Czech Acad. Scien. in Prague, 6 X 2015

# Can electron pairing promote the Kondo state ?

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http://kft.umcs.lublin.pl/doman/lectures

## Issues to be addressed:

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⇒ **Coulomb repulsion** / singlet-doublet transition /

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- **Role of electron correlations:**

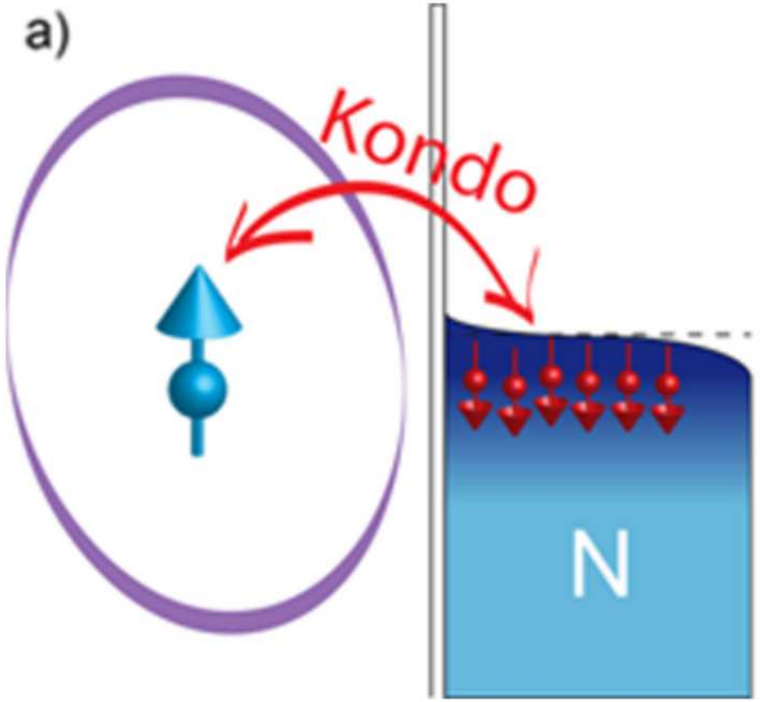
⇒ **Coulomb repulsion** **/ singlet-doublet transition /**

⇒ **spin exchange** **/ Kondo effect /**

- **Do they compete or cooperate ?**

## Kondo effect – reminder

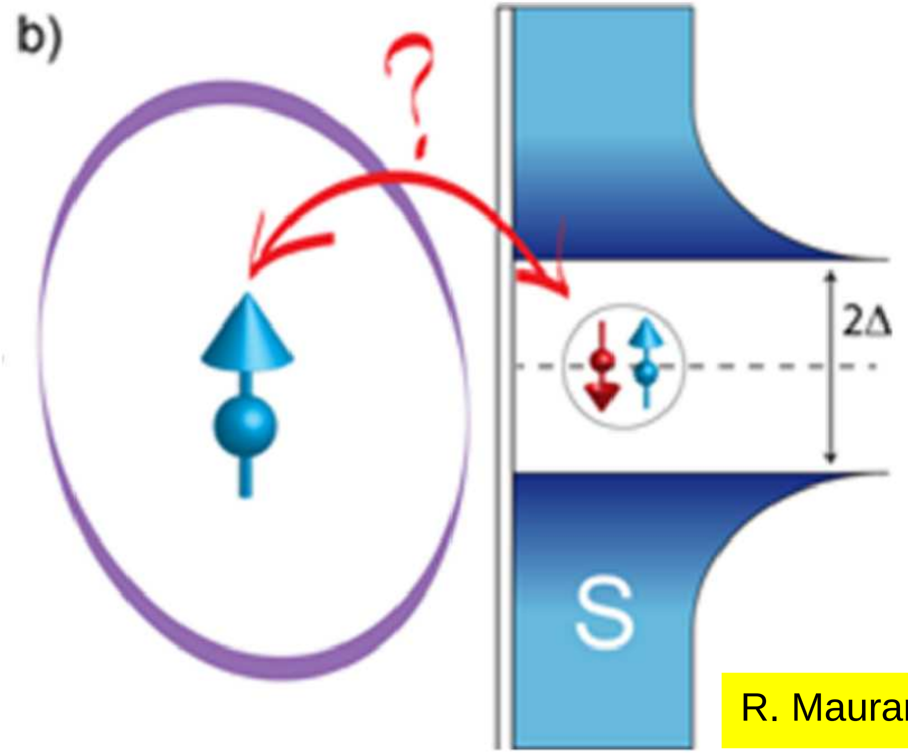
**Quantum impurity (dot) embedded in a metallic bath**




**forms the many-body Kondo state with itinerant electrons.**

**Kondo vs pairing** – ′ to screen or not to screen ?′

**Quantum impurity (dot) coupled to superconducting reservoir**




R. Maurand, Ch. Schönenberger, Physics **6**, 75 (2013).

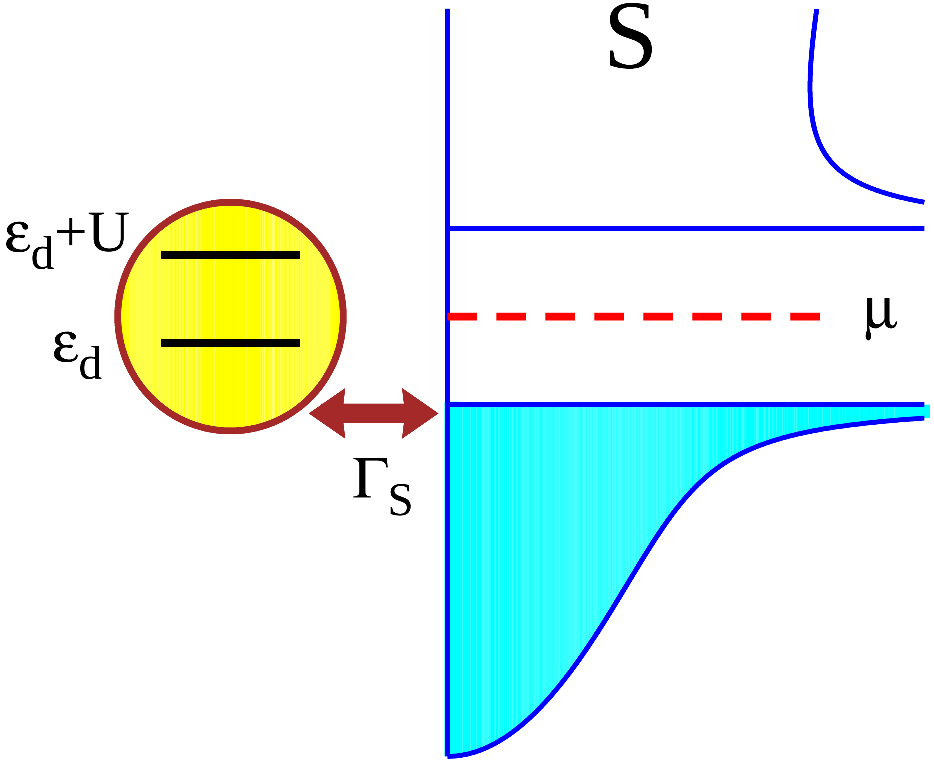
**encounters the following obstacles :**

⇒ **electron states near the Fermi level are depleted,**

⇒ **the induced pairing suppresses the Kondo state.**

## Quantum impurity (dot) in a superconducting host

# Electronic spectrum



## Microscopic model

## Anderson-type Hamiltonian

**Quantum impurity (dot)**

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U \; \hat{n}_{d\uparrow} \; \hat{n}_{d\downarrow}$$

**coupled with a superconductor**

$$\begin{aligned} \hat{H} \;\; &= \;\; \sum_{\sigma} \epsilon_d \hat{d}^{\dagger}_{\sigma} \hat{d}_{\sigma} + U \; \hat{n}_{d\uparrow} \; \hat{n}_{d\downarrow} + \hat{H}_S \\ &+ \;\; \sum_{\mathbf{k},\sigma} \left( V_{\mathbf{k}} \; \hat{d}^{\dagger}_{\sigma} \hat{c}_{\mathbf{k}\sigma} + V^{*}_{\mathbf{k}} \; \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{d}_{\sigma} \right) \end{aligned}$$

**where**

$$\hat{H}_S = \sum_{k,\sigma} \left( \varepsilon_k - \mu \right) \hat{c}^{\dagger}_{k\sigma} \hat{c}_{k\sigma} - \sum_{k} \left( \Delta \hat{c}^{\dagger}_{k\uparrow} \; \hat{c}^{\dagger}_{k\downarrow} + \mathrm{h.c.} \right)$$

## Uncorrelated QD – exactly solvable $U_d = 0$ case

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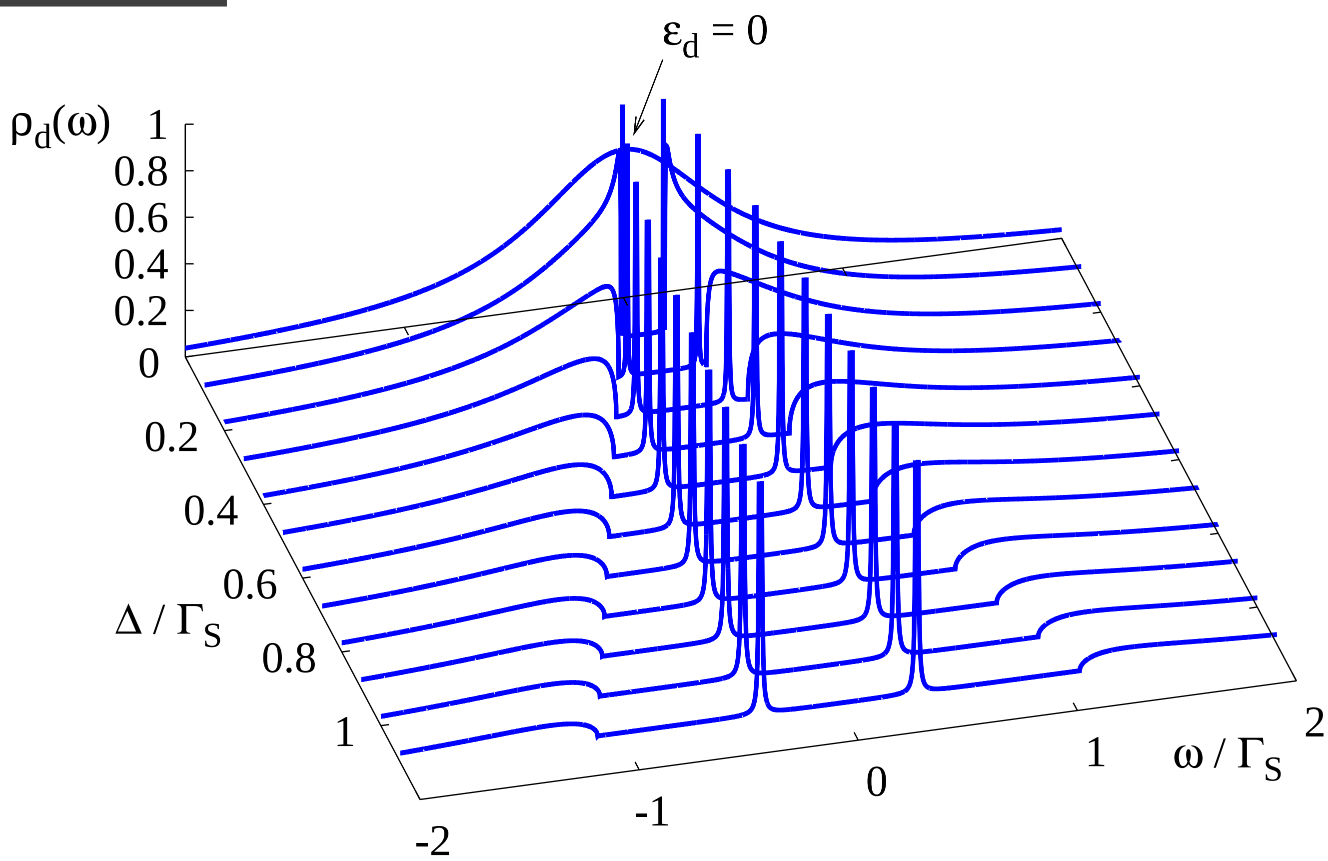
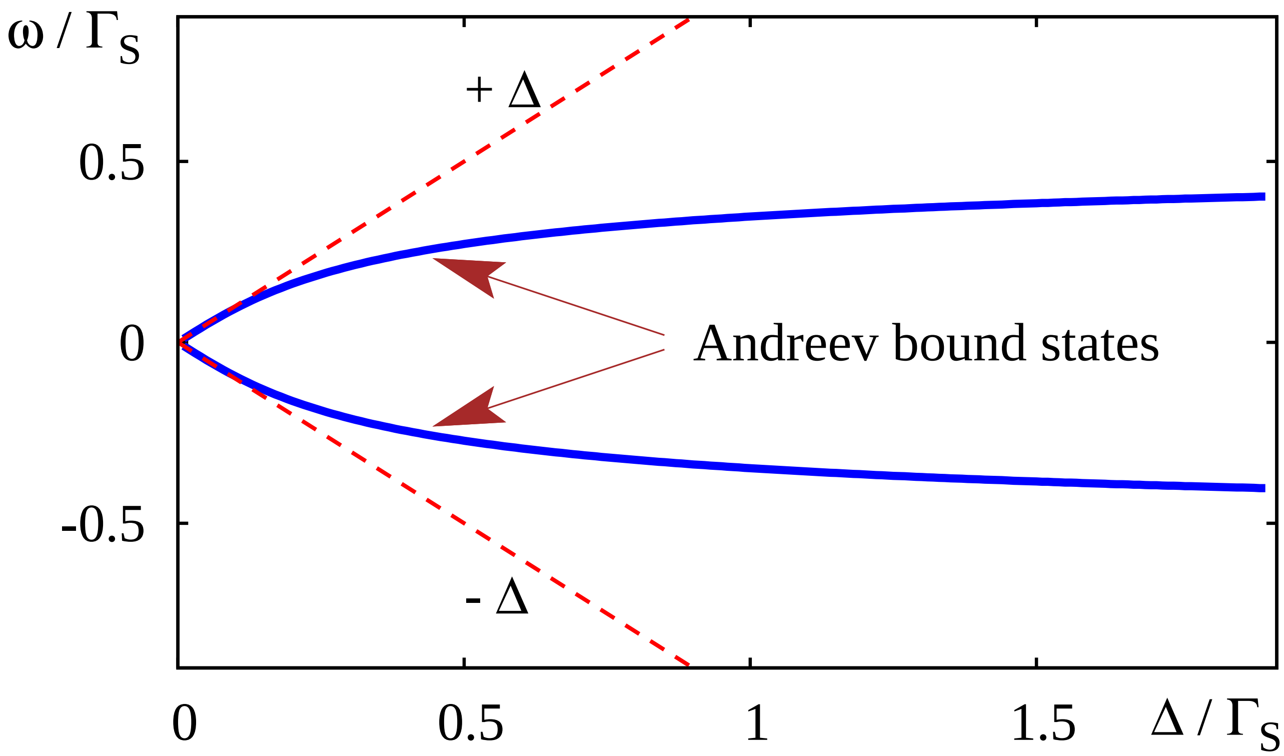



