Prague, 14 Nov. 2013

Subgap states of a quantum impurity coupled to superconducting reservoir

T. Domański

M. Curie–Skłodowska University

Lublin, Poland

http://kft.umcs.lublin.pl/doman/lectures

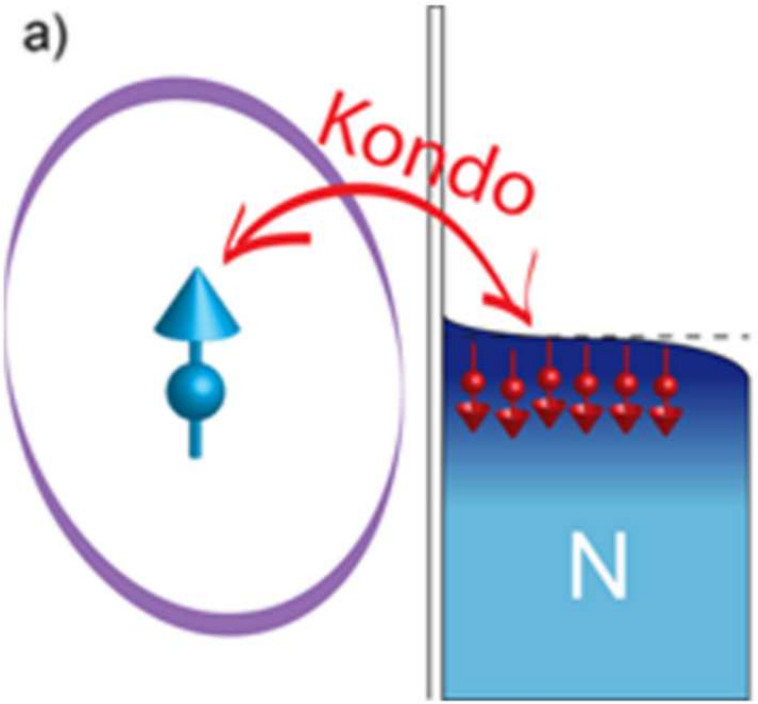
Motivation

Physical dilemma

′ to screen or not to screen ?′

Physical dilemma ′ to screen or not to screen ?′

Quantum impurity (dot) coupled to a metallic bath



can form the Kondo state with itinerant electrons (at $T < T_K$)

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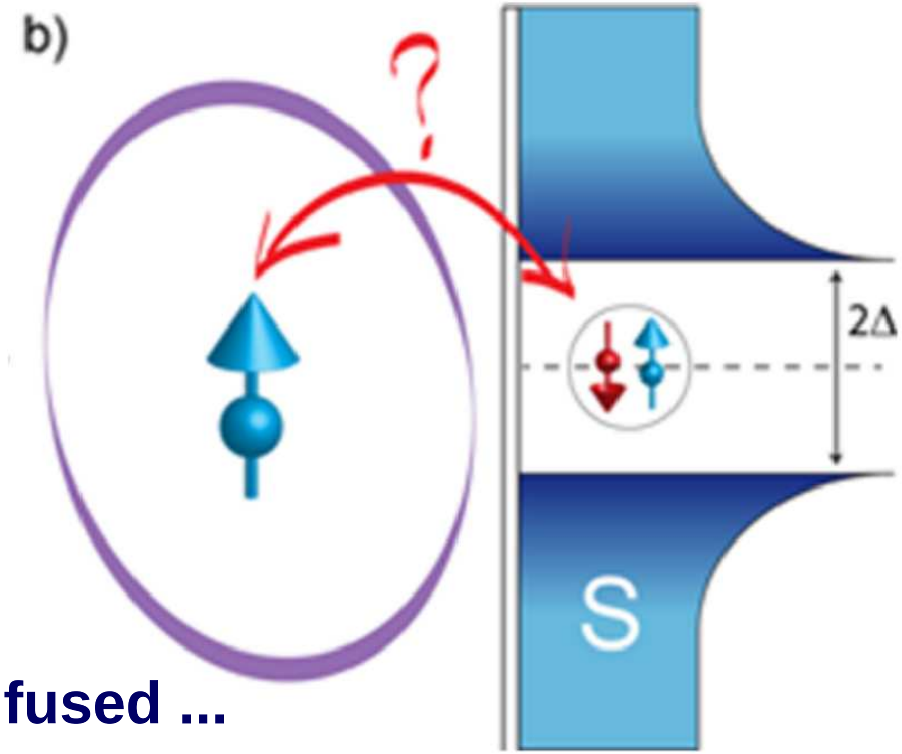

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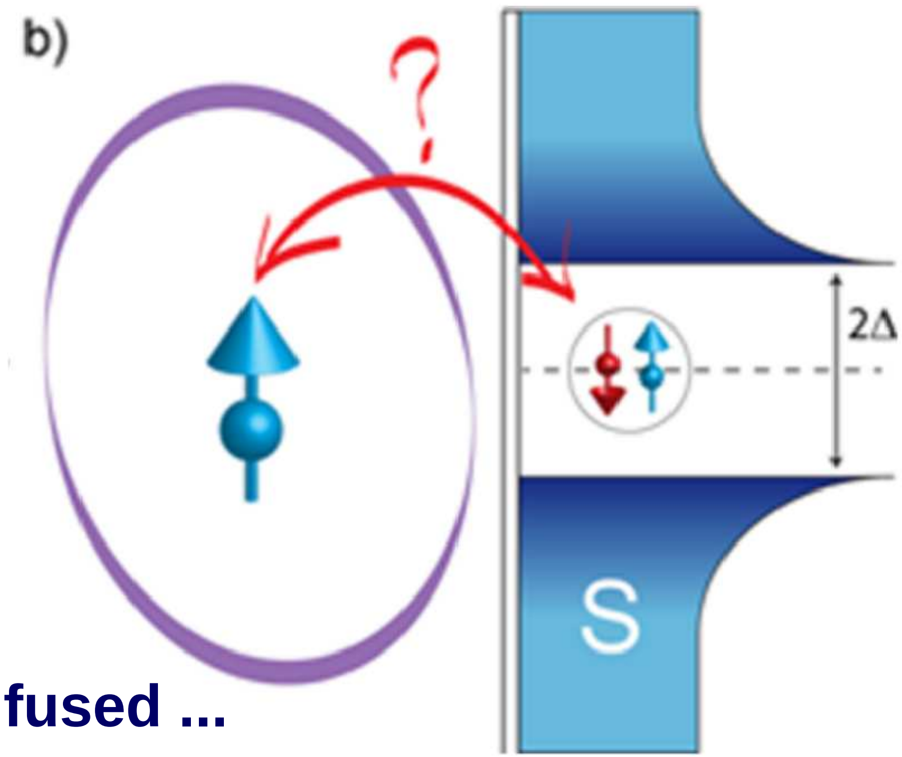

is a bit confused ...

The reasons :

⇒ **there are no available states at the Fermi level, and**

⇒ **QD absorbs a pairing (which competes with the Kondo physics).**

Physical dilemma ′ to screen or not to screen ?′

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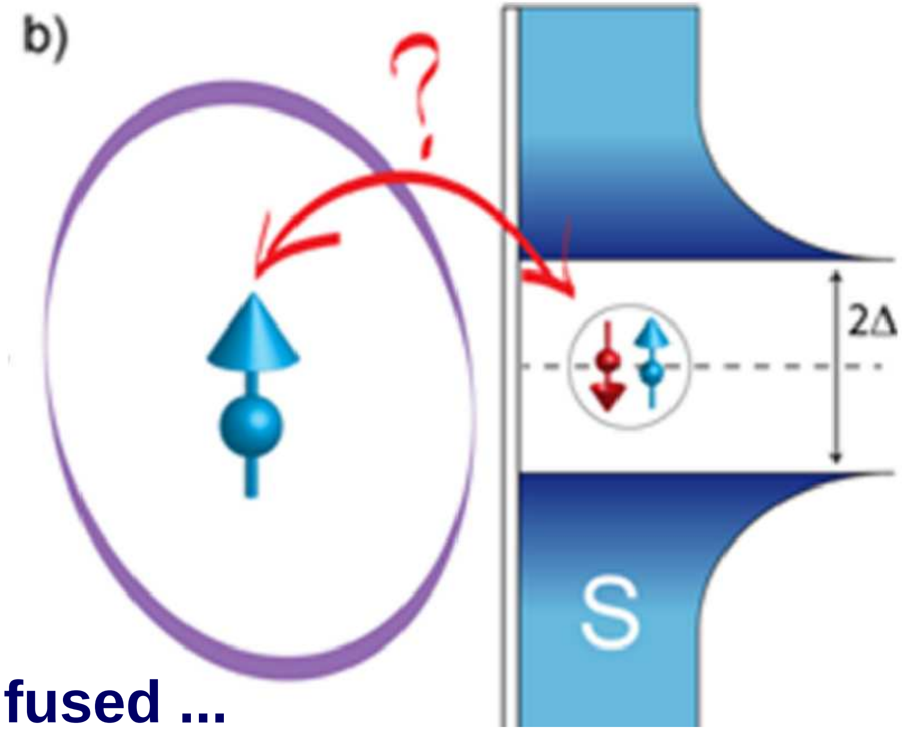

is a bit confused ...

Physics — *Physics* **6**, 75 (2013)

Viewpoint

To Screen or Not to Screen, That is the Question!

Romain Maurand and Christian Schönenberger

Physical dilemma

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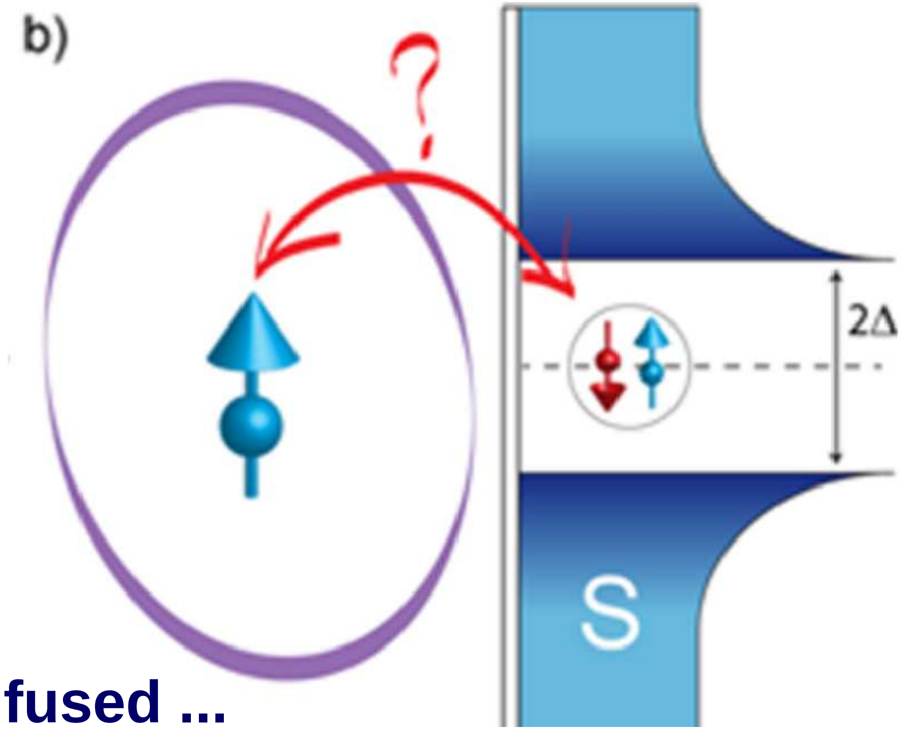

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Physics

Physics **6**, 75 (2013)

Viewpoint

This viewpoint appeared on July 2$^{\mathrm{nd}}$, 2013.

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Quantum impurity coupled to a superconducting medium

Schematic picture

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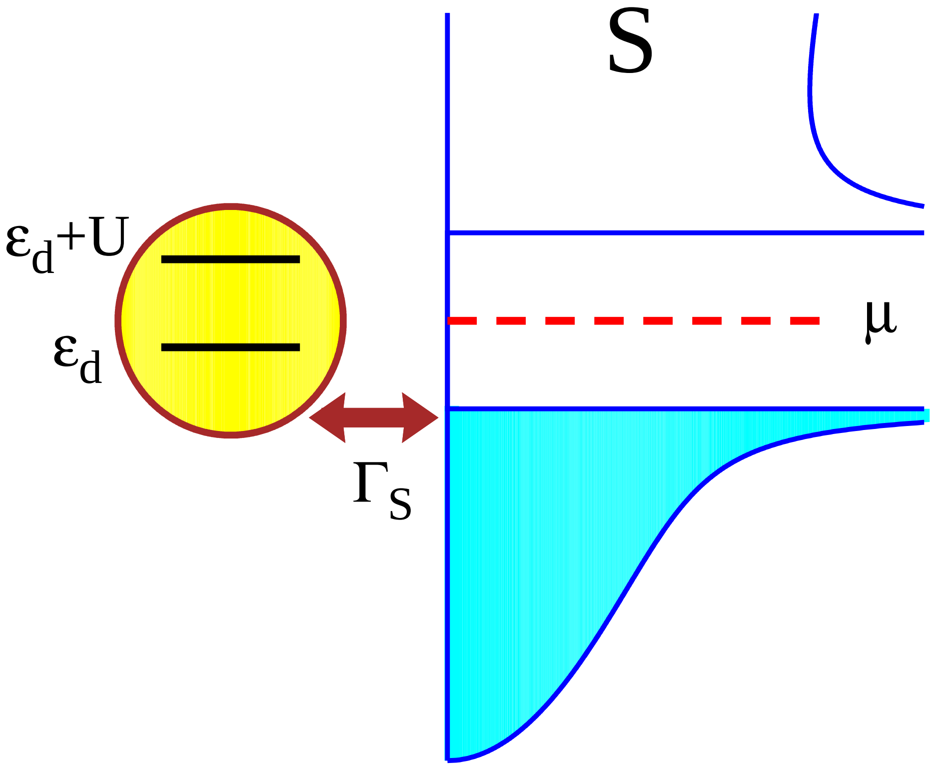
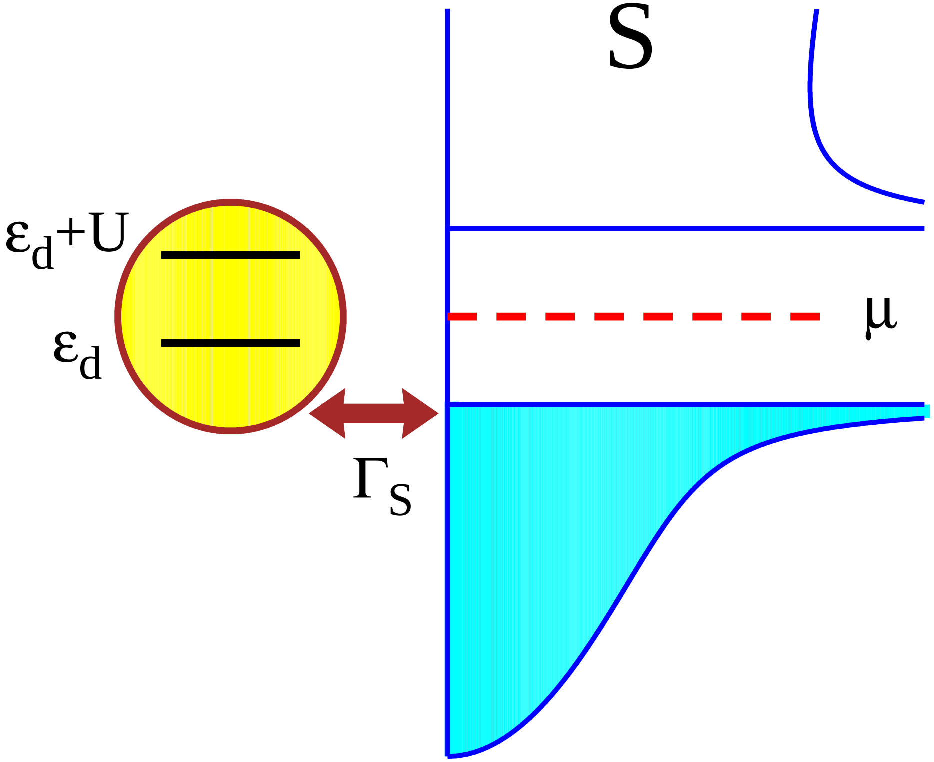


Schematic picture



$\Gamma_S(\omega) = 2\pi \sum_{\mathbf{k}} |V_{\mathbf{k}}|^2 \delta(\omega - \varepsilon_{\mathbf{k}})$ ← **hybridization coupling**

Microscopic model

Anderson-type Hamiltonian

The quantum impurity (dot)

Microscopic model **Anderson-type Hamiltonian**

The quantum impurity (dot)

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U \, \hat{n}_{d\uparrow} \, \hat{n}_{d\downarrow}$$

Microscopic model

Anderson-type Hamiltonian

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$$\begin{aligned} \hat{H} \;\; &= \;\; \sum_{\sigma} \epsilon_d \hat{d}^{\dagger}_{\sigma} \hat{d}_{\sigma} + U \; \hat{n}_{d\uparrow} \; \hat{n}_{d\downarrow} + \hat{H}_S \\ &+ \;\; \sum_{\mathbf{k},\sigma} \left(V_{\mathbf{k}} \; \hat{d}^{\dagger}_{\sigma} \hat{c}_{\mathbf{k}\sigma} + V^{*}_{\mathbf{k}} \; \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{d}_{\sigma} \right) \end{aligned}$$

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where

$$\hat{H}_S = \sum_{k,\sigma} \left(\varepsilon_k - \mu\right) \hat{c}^{\dagger}_{k\sigma} \hat{c}_{k\sigma} - \sum_k \left(\Delta \hat{c}^{\dagger}_{k\uparrow} \; \hat{c}^{\dagger}_{k\downarrow} + \mathrm{h.c.} \right)$$

Uncorrelated QD – the exactly solvable $U_d = 0$ case

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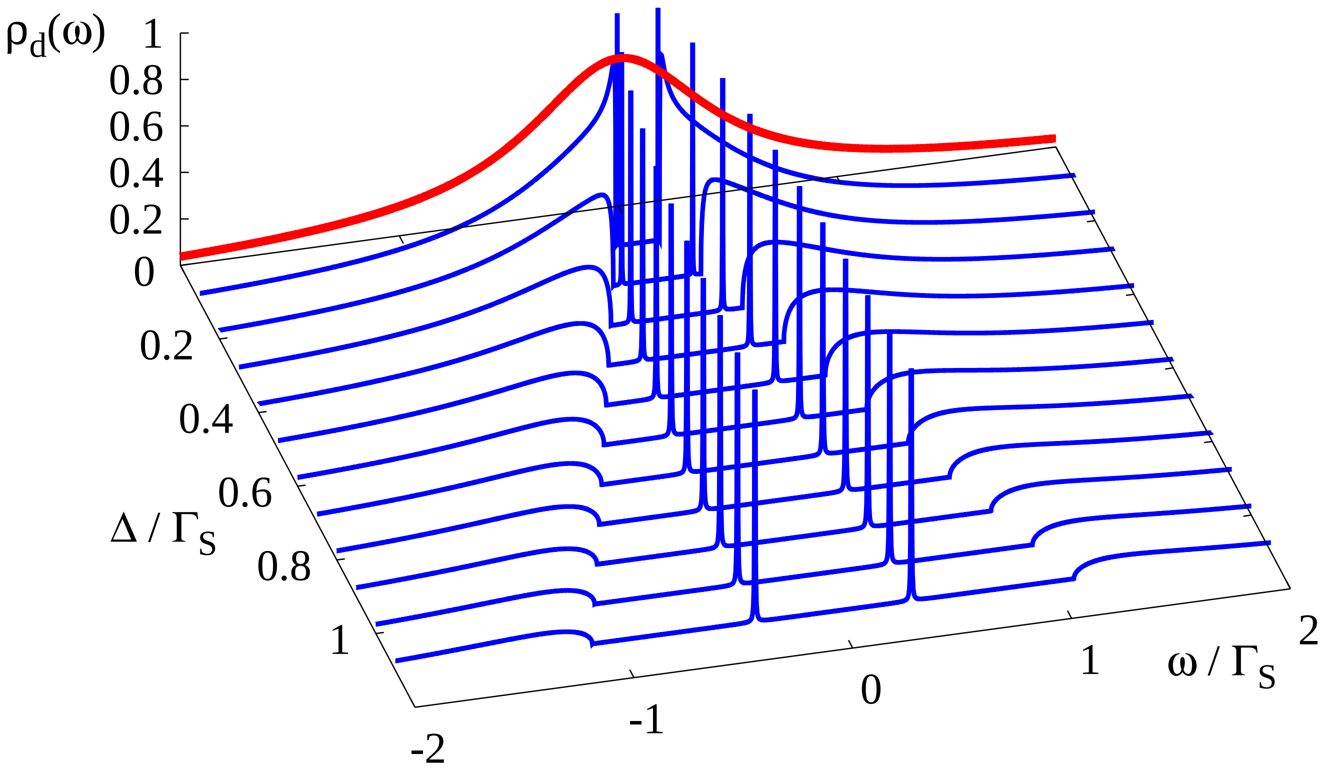

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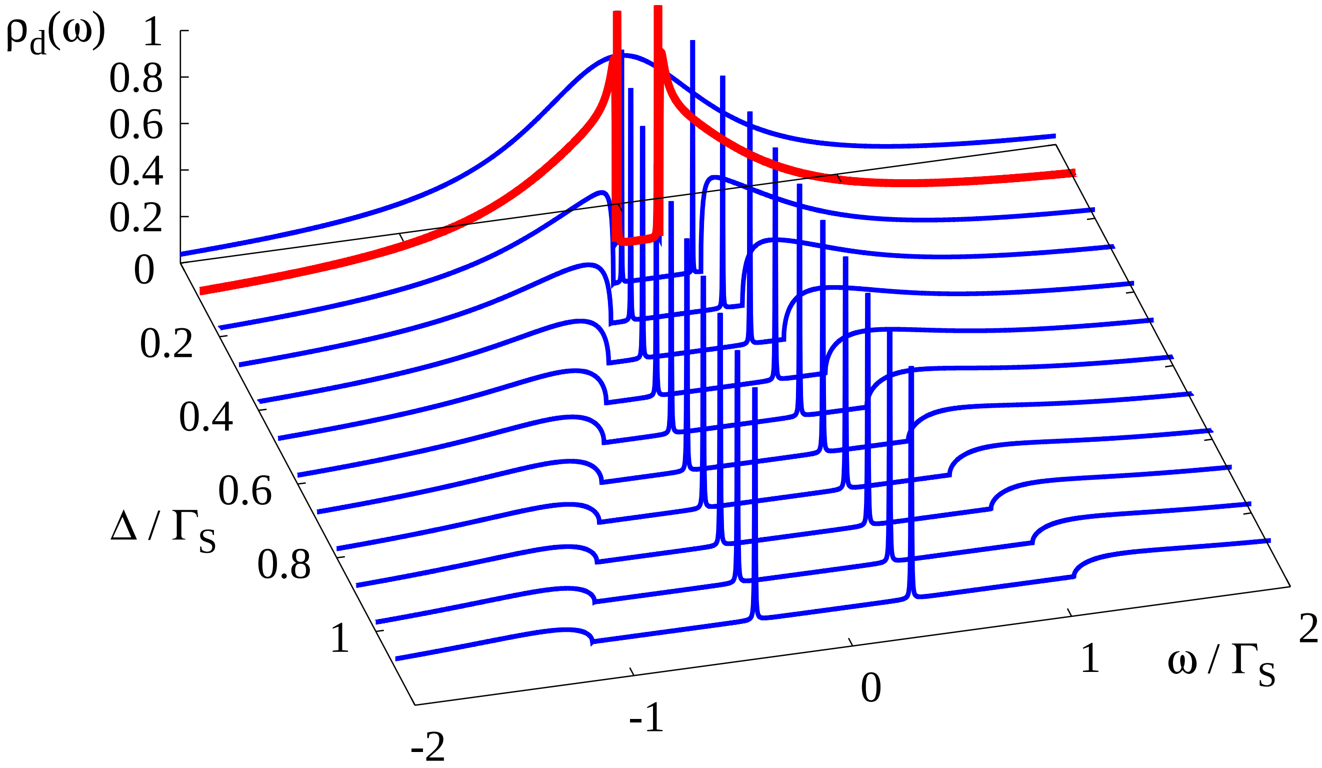

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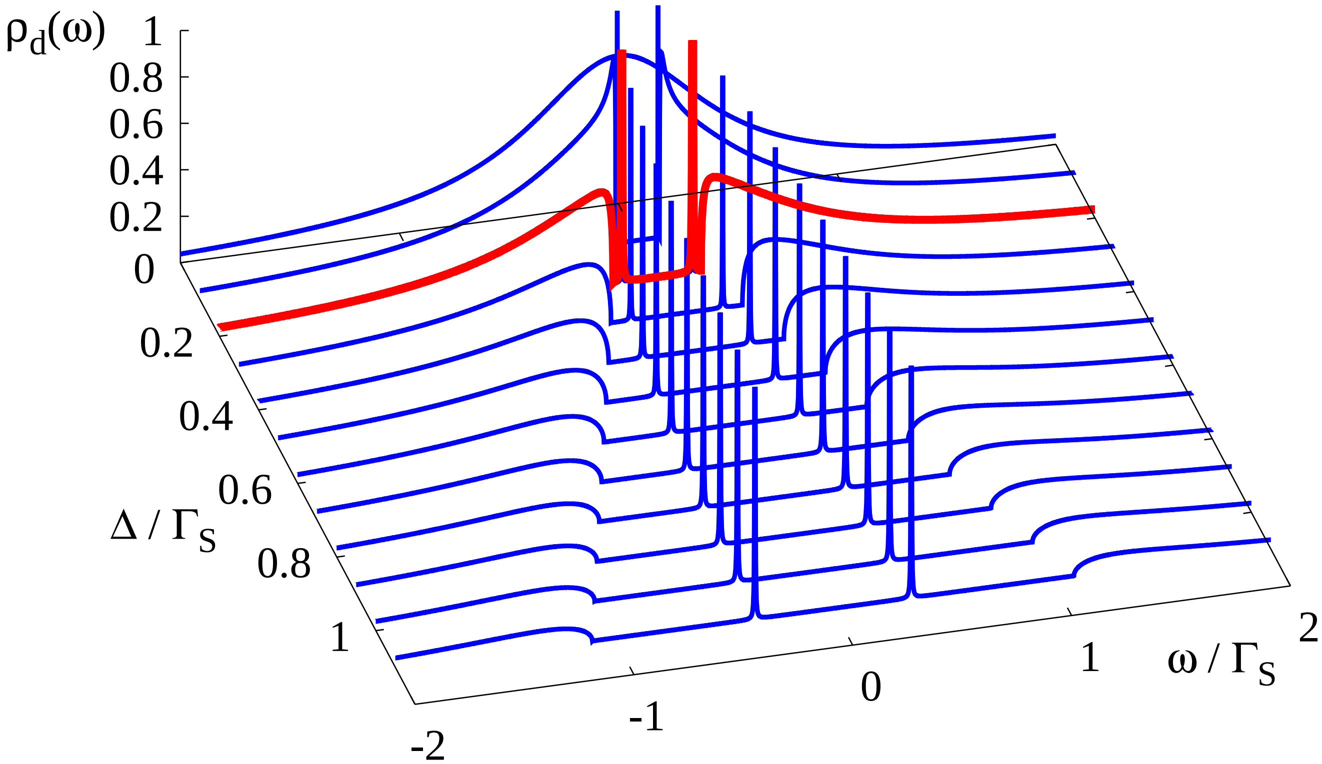

