Resztkowy efekt Meissnera powyżej $T_c$
w nadprzewodnikach wysokotemperaturowych

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Residual Meissner effect above $T_c$ in high-temperature superconductors

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Torque magnetometry evidence for diamagnetism above $T_c$
Torque magnetometry

evidence for diamagnetism above $T_c$

A few details:

**Used samples:**

- $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$
- $\text{Bi}_2\text{Sr}_{2-y}\text{La}_y\text{CuO}_6$
- $\text{Bi}_2\text{Sr}_2\text{Ca}_y\text{Cu}_2\text{O}_{8+\delta}$

**Methodology:**

Torque magnetisation was measured using the samples glued to the tip of a thin cantilever with $\vec{H}$ applied at a tilt angle $\theta = 10^0 - 15^0$ to the crystal $c$ axis.

**Who/where/when:**

L. Li, ... and N.P. Ong, Phys. Rev. B 81, 054510 (2010).
Princeton + Beijing + Tokyo + Ibaraki + Brookhaven
Torque magnetometry evidence for diamagnetism above $T_c$

$M_d(H) = M_{\text{eff}} - M_p$
where

$M_p = (A + BT)H$

$T_c = 24 K$

Magnetisation of La$_{2-x}$Sr$_x$CuO$_4$ sample with $x = 0.09$ content of Sr.
Torque magnetometry
evidence for diamagnetism above $T_c$

$M_d(H)$ measured in the strong magnetic field up to 33 T.

$T_c = 24$K

Magnetisation of La$_{2-x}$Sr$_x$CuO$_4$ sample with $x = 0.09$ content of Sr.
Torque magnetometry evidence for diamagnetism above $T_c$

Enhanced diamagnetism of Bi$_2$Sr$_{2-y}$La$_y$CuO$_6$ samples with $y = 0.44$ content of La atoms.

Van Vleck background $(A + BT)H$ is subtracted

$T_c = 30K$
Torque magnetometry evidence for diamagnetism above $T_c$. 

Magnetisation of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ and $\text{Bi}_{2}\text{Sr}_{2-y}\text{La}_y\text{CuO}_6$ samples, showing the onset of doping dependent diamagnetism above $T_C$. 
Torque magnetometry evidence for diamagnetism above $T_c$

\[ \begin{align*}
T^M & \iff \text{onset of the residual diamagnetism} \\
T_c & \iff \text{transition to superconducting state}
\end{align*} \]

L. Li, ... and N.P. Ong, Phys. Rev. B 81, 054510 (2010).
Issues to be addressed:
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Why does $T^M$ differ from $T_c$?
/ pairing vs coherence /
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- Why does $T^M$ differ from $T_c$?  
  / pairing vs coherence / 

- Meissner without Higgs mechanism?
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⇒ Diamagnetism above $T_c$
Issues to be addressed:

- **Why does $T^M$ differ from $T_c$?**
  
  / pairing vs coherence /

- **Meissner without Higgs mechanism?**
  
  - Diamagnetism above $T_c$
  
  $\Rightarrow$ Enhanced conductivity above $T_c$
Issues to be addressed:

- Why does $T^M$ differ from $T_c$?
  / pairing vs coherence /

- Meissner without Higgs mechanism?
  - Diamagnetism above $T_c$
  - Enhanced conductivity above $T_c$

- General remarks
Superconducting state – principal features
Superconducting state – principal features

Ideal d.c. conductance

![Graph showing YBa$_2$Cu$_3$O$_7$ with $T_c = 92K$.](image)
Superconducting state – principal features

ideal d.c. conductance

Normal conductors:

resistance \( R = \rho \frac{l}{S} \)

where \( \rho \equiv \frac{1}{\sigma} \)

and \( \sigma = \frac{ne^2}{m} \tau(T) \)

\( \tau(T) \) – relaxation time
Superconducting state – principal features

**ideal d.c. conductance**

Drude conductance

\[ \sigma(\omega) = \frac{ne^2\tau}{m} \frac{1}{1 - i\omega\tau} \]

obeys the f-sum rule

\[ \int_{-\infty}^{\infty} \text{Re} \sigma(\omega) = \pi \frac{ne^2}{m} \]

**a.c. conductance**

YBa_{2}Cu_{3}O_{7}

T_{C} = 92K
Superconducting state – principal features

**ideal d.c. conductance**

This f-sum rule

\[ \int_{-\infty}^{\infty} \text{Re} \sigma(\omega) = \pi \frac{ne^2}{m} \]

must be obeyed also below \( T_c \), where

\[ n = n_n + n_s \]

\( n_s \) – superfluid density
Superconducting state – properties (continued)
Superconducting state – properties (continued)

Ideal diamagnetism
/perefect screening of d.c. magnetic field/

\[ T > T_c \quad \rightarrow \quad T < T_c \]
Superconducting state – properties (continued)

⋆ Ideal diamagnetism /perfect screening of d.c. magnetic field/

Meissner effect is described by the London’s equation

\[ \vec{j} = - \frac{e^2 n_s(T)}{mc^2} \vec{A} \]

where the coefficient

\[ \frac{e^2 n_s(T)}{mc^2} \equiv \rho_s(T) = \frac{1}{\lambda^2} \]

\( \rho_s(T) – \text{superfluid stiffness} \)

\( \lambda(T) – \text{penetration depth} \)

\[ B(x) = B_0 e^{-x/\lambda} \]
Superconducting state – basic concepts
Superconducting state – basic concepts

→ ideal d.c. conductance
Superconducting state – basic concepts

⇒ ideal d.c. conductance

⇒ ideal diamagnetism (Meissner effect)
Superconducting state – basic concepts

⇒ ideal d.c. conductance

⇒ ideal diamagnetism (Meissner effect)

can be regarded as two sides of the same coin.
Superconducting state – basic concepts

⇒ ideal d.c. conductance

⇒ ideal diamagnetism (Meissner effect)

can be regarded as two sides of the same coin.

Both effects are caused by the superfluid fraction

\[ n_s(T) \]
Formal issues
Formal issues

The order parameter

\[ \chi \equiv \langle \hat{c}_{\downarrow}(\vec{r}_i)\hat{c}_{\uparrow}(\vec{r}_j) \rangle \]
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is a complex quantity

\[ \chi = |\chi| e^{i\theta} \]
Formal issues

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It has the following physical implications:
Formal issues

The order parameter

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It has the following physical implications:

$$|\chi| \neq 0 \rightarrow \text{amplitude causes the energy gap}$$
Formal issues

The order parameter

\[ \chi \equiv \langle \hat{c}_\downarrow(\vec{r}_i)\hat{c}_\uparrow(\vec{r}_j) \rangle \]

is a complex quantity

\[ \chi = |\chi| e^{i\theta} \]

It has the following physical implications:

- \(|\chi| \neq 0 \rightarrow \text{amplitude causes the energy gap}\)
- \(\nabla \theta \neq 0 \rightarrow \text{phase slippage induces supercurrents}\)
Critical temperature – amplitude vs phase transition
Critical temperature – amplitude vs phase transition

The complex order parameter

\[ \chi = |\chi| e^{i\theta} \]
The complex order parameter

$$\chi = |\chi| \, e^{i\theta}$$

can vanish at $T \to T_c$ by:
Critical temperature – amplitude vs phase transition

The complex order parameter

$$\chi = |\chi| e^{i\theta}$$

can vanish at $T \to T_c$ by:

1. closing the gap ........................................... [conventional BCS superconductors]

$$\lim_{T \to T_c} |\chi| = 0$$
Critical temperature – amplitude vs phase transition

The complex order parameter

\[ \chi = |\chi| e^{i\theta} \]

can vanish at \( T \to T_c \) by:

1. closing the gap ........................................... [conventional BCS superconductors ]

\[ \lim_{T \to T_c} |\chi| = 0 \]

2. randomizing the phase ..................... [ HTSC & disordered thin superconductors ]

\[ \lim_{T \to T_c} \langle \theta \rangle = 0 \]
Amplitude vs phase driven transition

scenario # 1
Amplitude vs phase driven transition

$\Rightarrow \text{amplitude transition / classical superconductors /}$

$k_B T_c \simeq \frac{\Delta(0)}{1.76}$
Amplitude vs phase driven transition

\[ k_B T_c \simeq \frac{\Delta(0)}{1.76} \]

Amplitude transition / classical superconductors /

Electrons’ pairing is responsible for the energy gap \( \Delta(T) \) in a single particle spectrum

\[ \Delta(T_c) = 0 \]

Appearance of the electron pairs is simultaneous with onset of their coherence.
Amplitude vs phase driven transition

scenario # 2
Amplitude vs phase driven transition scenario # 2

\[ T_c \sim \Delta(0) \]

\[ \Rightarrow \text{phase-driven transition / high } T_c \text{ cuprate oxides /} \]
Amplitude vs phase driven transition

\[ \Rightarrow \text{phase-driven transition} \quad / \text{high } T_c \text{ cuprate oxides} / \]

\[ T_c \sim \Delta(0) \]

Early experiments using the muon-spin relaxation indicated that in HTSC

\[ T_c \propto \rho_s(0) \]

/ Uemura scaling /

The superfluid stiffness \( \rho_s(T) \) is here defined by

\[ \rho_s(T) \equiv \frac{1}{\chi^2(T)} = \frac{4\pi e^2}{m^* c^2 n_s(T)} \]

Recently such scaling has been updated from transport measurements

$$\frac{1}{8} \rho_s = 4.4 \sigma_{dc} T_c$$

This new relation is valid for all samples ranging from the underdoped to overdoped region.

Amplitude vs phase driven transition

⇒ phase-driven transition / high $T_c$ cuprate oxides /

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This new relation is valid for all samples ranging from the underdoped to overdoped region.

Electron 'pre-pairing'

/experimental evidence/
Incoherent pairs above $T_c$

Experimental fact #1

Phase slippage detected in the large Nernst effect.

Incoherent pairs above $T_c$

Experimental fact # 1

Phase slippage detected in the large Nernst effect.

Incoherent pairs above $T_c$

**experimental fact #2**

Enhanced diamagnetic response revealed above $T_c$ by the high precision torque magnetometry.

$L. Li, \ldots \text{ and N.P. Ong, Phys. Rev. B} \ 81, \ 054510 (2010)$. / Princeton University /
Incoherent pairs above $T_c$

**Experimental fact #3**

Magnetization measurements for the needle shape La$_{2-x}$Sr$_x$CuO$_4$ single crystals.

Incoherent pairs above $T_c$

experimental fact # 4

Incoherent pairs above $T_c$  

Experimental fact # 5

Josephson current in YBaCuO - LaSrCuO - YBaCuO junction with LaSrCuO being in the pseudogap state well above $T_c$

*T. Kirzhner and G. Koren, Scientific Reports 4, 6244 (2014).*
Incoherent pairs above $T_c$

**Experimental fact # 6**

Pseudogap induced well above $T_c$ in ultrathin metallic slab deposited on $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$.

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**STM tip**

- 90 nm LSCO x=0.10

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**STM image**

- 600 nm

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**Figure b)**

- $R(\Omega)$ vs. $T\text{ (K)}$

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**Figure c)**

- Thin normal metal

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**Figure a)**

- STM tip

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Onset of the superfluid fraction observed in the c-axis optical measurements Re$\sigma_c(\omega)$. 

Incoherent pairs above $T_c$ 

$\Rightarrow$ Bogoliubov quasiparticles detected above $T_c$ in ARPES measurements for YBaCuO and LaSrCuO compounds


$\Rightarrow$ Bogoliubov-type interference patterns in pseudogap state as the *fingerprint* of phase incoherent d-wave superconductivity preserved up to $1.5T_c$

Diamagnetism

and its origin above $T_c$
Phenomenological model

\[ \hat{H} = \sum_{k,\sigma} \xi_k \hat{c}_{k\sigma}^\dagger \hat{c}_{k\sigma} + \sum_{q} E_q \hat{b}_q^\dagger \hat{b}_q 
+ \sum_{k,p} g_{k,p} \left( \hat{b}_{k+p}^\dagger \hat{c}_{k\downarrow} \hat{c}_{p\uparrow} + \hat{b}_{k+p} \hat{c}_{k\uparrow}^\dagger \hat{c}_{p\downarrow}^\dagger \right) \]

This Hamiltonian describes a two-component system consisting of:

\[ \hat{c}_{k\sigma}^{(\dagger)} \] itinerant fermions ........................................ (e.g. holes near the Mott insulator)
Phenomenological model

\[ \hat{H} = \sum_{k,\sigma} \xi_k \hat{c}_{k\sigma}^\dagger \hat{c}_{k\sigma} + \sum_q E_q \hat{b}_q^\dagger \hat{b}_q + \sum_{k,p} g_{k,p} \left( \hat{b}_{k+p}^\dagger \hat{c}_k^\dagger \hat{c}_p^\uparrow + \hat{b}_{k+p} \hat{c}_k^\dagger \hat{c}_p^\dagger \right) \]

This Hamiltonian describes a two-component system consisting of:

\( \hat{c}_{k\sigma}^{(\dagger)} \) itinerant fermions ................................................... (e.g. holes near the Mott insulator)

\( \hat{b}_q^{(\dagger)} \) preformed local pairs ......................................(RVB defines them on the bonds)
Phenomenological model

\[
\hat{H} = \sum_{k,\sigma} \xi_k \hat{c}^\dagger_{k\sigma} \hat{c}_{k\sigma} + \sum_q E_q \hat{b}^\dagger_q \hat{b}_q \\
+ \sum_{k,p} g_{k,p} \left( \hat{b}^\dagger_{k+p} \hat{c}_k \hat{c}_p \uparrow + \hat{b}_{k+p} \hat{c}^\dagger_k \hat{c}^\dagger_p \right)
\]

This Hamiltonian describes a two-component system consisting of:

\[\hat{c}^{(\dagger)}_{k\sigma}\] itinerant fermions ......................................................... (e.g. holes near the Mott insulator)

\[\hat{b}^{(\dagger)}_q\] preformed local pairs .............................................. (RVB defines them on the bonds)

interacting via:
This Hamiltonian describes a two-component system consisting of:

\[ \hat{H} = \sum_{k, \sigma} \xi_k \hat{c}_{k \sigma}^{\dagger} \hat{c}_{k \sigma} + \sum_{q} E_q \hat{b}_{q}^{\dagger} \hat{b}_{q} \]

\[ + \sum_{k, p} g_{k, p} \left( \hat{b}_{k+p}^{\dagger} \hat{c}_{k}^{\downarrow} \hat{c}_{p}^{\uparrow} + \hat{b}_{k+p} \hat{c}_{k}^{\uparrow} \hat{c}_{p}^{\downarrow} \right) \]

- Itinerant fermions \((\hat{c}_{k \sigma}^{\dagger})\) (e.g. holes near the Mott insulator)
- Preformed local pairs \((\hat{b}_{q}^{\dagger})\) (RVB defines them on the bonds)

interacting via:

\[ \hat{b}_{q}^{\dagger} \hat{c}_{q-k, \downarrow} \hat{c}_{k, \uparrow} + h.c. \] (the Andreev-type scattering)
Response to the external fields

Within the Kubo formalism a response to the electromagnetic field is characterized by the current-current correlation function

\[- \hat{T}_\tau \langle \hat{j}_q(\tau) \hat{j}_{-q}(0) \rangle\]

where

\[\langle \ldots \rangle = \text{Tr} \left\{ e^{-\beta \hat{H}} \ldots \right\} / \text{Tr} \left\{ e^{-\beta \hat{H}} \right\}\]

and \(\beta = 1/k_B T\).

We have diagonalized Hamiltonian by the continuous unitary transformation

\[\text{Tr} \left\{ e^{-\beta \hat{H}} \hat{O} \right\} = \text{Tr} \left\{ e^{\hat{S}(l)} e^{-\beta \hat{H}} \hat{O} e^{-\hat{S}(l)} \right\}\]

\[= \text{Tr} \left\{ e^{\hat{S}(l)} e^{-\beta \hat{H}} e^{-\hat{S}(l)} e^{\hat{S}(l)} \hat{O} e^{-\hat{S}(l)} \right\}\]

\[= \text{Tr} \left\{ e^{-\beta \hat{H}(l)} \hat{O}(l) \right\}\]

where

\[\hat{H}(l) = e^{\hat{S}(l)} \hat{H} e^{-\hat{S}(l)}\]

\[\hat{O}(l) = e^{\hat{S}(l)} \hat{O} e^{-\hat{S}(l)}\]
The initial current operator

\[ \hat{j}_{q,\sigma} = \sum_k v_{k+\frac{q}{2}} \hat{c}_{k,\sigma}^\dagger \hat{c}_{k+q,\sigma} \]

has been found (from the flow equation) as

\[ \hat{j}_{q,\uparrow} (l) = \sum_k v_{k+\frac{q}{2}} \left( A_{k,q}(l) \hat{c}_{k,\uparrow}^\dagger \hat{c}_{k+q,\uparrow} + B_{k,q}(l) \hat{c}_{-k,\downarrow} \hat{c}_{-(k+q),\downarrow} \right. \\
+ \left. \sum_p \left( D_{k,p,q}(l) \hat{b}_{k+p} \hat{c}_{k,\uparrow}^\dagger \hat{c}_{p-q,\downarrow} + F_{k,p,q}(l) \hat{b}_{k+p} \hat{c}_{p,\downarrow} \hat{c}_{k+q,\uparrow} \right) \right) \]

with the boundary conditions

\[ A_{k,q}(0) = 1 \quad \text{and} \quad B_{k,q}(0) = D_{k,p,q}(0) = F_{k,p,q}(0) = 0 \]

We next determined the asymptotic values \( \lim_{l \to \infty} A_{k,q}(l) \equiv \tilde{A}_{k,q} \)

from the coupled set of flow equations

\[ \frac{\partial}{\partial l} A_{k,q}(l) , \quad \frac{\partial}{\partial l} B_{k,q}(l) , \quad \frac{\partial}{\partial l} D_{k,p,q}(l) , \quad \frac{\partial}{\partial l} F_{k,p,q}(l) . \]
Technicalities

The current-current response function

\[- \hat{T}_\tau \langle \hat{j}_q(\tau) \hat{j}_{-q} \rangle \equiv \Pi(q, \tau)\]

consists the following contributions

\[\begin{align*}
&\Rightarrow \text{the usual bubble diagram} \\
&\text{where each vertex must be determined from the flow equations.}
\end{align*}\]
Diamagnetic response above $T_c$

The anomalous contributions

resemble the Aslamasov-Larkin diagram

They substantially enhance the conductance/diamagnetism above $T_c$. 
Onset of diamagnetism above $T_c$
Onset of diamagnetism above $T_c$

Onset of the diamagnetism is driven by the low-momentum preformed pairs.
Onset of diamagnetism above $T_c$

Onset of the diamagnetism is driven by the low-momentum preformed pairs.

Qualitative changes of the preformed pairs’ dispersion.
Onset of diamagnetism above $T_c$

Onset of the diamagnetism is driven by the low-momentum preformed pairs.

Effective mass $m_B$ of the preformed pairs

Onset of the diamagnetism is driven by the low-momentum preformed pairs.


Onset of diamagnetism above $T_c$

Onset of the diamagnetism is driven by the low-momentum preformed pairs.

$J_x(q \to 0, 0) = -\frac{e^2}{m} n_s(T) A_x(q \to 0, 0)$


Dynamic conductivity above $T_c$
Dynamic conductivity above $T_c$

Distribution of the low-momentum preformed pairs

$n^B_q$ vs. $q_xa$ for different $T/D$ values: $0.02$, $0.05$, and $0.10$. The graph shows the distribution of low-momentum preformed pairs for each $T/D$ value.
Dynamic conductivity above $T_c$

Qualitative effects: optical gap + Drude peak

causd by: gaped spectrum + accumulation of low-momentum bosons
The d.c. resistivity of Bi2223 cuprate superconductors.

The fluctuation conductivity appears already at $T^* \approx 2.2T_c$. 

Dynamic conductivity above $T_c$
Dynamic conductivity above $T_c$

Summary

- Experimental evidence for the pre-existing pairs above $T_c$
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  $\Rightarrow$ residual diamagnetism

  / torque magnetometry, magnetization of needle samples /
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  / enhanced d.c. conductivity, Drude peak in a.c. conductivity/
Summary

- Experimental evidence for the pre-existing pairs above $T_c$

  $\Rightarrow$ **residual diamagnetism**
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  $\Rightarrow$ **fluctuation conductivity**
  / enhanced d.c. conductivity, Drude peak in a.c. conductivity/

  $\Rightarrow$ **Bogoliubov quasiparticles**
  / ARPES, Josephson effect, FT-STM /
Summary

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  \[ \Rightarrow \text{residual diamagnetism} \]
  \[ / \text{torque magnetometry, magnetization of needle samples} / \]

  \[ \Rightarrow \text{fluctuation conductivity} \]
  \[ / \text{enhanced d.c. conductivity, Drude peak in a.c. conductivity/} \]

  \[ \Rightarrow \text{Bogoliubov quasiparticles} \]
  \[ / \text{ARPES, Josephson effect, FT-STM} / \]

- These superconducting-like features are due to:
Experimental evidence for the pre-existing pairs above $T_c$

- Residual diamagnetism
  - Torque magnetometry, magnetization of needle samples

- Fluctuation conductivity
  - Enhanced d.c. conductivity, Drude peak in a.c. conductivity

- Bogoliubov quasiparticles
  - ARPES, Josephson effect, FT-STM

These superconducting-like features are due to:

- The short-range correlations between the pairs