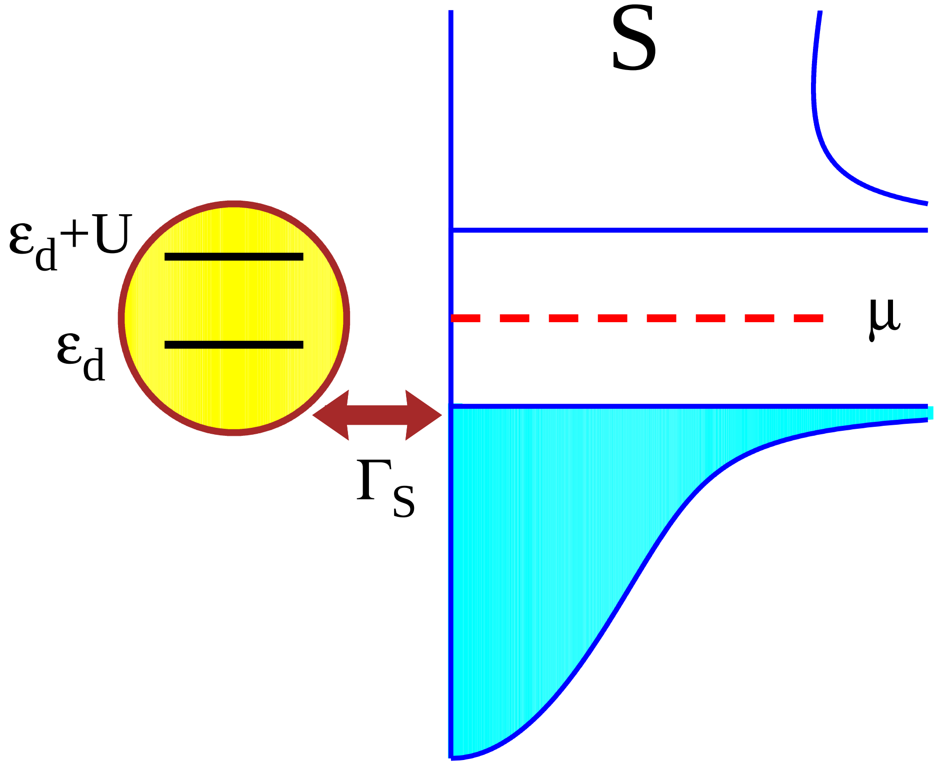
Any material contacted with a bulk superconductor absorbs the Cooper pairs



Size of the quantum dots is usually smaller than ξ_N

Prototype model – single Anderson impurity

The single quantum impurity (dot) coupled to superconducting reservoir



ε_d – energy level, U – Coulomb potential, Γ_S – hybridization

Microscopic model

Anderson-type Hamiltonian

Quantum impurity (dot)

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U \; \hat{n}_{d\uparrow} \; \hat{n}_{d\downarrow}$$

coupled with a superconductor

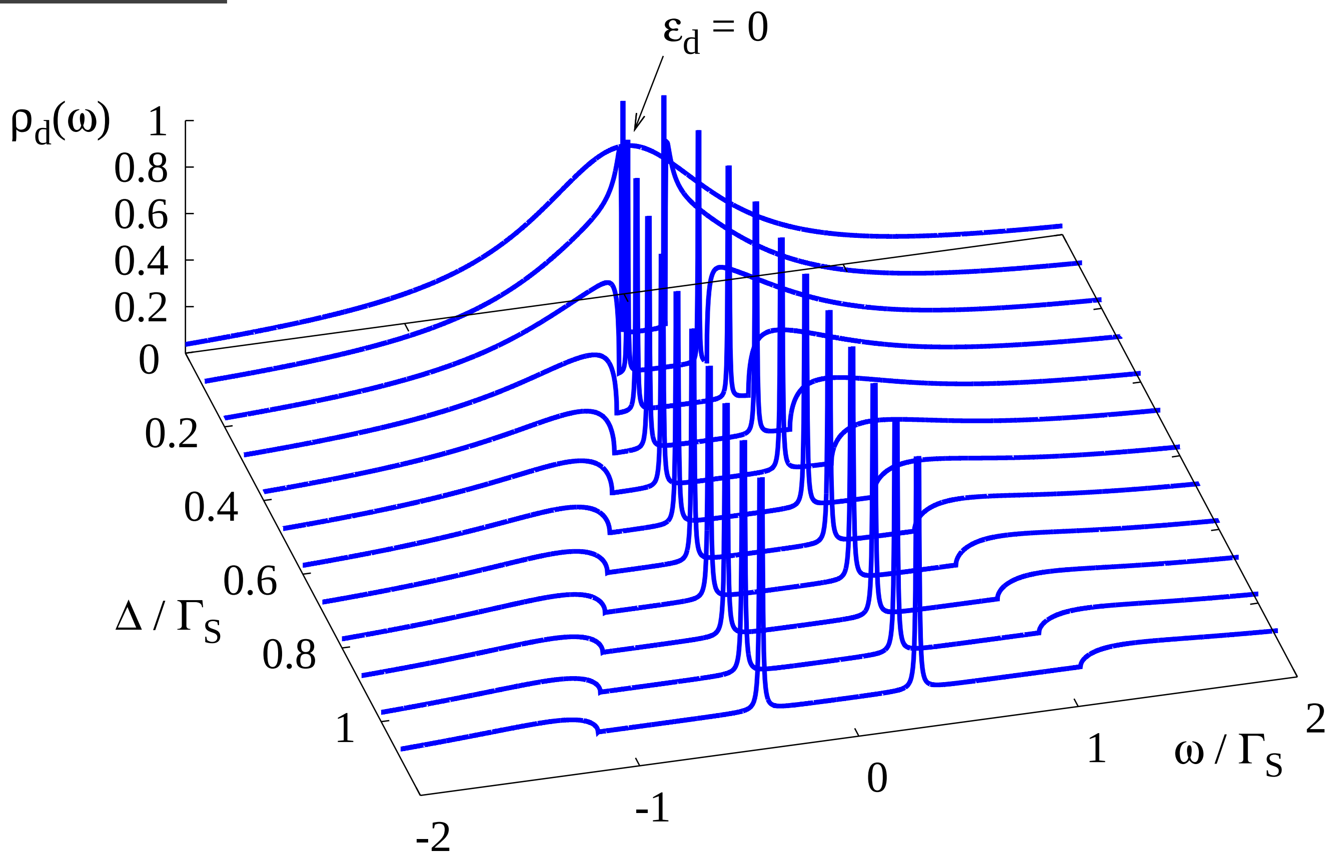
$$\begin{aligned} \hat{H} \;&=\; \sum_{\sigma} \epsilon_d \hat{d}^{\dagger}_{\sigma} \hat{d}_{\sigma} + U \; \hat{n}_{d\uparrow} \; \hat{n}_{d\downarrow} + \hat{H}_S \\ &+\; \sum_{\mathbf{k},\sigma} \left(V_{\mathbf{k}} \; \hat{d}^{\dagger}_{\sigma} \hat{c}_{\mathbf{k}\sigma} + V^{*}_{\mathbf{k}} \; \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{d}_{\sigma} \right) \end{aligned}$$

where

$$\hat{H}_S = \sum_{k,\sigma} \left(\varepsilon_k - \mu\right) \hat{c}^{\dagger}_{k\sigma} \hat{c}_{k\sigma} - \sum_{k} \left(\Delta \hat{c}^{\dagger}_{k\uparrow} \; \hat{c}^{\dagger}_{k\downarrow} + \mathrm{h.c.} \right)$$

Uncorrelated QD

$U_d = 0$ (exactly solvable case)



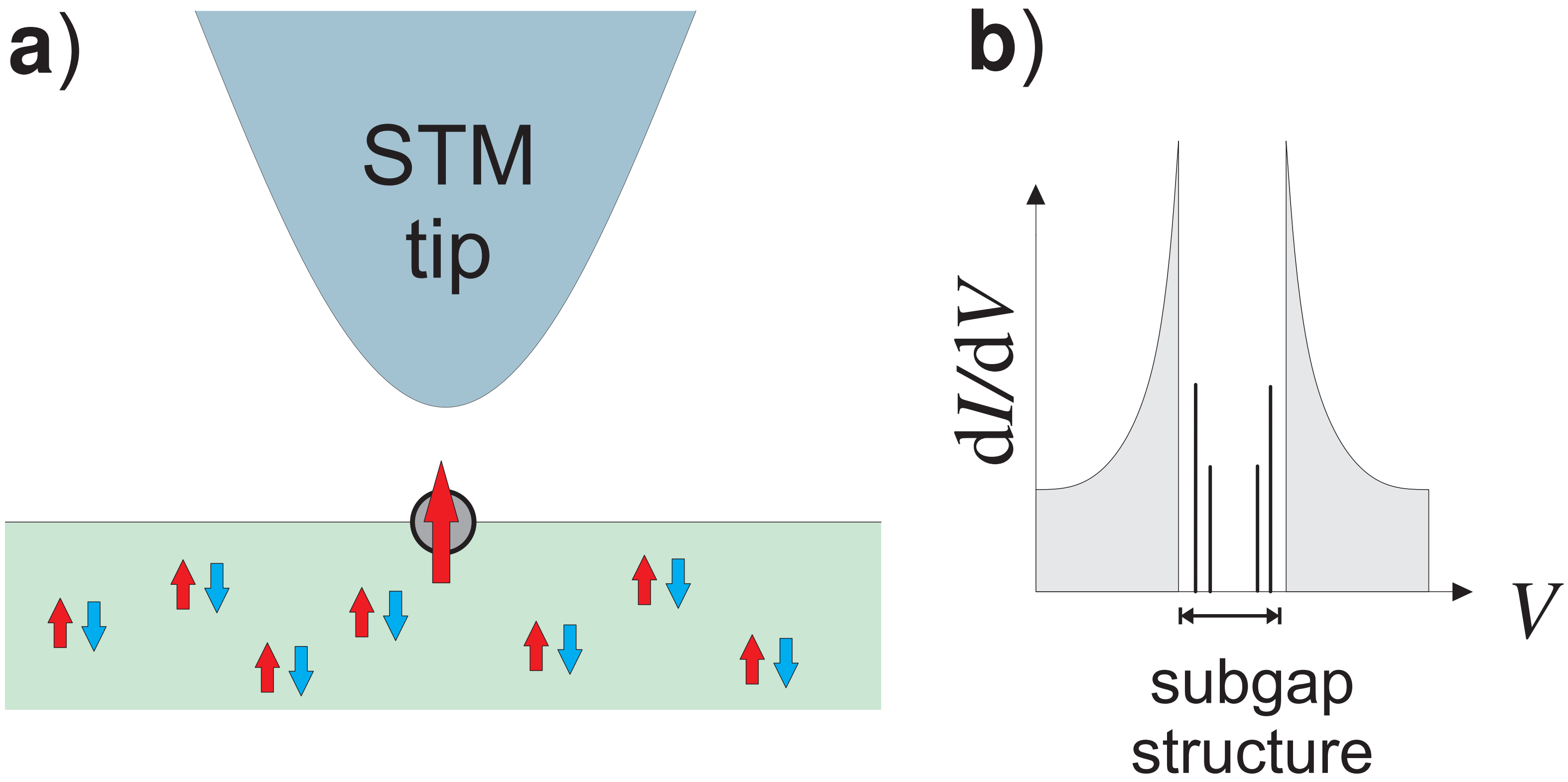
In-gap (Andreev/Shiba) bound states :

⇒ always appear in pairs,

⇒ appear symmetrically at finite energies.

