Interplay of superconductivity and magnetism in nanostructures

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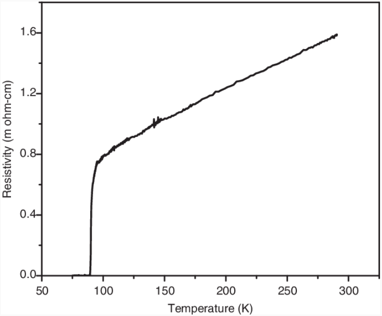
M. Curie-Skłodowska Univ.

Symposium on Spintronics and Quantum Information (Poznań) **21.10.21**

PROPERTIES OF BULK SUPERCONDUCTORS

Perfect conductor



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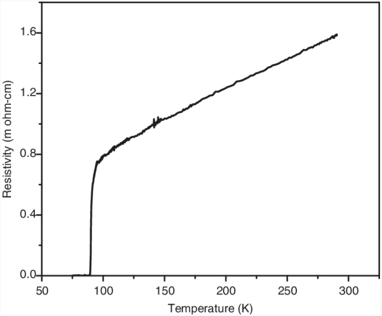

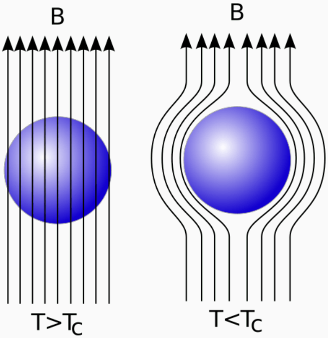

Perfect diamagnet

BCS (non-Fermi liquid) ground state :

$$|\mathbf{BCS}\rangle = \prod_{k} \left(u_k + v_k \, \hat{c}^{\dagger}_{k\uparrow} \, \hat{c}^{\dagger}_{-k\downarrow} \right) \, |\mathbf{vacuum}\rangle$$

ELECTRON PAIRING

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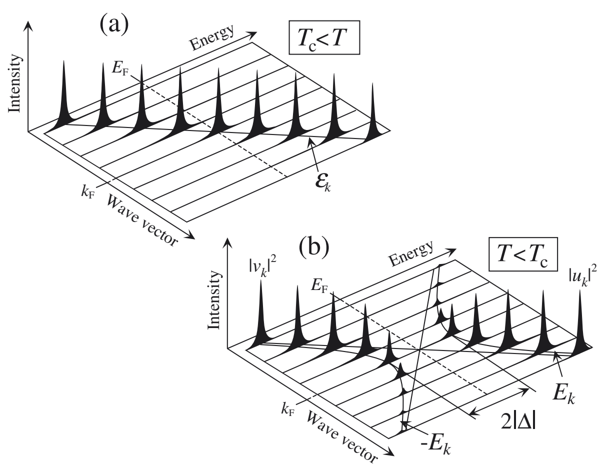
Bogoliubov quasiparticle = superposition of a particle and hole

$$\begin{aligned} \hat{\gamma}_{k\uparrow} &= u_k \hat{c}_{k\uparrow} + v_k \hat{c}^\dagger_{-k\downarrow} \\ \hat{\gamma}^\dagger_{-k\downarrow} &= -v_k \hat{c}_{k\uparrow} + u_k \hat{c}^\dagger_{-k\downarrow} \end{aligned}$$

BOGOLIUBOV QUASIPARTICLES

Quasiparticle spectrum of conventional superconductors consists of two Bogoliubov (p/h) branches, gaped around E_F



Pairing vs magnetism

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are they friends or foes ?

DESTRUCTIVE INFLUENCE OF MAGNETIC FIELD

Magnetism and electron pairing seem to be antagonistic ...

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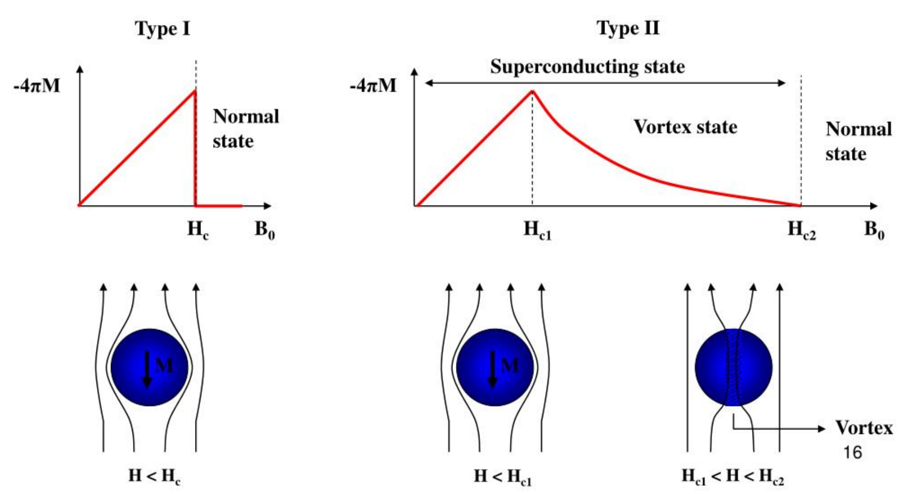

Magnetic field can penetrate type-II superconductors (vortex-structure)

Nanoscopic superconductors

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1. impurity attached to bulk superconductor

PAIRING MECHANISM: PROXIMITY EFFECT

- **Quantum impurity/dot (QD) coupled to bulk superconductor:**

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PAIRING MECHANISM: PROXIMITY EFFECT

- Quantum impurity/dot (QD) coupled to bulk superconductor:

$\Longrightarrow$ **develops electron pairing**

- which is spectroscopically manifested by:

$\Longrightarrow$ **in-gap bound states**

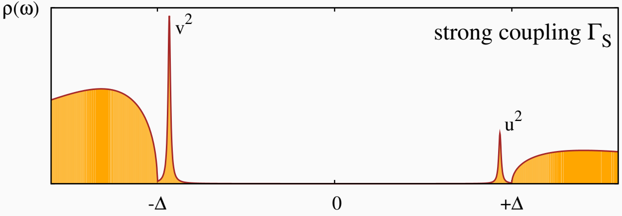
- driven by:

$\Longrightarrow$ **leakage of Cooper pairs on QD** (Andreev)

$\Longrightarrow$ **exchange int. of QD with SC** (Yu-Shiba-Rusinov)

IN-GAP STATES

Spectrum of a single impurity coupled to bulk superconductor:



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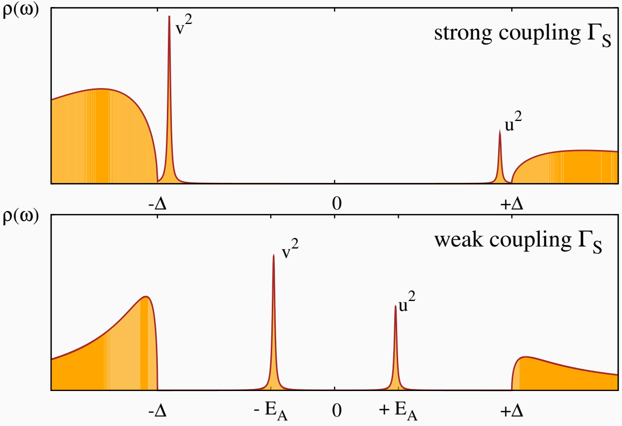

Quasiparticle states appearing in the subgap region $-\Delta < \omega < \Delta$.