**In-gap resonances: Andreev bound states.**

J. Barański and T. Domański, J. Phys.: Condens. Matter **25**, 435305 (2013).

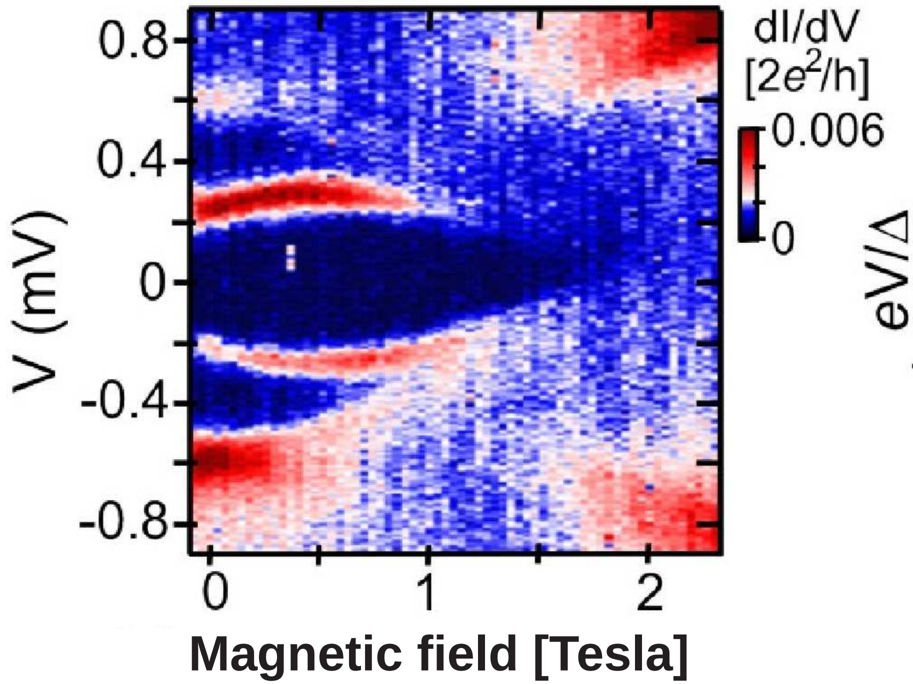
**Uncorrelated QD** – **exactly solvable $U_d = 0$ case**

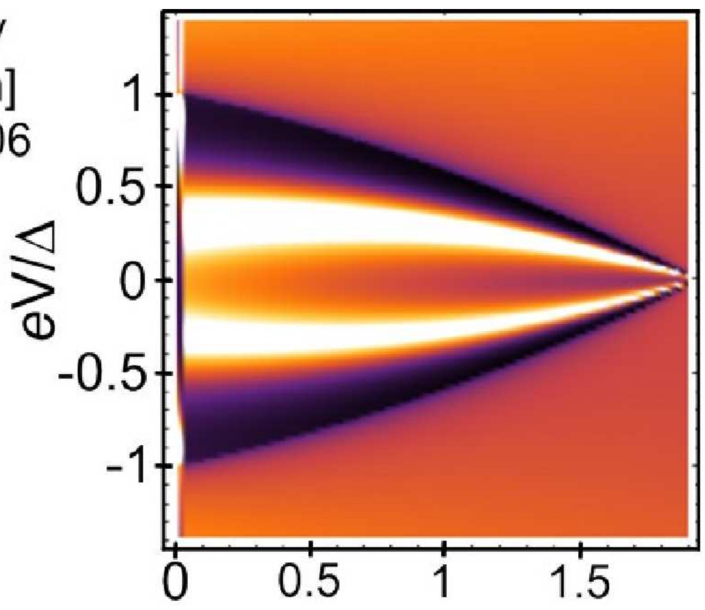



**Energies of the in-gap resonances (Andreev bound states)**

J. Barański and T. Domański, J. Phys.: Condens. Matter **25**, 435305 (2013).

## Subgap states – experimental data



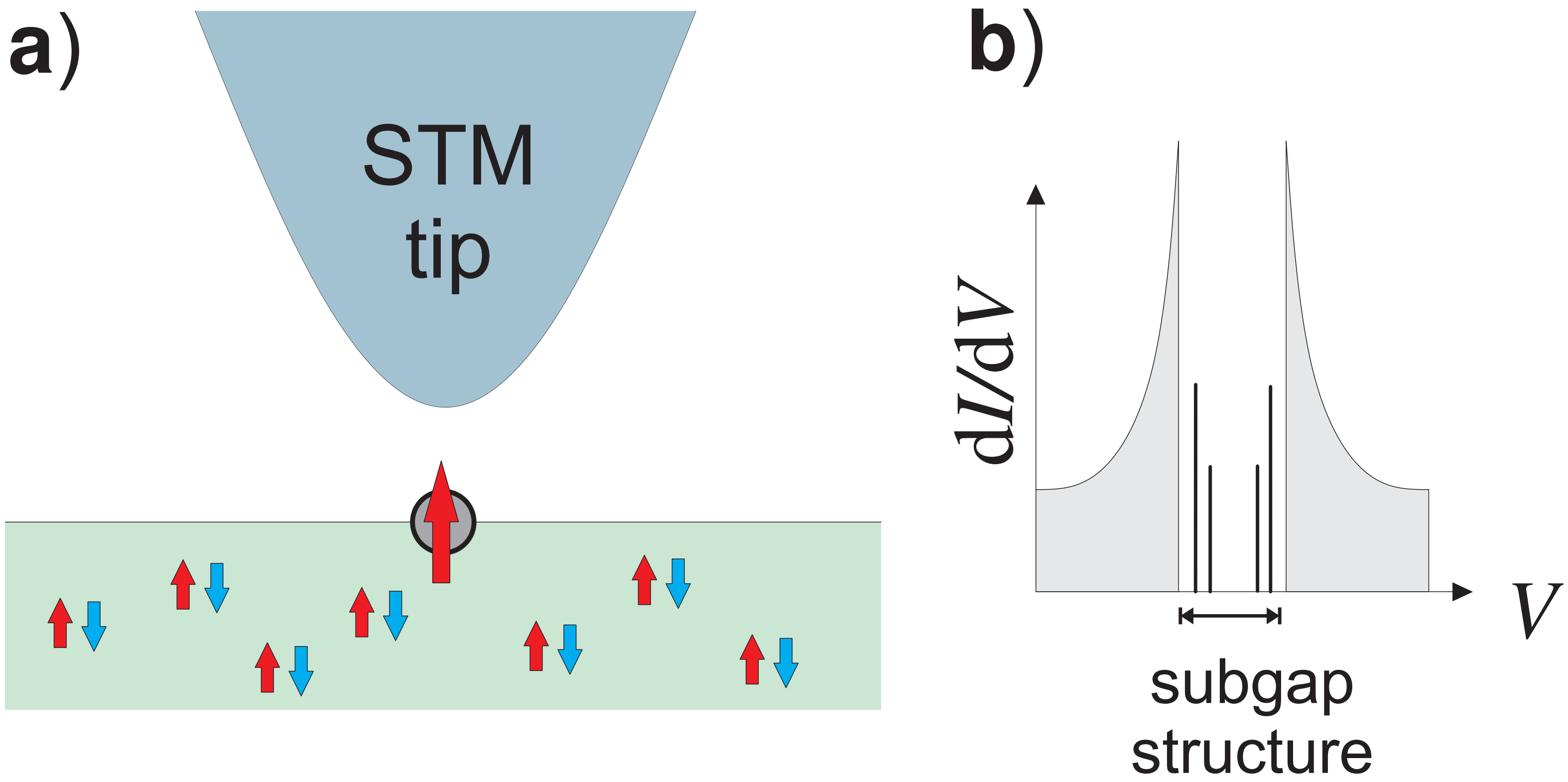




**Differential conductance of nanotubes coupled to vanadium (S) and gold (N)**

**/ external magnetic field changes the magnitude of pairing gap $\Delta(B)$ /**

Eduardo J.H. Lee, ..., S. De Franceschi, Nature Nanotechnology **9**, 79 (2014).

# Subgap states of multilevel impurities




**a) STM scheme and b) differential conductance for a multilevel quantum impurity adsorbed on a superconductor surface.**

R. Žitko, O. Bodensiek, and T. Pruschke, Phys. Rev. B **83**, 054512 (2011).

## Correlated QD – singlet/doublet configurations

Deep in a subgap regime $|\omega| \ll \Delta$ quantum impurity can described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left( \Delta_d \, \hat{d}^{\dagger}_{\uparrow} \hat{d}^{\dagger}_{\downarrow} + \mathrm{h.c.} \right)$$

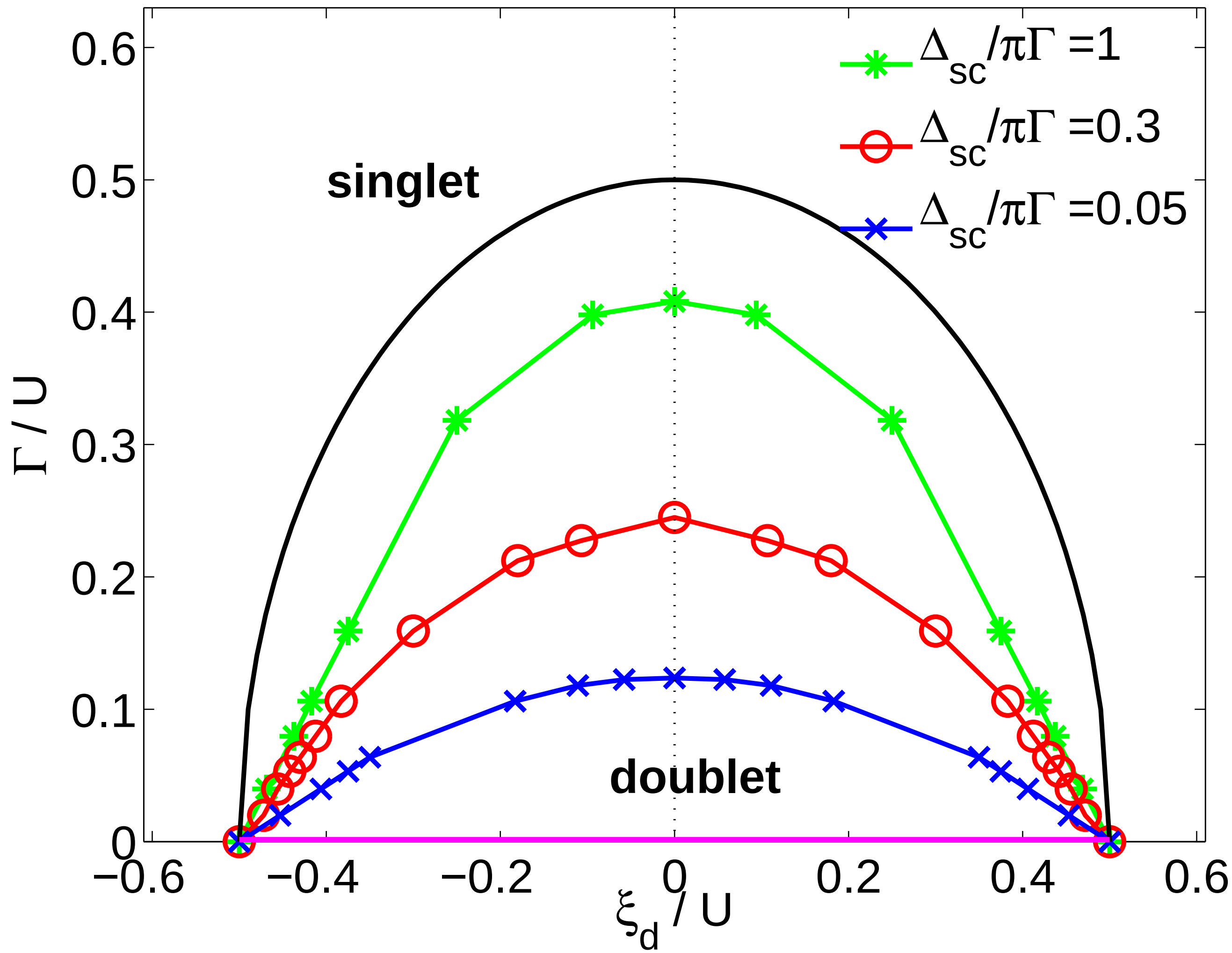
where the induced pairing gap $\Delta_d = \Gamma_S/2$.

Eigen-states of this problem:

| | | |
|---|---|---|
| $\lvert\uparrow\rangle$ and $\lvert\downarrow\rangle$ | $\Leftarrow$ | doublet (spin $\frac{1}{2}$) |
| $\left. \begin{array}{l} u\,\lvert 0\rangle - v\,\lvert\uparrow\downarrow\rangle \\ v\,\lvert 0\rangle + u\,\lvert\uparrow\downarrow\rangle \end{array} \right\}$ | $\Leftarrow$ | singlets (spin 0) |

Doublet-singlet quantum phase transitions occur upon varying $\varepsilon_d$, $U_d$ or $\Gamma_S$.