J. Barański and T. Domański, J. Phys.: Condens. Matter **25**, 435305 (2013).

Subgap states of multilevel quantum impurities



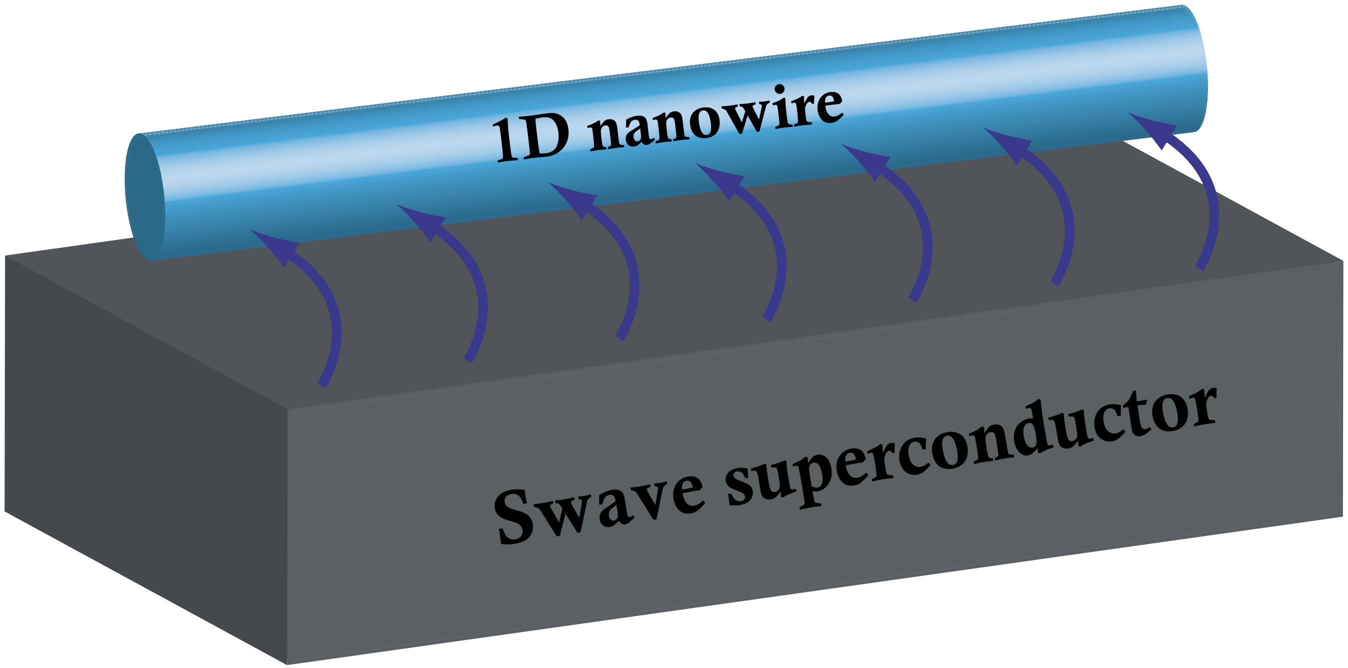
a) STM scheme and b) differential conductance for multilevel quantum impurity adsorbed on surface of bulk superconductor.

R. Žitko, O. Bodensiek, and T. Pruschke, Phys. Rev. B **83**, 054512 (2011).

Andreev vs Majorana states – 'A story of mutation'

Andreev vs Majorana states – 'A story of mutation'

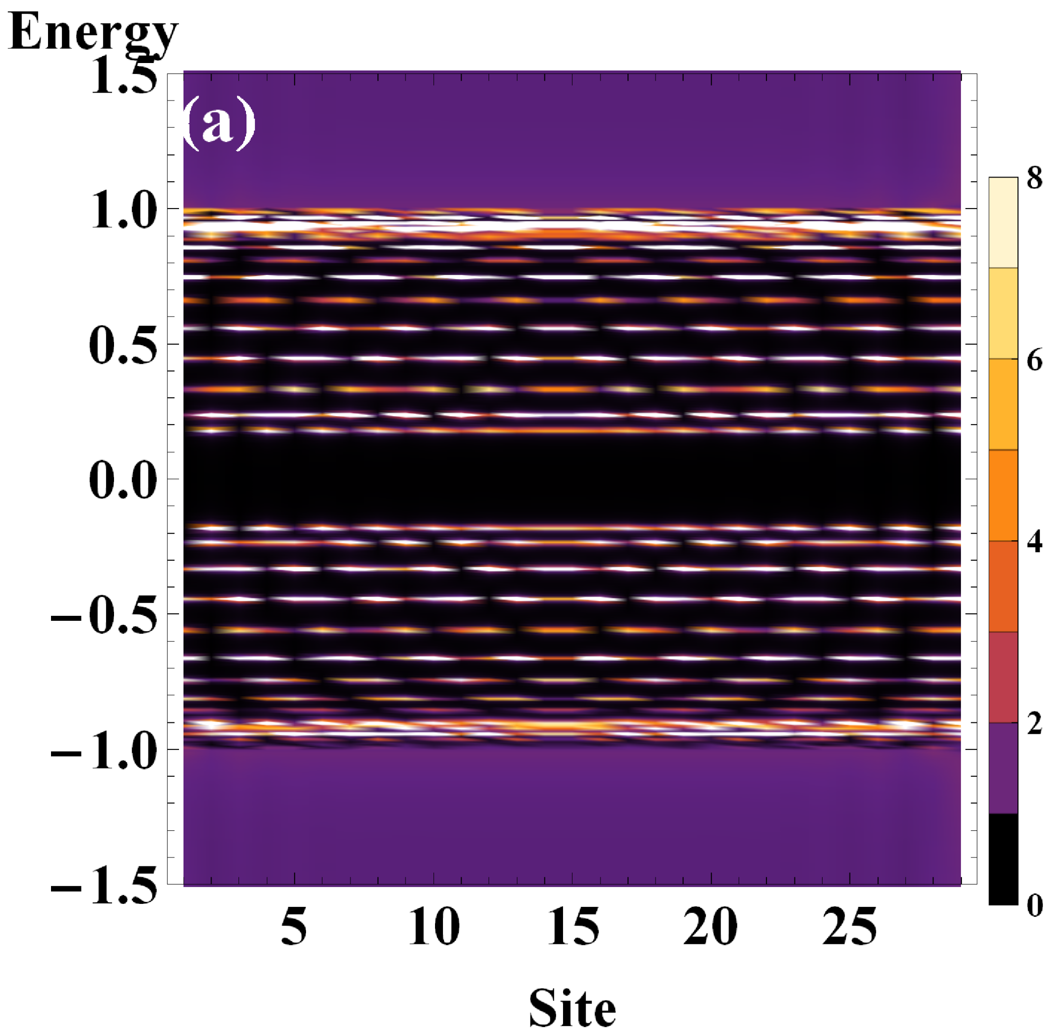
Let us consider:



1D quantum wire deposited on s-wave superconductor

D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B **88**, *165401 (2013).*

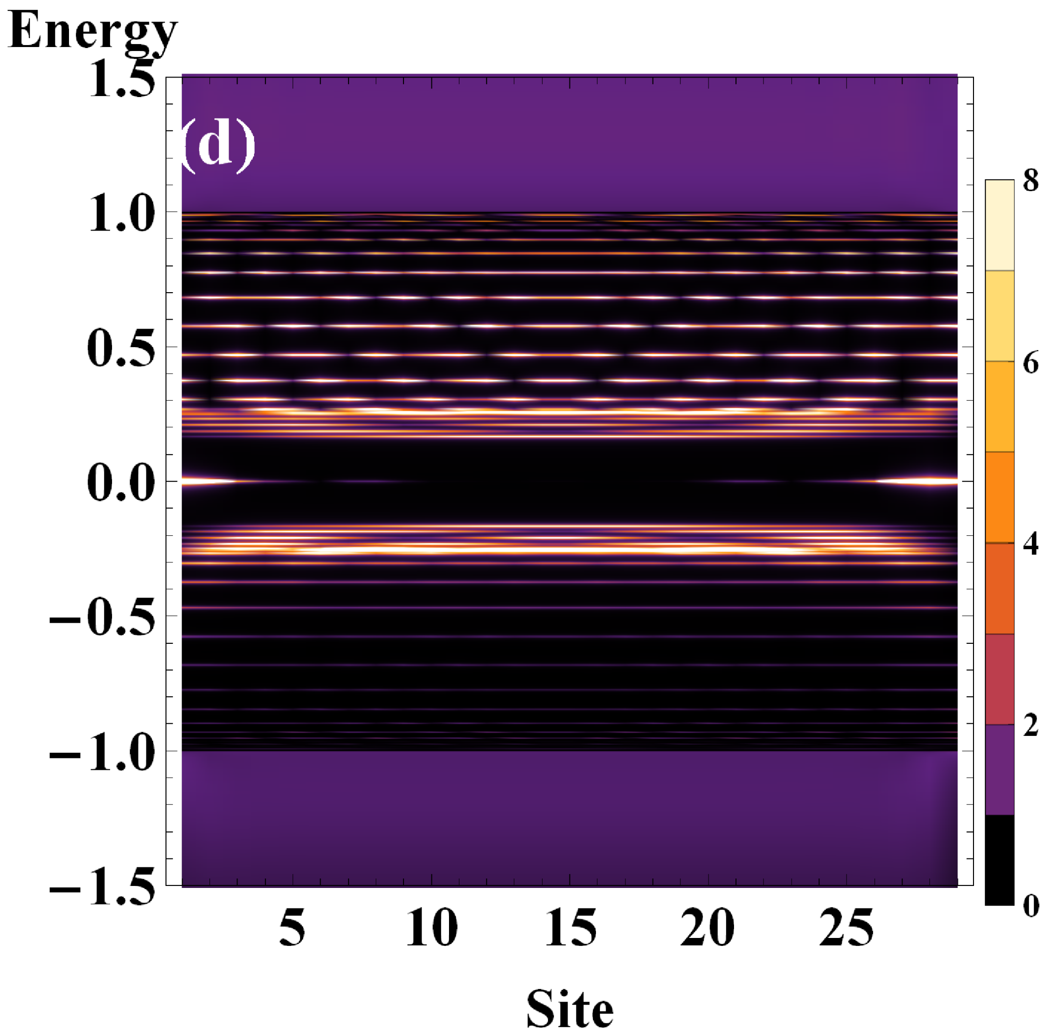
Andreev vs Majorana states – 'A story of mutation'



Electronic spectrum comprises a series of Andreev states.

D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B **88***, 165401 (2013).*

Andreev vs Majorana states – 'A story of mutation'



Spin-orbit + Zeeman interactions induce the Majorana edge modes.