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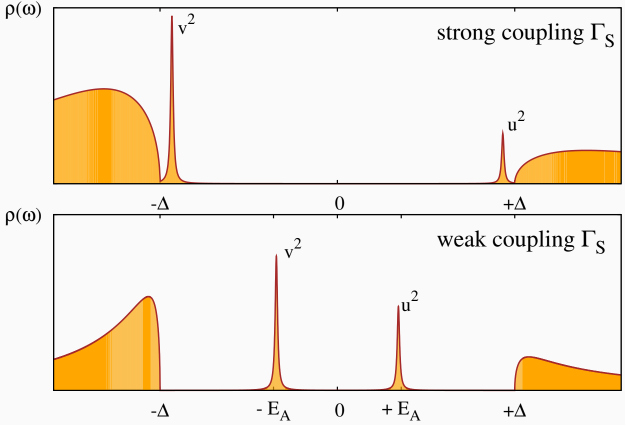

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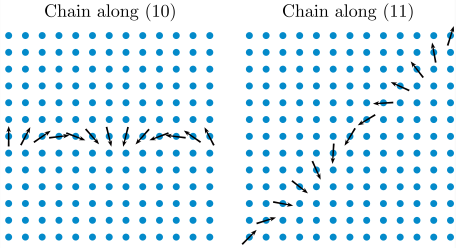
Yu-Shiba-Rusinov (Andreev) bound states

MAGNETIC OBJECTS IN SUPERCONDUCTORS

More complex objects in superconductors, like magnetic chains



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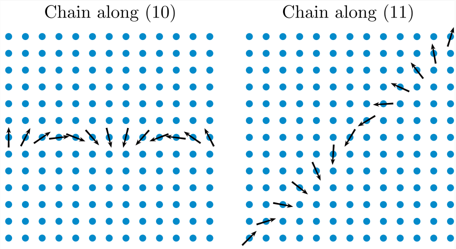
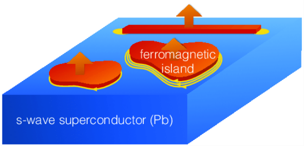


or magnetic islands



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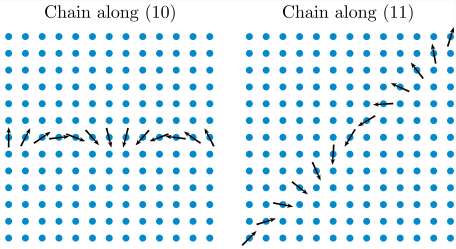

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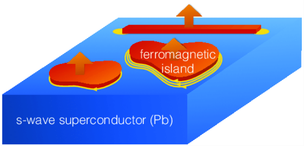

arrange their in-gap bound states into Shiba-bands.

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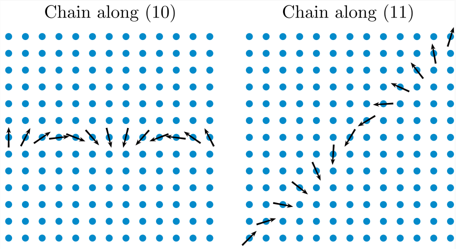

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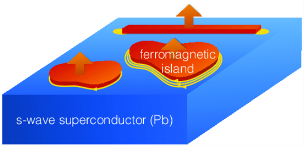


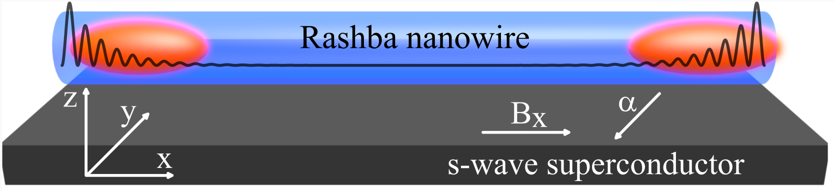
arrange their in-gap bound states into Shiba-bands.

Specific magnetic textures of these chains and/or islands can induce topologically non-trivial superconducting state, hosting the Majorana-type boundary modes !

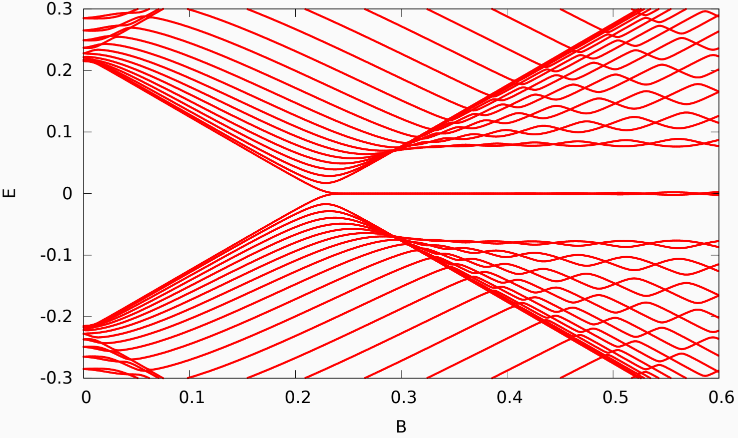
A few examples ...

1. RASHBA NANOWIRE

Pairing of identical spin electrons is driven by the spin-orbit (Rashba) interaction in presence of magnetic field, using the semiconducting nanowires proximitized to conventional (*s-wave*) superconductor.

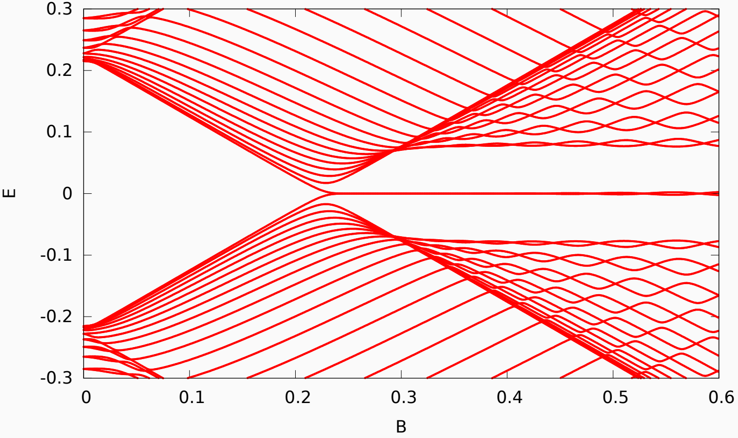


Bound states of the proximitized Rashba nanowire



TRANSITION TO TOPOLOGICAL PHASE

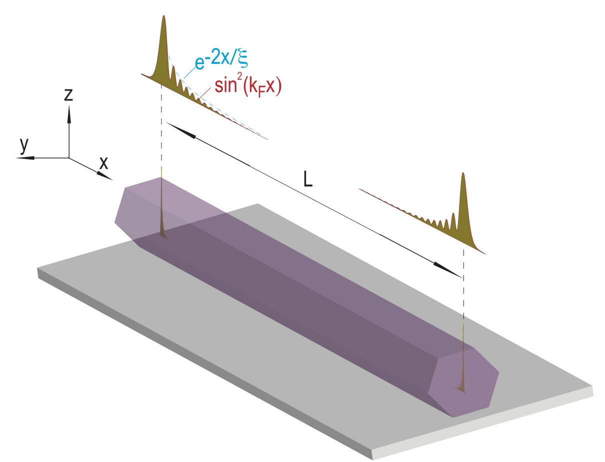
Bound states of the proximitized Rashba nanowire



closing/reopening of a soft gap $\Longleftrightarrow$ topological transition

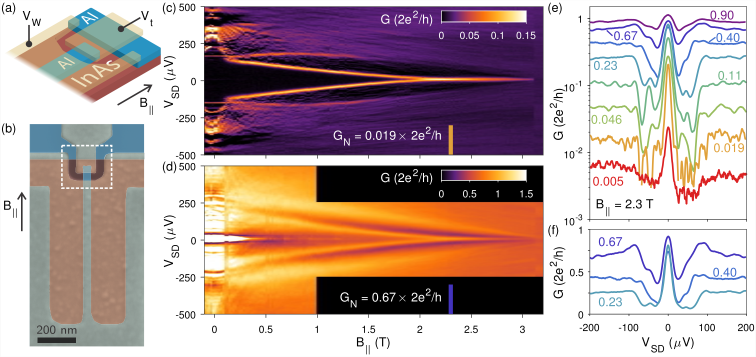
M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