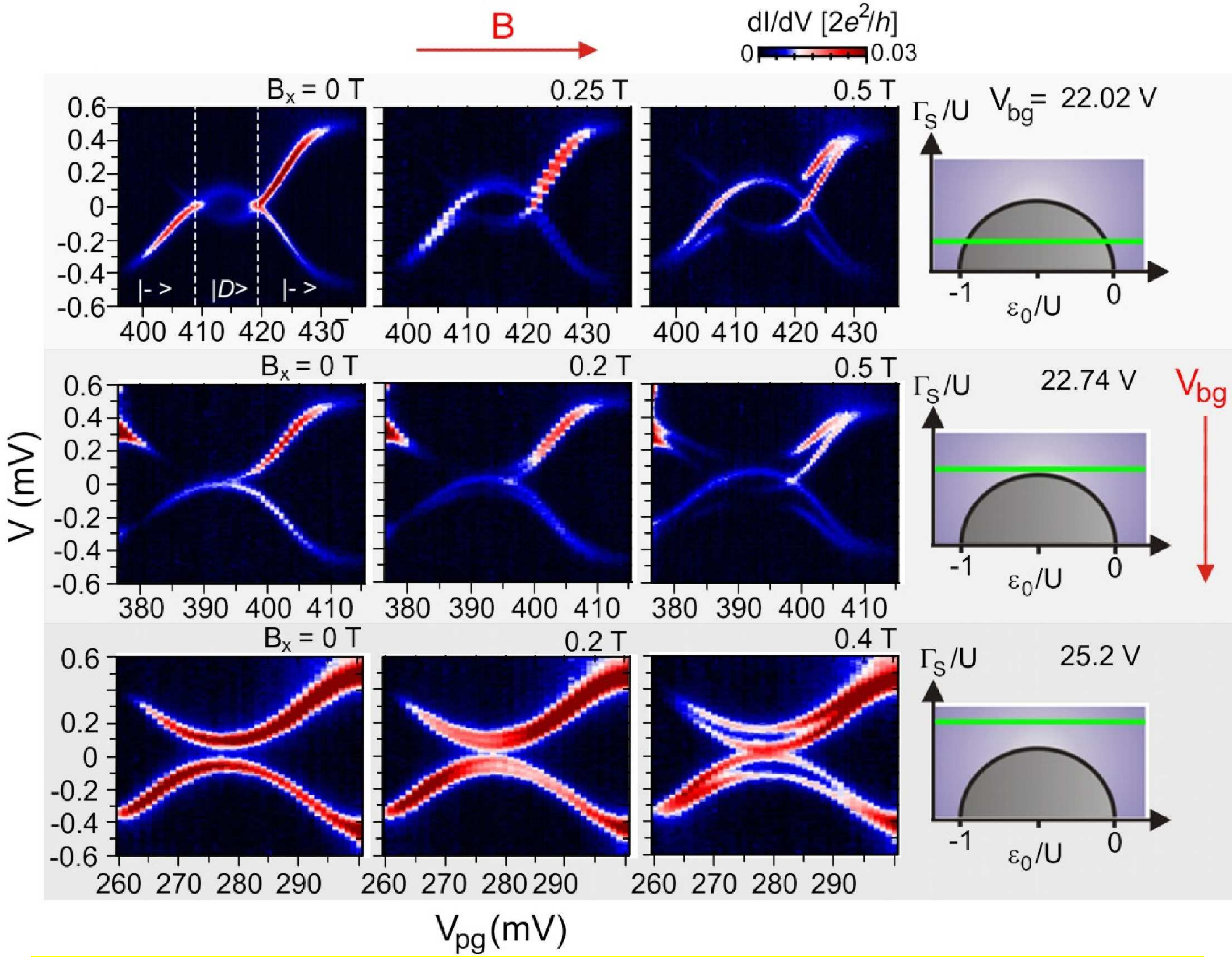
## Singlet-doublet transition – NRG results for arbitrary $\Delta$




$$\xi_d \equiv \varepsilon_d + \frac{1}{2}U$$

*J. Bauer, A. Oguri, and A.C. Hewson, J. Phys.: Condens. Matter* **19**, *486211 (2007).*

# Singlet-doublet transition – experimental data

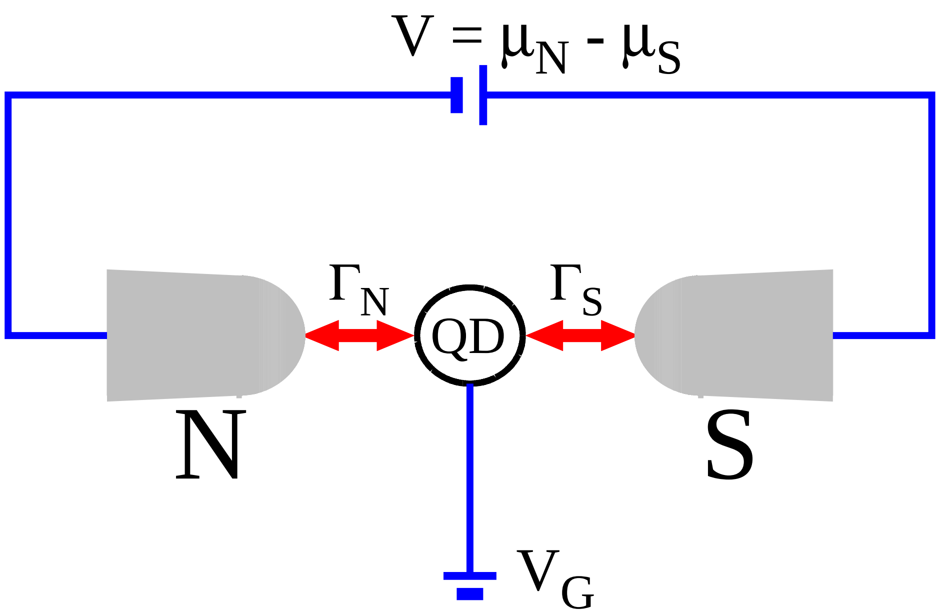



Eduardo J.H. Lee, ..., S. De Franceschi, Nature Nanotechnology **9**, 79 (2014).

# N–QD–S heterojunction

## Physical situation

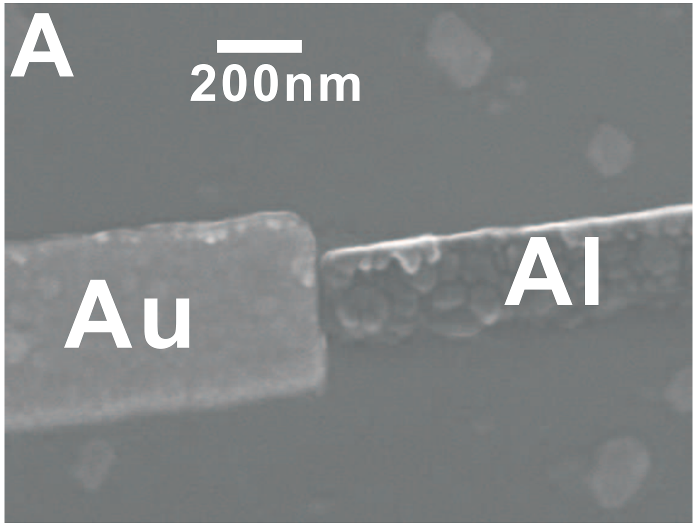
**To probe the subgap states one can study the electron transport through a quantum dot (QD) coupled between the normal (N) and superconducting (S) electrodes**




metallic lead　　QD　　superconductor

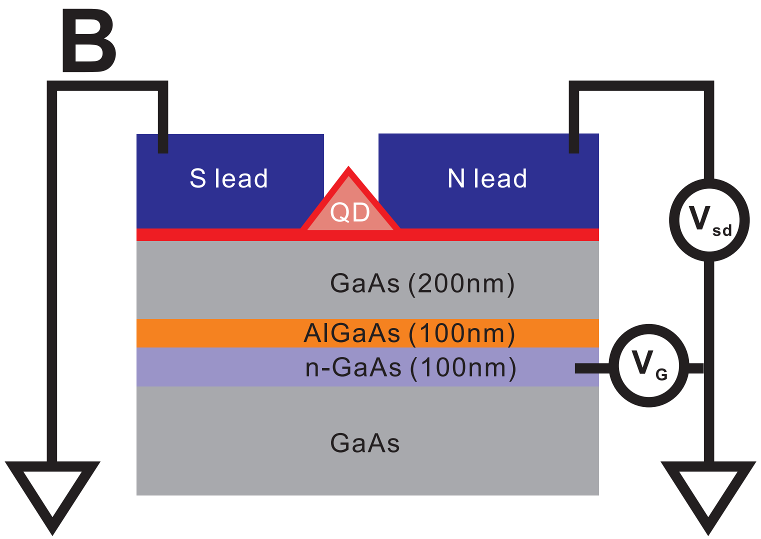
**This N–QD–S setup has been practically studied in several recent experiments.**

# Andreev spectroscopy – experimental realization # 1




$T_c \simeq 1$K

$\Delta \simeq 152\mu$eV



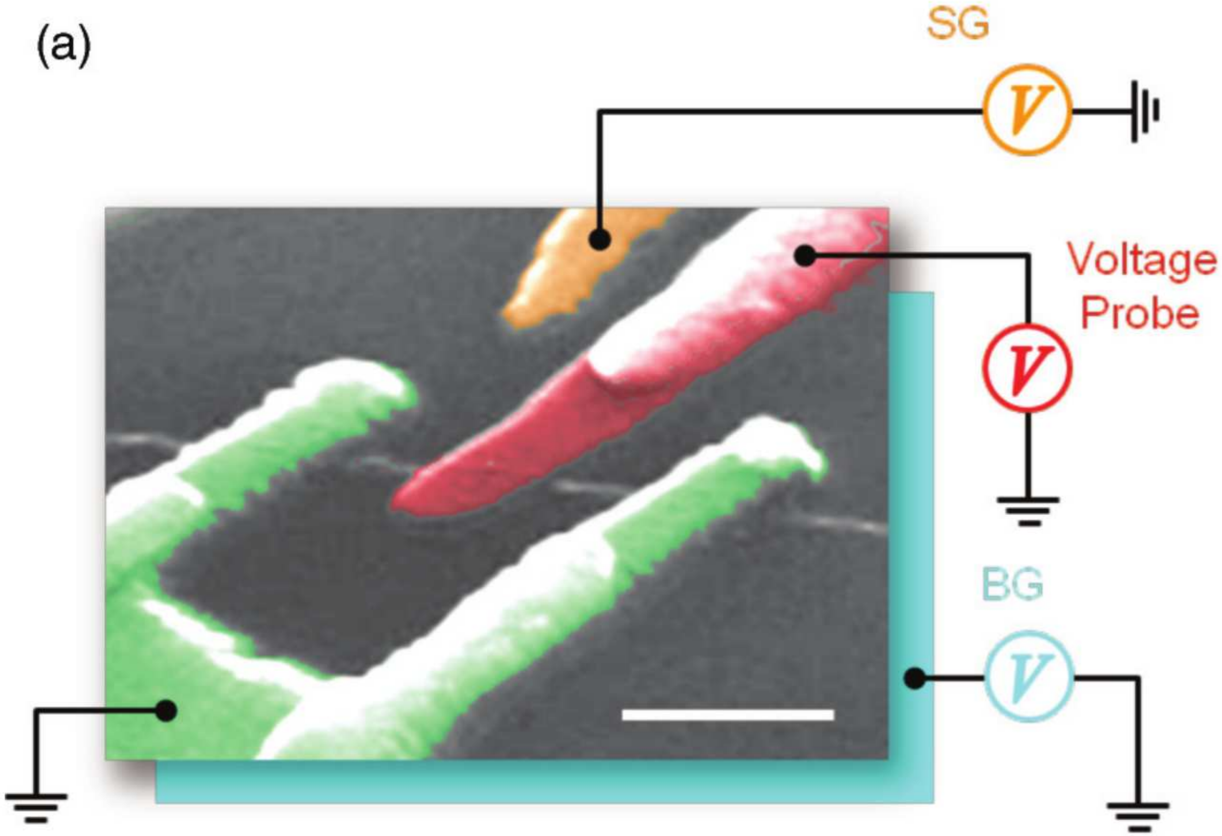

**QD : self-assembled InAs**

**diameter ∼ 100 nm**

**backgate : Si-doped GaAs**

*R.S. Deacon* et al, *Phys. Rev. Lett.* **104**, *076805 (2010).*

# Andreev spectroscopy – experimental realization # 2




**QD** : **carbon nanotube**

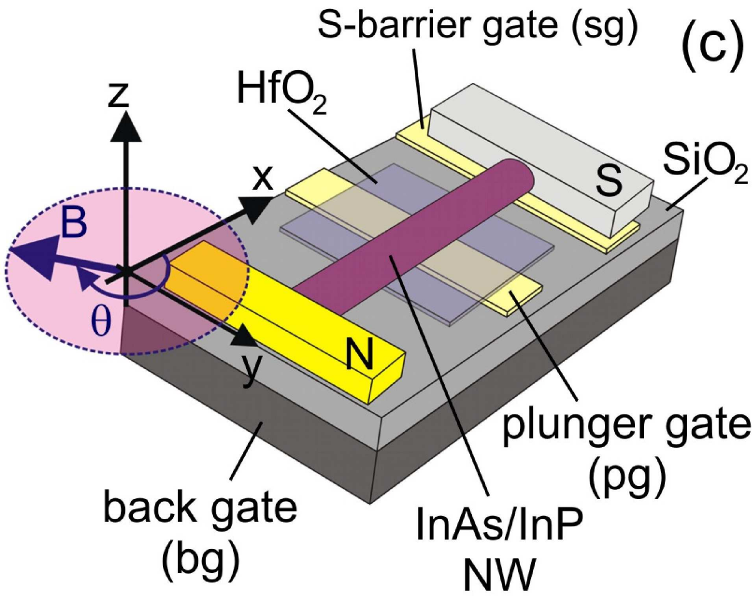
**S** : **aluminum**

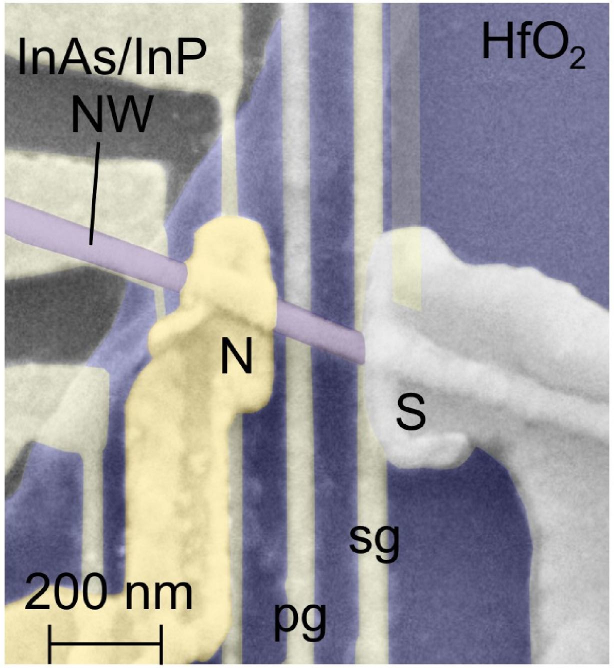
$U \simeq 2$ **meV**

$\Delta \simeq 0.15$**meV**

*J.-D. Pillet, R. Žitko, and M.F. Goffman, Phys. Rev. B* **88**, *045101 (2013).*