D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B ***88****, 165401 (2013).*

Towards more realistic situation

/ Rashba chain + pairing /

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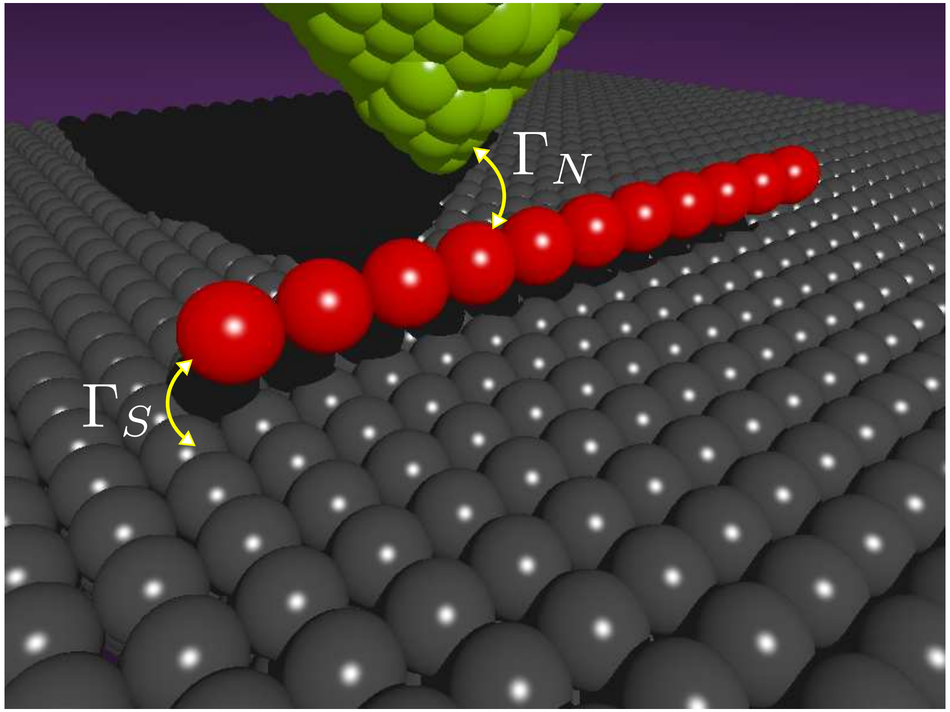
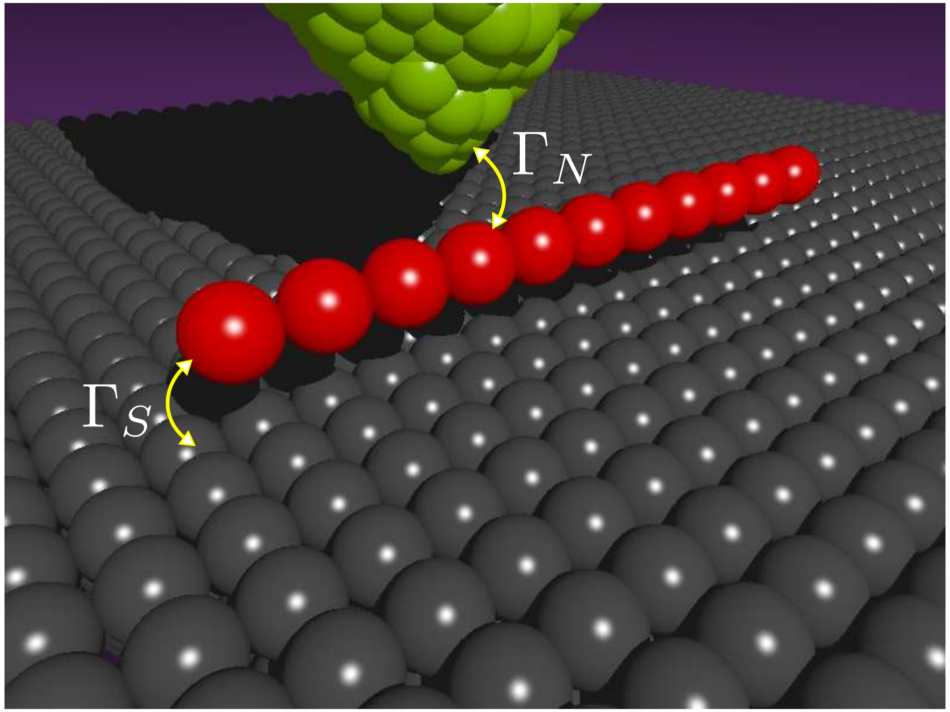


Scheme of STM configuration

Towards more realistic situation

/ Rashba chain + pairing /



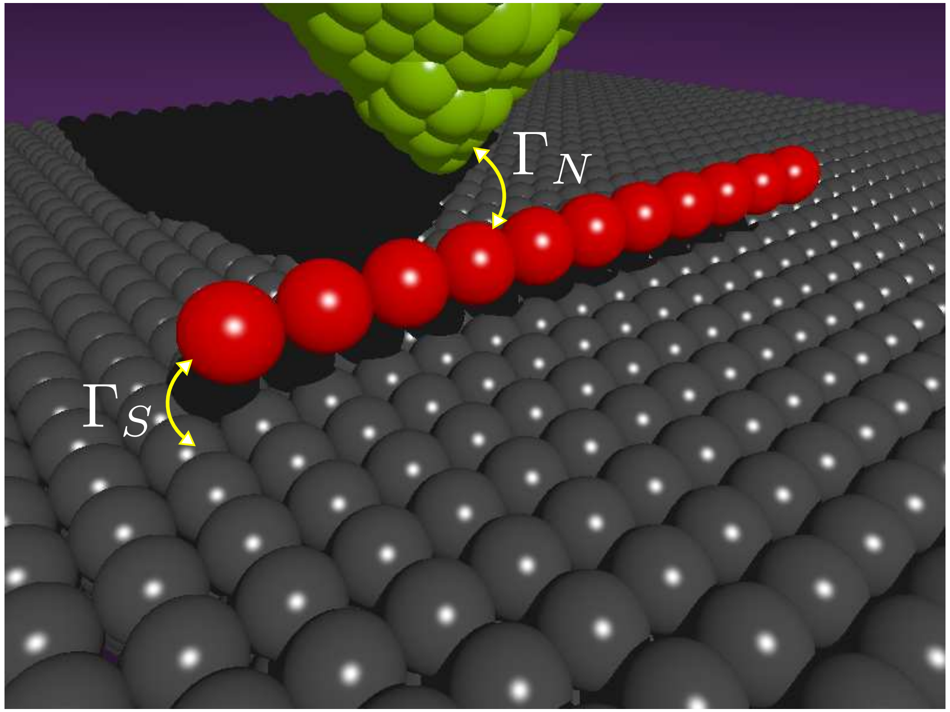
Scheme of STM configuration

$$\hat{H} = \hat{H}_{tip} + \hat{H}_{chain} + \hat{H}_S + \hat{V}_{hybr}$$

We studied this model, focusing on the deep subgap regime $|E| \ll \Delta_{sc}$.

Towards more realistic situation

/ Rashba chain + pairing /



Scheme of STM configuration

where

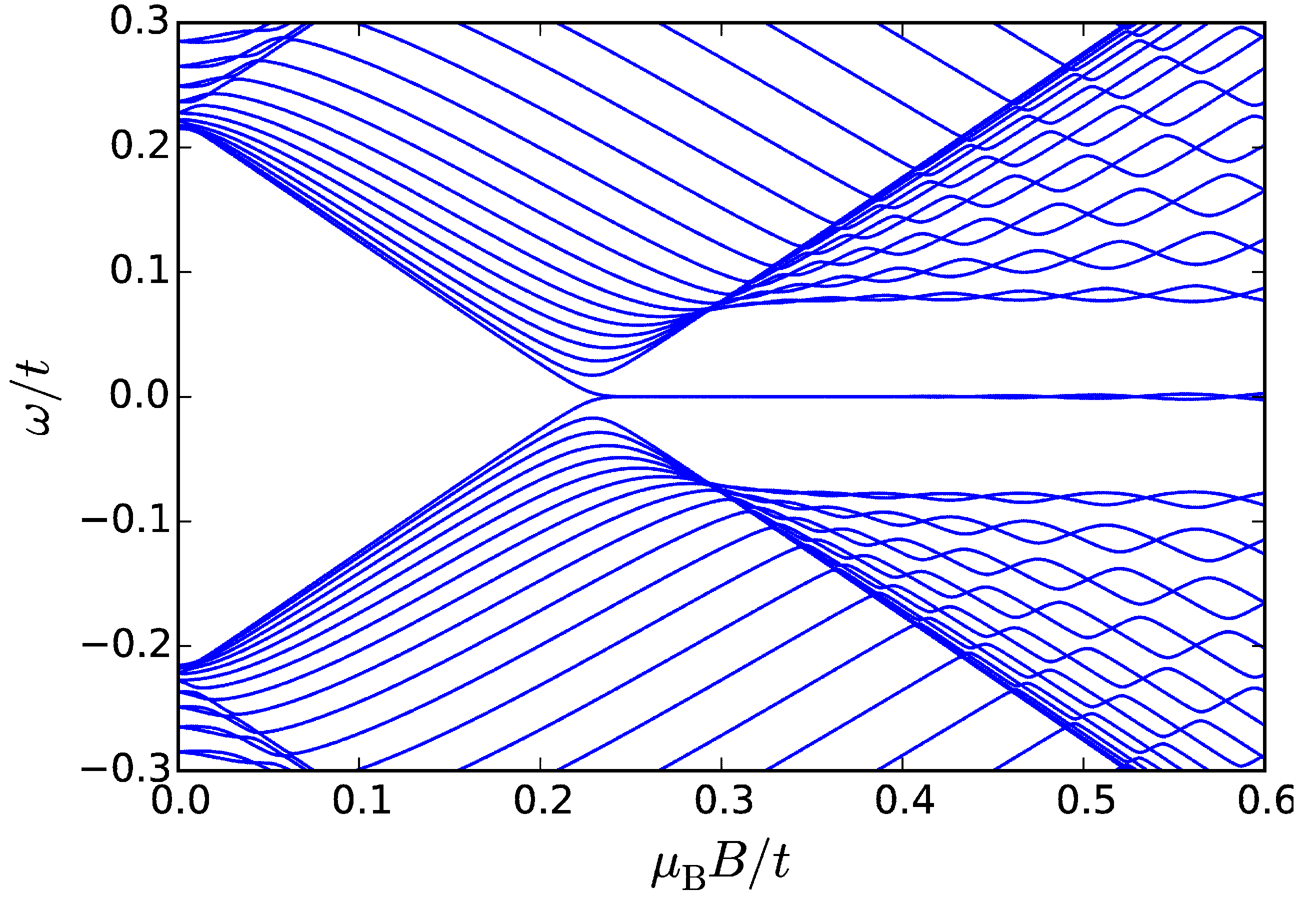
$$\hat{H}_{chain} = \sum_{i,j,\sigma} (t_{ij} - \delta_{ij}\mu) \hat{d}^{\dagger}_{i,\sigma} \hat{d}_{j,\sigma} + \hat{H}_{Rashba} + \hat{H}_{Zeeman}$$

M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, Phys. Rev. B **95**, *045429 (2017).*

Majorana states – of the Rashba chain

Majorana states – of the Rashba chain

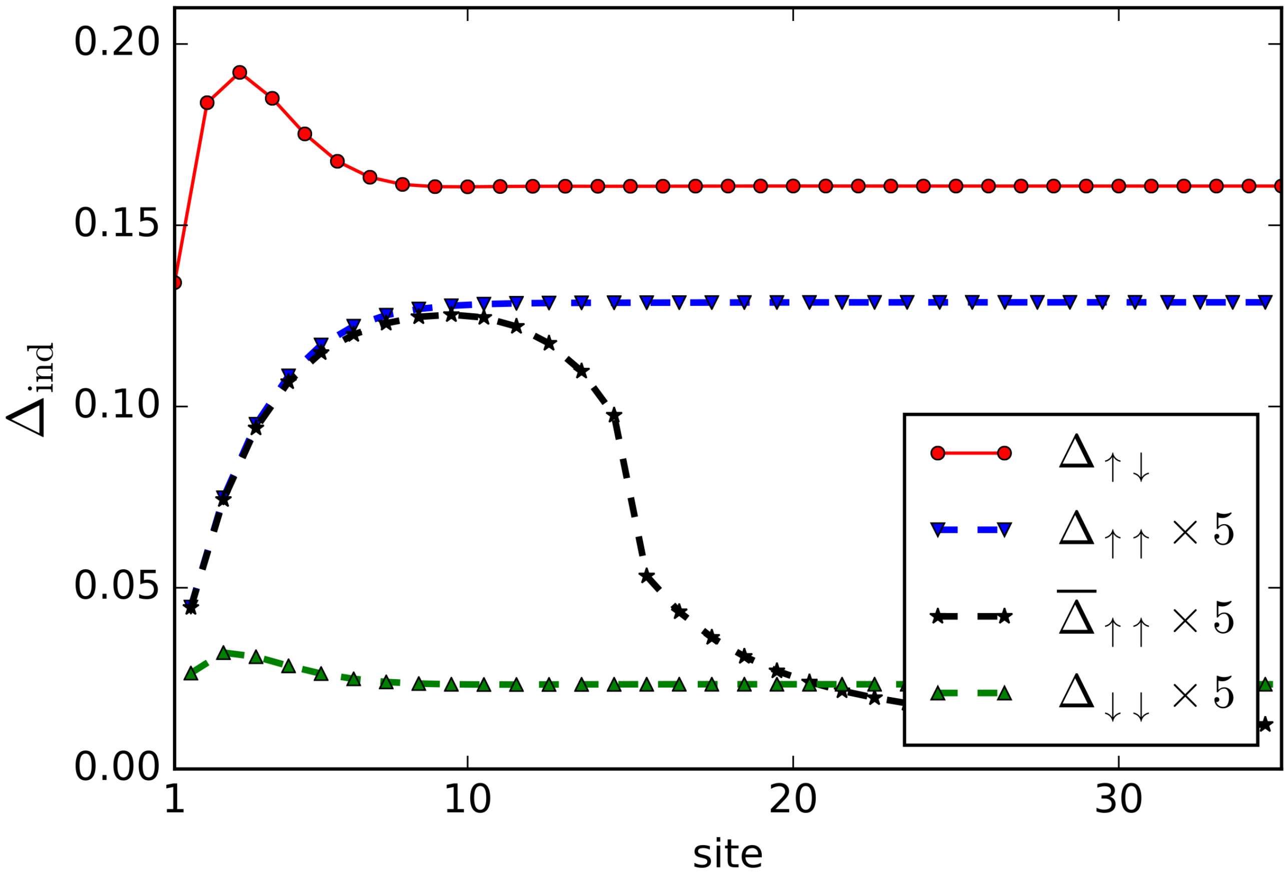
Mutation of Andreev states into zero-energy (Majorana) mode