Majorana qps are localized near the edges



R. Aguado, Riv. Nuovo Cim. 40, 523 (2017).

Litographically fabricated Al nanowire contacted to InAs

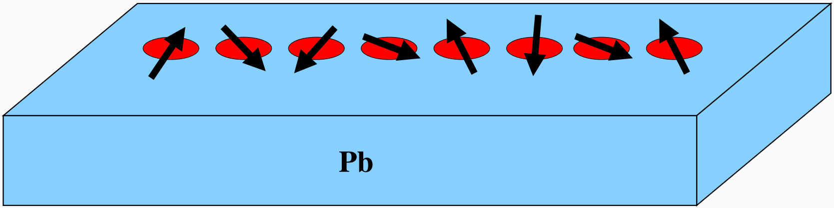


F. Nichele, ..., and Ch. Marcus, Phys. Rev. Lett. **119**, 136803 (2017).

/ Niels Bohr Institute, Copenhagen, Denmark /

2. SELFORGANIZED MAGNETIC CHAINS

Magnetic atoms (like Fe) on a surface of s-wave superconductor (for example Pb) arrange themselves into a spiral order, which selfsustains the topological superconducting phase (topolofilia)



MAGNETIC CHAIN ON SUPERCONDUCTOR

Itinerant electrons in the chain of magnetic impurities placed on a surface of isotropic superconductor can be described by the Hamiltonian:

$$H = - t \sum_{i,\sigma} \left(\hat{c}^{\dagger}_{i,\sigma} \hat{c}_{i+1,\sigma} + \text{H.c.} \right) - \mu \sum_{i,\sigma} \hat{c}^{\dagger}_{i,\sigma} \hat{c}_{i,\sigma} + J \sum_{i} \vec{S}_i \cdot \hat{\vec{s}}_i + \sum_{i} \left(\Delta \hat{c}^{\dagger}_{i\uparrow} \hat{c}^{\dagger}_{i\downarrow} + \text{H.c.} \right)$$

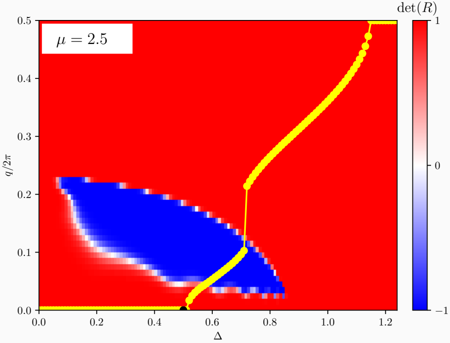
Here $\vec{S}_i$ are the classical magnetic moments and $\hat{\vec{s}}_i = \frac{1}{2} \sum_{\alpha,\beta} \hat{c}^{\dagger}_{i,\alpha} \vec{\sigma}_{\alpha\beta} \hat{c}_{i,\beta}$ denote the spins of mobile electrons

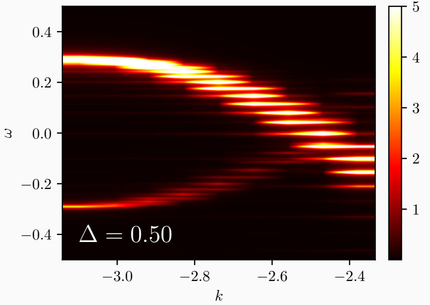
$\Longrightarrow$ **J is the coupling between magnetic atoms and itinerant electrons**

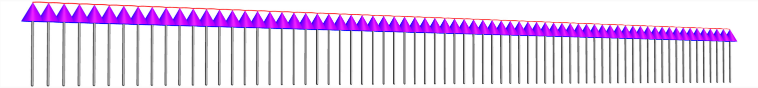
$\Longrightarrow$ **Δ is the proximity induced on-site pairing**

HELICAL SELFORGANISATION & TOPOFILIA

A. Gorczyca-Goraj, T. Domański & M.M. Maśka, Phys. Rev. B 99, 235430 (2019).







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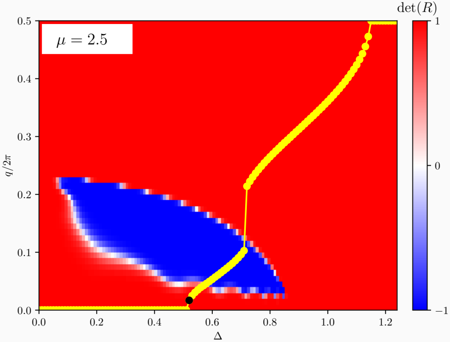

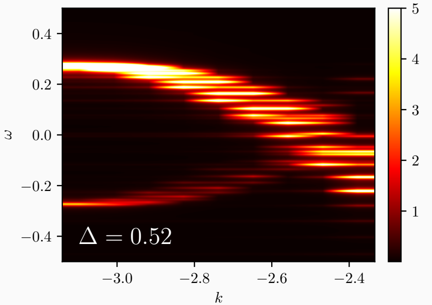

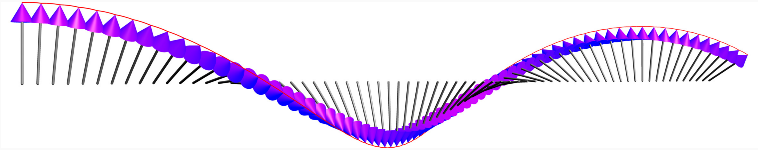

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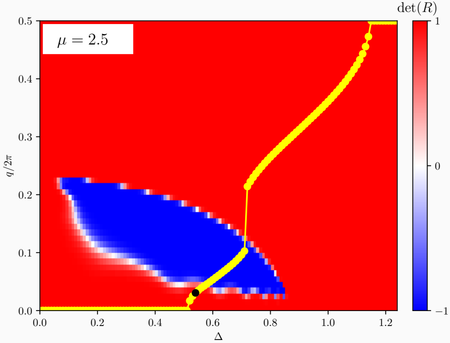

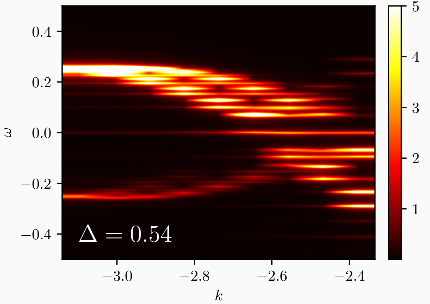

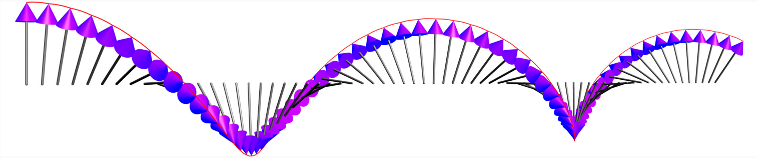

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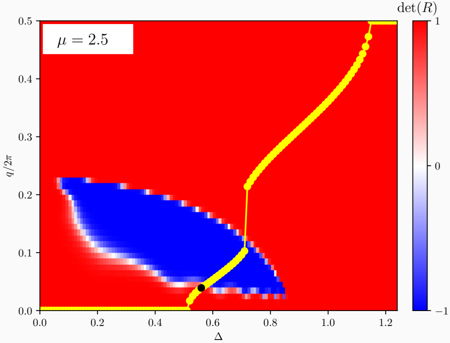

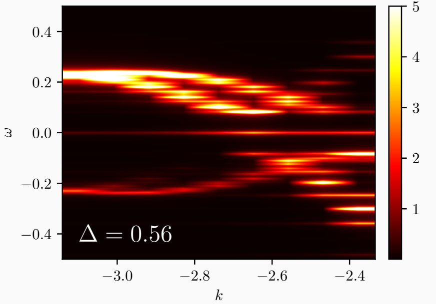

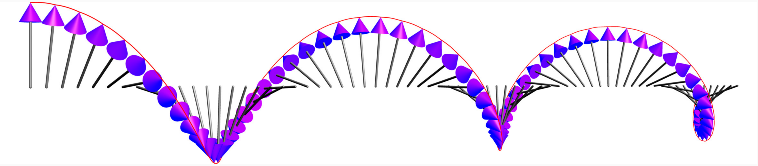

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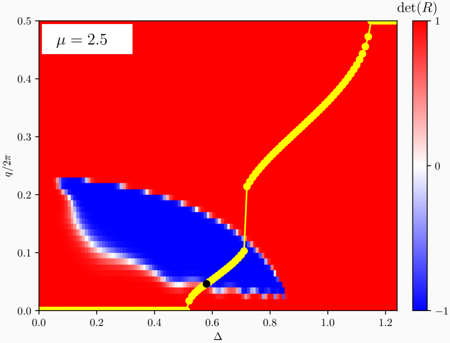

