

*Poznań, 28 June 2011*

**Interplay between correlations and  
superconductivity in the electron  
transport through the quantum dots**

T. DOMAŃSKI

**M. Curie-Skłodowska University,  
Lublin, Poland**

## Outline

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### Physical setup

*/ metal - QD - superconductor /*

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### Relevant issues

*/ correlations vs superconductivity /*

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*⇒ quantum interference in the multiple QDs*

*⇒ QD in the multiterminal structures*



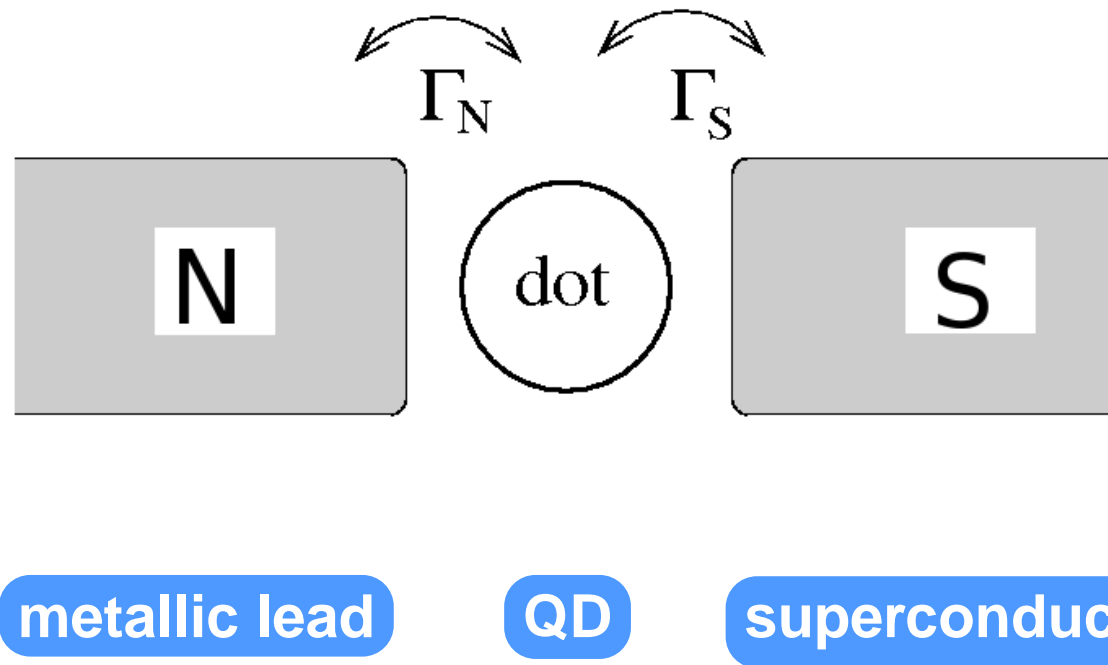
# Physical setup

## Physical situation

**We consider the quantum dot (QD) in the following setup**

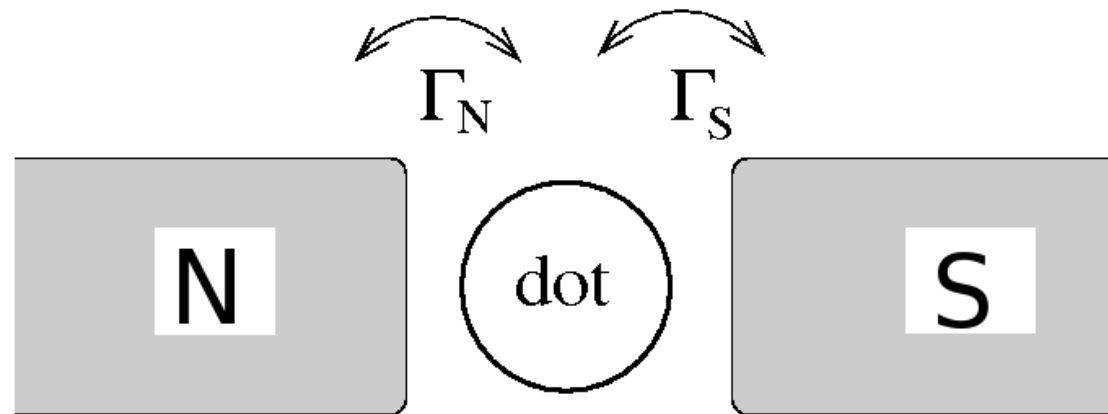
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metallic lead

QD

superconductor

This represents a particular version of the SET.

**Relevant issues**

**Physical aspects :**

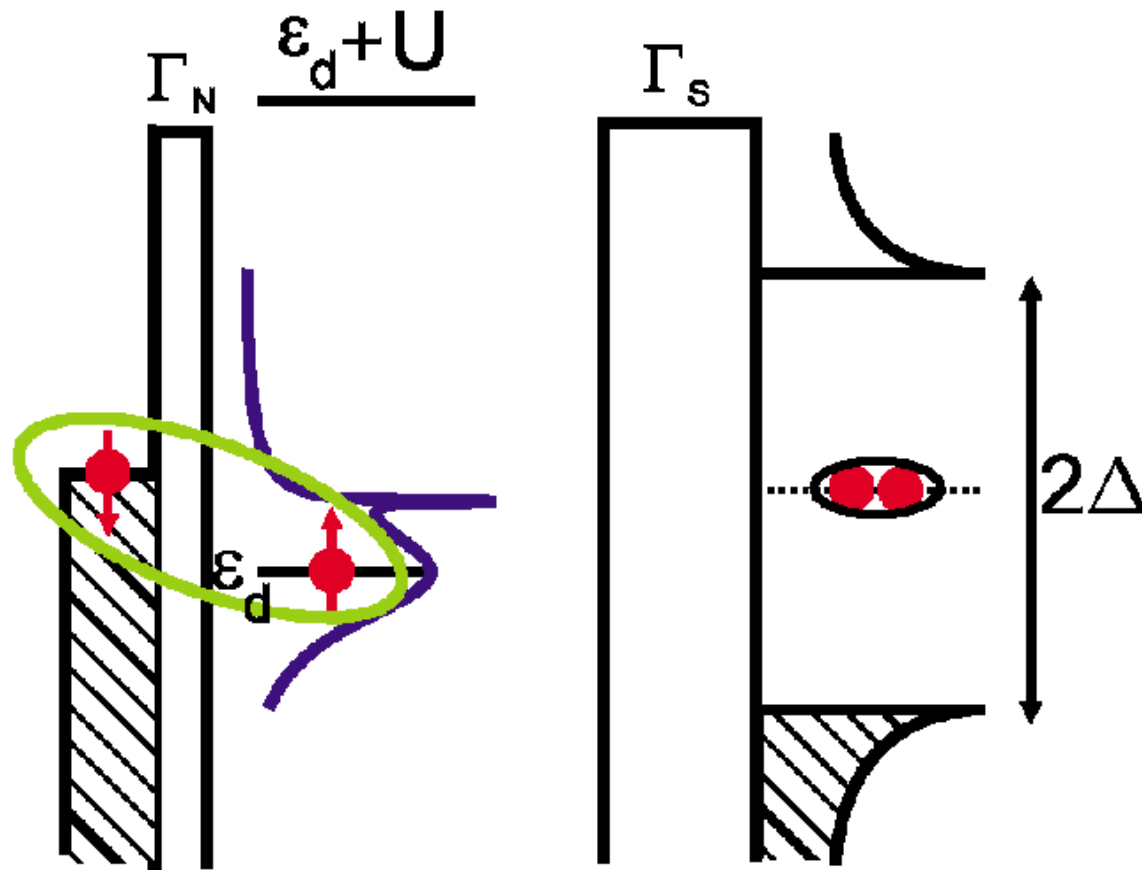
# 1

## **Physical aspects :** # 1

**Hybridization of the QD to metallic lead causes:**

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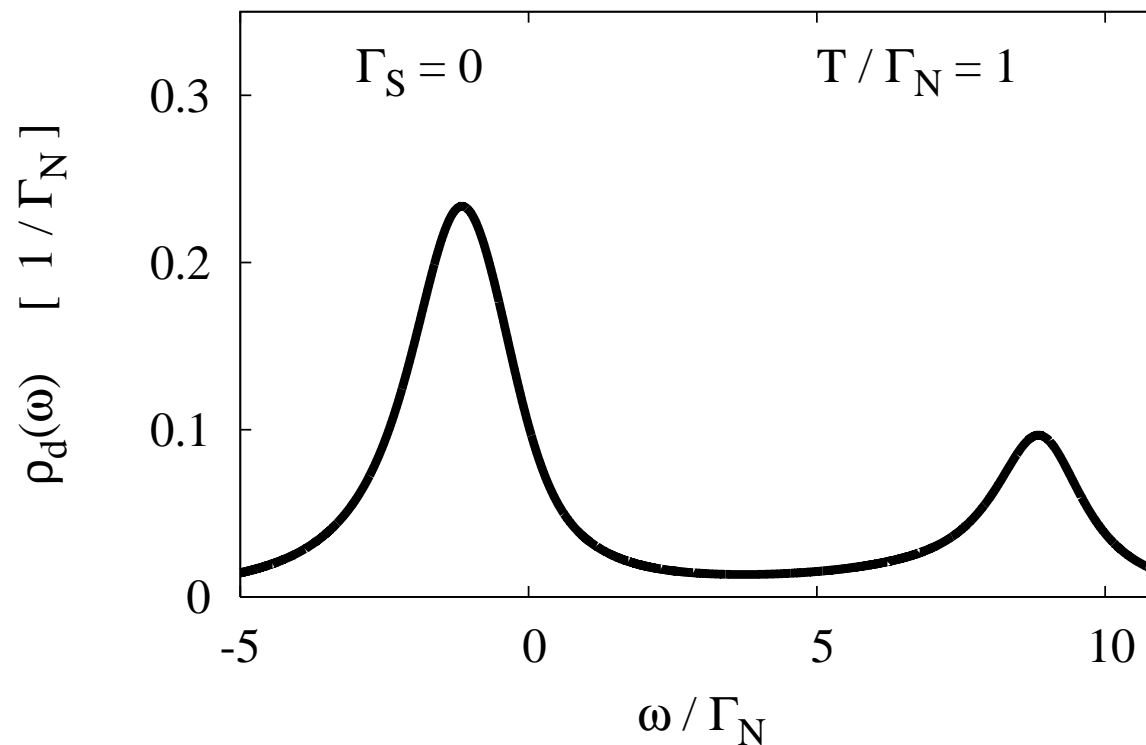
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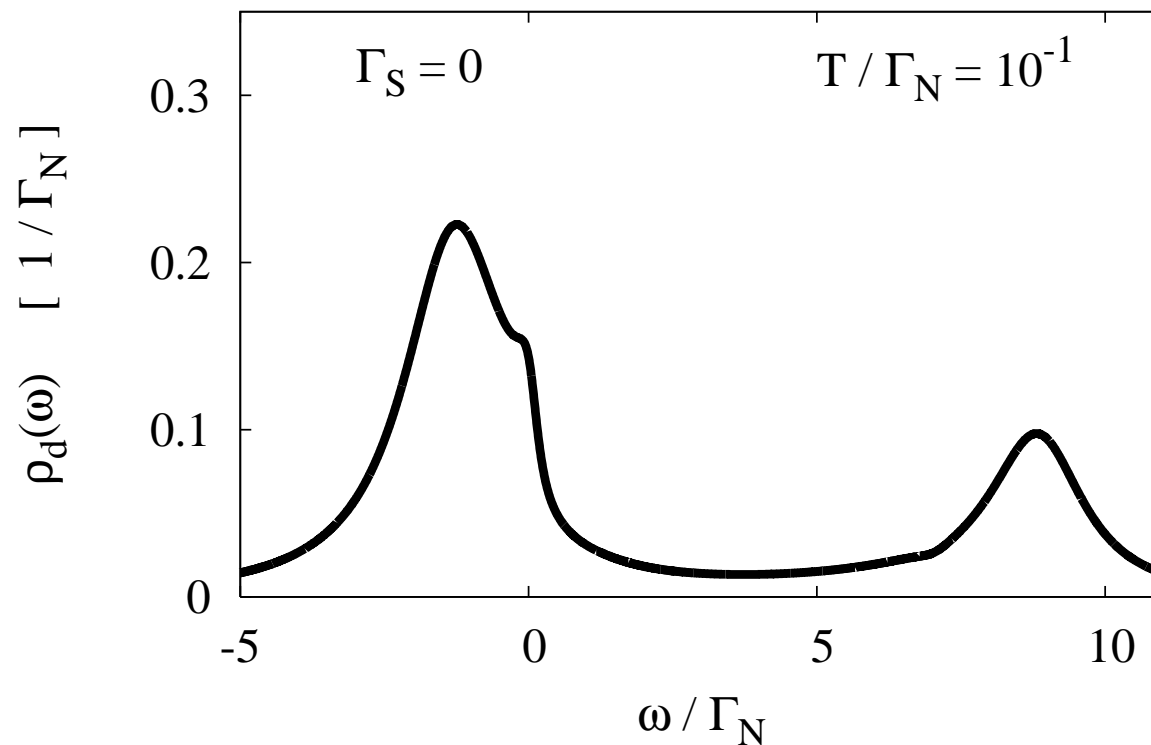
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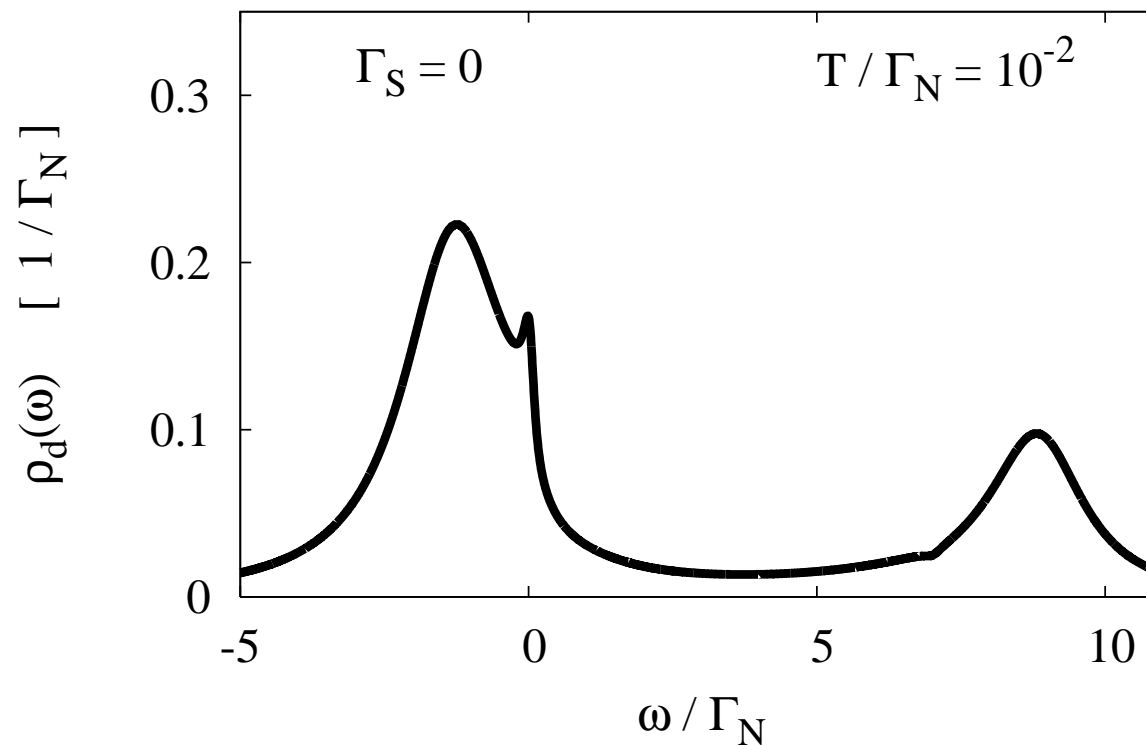
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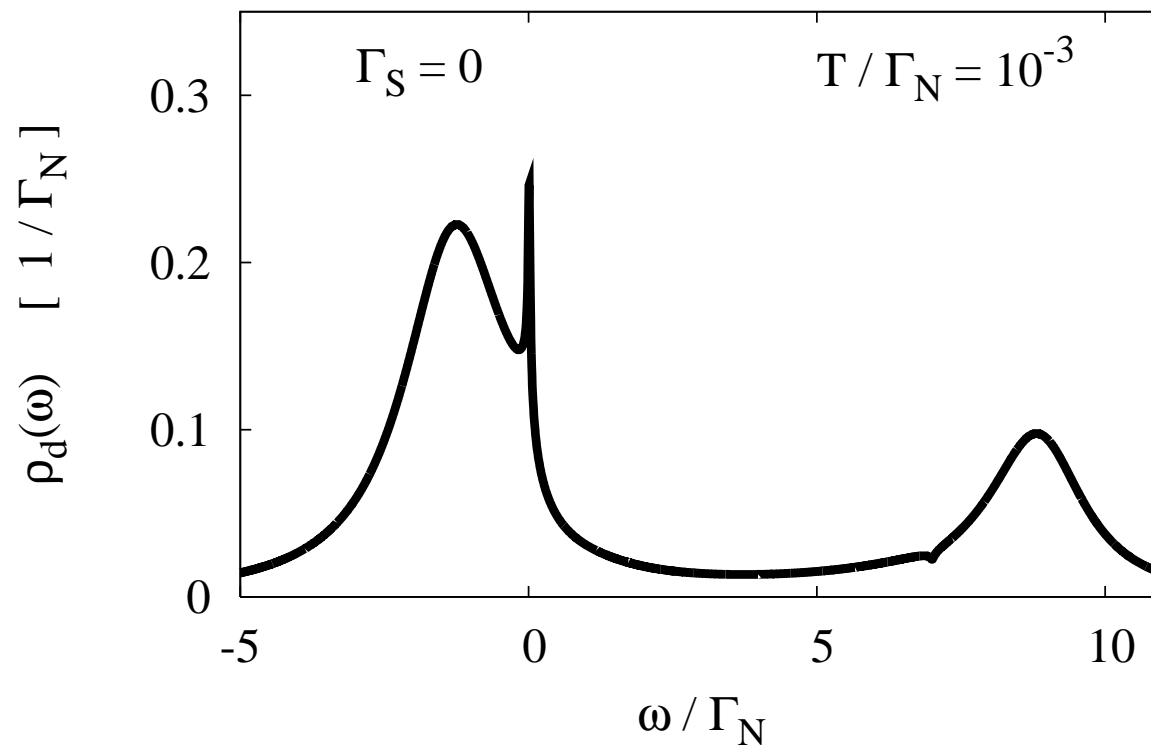
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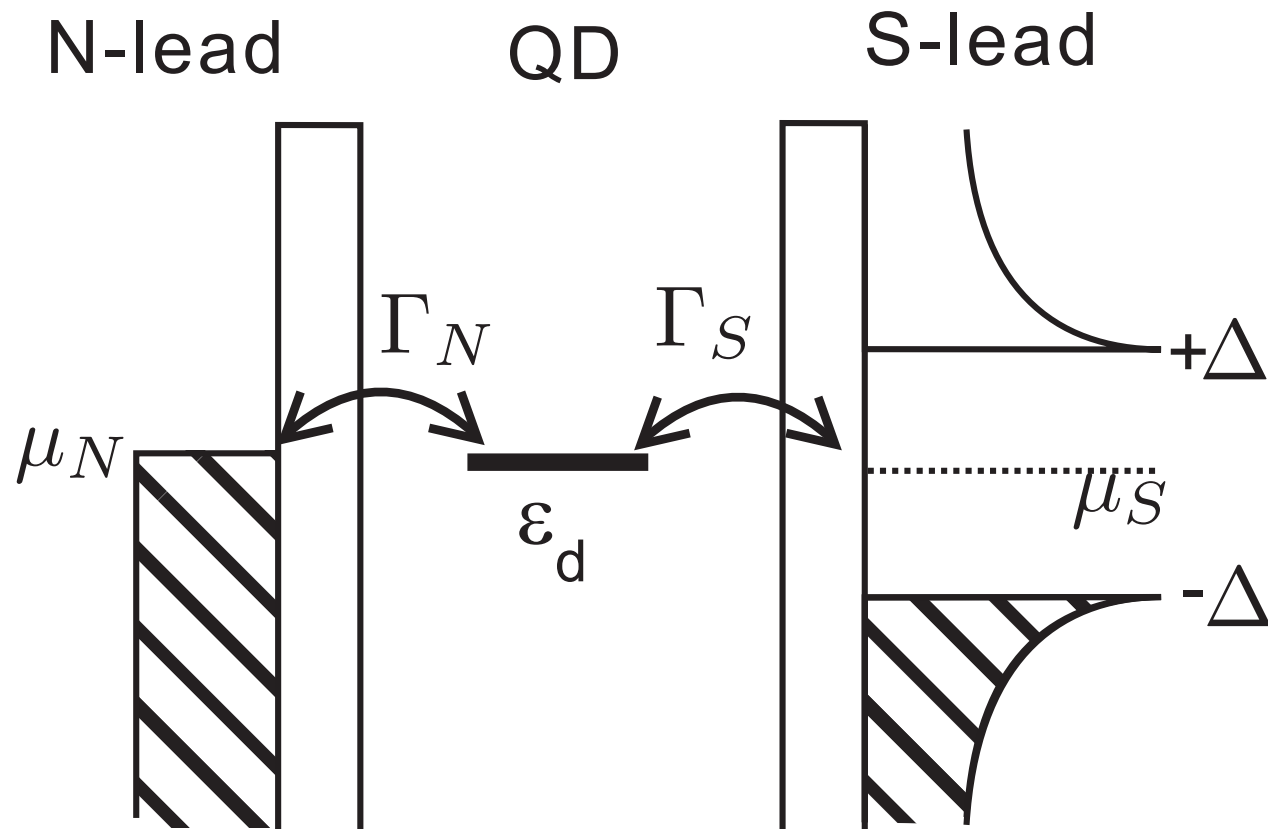
- ★ a broadening of the QD levels and
- ★ appearance of the Kondo resonance at  $T \leq T_K$ .

**Physical aspects :**

# 2

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### Hybridization of the QD to superconducting lead

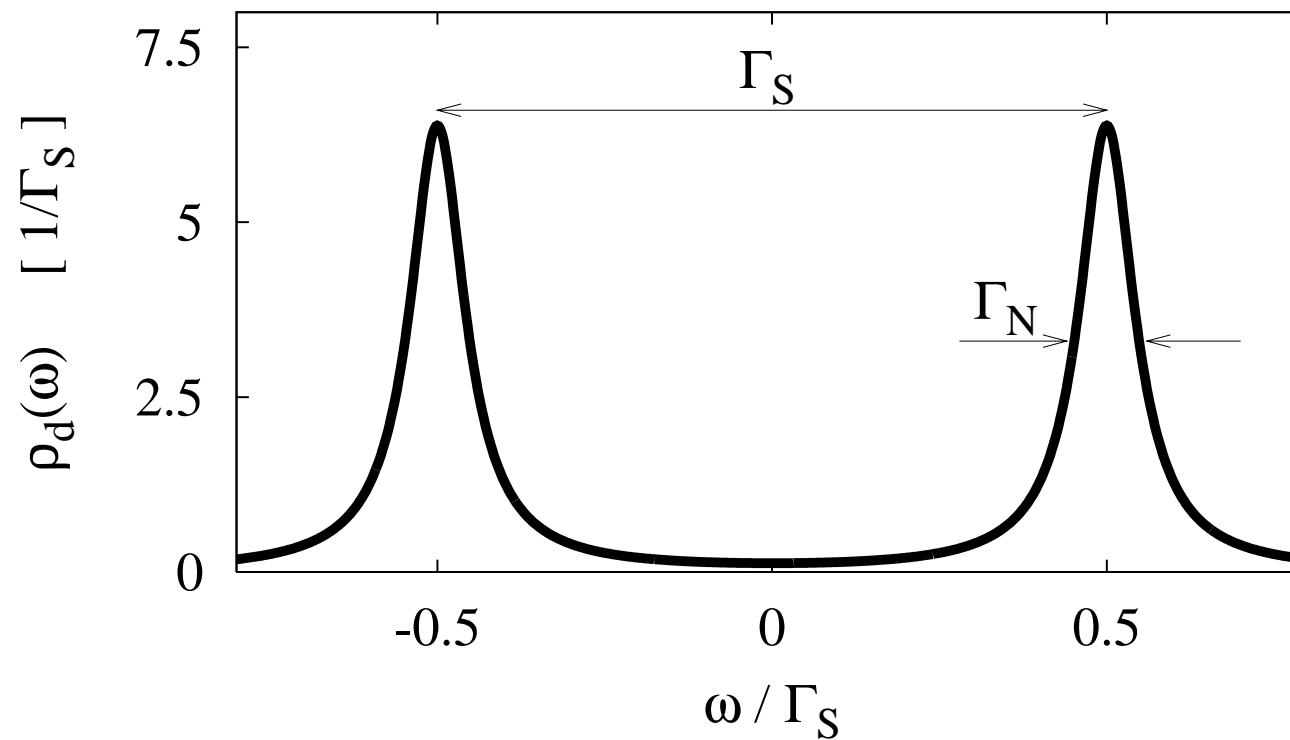


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Hybridization of the QD to superconducting lead  
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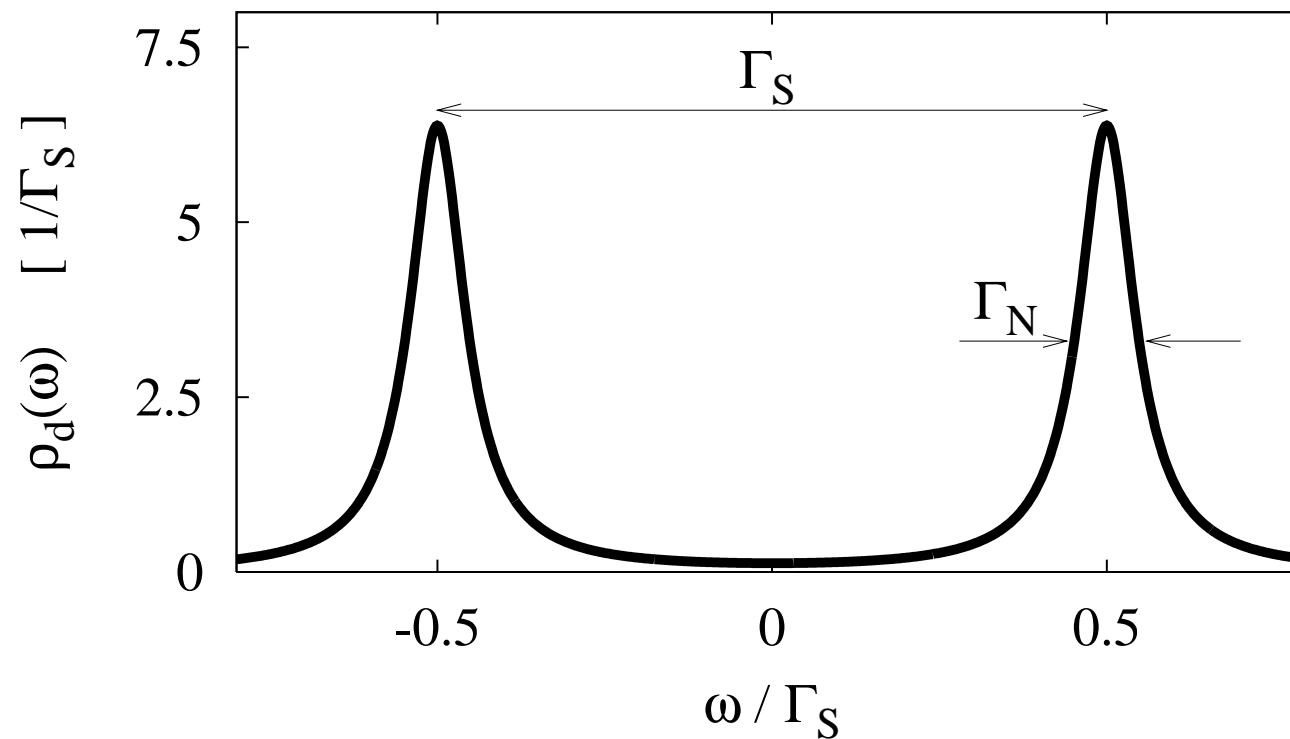
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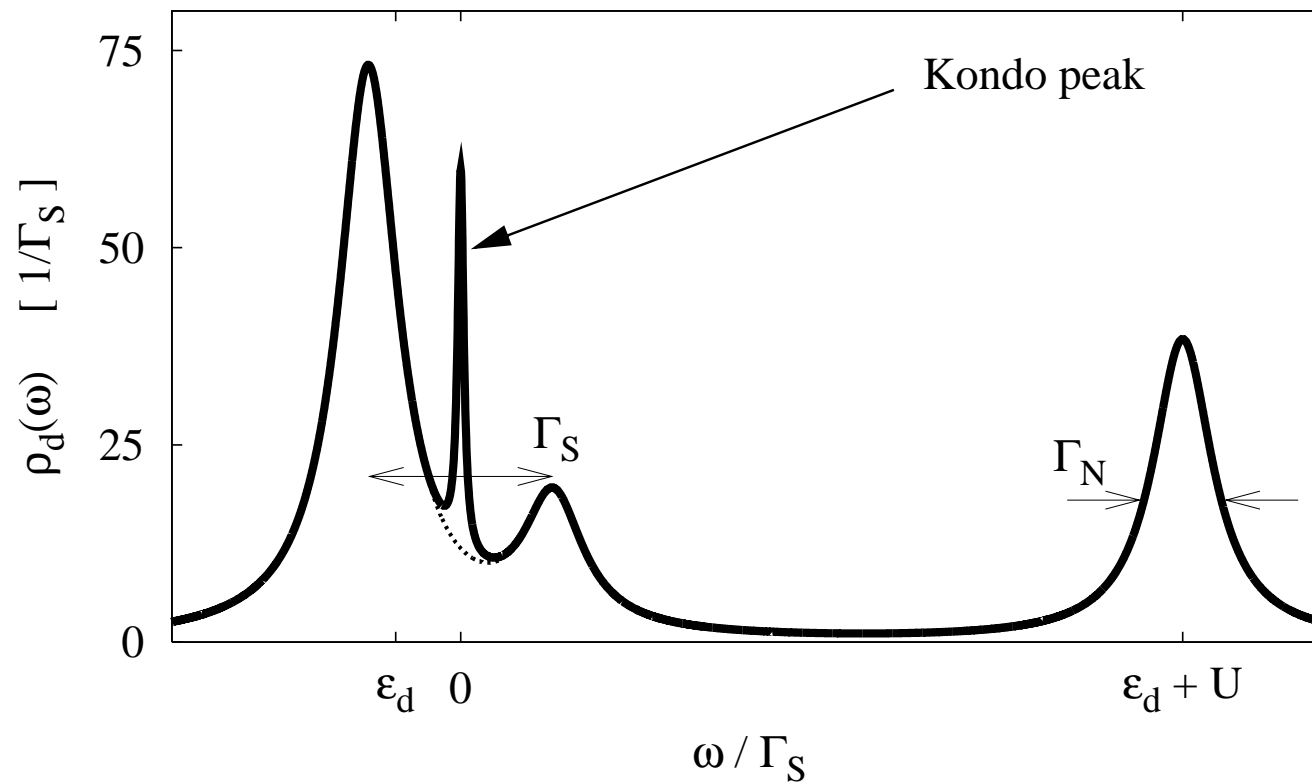
QD spectrum obtained for  $\epsilon_d = 0, U = 0$

**Physical aspects :**

# 1 + 2

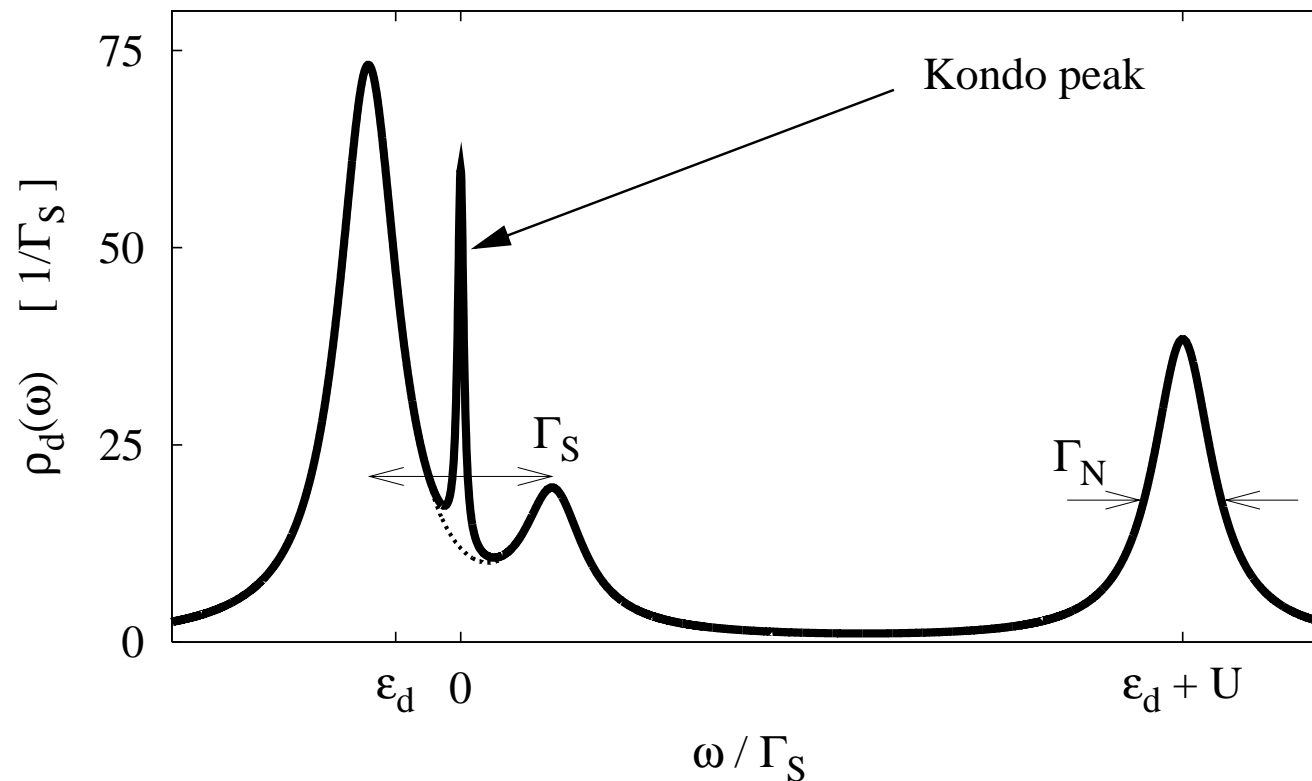
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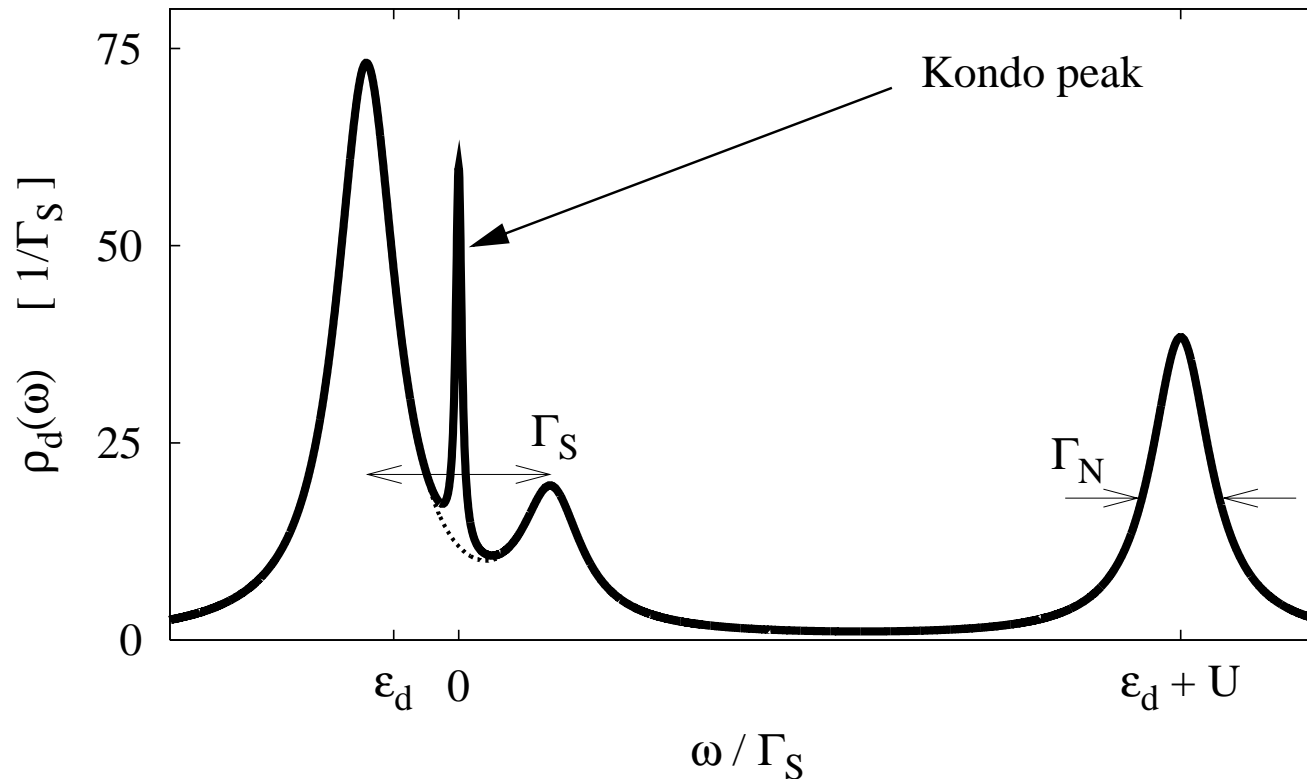
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which is a subject of the present study.

**Questions:**

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Any particular features ?

## **Microscopic model**

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$$\begin{aligned}\hat{H} = & \sum_{\sigma} \epsilon_d \hat{d}_{\sigma}^{\dagger} \hat{d}_{\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} + \hat{H}_N + \hat{H}_S \\ & + \sum_{\mathbf{k}, \sigma} \sum_{\beta=N, S} \left( V_{\mathbf{k}\beta} \hat{d}_{\sigma}^{\dagger} \hat{c}_{\mathbf{k}\sigma\beta} + V_{\mathbf{k}\beta}^{*} \hat{c}_{\mathbf{k}\sigma, \beta}^{\dagger} \hat{d}_{\sigma} \right)\end{aligned}$$

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and

$$\hat{H}_S = \sum_{k,\sigma} (\epsilon_{k,S} - \mu_S) \hat{c}_{k\sigma S}^{\dagger} \hat{c}_{k\sigma S} - \sum_k \left( \Delta \hat{c}_{k\uparrow S}^{\dagger} \hat{c}_{k\downarrow S}^{\dagger} + \text{h.c.} \right)$$

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For justification see e.g.

*Y. Tanaka, N. Kawakami, and A. Oguri, J. Phys. Soc. Jpn. **76**, 074701 (2007).*

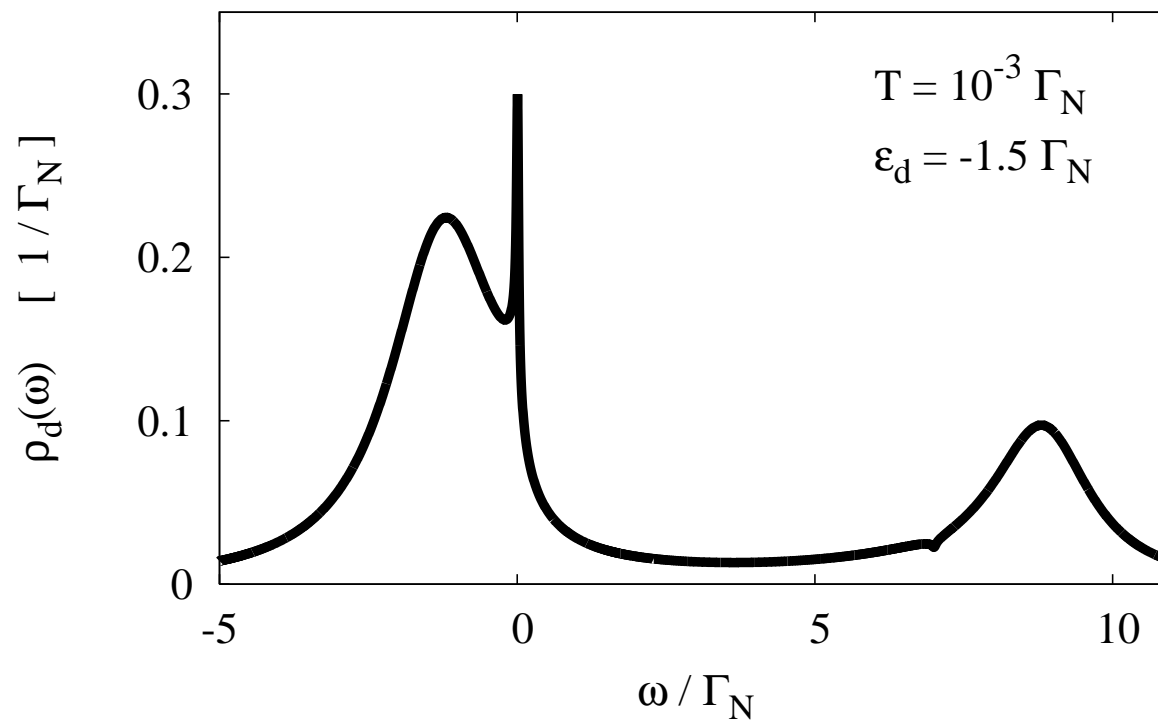
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Spectral function obtained below  $T_K$  for  $U = 10\Gamma_N$

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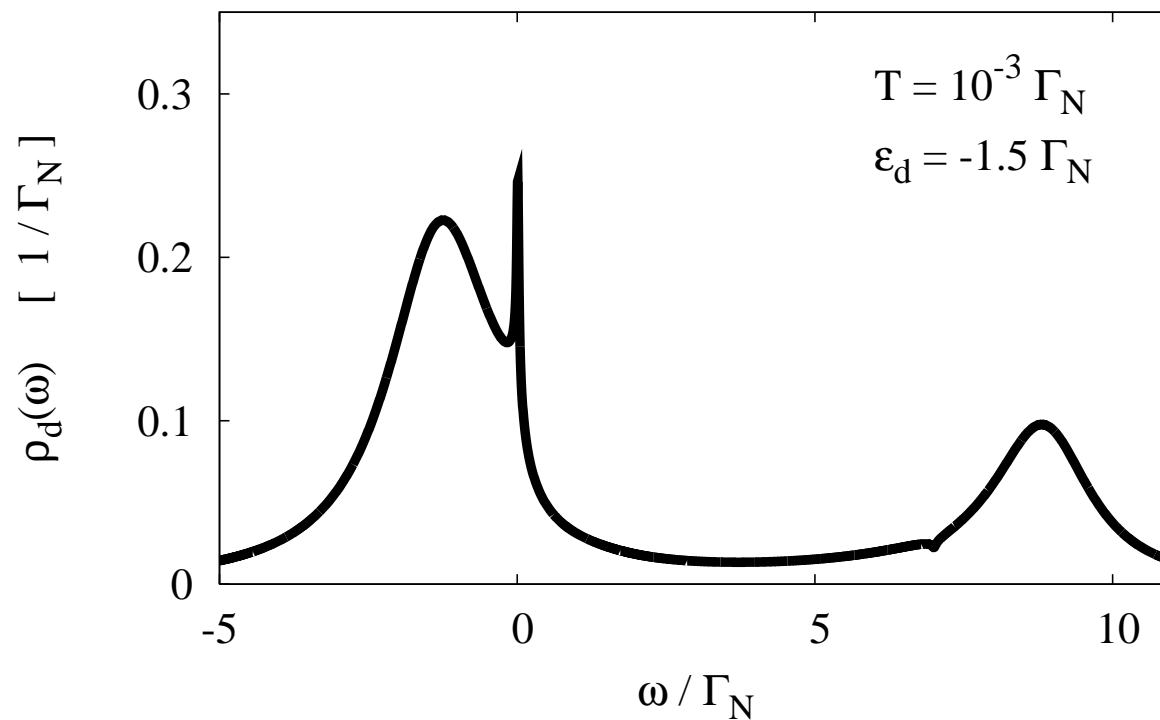
Spectral function obtained below  $T_K$  for  $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 0$$

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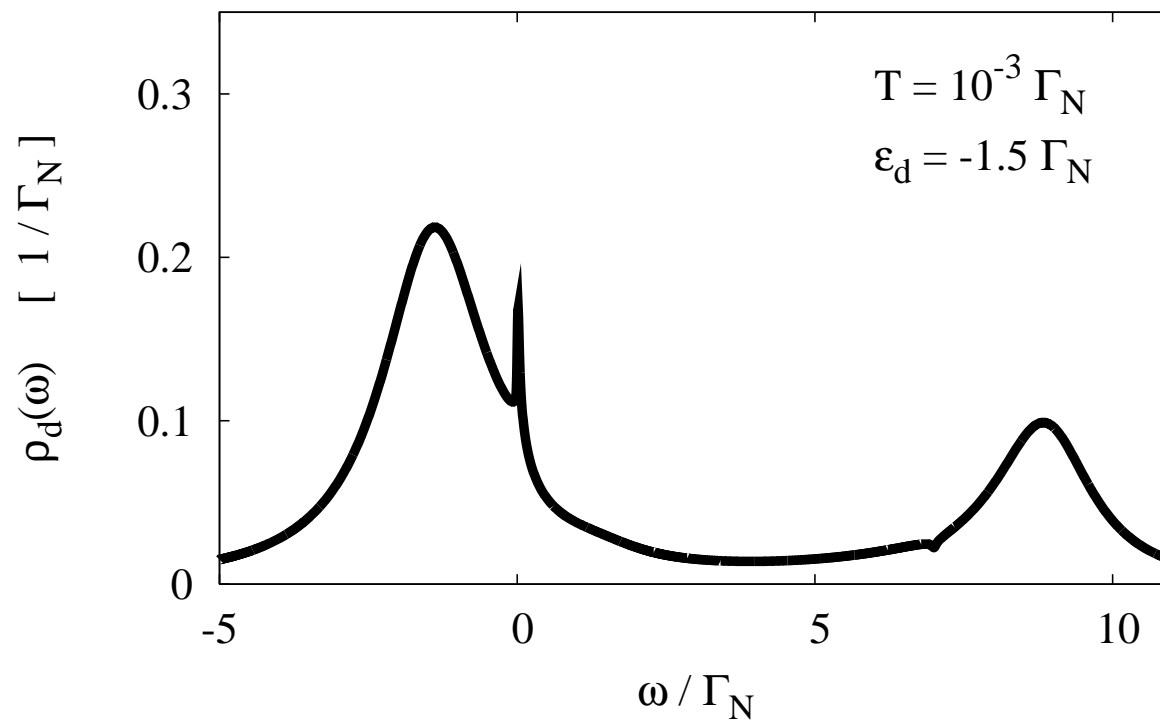
Spectral function obtained below  $T_K$  for  $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 1$$

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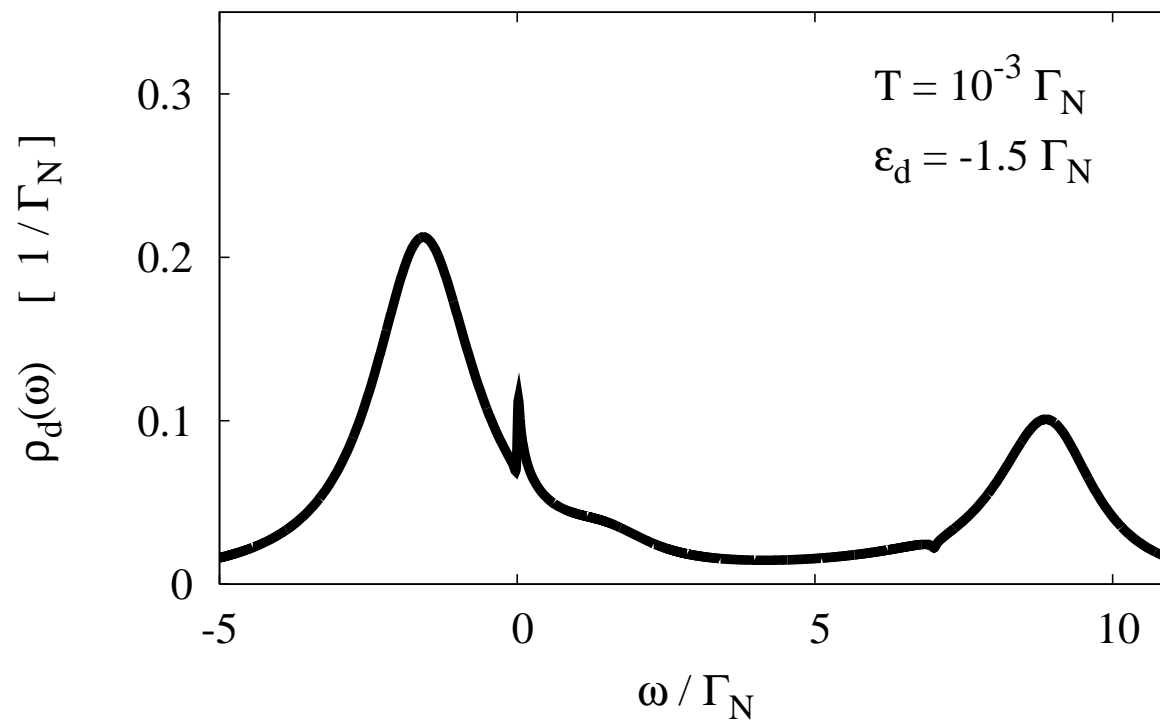
Spectral function obtained below  $T_K$  for  $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 2$$

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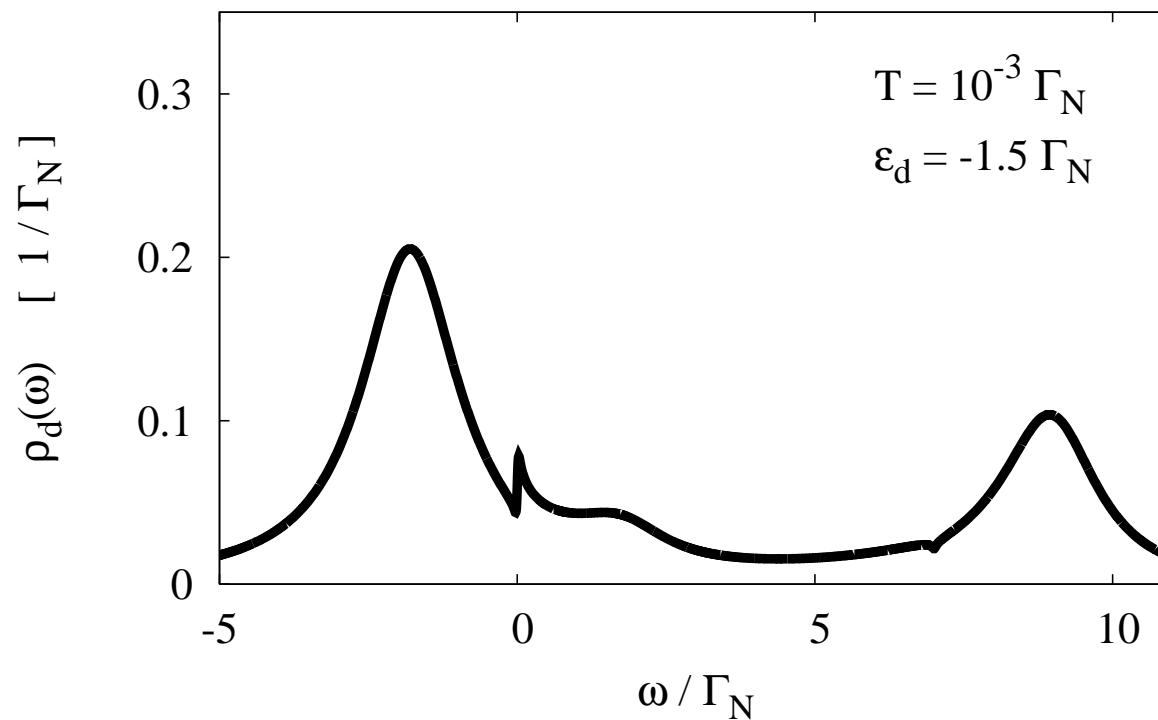


$$\Gamma_S/\Gamma_N = 3$$



## Correlated QD – influence of $\Gamma_S/\Gamma_N$ ratio

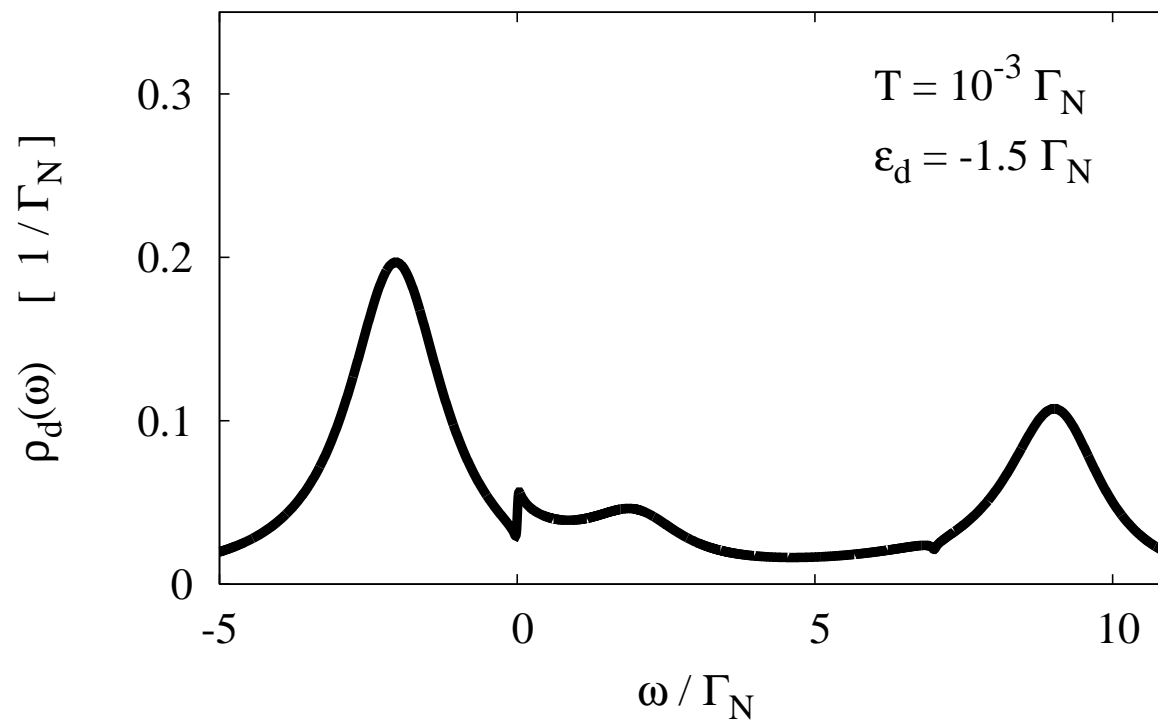
Spectral function obtained below  $T_K$  for  $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 4$$

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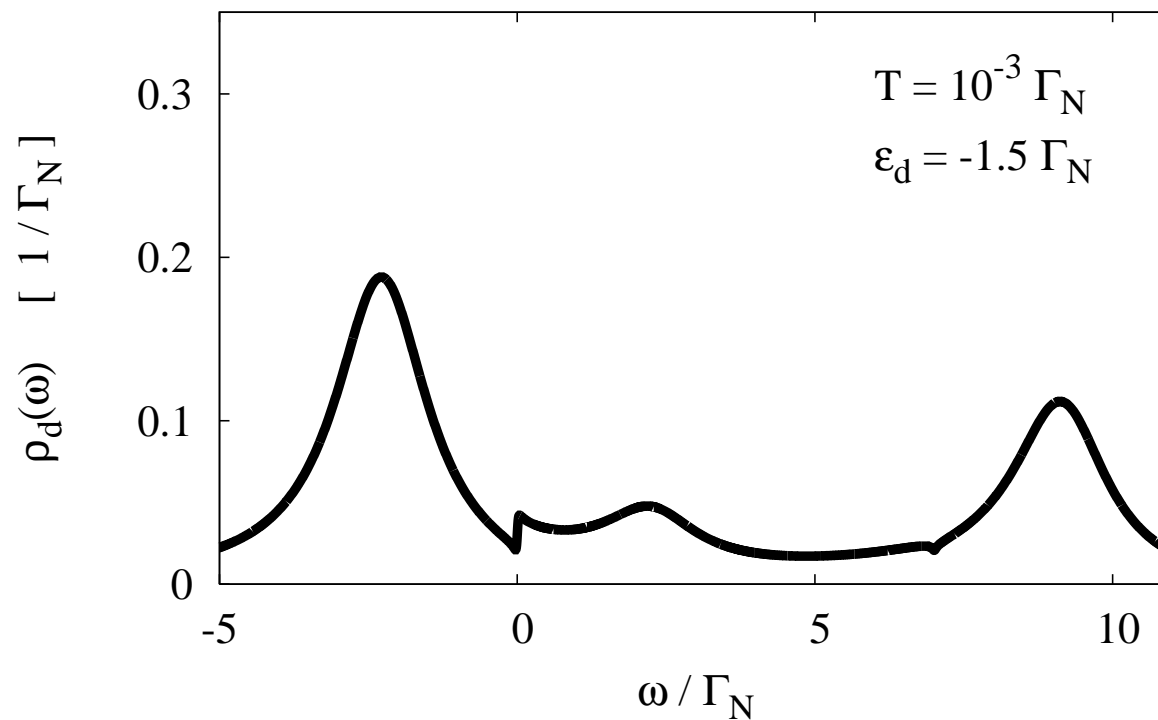
Spectral function obtained below  $T_K$  for  $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 5$$

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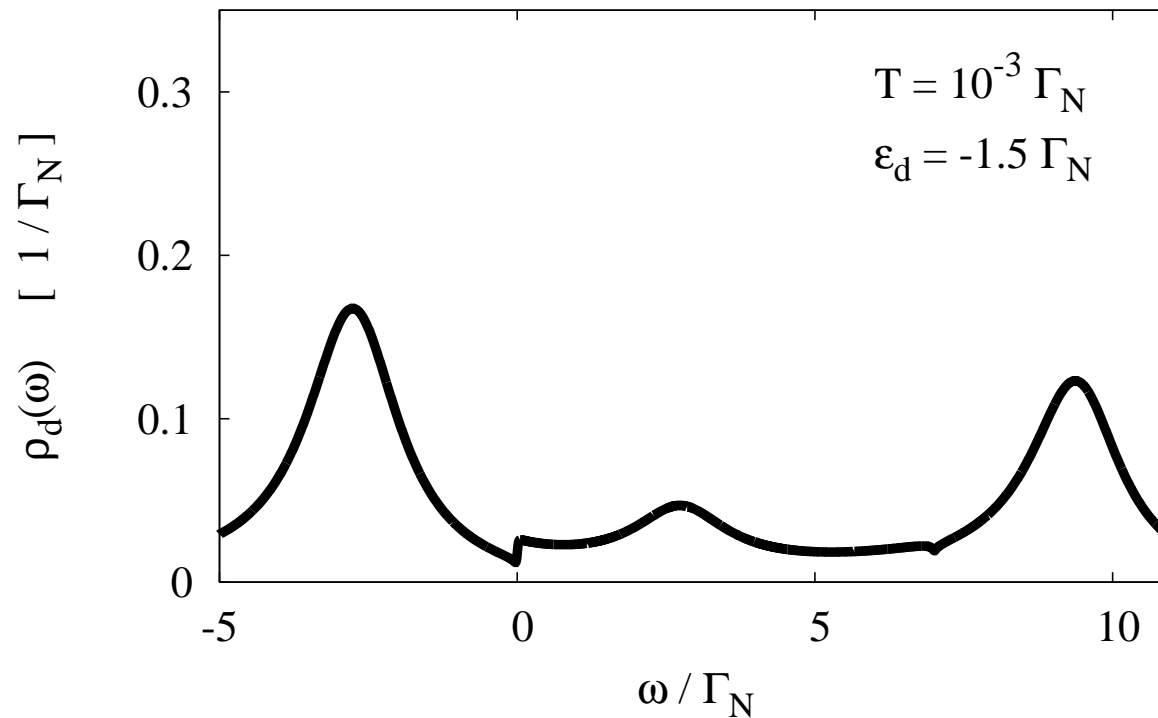
Spectral function obtained below  $T_K$  for  $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 6$$

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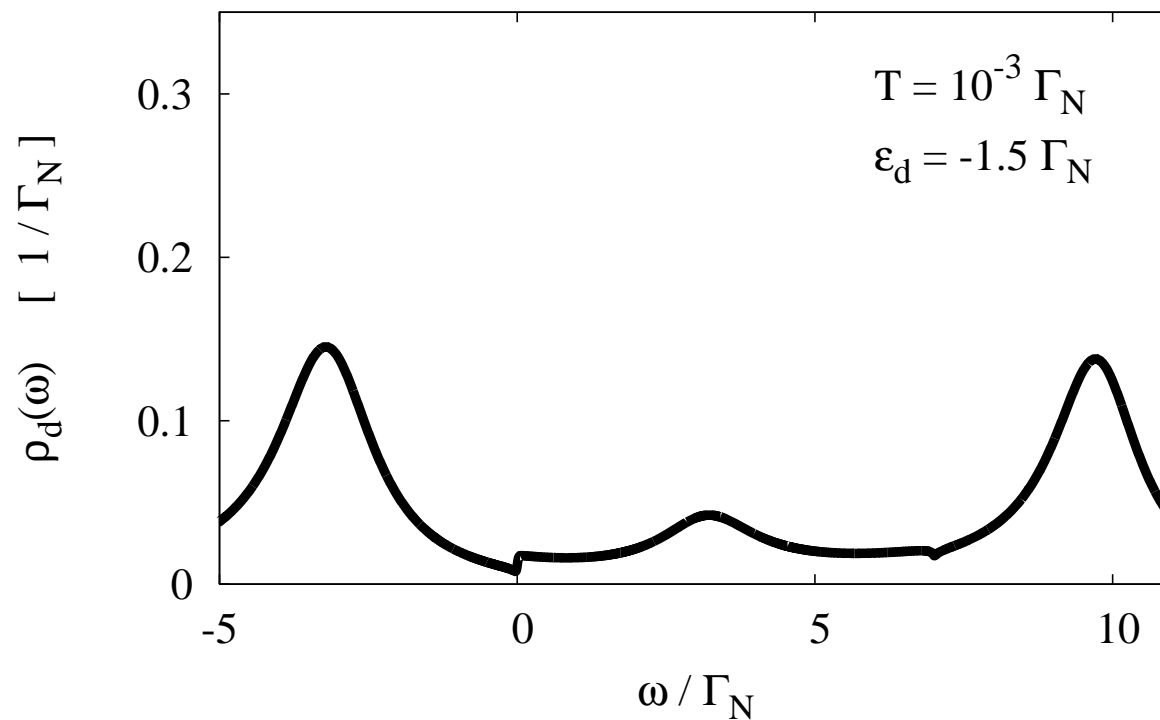
Spectral function obtained below  $T_K$  for  $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 8$$

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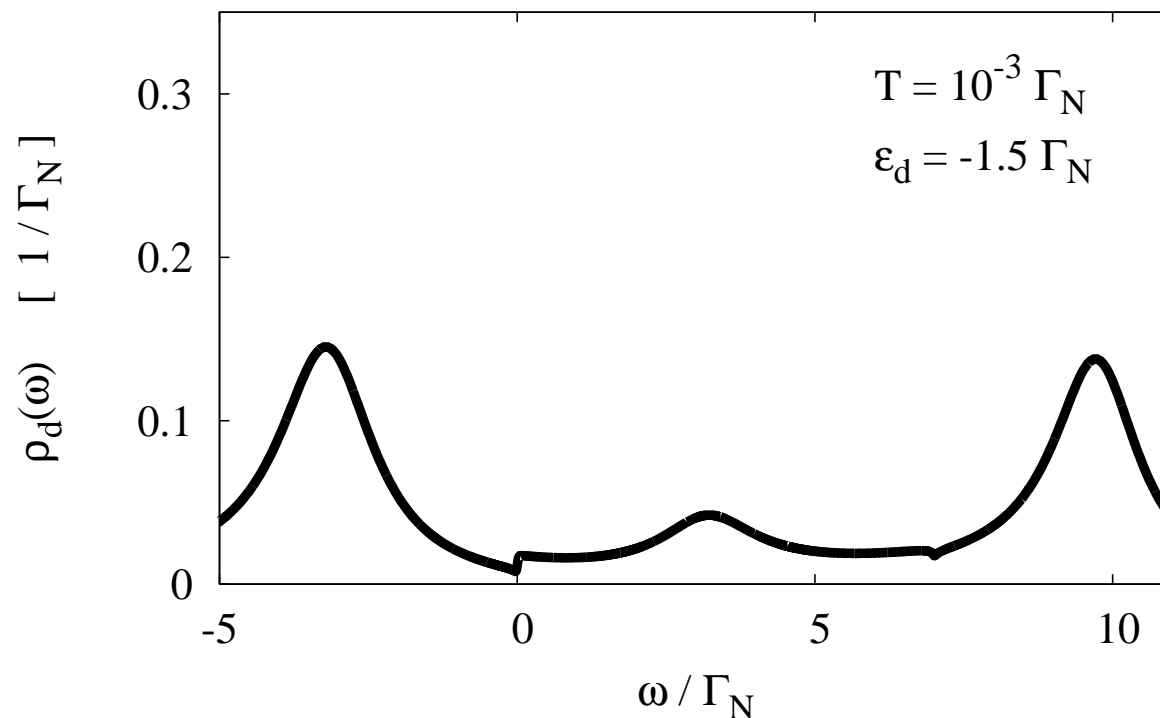
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$$\Gamma_S/\Gamma_N = 10$$

## Correlated QD – influence of $\Gamma_S/\Gamma_N$ ratio

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Superconductivity suppresses the Kondo resonance

# Transport properties

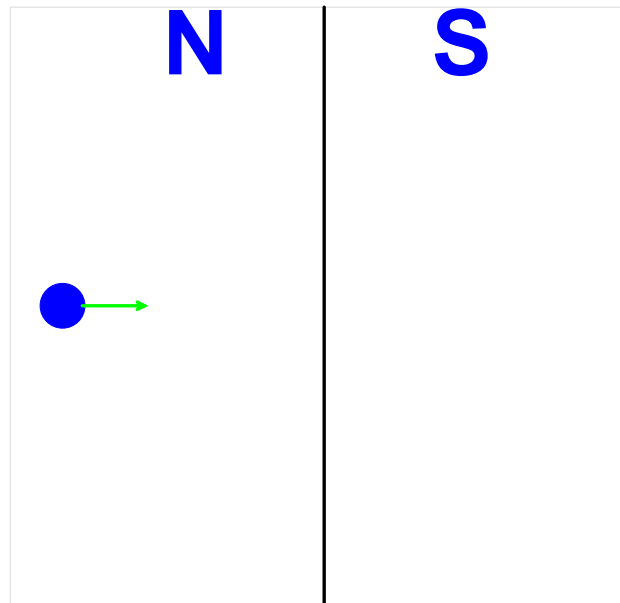
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Besides the usual electron tunnelling (for  $|eV| \geq \Delta$ ) there is also a contribution from the charge transfer between **N** and **S** electrodes via anomalous channel



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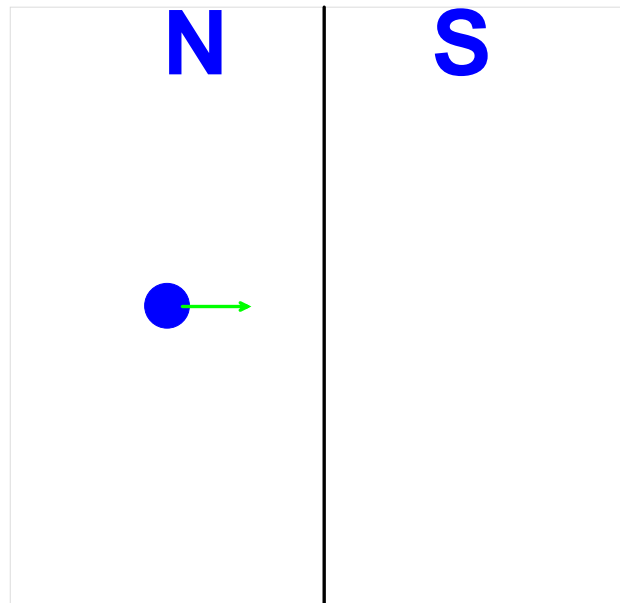
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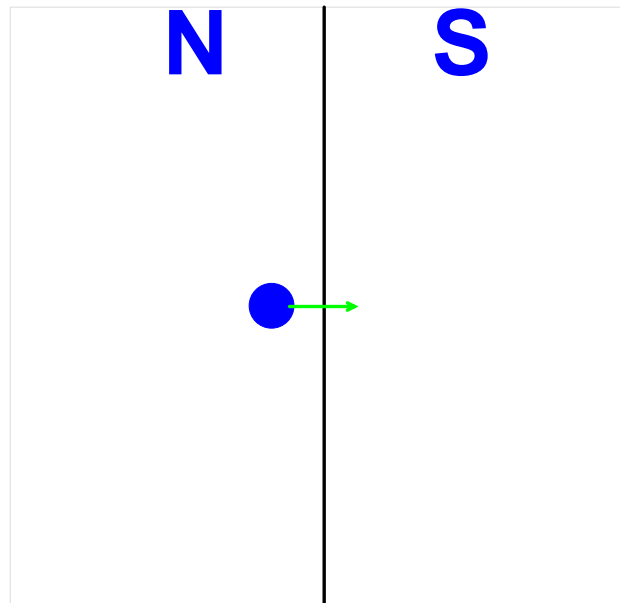
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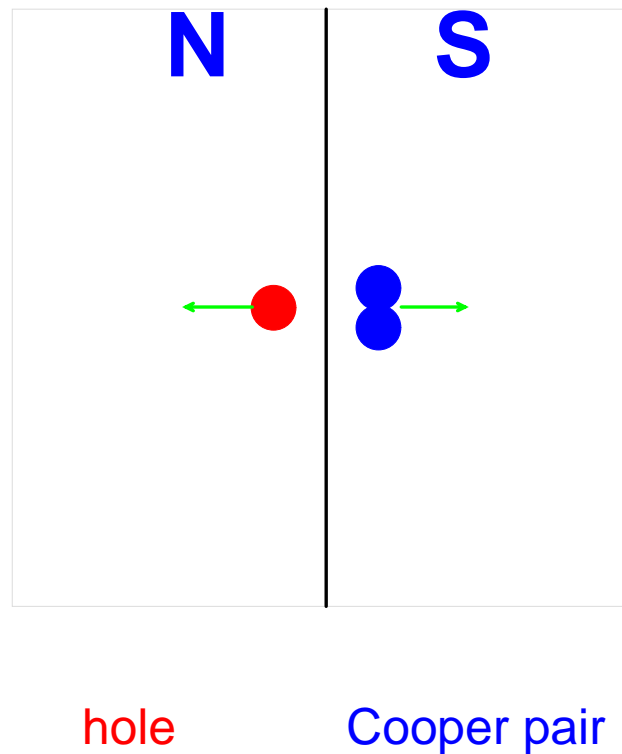
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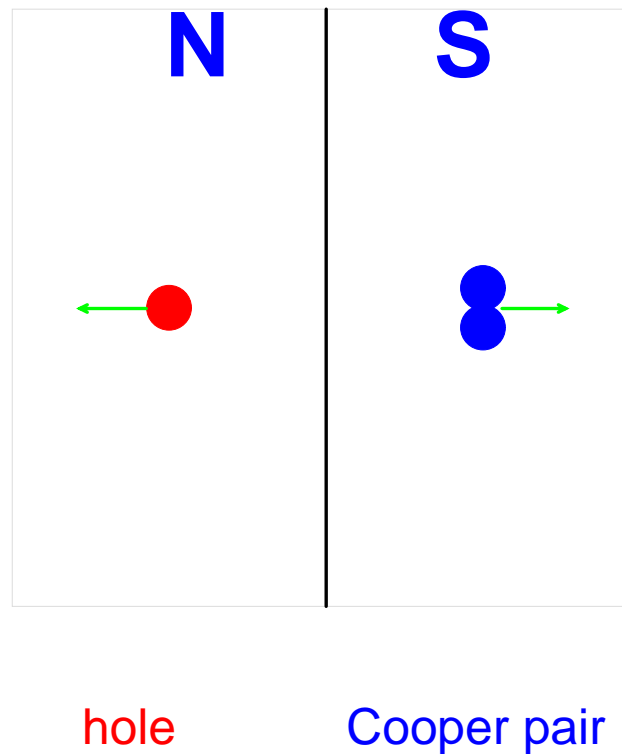
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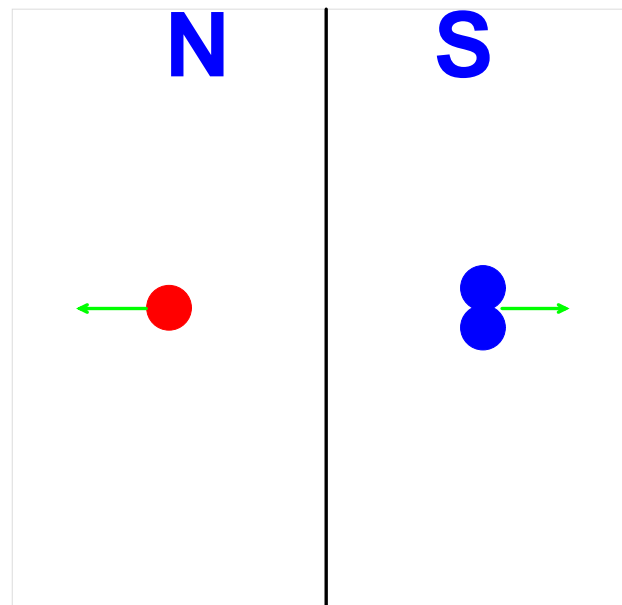
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hole

Cooper pair

This process is called Andreev reflection.

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with the transmittance  $T_1(\omega)$  is equal

$$\Gamma_N \Gamma_S \left( |G_{11}^r(\omega)|^2 + |G_{12}^r(\omega)|^2 - \frac{2\Delta}{|\omega|} \text{Re} G_{11}^r(\omega) G_{12}^r(\omega) \right)$$

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of the Andreev current  $J_A$  appearing at sub-gap voltages !

## **Correlated QD** – influence of $\Gamma_S/\Gamma_N$ ratio

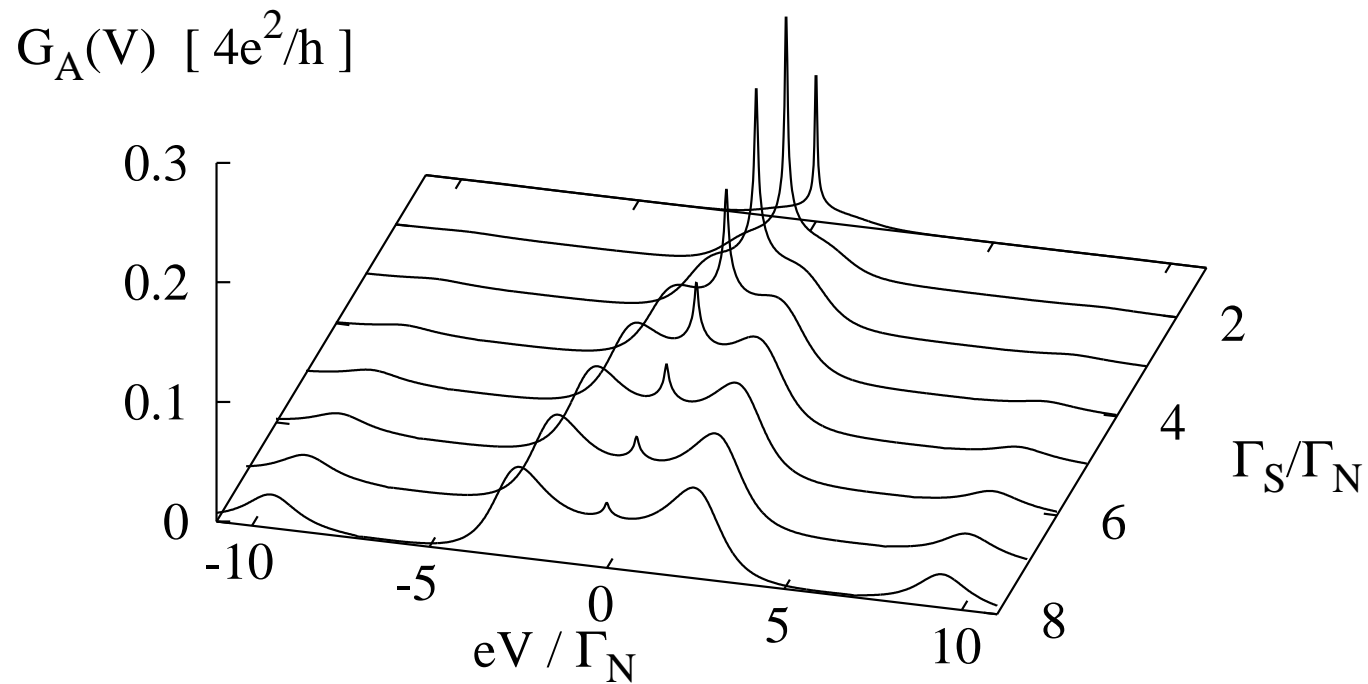
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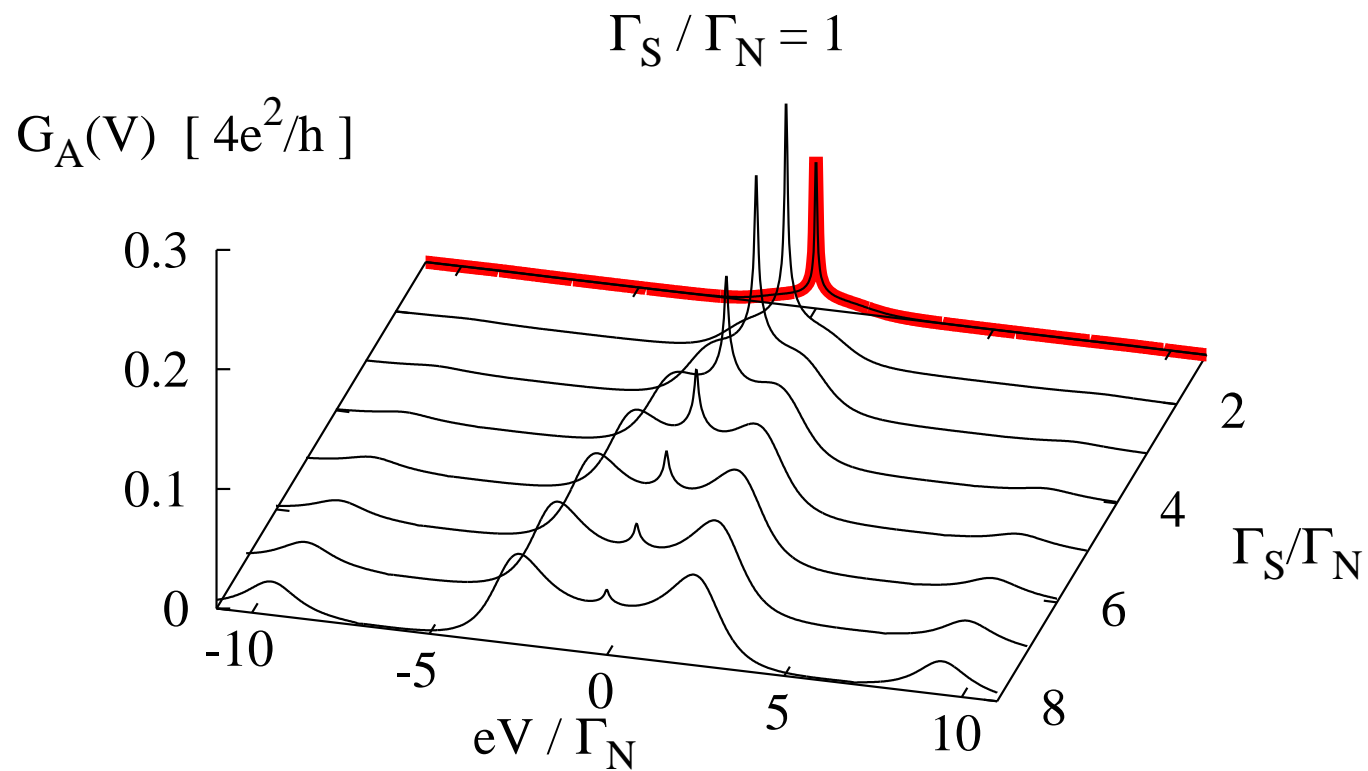
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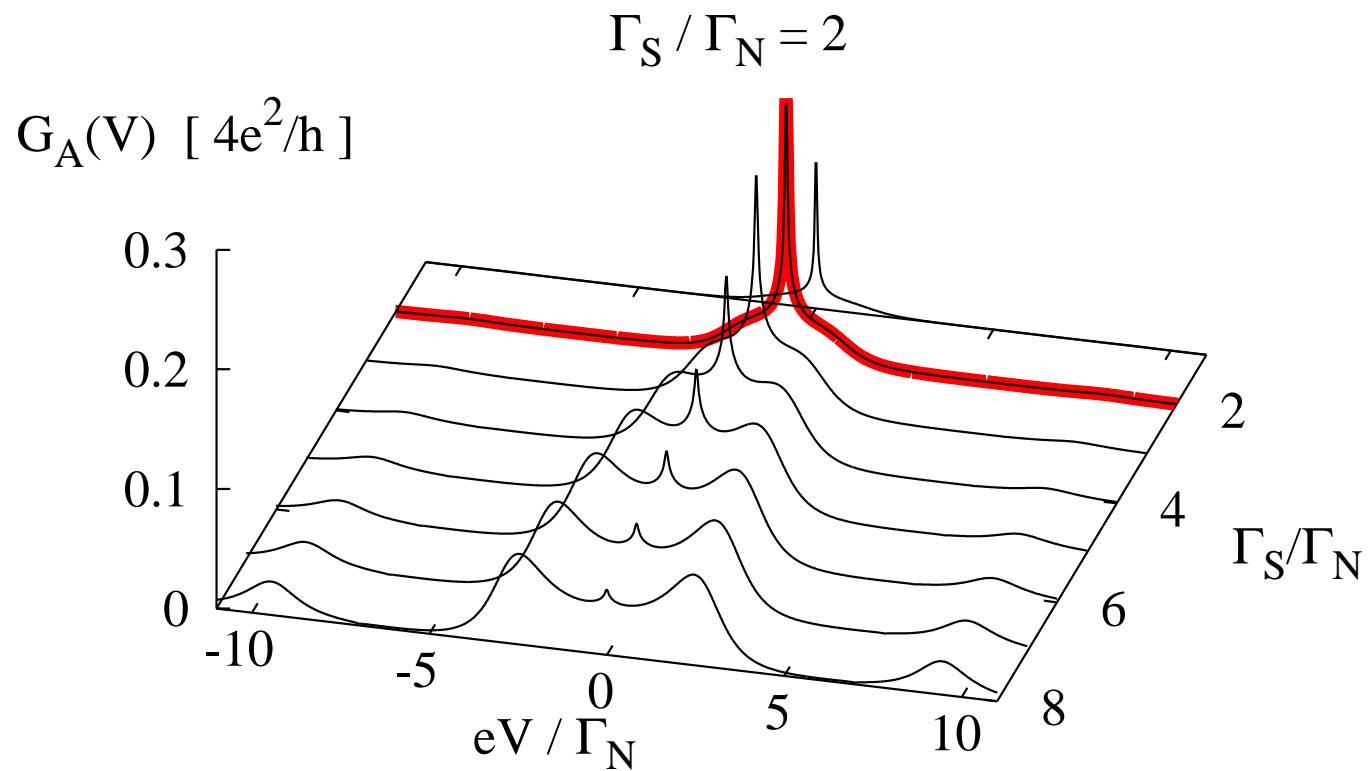
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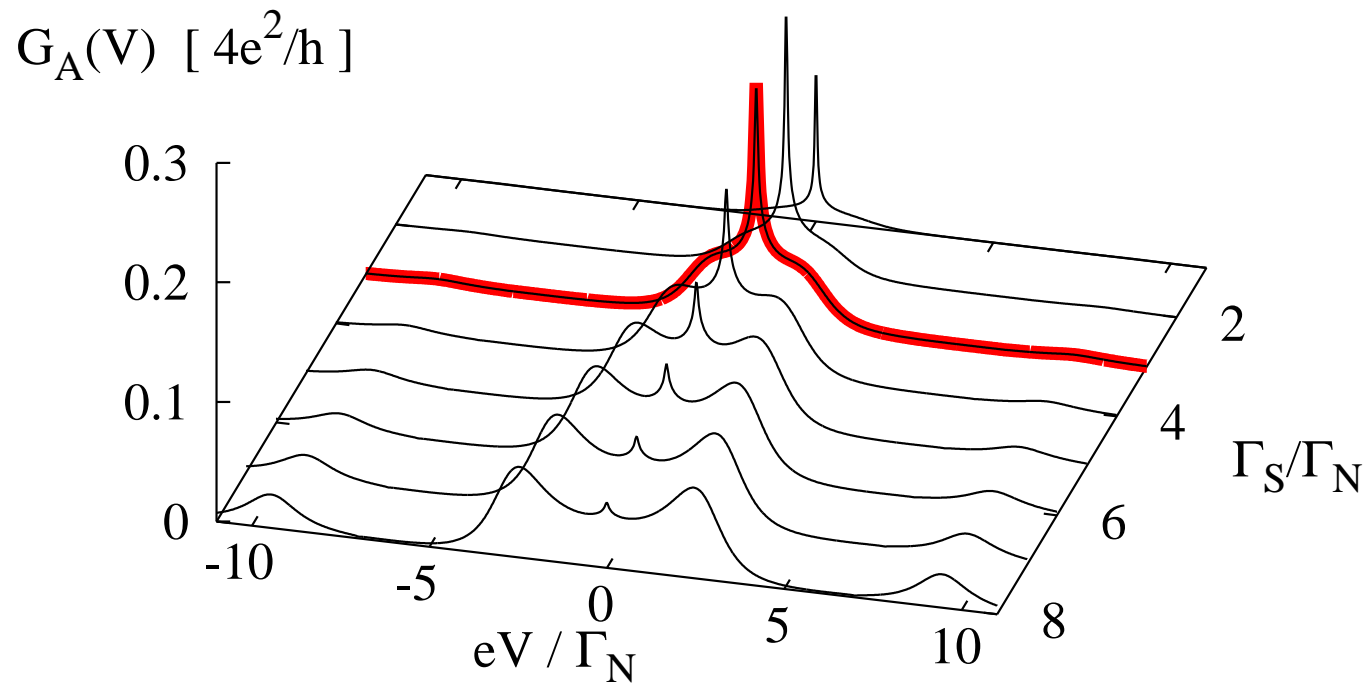


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$$U = 10\Gamma_N$$

$$\Gamma_S / \Gamma_N = 3$$



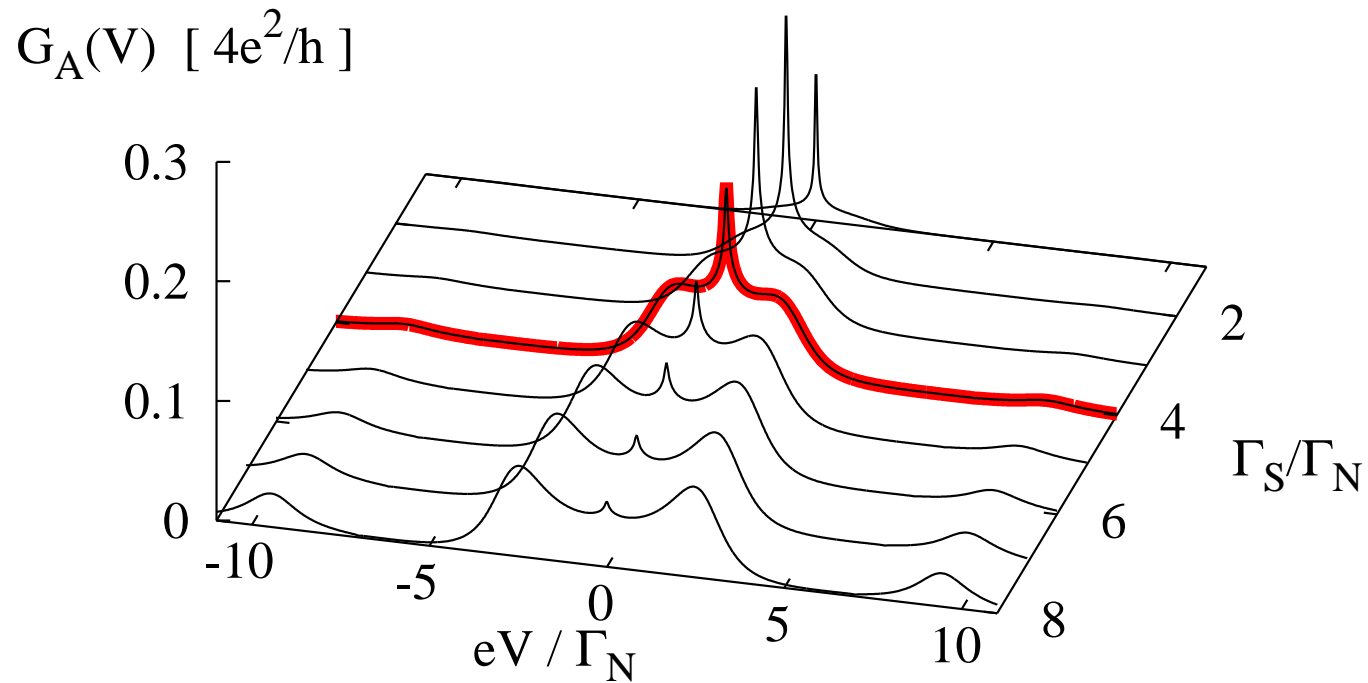


## Correlated QD – influence of $\Gamma_S/\Gamma_N$ ratio

Andreev conductance  $G_A(V)$  for:

$$U = 10\Gamma_N$$

$$\Gamma_S / \Gamma_N = 4$$

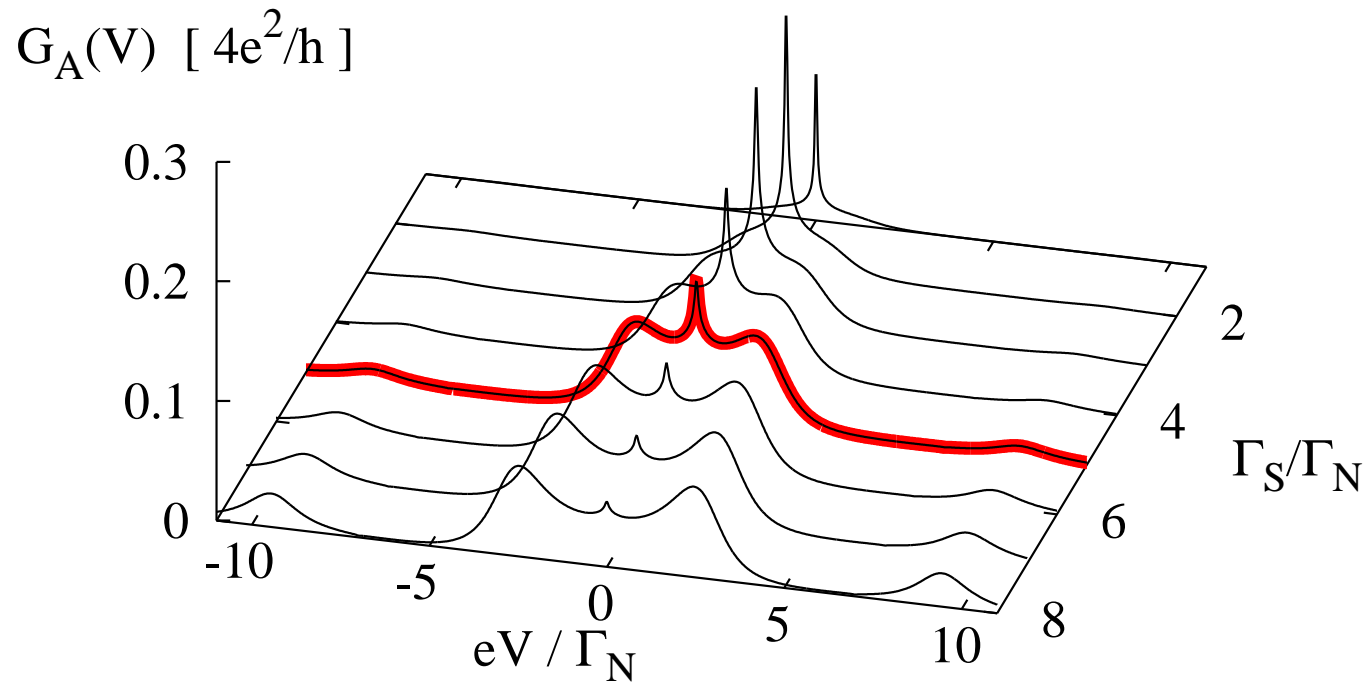


## Correlated QD – influence of $\Gamma_S/\Gamma_N$ ratio

Andreev conductance  $G_A(V)$  for:

$$U = 10\Gamma_N$$

$$\Gamma_S / \Gamma_N = 5$$

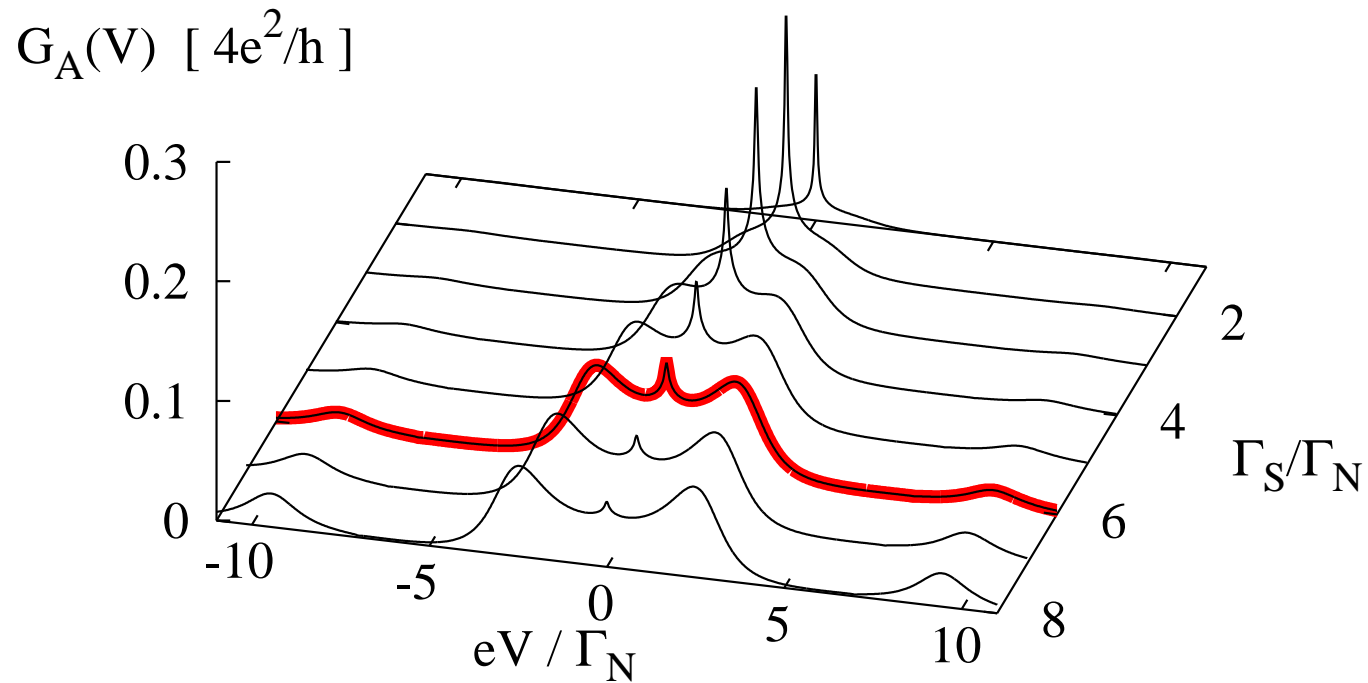


## Correlated QD – influence of $\Gamma_S/\Gamma_N$ ratio

Andreev conductance  $G_A(V)$  for:

$$U = 10\Gamma_N$$

$$\Gamma_S / \Gamma_N = 6$$

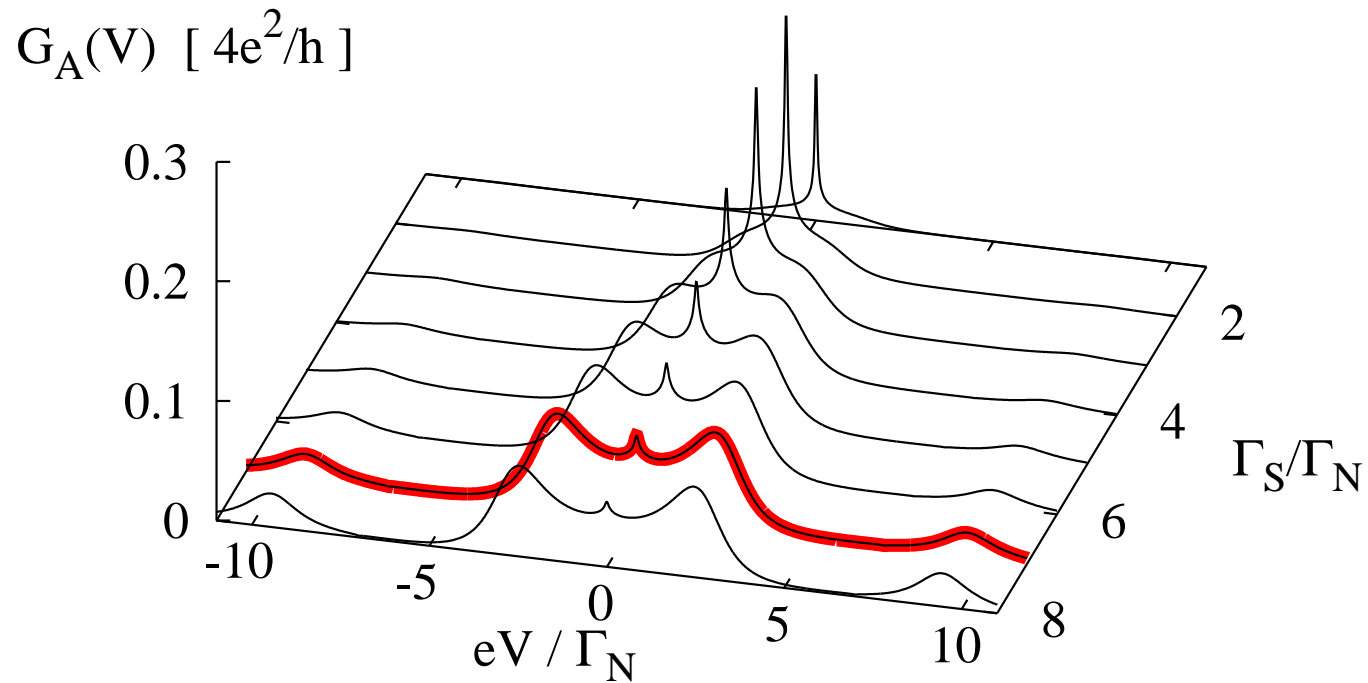


## Correlated QD – influence of $\Gamma_S/\Gamma_N$ ratio

Andreev conductance  $G_A(V)$  for:

$$U = 10\Gamma_N$$

$$\Gamma_S / \Gamma_N = 7$$

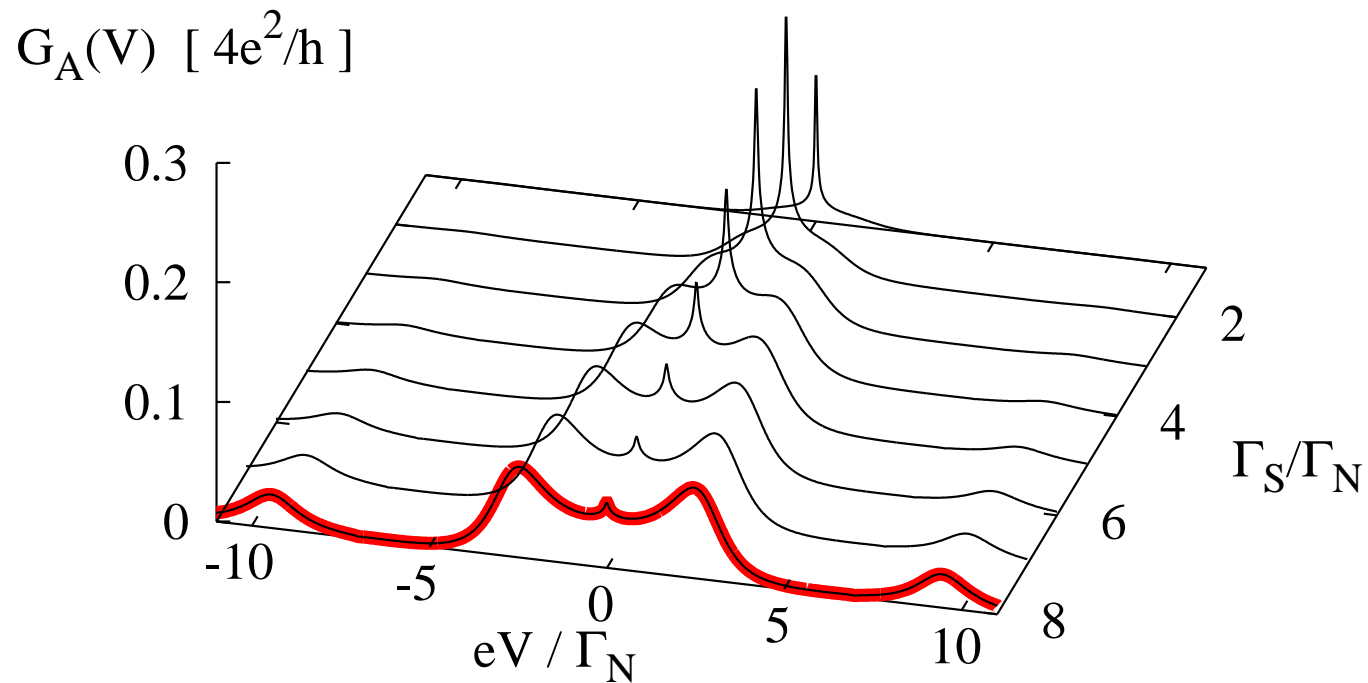


## Correlated QD – influence of $\Gamma_S/\Gamma_N$ ratio

Andreev conductance  $G_A(V)$  for:

$$U = 10\Gamma_N$$

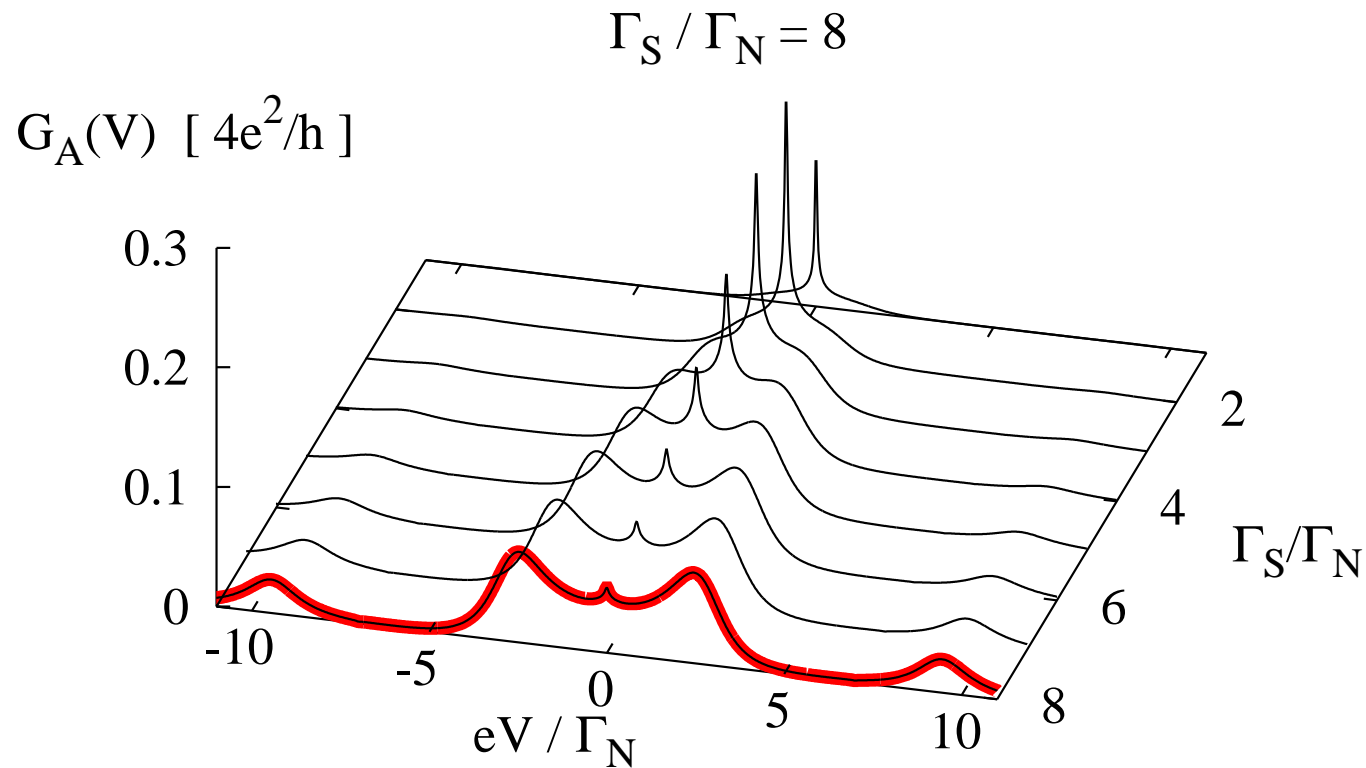
$$\Gamma_S / \Gamma_N = 8$$



## Correlated QD – influence of $\Gamma_S/\Gamma_N$ ratio

Andreev conductance  $G_A(V)$  for:

$$U = 10\Gamma_N$$



**Kondo resonance enhances zero-bias  
Andreev conductance for  $\Gamma_S \sim \Gamma_N$  !**

**Experimental realization**

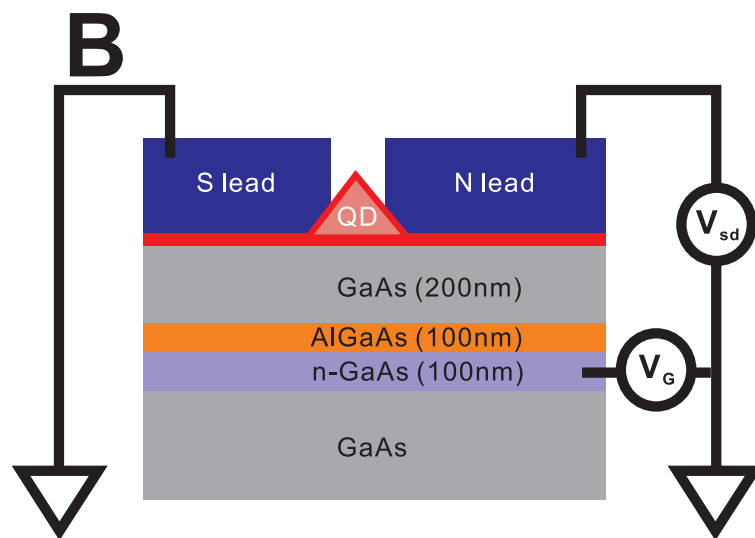
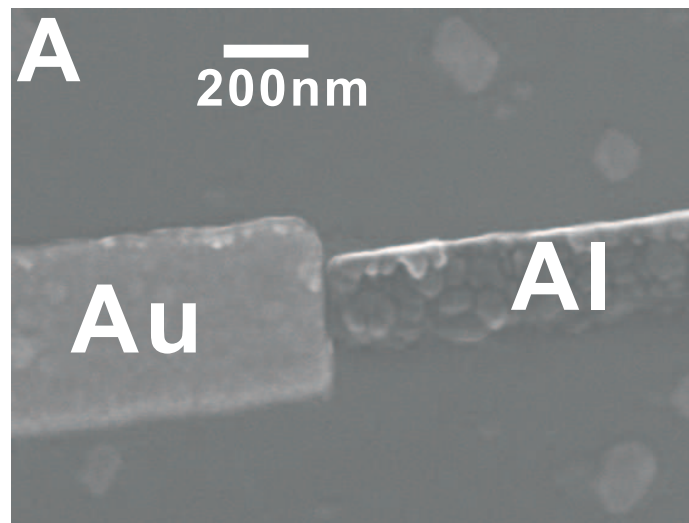
**Experimental setup**

**/ University of Tokyo /**



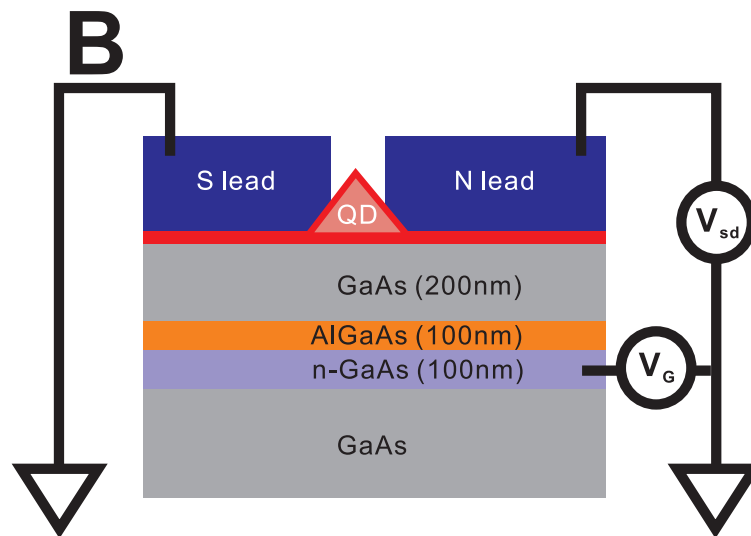
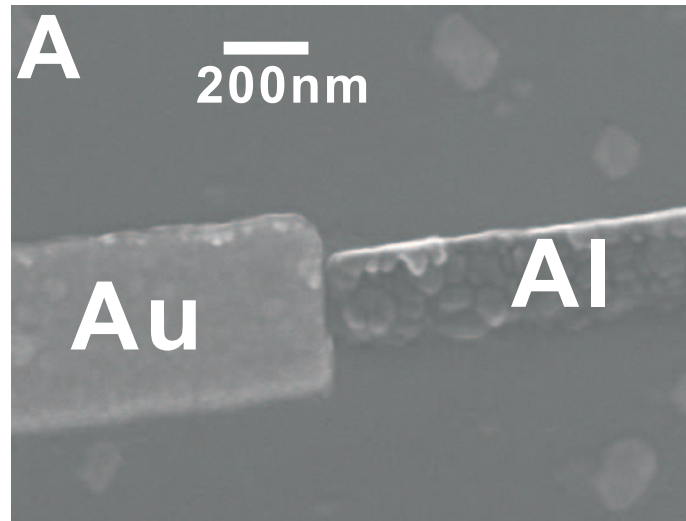
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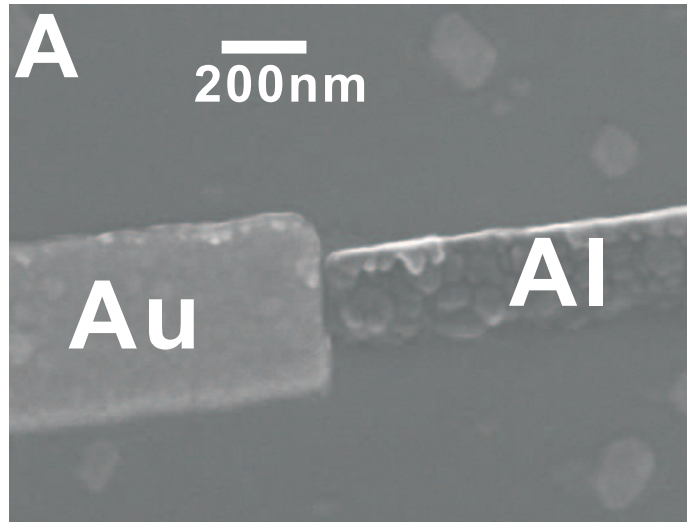
**QD** : self-assembled InAs

**diameter**  $\sim$  100 nm

**backgate** : Si-doped GaAs

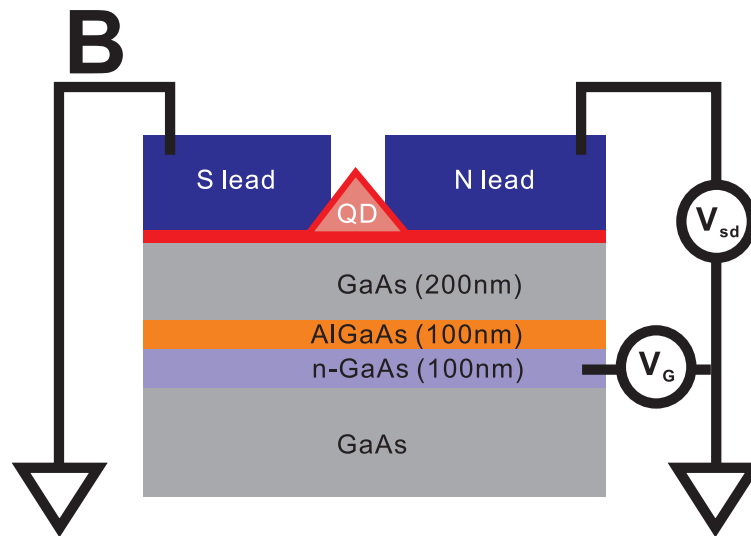
## Experimental setup

/ University of Tokyo /



$$T_K \simeq 1.2\text{K}$$

$$\Delta \simeq 152\mu\text{eV}$$



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**diameter**  $\sim 100$  nm

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*R.S. Deacon et al, Phys. Rev. Lett. **104**, 076805 (2010).*

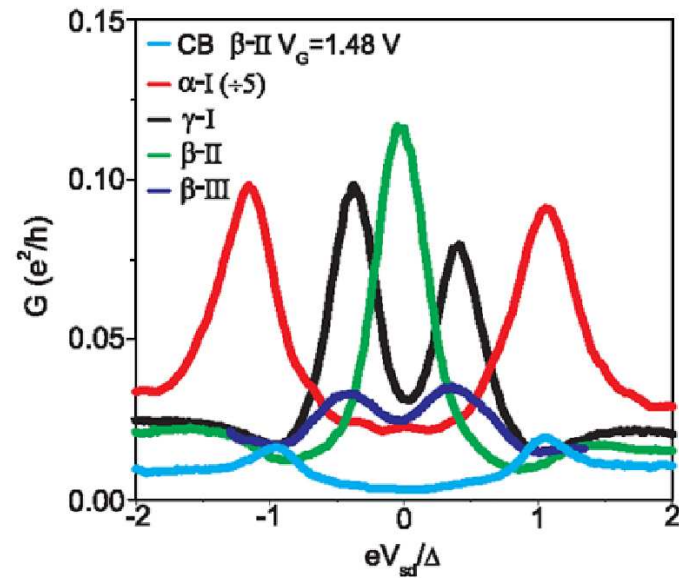
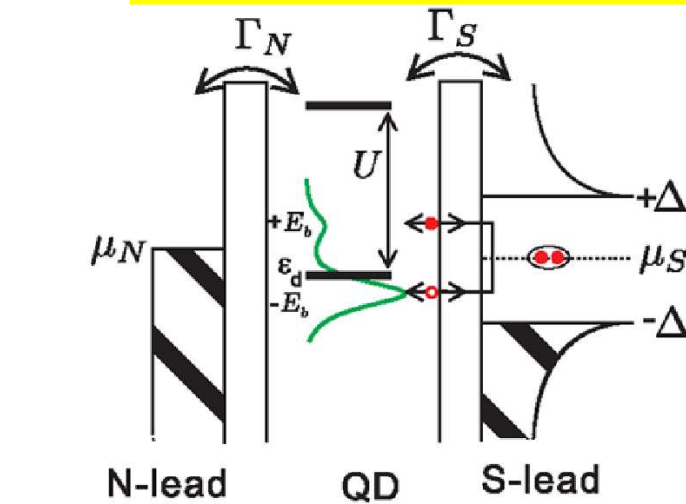
**Andreev features**

**/ due to the proximity effect /**

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/ due to the proximity effect /

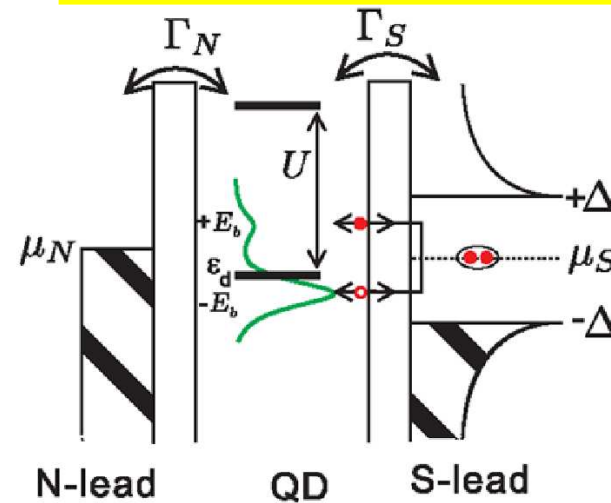
*R.S. Deacon et al, Phys. Rev. Lett. 104, 076805 (2010).*



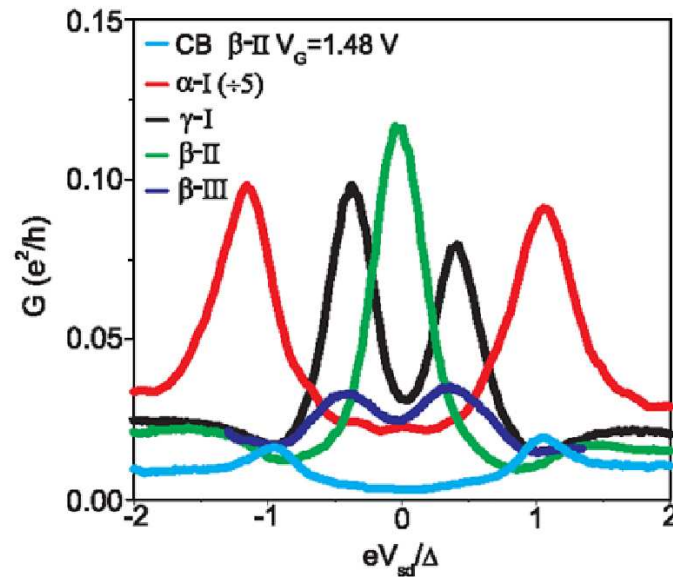
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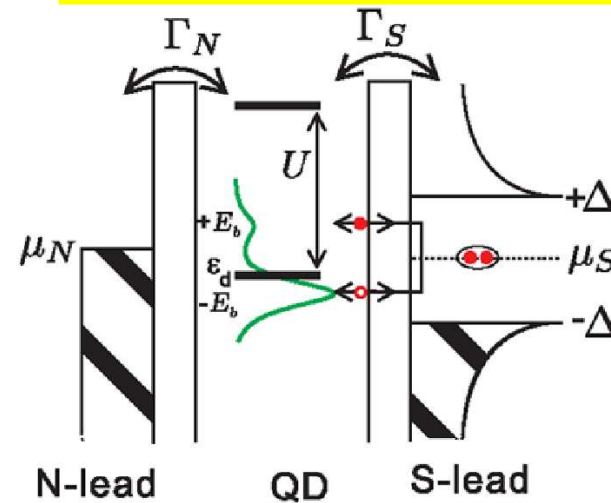
$$\epsilon_d \sim \mu_{N,S}$$



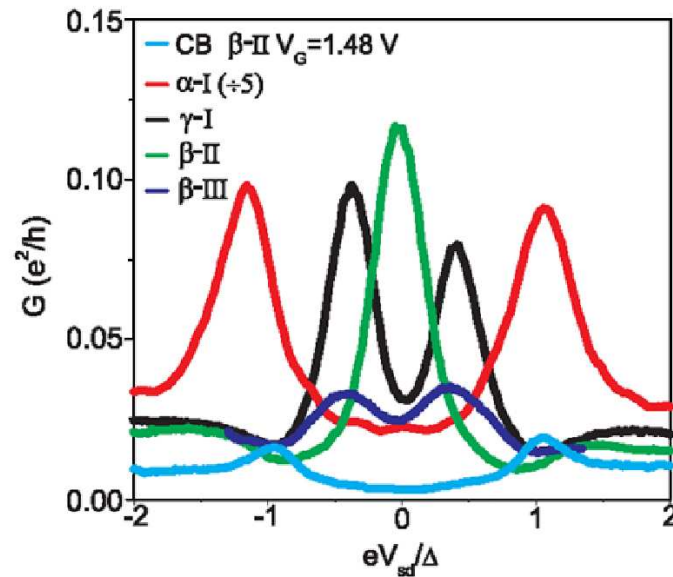
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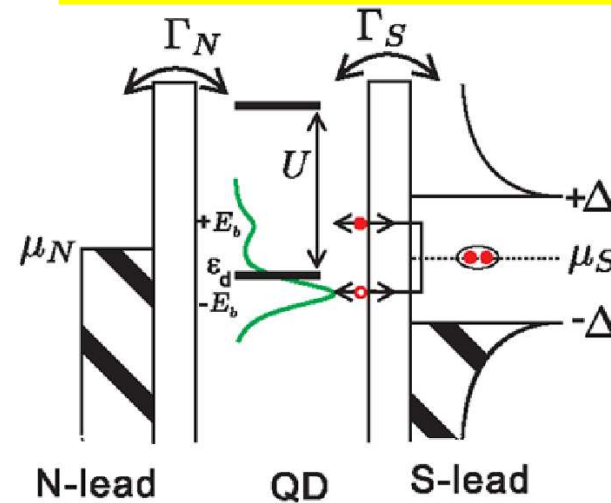
Sample  $\alpha$ -I

$$\Gamma_N \sim 12 \Gamma_S$$

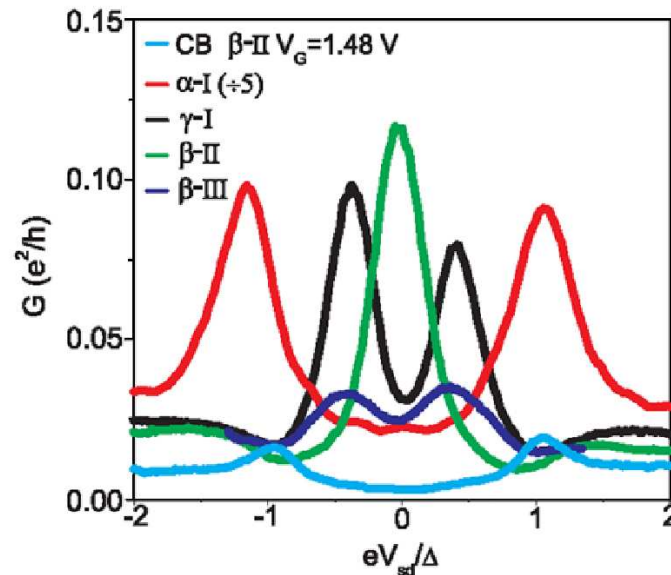
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*R.S. Deacon et al, Phys. Rev. Lett. 104, 076805 (2010).*



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Sample  $\alpha$ -I

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Samples  $\gamma$ -I,  $\beta$ -III

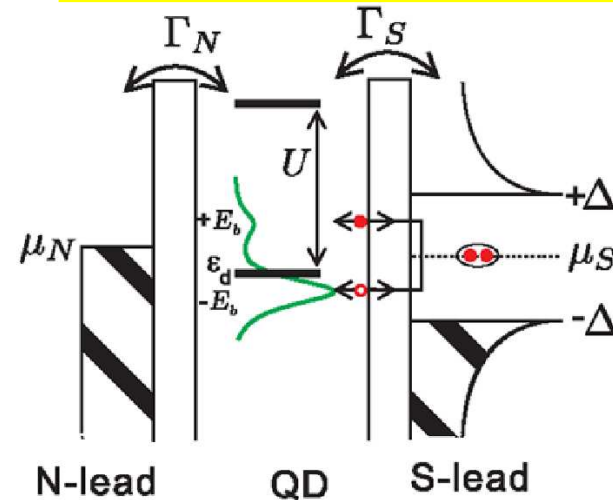
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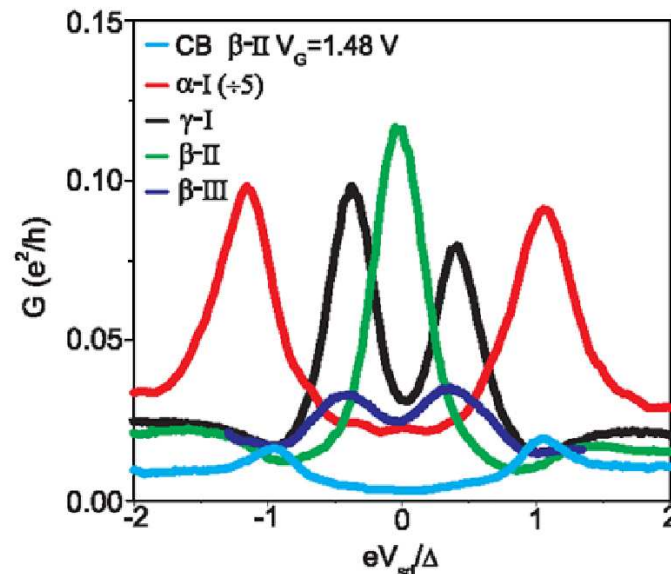
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R.S. Deacon et al, Phys. Rev. Lett. **104**, 076805 (2010).



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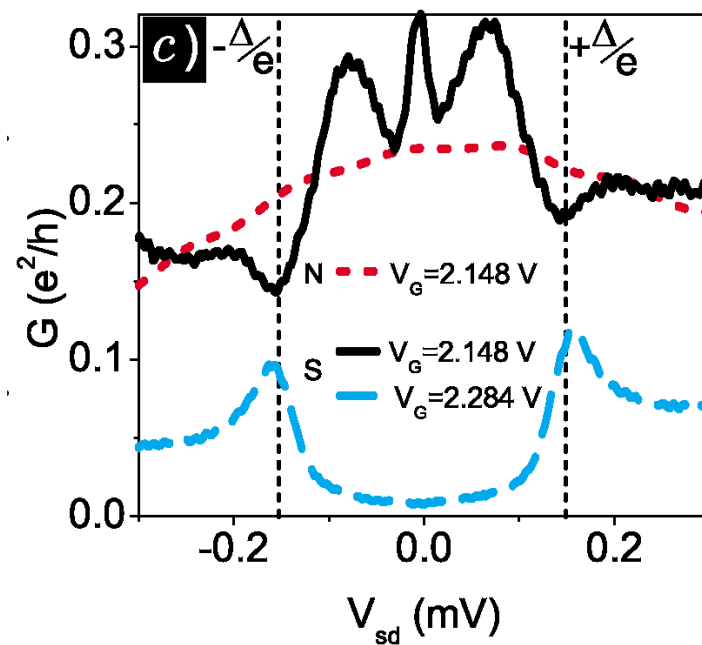
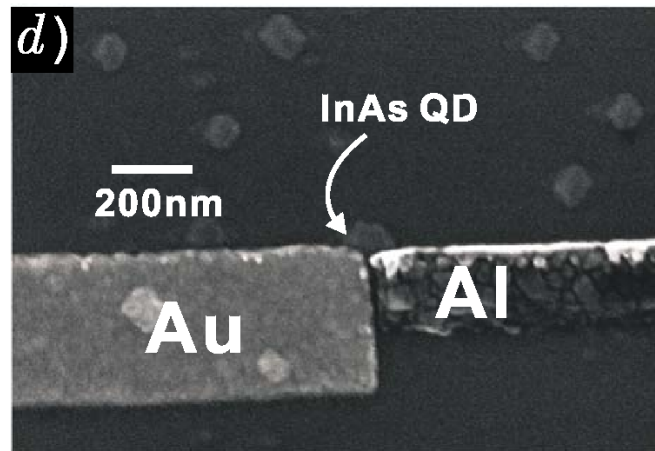
Samples  $\gamma$ -I,  $\beta$ -III

$$\Gamma_S \sim 40 \Gamma_N$$

"We attribute the subgap peaks to resonant Andreev transport ... through electron-hole mixing of the QD energy level."

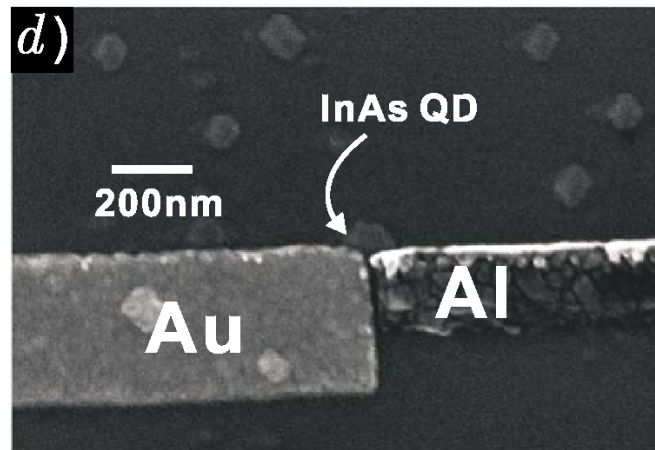
## Interplay with the Kondo effect

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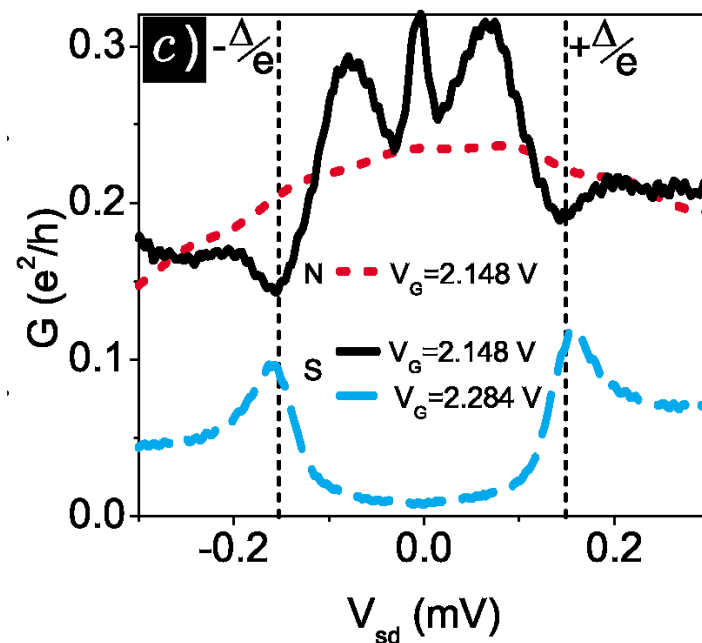


*R.S. Deacon et al, Phys. Rev. B* **81**, 121308(R) (2010).

## Interplay with the Kondo effect

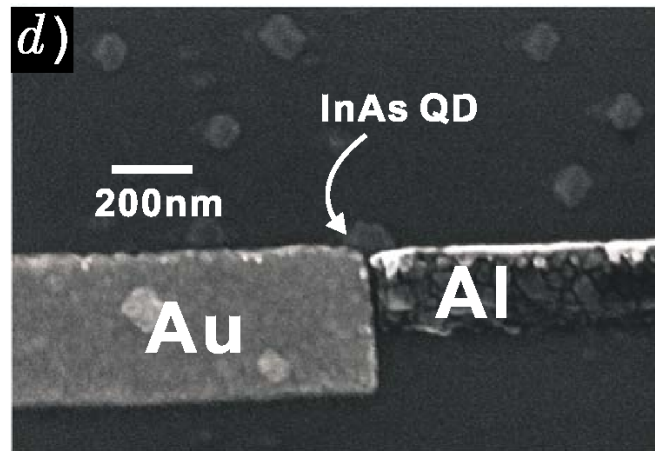


*"The zero-bias conductance peak is consistent with Andreev transport enhanced by the Kondo singlet state"*

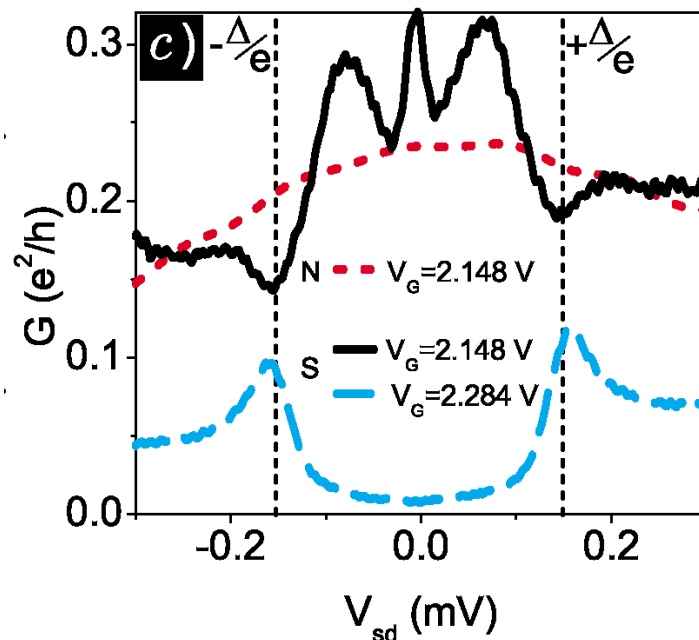


R.S. Deacon et al, *Phys. Rev. B* **81**, 121308(R) (2010).

## Interplay with the Kondo effect



*"The zero-bias conductance peak is consistent with Andreev transport enhanced by the Kondo singlet state"*



*"We note that the feature exhibits excellent qualitative agreement with a recent theoretical treatment by Domanski et al"*

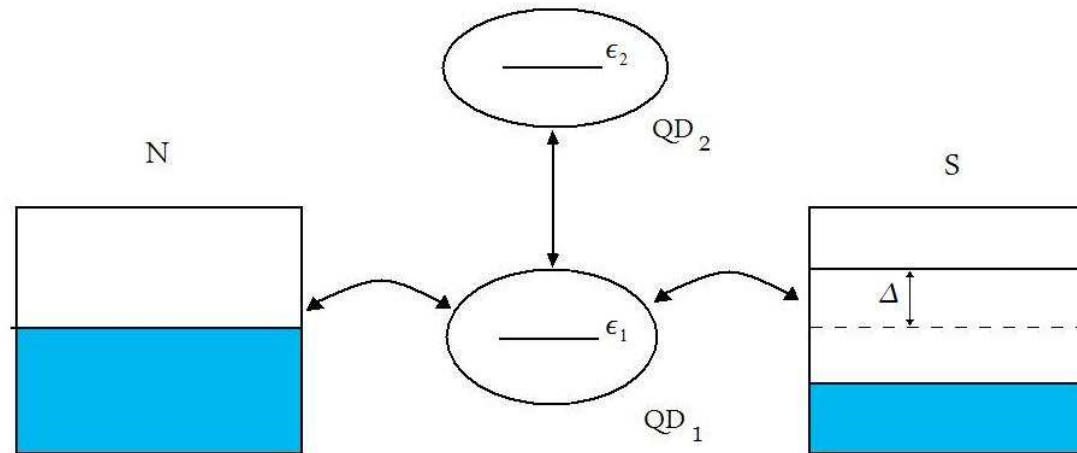
**Further outlook**

## Double QD

– between a metal and superconductor

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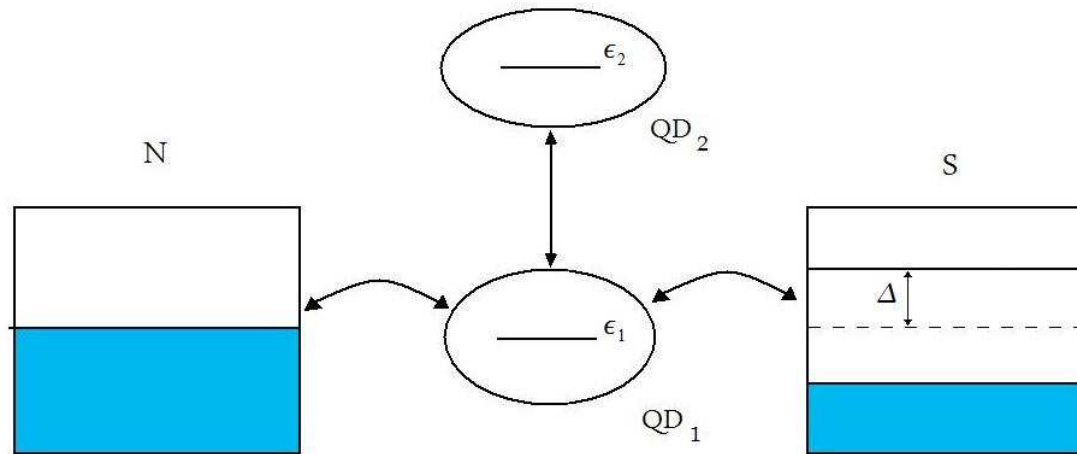
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## Double QD

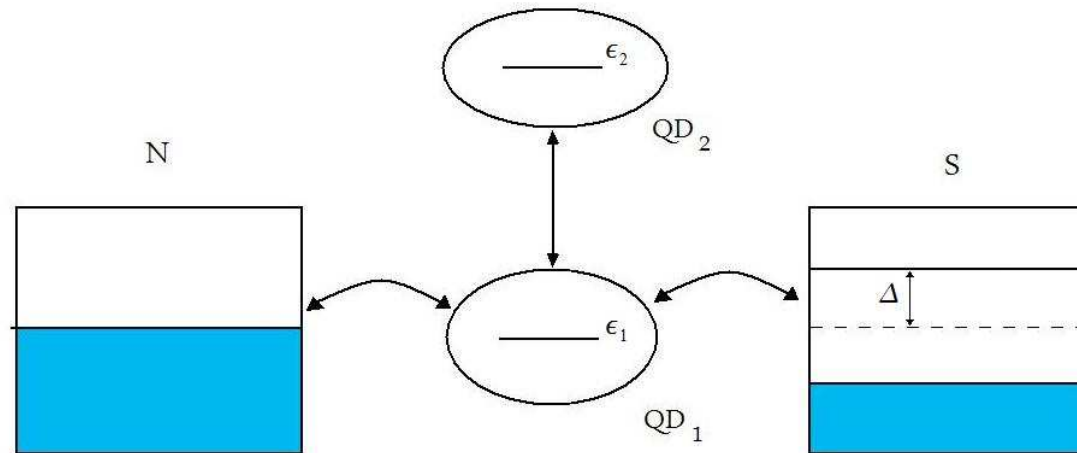
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**Side-coupled QD (T-shape configuration)**

## Double QD

– between a metal and superconductor

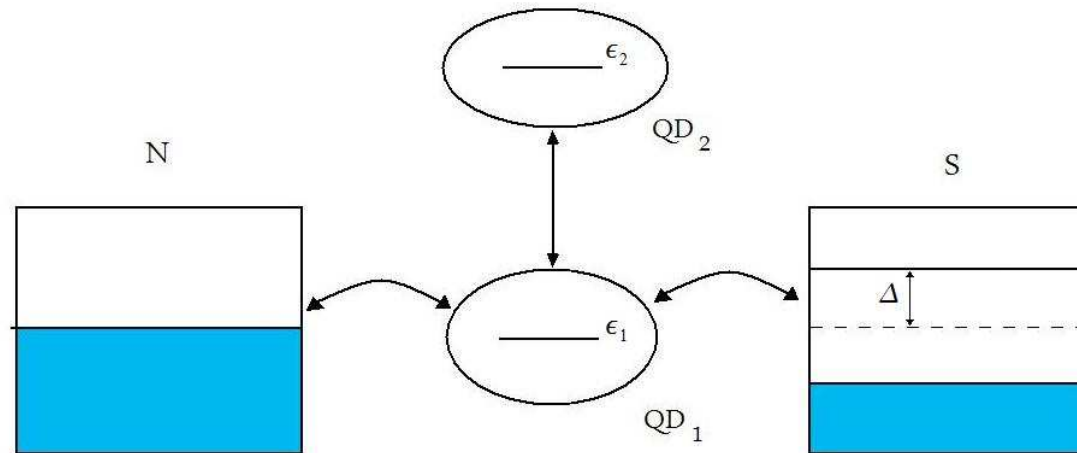


### Side-coupled QD (T-shape configuration)

Relevant issues:

## Double QD

– between a metal and superconductor



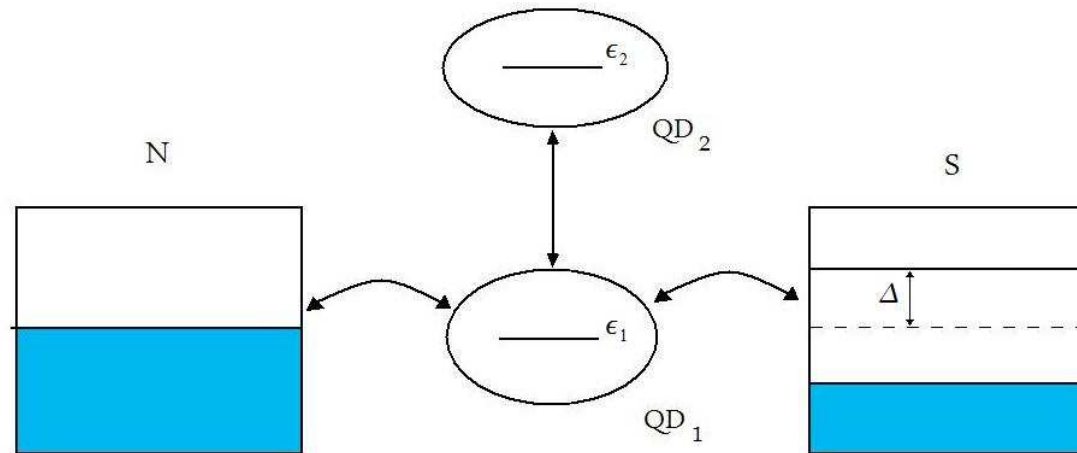
### Side-coupled QD (T-shape configuration)

#### Relevant issues:

⇒ induced on-dot pairing ..... (due to  $\Gamma_S$ )

## Double QD

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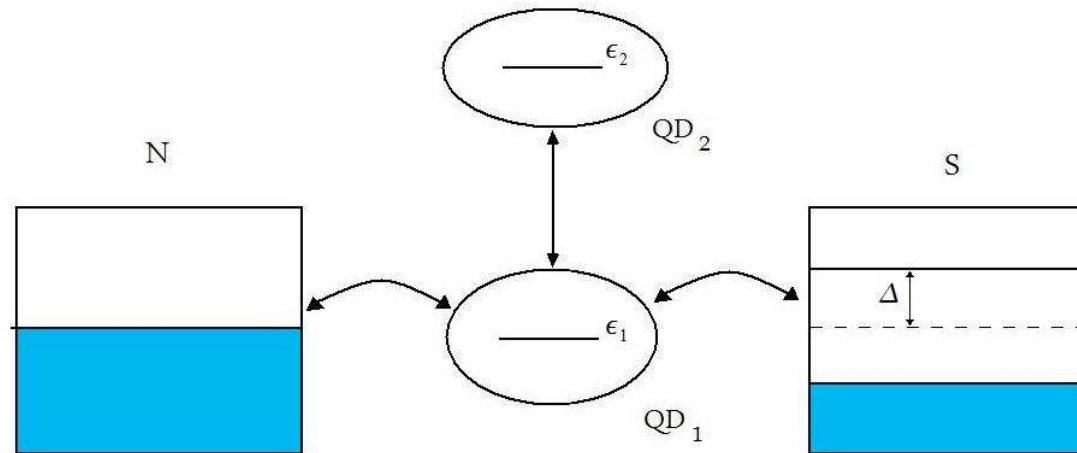
### Side-coupled QD (T-shape configuration)

#### Relevant issues:

- $\Rightarrow$  induced on-dot pairing ..... (due to  $\Gamma_S$ )
- $\Rightarrow$  Coulomb blockade & Kondo effect ..... (due to  $U$ )

## Double QD

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### Side-coupled QD (T-shape configuration)

#### Relevant issues:

- ⇒ induced on-dot pairing ..... (due to  $\Gamma_S$ )
- ⇒ Coulomb blockade & Kondo effect ..... (due to  $U$ )
- ⇒ quantum interference ..... (due to  $t$ )

## Quantum interference

– effect of  $t$

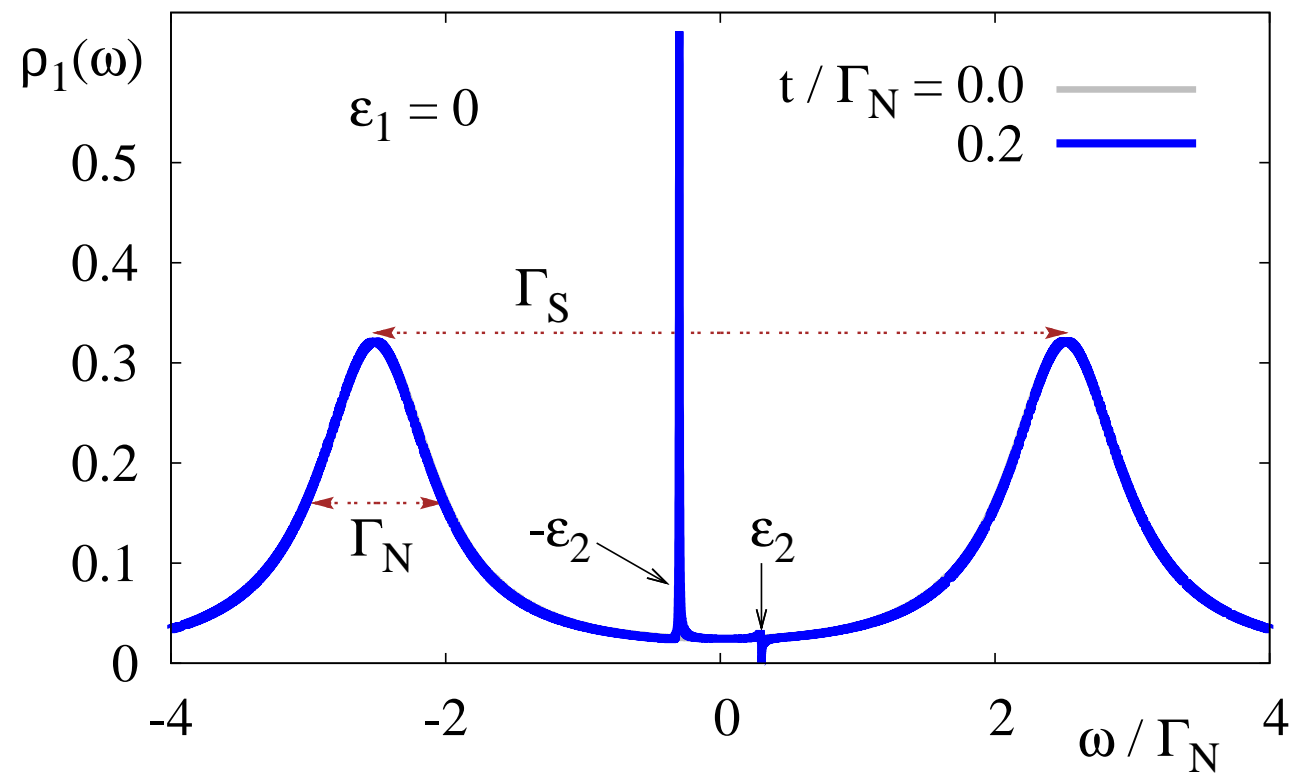
## Quantum interference – effect of $t$

Spectral function obtained at  $QD_1$  for  $\varepsilon_1 = 0$

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Spectral function obtained at  $QD_1$  for  $\varepsilon_1 = 0$



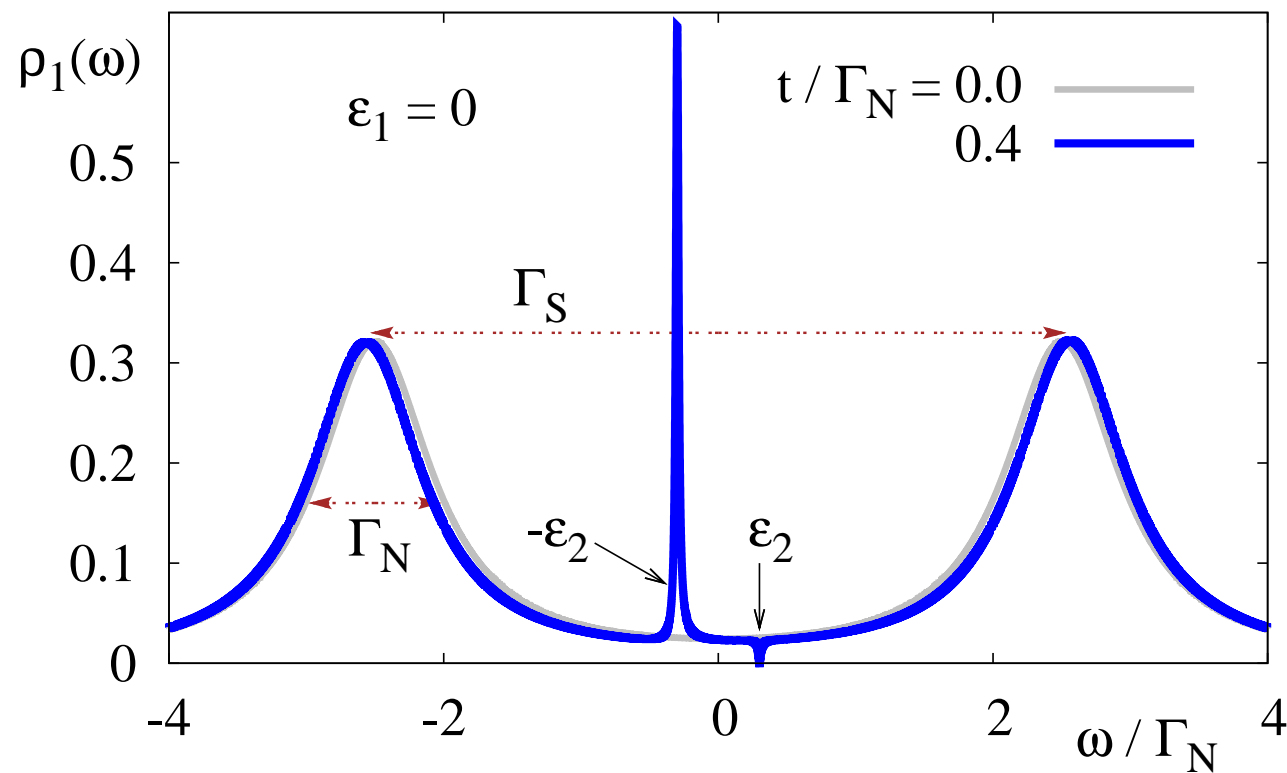
$$t / \Gamma_N = 0.2$$



## Quantum interference

– effect of  $t$

Spectral function obtained at  $QD_1$  for  $\varepsilon_1 = 0$

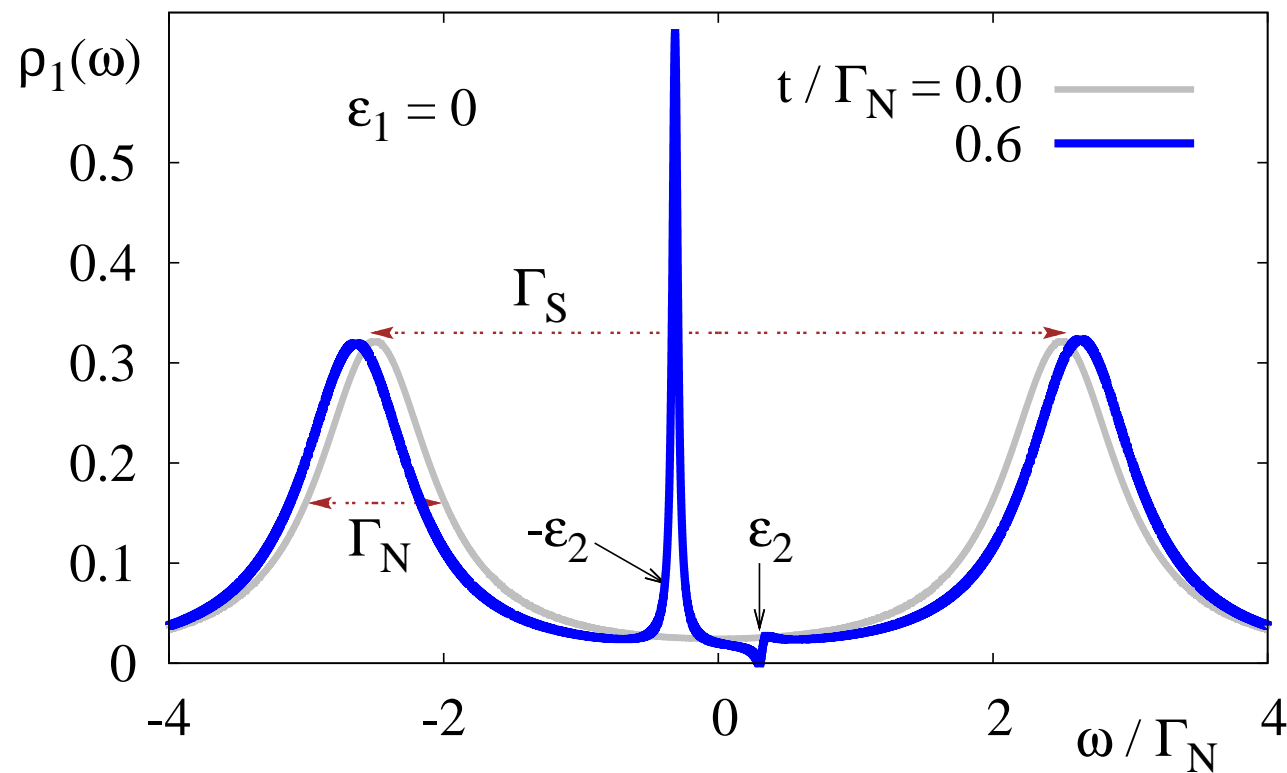


$$t / \Gamma_N = 0.4$$

## Quantum interference

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Spectral function obtained at  $QD_1$  for  $\varepsilon_1 = 0$

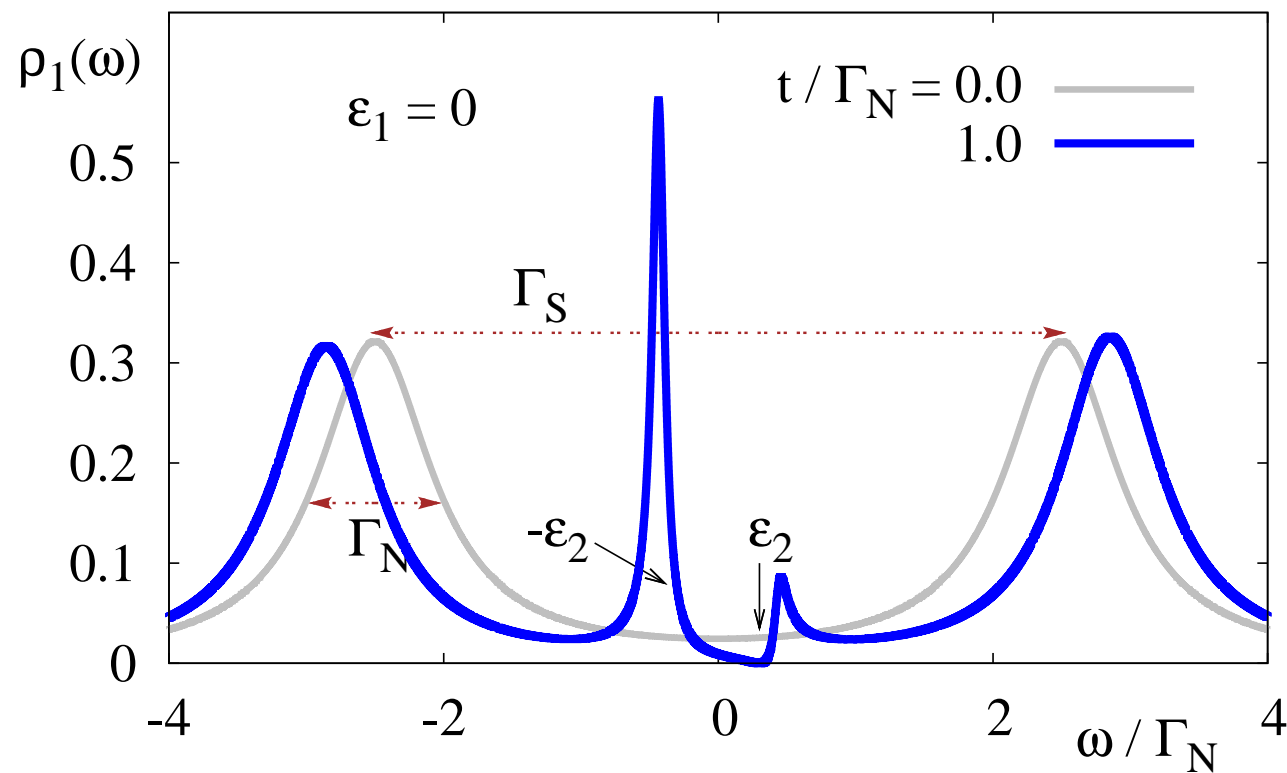


$$t / \Gamma_N = 0.6$$

## Quantum interference

– effect of  $t$

Spectral function obtained at  $QD_1$  for  $\varepsilon_1 = 0$



$$t / \Gamma_N = 1.0$$

**Fano-type resonance**

**in Andreev conductance**

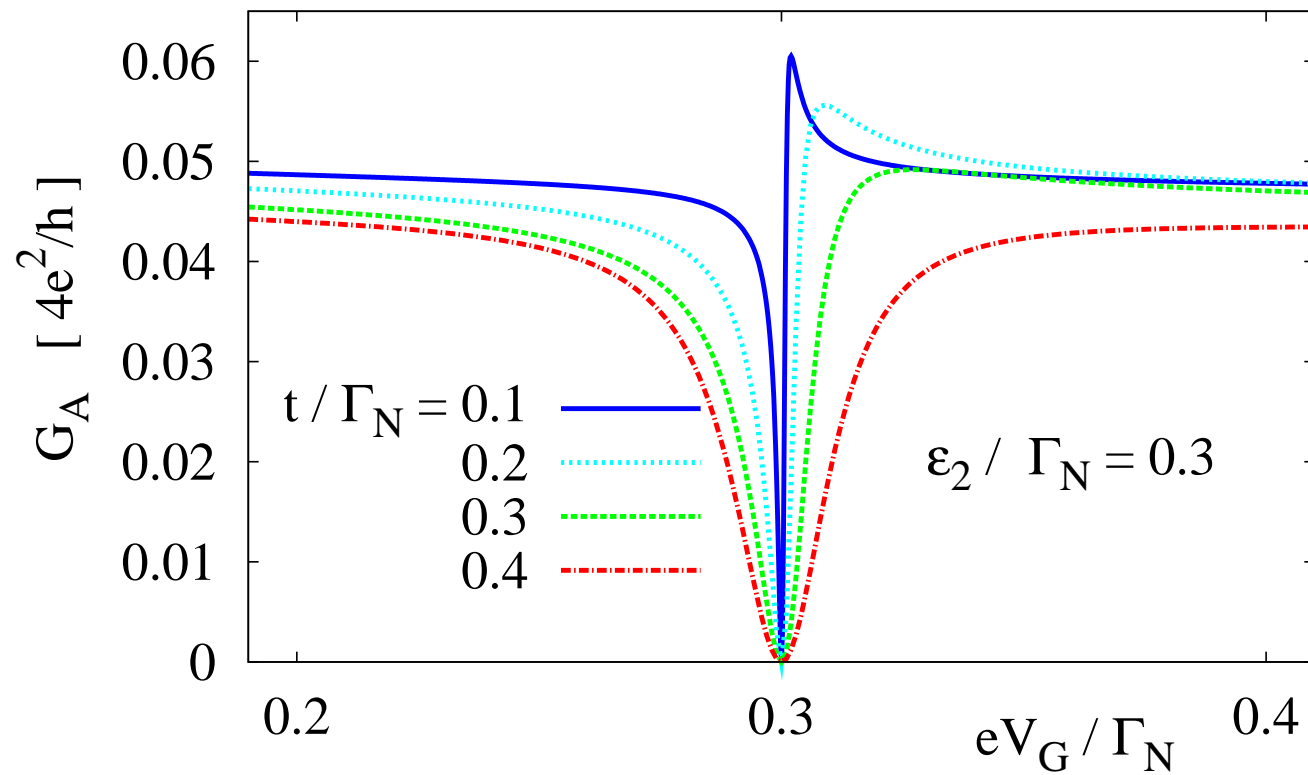
# **Fano-type resonance** in Andreev conductance

Differential conductance of the Andreev current

# Fano-type resonance

# in Andreev conductance

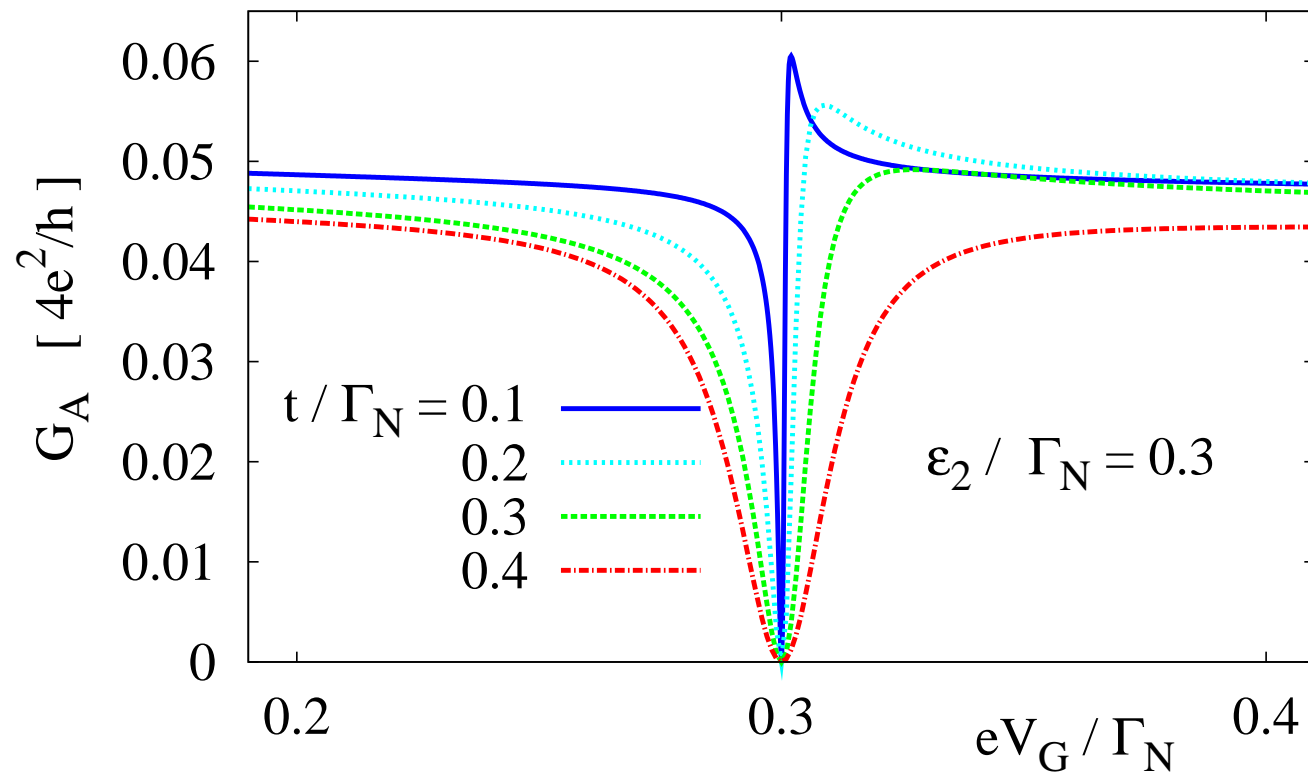
## Differential conductance of the Andreev current



# Fano-type resonance

# in Andreev conductance

## Differential conductance of the Andreev current



The gate-voltage dependence of  $G_A$  obtained for  $U = 0$

*J. Barański and T. Domański, (2011) submitted.*

## **Fano vs Kondo** – competition

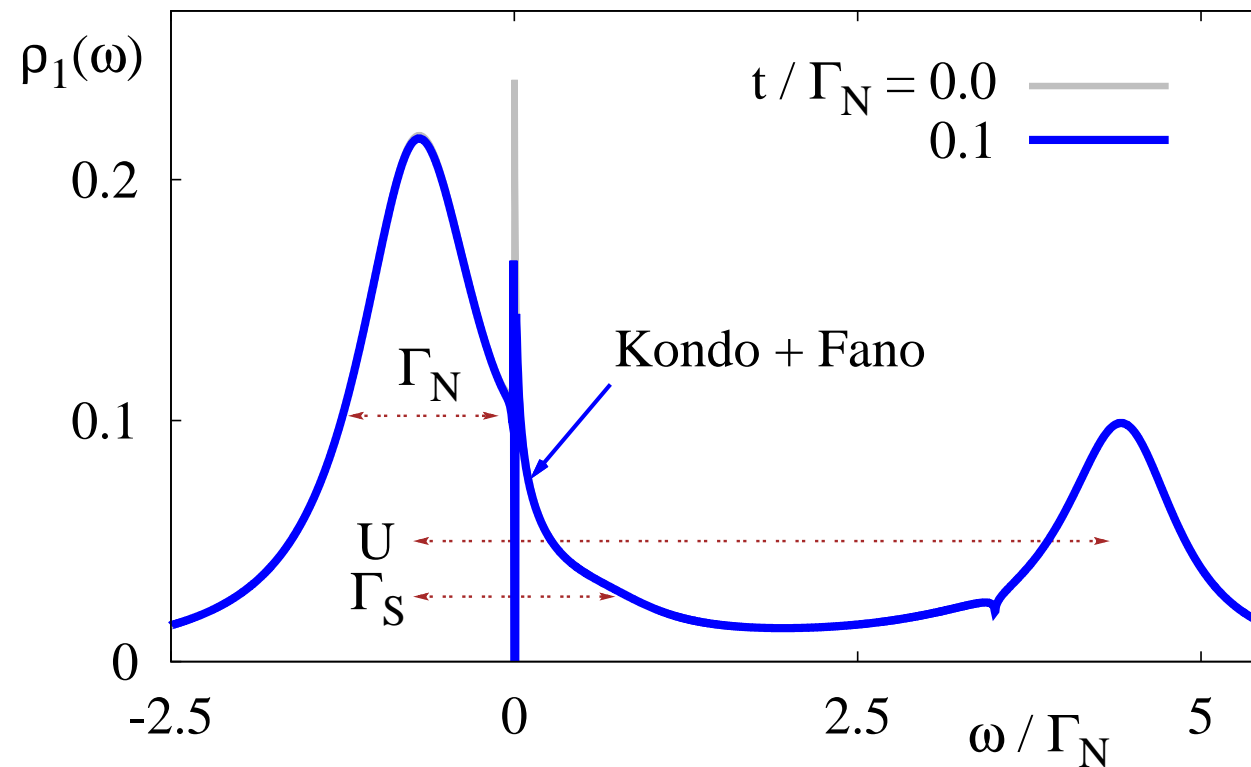


## **Fano vs Kondo** – competition

Spectral function of  $QD_1$  obtained for  $\varepsilon_2 = 0$

## Fano vs Kondo – competition

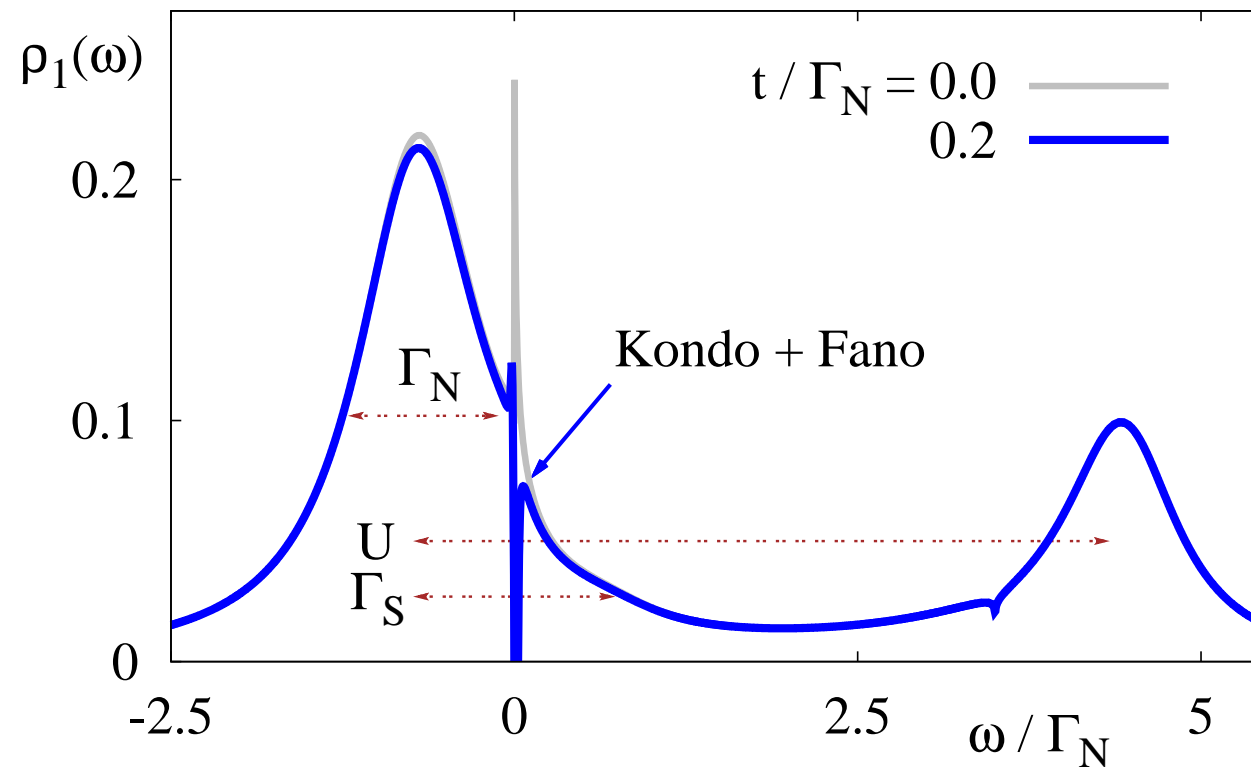
Spectral function of  $QD_1$  obtained for  $\varepsilon_2 = 0$



$$t / \Gamma_N = 0.1$$

## Fano vs Kondo – competition

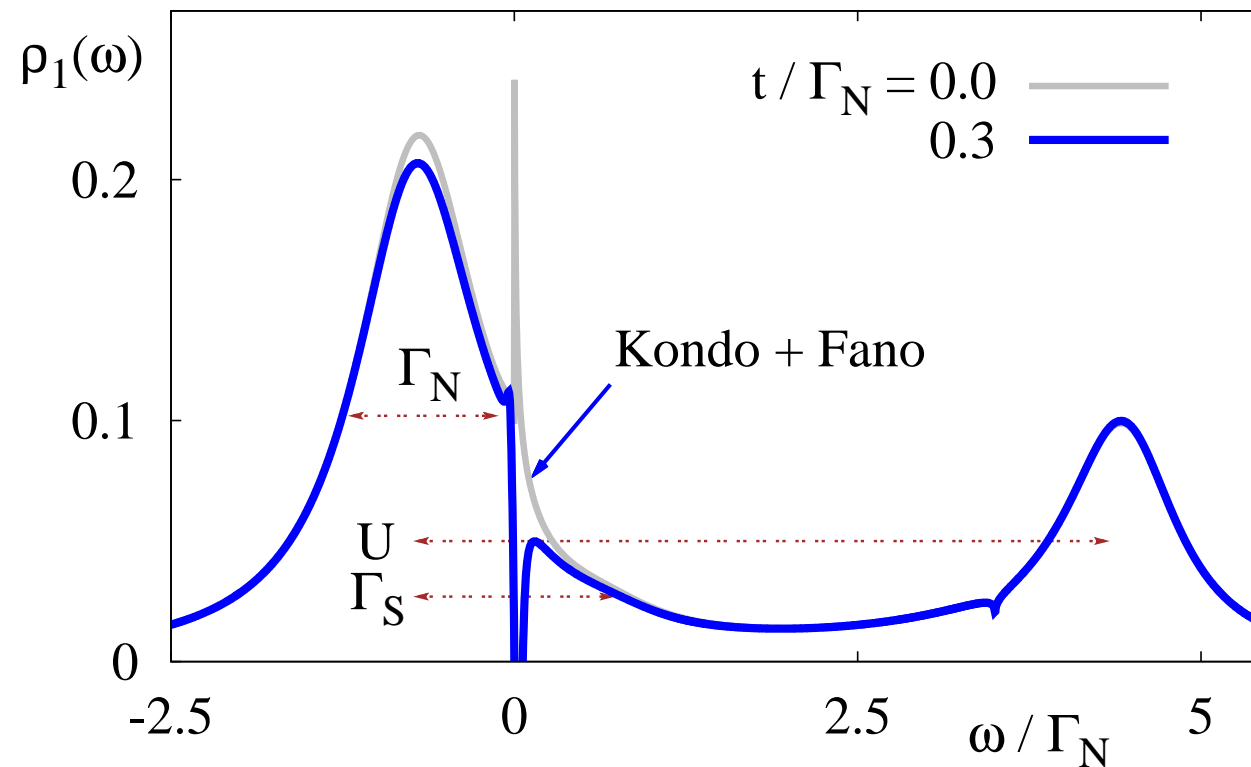
Spectral function of  $QD_1$  obtained for  $\varepsilon_2 = 0$



$$t / \Gamma_N = 0.2$$

## Fano vs Kondo – competition

Spectral function of  $QD_1$  obtained for  $\varepsilon_2 = 0$



$$t / \Gamma_N = 0.3$$

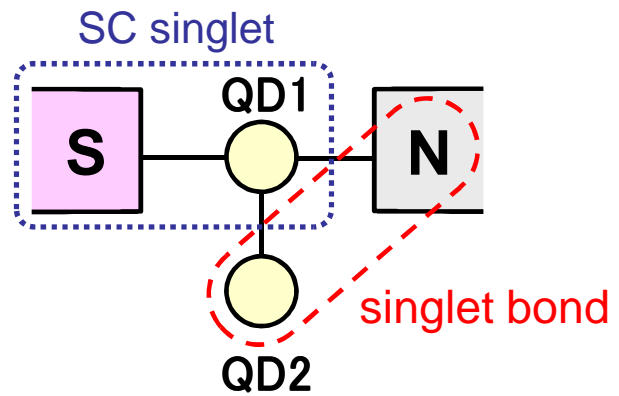
**Double QD**

– singlet states

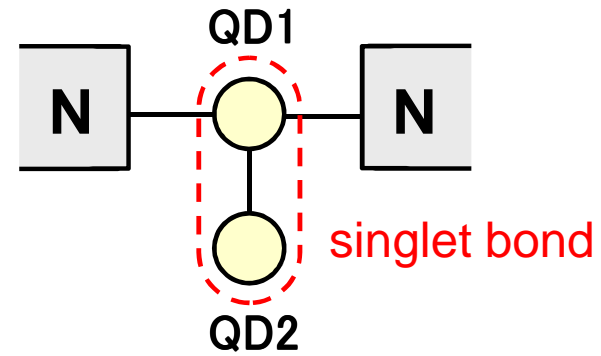
## Double QD

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(a)



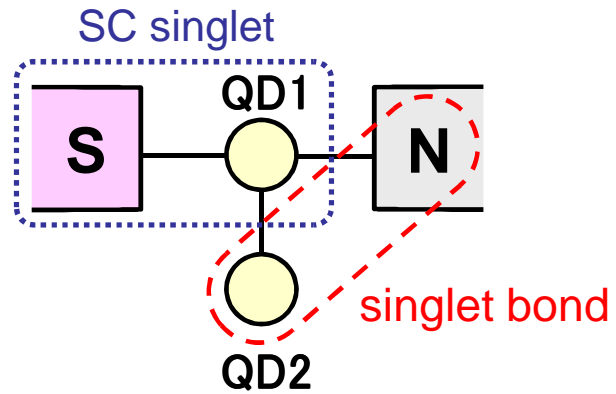
(b)



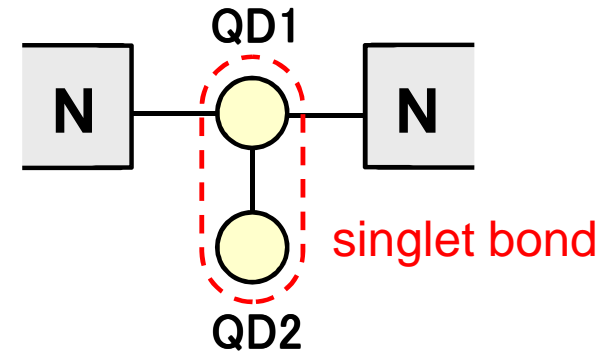
## Double QD

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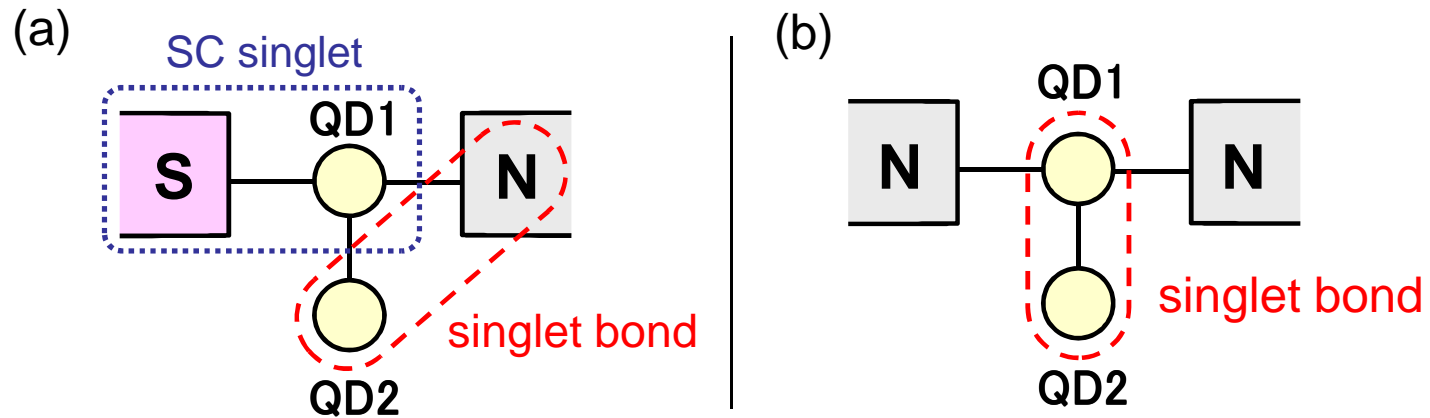
(b)



Various kinds of possible singlet states

## Double QD

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**Various kinds of possible singlet states**

*Y. Tanaka, N. Kawakami, and A. Oguri, Phys. Rev. B **82**, 094514 (2008).*



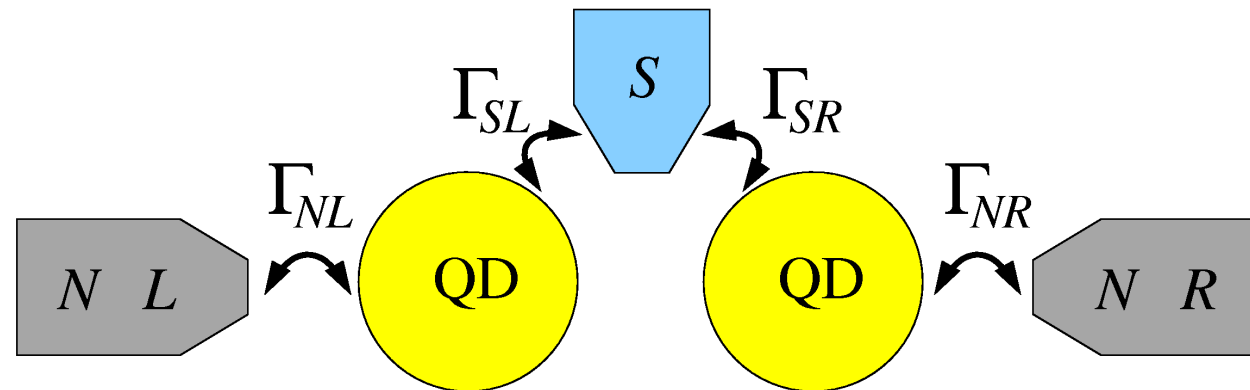
**Double-QD**

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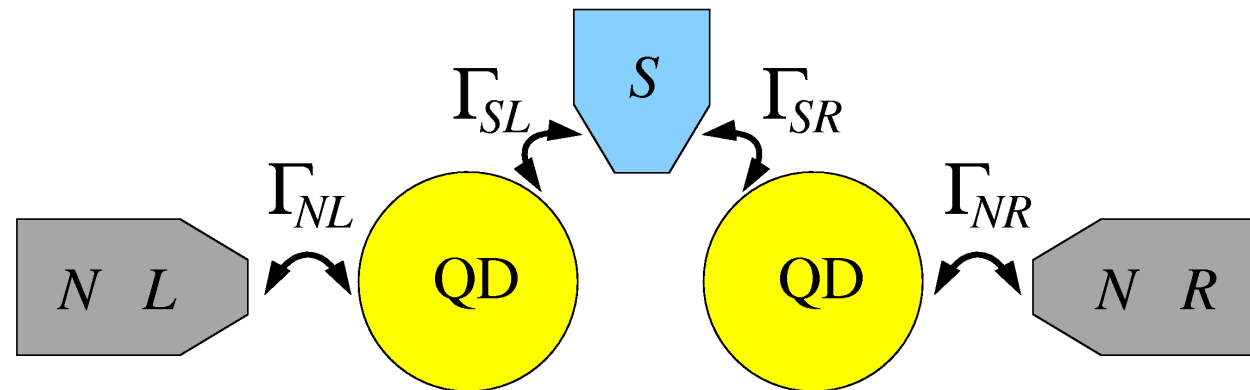
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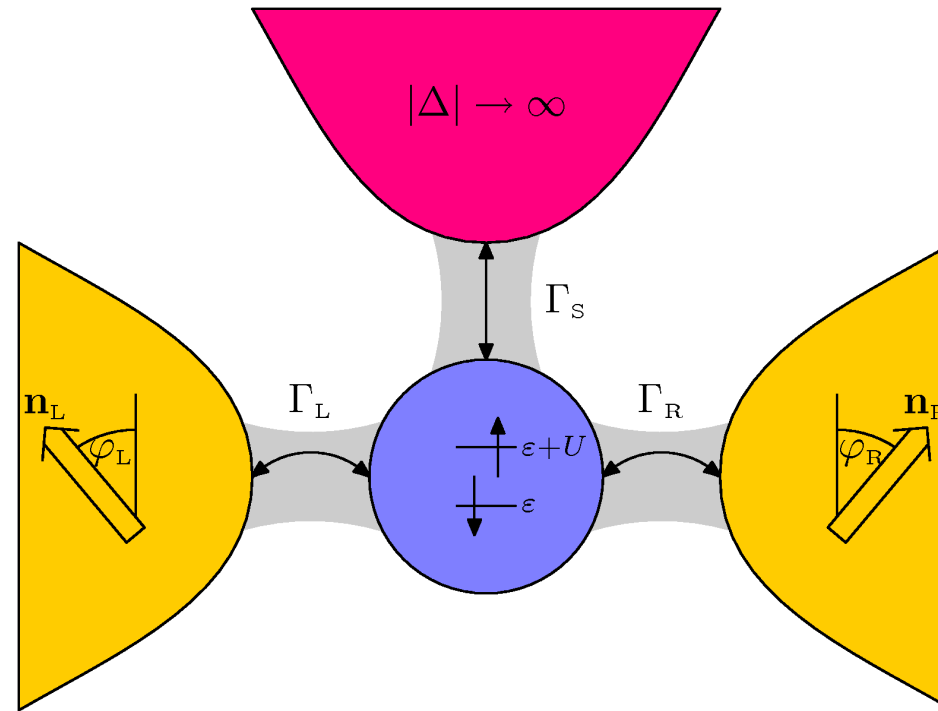
*J. Eldridge, M.G. Pala, M. Governale, J. König, Phys. Rev. B* **82**, 184507 (2010)

**QD spin-valve**

– using a superconducting lead

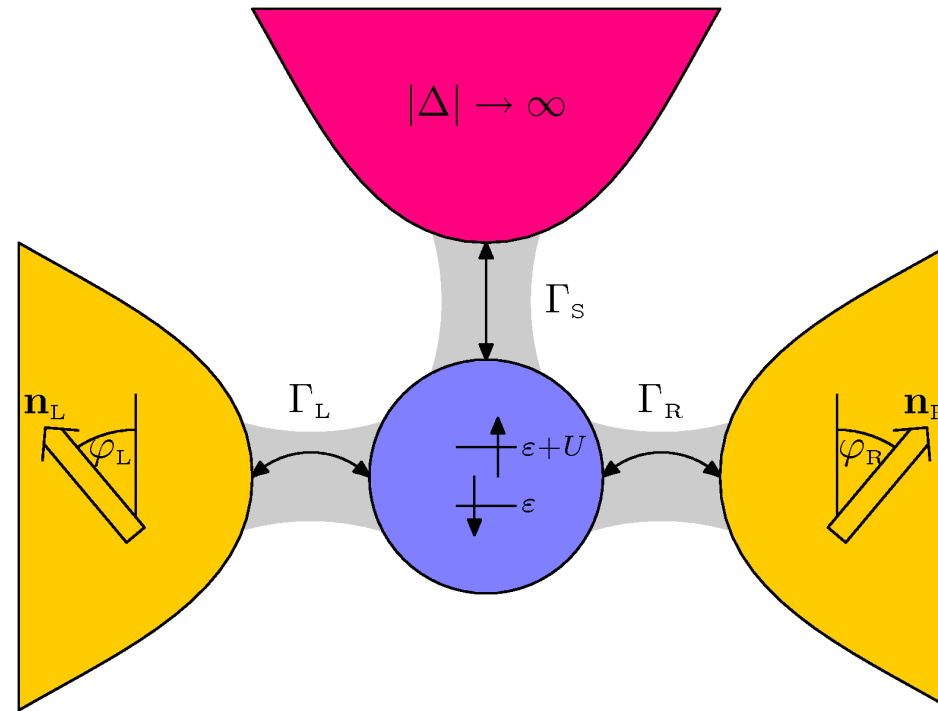
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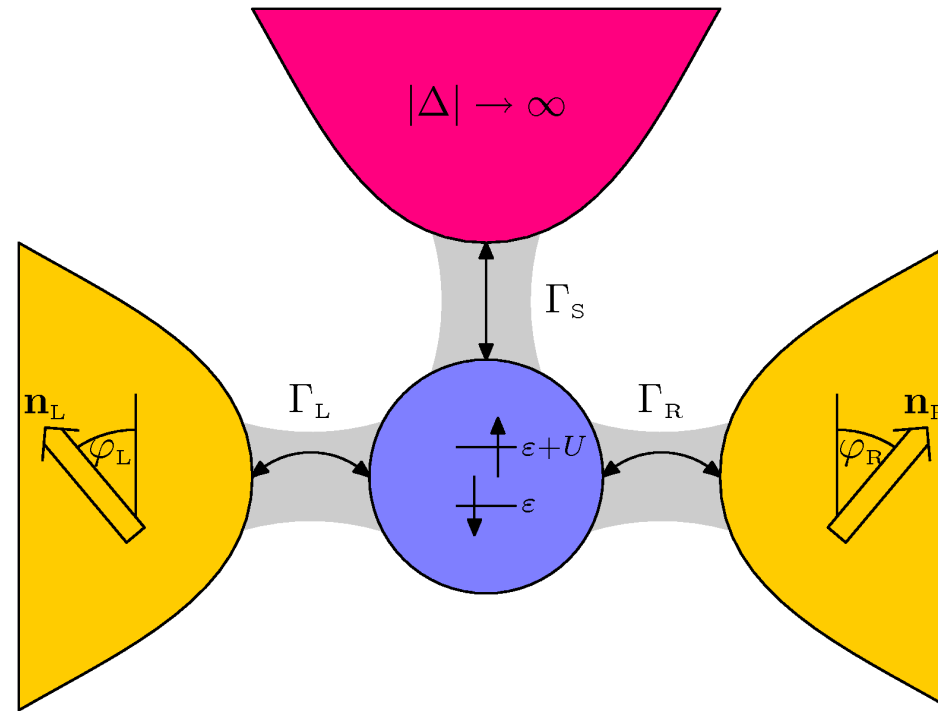
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Spintronic transport via the Andreev reflections

## QD spin-valve

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**Spintronic transport via the Andreev reflections**

*B. Sothmann, D. Futterer, M. Governale, J. König, Phys. Rev. B **82**, 094514 (2010).*

# Summary



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<http://kft.umcs.lublin.pl/doman/lectures>