## Andreev spectroscopy – experimental realization # 3




**QD : semiconducting InAs/InP nanowire**

**S : vanadium**

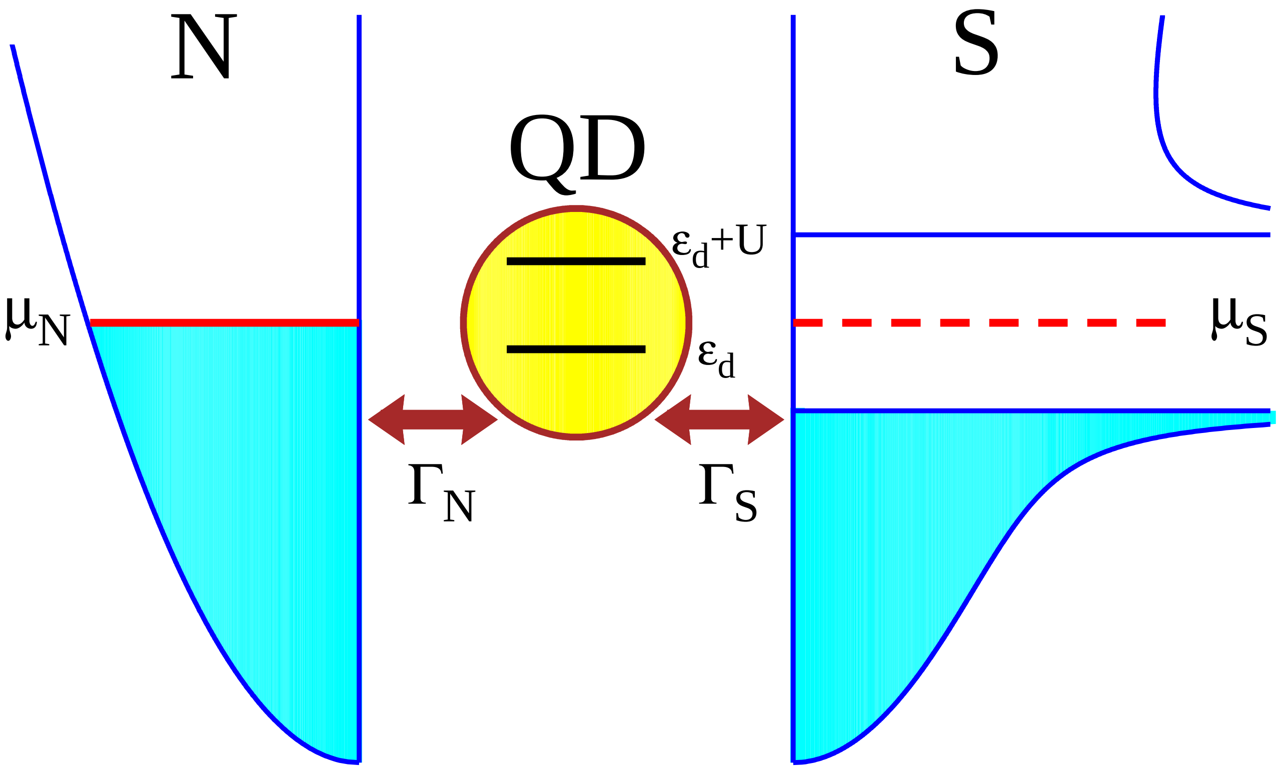
$\Delta \simeq 0.55\text{meV}$ $U \simeq 3 - 10\Delta$

**N : gold**

*E.J.H. Lee* et al, *Phys. Rev. Lett.* **109**, *186802 (2012).*

# Electronic spectrum

## Spectrum of the N-QD-S heterostructure components



## Theoretical background: – various many-body techniques

| | |
|---|---|
| EOM | R. Fazio and R. Raimondi (1998) |
| slave bosons | P. Schwab and R. Raimondi (1999) |
| NCA | A.A. Clerk, V. Ambegaokar, and S. Hershfield (2000) |
| IPT | J.C. Cuevas, A. Levy Yeyati, and A. Martin-Rodero (2001) |
| constr. sb | M. Krawiec and K.I. Wysokiński (2004) |
| NRG | Y. Tanaka, N. Kawakami, and A. Oguri, (2007) |
| NRG | J. Bauer, A. Oguri, and A.C. Hewson, (2007) |
| rev. EOM | T. Domański, A. Donabidowicz (2007) |
| f-RG | C. Karrasch, A. Oguri, and V. Meden (2008) |
| QMC | A. Koga (2013) |
| NRG | R. Žitko et al (2014) |
| SOPT+NRG | M. Žonda, V. Pokorný, V. Janiš, and T. Novotný (2015) |
| SW+NRG+SOPT | T.D., I. Weymann, M. Barańska, G. Górski (2015) |

## Specific issue – for superconductivity in nanosystems

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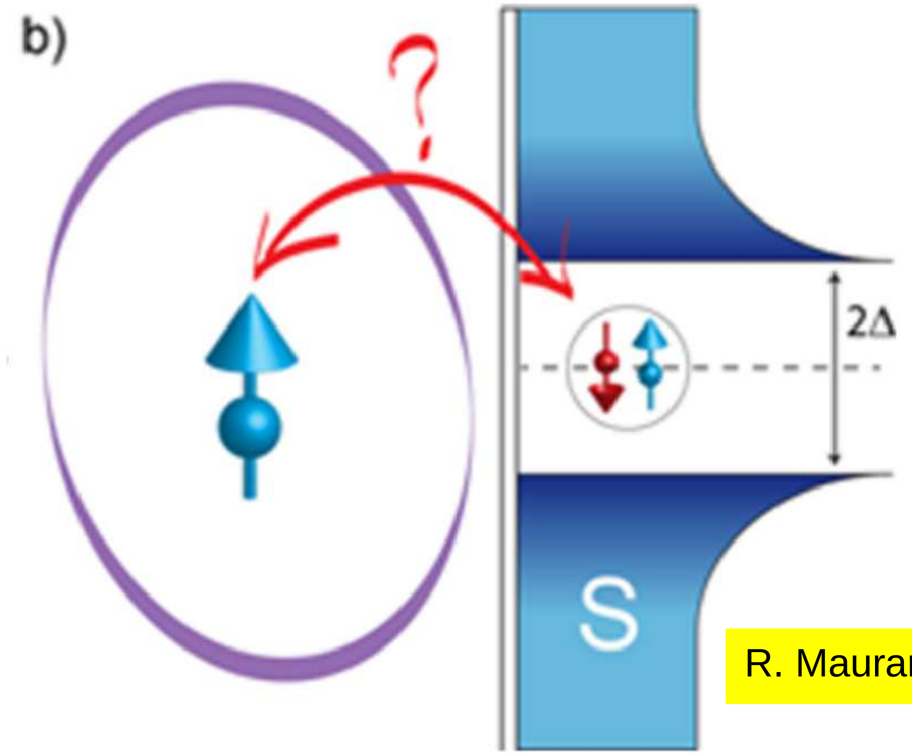
## Pairing vs Kondo state ('to screen or not to screen')

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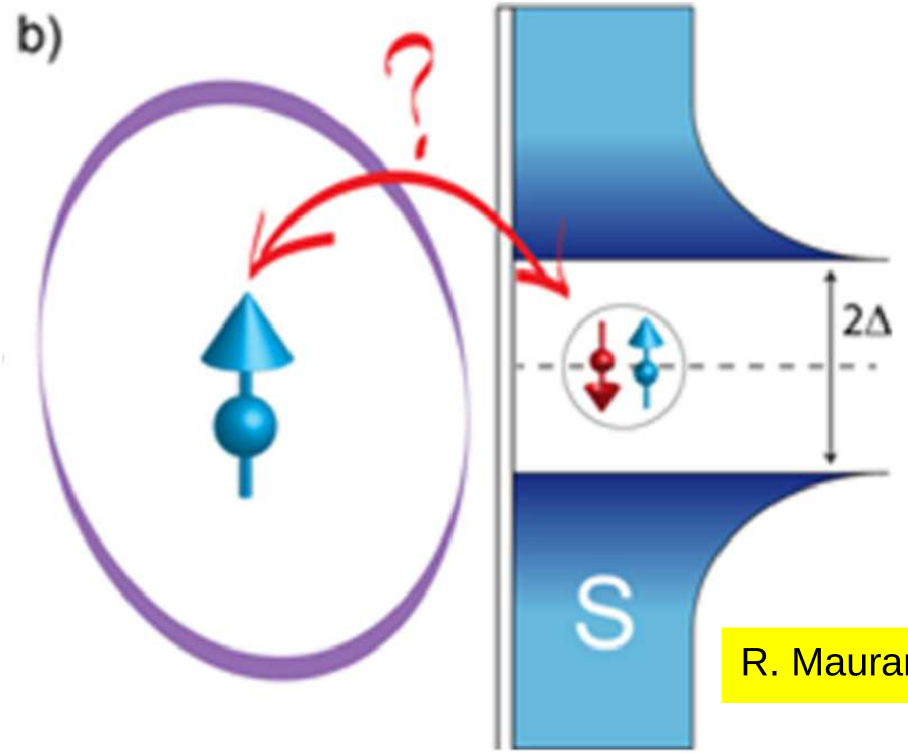


R. Maurand, Ch. Schönenberger, Physics **6**, 75 (2013).

⇒ **states near the Fermi level are depleted**

⇒ **electron pairing vs the Kondo state** **(nontrivial relation)**

## Singlet-doublet transition – impact on the Kondo state

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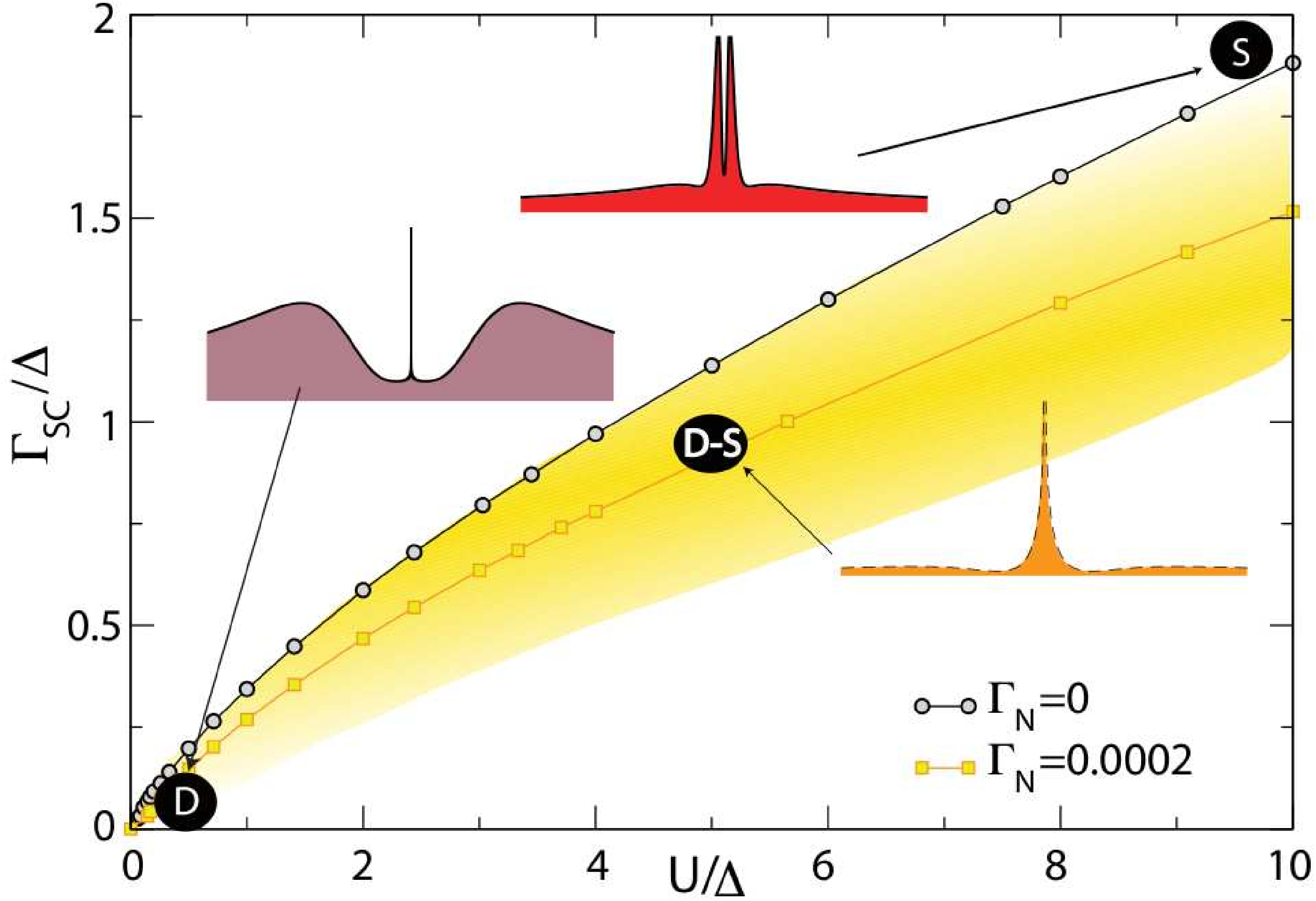
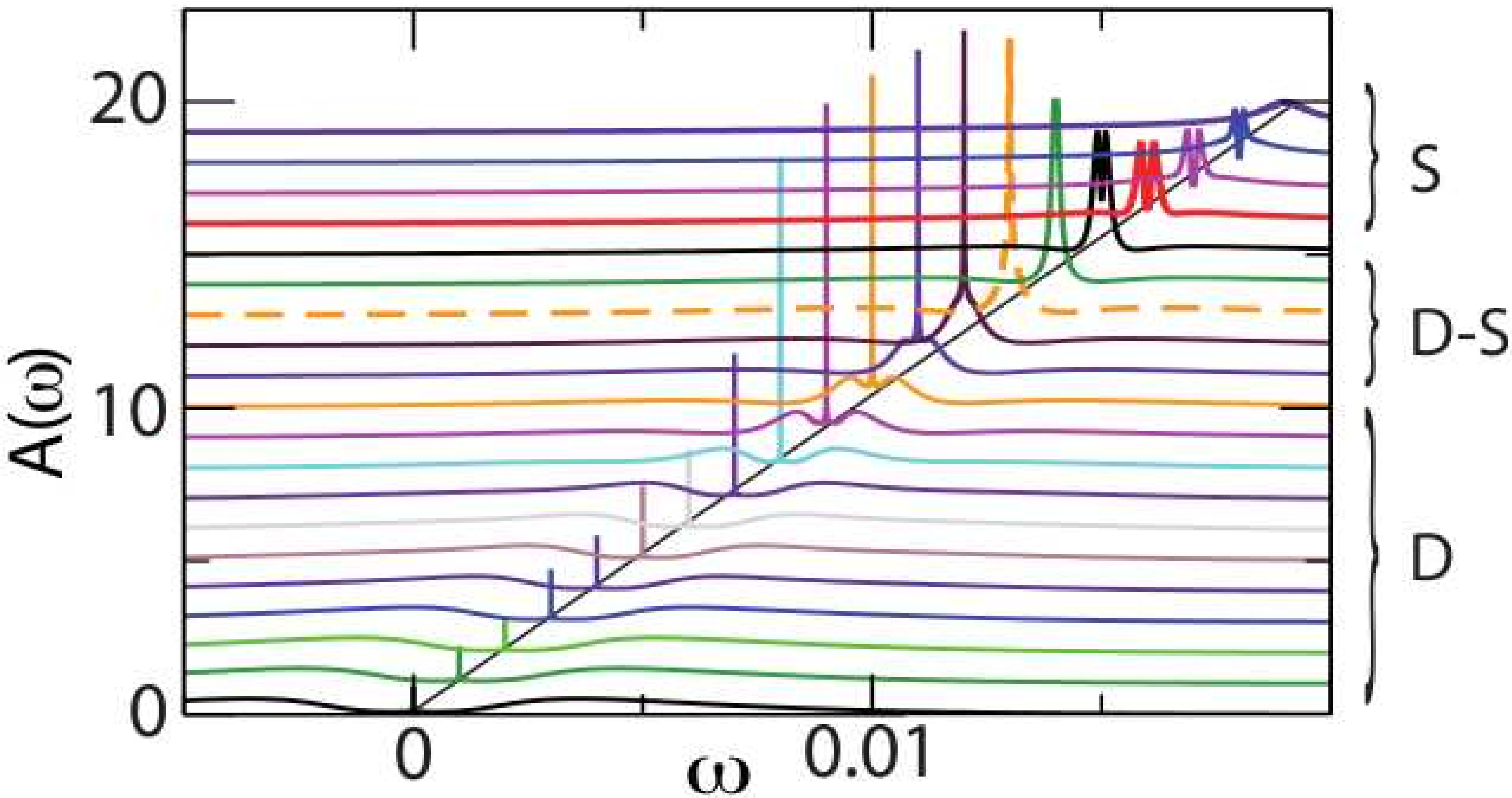