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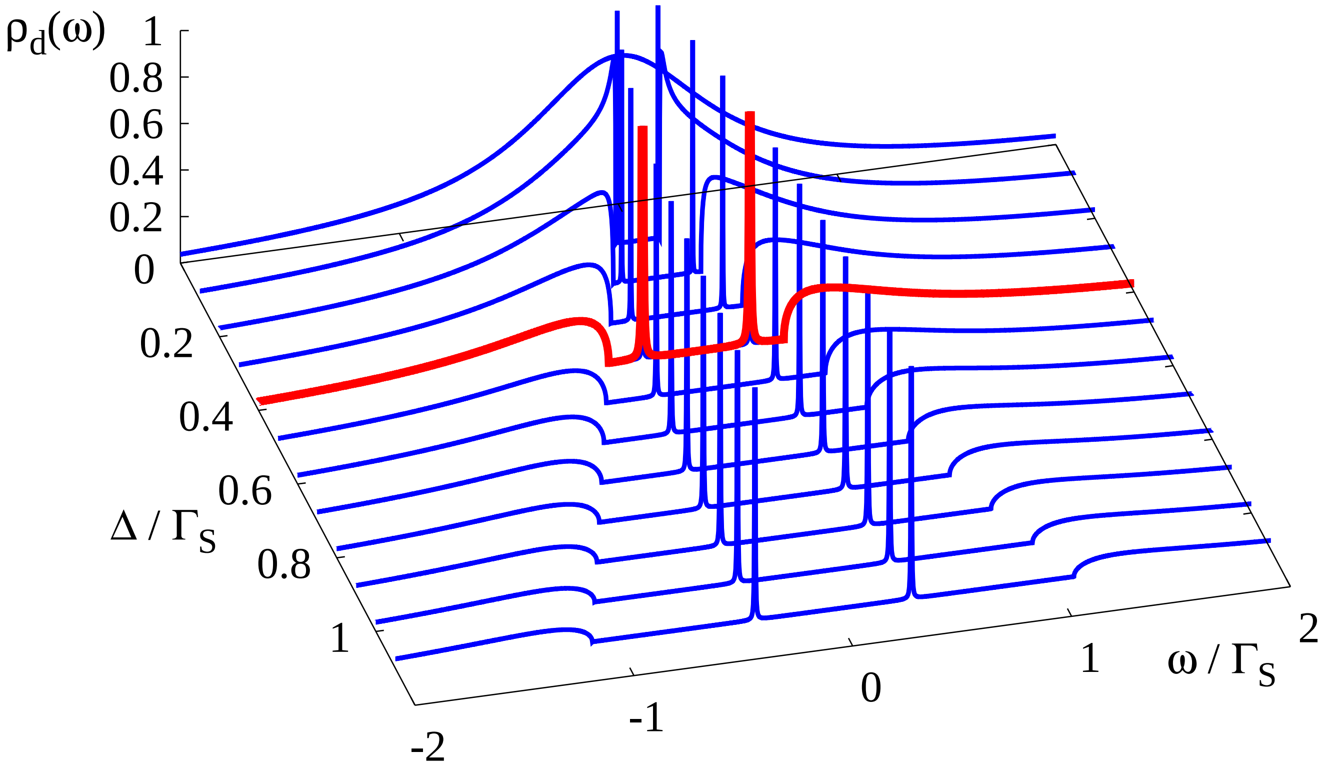

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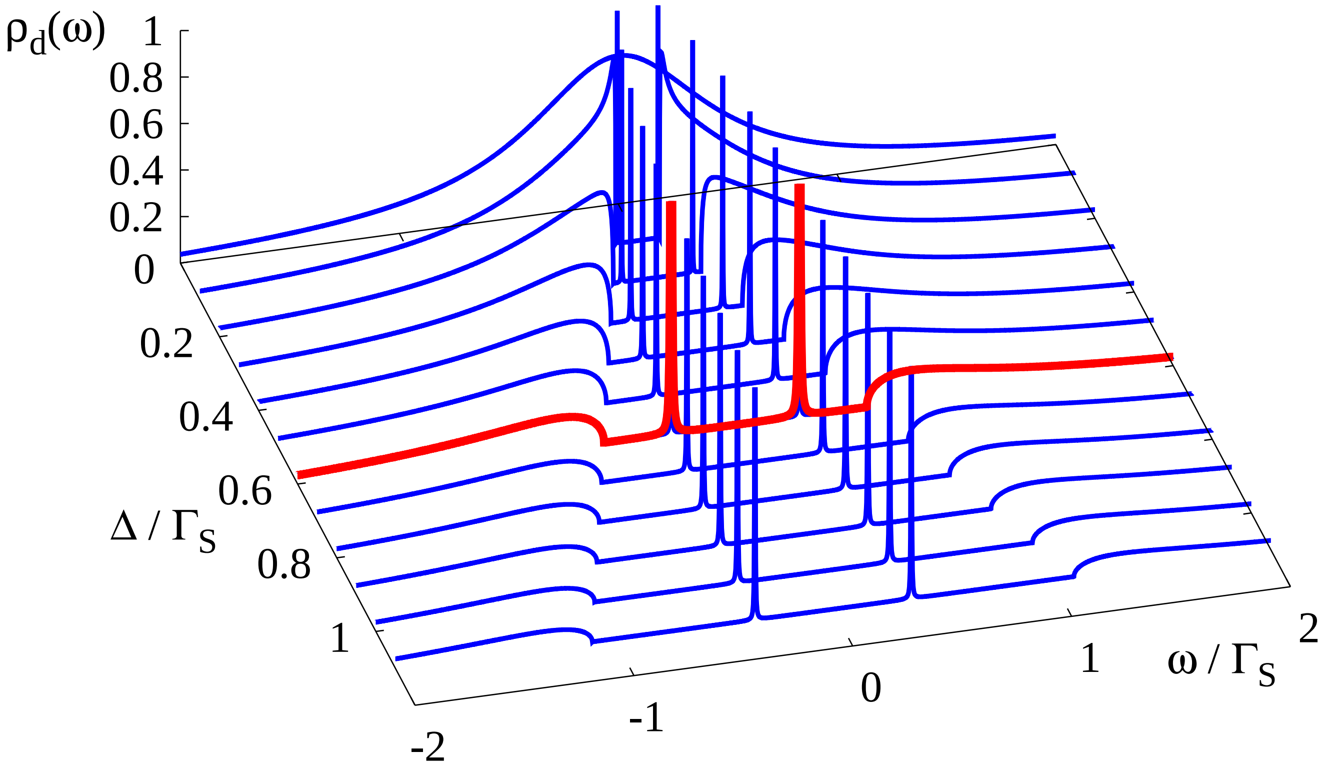

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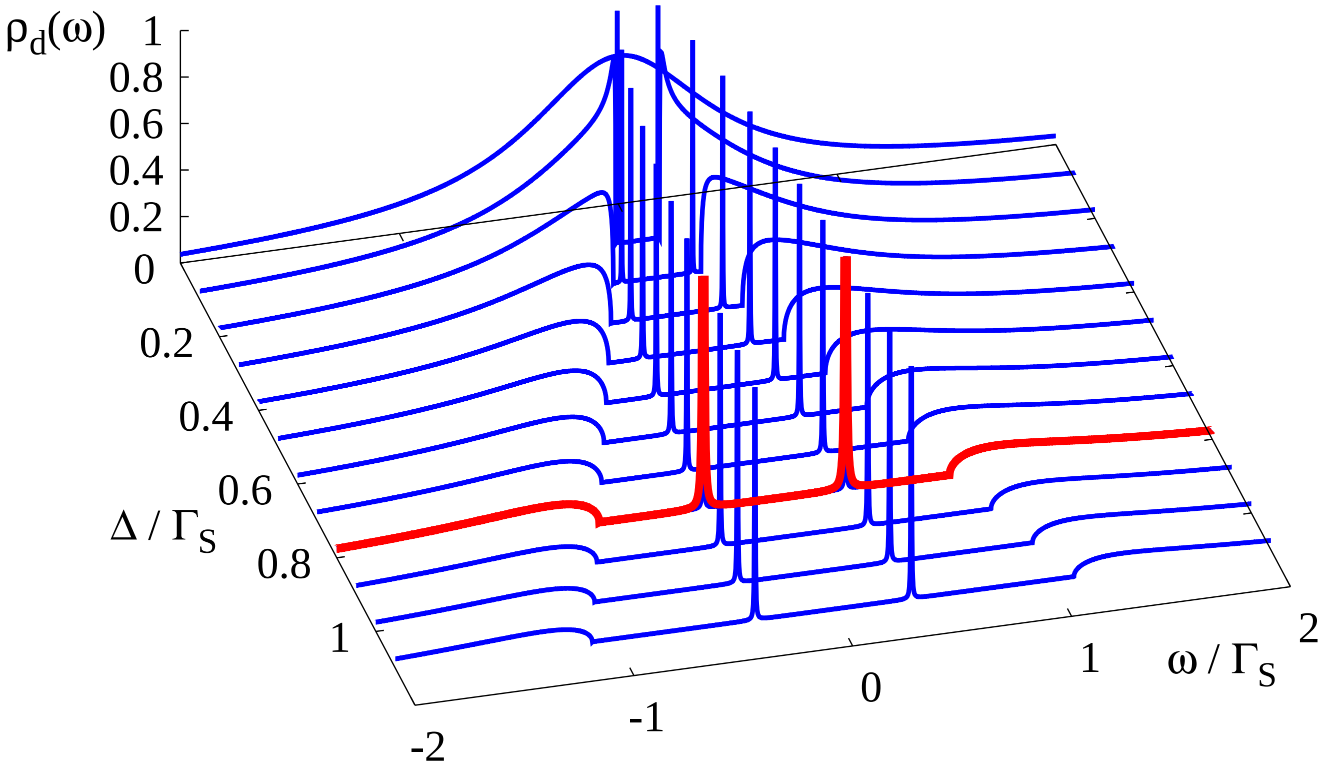

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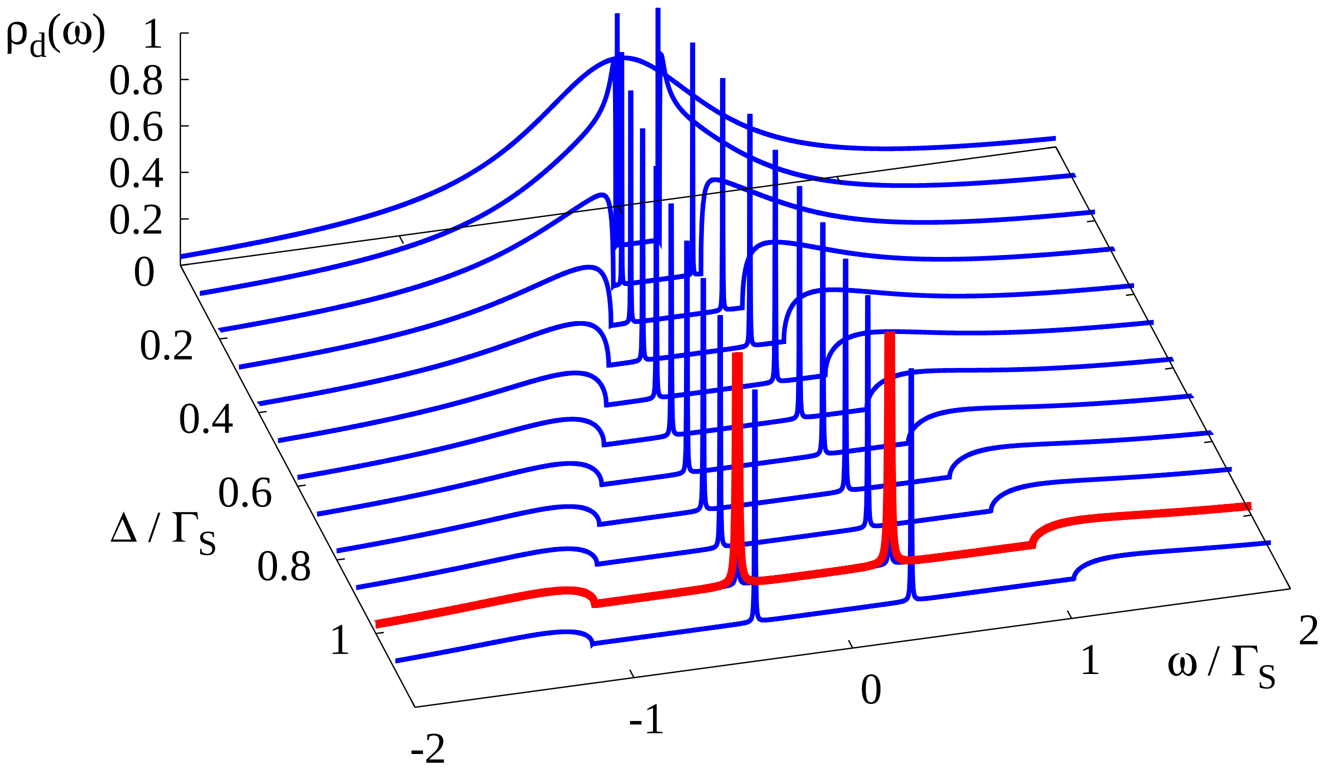

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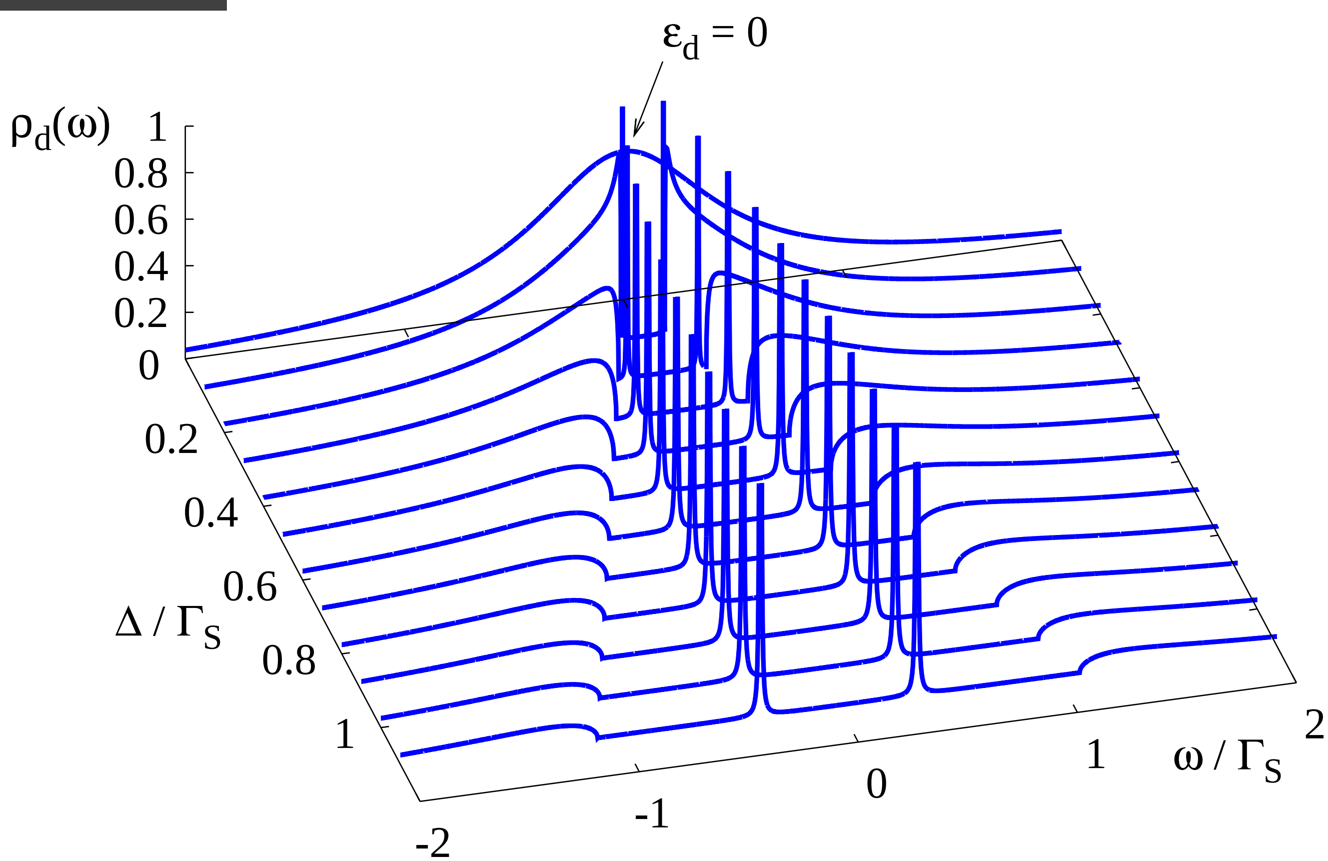

Appearance of the in-gap resonances (Andreev bound states)

J. Barański and T. Domański, J. Phys.: Condens. Matter **25**, 435305 (2013).

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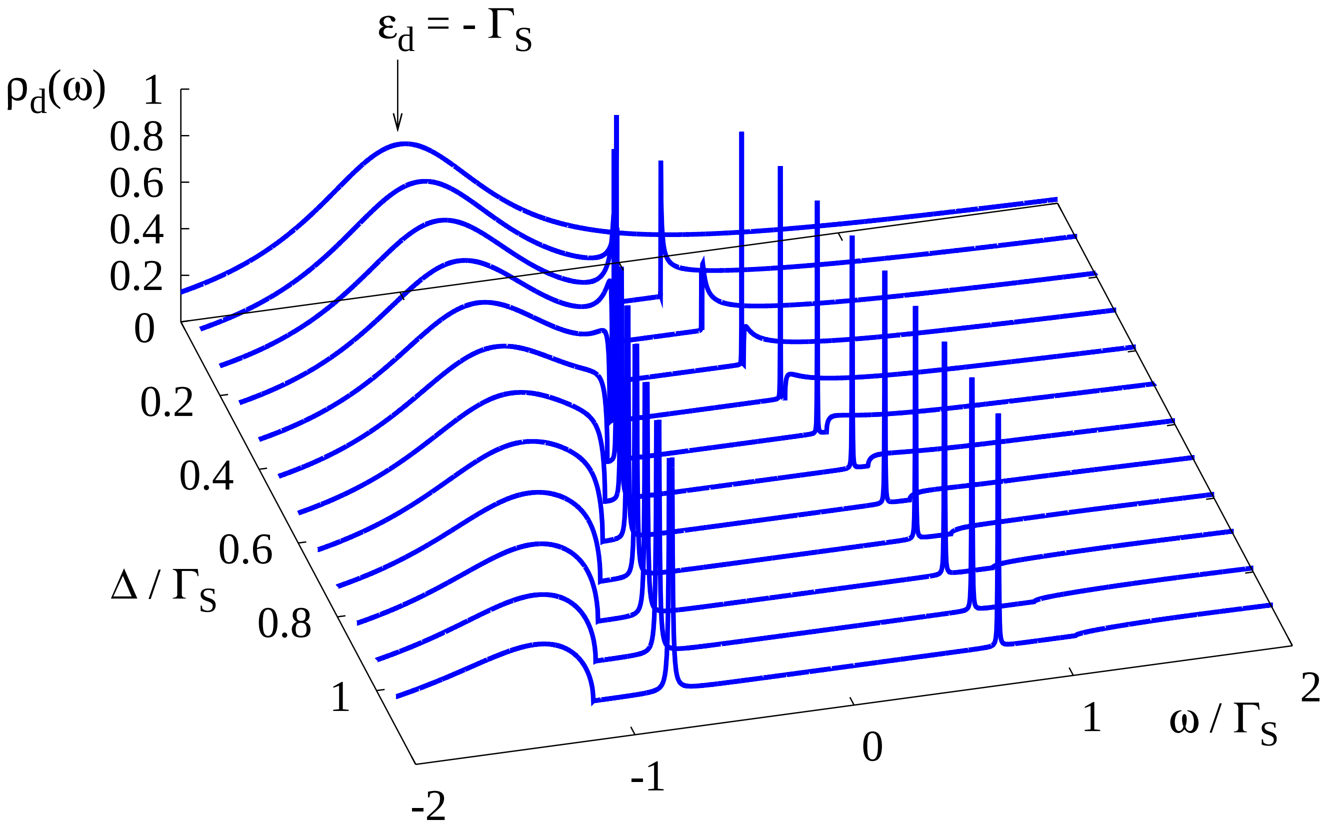

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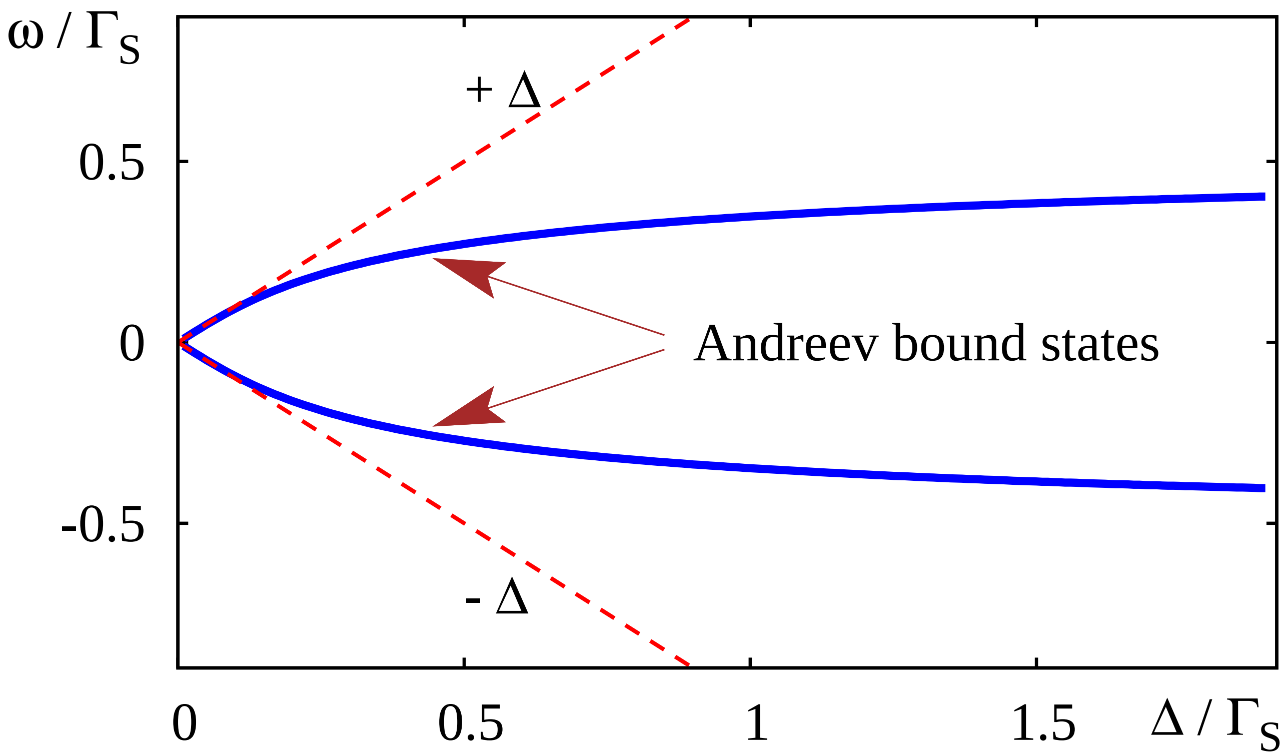

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Influence of the correlations – singlet/doublet configurations

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$\vert\uparrow\rangle$ and $\vert\downarrow\rangle$	$\Leftarrow$	doublet states (spin $\frac{1}{2}$)
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There can occur doublet-singlet quantum phase transition by varrying ε_d, U_d or Γ_S.

