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Quantum fluctuations of the ultracold atom-molecule mixtures

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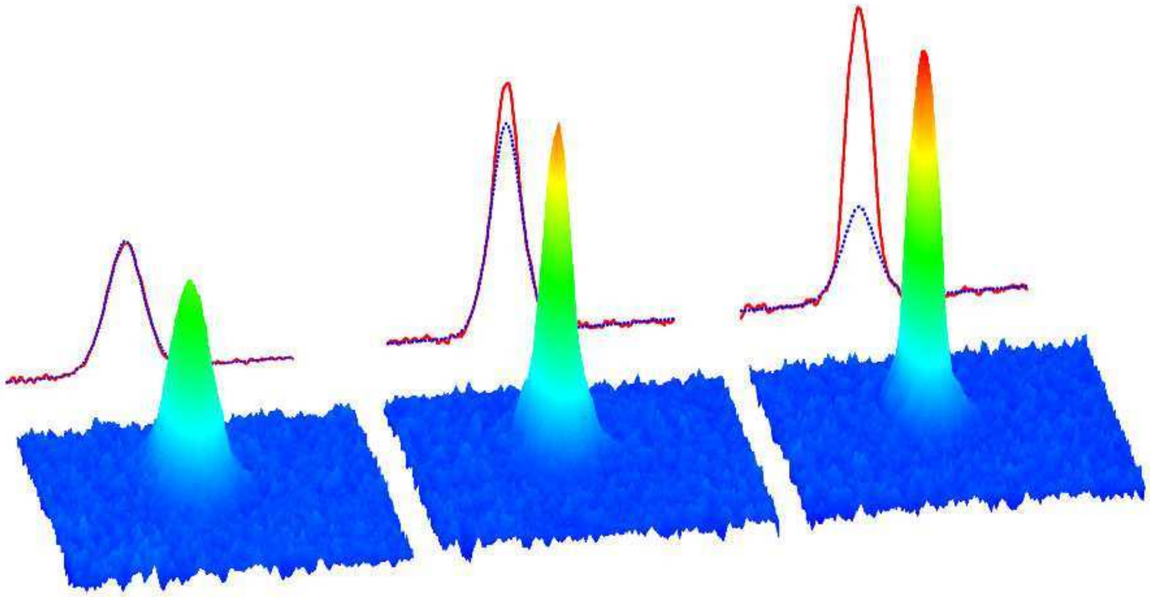

Superfluidity of the ultracold fermion atoms [1,2].

$T_c \sim 0.15\, T_F$ ($\sim$ *10 nK*)

[1] M.W. Zwierlein *et al*, Phys. Rev. Lett. **92**, 120403 (2004).

[2] C.A. Regal *et al*, Phys. Rev. Lett. **92**, 040403 (2004).

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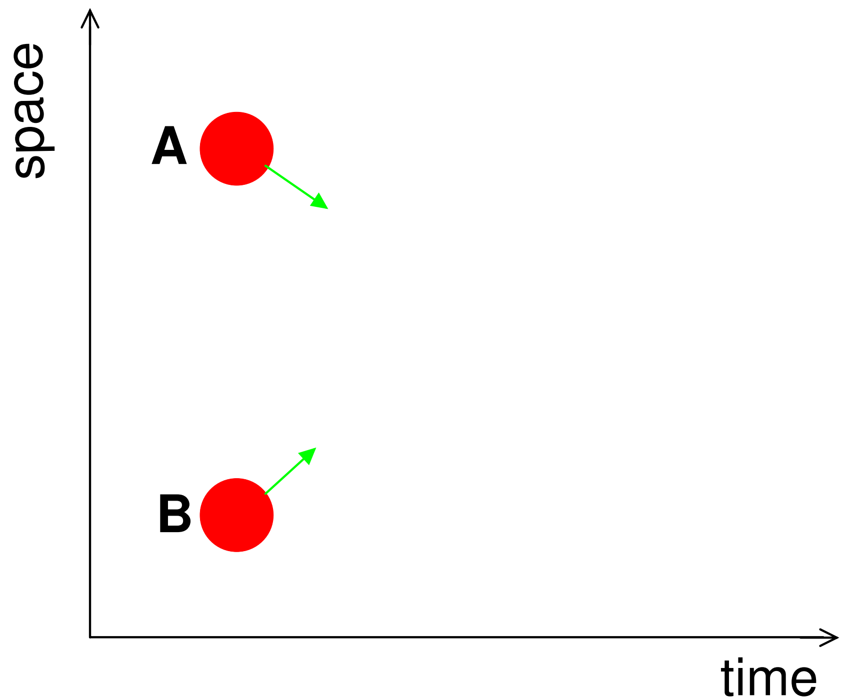
- **Fluctuations induced by a sweep**

  /atom-molecule oscillations, etc./

Let us consider two fermion atoms:



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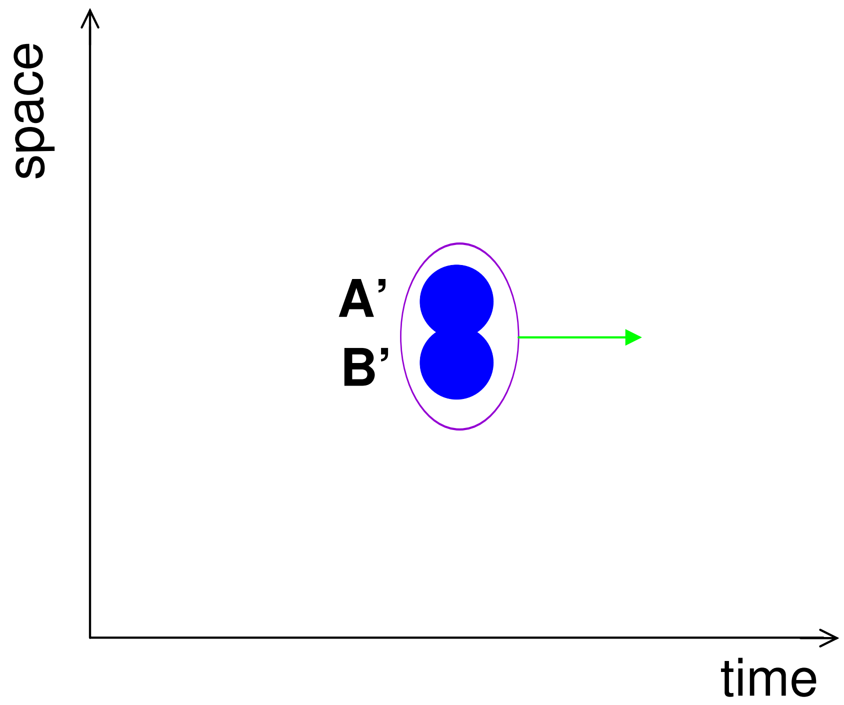

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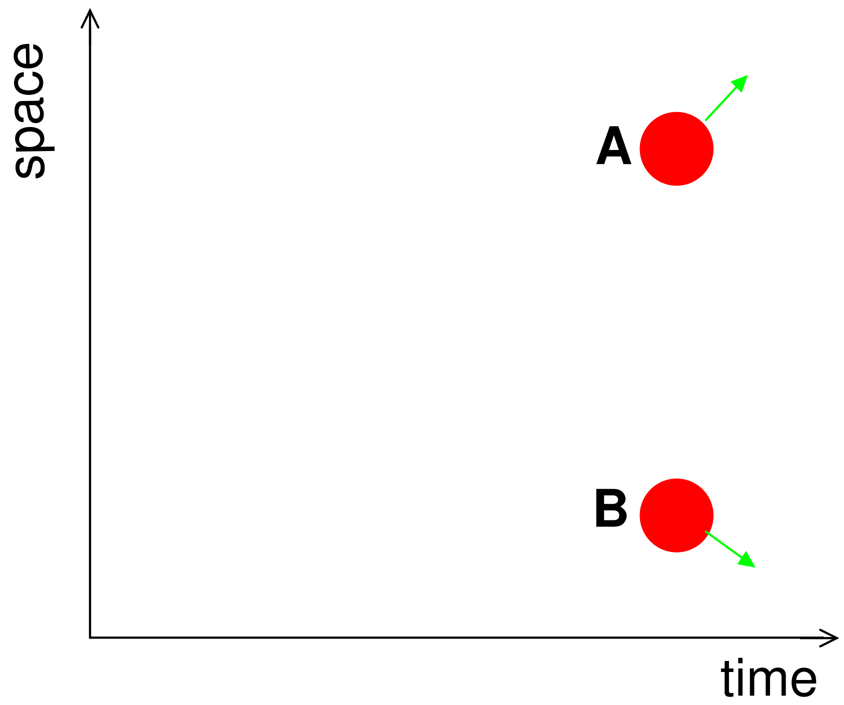
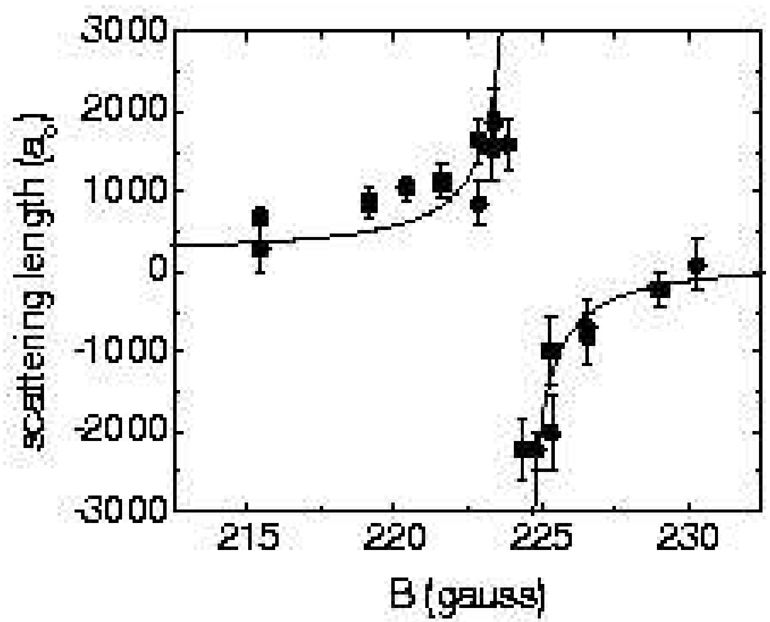


Effective scattering potential of these atoms depends on the external magnetic field and has a *resonant character*.



Example:

Feshbach resonance observed experimentally [3] for a mixture of ^{40}K atoms in two hyperfine states $\left|\frac{9}{2},-\frac{9}{2}\right\rangle$ and $\left|\frac{9}{2},-\frac{9}{2}\right\rangle$.

[3] C.A. Regal and D.S. Jin, Phys. Rev. Lett. **90**, 230404 (2003).

On a microscopic level this *resonant interaction* between atoms can be described by the following Hamiltonian [4]

$$H = \sum_{\mathbf{k},\sigma} \left(\frac{\hbar^2 \mathbf{k}^2}{2m} - \mu \right) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{4\pi\hbar^2\, a_{bg}}{m} \sum_{\mathbf{k},\mathbf{p},\mathbf{q}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}-\mathbf{k}\downarrow} c_{\mathbf{q}-\mathbf{p}\downarrow} c_{\mathbf{p}\uparrow}$$

$$+ g \sum_{\mathbf{k},\mathbf{q}} \left(b^\dagger_{\mathbf{q}} c_{\mathbf{k},\downarrow} c_{\mathbf{q}-\mathbf{k},\uparrow} + \mathit{h.c.} \right) + \sum_{\mathbf{q}} \left(\frac{\hbar^2 \mathbf{q}^2}{4m} + \delta - 2\mu \right) b^\dagger_{\mathbf{q}} b_{\mathbf{q}}$$

$c^{(\dagger)}_{\mathbf{k}\sigma}$	–	fermion atoms in two states $\sigma = \uparrow$ or $\sigma = \downarrow$
a_{bg}	–	background scattering length
g	–	atom molecule coupling
δ	–	detuning from the resonance
$b^{(\dagger)}_{\mathbf{q}}$	–	diatomic molecules (hard-core bosons)

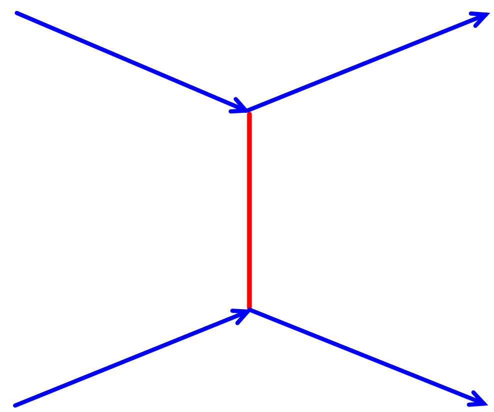
[4] E. Timmermans *et al*, Phys. Rep. **315**, 199 (1999); for a recent review see also R.A. Duine and H.T.C. Stoof, Phys. Rep. **396**, 115 (2004).

Detuning parameter depends on the applied magnetic field

$$\delta = \Delta\mu_{mag} \left(B - B_0\right)$$

so, within the lowest order perturbation theory [5] the effective scattering length becomes *resonant*

$$a = a_{bg} - \frac{g^2}{\delta} \; \frac{m}{4\pi\hbar^2}$$

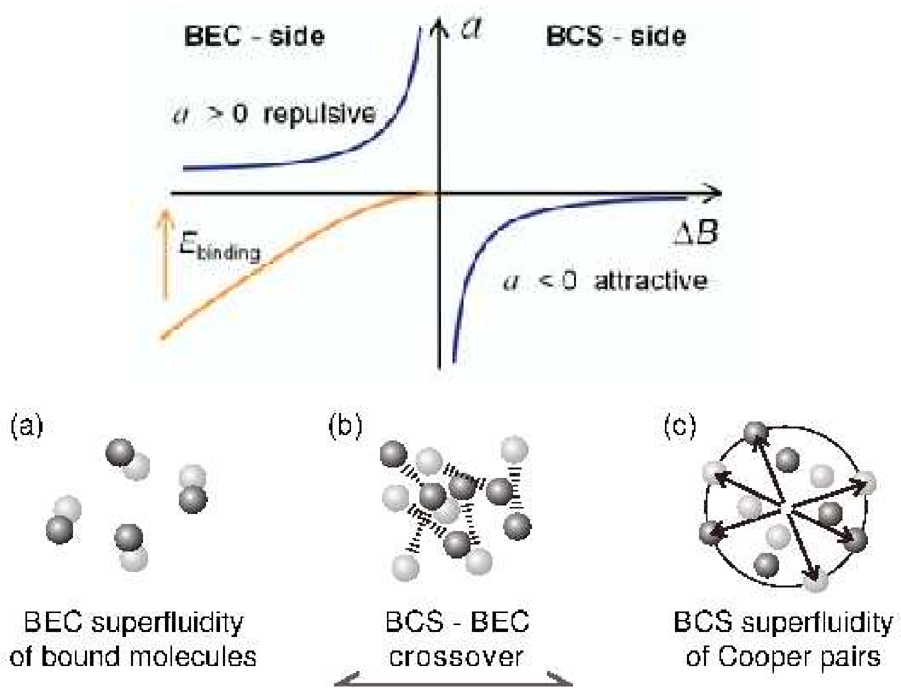


In the selfconsistent treatment this divergence gets smeared [6].

[5] M.W.J. Romans and H.T.C. Stoof, cond-mat/0506282.

[6] T. Domański, Phys. Rev. A **68**, 013603 (2003).

By changing the magnetic field experimentalists can switch between qualitatively different physical limits:



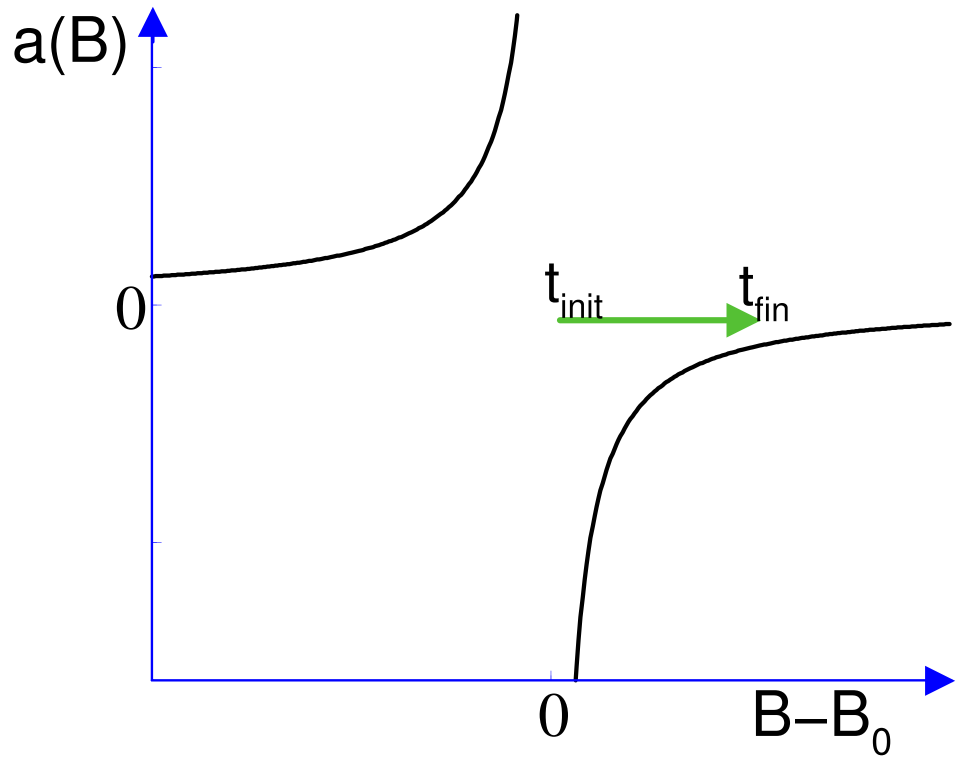
Switching from the BCS to BEC limit [7] and vice versa [8] has been done using various profiles of time-dependent sweeps.

[7] K.E. Strecker *et al*, Phys. Rev. Lett. **91**, 080406 (2003);

M. Greiner *et al*, Nature **426**, 537 (2003).

[8] M. Bartenstein *et al*, Phys. Rev. Lett. **92**, 203201 (2004).

Motivated by these experiments we address here the following situation:



★ *initially, at $t \leq 0$, the system is tuned exactly to the Feshbach resonance ($B = B_0$),*

★ *for $t > 0$ it is switched towards the BCS regime with the residual attractive scattering $a < 0$.*

For simplicity we neglect:

- the weak background scattering a_{bg},
- and omit the finite momentum molecular states.

In this *single mode approach* Hamiltonian reduces to [9,10]

$$H = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + E_{\mathbf{0}}(t) b^{\dagger}_{\mathbf{0}} b_{\mathbf{0}} + g \sum_{\mathbf{k}} \left(b^{\dagger}_{\mathbf{0}} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} + \textit{h.c.} \right)$$

where $\xi_{\mathbf{k}} = \frac{\hbar^2 \mathbf{k}^2}{2m} - \mu$ and $E_{\mathbf{q}}(t) = \frac{\hbar^2 \mathbf{q}^2}{2m} + \delta(t) - 2\mu$.

[9] A.V. Andreev, V. Gurarie, L. Radzihovsky, Phys. Rev. Lett. **93**, 130402 (2004).

[10] R.A. Barankov and L.S. Levitov, Phys. Rev. Lett. **93**, 130403 (2004).

For studying the time-dependent Hamiltonian it is convenient to use the pseudospin notation introduced by Anderson [11]

$$\begin{aligned} \sigma_{\mathbf{k}}^{+} &\equiv c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \\ \sigma_{\mathbf{k}}^{-} &\equiv c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger} \\ \sigma_{\mathbf{k}}^{z} &\equiv 1 - c_{\mathbf{k}\uparrow}^{\dagger} c_{\mathbf{k}\uparrow} - c_{\mathbf{k}\downarrow}^{\dagger} c_{\mathbf{k}\downarrow} \end{aligned}$$

such that

$$H = -\sum_{\mathbf{k}} \xi_{\mathbf{k}} \sigma_{\mathbf{k}}^{z} + g \sum_{\mathbf{k}} (b_{\mathbf{0}} \sigma_{\mathbf{k}}^{-} + b_{\mathbf{0}}^{\dagger} \sigma_{\mathbf{k}}^{+}) + E_{\mathbf{0}} b_{\mathbf{0}}^{\dagger} b_{\mathbf{0}}$$

Heisenberg equations of motion for the operators are [9,10]

$$\begin{aligned} i\hbar \frac{\partial \sigma_{\mathbf{k}}^{+}}{\partial t} &= 2\xi_{\mathbf{k}} \sigma_{\mathbf{k}}^{+} + g b_{\mathbf{0}} \sigma_{\mathbf{k}}^{z} \\ i\hbar \frac{\partial \sigma_{\mathbf{k}}^{z}}{\partial t} &= 2g \left(b_{\mathbf{0}}^{\dagger} \sigma_{\mathbf{k}}^{+} - b_{\mathbf{0}} \sigma_{\mathbf{k}}^{-} \right) \\ i\hbar \frac{\partial b_{\mathbf{0}}}{\partial t} &= E_{\mathbf{0}} b_{\mathbf{0}} + g \sum_{\mathbf{k}} \sigma_{\mathbf{k}}^{+} \end{aligned}$$

[11] P.W. Anderson, Phys. Rev. **112**, 1900 (1958).

We calculated numerically the time-dependent order parameters

$$b(t) = \langle b_{\mathbf{0}} \rangle$$
$$\chi(t) = \sum_{\mathbf{k}} \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle$$

which in the stationary case have been studied in detail for the boson-fermion model by R. Micnas et al [12].

We focused on the following processes:

- ⋆ **a sudden sweep [13],**
- ⋆ **a smooth detuning $\delta \propto t$ [14].**

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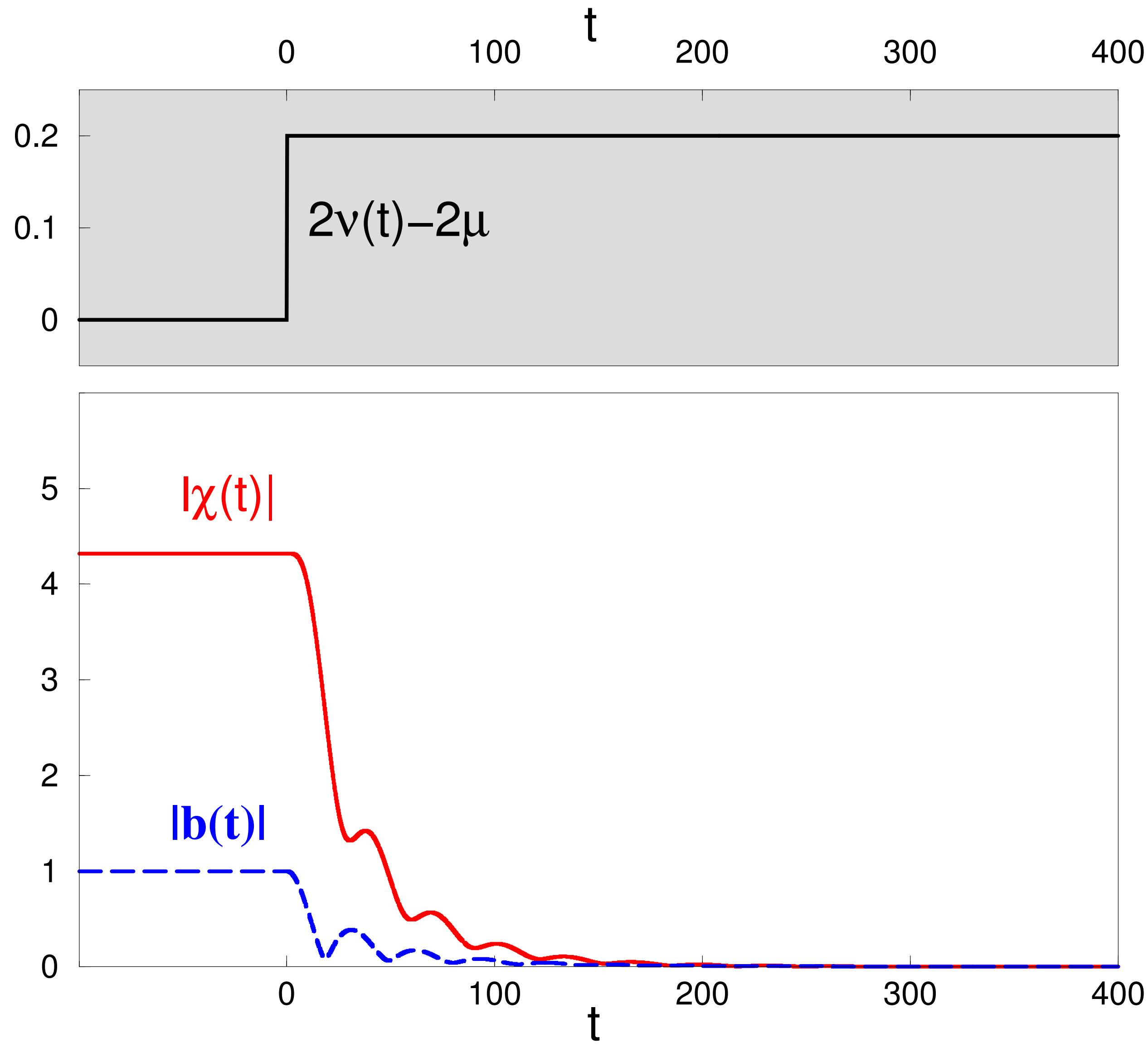
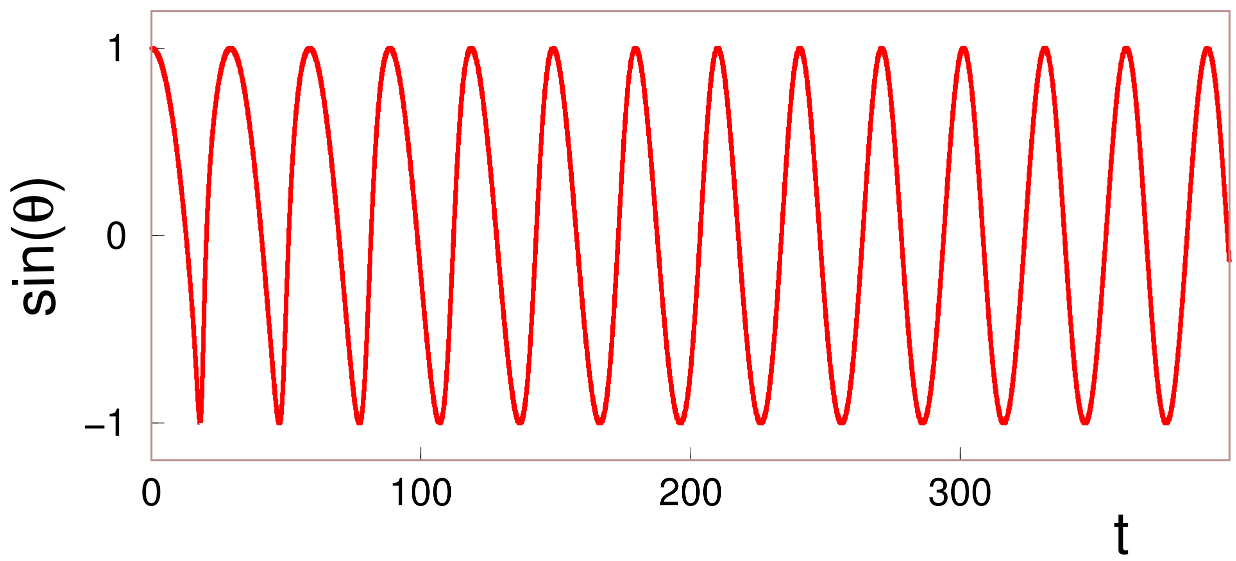
[12] R. Micnas, J. Ranninger, S. Robaszkiewicz, Rev. Mod. Phys. **62**, 113 (1990).

[13] M.H. Szymańska, B.D. Simons, K. Burnet, Phys. Rev. Lett. **94**, 170402 (2005).

[14] M. Haque, H.T.C. Stoof, Phys. Rev. A **71**, 063603 (2005).



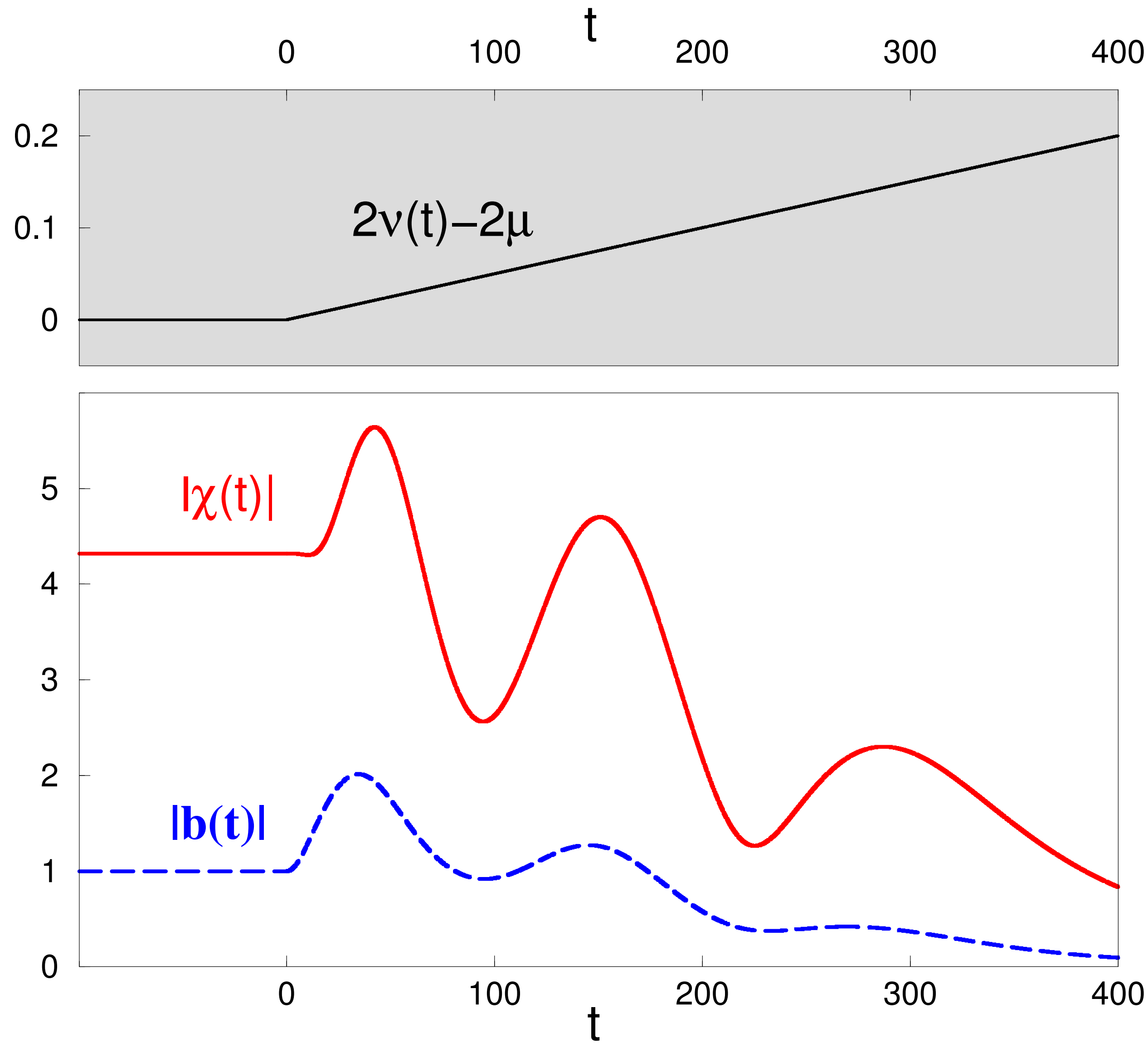
Evolution of the order parameters $\chi(t)$ and $b(t)$ caused by a <u>sudden</u> sweep across the Feshbach resonance



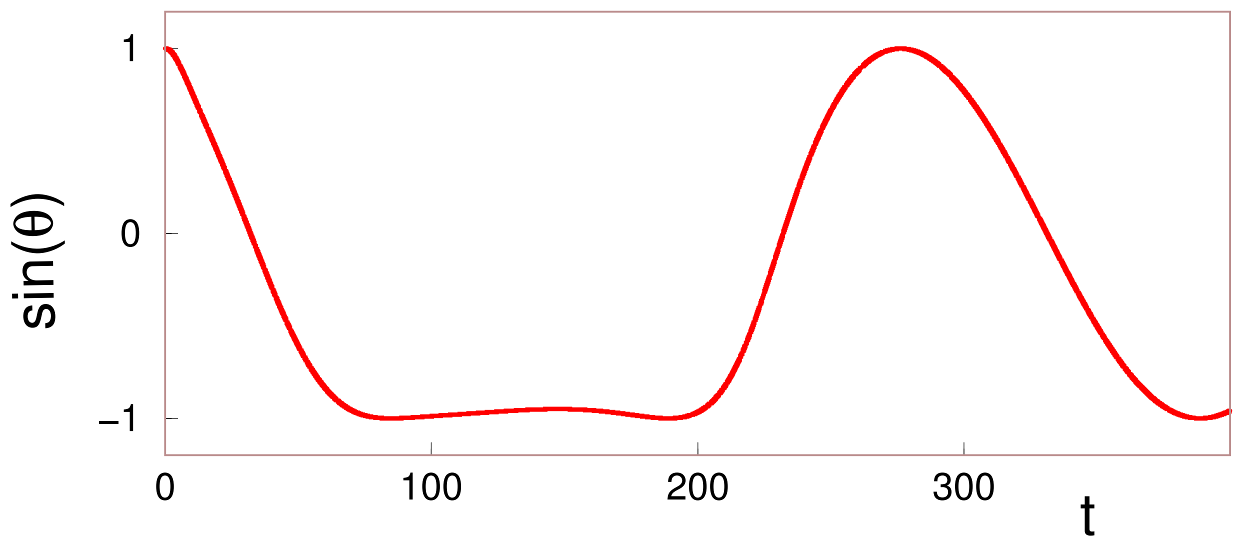
Phase variation of the complex order parameter

$$b(t) \ = \ |b(t)| \, e^{i\theta(t)}$$

in case of a <u>sudden</u> sweep.



Evolution of the order parameters $\chi(t)$ and $b(t)$ caused by a smooth sweep across the Feshbach resonance



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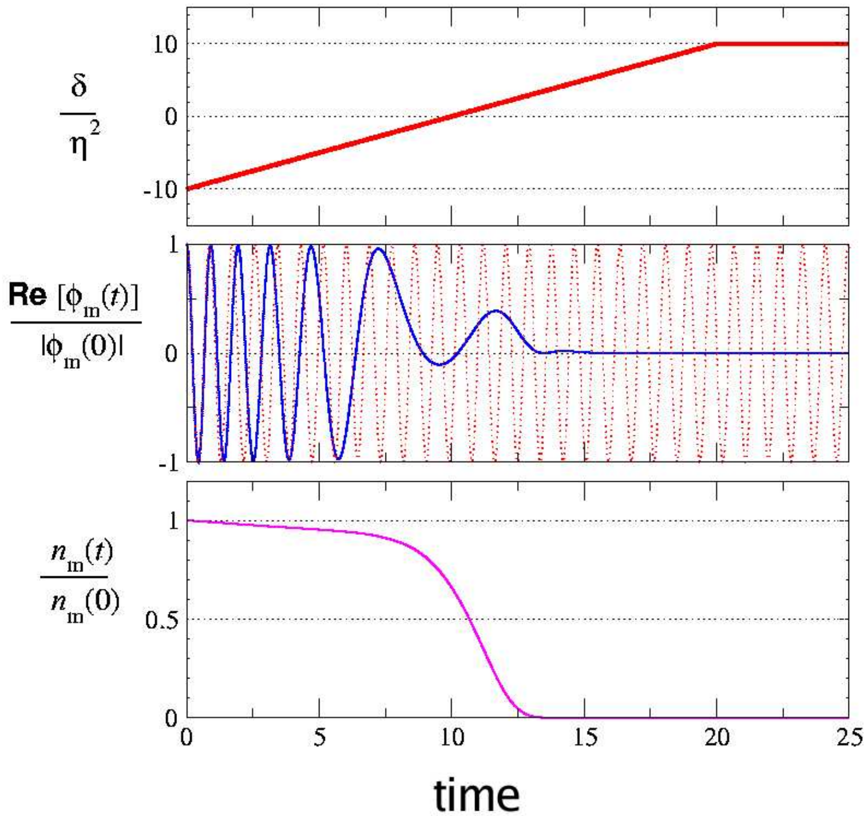

Decay of the boson order parameter upon a <u>smooth</u> switching between the BEC and BCS regions /results were obtained [15] on a basis of the generalized Gross–Pitaevskii equation/.

[15] M. Haque, H.T.C. Stoof, Phys. Rev. A **71**, 063603 (2005).

Conclusions

$\Rightarrow$ **Any (sudden or smooth) changes of the magnetic field lead to substantial fluctuations of the order parameters.**

- Amplitude fluctuations die out with a passing time.
- Phase fluctuations depend on a specific profile of the detuning process (can be regular or not).
- These fluctuations are acompanied by variations of the atom-molecule populations.

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