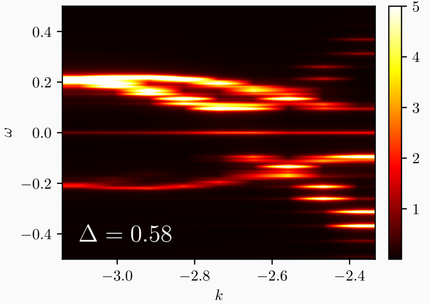

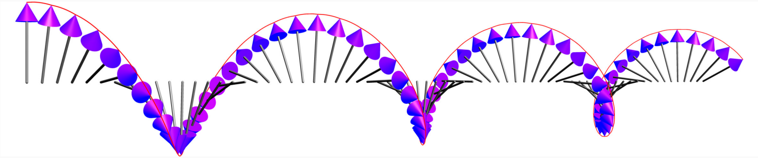

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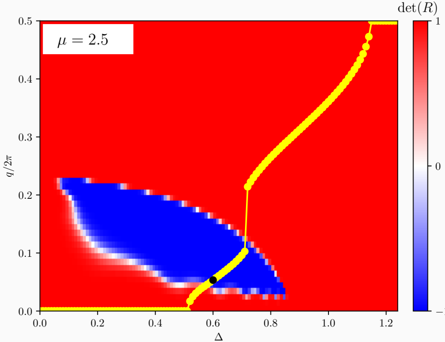

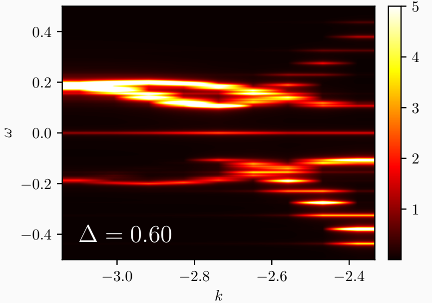

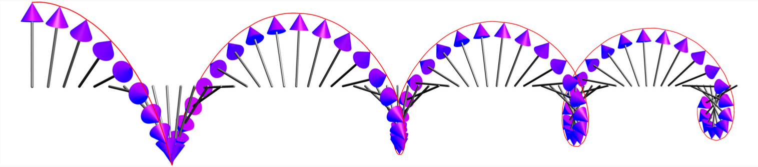

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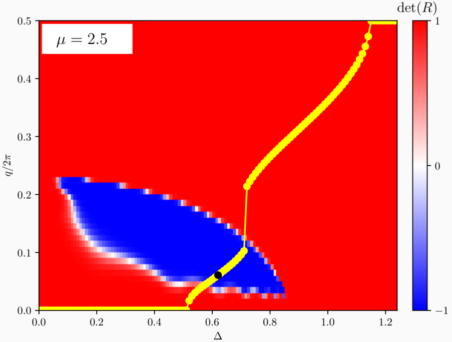

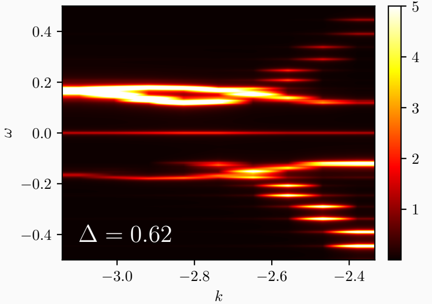

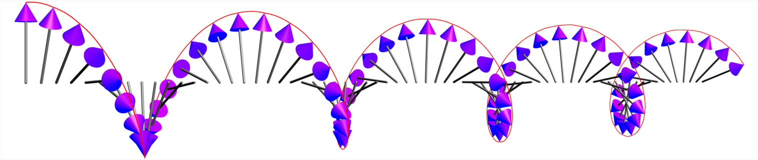

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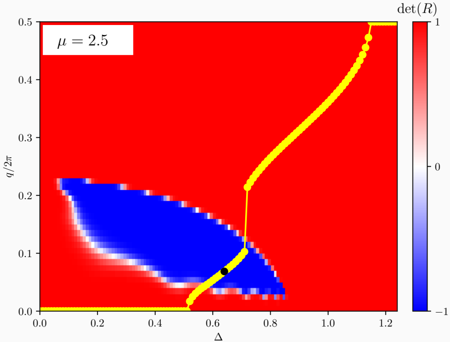

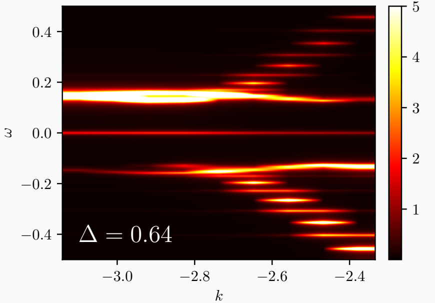

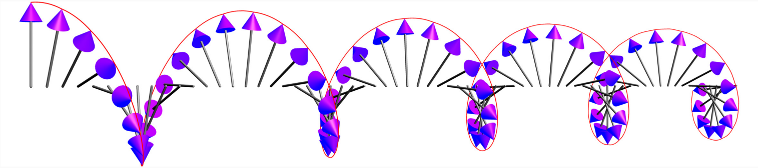

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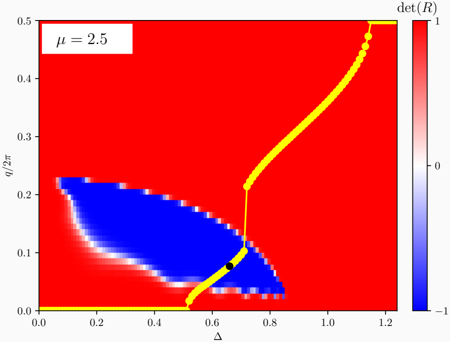

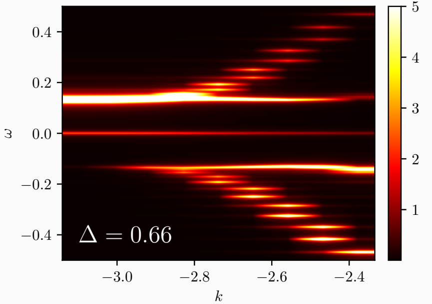

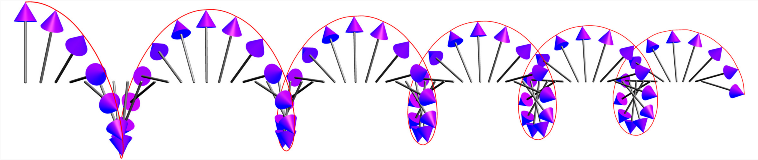

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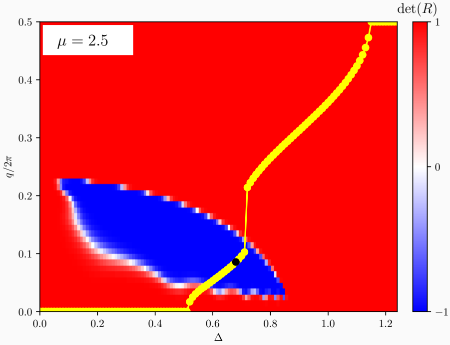