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, Phys. Rev. B **95***, 045429 (2017).*

Majorana states – of the Rashba chain

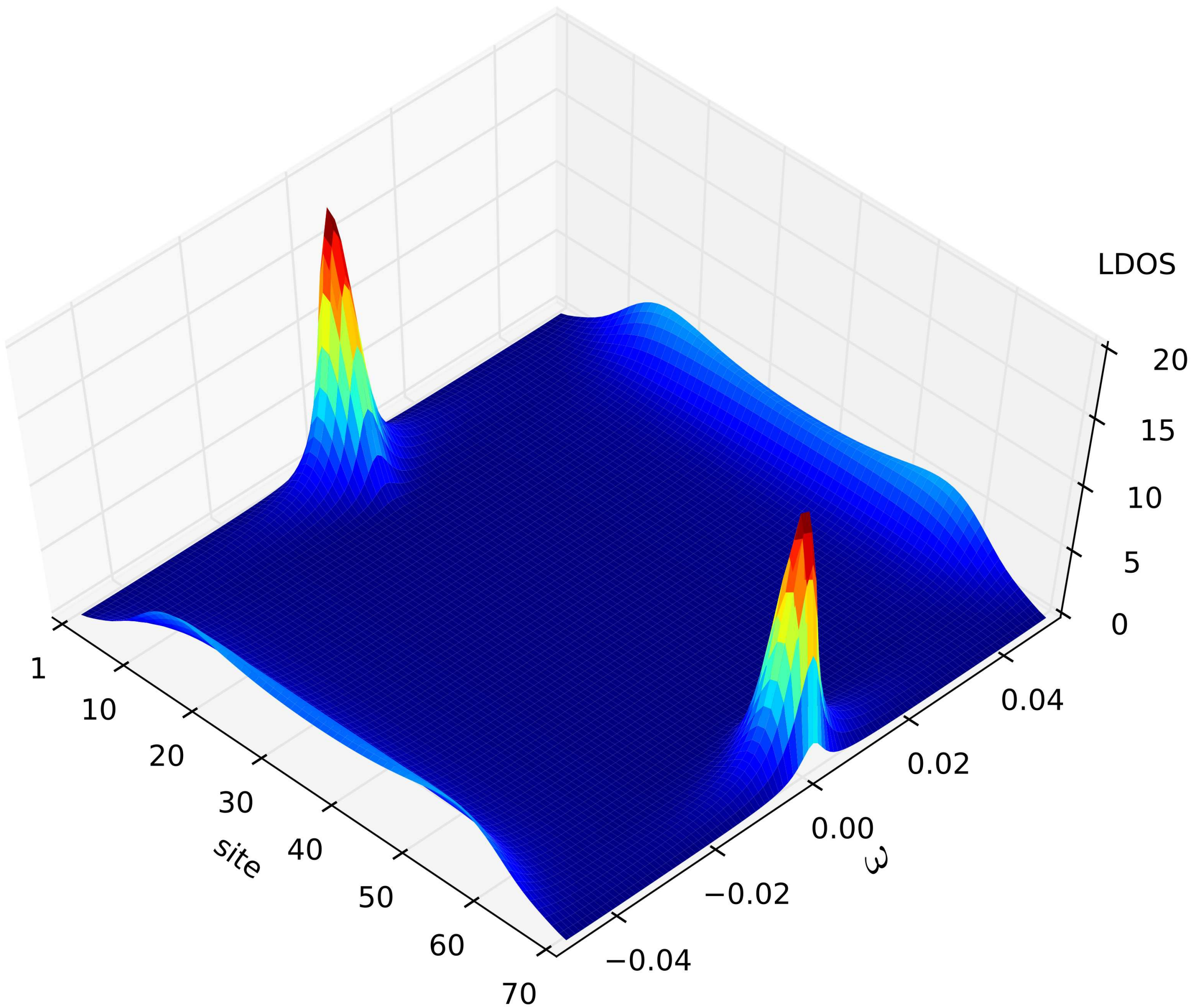
Spatial variation of the induced pairings $\Delta_{\sigma,\sigma'} = \langle \hat{d}_{i,\sigma} \hat{d}_{i+1,\sigma'} \rangle$



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, Phys. Rev. B **95***, 045429 (2017).*

Majorana states – of the Rashba chain

Spectrum with the edge Majorana quasiparticles



Breaking the Rashba chain

by a reduced hopping at site $i = N/2$

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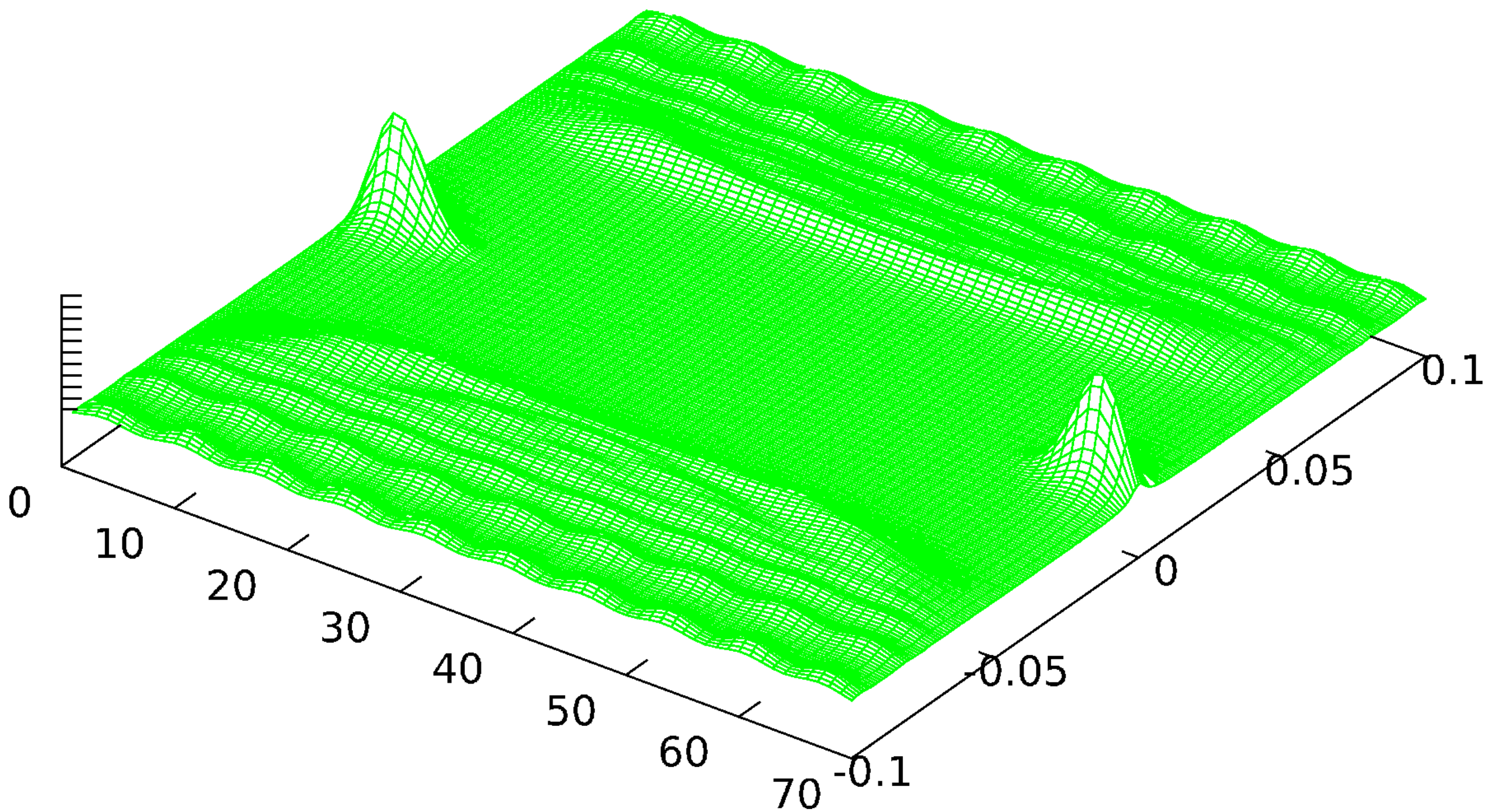
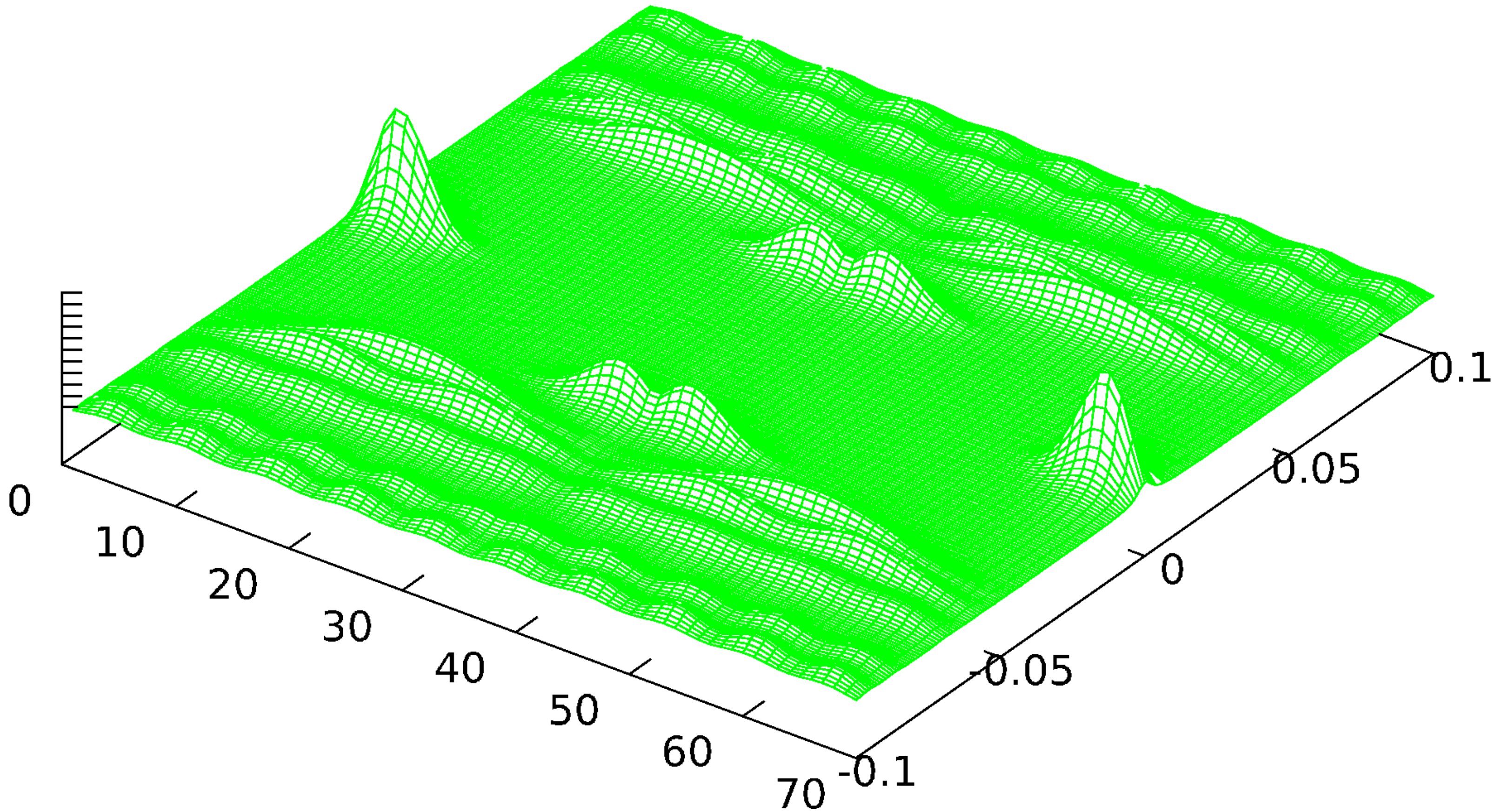