Correlated quantum dot – exact solution for $\Gamma_S \gg \Delta$

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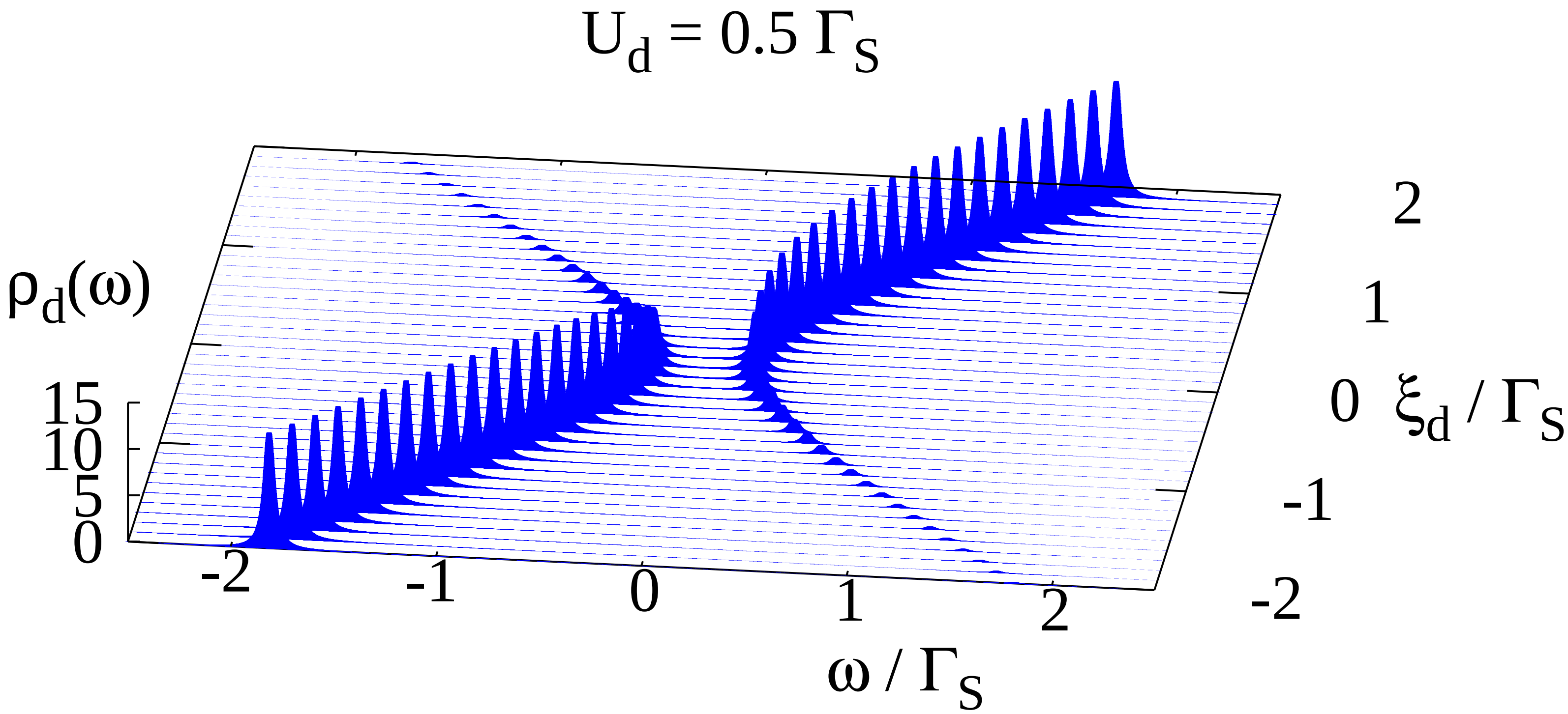
Subgap spectrum vs energy ω and $\xi_d = \varepsilon_d + \frac{1}{2} U_d$



2 in-gap states

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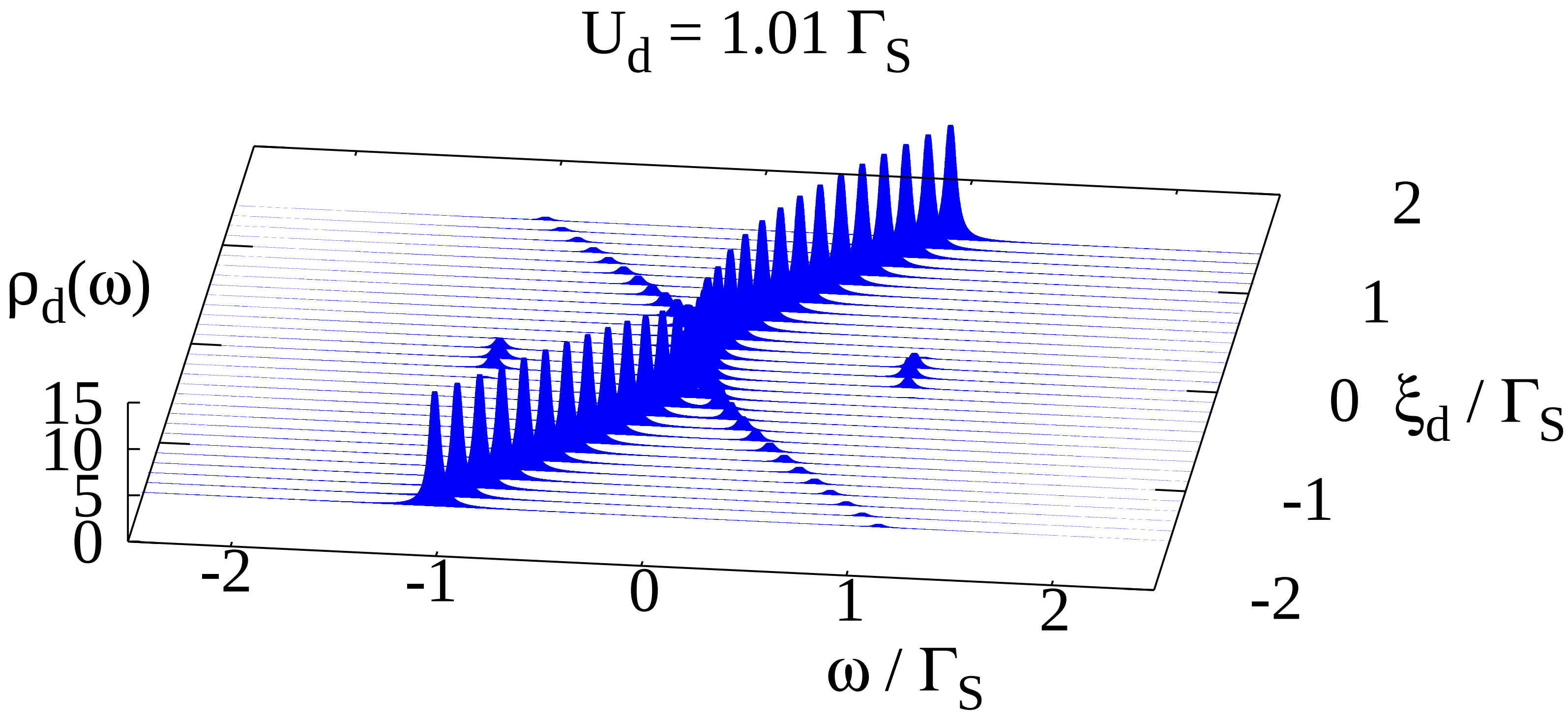

nearby a quantum phase transition

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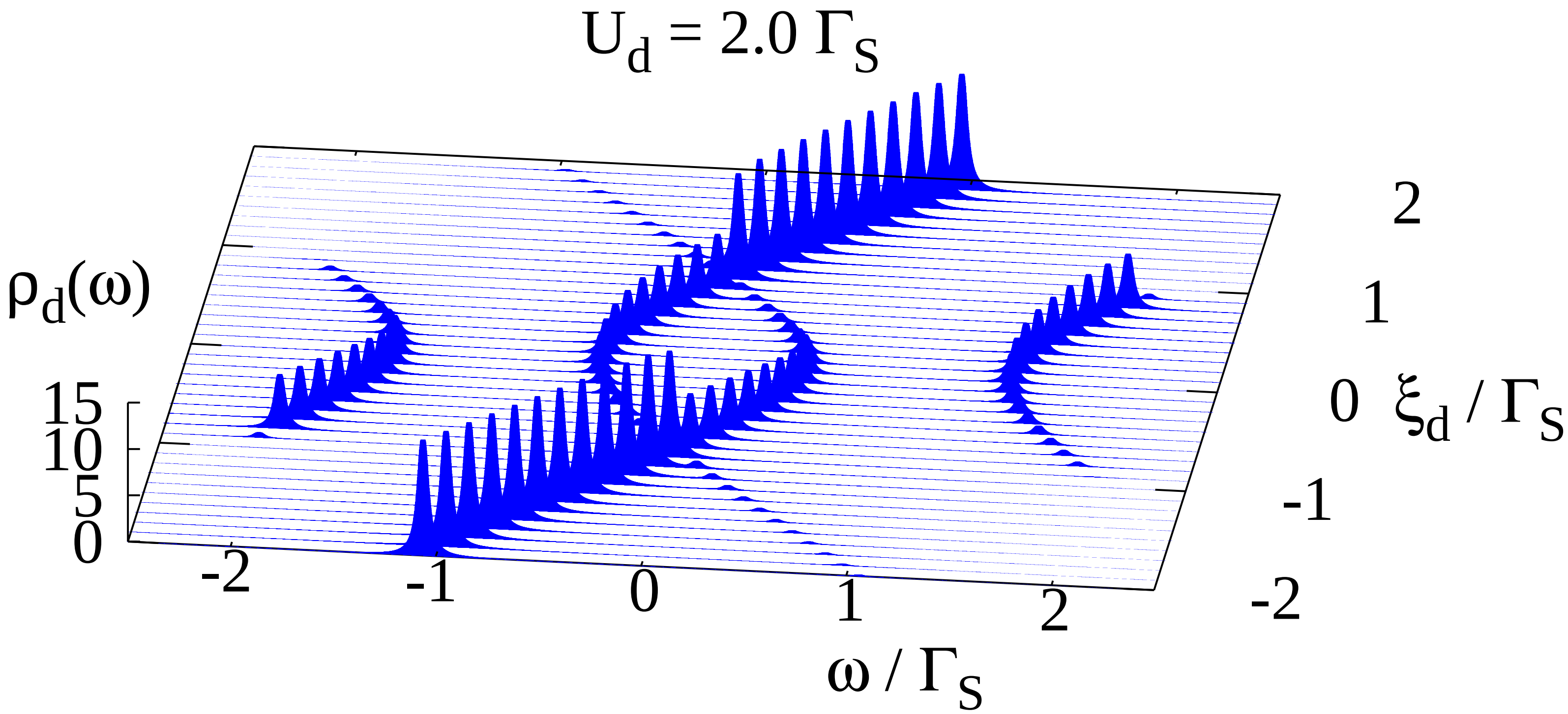

4 in-gap states

Andreev spectroscopy

Physical situation

N-QD-S scheme

To probe the subgap states one can study the electron transport through a quantum dot (QD) coupled between the normal (N) and superconducting (S) electrodes

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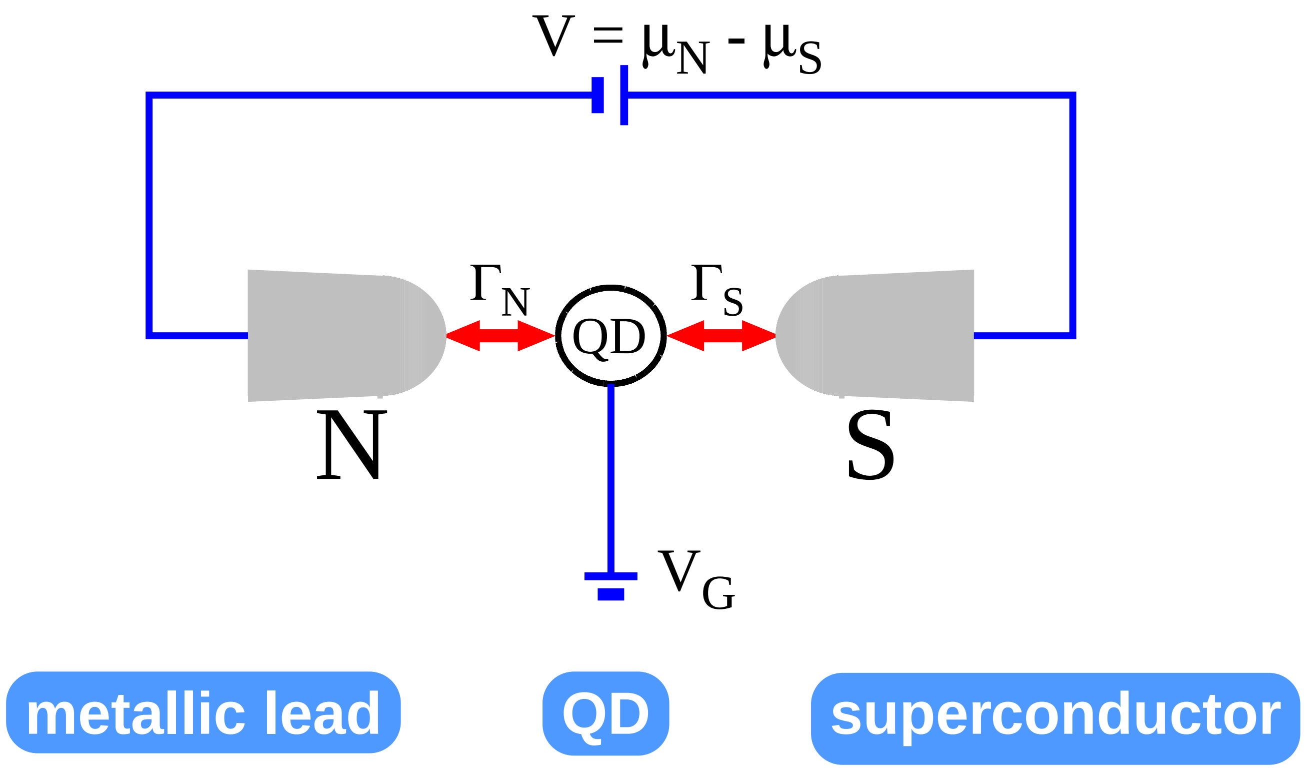

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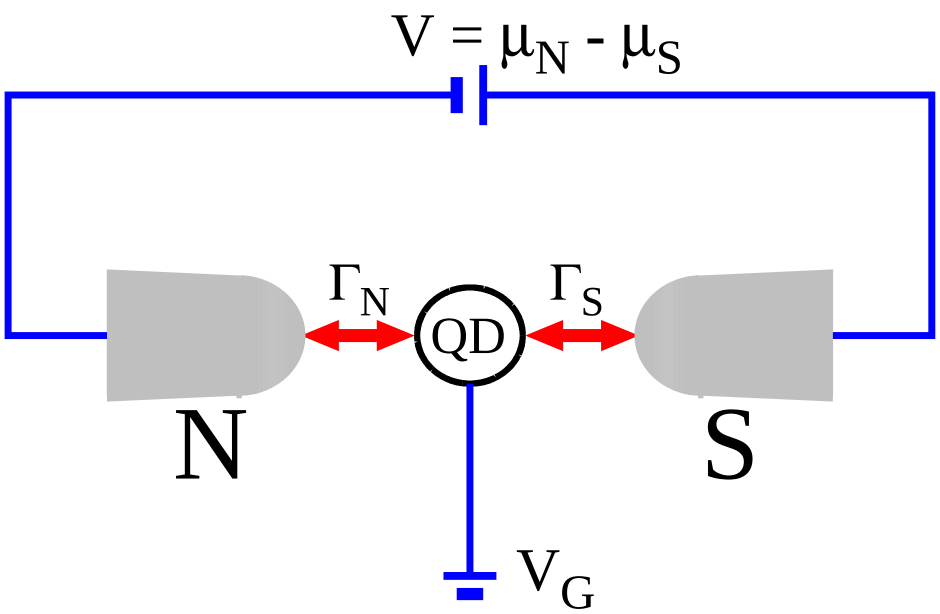

metallic lead

QD

superconductor

This setup can be thought of as a particular version of the SET.

Andreev spectroscopy – experimental realization # 1

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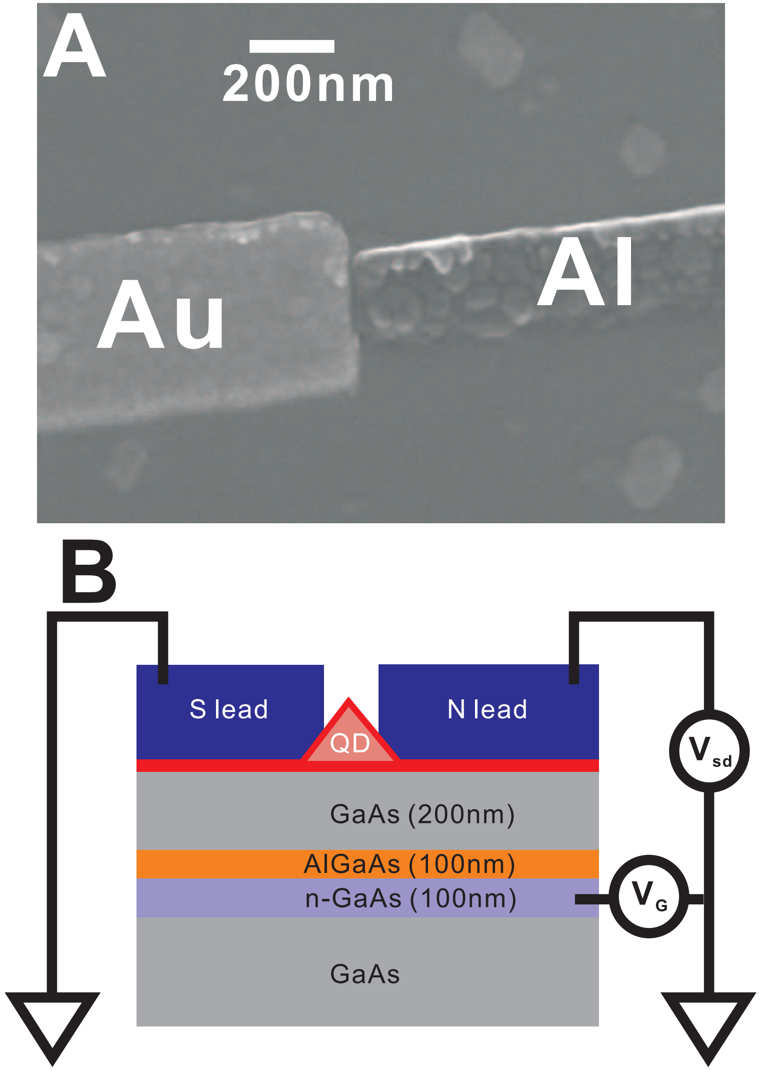

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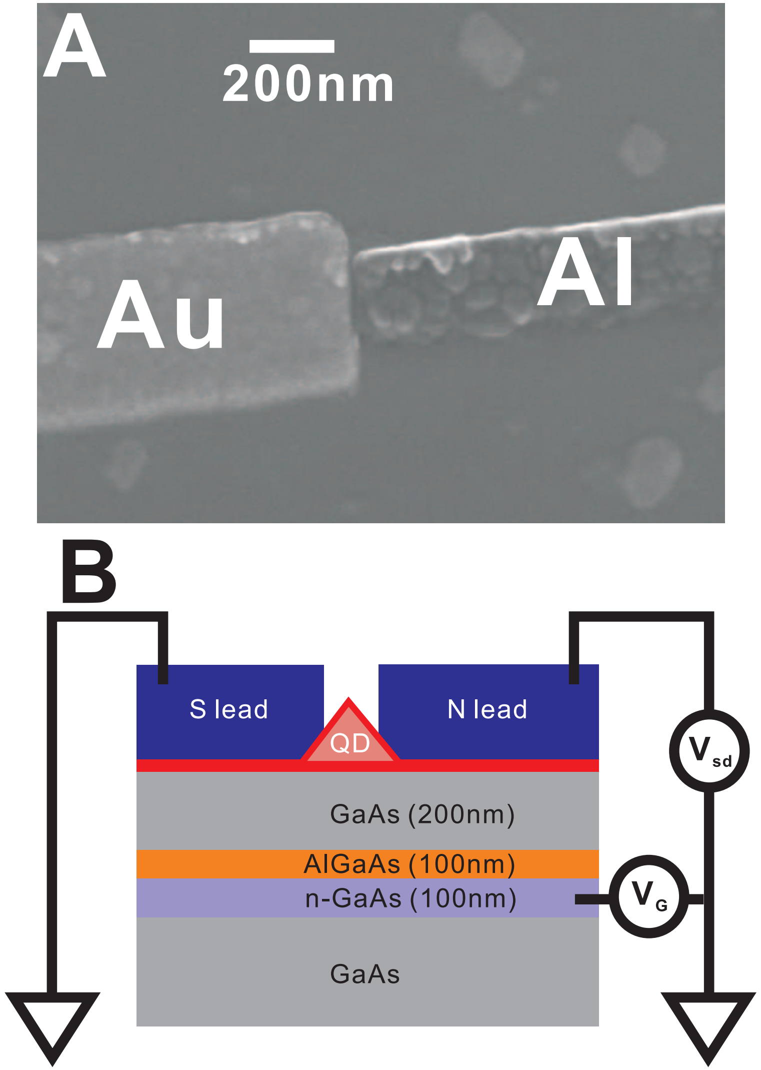

QD : self-assembled InAs

diameter ~ 100 nm

backgate : Si-doped GaAs

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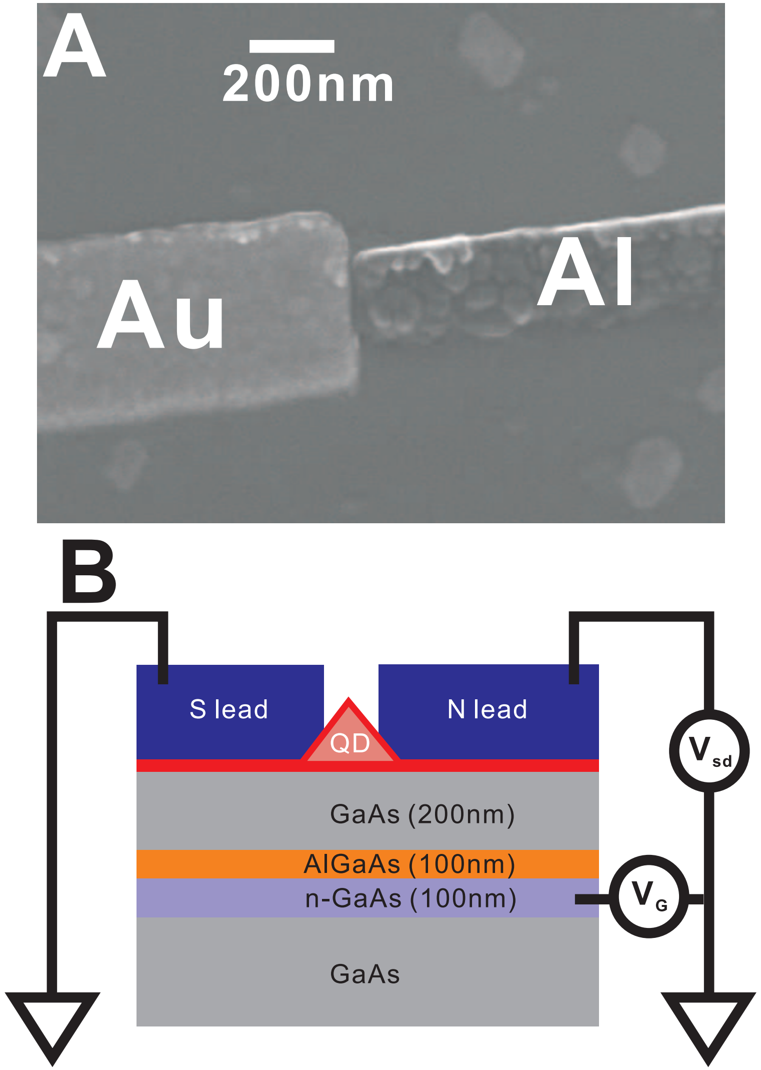

$T_c \simeq 1$K

$\Delta \simeq 152\mu$eV

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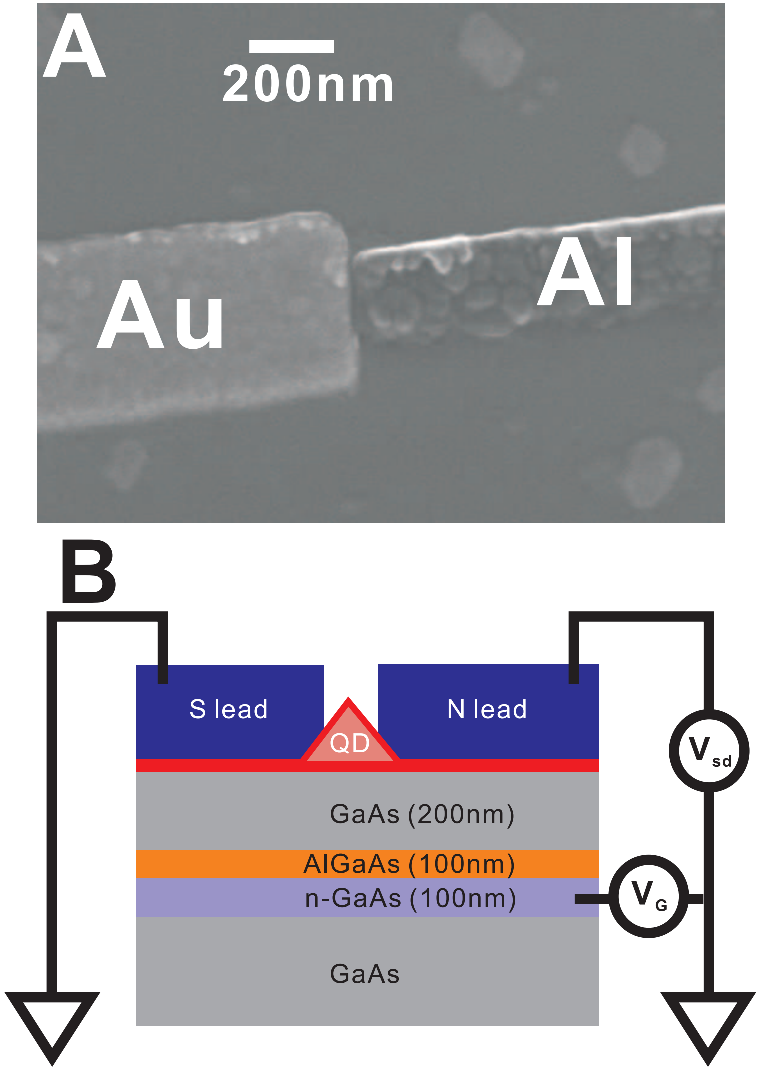

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R.S. Deacon et al, *Phys. Rev. Lett.* **104**, *076805 (2010).*

Andreev spectroscopy – experimental realization # 2

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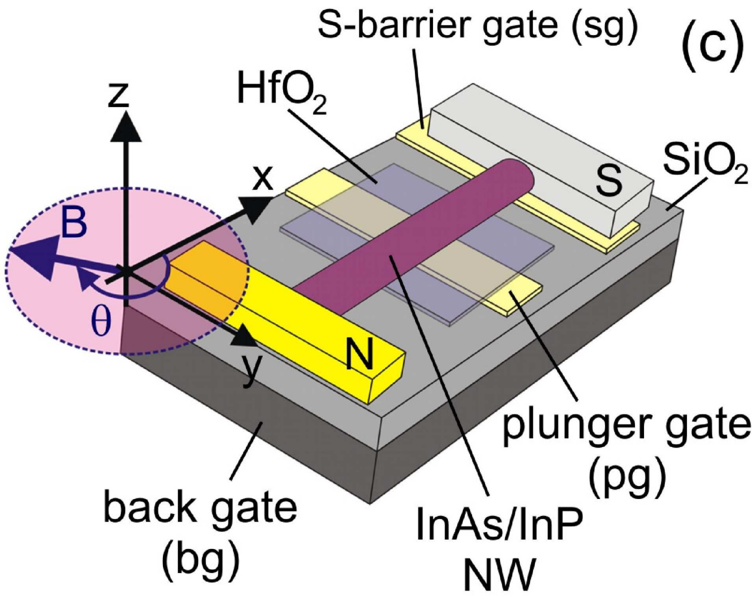