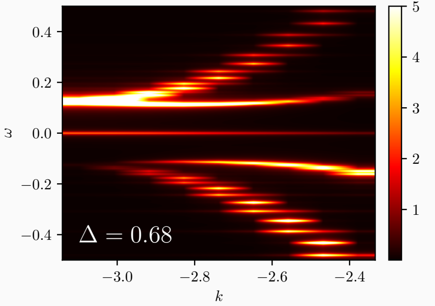

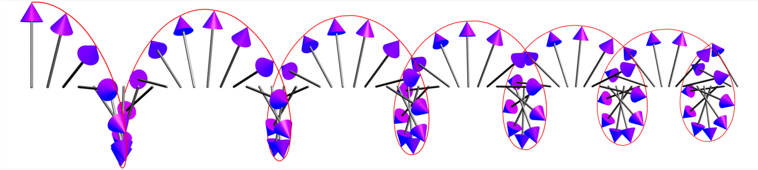

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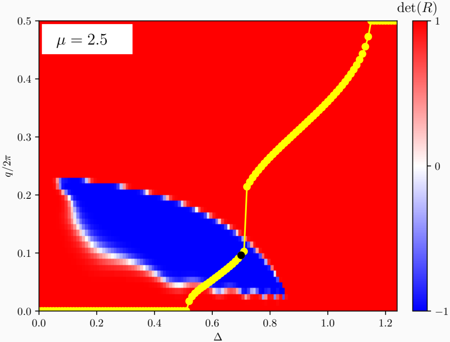

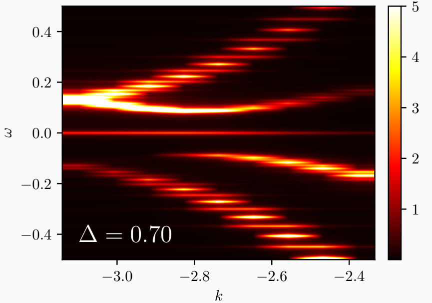

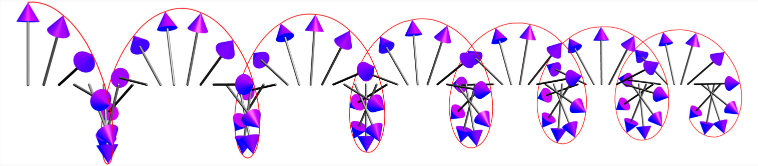

HELICAL SELFORGANISATION & TOPOFILIA

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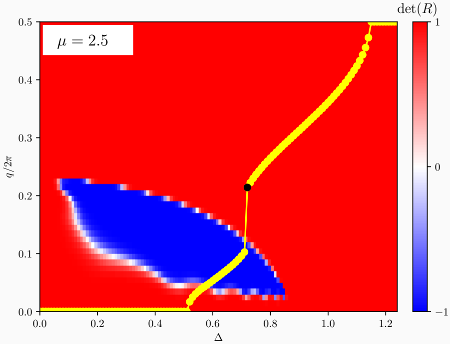
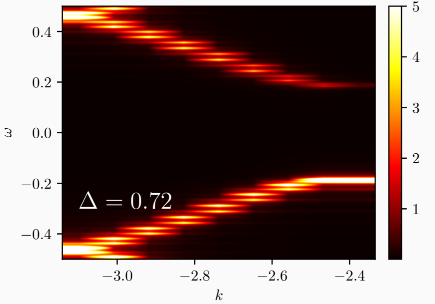
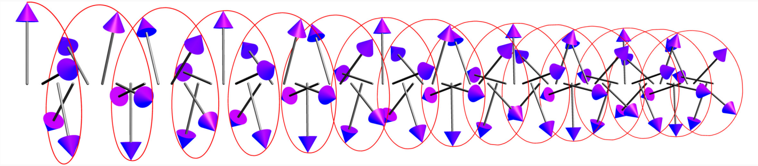
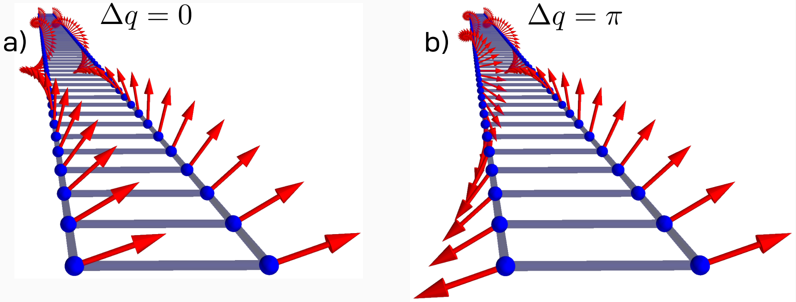






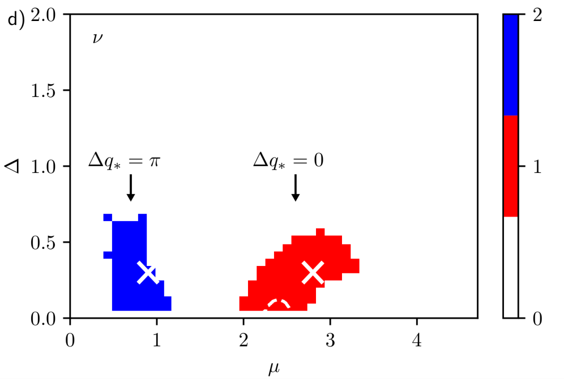
3. MAGNETIC LADDER ON SUPERCONDUCTOR

Spiral order of a magnetic ladder deposited on conventinal superconductor.



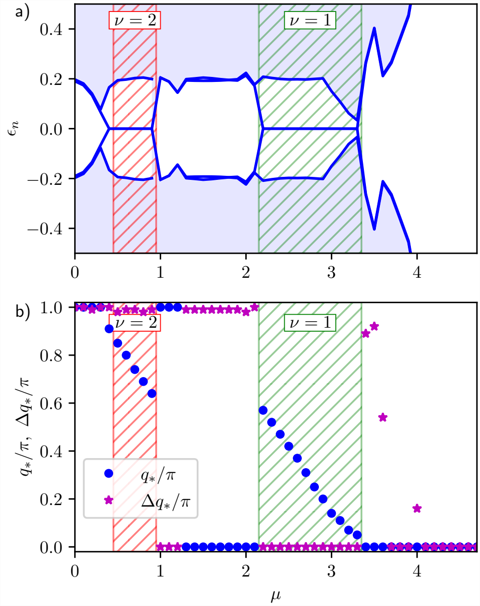
M.M. Maśka, N. Sedlmayr, A. Kobiałka, T. Domański, Phys. Rev. B 103, 235419 (2021).

In thermodynamic limit ($N \to \infty$) we have determined the topological invariant $\mathbb{Z}$ of this system, which belongs to class AIII.



Regions of the topological superconducting phase are characterized by either antiparallel or parallel spiral arrangements of the magnetic ladder.

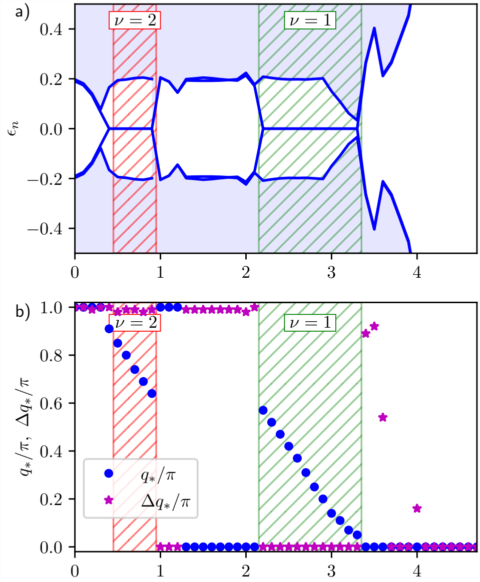
UNCONVENTIONAL TOPOLOGICAL TRANSITIONS



Variation of eigenenergies ε_n against μ for $\Delta = 0.3$

Variation of $q_\star$ and $\Delta q_\star$

UNCONVENTIONAL TOPOLOGICAL TRANSITIONS

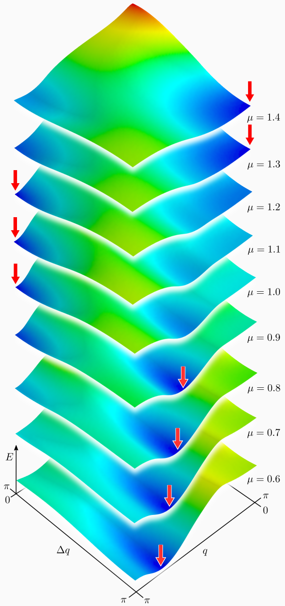


Variation of eigenenergies ε_n against μ for $\Delta = 0.3$

Variation of $q_\star$ and $\Delta q_\star$

Discontinuous transitions to/from topological phase without gap closing!