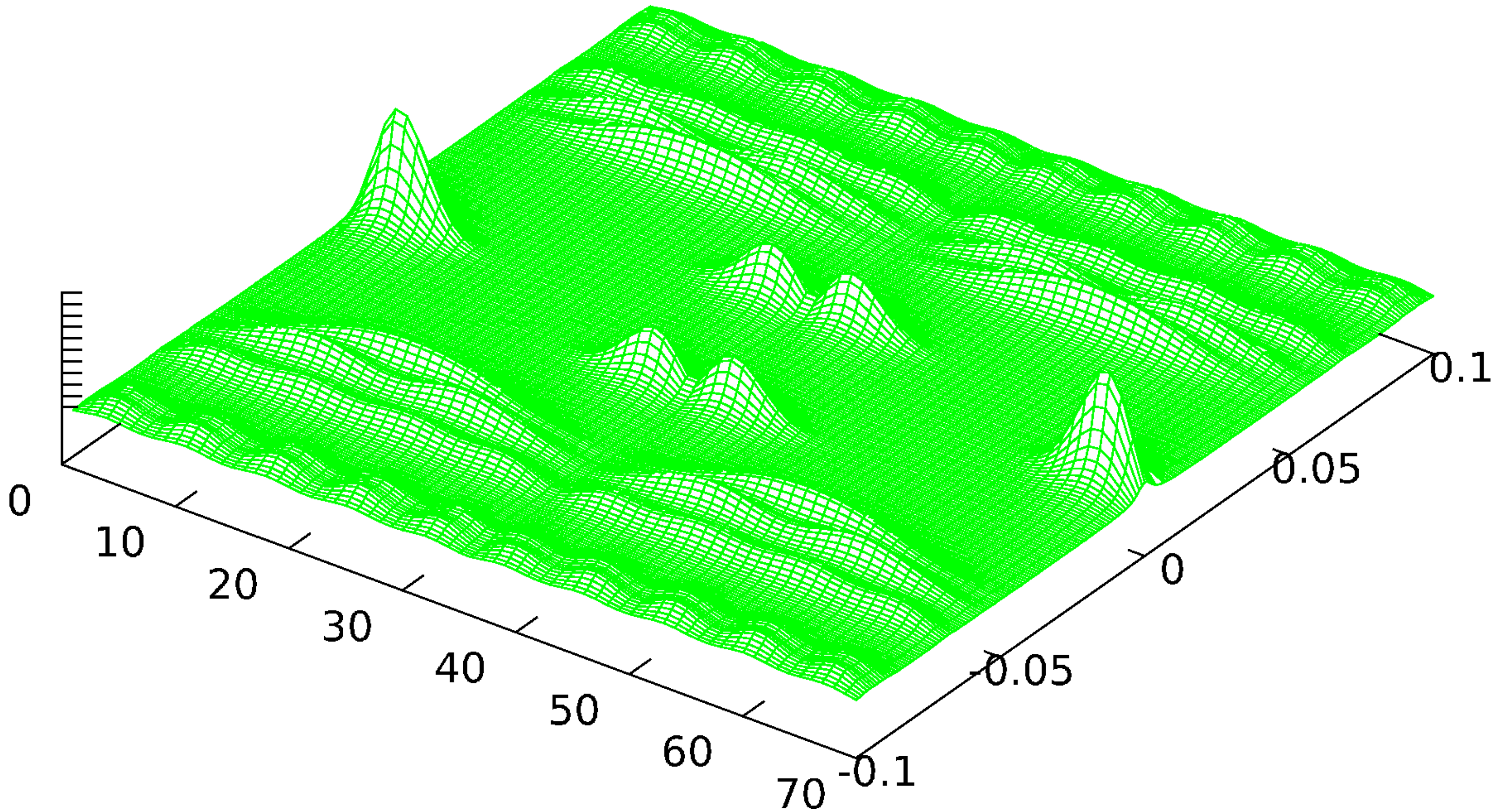
$t_i/t = 1$

Breaking the Rashba chain by a reduced hopping at site $i = N/2$



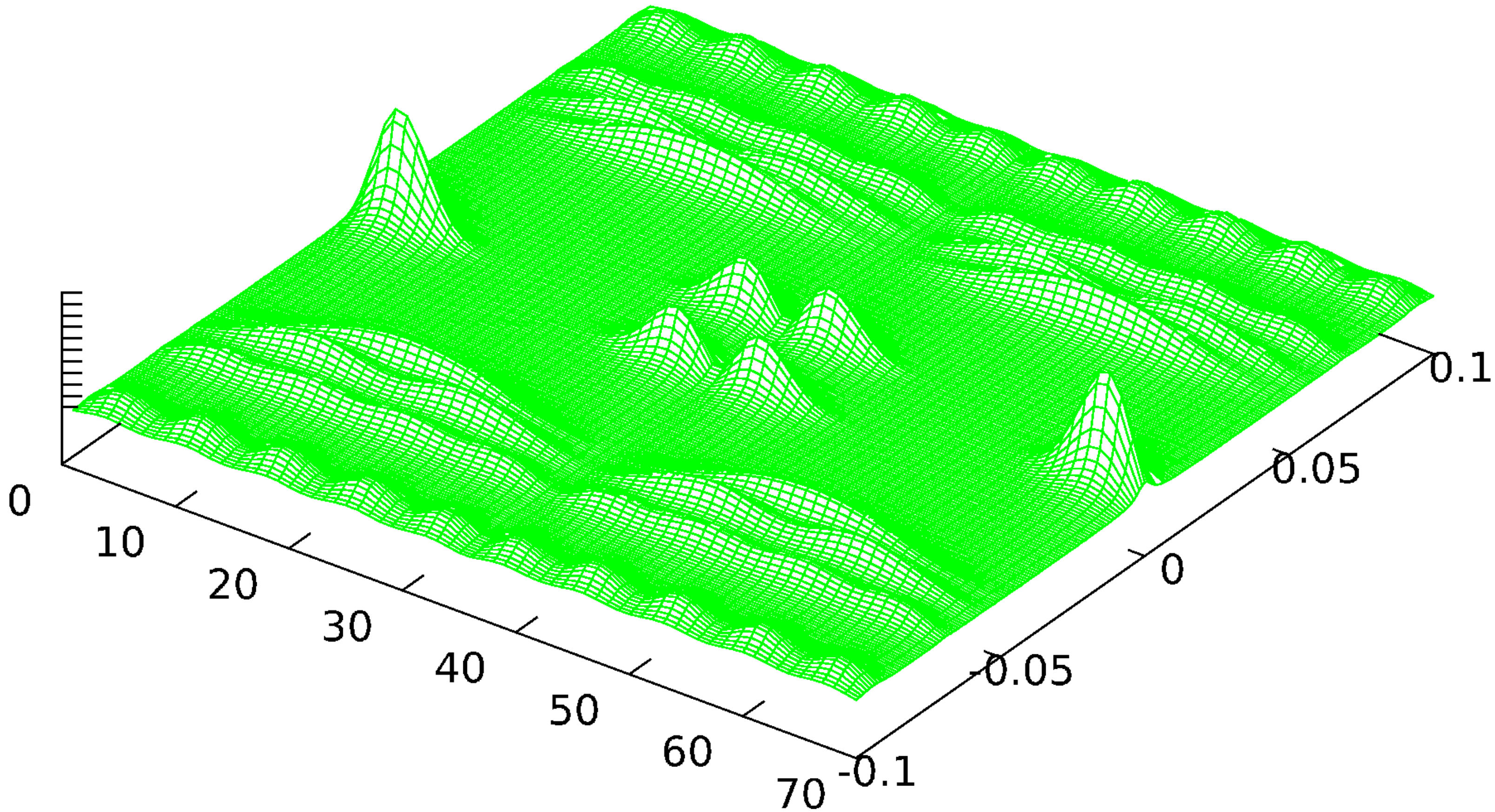
$t_i/t = 0.8$

Breaking the Rashba chain by a reduced hopping at site $i = N/2$



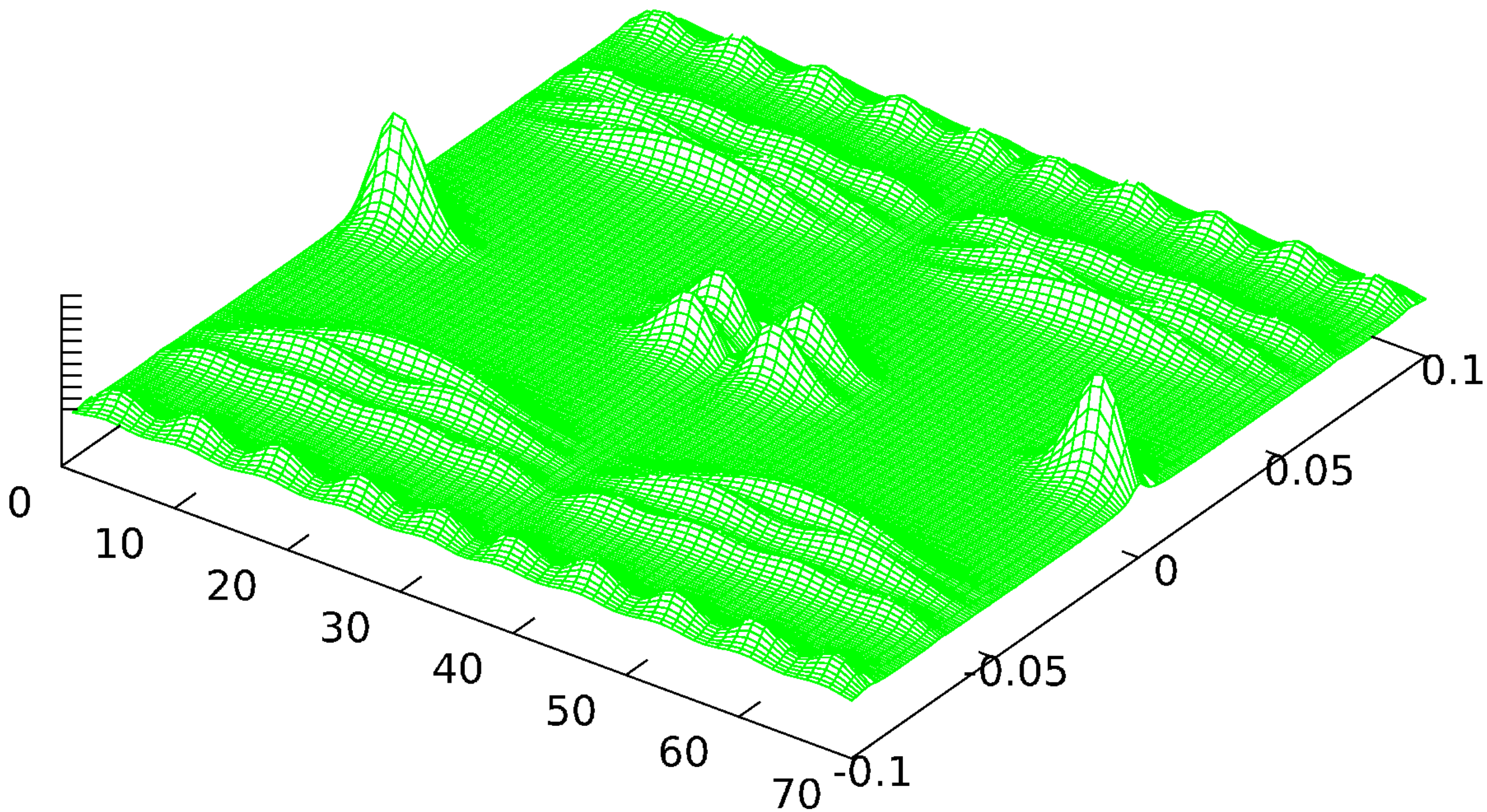
$t_i/t = 0.6$

Breaking the Rashba chain by a reduced hopping at site $i = N/2$



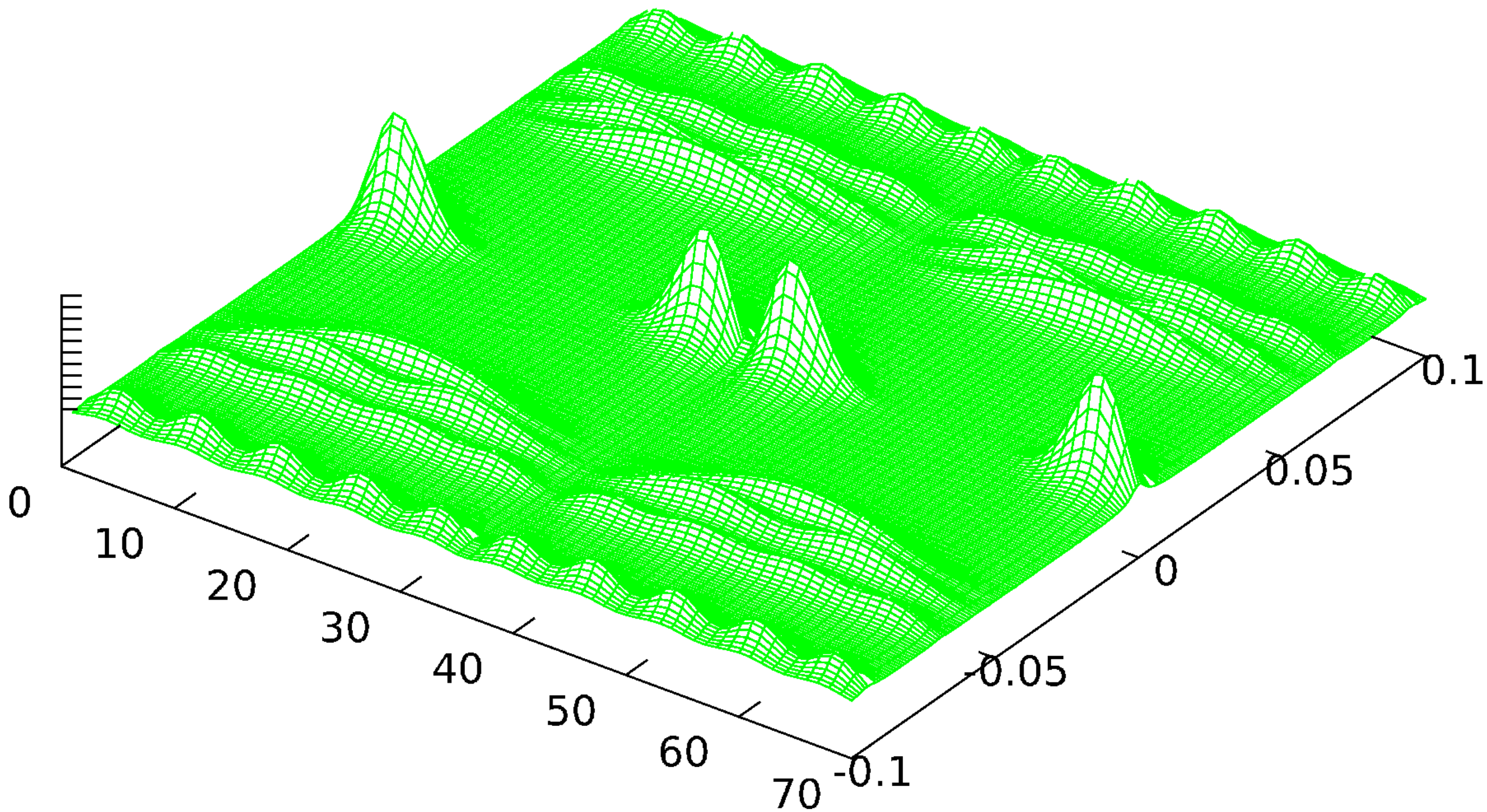