R. Žitko, J.S. Lim, R. López, and R. Aguado, Phys. Rev. B **91**, 045441 (2015).

**Singlet-doublet transition** – **impact on the Kondo state**

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## Correlated quantum dot – exact solution for $\Gamma_N \longrightarrow 0$

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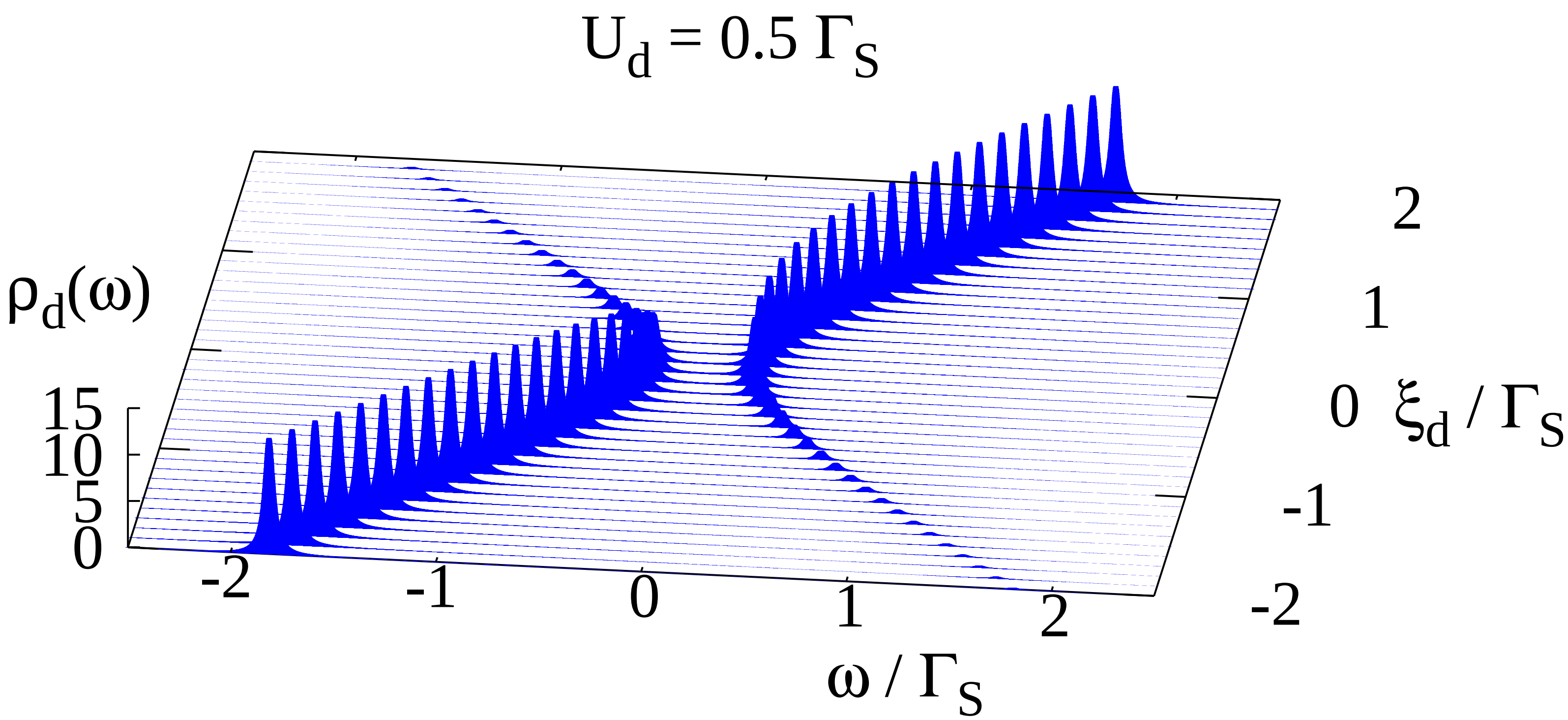
**Subgap spectrum $\rho_d(\omega)$ for varying $\xi_d \equiv \varepsilon_d + \frac{1}{2}U_d$**




**BCS-like states**

## Correlated quantum dot – exact solution for $\Gamma_N \to 0$

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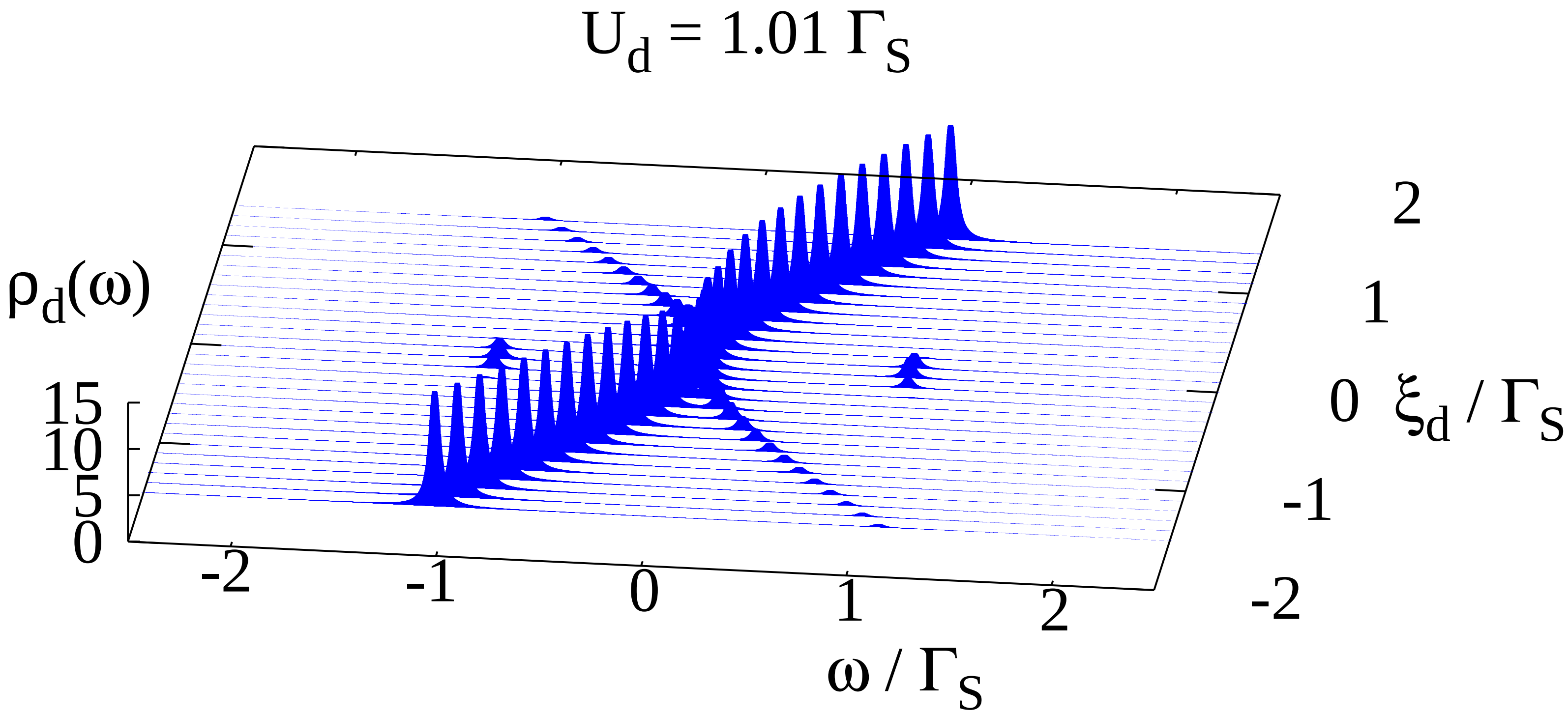


**near the quantum phase transition**

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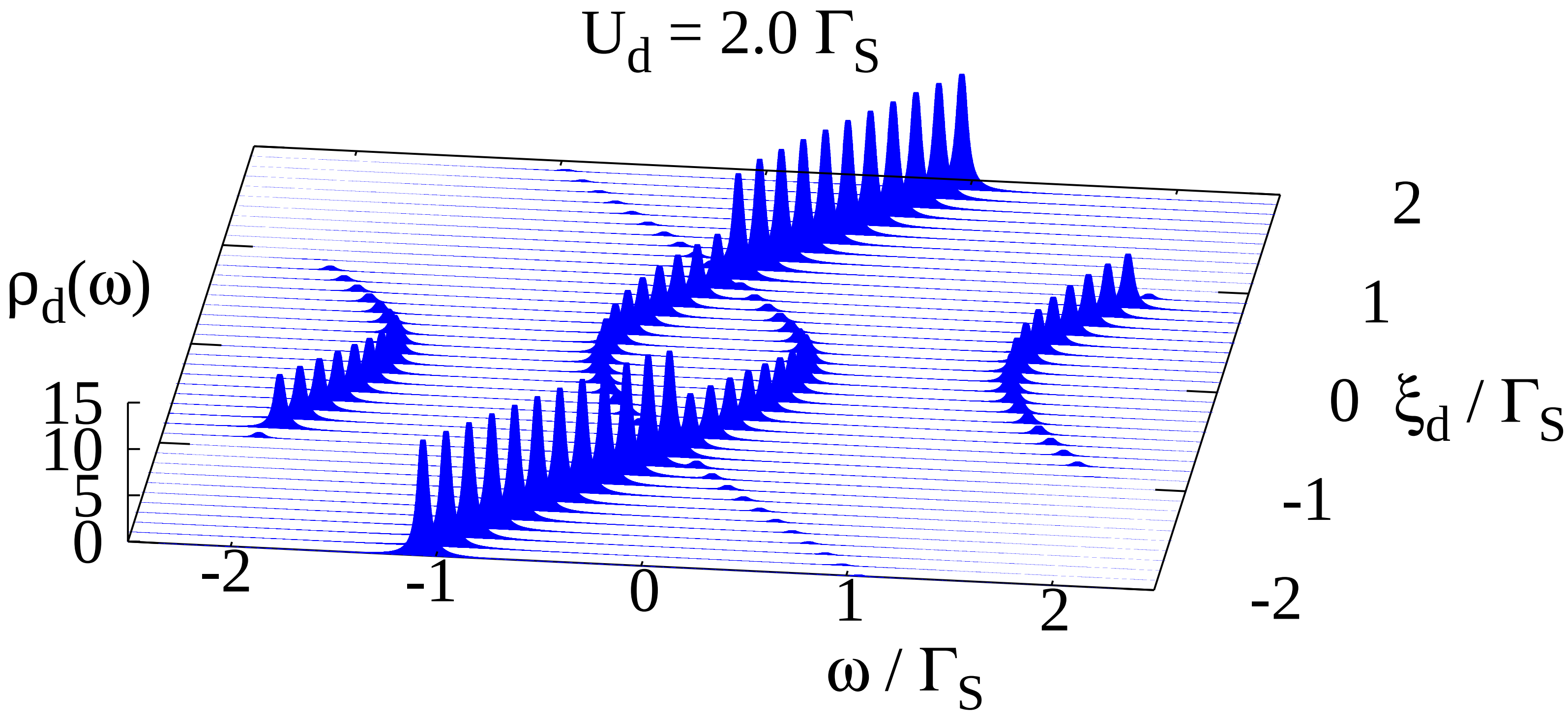
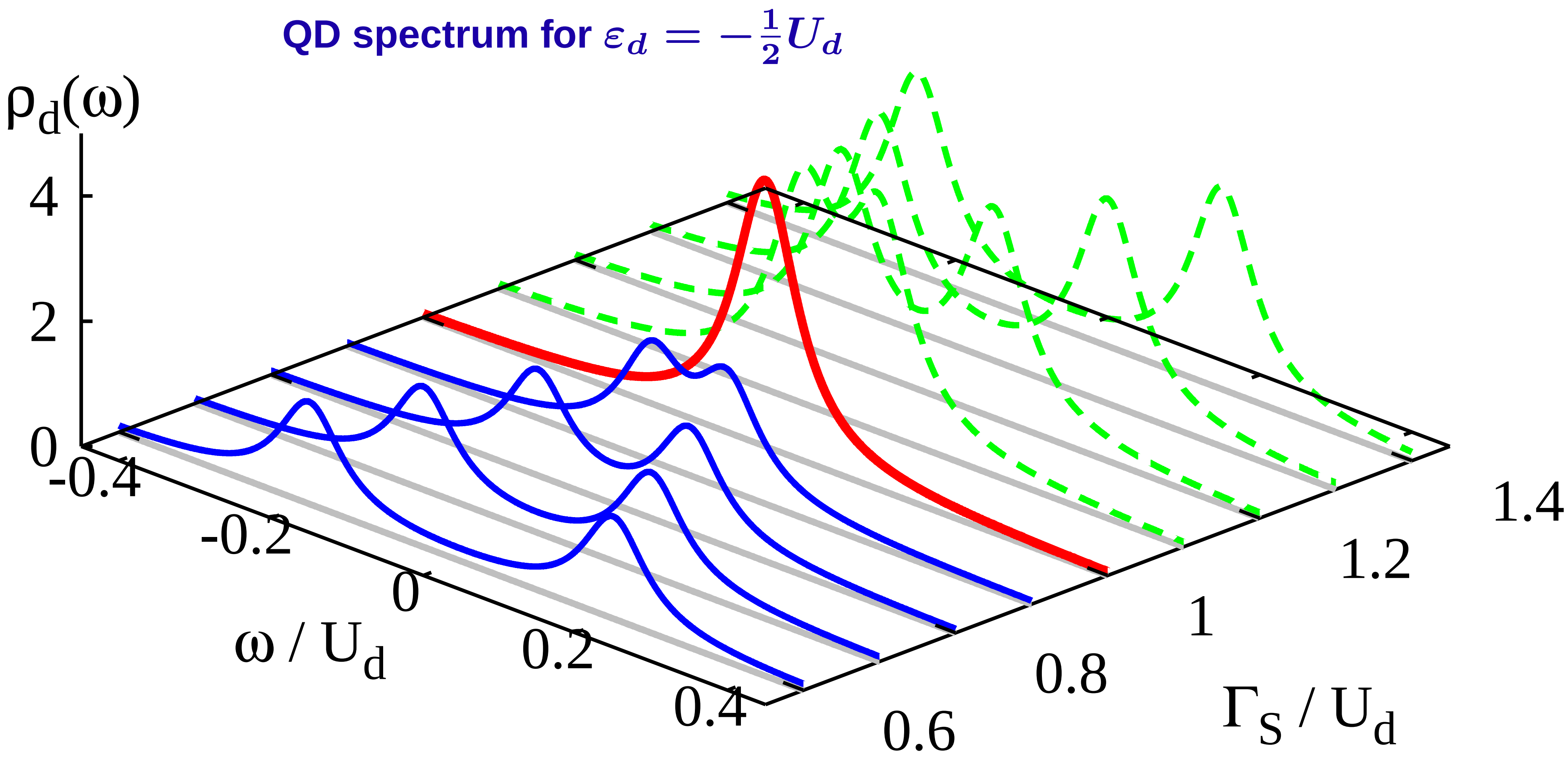