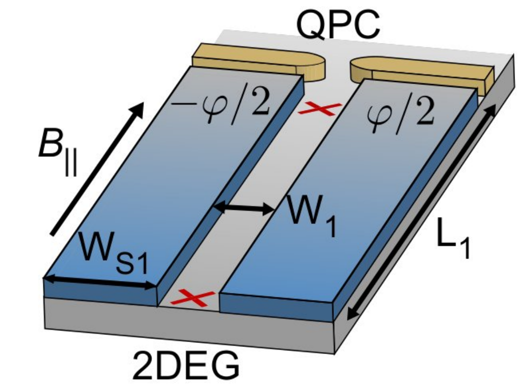
DISCONTINUOUS TRANSITIONS



Total energy as function of q and Δq obtained for $\Delta = 0.3t$ and several μ.

Red arrows indicate the minimum energy.

Two-dimensional electron gas of InAs epitaxially covered by a thin Al layer



Width:

W_1 = 80 nm

Length:

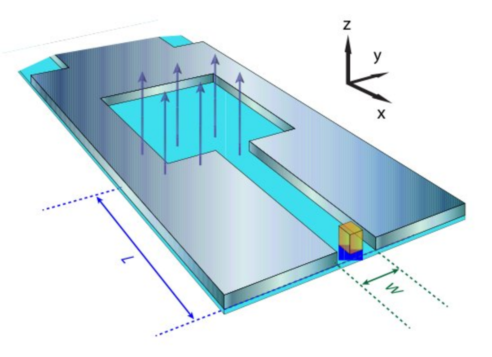
L_1 = 1.6 μm

A. Fornieri, ..., Ch. Marcus and F. Nichele, Nature 569, 89 (2019).

Niels Bohr Institute (Copenhagen, Denmark)

PLANAR JOSEPHSON JUNCTIONS

Two-dimensional HgTe quantum well coupled to 15 nm thick Al film



Width:

W = 600 nm

Length:

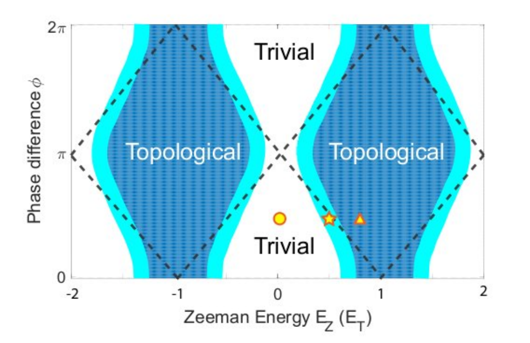
L = 1.0 μm

H. Ren, ..., L.W. Molenkamp, B.I. Halperin & A. Yacoby, Nature 569, 93 (2019).

Würzburg Univ. (Germany) + Harvard Univ. (USA)

PLANAR JOSEPHSON JUNCTIONS

Diagram of the trivial and topological superconducting state with respect to (1) phase difference ϕ and (2) in-plane magnetic field

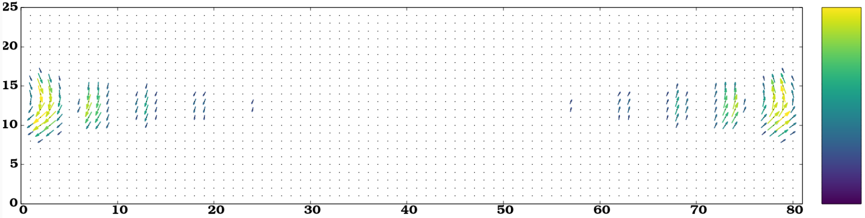


H. Ren, ..., L.W. Molenkamp, B.I. Halperin & A. Yacoby, Nature 569, 93 (2019).

Würzburg Univ. (Germany) + Harvard Univ. (USA)

TOPOGRAPHY OF MAJORANA MODES

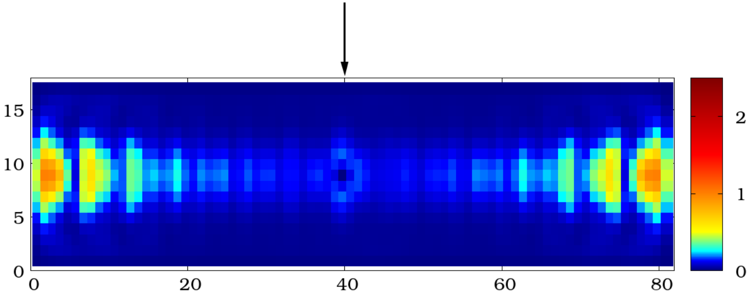
Spatial profile of the zero-energy ($E_n = 0$) Majorana quasiparticles in a homogeneous metallic strip embedded into Josephson junction.



Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

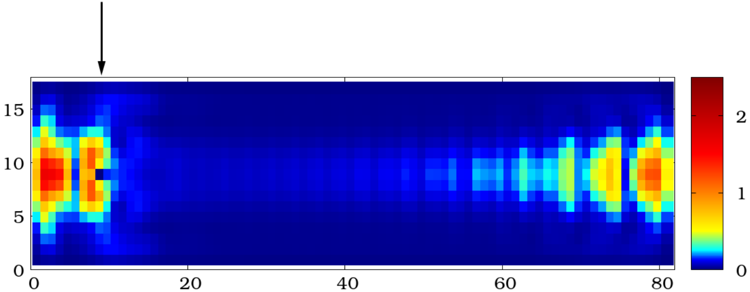
LOCAL DEFECT IN JOSEPHSON JUNCTION

Spatial profile of the Majorana modes in presence of the strong electrostatic defect placed in the center.



Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

Spatial profile of the Majorana modes in presence of the strong electrostatic defect placed near the edge.



Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

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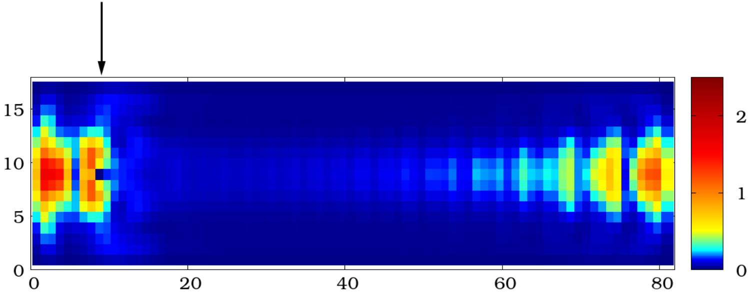


Sz. Głodzik, N. Sedlmayr & T. Domański, PRB <u>102</u>, 085411 (2020).

"Benefits of Weak Disorder in One-Dimensional Topological Superconductors"

A. Haim & A. Stern, Phys. Rev. Lett. 122, 126801 (2019).

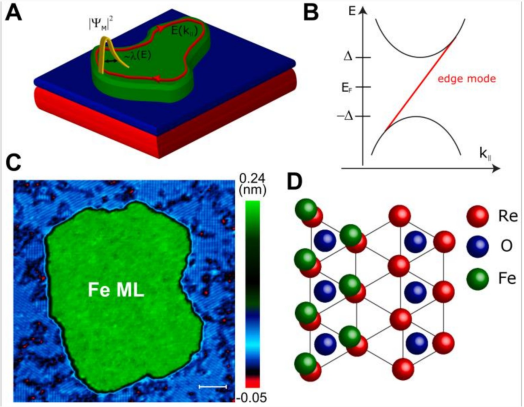
Two-dimensional topological textures

Two-dimensional topological textures

(platform for chiral Majorana modes)