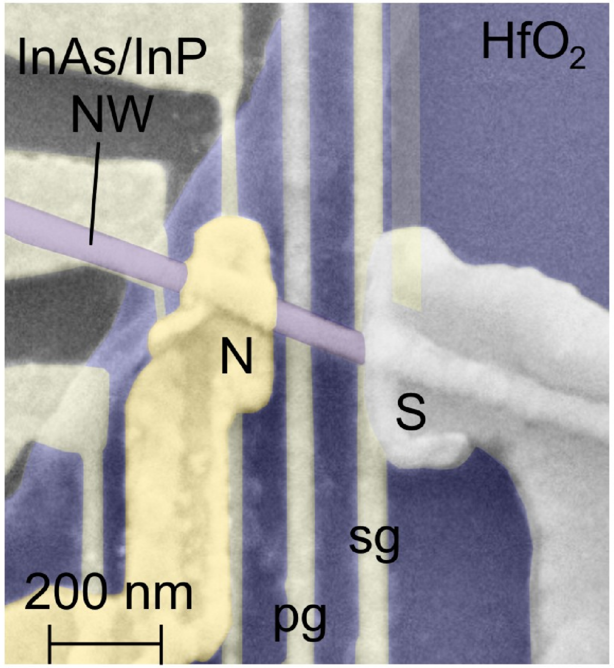

QD : semiconducting InAs/InP nanowire

S : vanadium

$\Delta \simeq 0.55\text{meV}$

$U \simeq 3 - 10\Delta$

N : gold

S. Di Franceschi and coworkers, arXiv:1302.2611 (2013).

Andreev spectroscopy – experimental realization # 3

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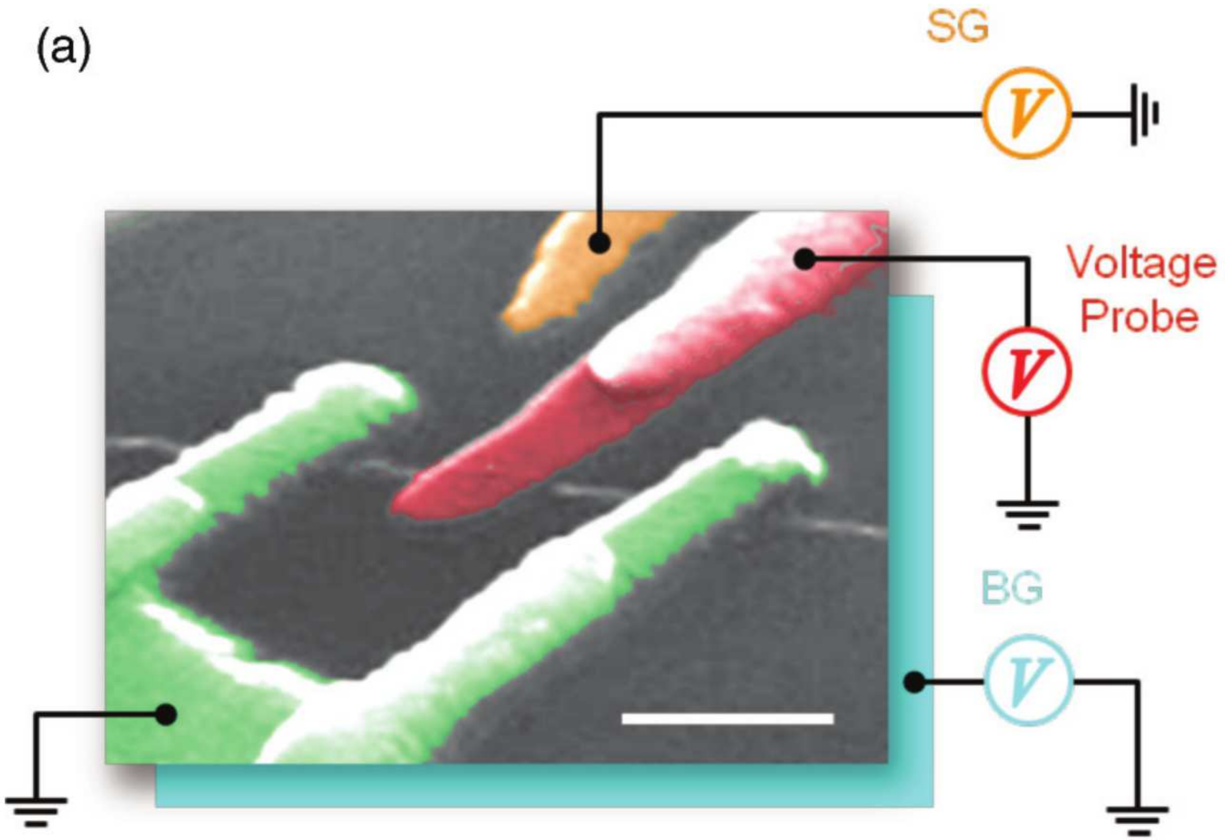

QD : carbon nanotube

$U \simeq 2$ meV

$\Delta \simeq 0.15$meV

J.-D. Pillet, R. Žitko, and M.F. Goffman, Phys. Rev. B **88**, *045101 (2013).*

Andreev spectroscopy – experimental realizations

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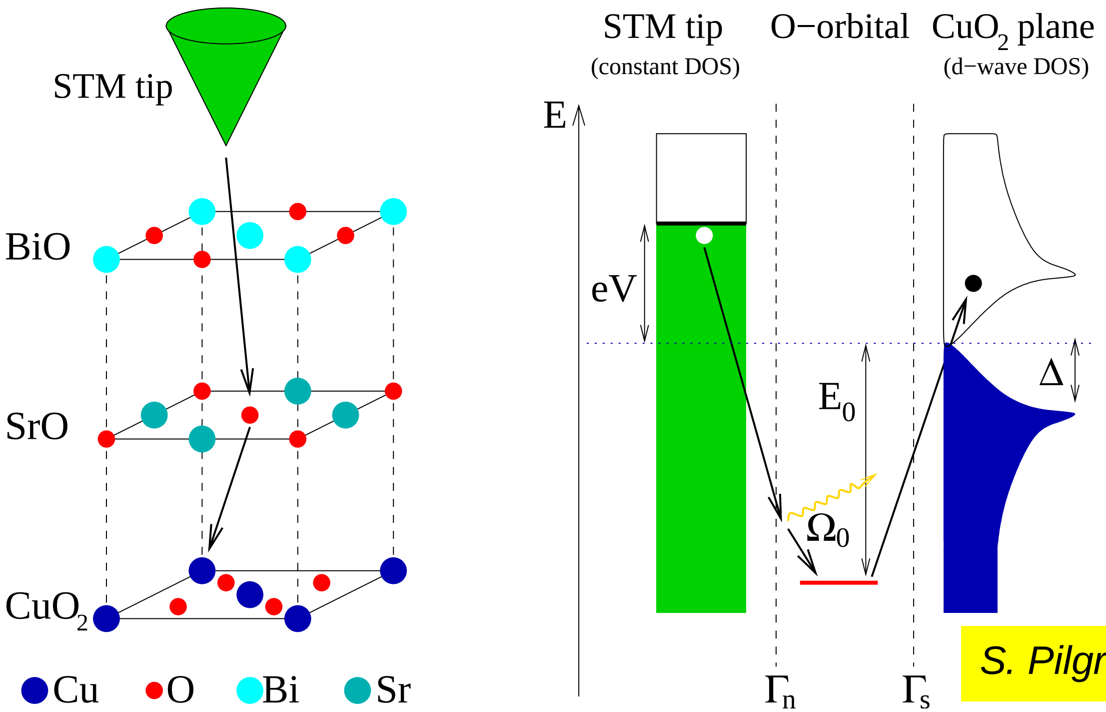

S. Pilgram et al, *Phys. Rev. Lett.* **97**, *117003 (2006).*

In such STM configuration the apex oxygen plays a role analogous to the QD in the N-QD-S setup.

Physical situation – energy spectrum

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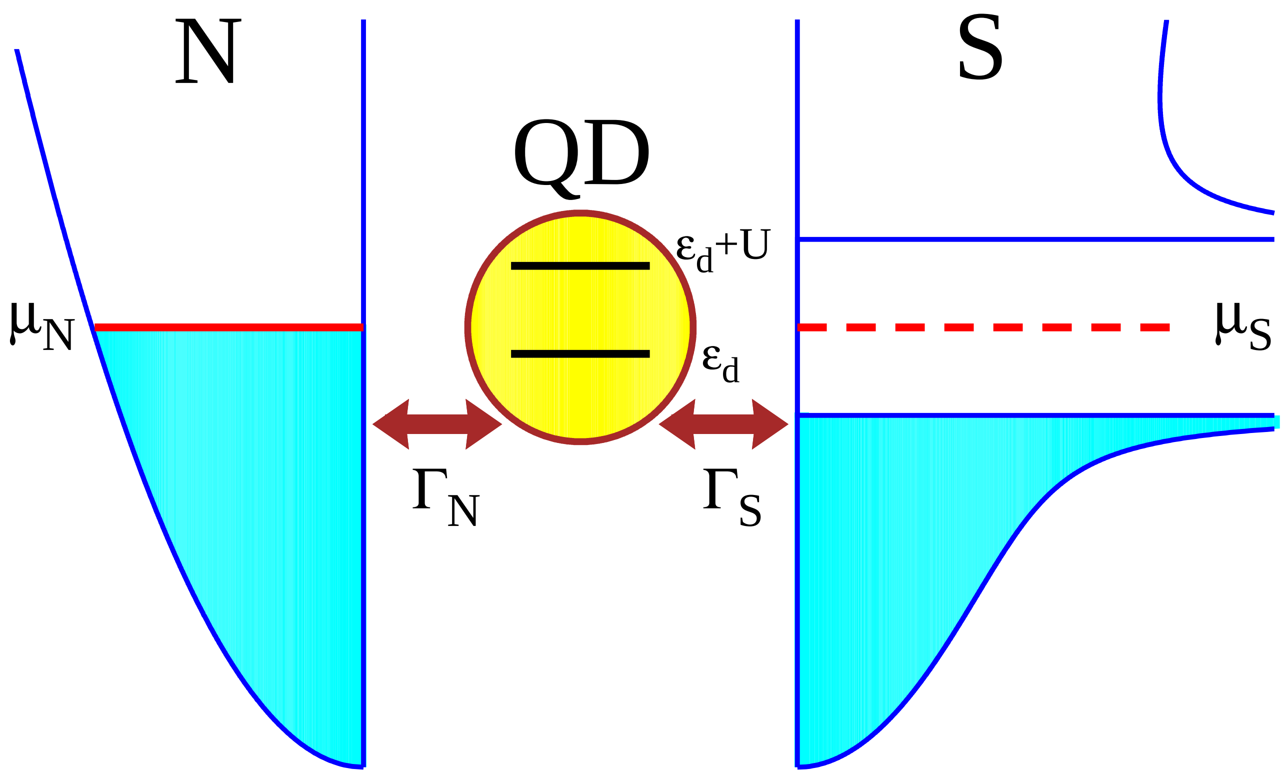

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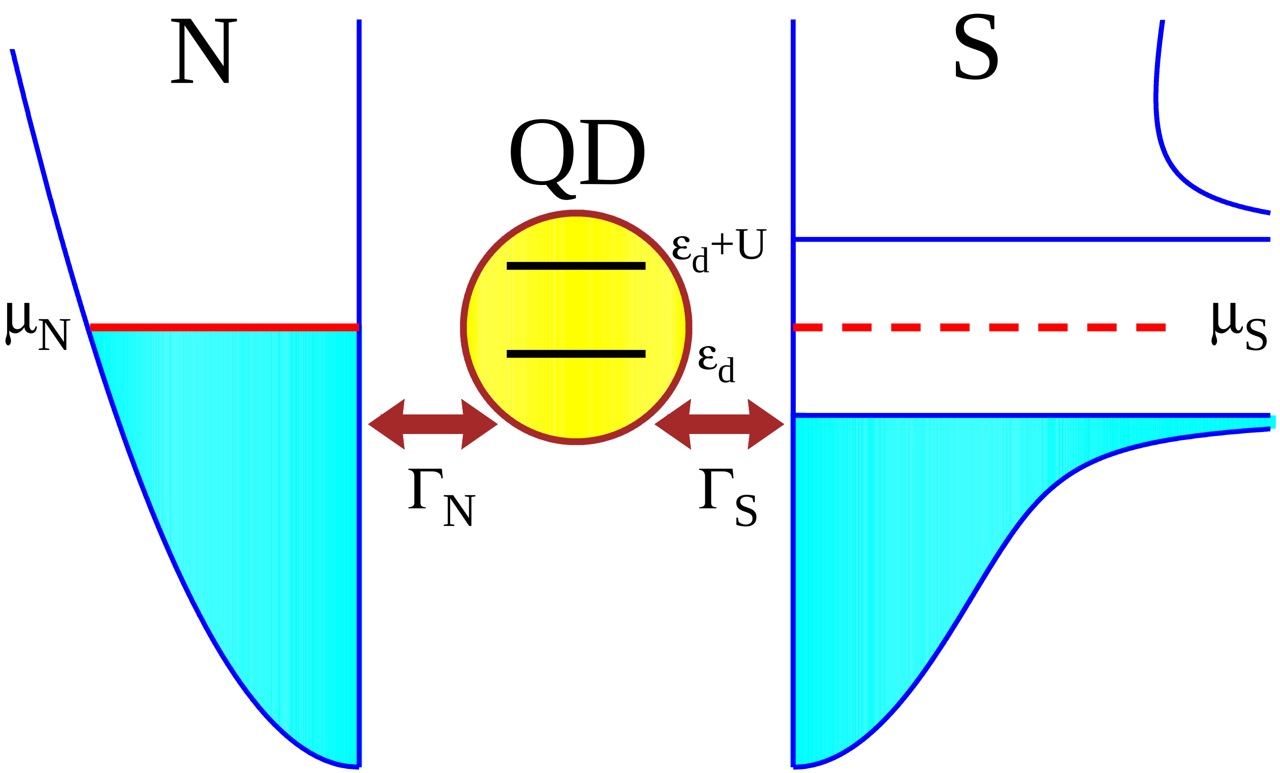

External bias $eV = \mu_N - \mu_S$ induces the current(s) through QD.

Microscopic model

The correlation effects

Microscopic model

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$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U \, \hat{n}_{d\uparrow} \, \hat{n}_{d\downarrow}$$

Microscopic model

The correlation effects

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are expected to affect the transport properties of the system

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Microscopic model

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where

$$\hat{H}_N = \sum_{k,\sigma} \left(\varepsilon_{k,N} - \mu_N \right) \hat{c}^{\dagger}_{k\sigma N} \hat{c}_{k\sigma N}$$

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Formal aspects

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$$G_d(\tau,\tau') = - \begin{pmatrix} \hat{T}_\tau \langle \hat{d}_\uparrow(\tau) \hat{d}_\uparrow^\dagger(\tau') \rangle & \hat{T}_\tau \langle \hat{d}_\uparrow(\tau) \hat{d}_\downarrow(\tau') \rangle \\ \hat{T}_\tau \langle \hat{d}_\downarrow^\dagger(\tau) \hat{d}_\uparrow^\dagger(\tau') \rangle & \hat{T}_\tau \langle \hat{d}_\downarrow^\dagger(\tau) \hat{d}_\downarrow(\tau') \rangle \end{pmatrix}$$

Formal aspects

To describe an interplay between the proximity effect and electron correlations we have to determine the matrix Green's function (Nambu representation)

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$$J_A(V) = \frac{2e}{h} \int d\omega \, T_A(\omega) \, [f(\omega+eV,T) - f(\omega-eV,T)]$$

with the transmittance

$$T_1(\omega) = \Gamma_N \Gamma_S \left(\left| G^r_{11}(\omega) \right|^2 + \left| G^r_{12}(\omega) \right|^2 - \frac{2\Delta}{|\omega|} \mathrm{Re} G^r_{11}(\omega) G^r_{12}(\omega) \right)$$

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Relevant problems : issue # 1

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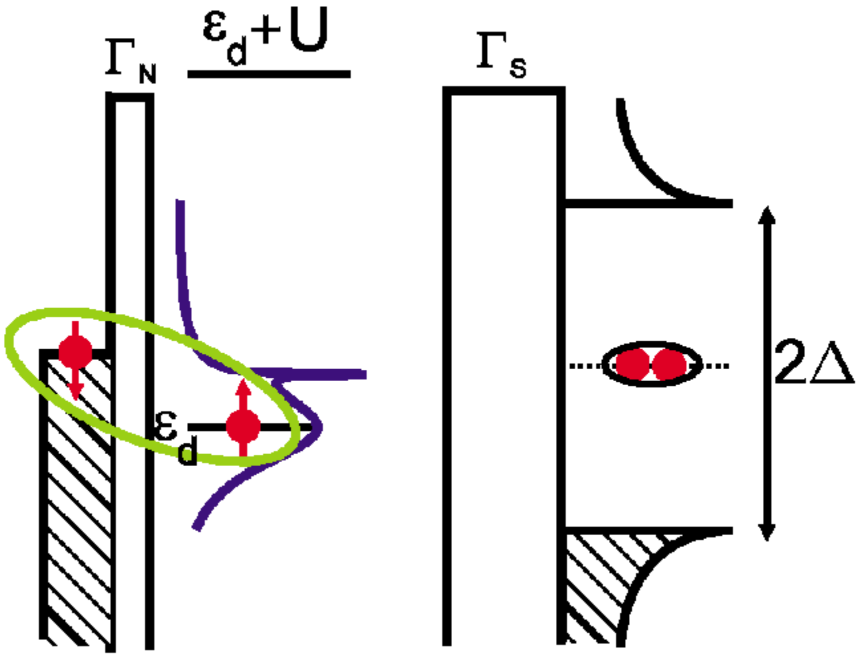

- ★ broadens the QD levels
- ★ induces the Kondo resonance below T_K.

Relevant problems : issue # 2

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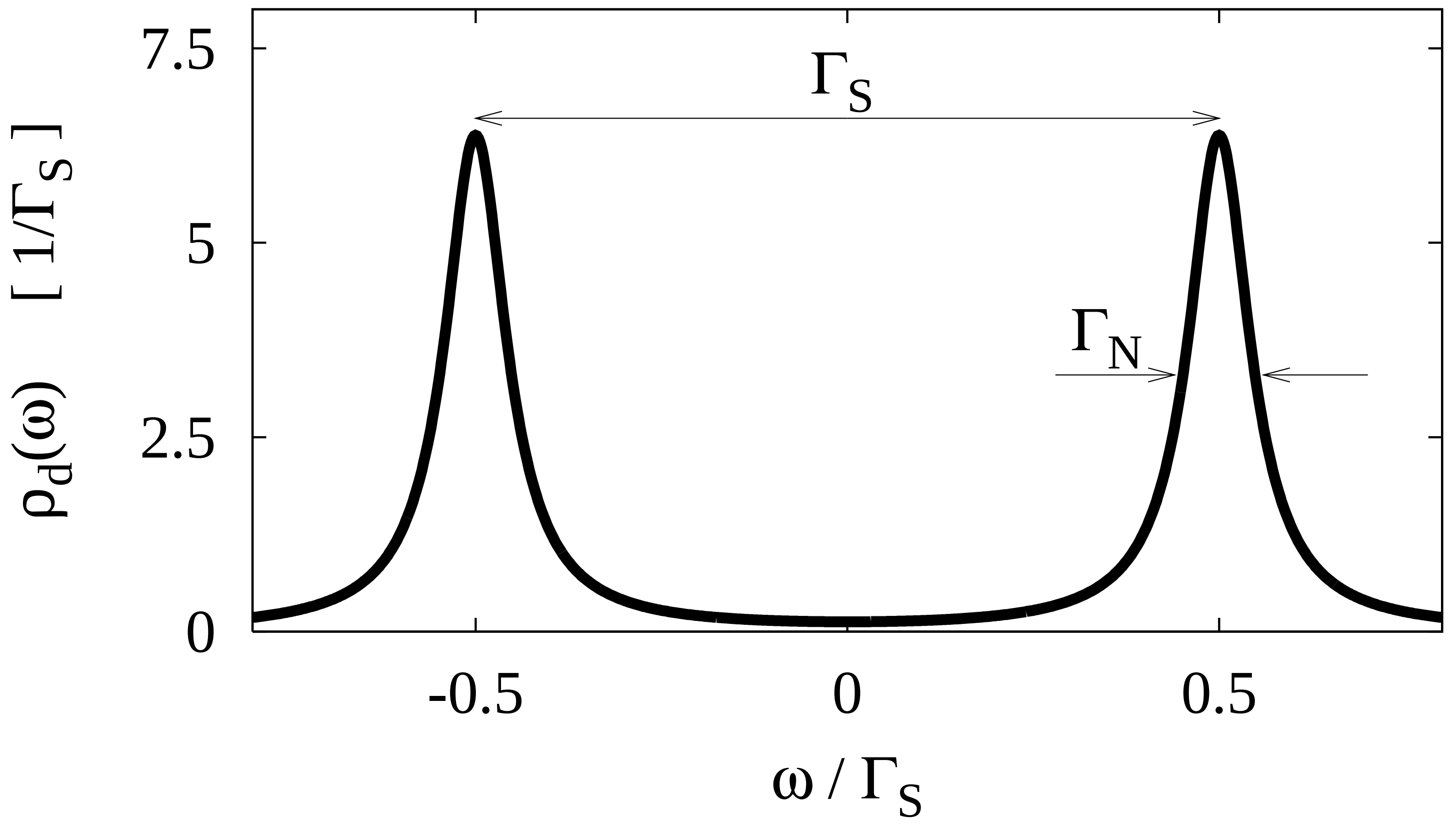

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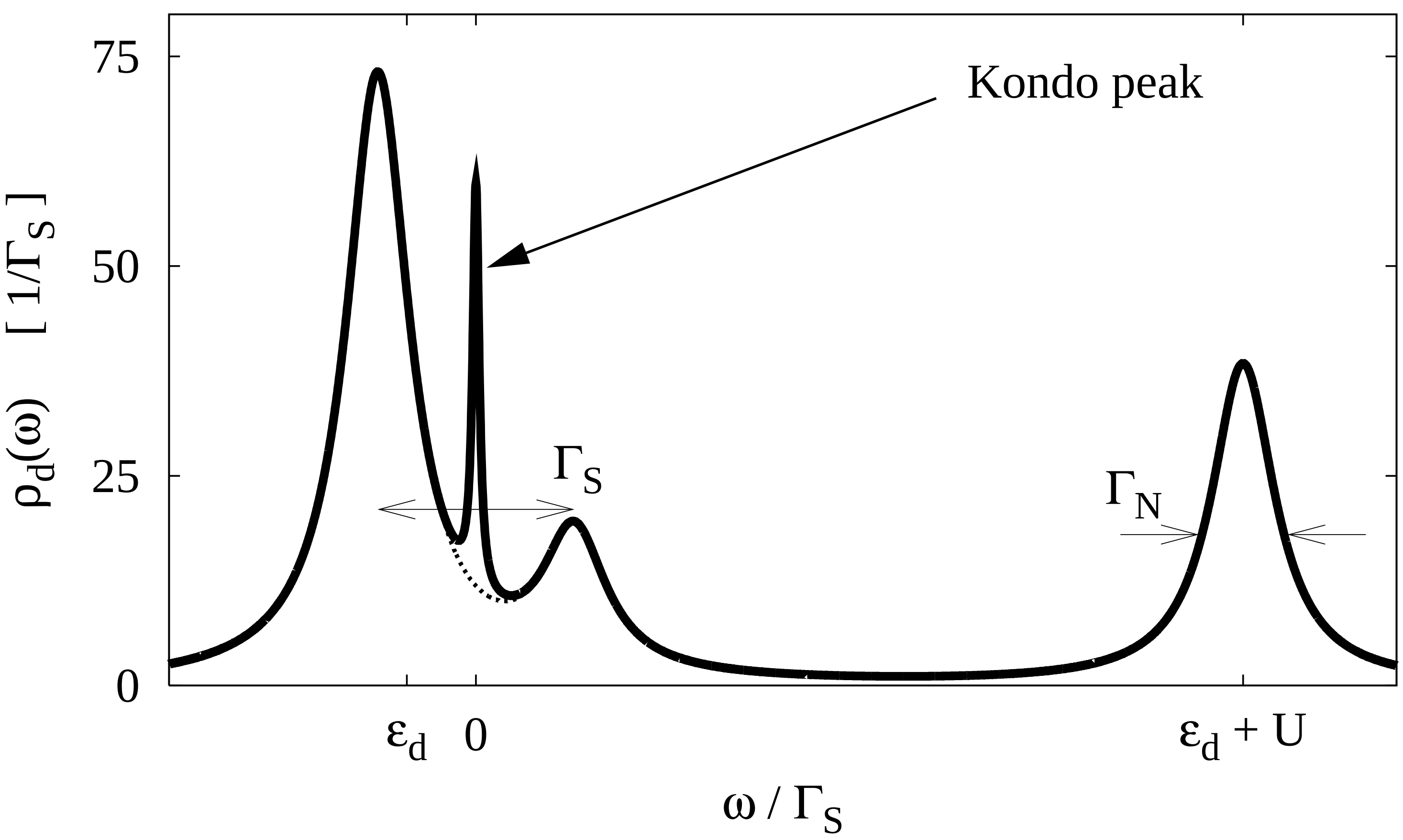

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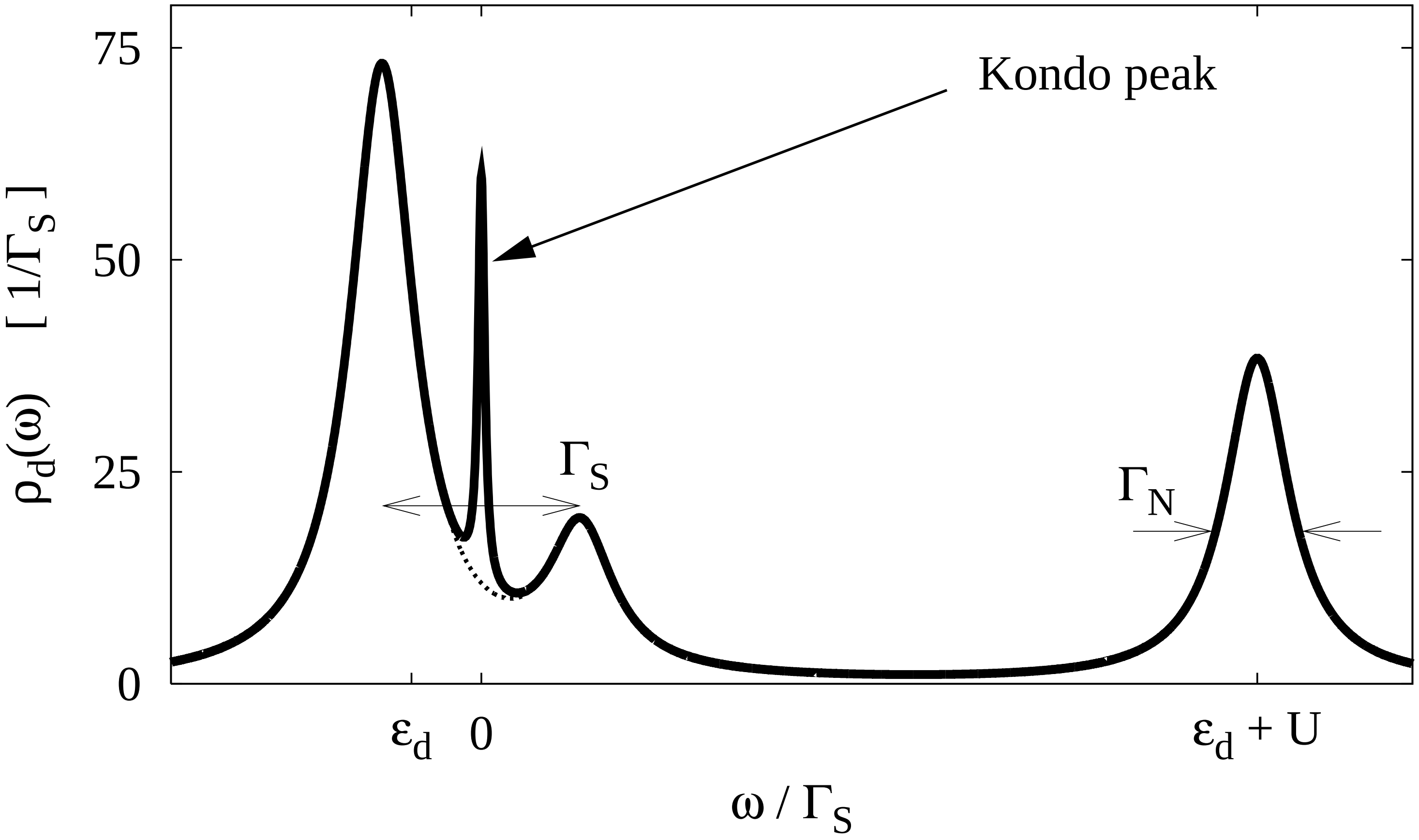