**crossings of the in-gap states**

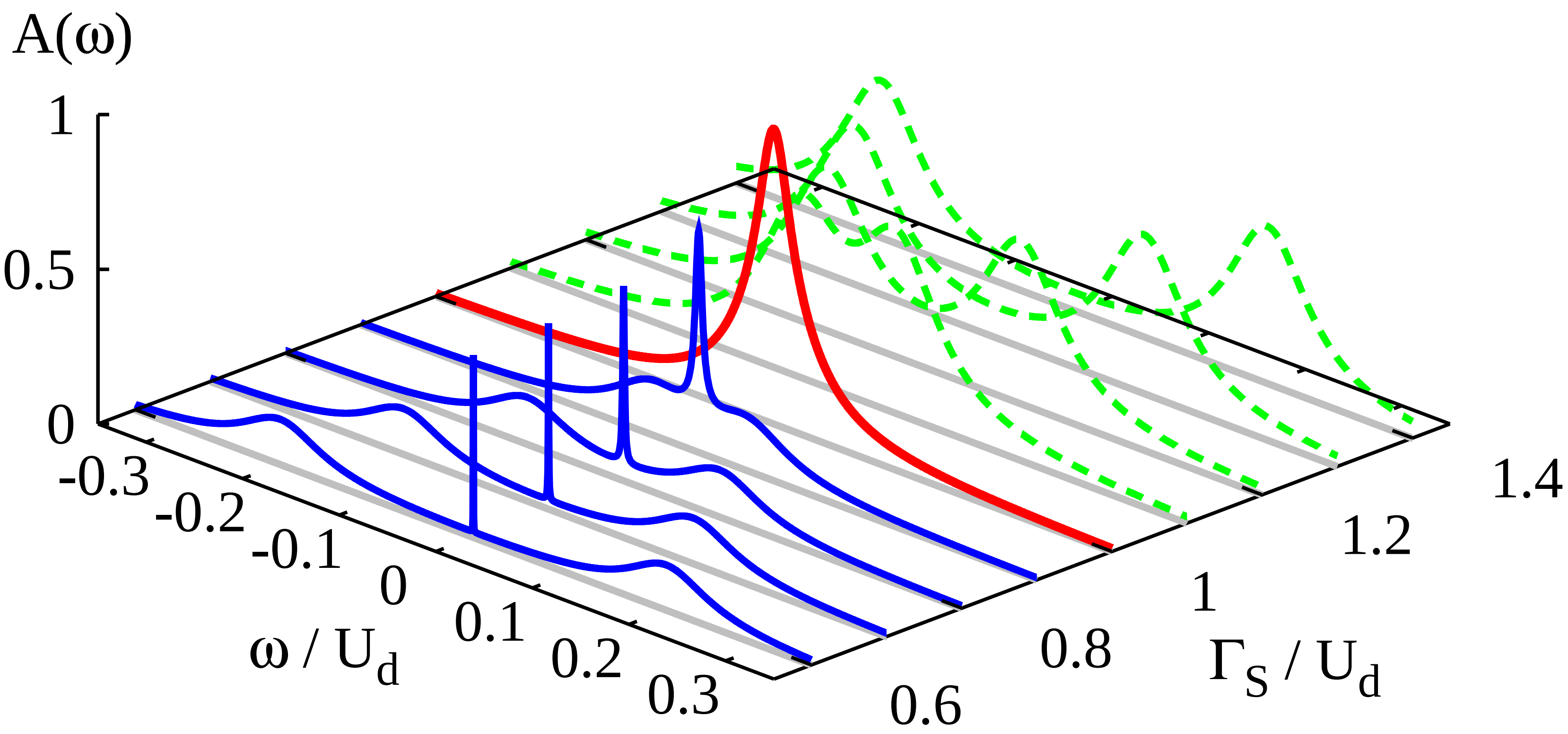
**Correlated quantum dot** – **exact solution for $\Gamma_N \to 0$**




**Quantum phase transition from the doublet to singlet states**

## Correlated quantum dot – $\Gamma_N \ll \Gamma_S$

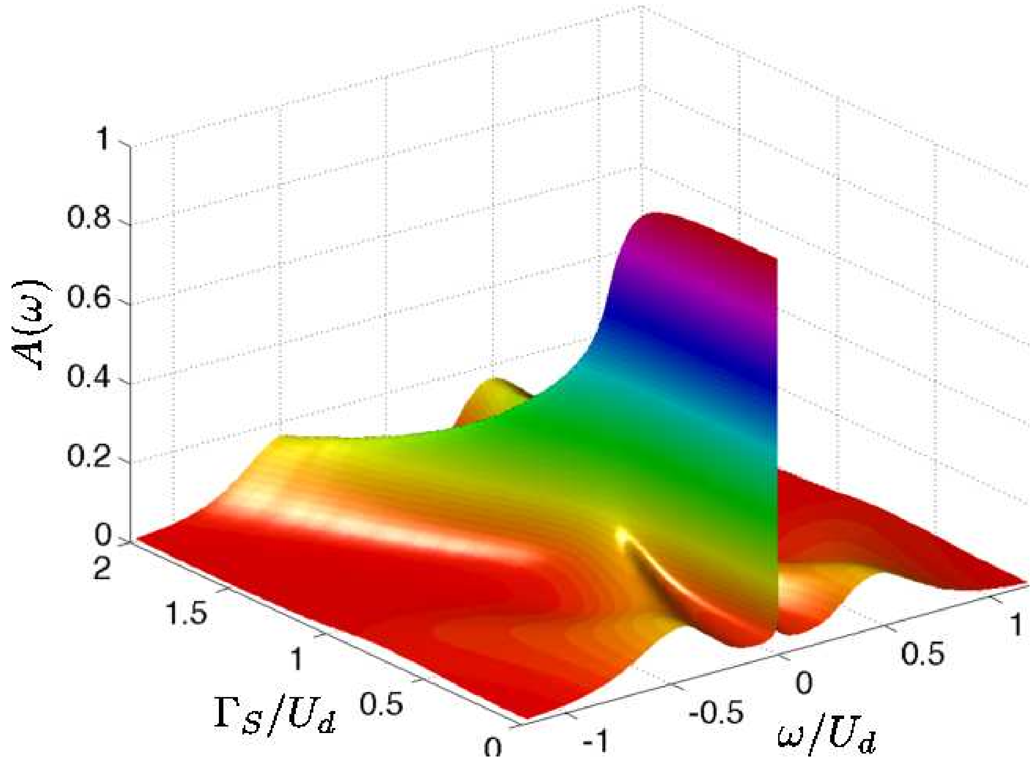
**The half-filled quantum dot ($\varepsilon_d = -\frac{1}{2}U_d$)**




**Kondo peak is present only in the spinful (doublet) state**

**Results obtained from the generalized Schrieffer-Wolff transformation**

## Correlated quantum dot – NRG results



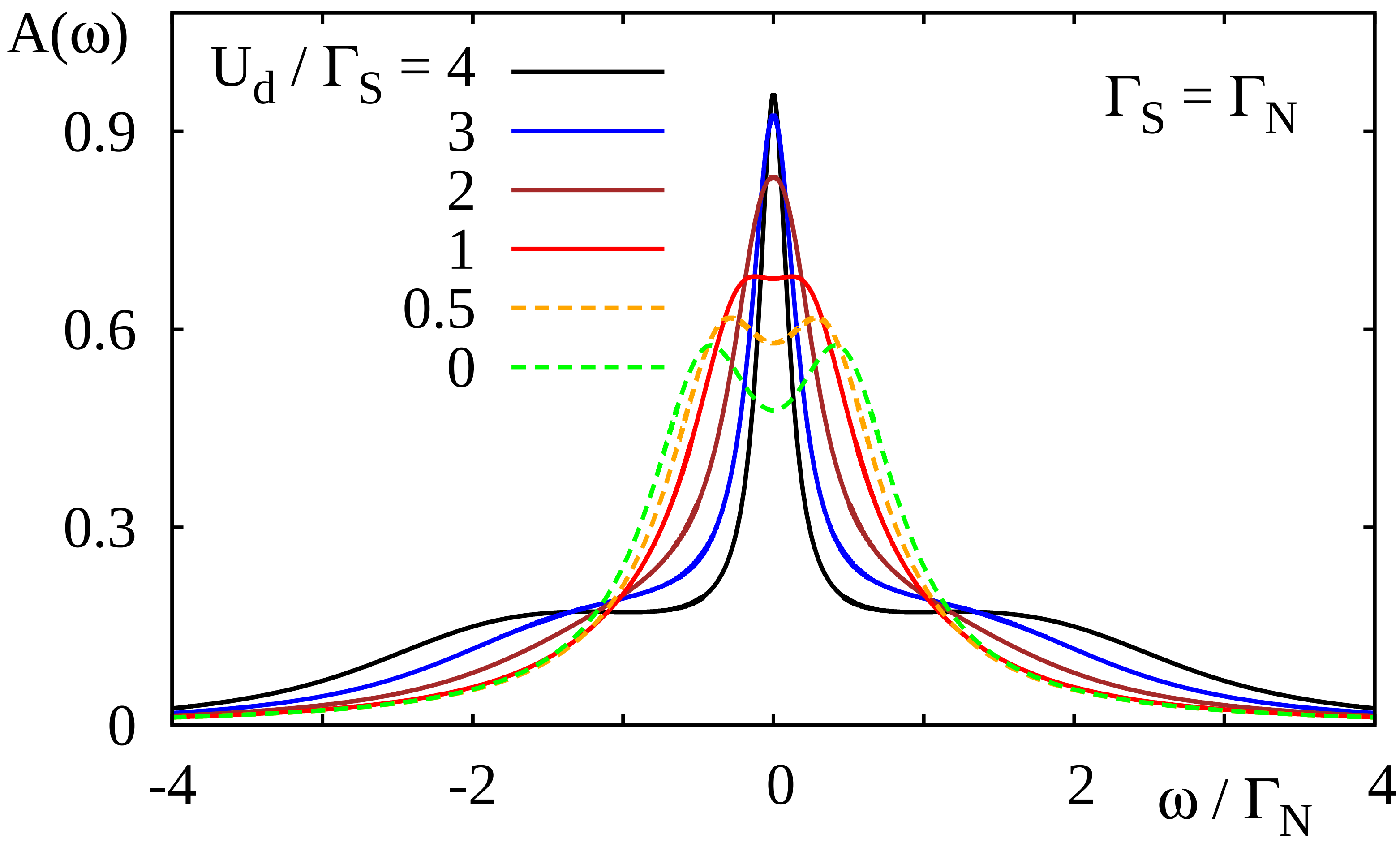

**Very intriguing observation: $T_K$ is enhanced with increasing $\Gamma_S$**

**Results obtained from the NRG calculations (Budapest code).**

T. Domański, I. Weymann, M. Barańska & G. Górski, arXiv:1507.01851 (2015).

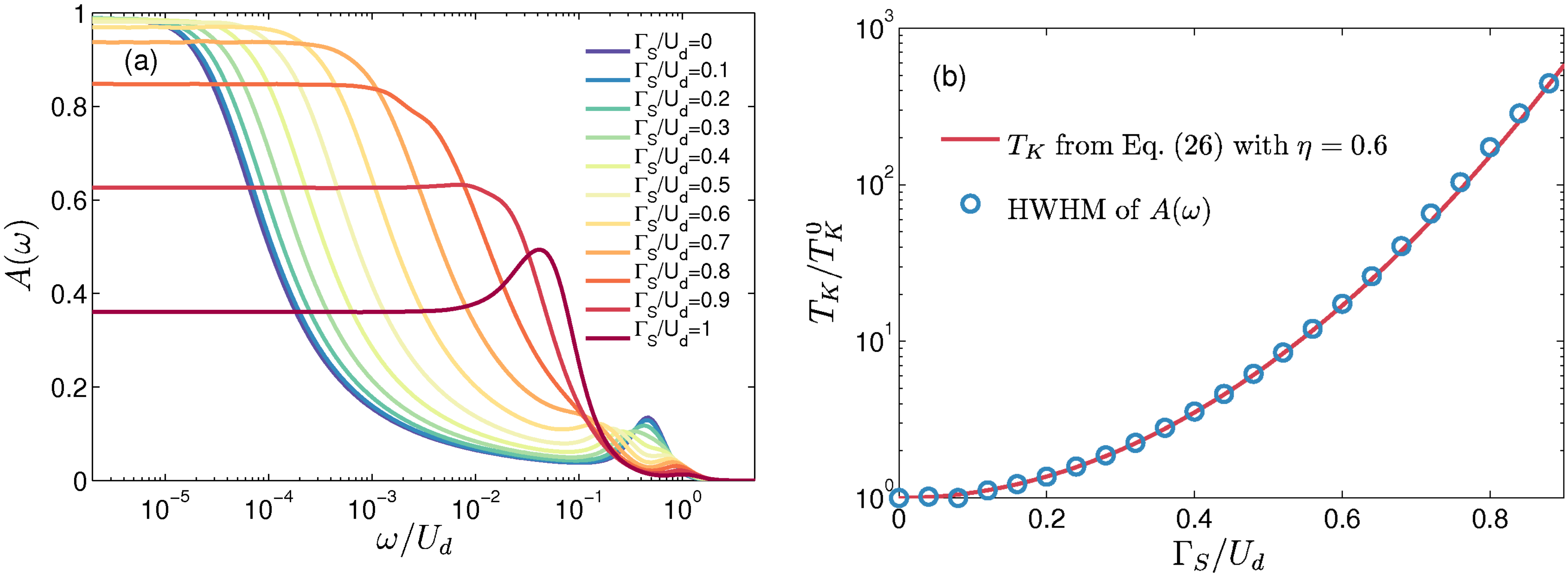
## Correlated quantum dot – arbitrary $\Gamma_N$