PROPAGATING MAJORANA EDGE MODES

Magnetic island of Fe atoms deposited on the superconducting Re surface



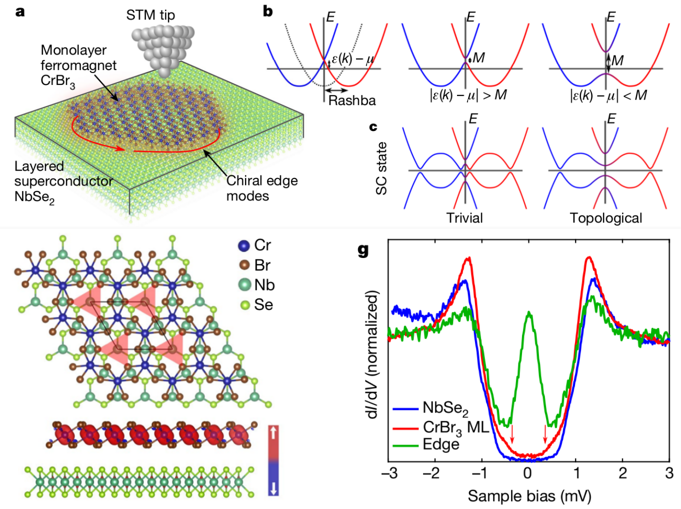
Chern number

C = 20

A. Palacio-Morales, ... & R. Wiesendanger, Science Adv. 5, eaav6600 (2019).

University of Hamburg (Germany)

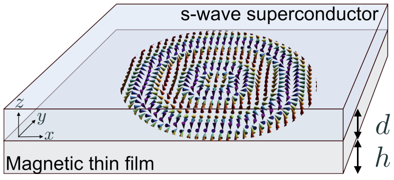
Ferromagnetic island $CrBr_3$ deposited on superconducting $NbSe_2$



S. Kezilebieke ... Sz. Głodzik ... P. Lilienroth, Nature 424, 588 (2020).

MAGNETIC SKYRMION-TYPE TEXTURES

Scenario for topological superconductivity induced in 2D magnetic thin film hosting a skyrmion deposited on conventional s-wave superconductor



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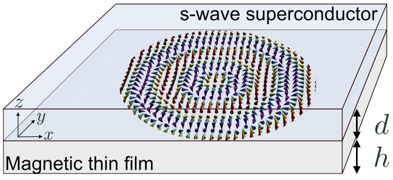

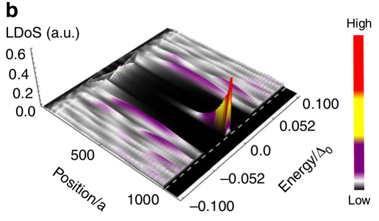

M. Garnier, A. Mesaros, P. Simon, Comm. Phys. 2, 126 (2019).

CONCLUSIONS

Magnetism and superconductivity of nanoscopic systems:

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$\Longrightarrow$ **constructively cooperate**

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Magnetism and superconductivity of nanoscopic systems:

$\Rightarrow$ **constructively cooperate**

$\Rightarrow$ **inducing novel topological phases**

CONCLUSIONS

Magnetism and superconductivity of nanoscopic systems:

$\Longrightarrow$ **constructively cooperate**

$\Longrightarrow$ **inducing novel topological phases**

Topological superconductors, hosting the Majorana boundary modes, can be used for constructing stable qubits & quantum computations.

COAUTHORS

⇒ **Maciek Maśka**

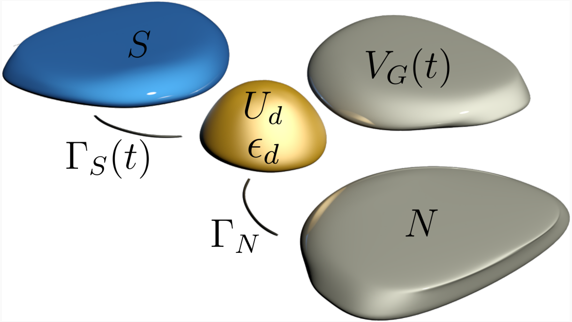
(Technical University, Wrocław)

⇒ **Nick Sedlmayr**

(M. Curie-Skłodowska University, Lublin)

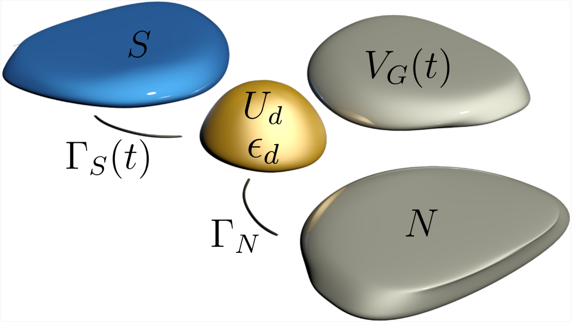
⇒ **Aksel Kobiałka**

(M. Curie-Skłodowska University, Lublin)



Quantum quench protocols:

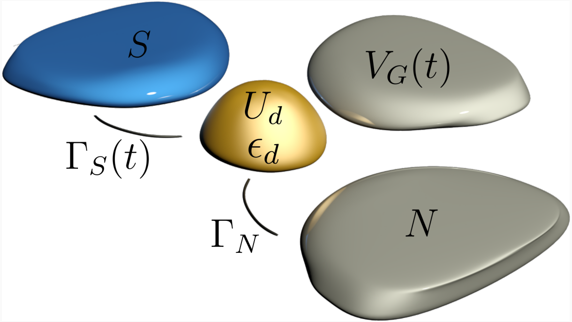
DYNAMICS OF BOUND STATES



Quantum quench protocols:

$\Rightarrow$ **sudden coupling to superconductor $0 \to \Gamma_S$**

DYNAMICS OF BOUND STATES



Quantum quench protocols:

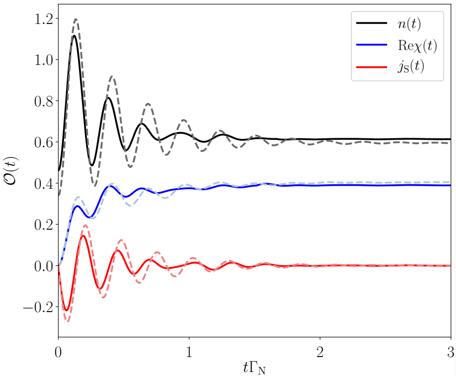
⟹ **sudden coupling to superconductor** $0 \rightarrow \Gamma_S$

⟹ **abrupt application of gate potential** $0 \rightarrow V_G$

K. Wrześniewski, B. Baran, R. Taranko, T. Domański & I. Weymann, PRB 103, 155420 (2021).

BUILDUP OF IN-GAP STATES

Time-dependent observables driven by the quantum quench $0 \rightarrow \Gamma_S$



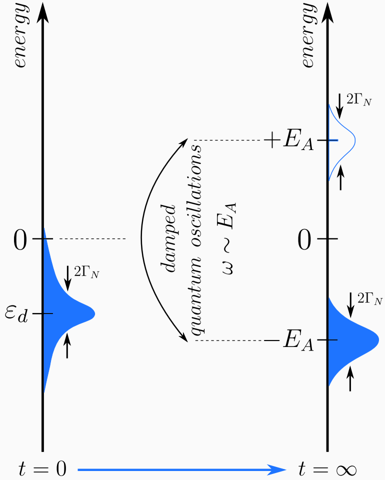
solid lines - time dependent NRG

dashed lines - Hartree-Fock-Bogolubov

K. Wrześniewski, B. Baran, R. Taranko, T. Domański & I. Weymann, PRB 103, 155420 (2021).

BUILDUP OF IN-GAP STATES

Rabbi-type oscillations observable in development of the in-gap states



K. Wrześniewski, B. Baran, R. Taranko, T. Domański & I. Weymann, PRB 103, 155420 (2021).

SINGLET/DOUBLET CONFIGURATIONS

The proximitized quantum dot can described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left(\Delta_d \, \hat{d}^{\dagger}_{\uparrow} \hat{d}^{\dagger}_{\downarrow} + \text{h.c.} \right)$$

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Eigen-states of this problem are represented by:

$|\uparrow\rangle$ **and** $|\downarrow\rangle$ $\Leftarrow$ **doublet states (spin $\frac{1}{2}$)**

$$\left. \begin{array}{l} u\,|0\rangle - v\,|\uparrow\downarrow\rangle \\ v\,|0\rangle + u\,|\uparrow\downarrow\rangle \end{array} \right\} \Leftarrow$$

singlet states (spin 0)

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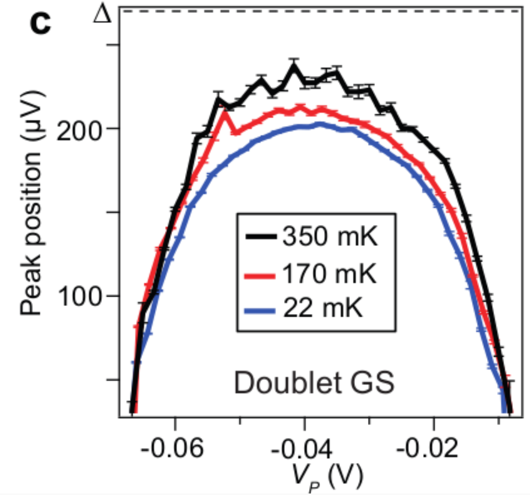
Upon varrying the parameters ε_d, U_d or Γ_S there can be induced transition between these doublet/singlet states.

SINGLET-DOUBLET TRANSITION: EXPERIMENT

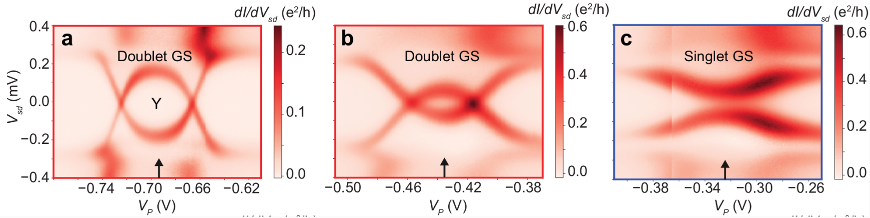


J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, Commun. Phys. 3, 125 (2020).

SINGLET VS DOUBLET: EXPERIMENT

Differential conductance vs source-drain bias V_{sd} (vertical axis) and gate potential V_p (horizontal axis) measured for various Γ_S/U

$U \gg \Gamma_s$ $\quad$ $U \geq \Gamma_s$ $\quad$ $U < \Gamma_s$



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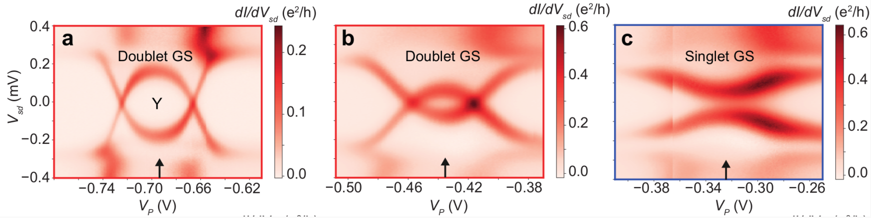
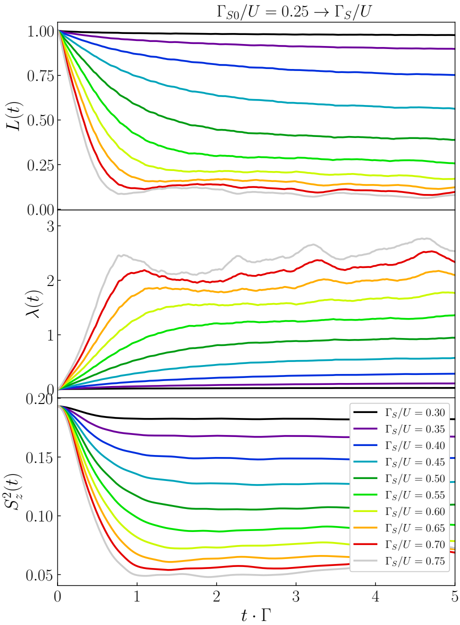


J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, Commun. Phys. 3, 125 (2020).

Crossings of bound states correspond to singlet-doublet transition.

DYNAMICAL SINGLET-DOUBLET TRANSITION



Loschmidt echo

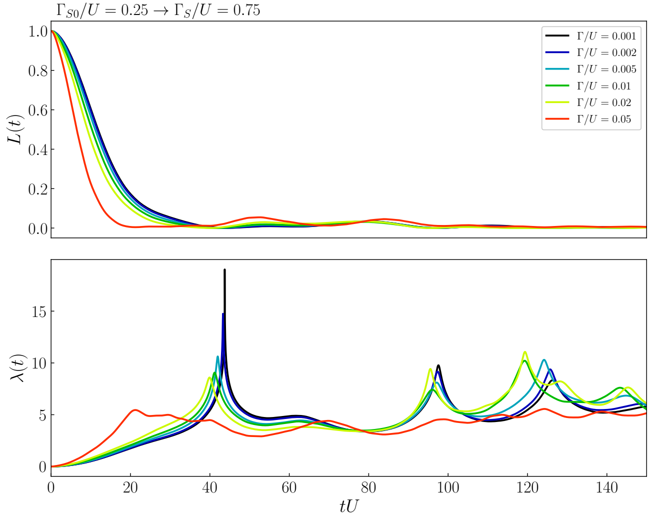
$$L(t) \equiv |\langle \Psi(0) | \Psi(t) \rangle|^2$$

Return rate

$$\lambda(t) \equiv -\frac{1}{N} \ln \{L(t)\}$$

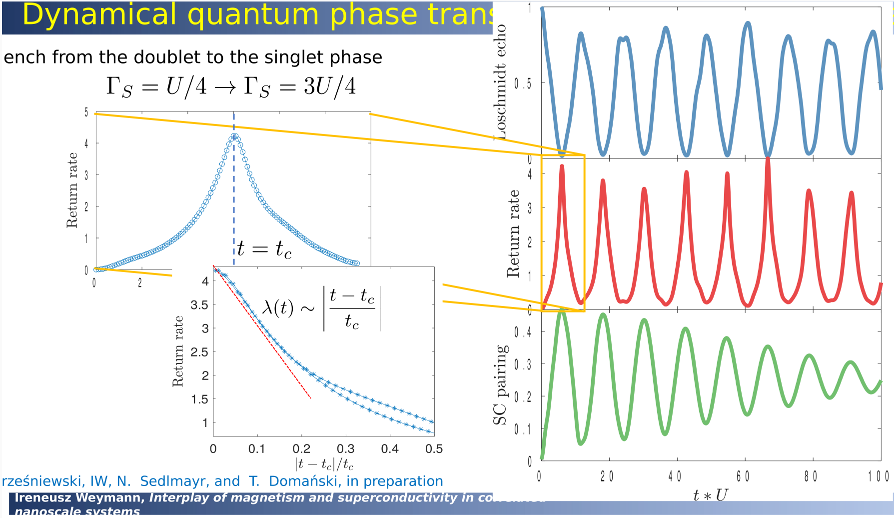
The squared magnetic moment $\langle S_z^2(t) \rangle$

tNRG RESULTS: $\Gamma_S = U/4 \longrightarrow \Gamma_S = 3U/4$



Loschmidt echo $L(t)$ and return rate $\lambda(t)$ obtained for various $\Gamma_N \equiv \Gamma$

FINITE-SIZE SCALING



Finite-size scaling analysis near the critical-time point.

SUMMARY

Quantum quench:

- **leads to development of bound states** (or their rescaling)

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- **can exhibit dynamical transition** **(upon varying ground states)**

These phenomena are detectable in transport properties !

ACKNOWLEDGEMENTS

- dynamical singlet-doublet transition

$\Longrightarrow$ I. Weymann (Poznań), K. Wrześniewski (Poznań),

N. Sedlmayr (Lublin),

- transient phenomena, Floquet formalism

$\Longrightarrow$ R. Taranko (Lublin), B. Baran (Lublin),

- time-resolved leakage of Majorana qps

$\Longrightarrow$ J. Barański (Dęblin)