/ interplay between the Kondo effect and superconductivity /

Transport channels

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Qualitative features in the differential conductance $G(V) = \frac{\partial J(V)}{\partial V}$

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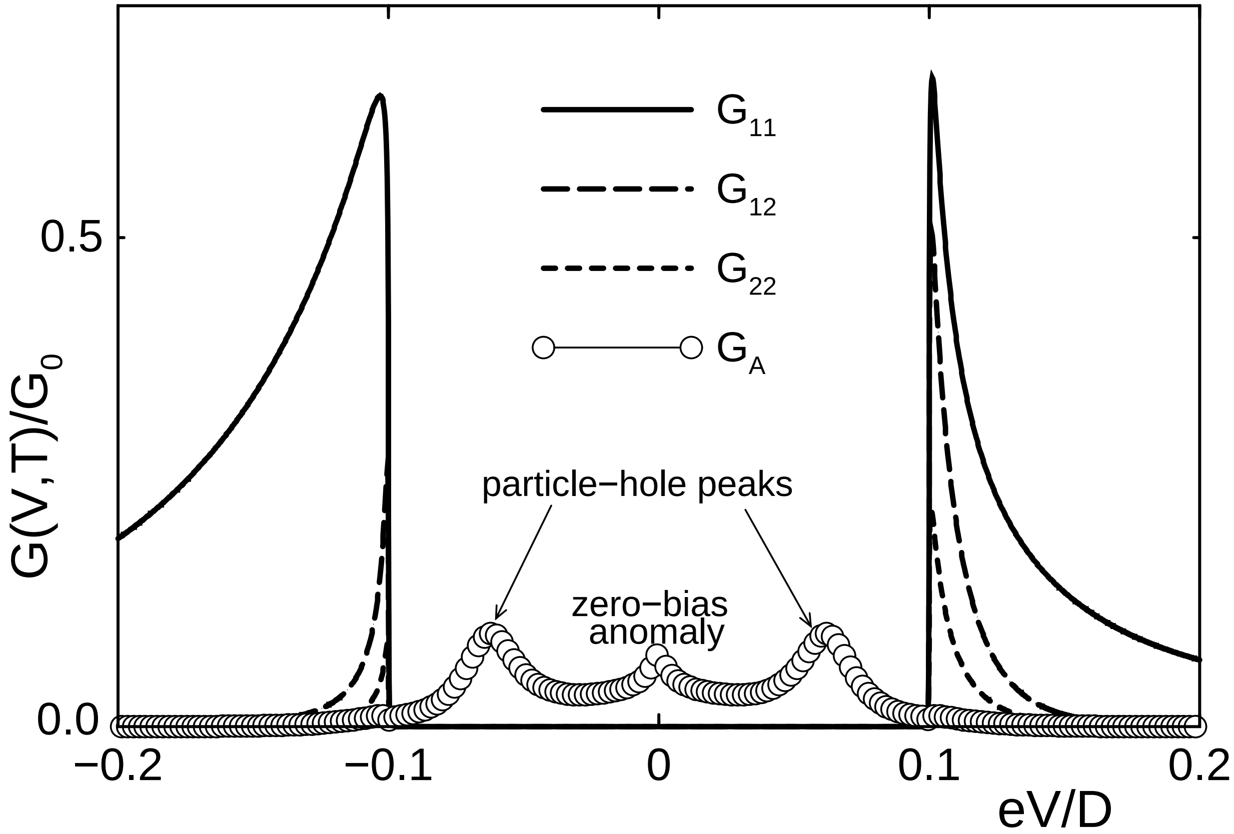

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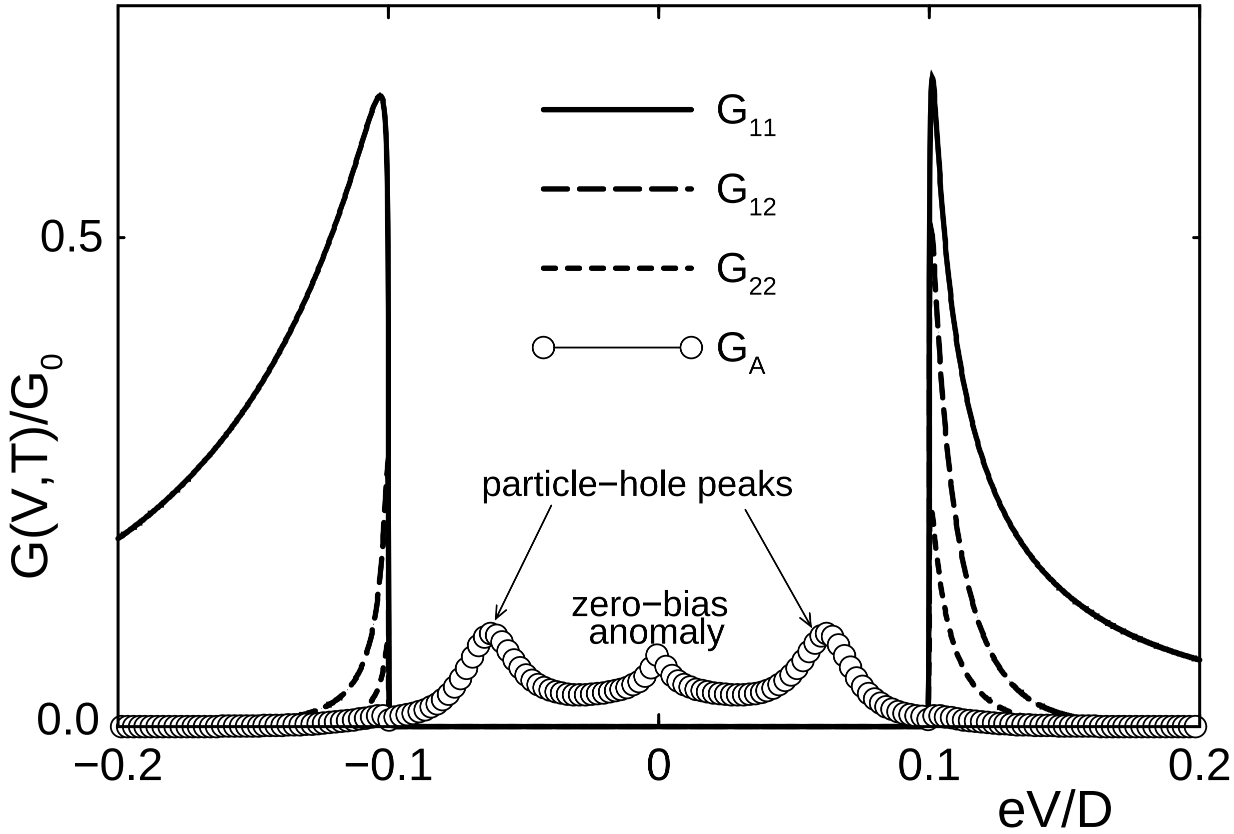

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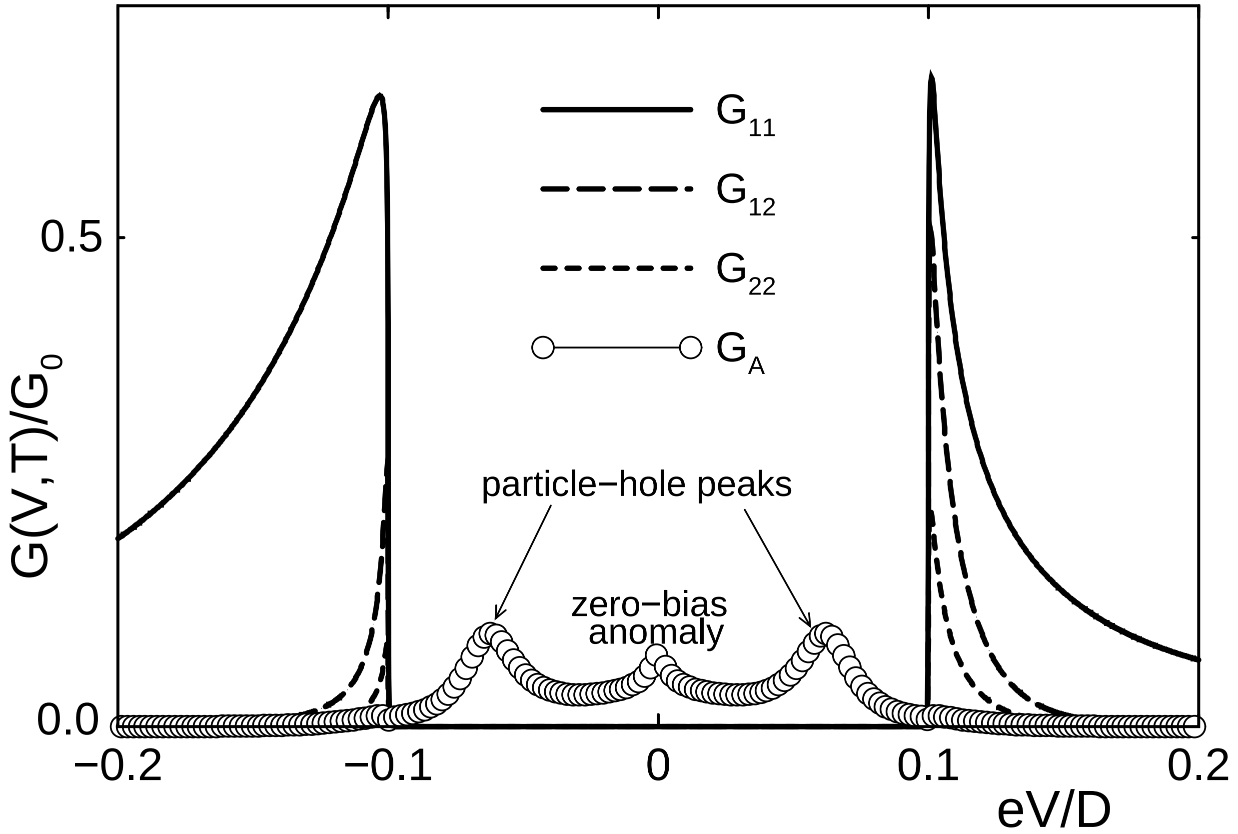

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We shall now focus on the subgap Andreev conductance.

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Spectral function obtained below T_K for $U = 10\Gamma_N$

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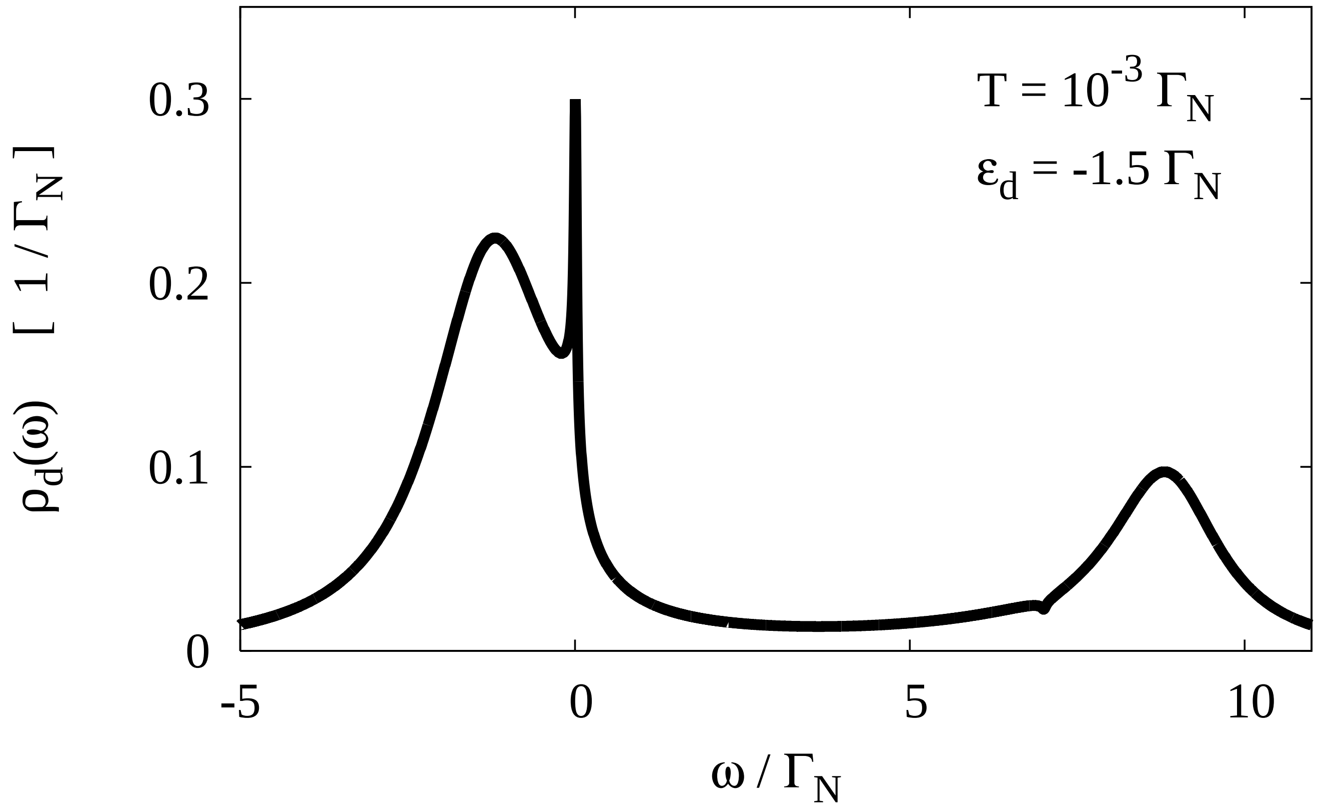

$$\Gamma_S/\Gamma_N = 0$$

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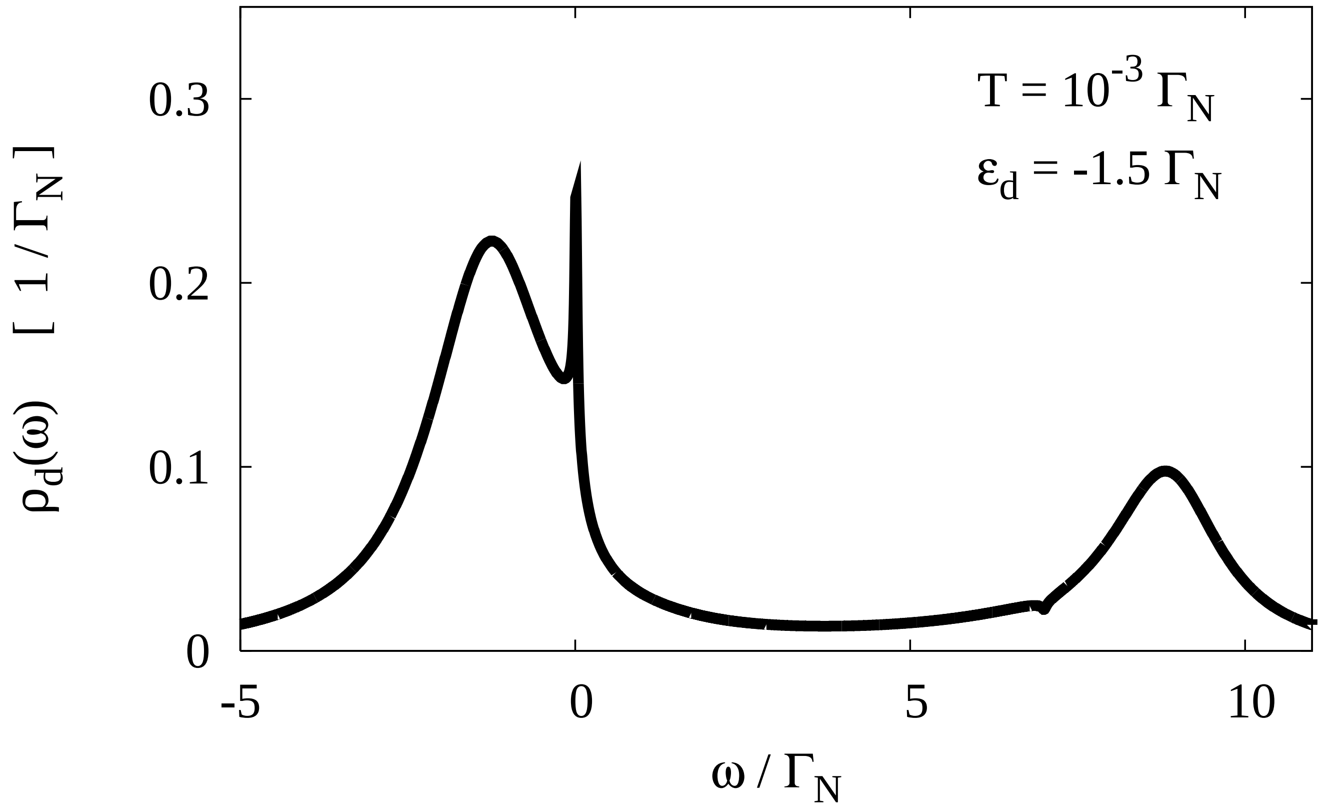

$$\Gamma_S/\Gamma_N = 1$$

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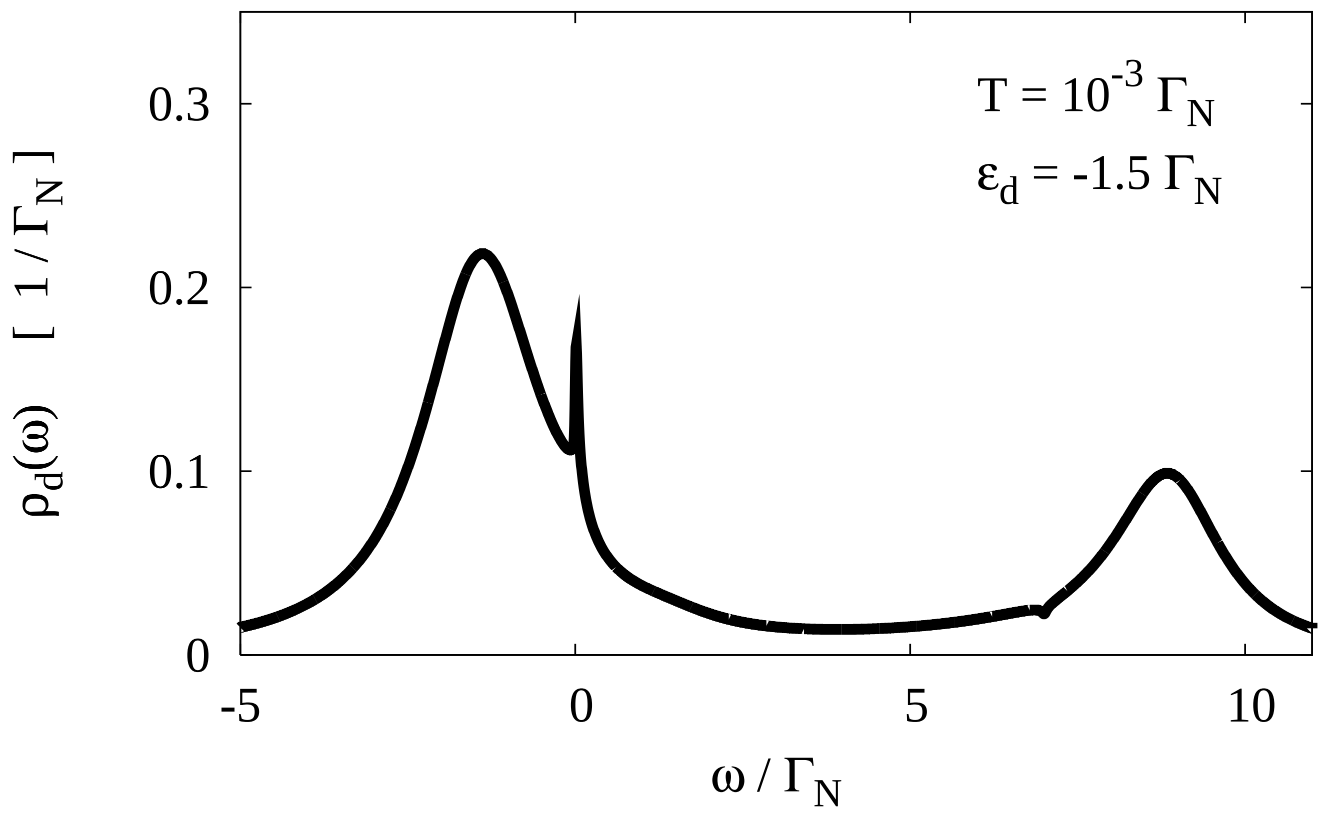

$$\Gamma_S/\Gamma_N = 2$$

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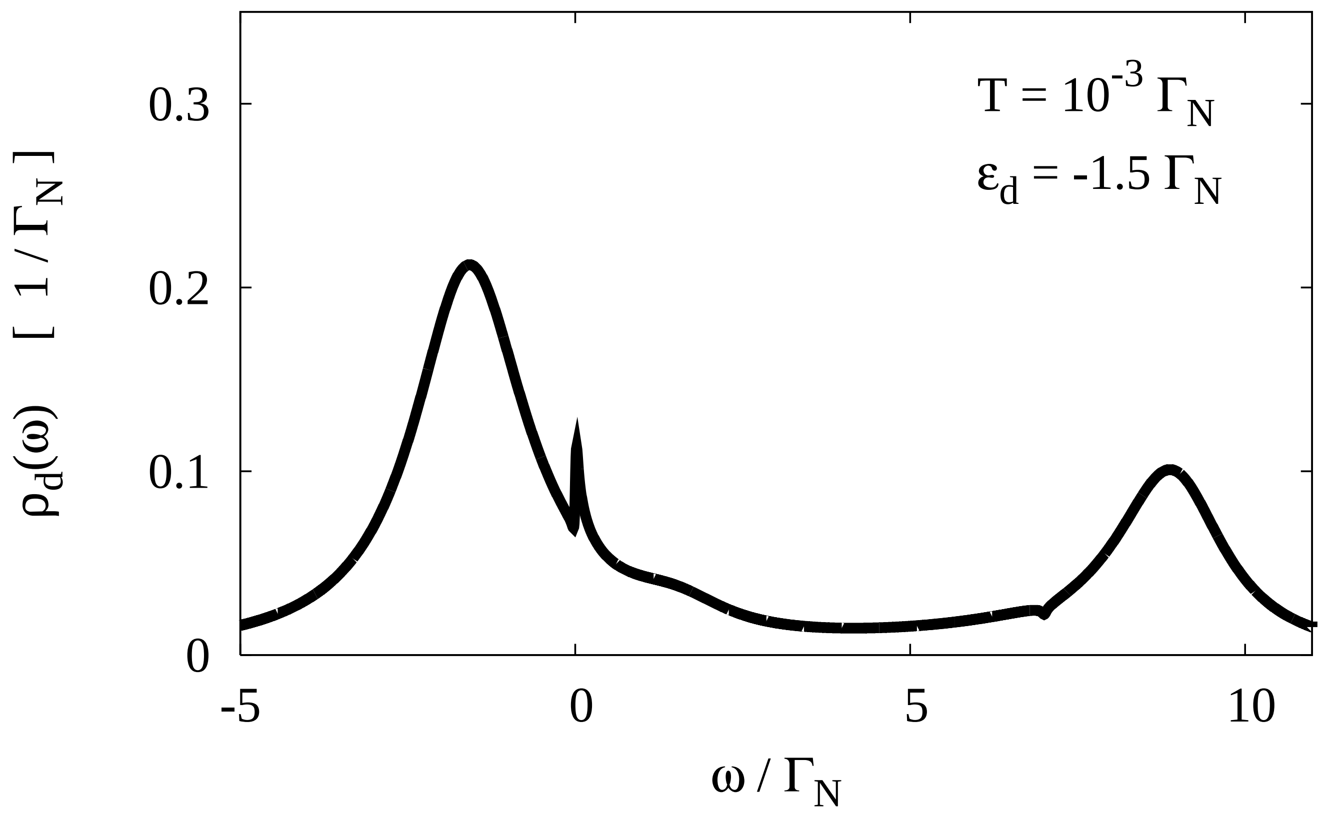
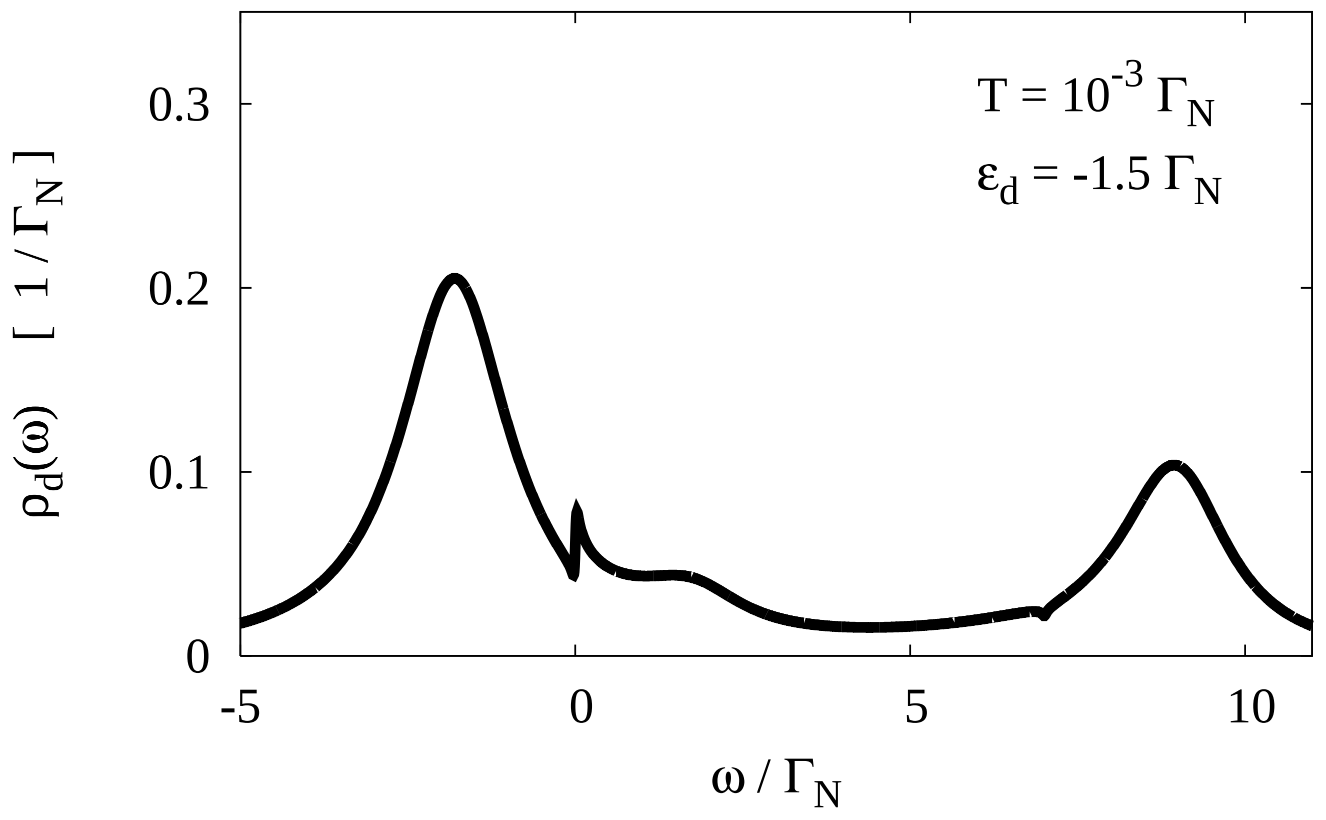


$\Gamma_S/\Gamma_N = 3$

Spectral function obtained below T_K for $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 4$$

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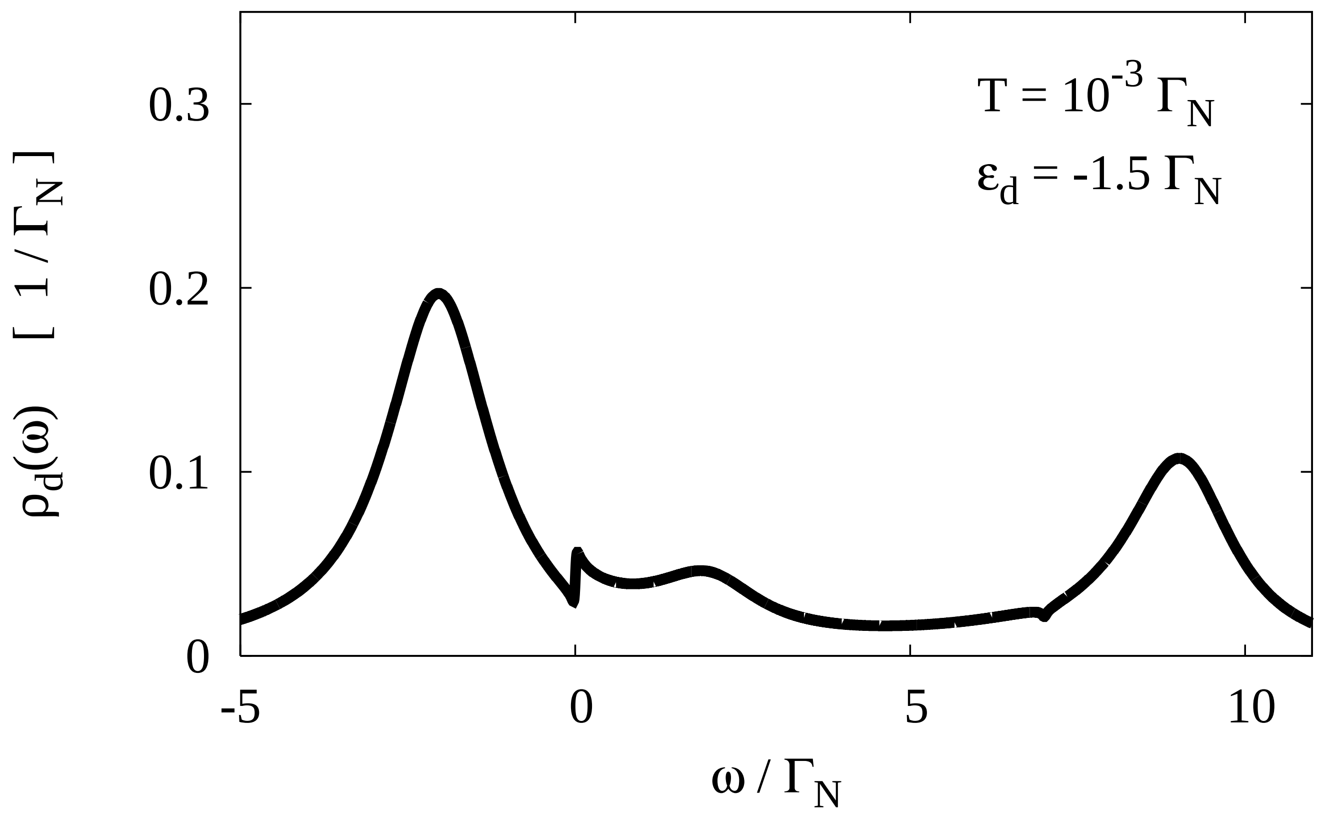

$\Gamma_S/\Gamma_N = 5$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

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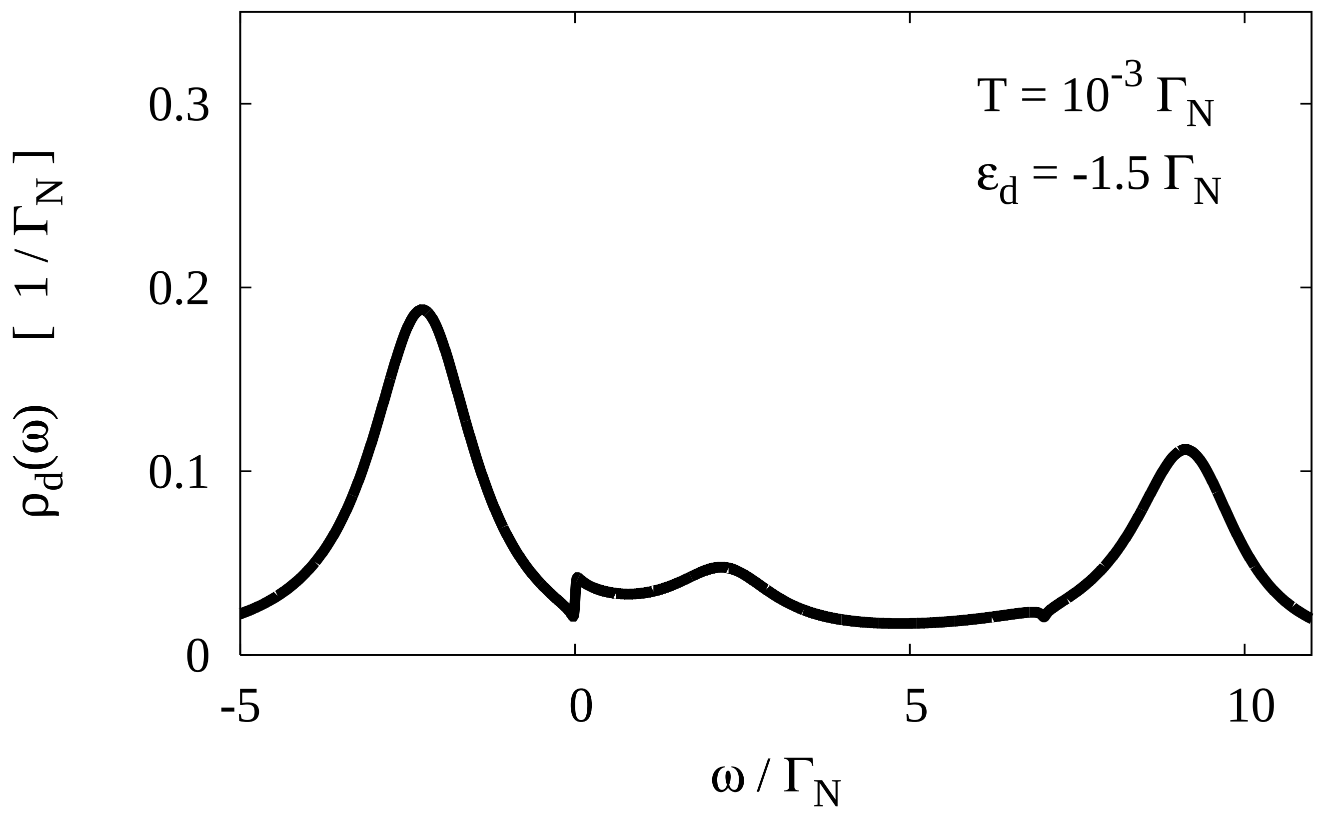

$$\Gamma_S/\Gamma_N = 6$$

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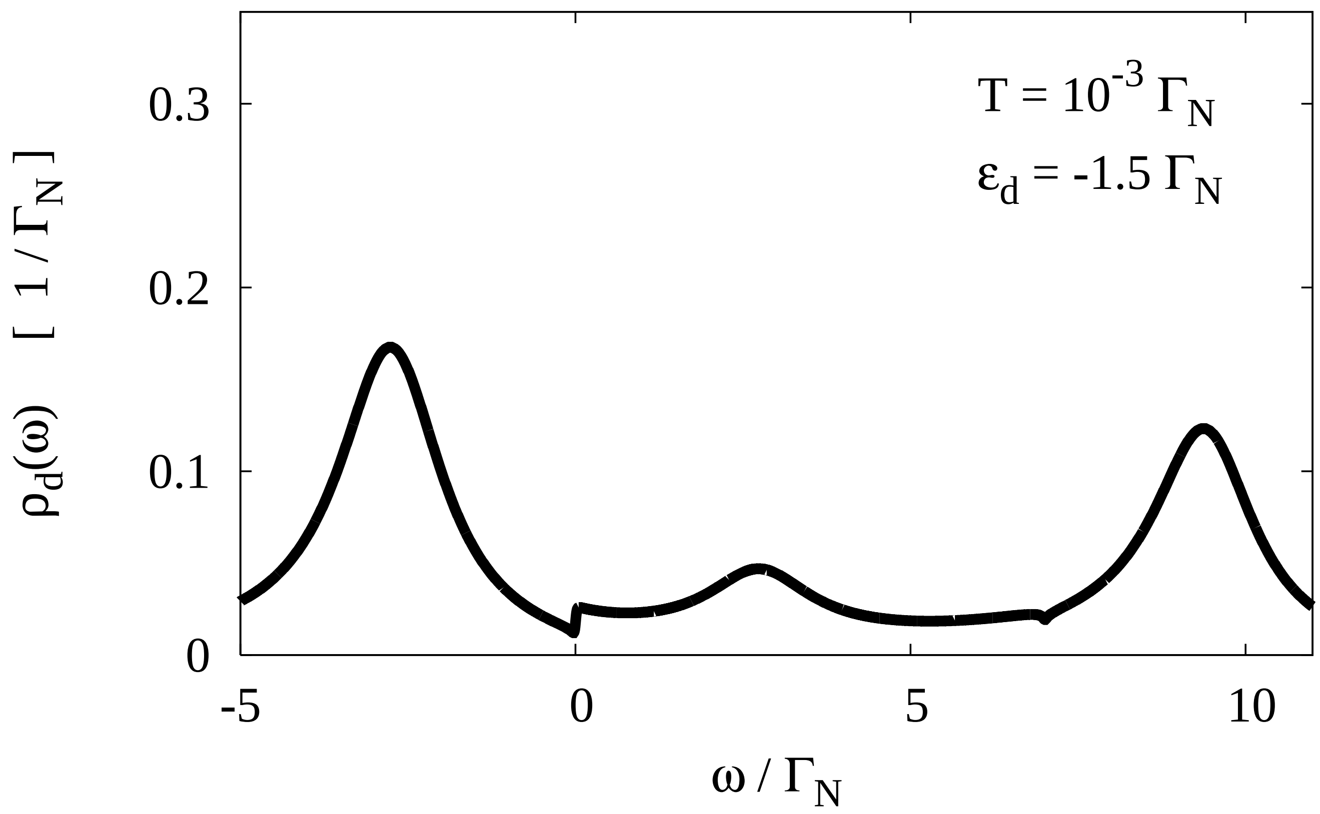

$$\Gamma_S/\Gamma_N = 8$$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

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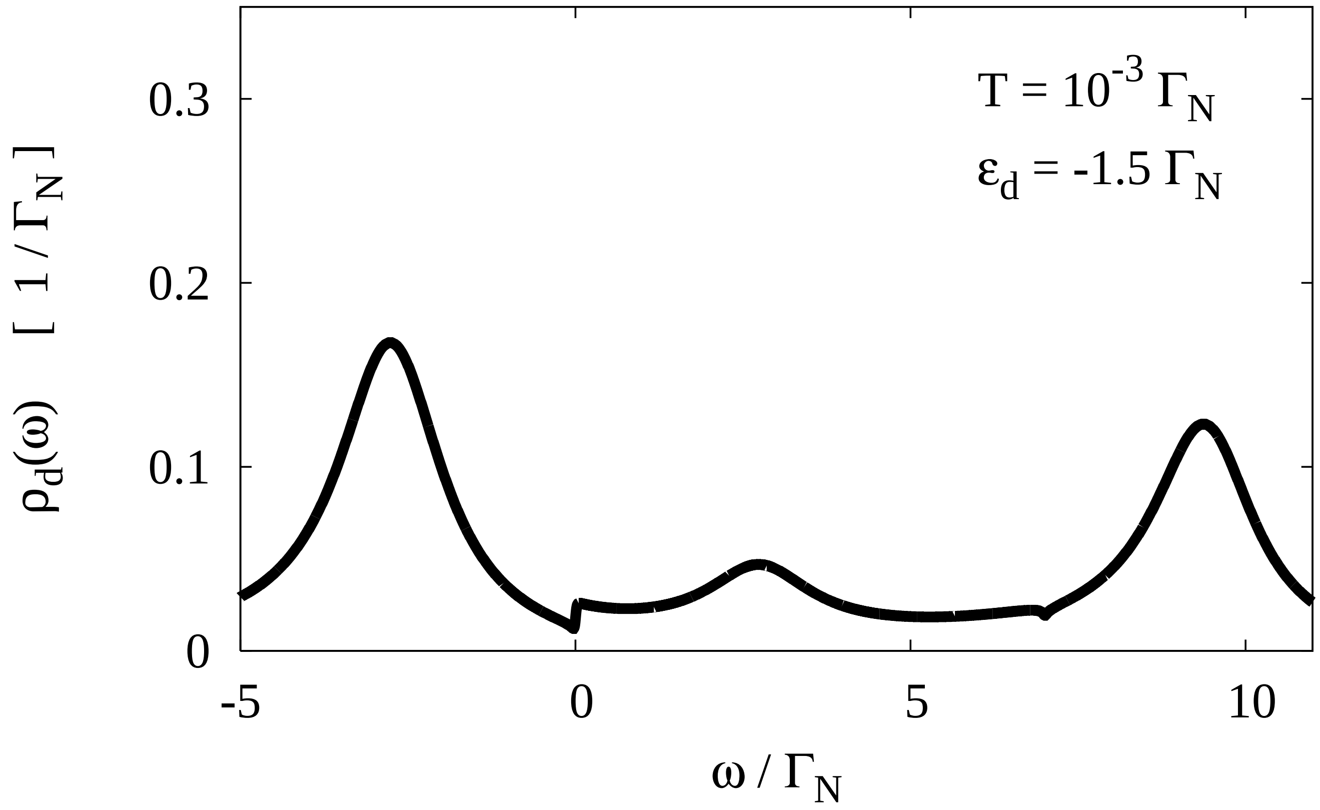

Superconductivity suppresses the Kondo resonance

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Andreev conductance $G_A(V)$ for: $U = 10\Gamma_N$

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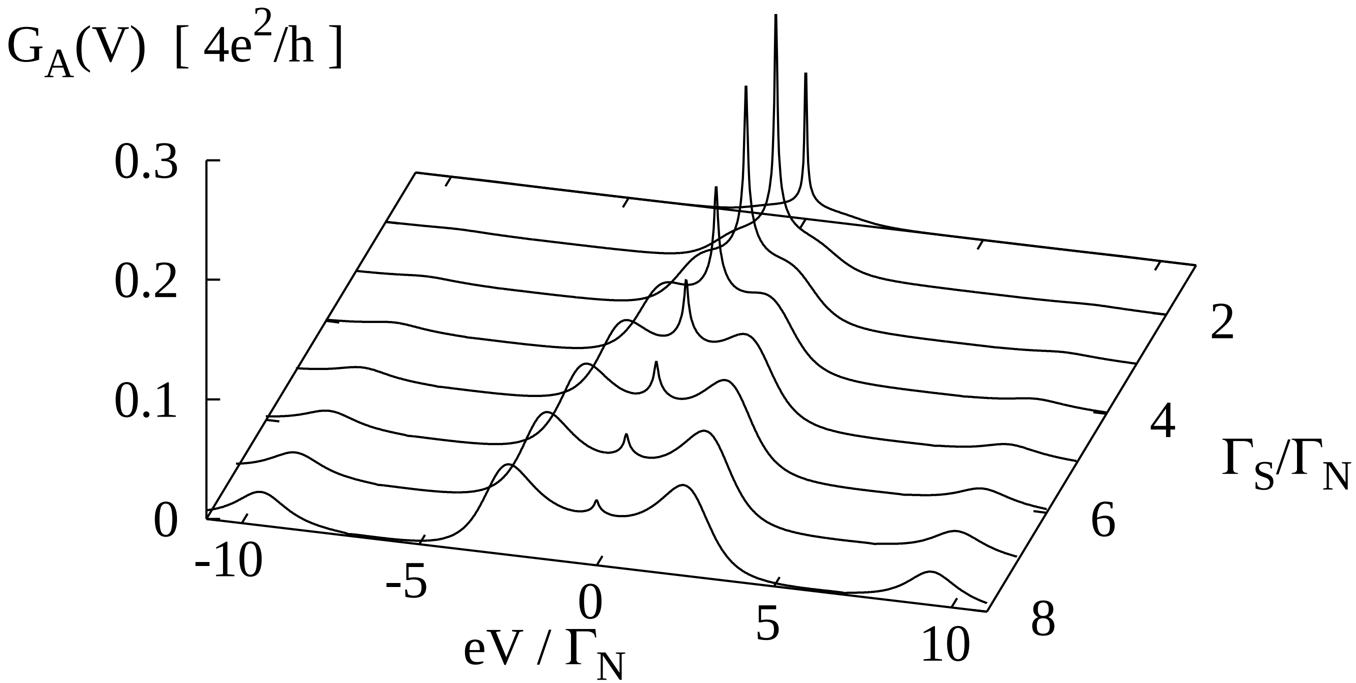

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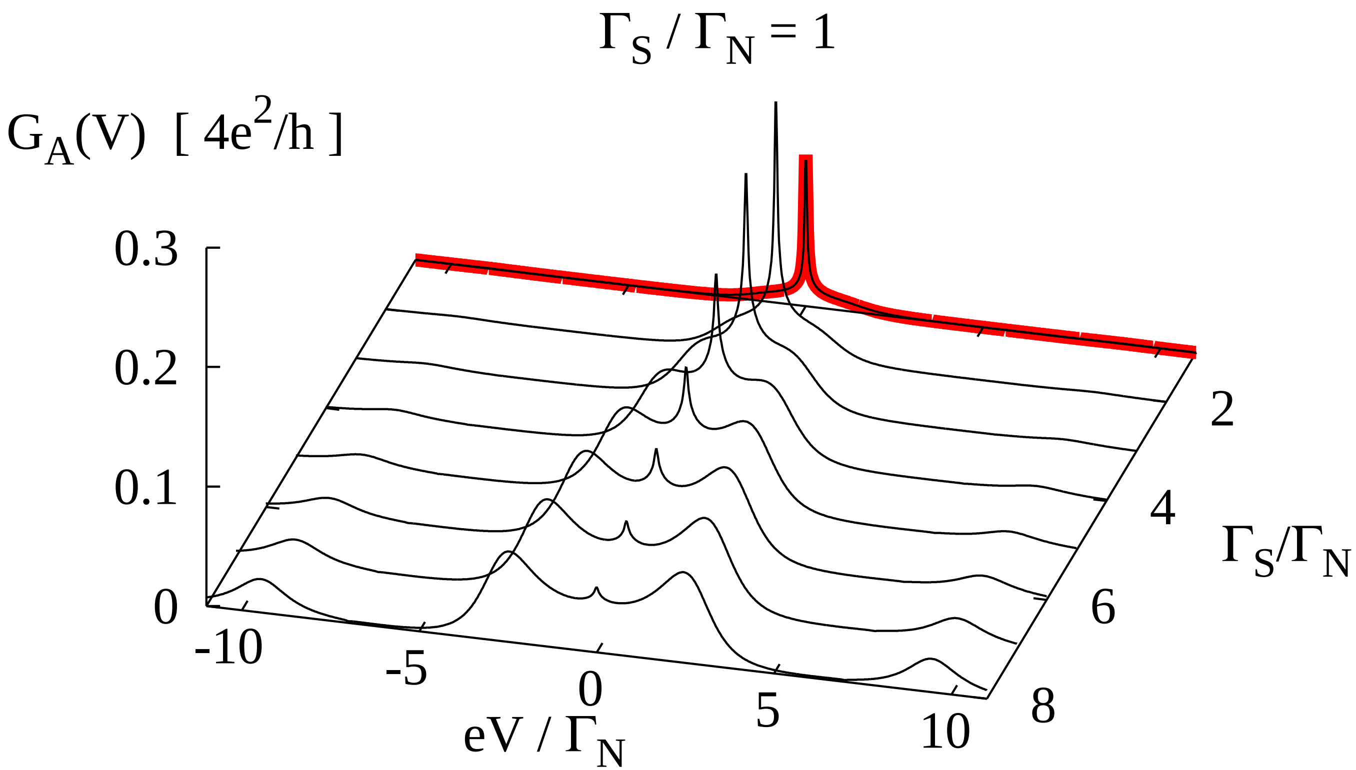

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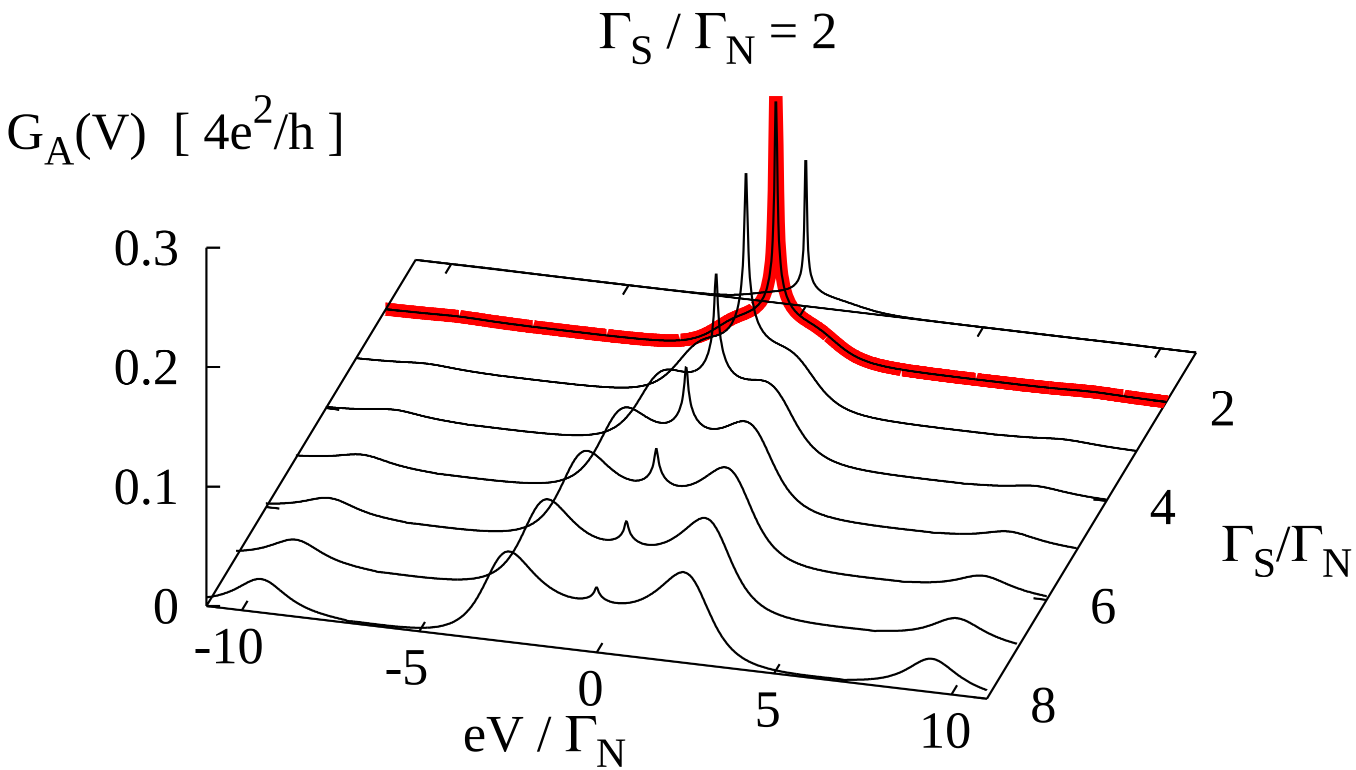

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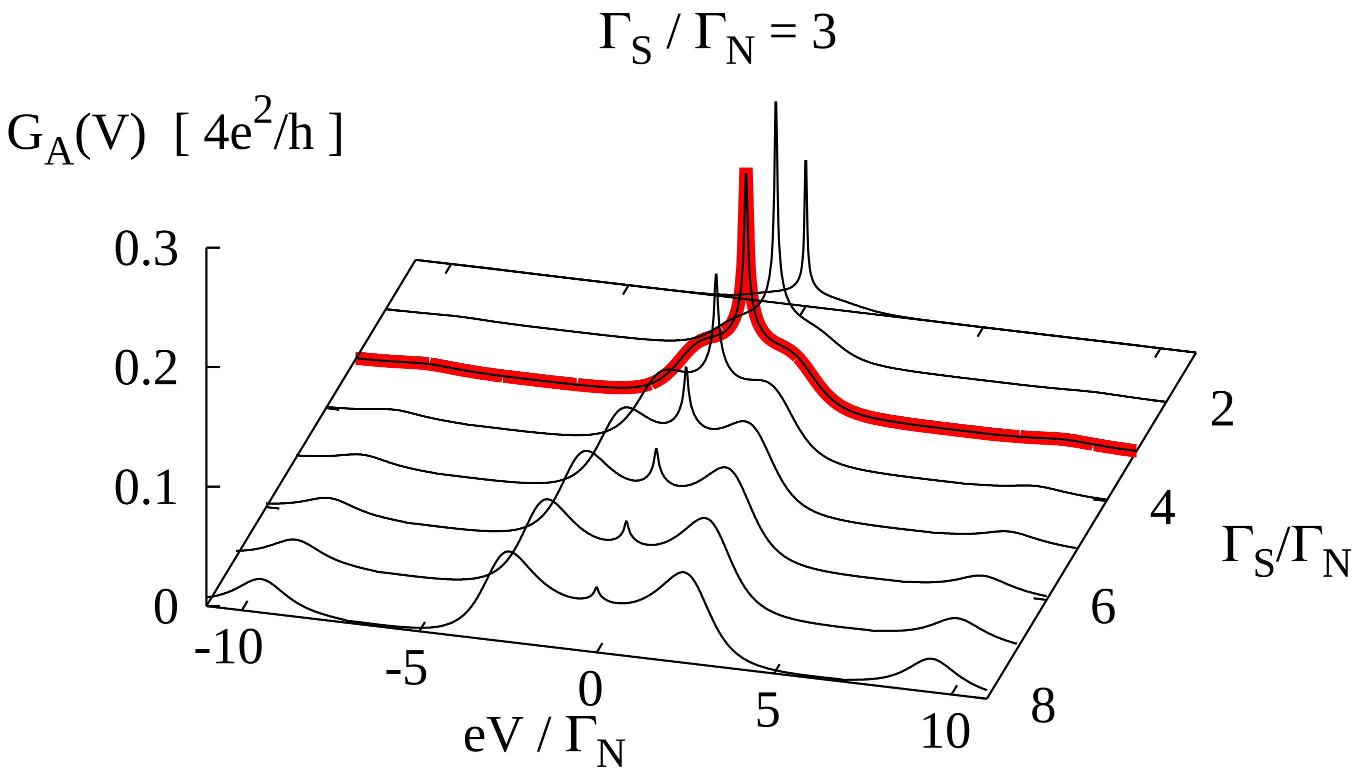

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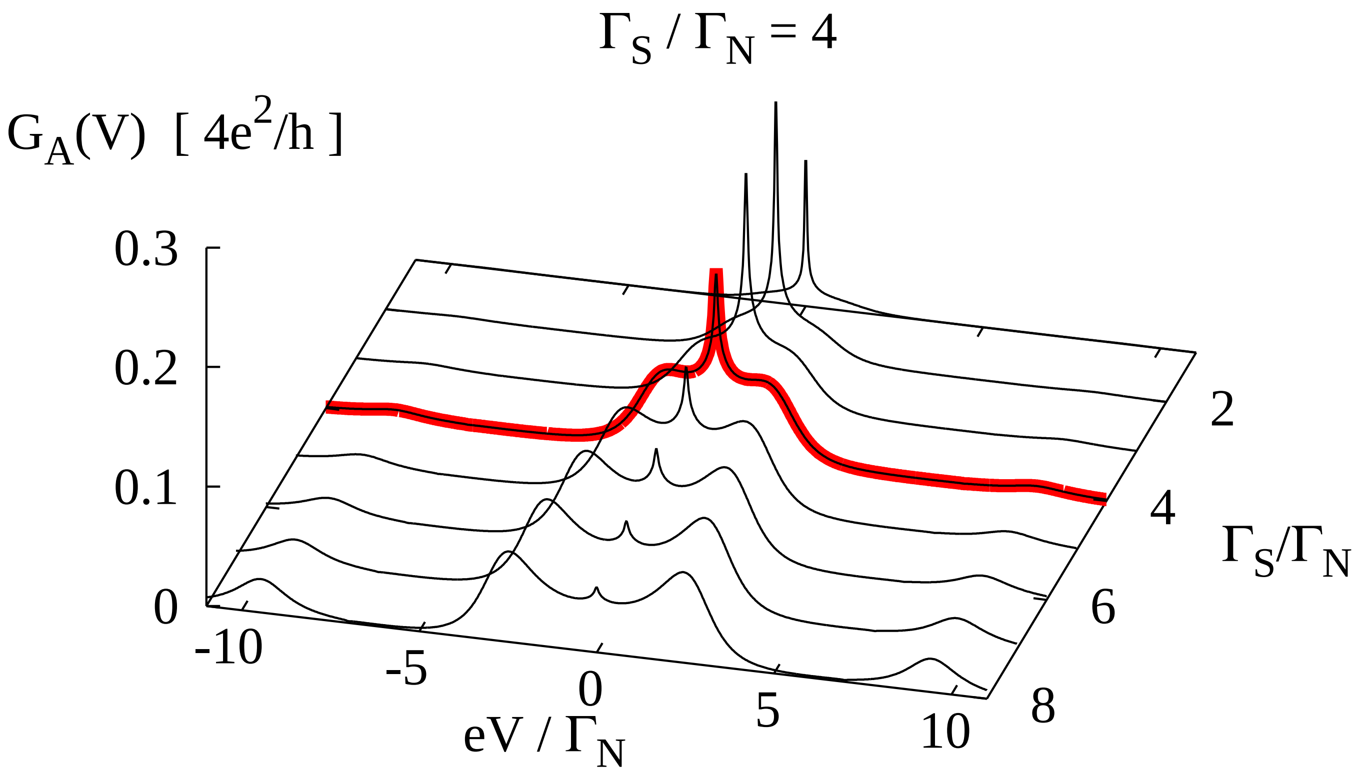

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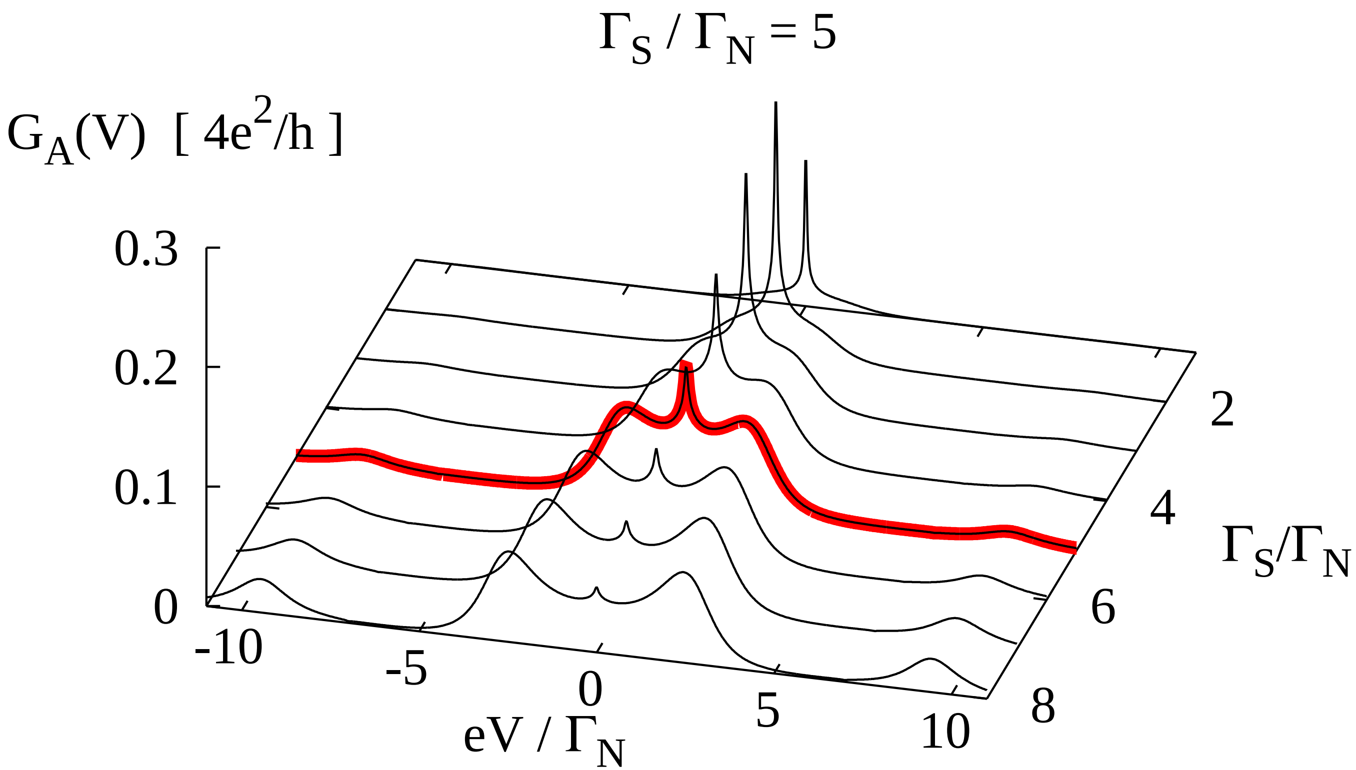

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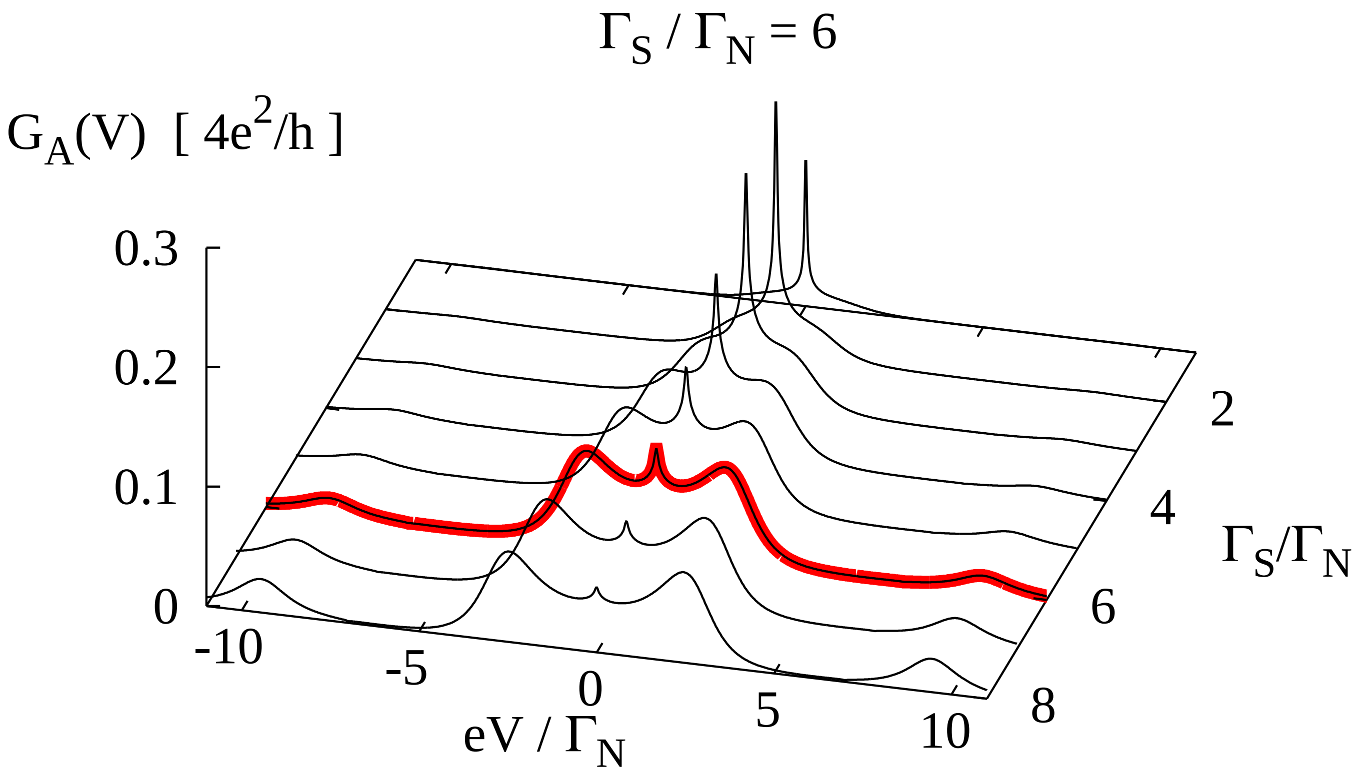

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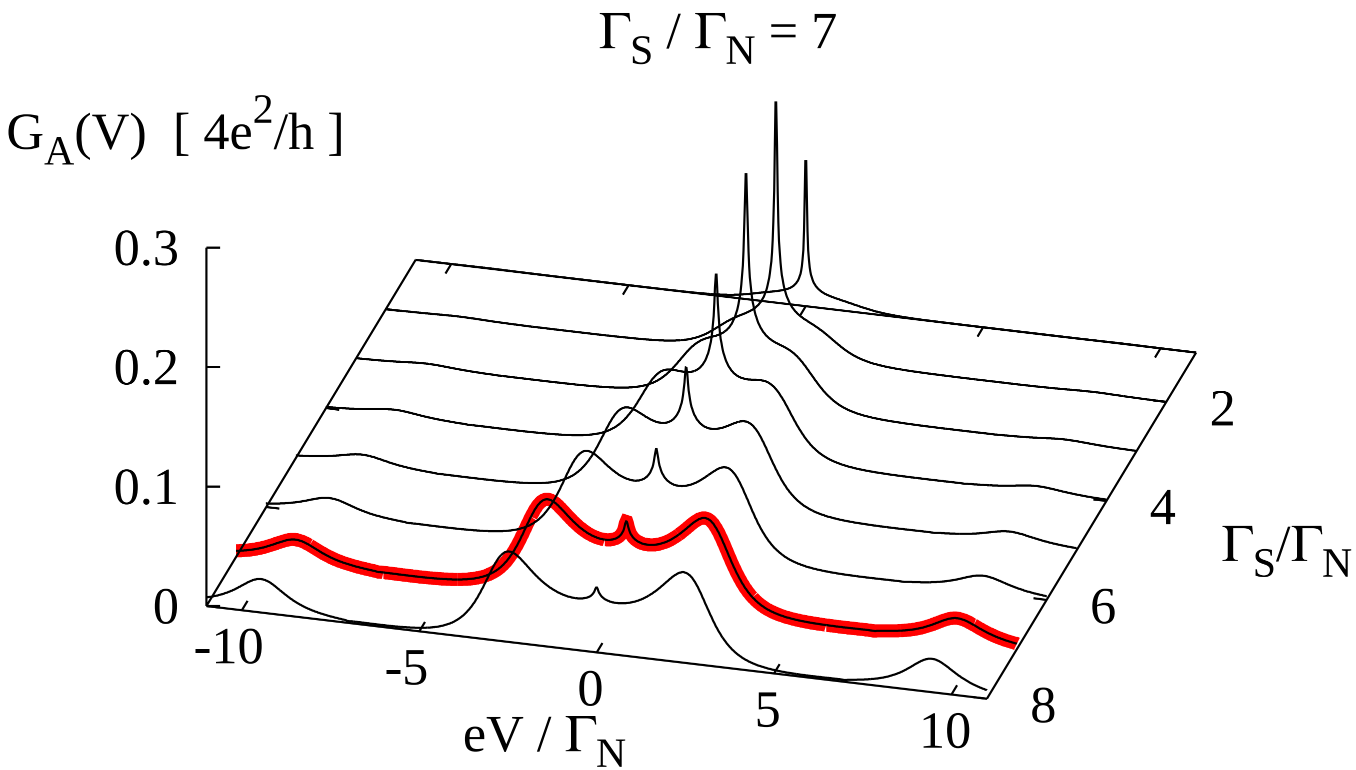

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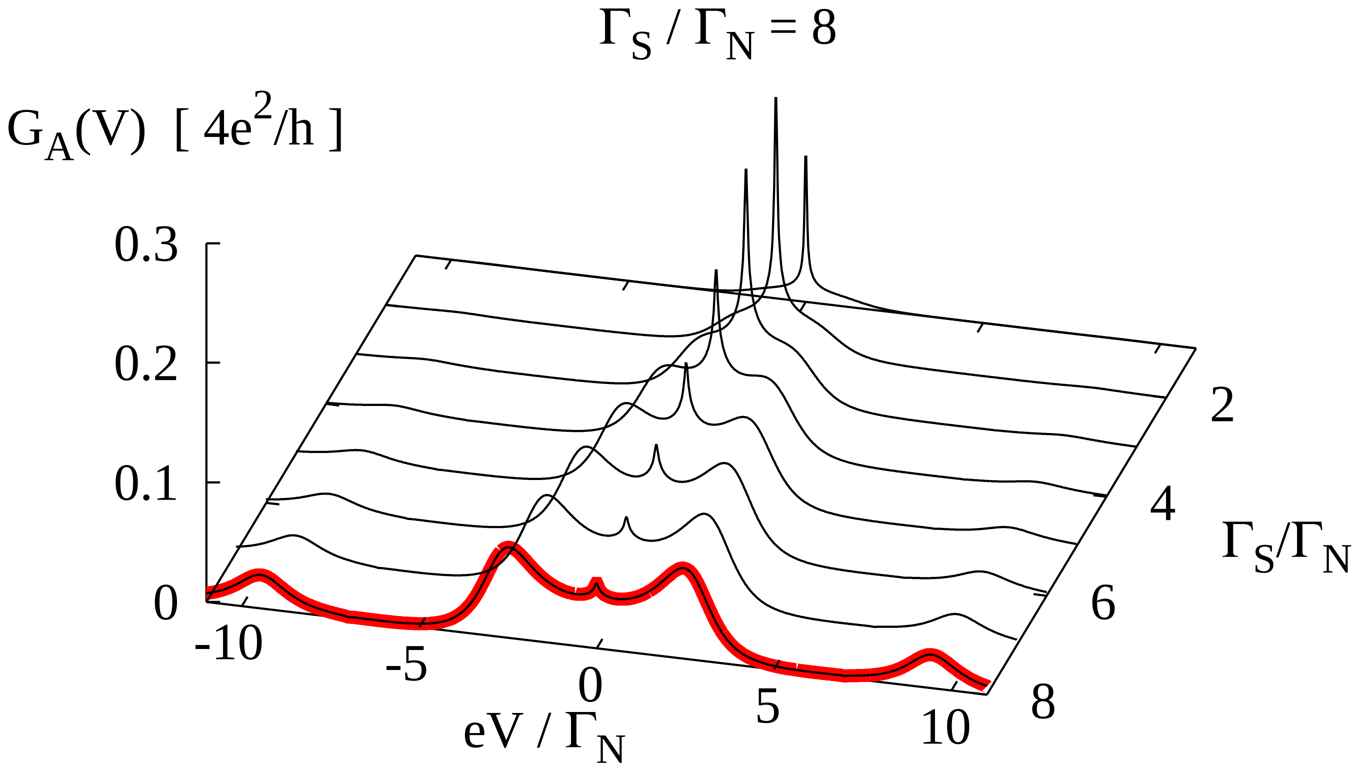

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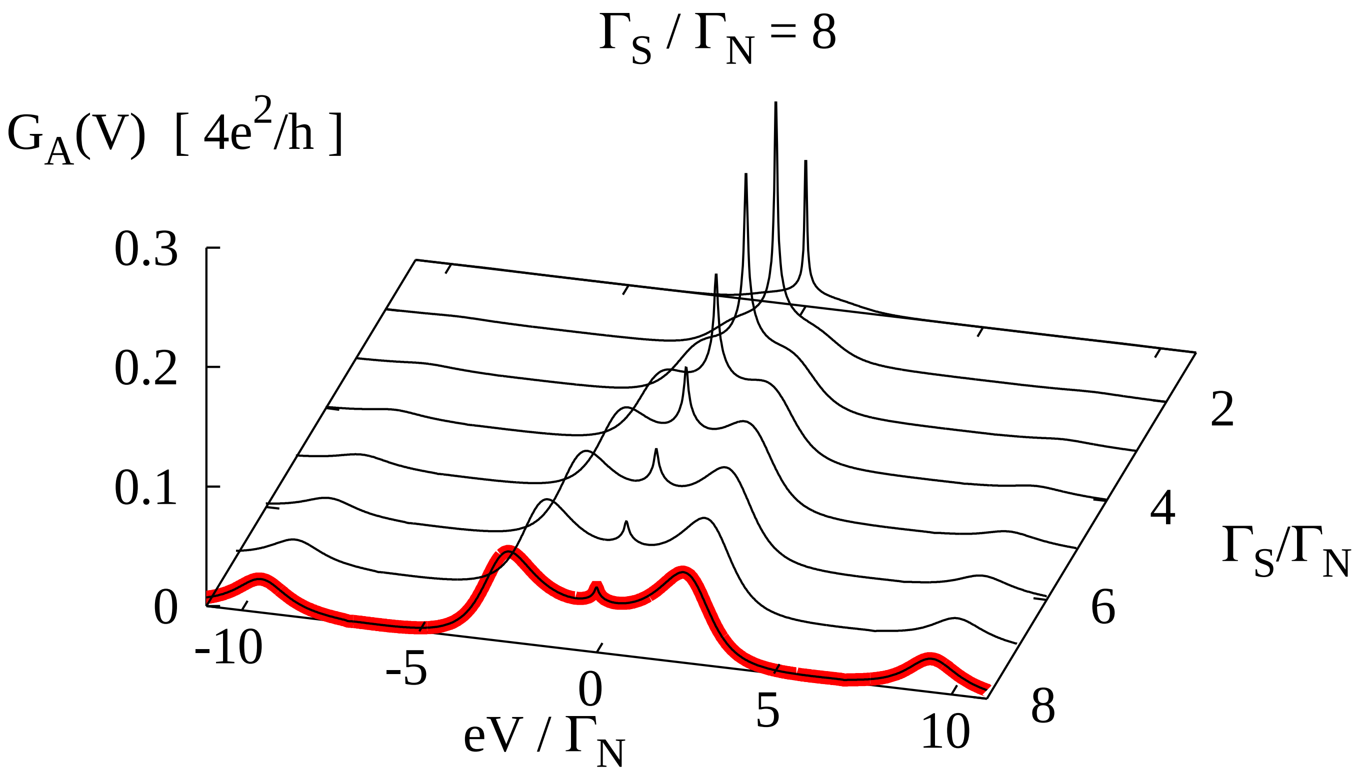

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Kondo resonance slightly enhances the zero-bias Andreev conductance, especially for $\Gamma_S \sim \Gamma_N$!

Interplay with the Kondo effect

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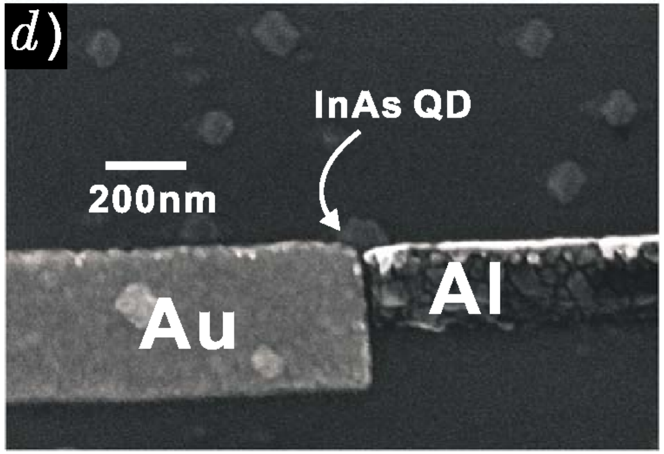

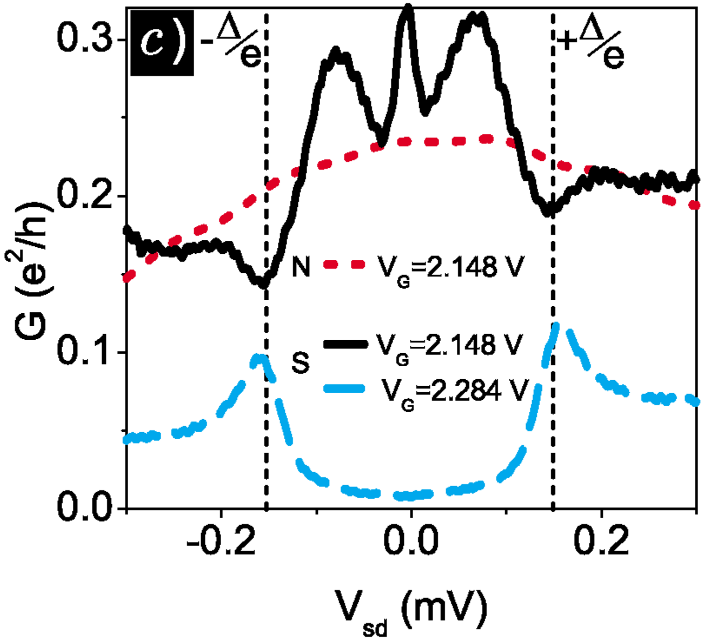

R.S. Deacon et al, *Phys. Rev. B* **81**, *121308(R) (2010).*

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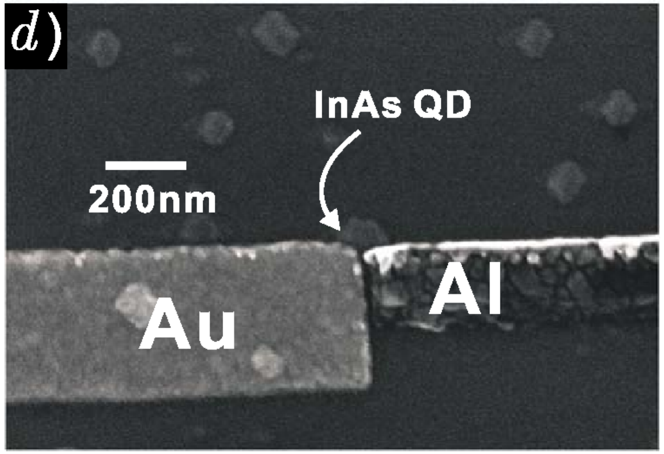

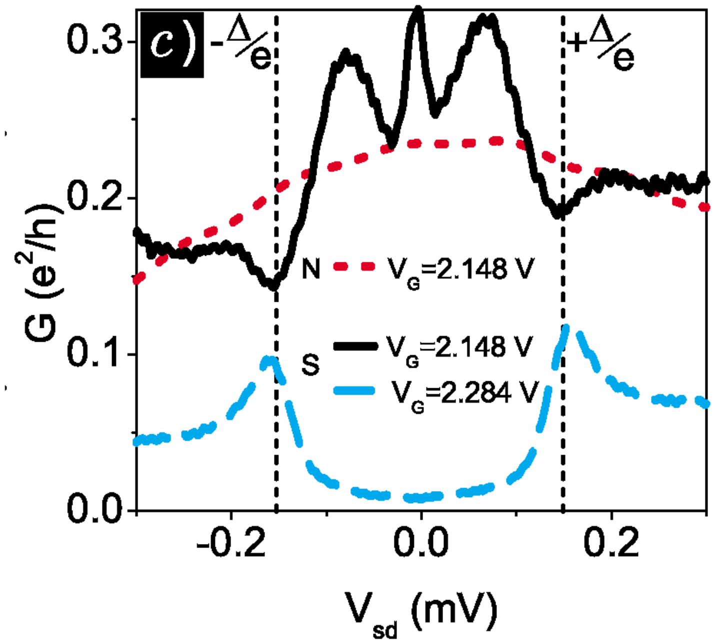

"The zero-bias conductance peak is consistent with Andreev transport enhanced by the Kondo singlet state"

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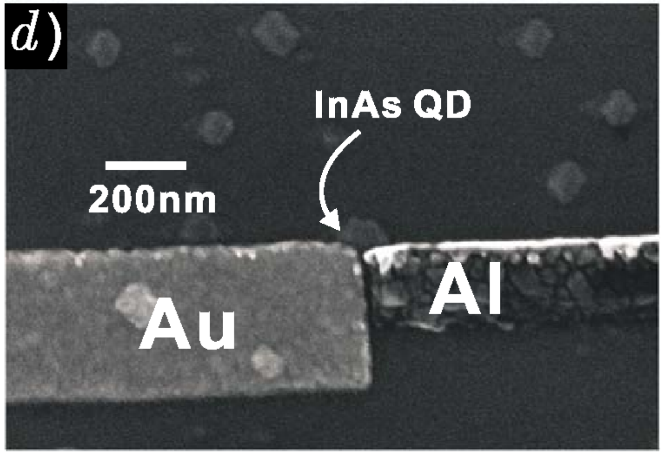

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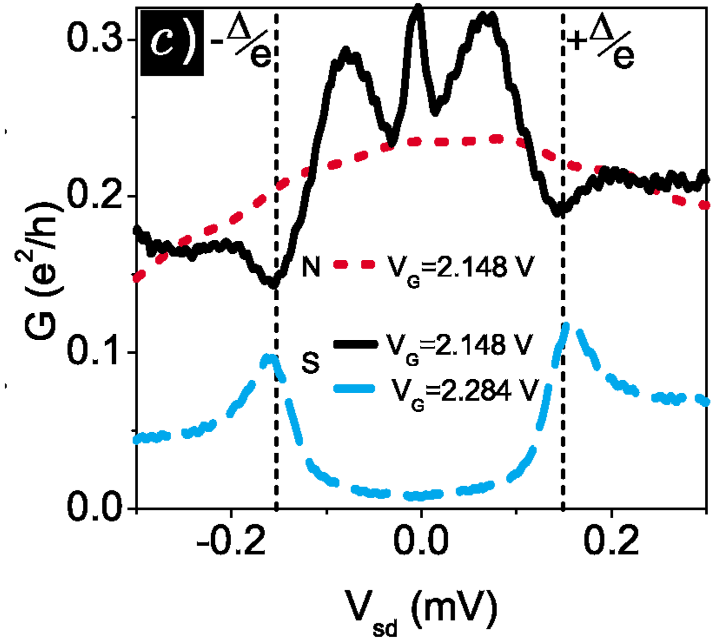

"We note that the feature exhibits excellent qualitative agreement with a recent theoretical treatment by Domanski et al"

R.S. Deacon et al, *Phys. Rev. B* **81**, *121308(R) (2010).*

Direct vs crossed Andreev reflections

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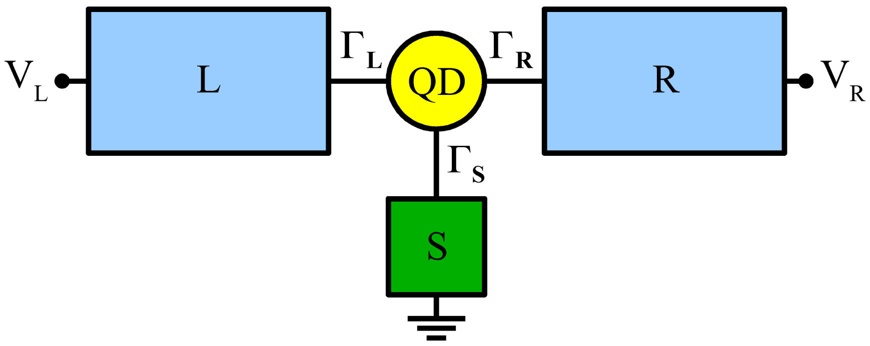

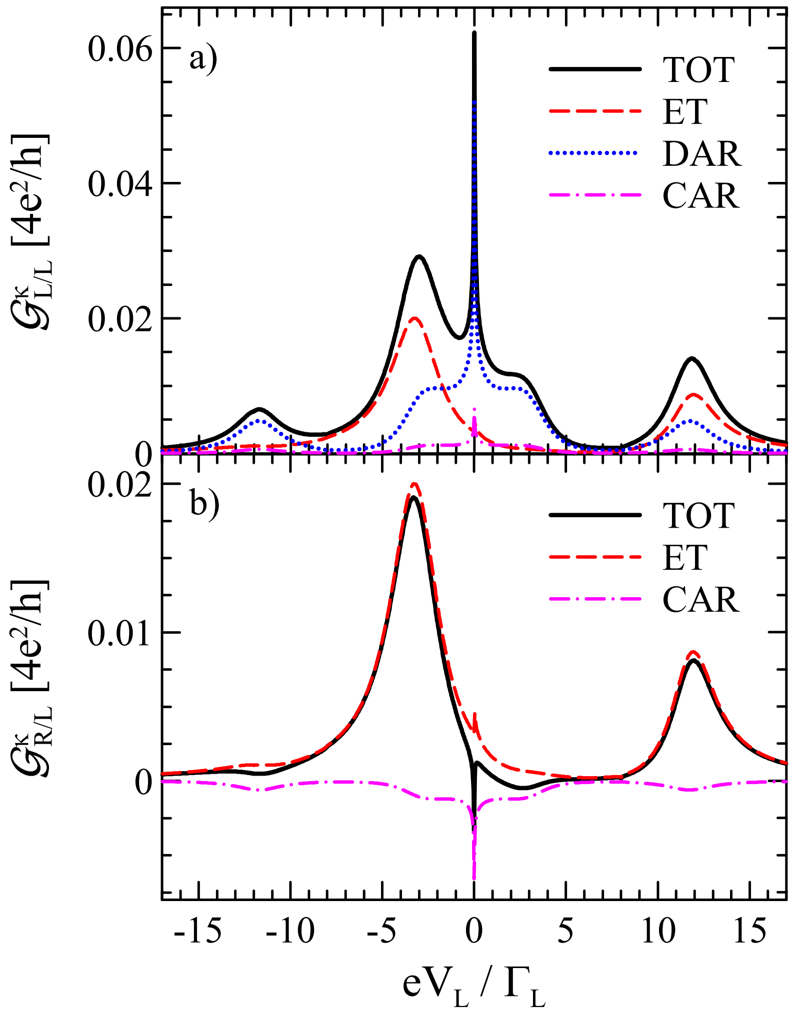

L, R – normal electrods

S – superconducting electrode

Kondo effect inverts a sign of the CAR conductance $G_{R/L}$

G. Michałek, B.R. Bułka, T. Domański, and K.I. WYsokiński, Phys. Rev. B (2013) in print.

Further related topics – Cooper pair splitters

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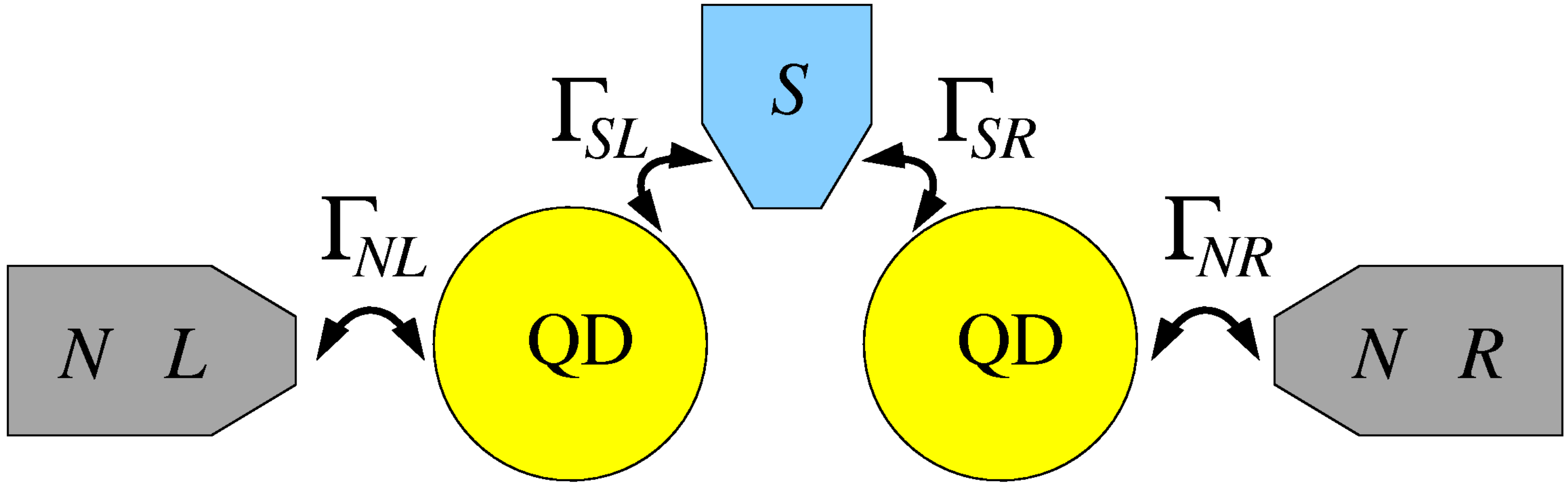

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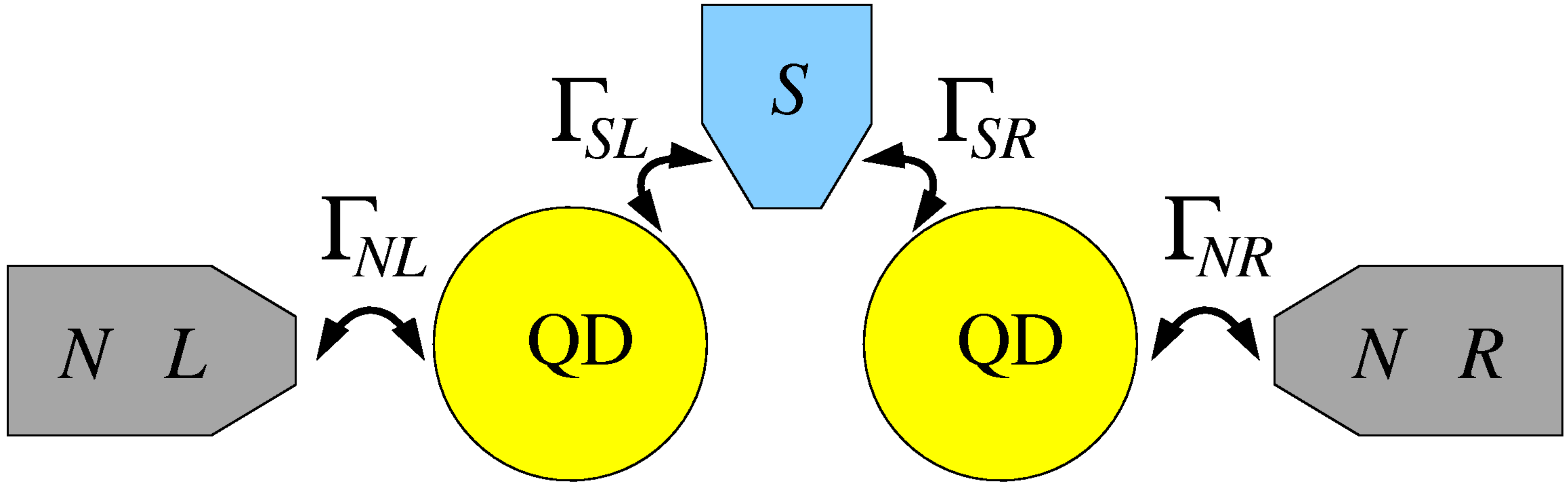

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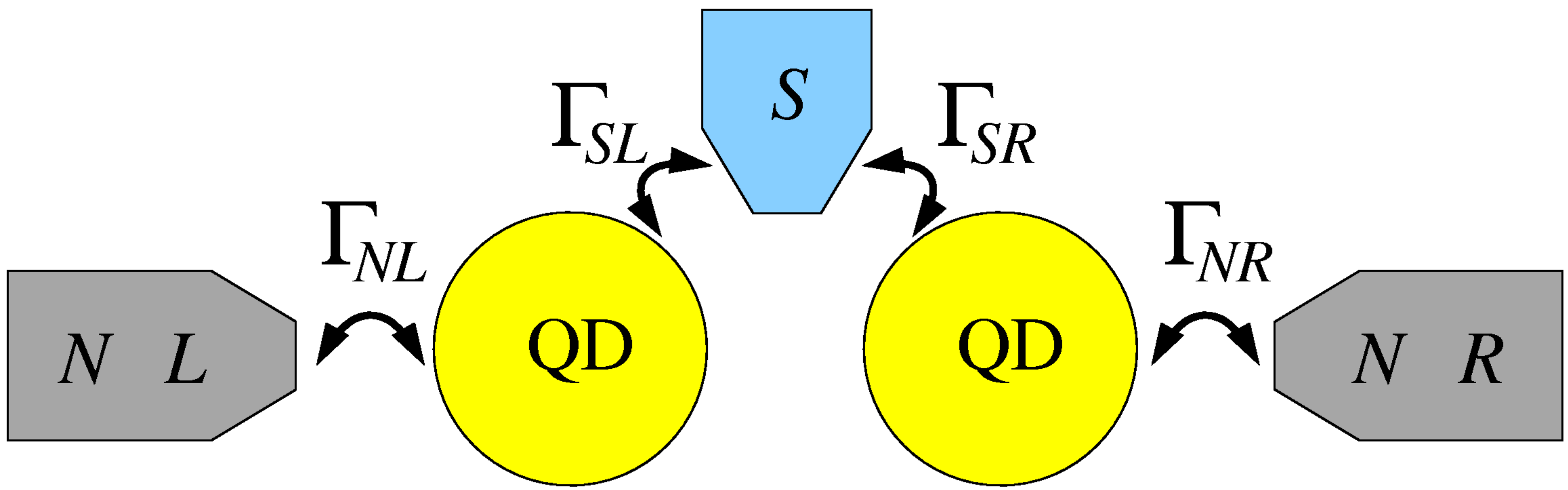

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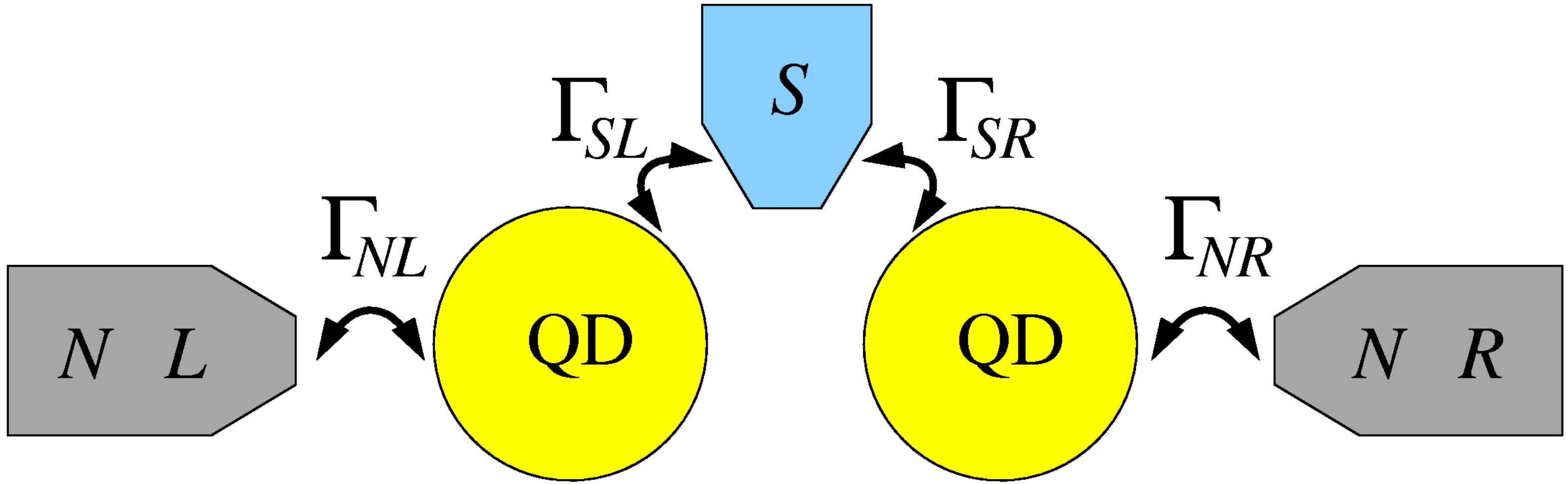

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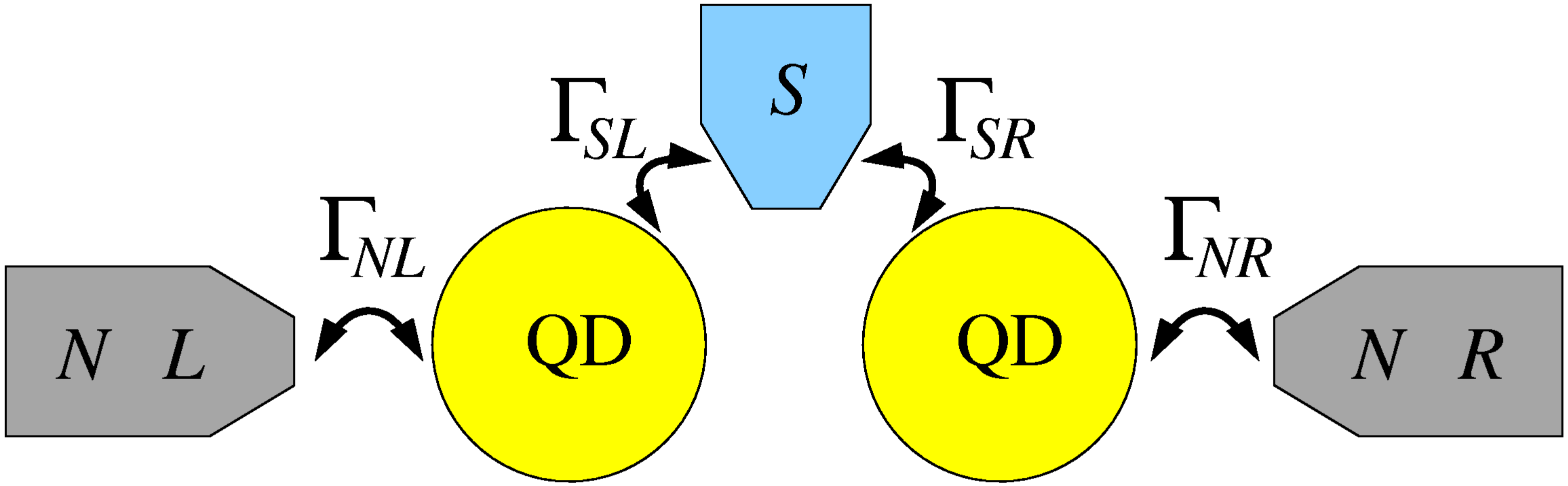

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J. Schindele, A. Baumgartner, C. Schönenberger, Phys. Rev. Lett. **109**, *157002 (2012).*

Further related topics

– **Majorana fermions**

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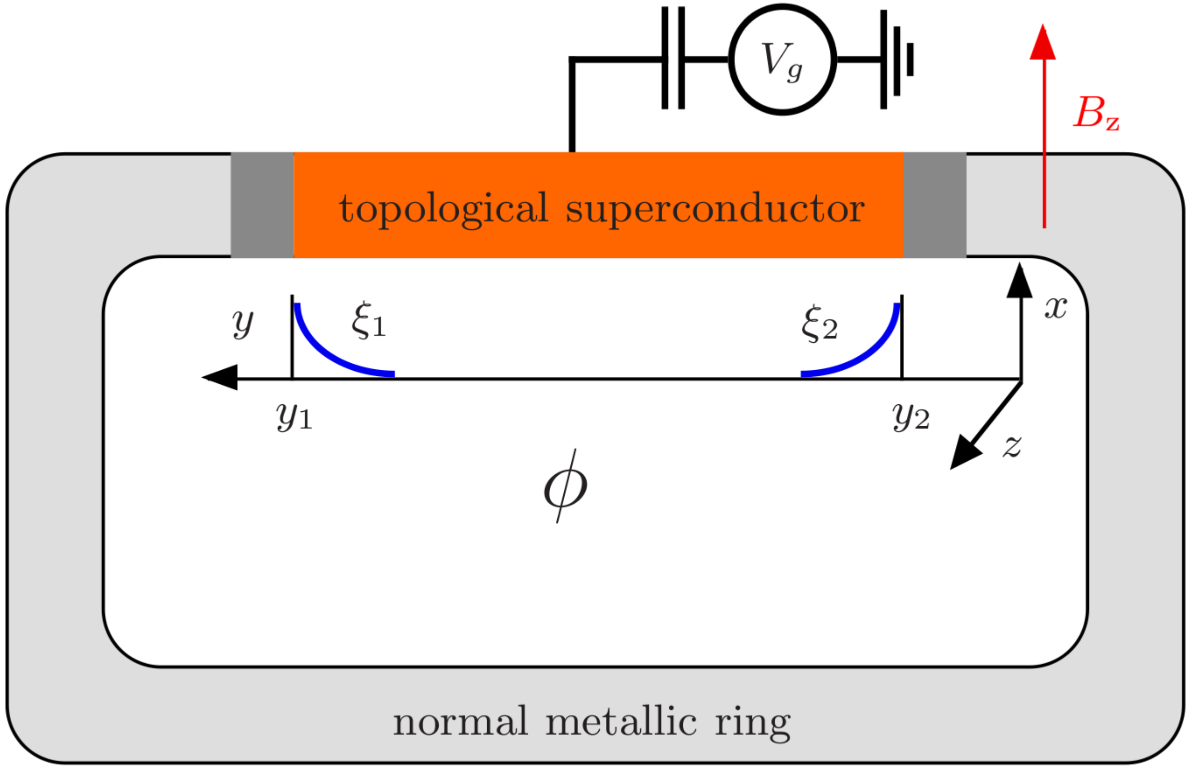

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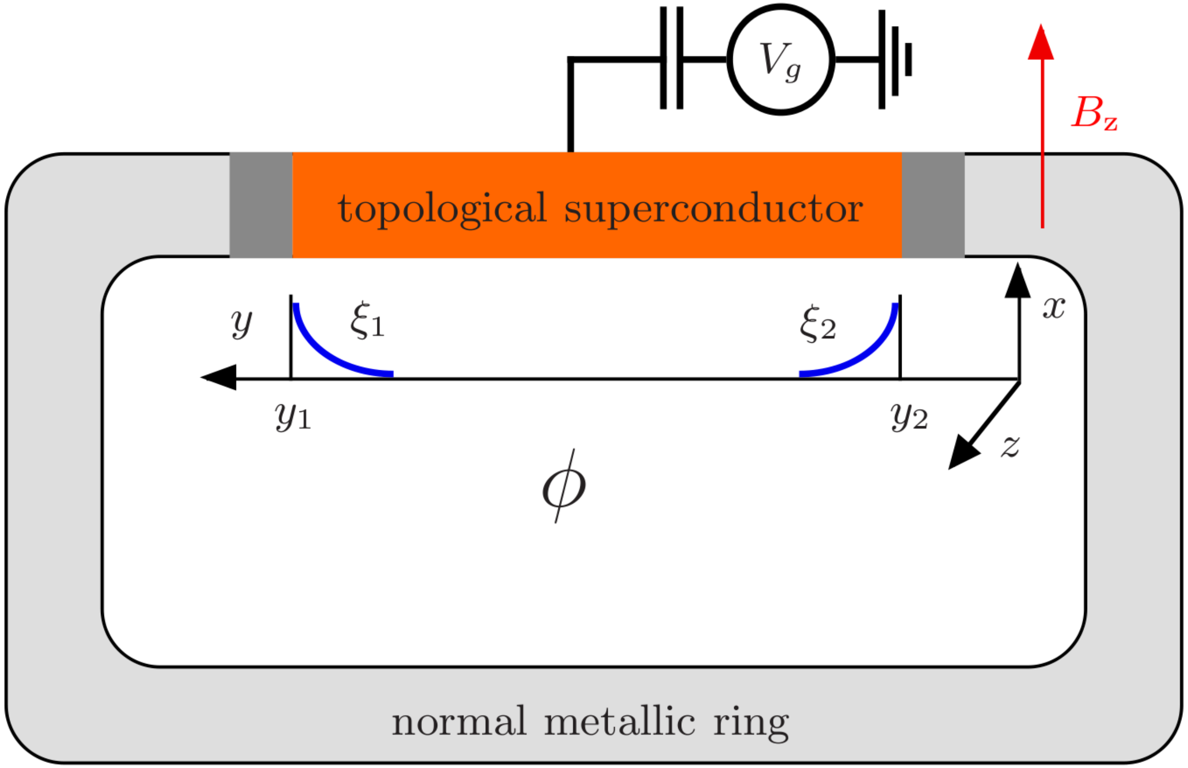

Majorana-type fermions in hybrid normal–superconducting rings

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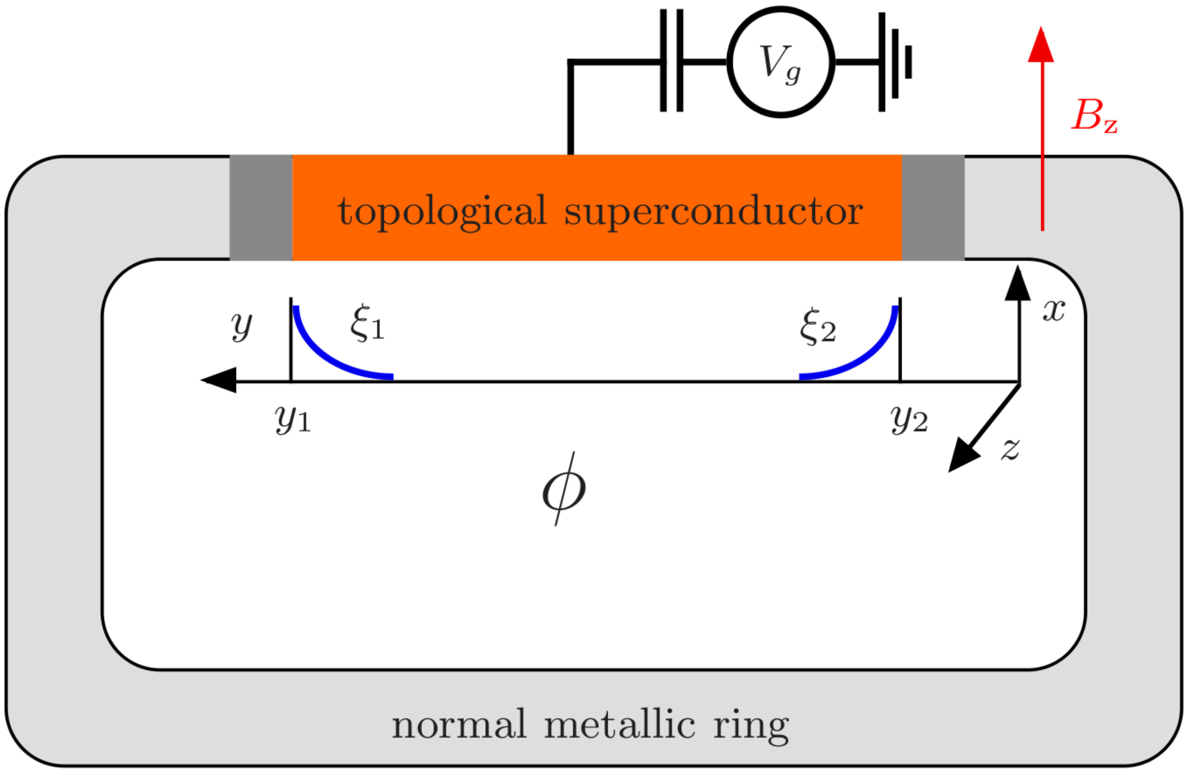

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Ph. Jacquod and M. Büttiker, arXiv:1306.6343 (preprint).

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http://kft.umcs.lublin.pl/doman/lectures