$t_i/t = 0.4$

Breaking the Rashba chain by a reduced hopping at site $i = N/2$



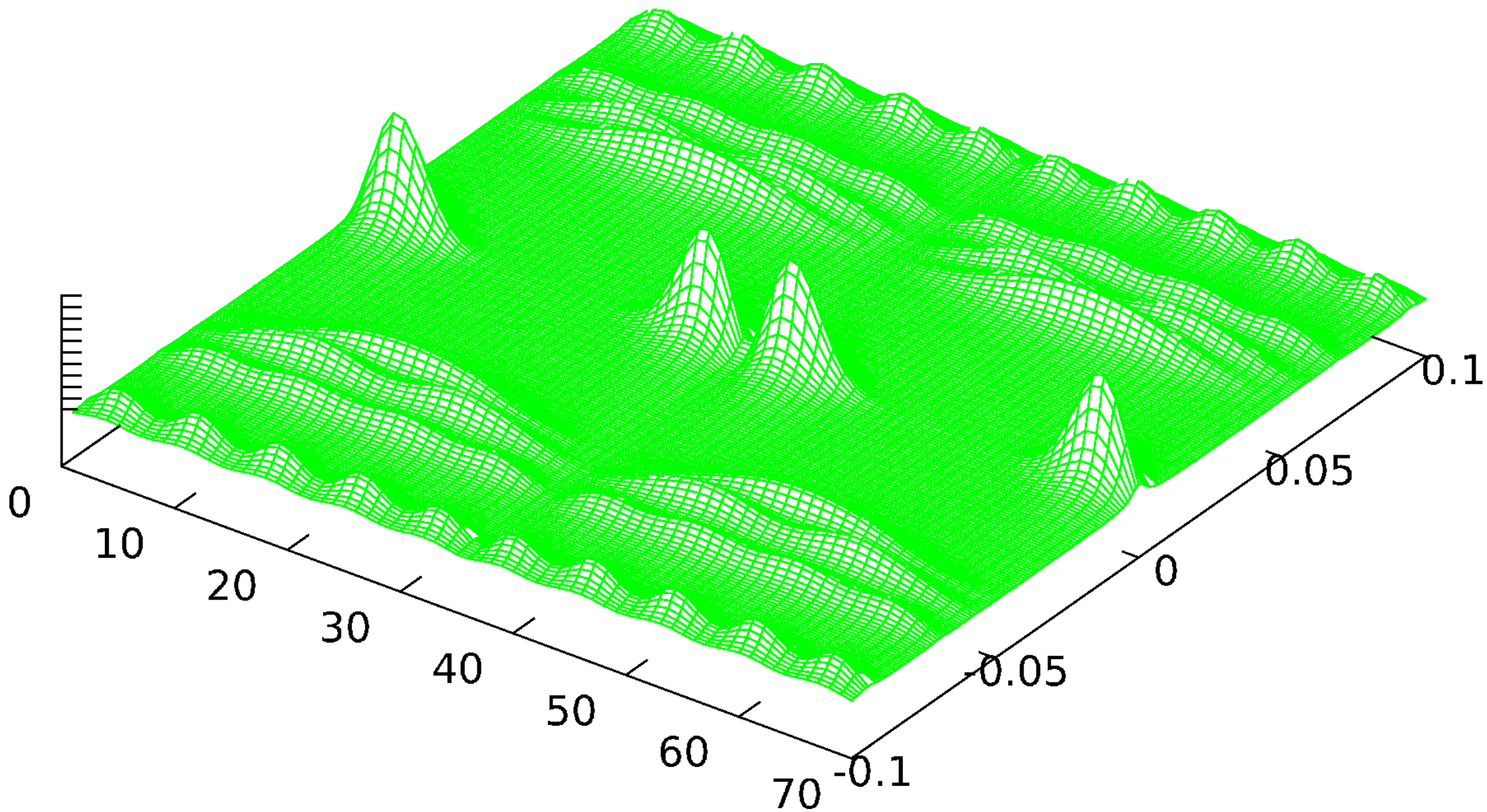
$t_i/t = 0.2$

Breaking the Rashba chain by a reduced hopping at site $i = N/2$



$t_i/t = 0$

Breaking the Rashba chain by a reduced hopping at site $i = N/2$

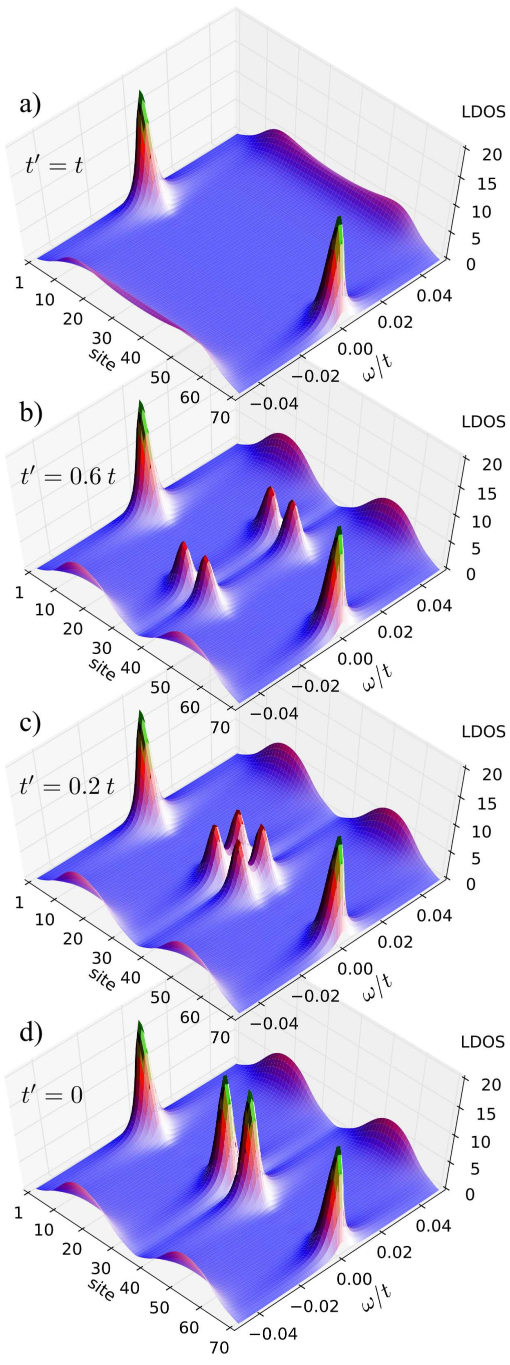


Number of Majorana quasiparticles doubled !