### The half-filled quantum dot ($\varepsilon_d = -\frac{1}{2}U_d$)




**Results obtained from the 2-nd order perturbative treatment of $U_d$.**

# Correlated quantum dot – Kondo temperature $T_K$



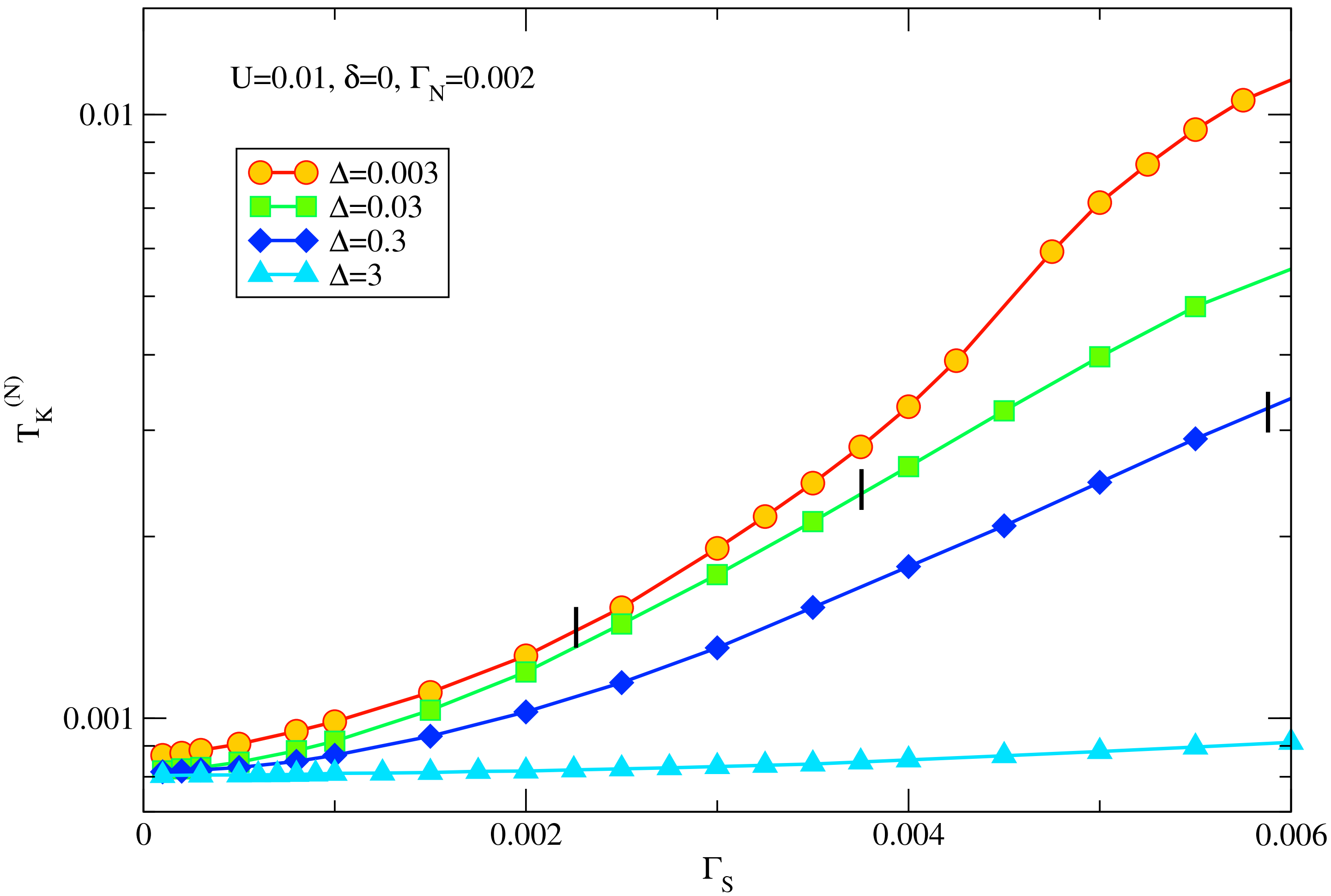

**$T_K$ estimated from the NRG and the extended Schrieffer-Wolff transformation**

$$T_K \simeq 0.3\sqrt{\Gamma_N U_d}\,\exp\left[\frac{\pi\varepsilon_d(\varepsilon_d+U_d)+(\Gamma_S/2)^2}{\Gamma_N U_d}\right]$$

T. Domański et al, arXiv:1507.01851 (2015).

**Correlated quantum dot** – **Kondo temp. $T_K$ for finite $\Delta_{sc}$**

## R. Žitko (private information)




**Results obtained by the NRG calculations (Lubljana code).**

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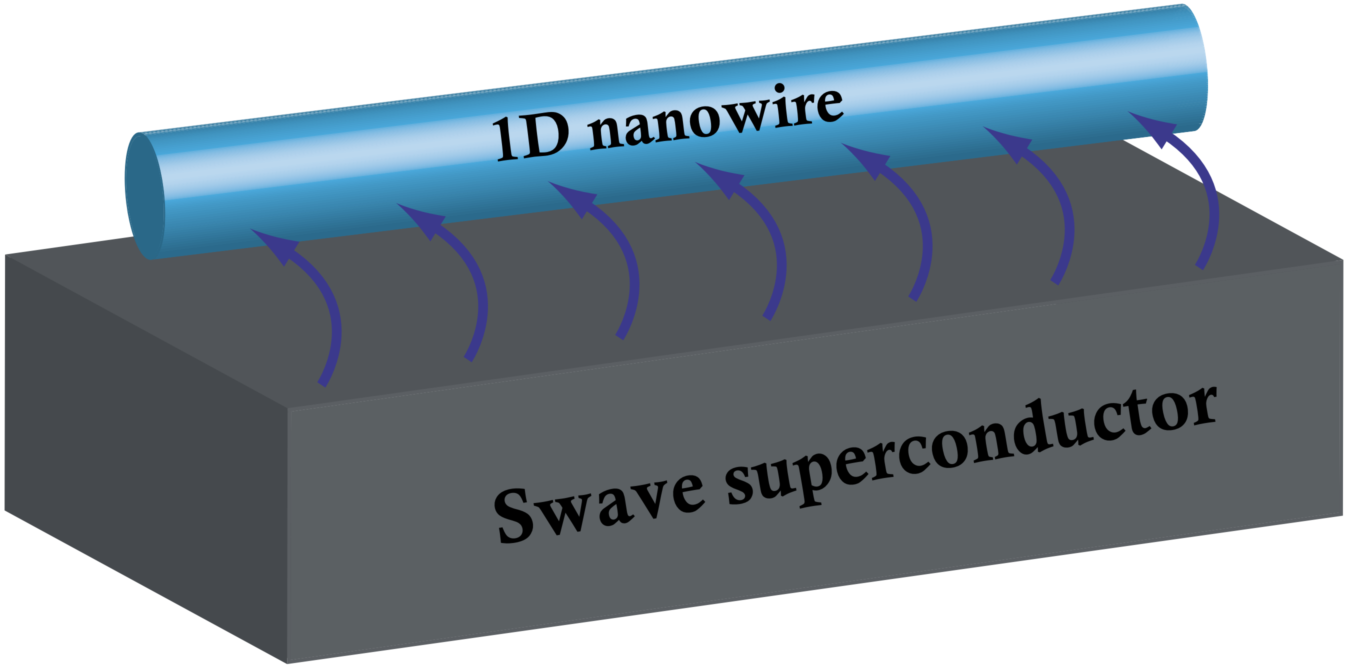
**http://kft.umcs.lublin.pl/doman/lectures**

**Further related issues:**

$\Rightarrow$ **Majorana qps in qunatum wires**

# Andreev vs Majorana states – a story of mutation

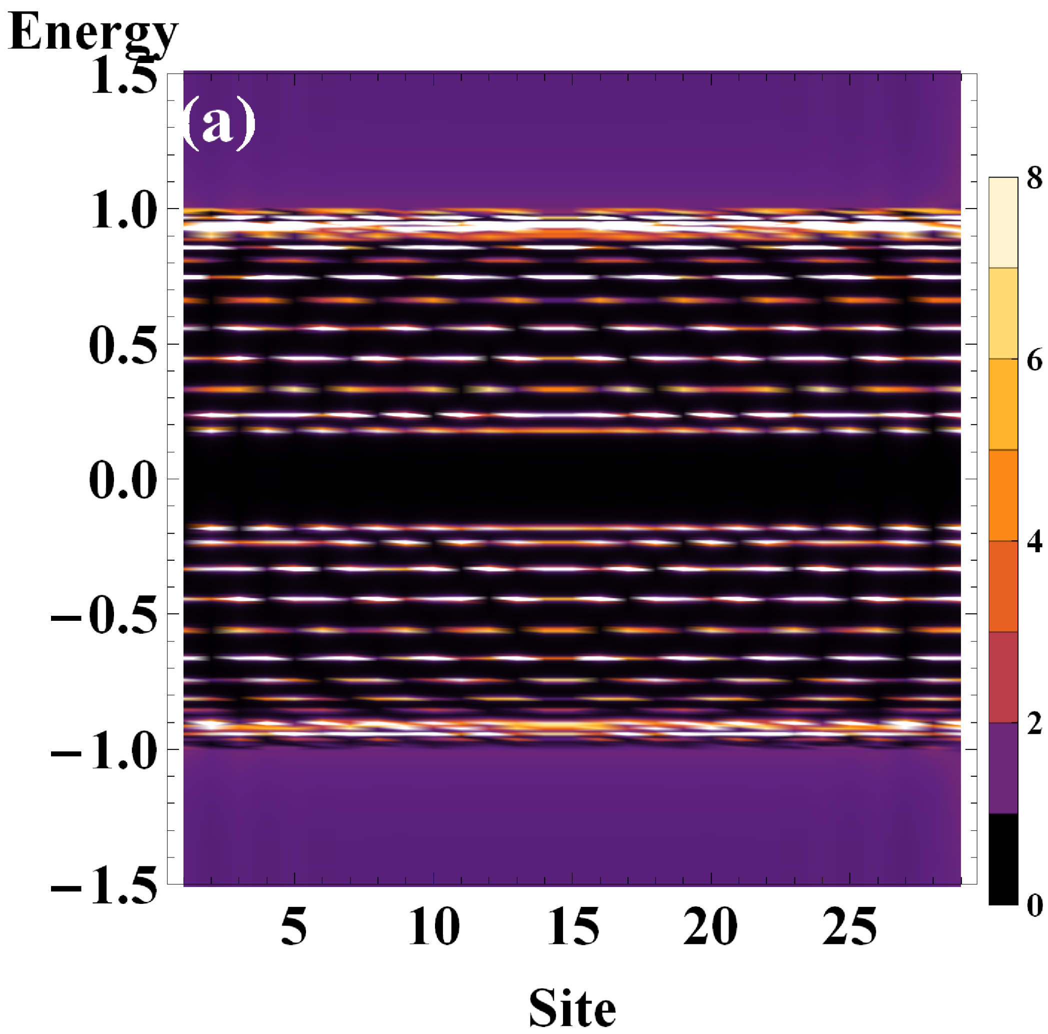
## Andreev vs Majorana states – a story of mutation




**Quantum wire deposited on s-wave superconductor**

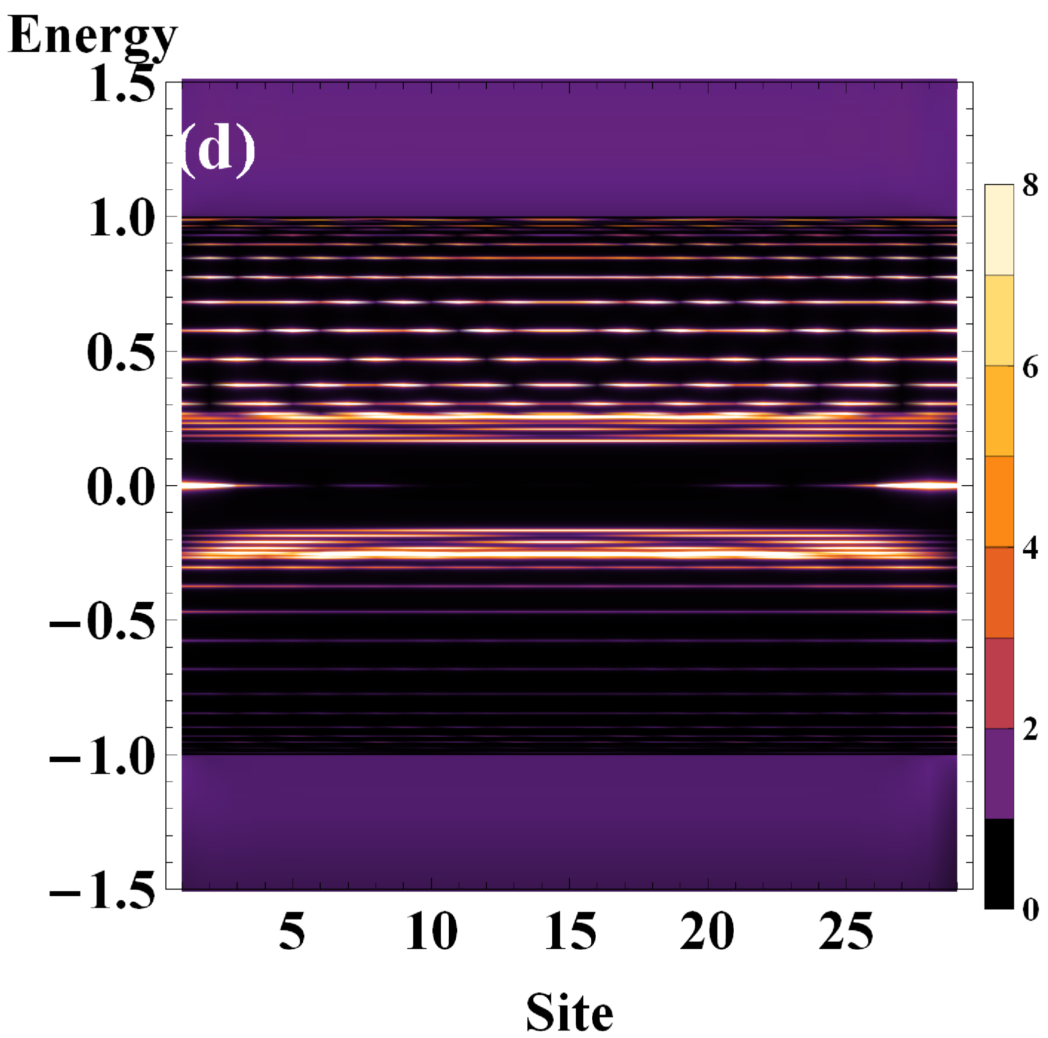
*D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B* **88**, *165401 (2013).*

## Andreev vs Majorana states – a story of mutation



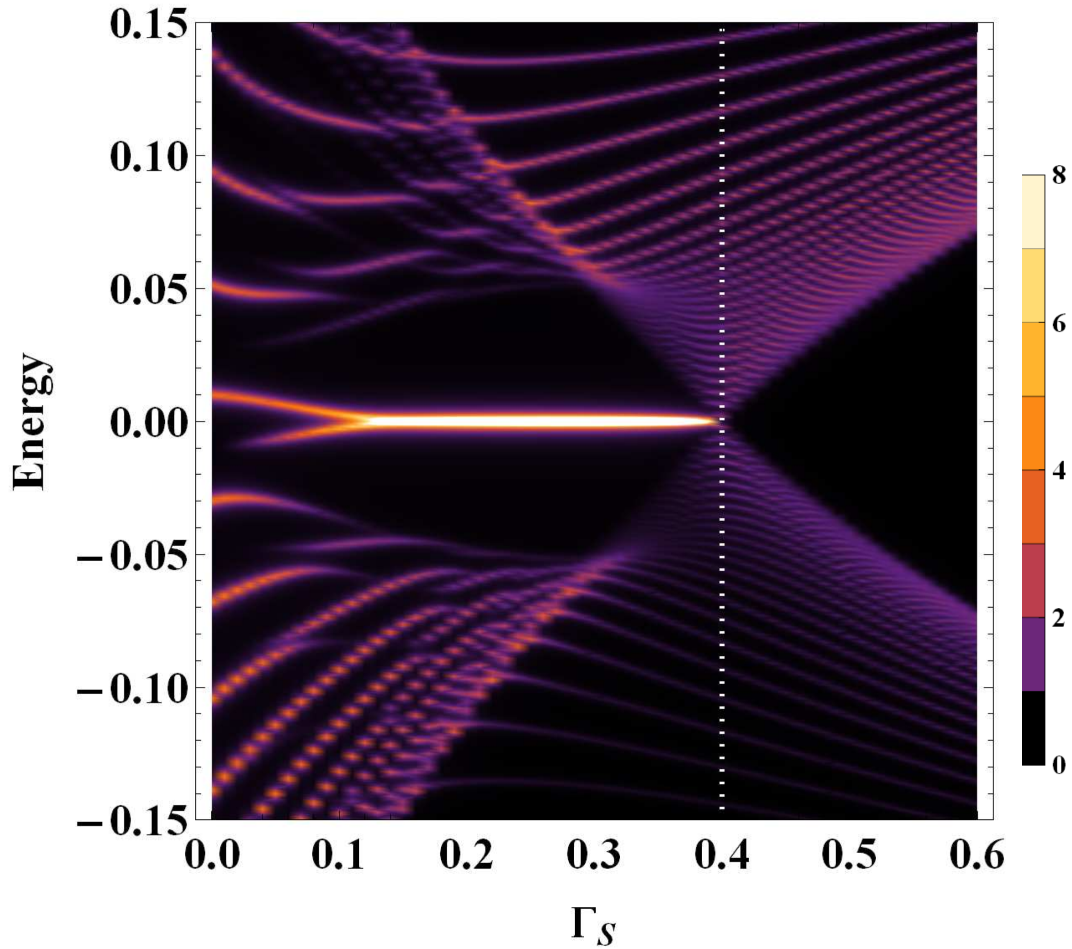

Spectrum of a quantum wire has a series of Andreev states.