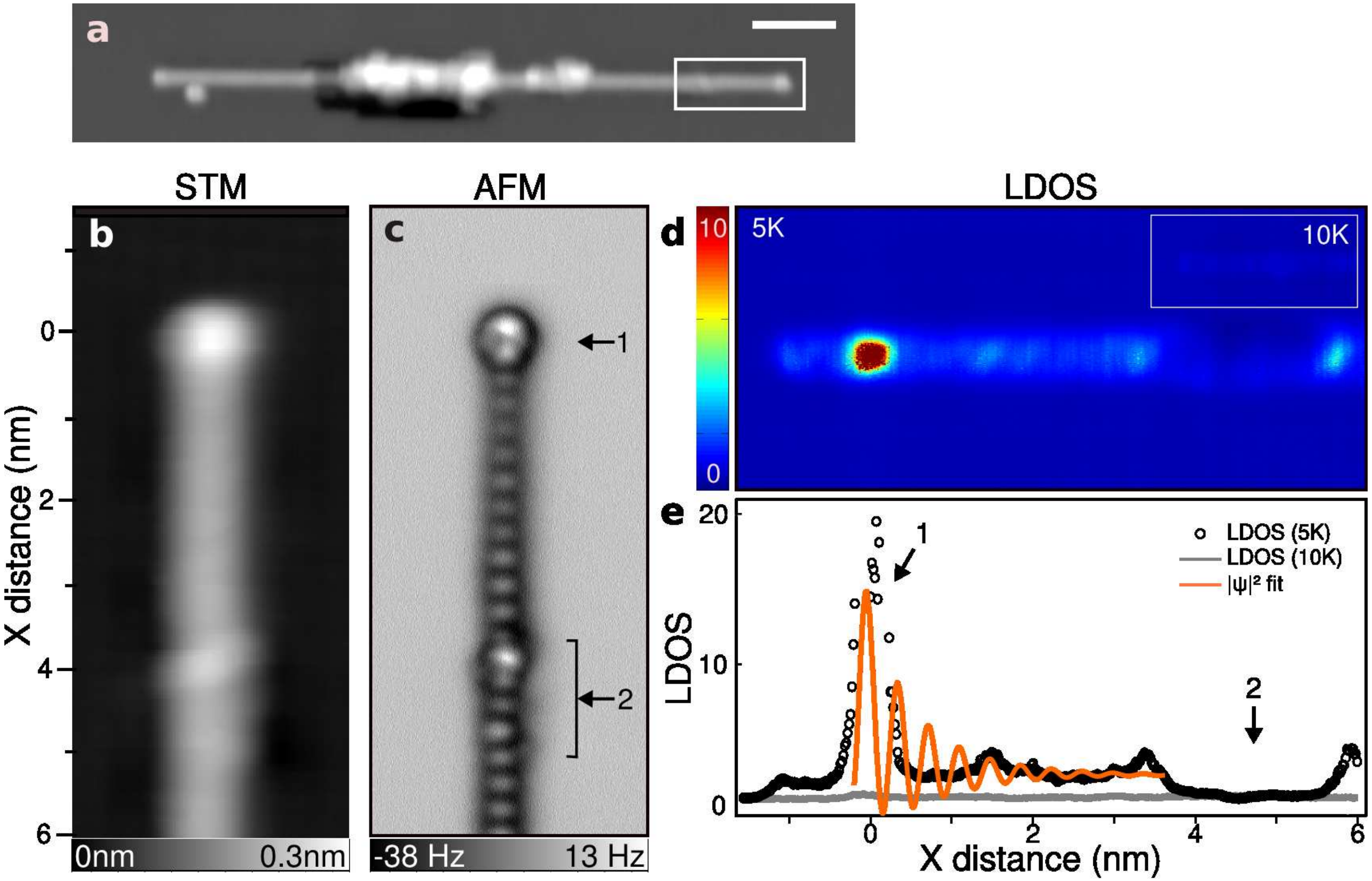
Fussion/splitting of Majorana states

Partitioning
the Rashba chain
into two pieces
via reduced hopping
t_j at site $j = N/2$

M. Maśka et al, Phys. Rev. B **95**, *045429 (2017).*



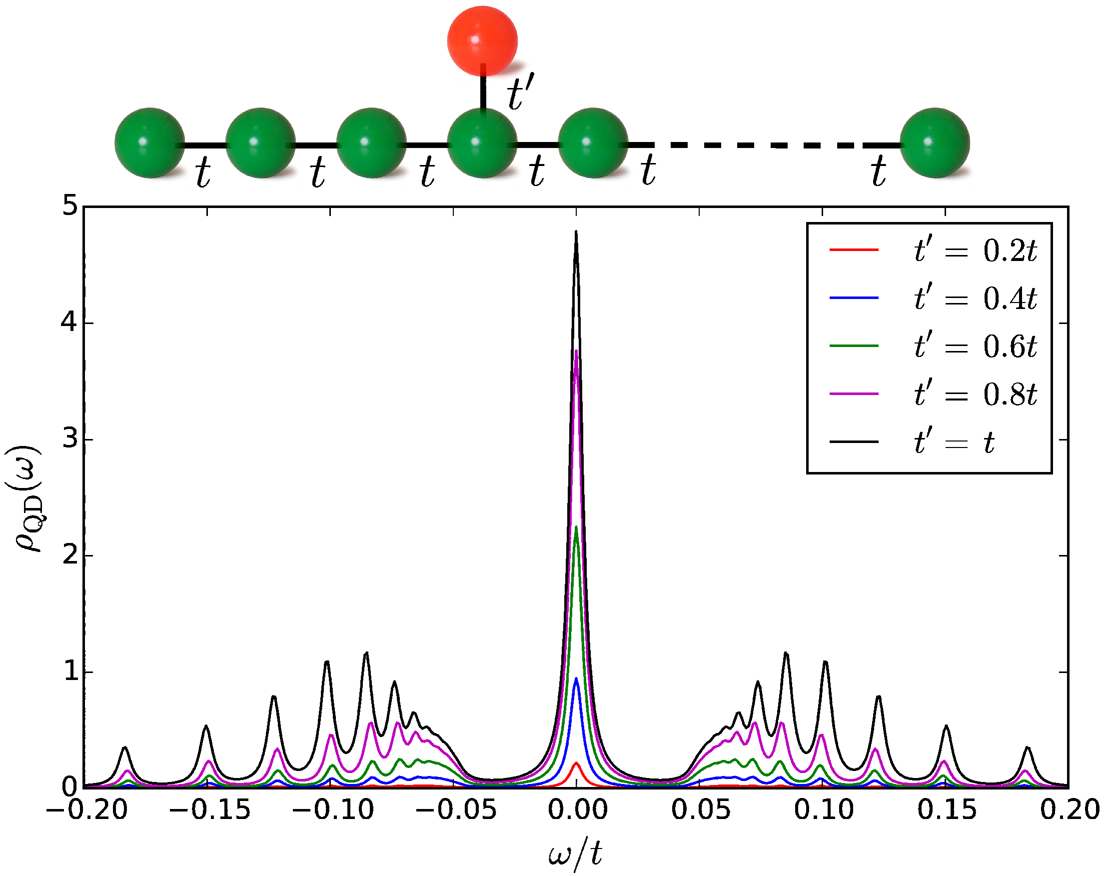
Majoranas at quantum defects – experimental relevance



R. Pawlak, **M. Kisiel**, ..., and E. Meyer, npj Quantum Information **2**, 16035 (2016).

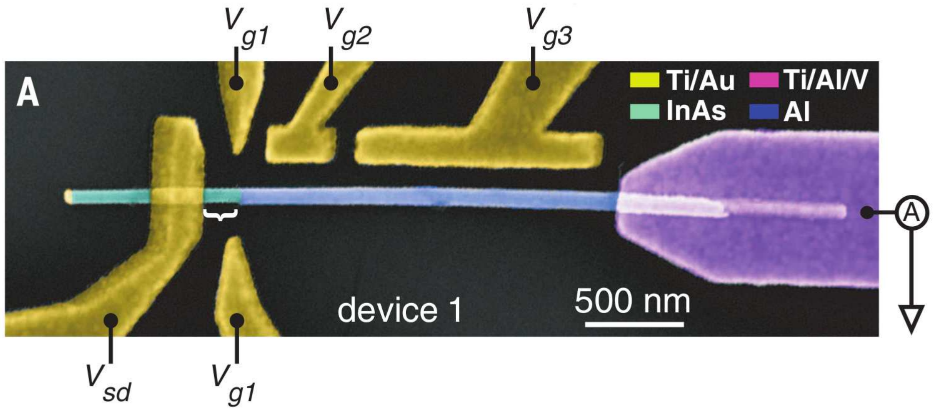
Majorana states – proximity effect

Majorana state can ’leak’ into a normal quantum impurity



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, Phys. Rev. B **95**, *045429 (2017).*

Proximity effect – experimental realization (27 Dec 2016)

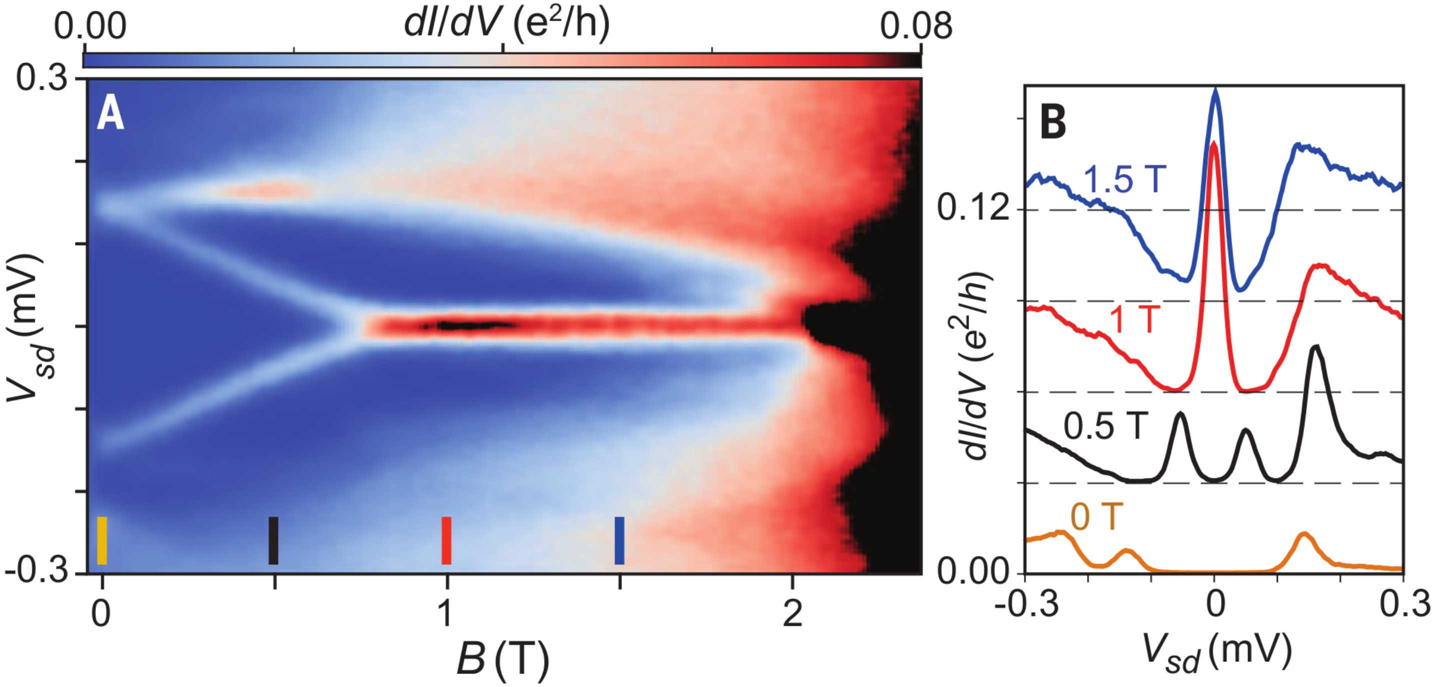


TOPOLOGICAL MATTER

Majorana bound state in a coupled quantum-dot hybrid-nanowire system

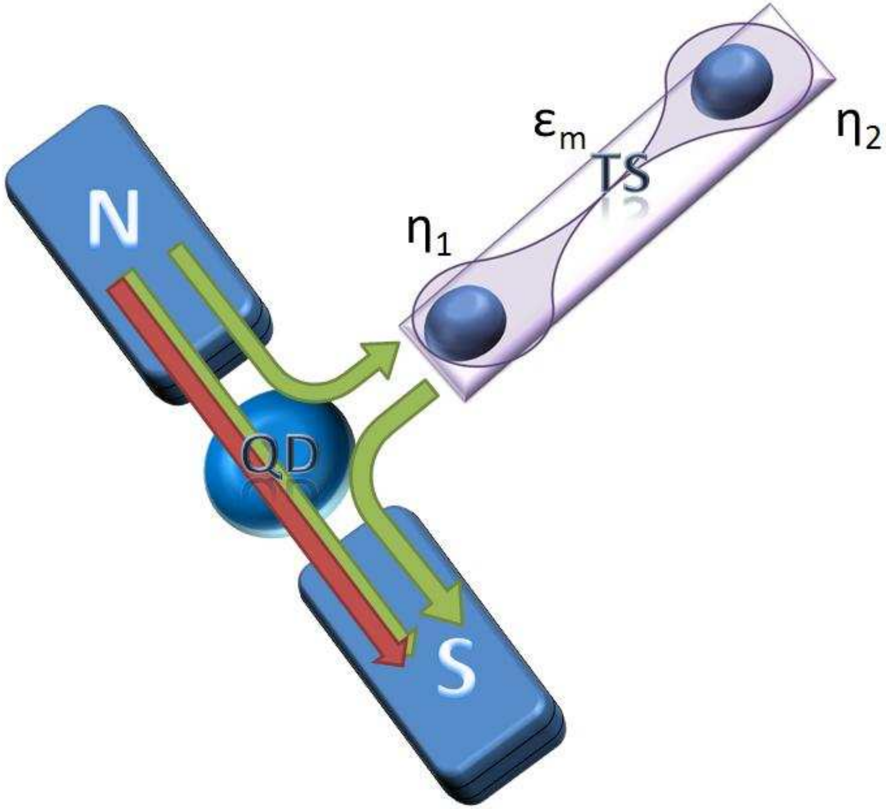
M. T. Deng,[1,2] S. Vaitiekėnas,[1,3] E. B. Hansen,[1] J. Danon,[1,4] M. Leijnse,[1,5] K. Flensberg,[1] J. Nygård,[1] P. Krogstrup,[1] C. M. Marcus[1*]

Science **354**, *1557 (2016).*



Fractionality of Majoranas

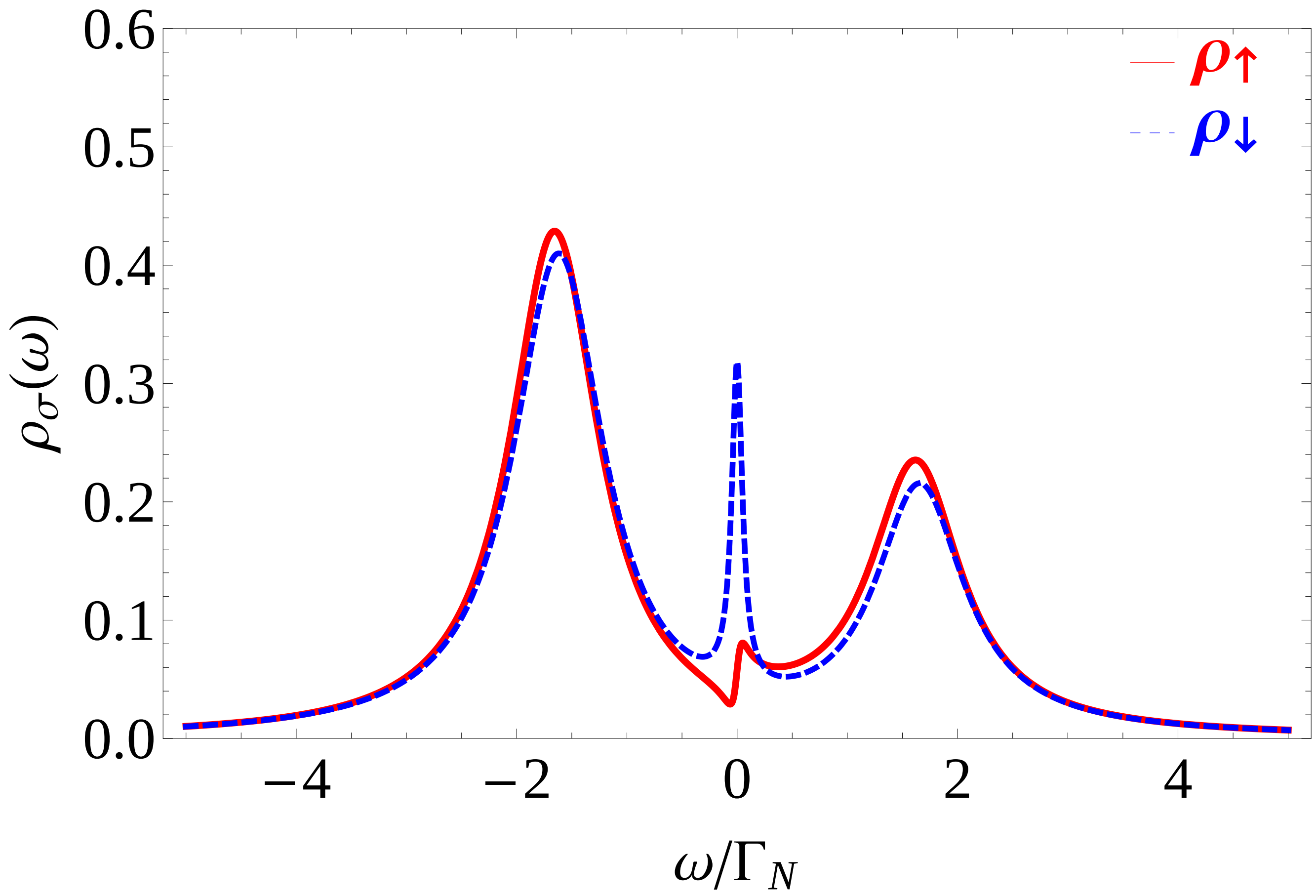
Let’s consider QD with the side-attached Majorana quasiparticle.



J. Barański, A. Kobiałka and T. Domański, J. Phys.: Condens. Matt. **29**, *075603 (2017).*

Fractionality of Majoranas

Fractional (Fano-type) interference in the QD spectrum.



J. Barański, A. Kobiałka and T. Domański, J. Phys.: Condens. Matt. **29**, *075603 (2017).*

Summary on Majorana qps: – message to take home

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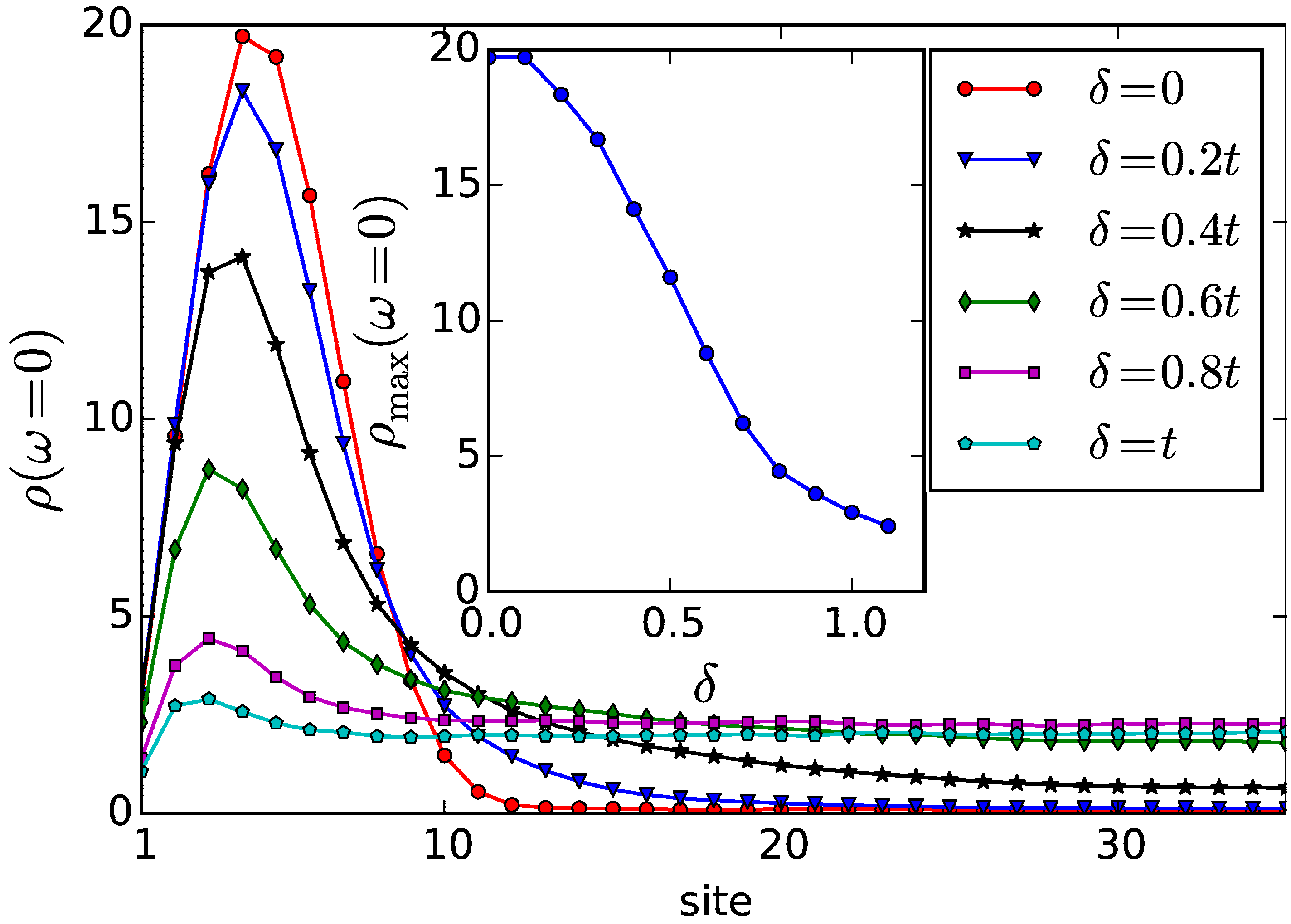
- topologically protected

$\Rightarrow$ should be immune to decoherence ... yet be cautious about that !

Majorana states – effect of disorder

Majorana quasiparticles are not truly immune to disorder !



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, Phys. Rev. B **95***, 045429 (2017).*

Summary on Majorana qps: – possible applications

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