# Andreev vs Majorana states – a story of mutation




Spin-orbit coupling can induce the Majorana-type quasiparticles.

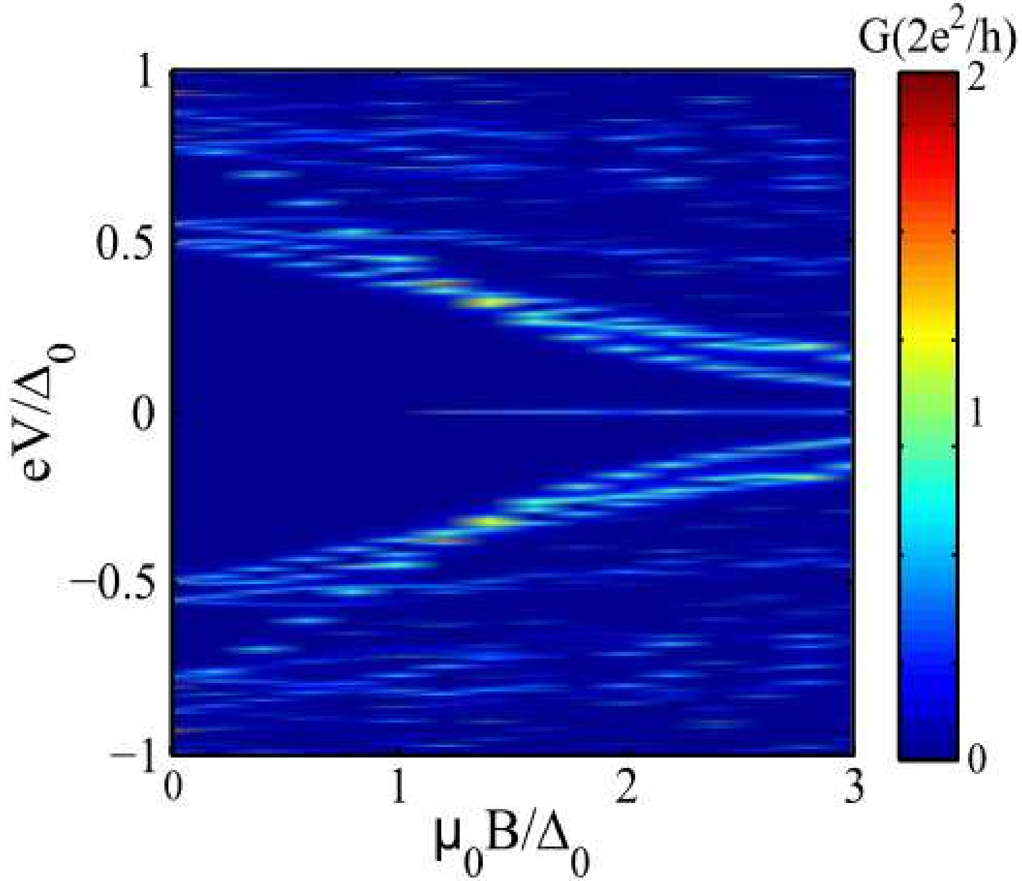
## Andreev vs Majorana states – a story of mutation




**Majorana quasiparticles appear at the edges of a quantum wire.**

*D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B* **88**, *165401 (2013).*

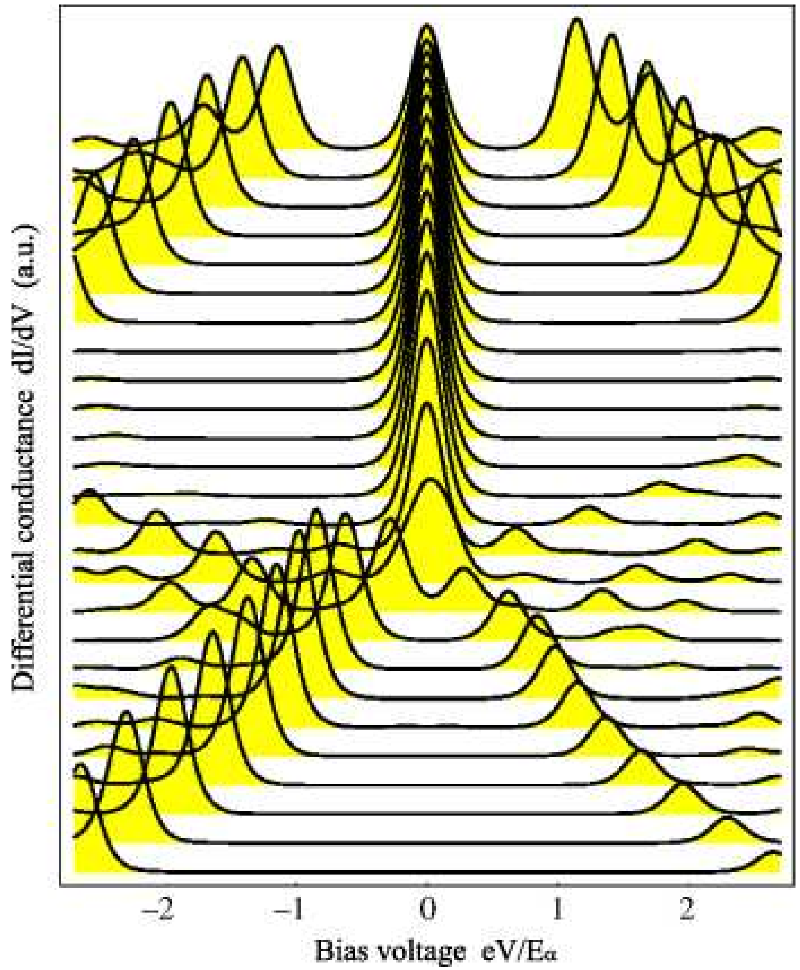
# Andreev vs Majorana states – a story of mutation




**Quasiparticles at the edge of a quantum wire for varying magnetic field.**

*J. Liu, A.C. Potter, K.T. Law, and P.A. Lee, Phys. Rev. Lett.* **109**, *267002 (2012).*

## Andreev vs Majorana states – a story of mutation




**Quasiparticles at the edge of a quantum wire for varying magnetic field.**

*T.D. Stanescu, R.M. Lutchyn, and S. Das Sarma, Phys. Rev. B* **84**, *144522 (2011).*

## Experimental results – for Majorana quasiparticles

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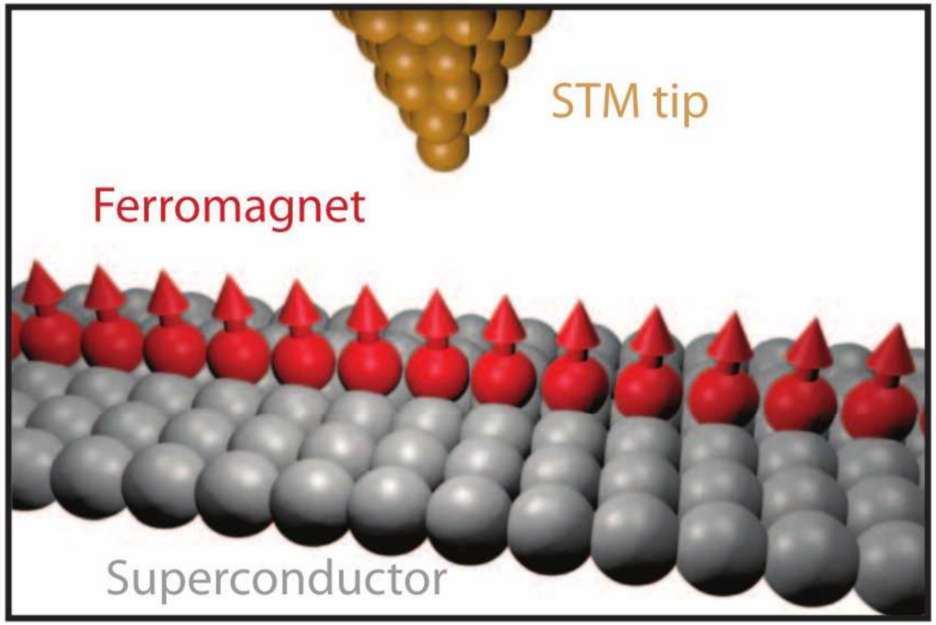
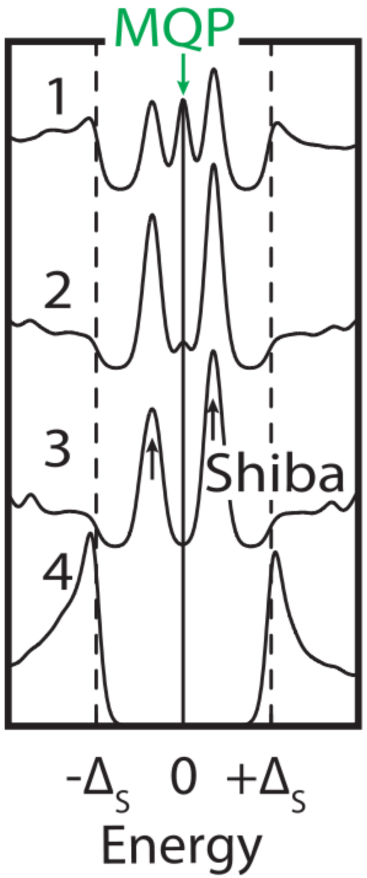
**A chain of iron atoms deposited on a surface of superconducting lead**




**STM measurements provided evidence for:**

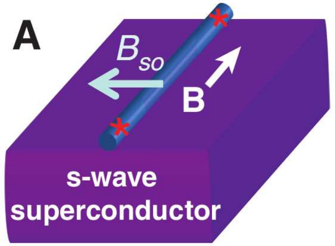
⇒ **Majorana bound states at the edges of a chain.**

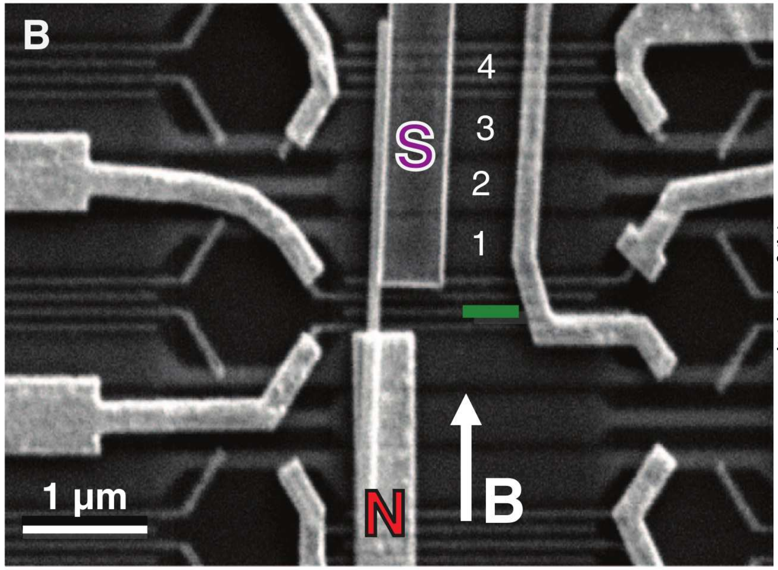
S. Nadj-Perge, ..., and A. Yazdani, Science **346**, 602 (2014).

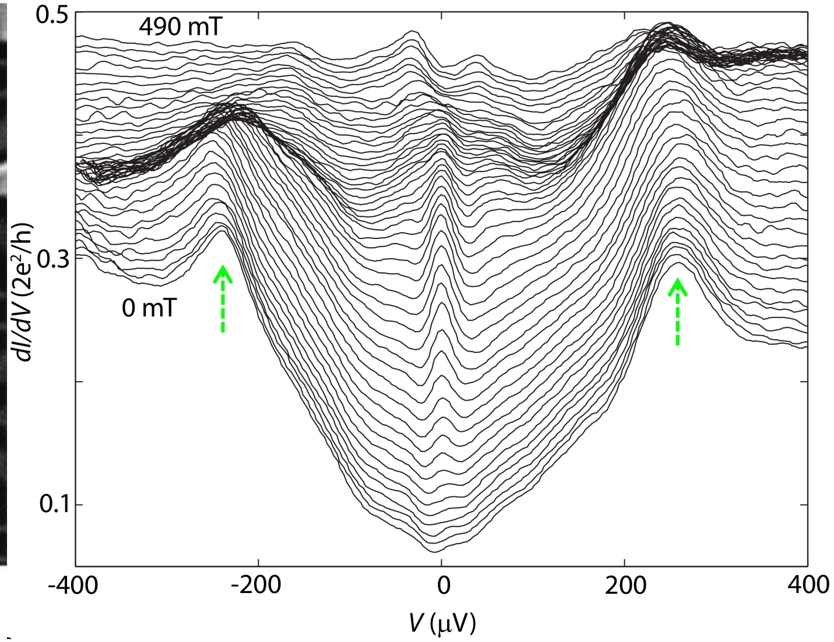
**/ Princeton University, Princeton (NJ), USA /**

## Experimental results – for Majorana quasiparticles

**InSb nanowire between a metal (gold) and a superconductor (Nb-Ti-N)**




**$dI/dV$ measured at 70 mK for varying magnetic field B indicated:**

$\Rightarrow$ **a zero-bias enhancement due to Majorana state**

V. Mourik, ..., and L.P. Kouwenhoven, Science **336**, 1003 (2012).

**/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /**