NCBJ Wa-wa, 23 XI 2016 r.

Realizacja fermionów Majorany

w heterostrukturach nadprzewodnikowych

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$\Longleftrightarrow$

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Nobel 2016

E. Majorana

Tematyka wykładu

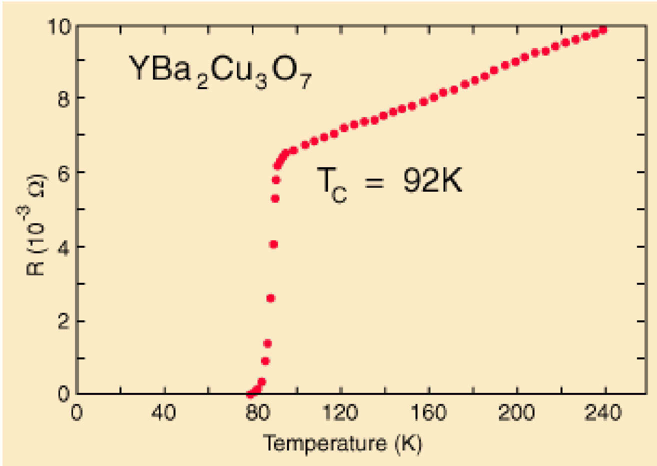
Egzotyka nadprzewodnictwa:

$\Rightarrow$ **rola wymiarowości (Nobel 2016)**

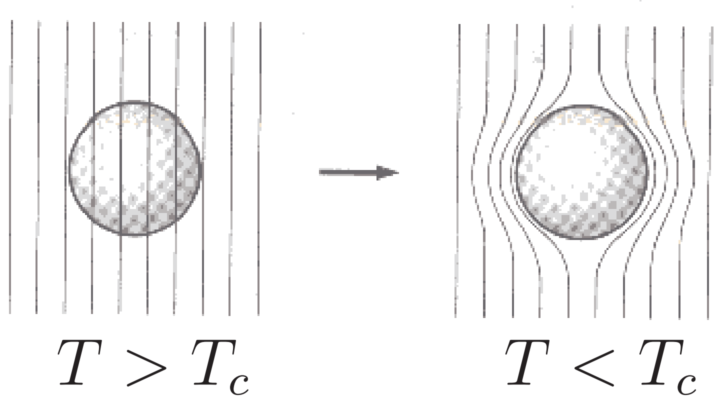
$\Rightarrow$ **rola topologii (kwazicząstki Majorany)**

Superconducting state – basic concepts

★ **ideal d.c. conductance**



★ **ideal diamagnetism** /perfect screening of d.c. magnetic field/



Superconducting state – basic concepts

⇒ ideal d.c. conductance

⇒ ideal diamagnetism (Meissner effect)

are caused by the superfluid electron pairs

$$n_s(T)$$

Formal issues

The order parameter

$$\chi \equiv \langle \hat{c}_{\sigma_1}(\vec{r}_1)\hat{c}_{\sigma_2}(\vec{r}_2)\rangle$$

is a complex quantity

$$\chi = |\chi|\, e^{i\theta}$$

which has the following implications:

Formal issues

The order parameter

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is a complex quantity

$$\chi = |\chi| \, e^{i\theta}$$

which has the following implications:

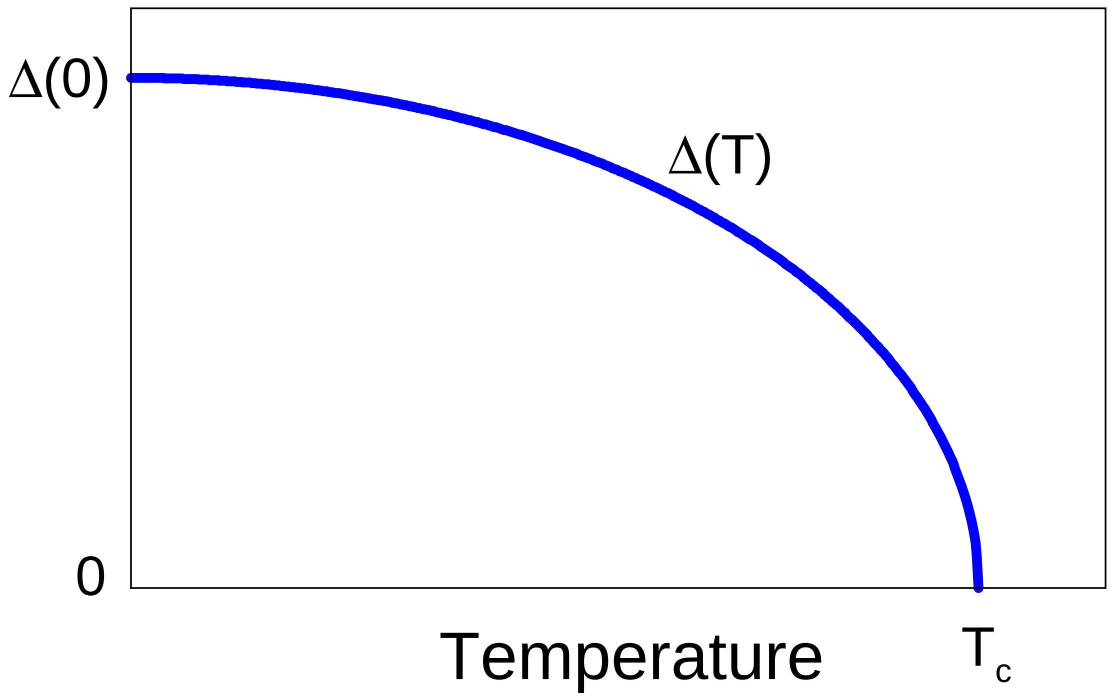
$|\chi| \neq 0 \longrightarrow$ **amplitude causes the energy gap**

$\nabla\theta \neq 0 \longrightarrow$ **phase slippage induces supercurrents**

Amplitude driven transition

BCS scenario

⇒ **classical superconductors** $k_B\,T_c \simeq \frac{\Delta(0)}{1.76}$



Electrons' pairing is responsible for the energy gap $\Delta(T)$ *in a single particle spectrum*

$$\lim_{|r_1-r_2|\to\infty}\langle e^{i(\theta(r_1)-\theta(r_2))}\rangle \neq 0$$

long-range coherence (below T_c)

Lower dimensions – problems

- N.D. Mermin & H. Wagner, Phys. Rev. Lett. 17, 1133 (1966)
- F. Wegner, Zeitschrift für Physik 206, 465 (1967)

In dimensions

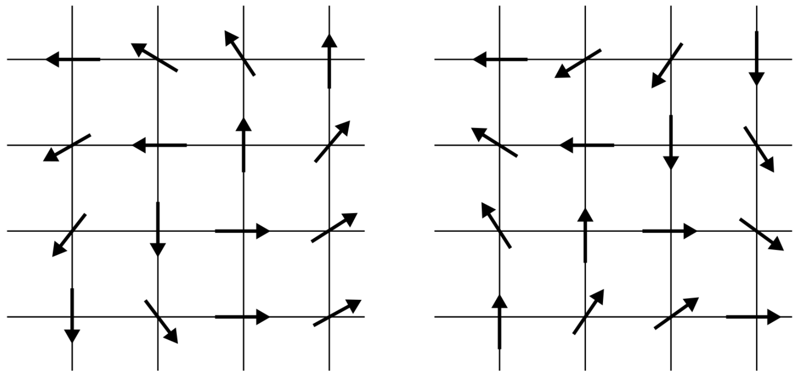
$$dim \leq 2$$

any long-range coherence is absent !

Topology – role of vortices

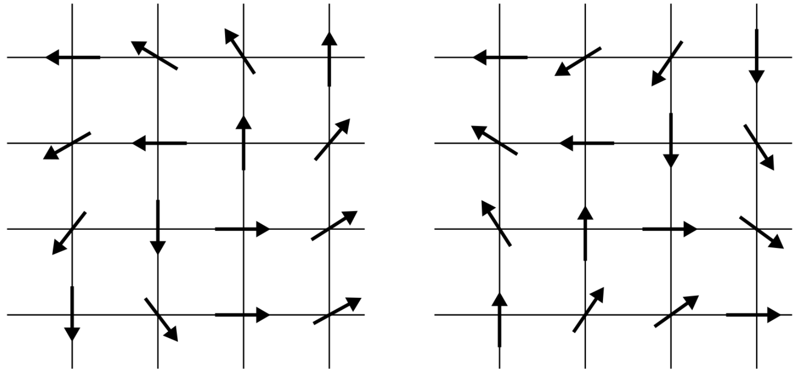
Topology – role of vortices

★ **vortex and antivortex**

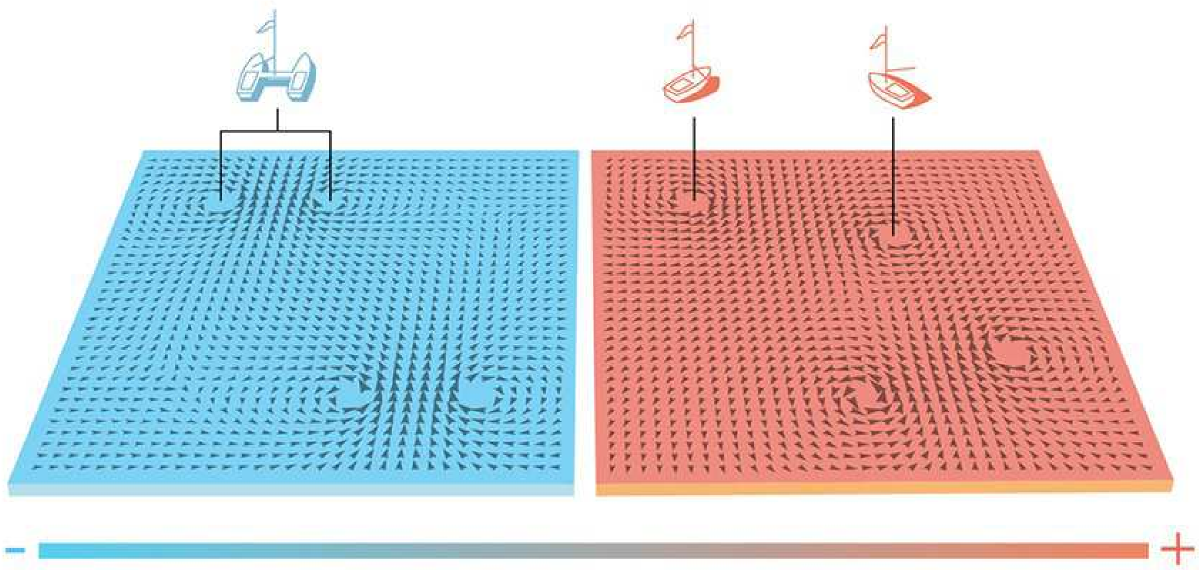


Topology – role of vortices

- vortex and antivortex



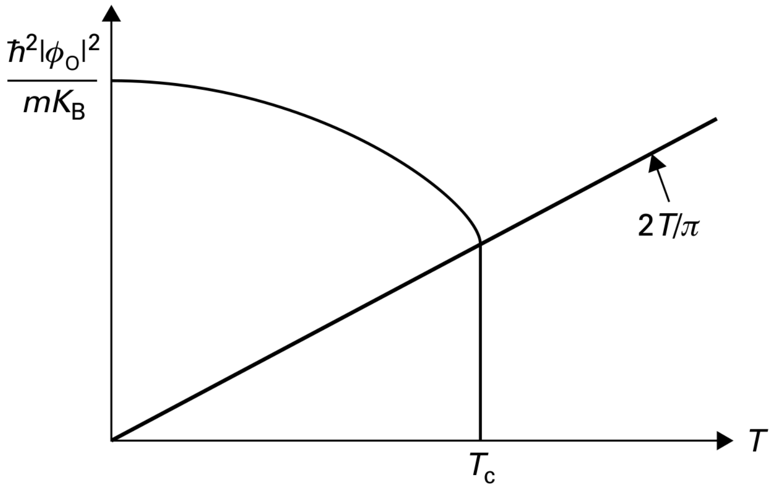
- ordering of vortices



$$\lim_{|r_1 - r_2| \to \infty} \langle e^{i(\theta(r_1) - \theta(r_2))} \rangle \sim \left| \frac{1}{r_1 - r_2} \right|^{\frac{T}{4T_c}}$$

Kosterlitz-Thouless phase transition (dim=2)

★ **Superfluid stifness**



J.M. Kosterlitz & D.J. Thouless (1973)

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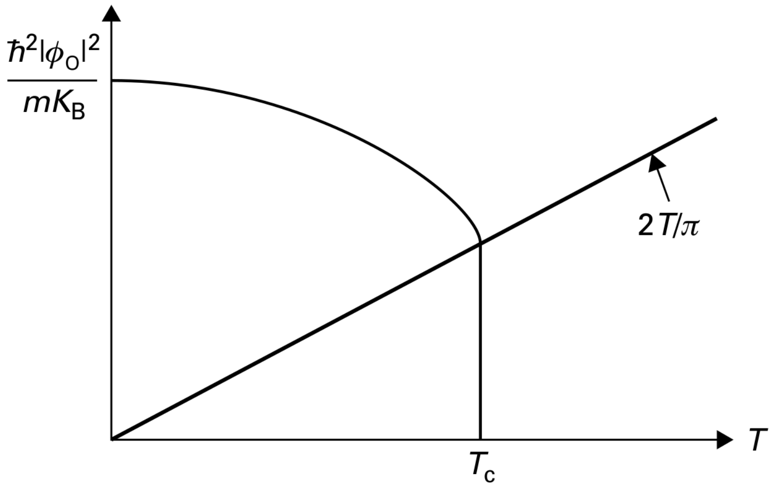
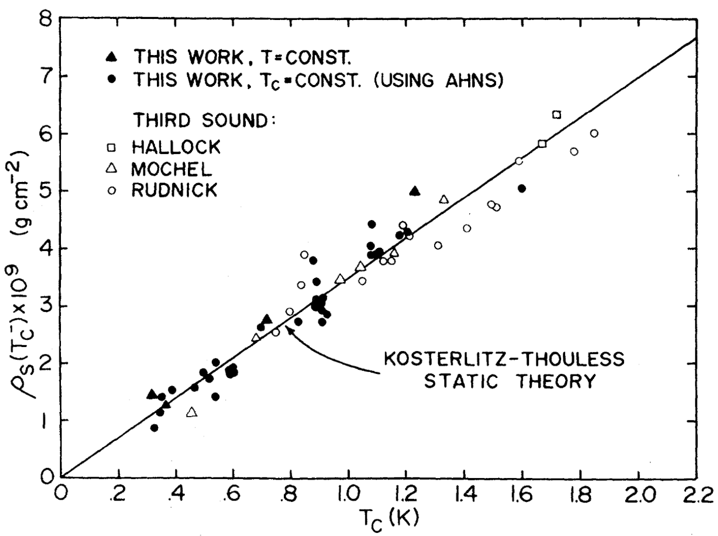


★ **Experimental data**



(results for ^{4}He 2-dim samples)

J.M. Kosterlitz & D.J. Thouless (1973)

D.J. Bishop & J.D. Reppy (1978)

Phase driven transition

unconventional superconductivity

Phase driven transition

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⇒ **phase-driven transition** */ high T_c cuprate oxides /* $T_c \not\sim \Delta(0)$

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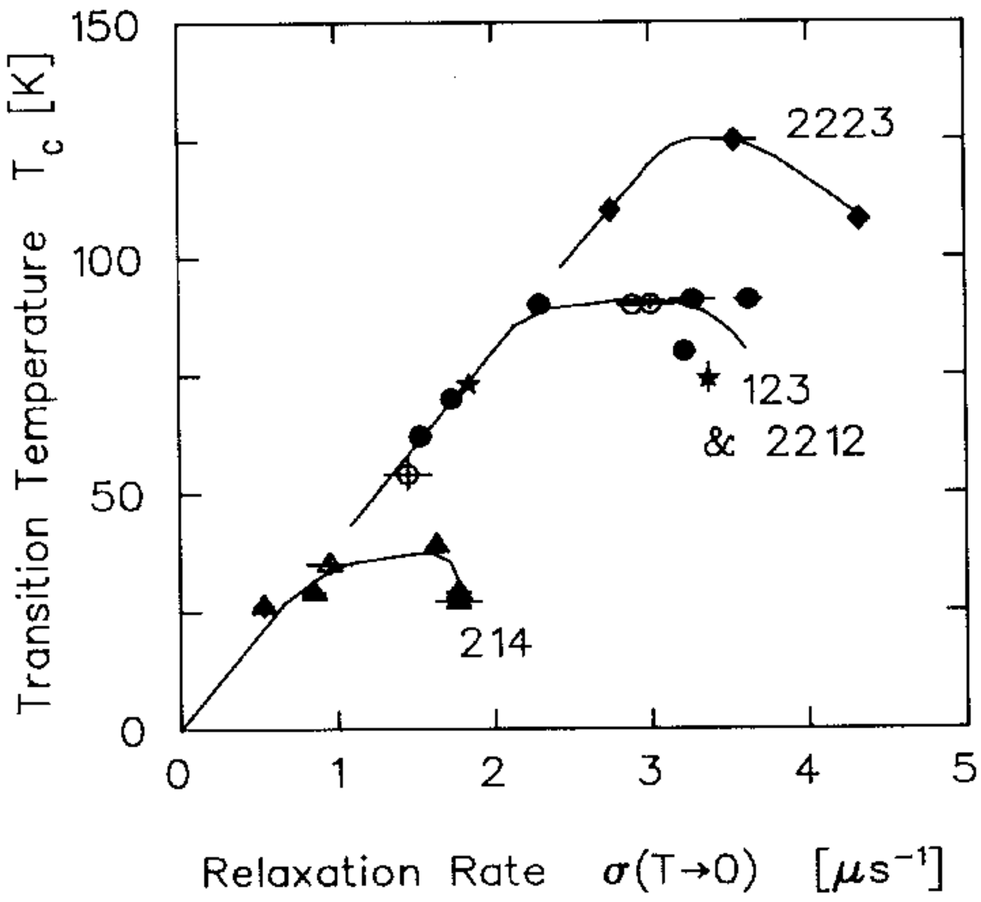


Y.J. Uemura et al, Phys. Rev. Lett. **62**, 2317 (1989).

Early experiments using the muon-spin relaxation indicated that in HTSC

$$T_c \propto \rho_s(0)$$

/ Uemura scaling /

The superfluid stiffness $\rho_s(T)$ is here defined by

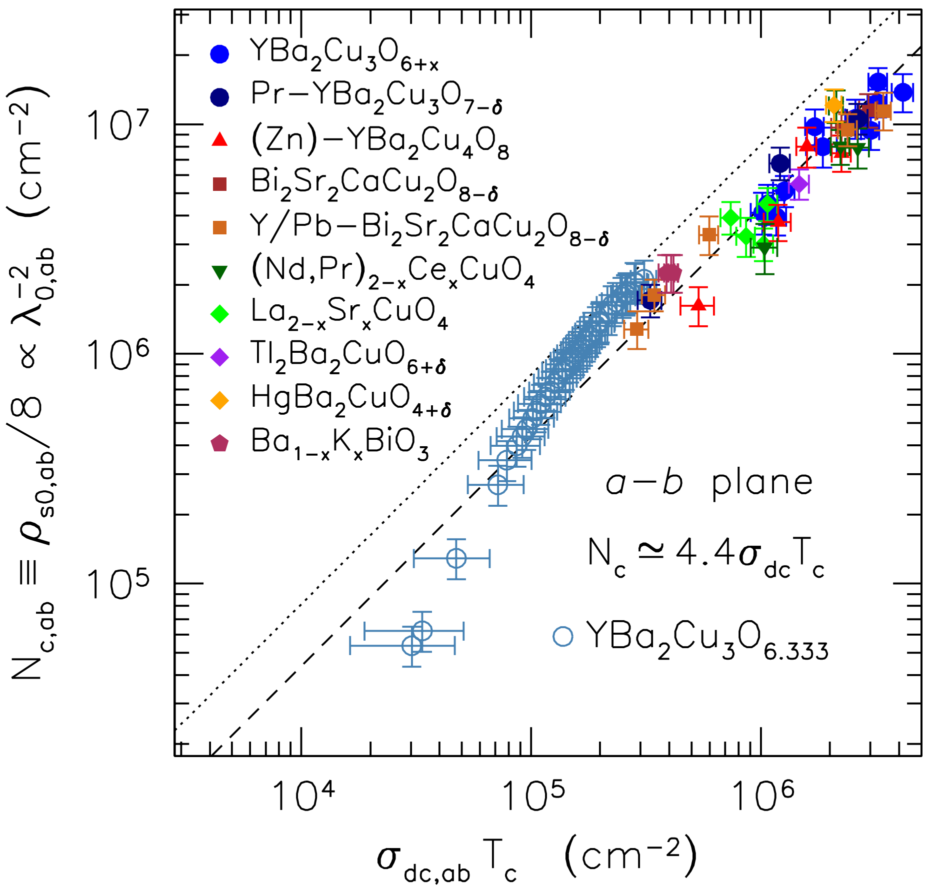
$$\rho_s(T) \equiv \frac{1}{\lambda^2(T)} = \frac{4\pi e^2}{m^* c^2} n_s(T)$$

Phase driven transition

unconventional superconductivity

⇒ **phase-driven transition** */ high T_c cuprate oxides /*

$T_c \not\sim \Delta(0)$



Recently such scaling has been updated from transport measurements

$$\frac{1}{8}\rho_s = 4.4\sigma_{dc}\, T_c$$

/ Homes scaling /

This new relation is valid for all samples ranging from the underdoped to overdoped region.

C.C. Homes, Phys. Rev. B ***80****, 180509(R) (2009).*

/ ab - plane /

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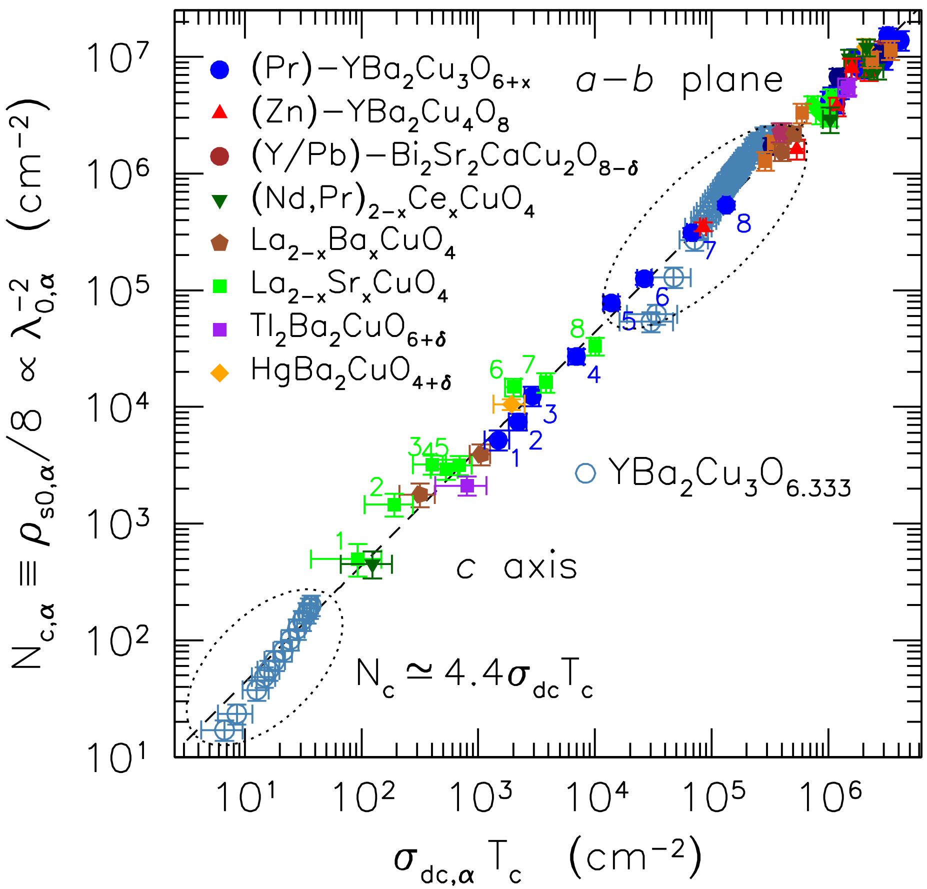

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/ c - axis /

What about dim=1 systems ?

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★ **AF Heisenberg chain of 1/2 spins**

 $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

F.D.M. Haldane, Phys. Lett. A 93, 464 (1983)

F.D.M. Haldane, Phys. Rev. Lett. 50, 1153 (1983)

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F.D.M. Haldane, Phys. Lett. A 93, 464 (1983)

F.D.M. Haldane, Phys. Rev. Lett. 50, 1153 (1983)

★ **Zero-energy edge states**

This story unfortunately is too long ...

Back to the main issue of the seminar:

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- **Superconductivity in nanostructures:**

⇒ **electron pairing** .... **/ due to proximity effect /**

⇒ **subgap quasiparticles** .... **/ Andreev (Shiba) states /**

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⇒ **zero-energy mode** .... **/ experimental facts /**

⇒ **microscopic description** .... **/ Kitaev vs realistic scenario /**

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- **Superconductivity in Rashba chain:**

⇒ **zero-energy mode** ....**/ experimental facts /**

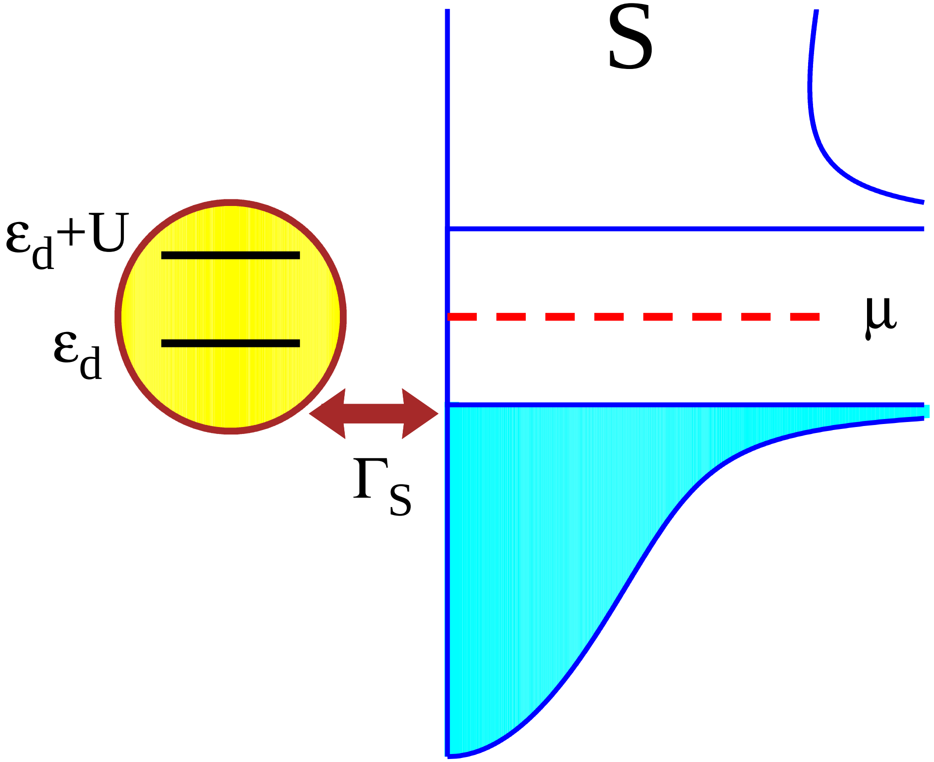
⇒ **microscopic description** ....**/ Kitaev vs realistic scenario /**

- **Majorana madness ...**

Superconductivity in dim=0 systems

$\Rightarrow$ **Shiba/Andreev bound states**

Electronic spectrum



Microscopic model

Anderson-type Hamiltonian

Quantum impurity (dot)

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^\dagger_\sigma \, \hat{d}_\sigma \; + \; U \; \hat{n}_{d\uparrow} \; \hat{n}_{d\downarrow}$$

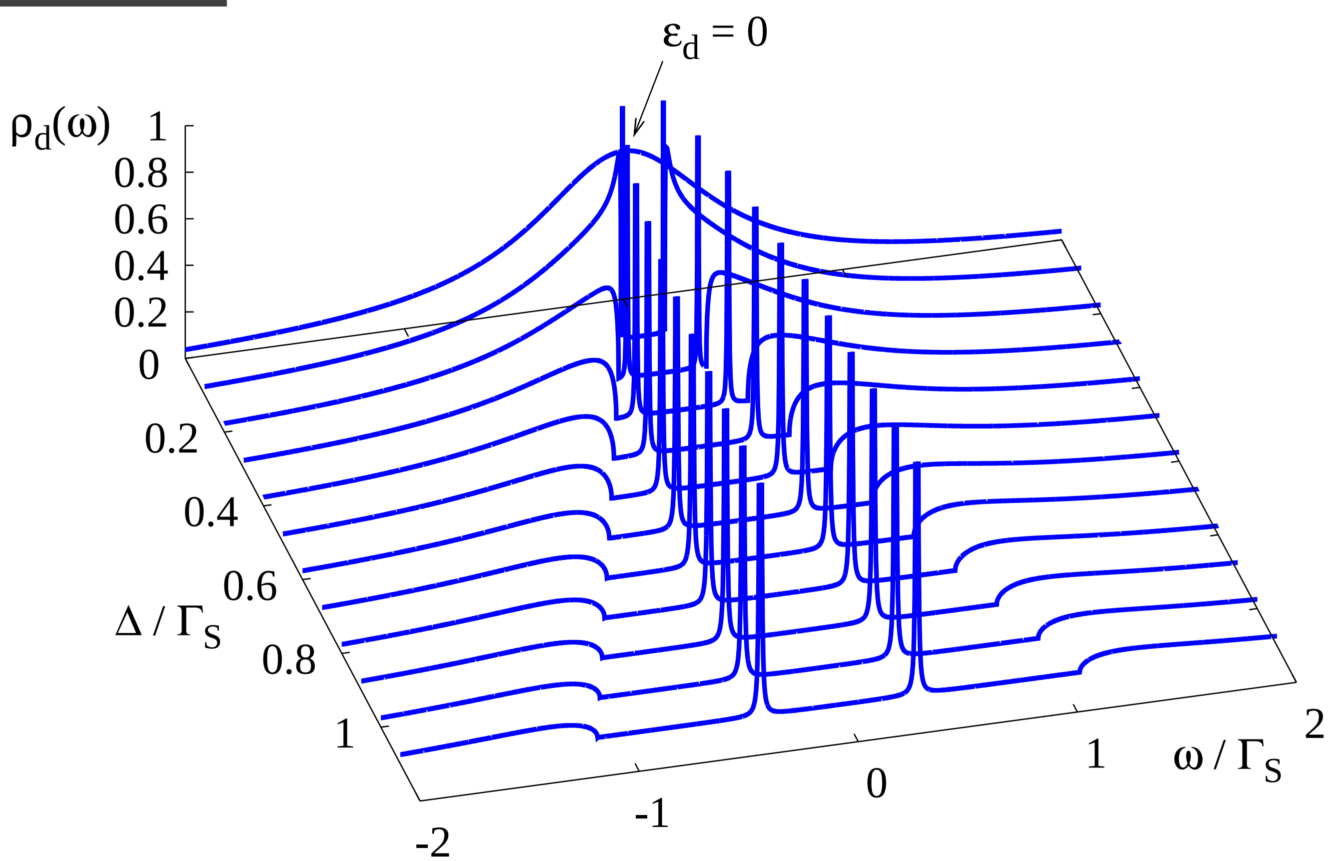
coupled with a superconductor

$$\begin{aligned} \hat{H} \;&=\; \sum_{\sigma} \epsilon_d \hat{d}^\dagger_\sigma \hat{d}_\sigma + U \; \hat{n}_{d\uparrow} \; \hat{n}_{d\downarrow} + \hat{H}_S \\ &+\; \sum_{\mathrm{k},\sigma} \left(V_{\mathrm{k}} \; \hat{d}^\dagger_\sigma \hat{c}_{\mathrm{k}\sigma} + V^*_{\mathrm{k}} \; \hat{c}^\dagger_{\mathrm{k}\sigma} \hat{d}_\sigma \right) \end{aligned}$$

where

$$\hat{H}_S = \sum_{k,\sigma} \left(\varepsilon_k - \mu\right) \hat{c}^\dagger_{k\sigma} \hat{c}_{k\sigma} - \sum_k \left(\Delta \hat{c}^\dagger_{k\uparrow} \; \hat{c}^\dagger_{k\downarrow} + \mathrm{h.c.} \right)$$

Uncorrelated QD $U_d = 0$ **(exactly solvable case)**

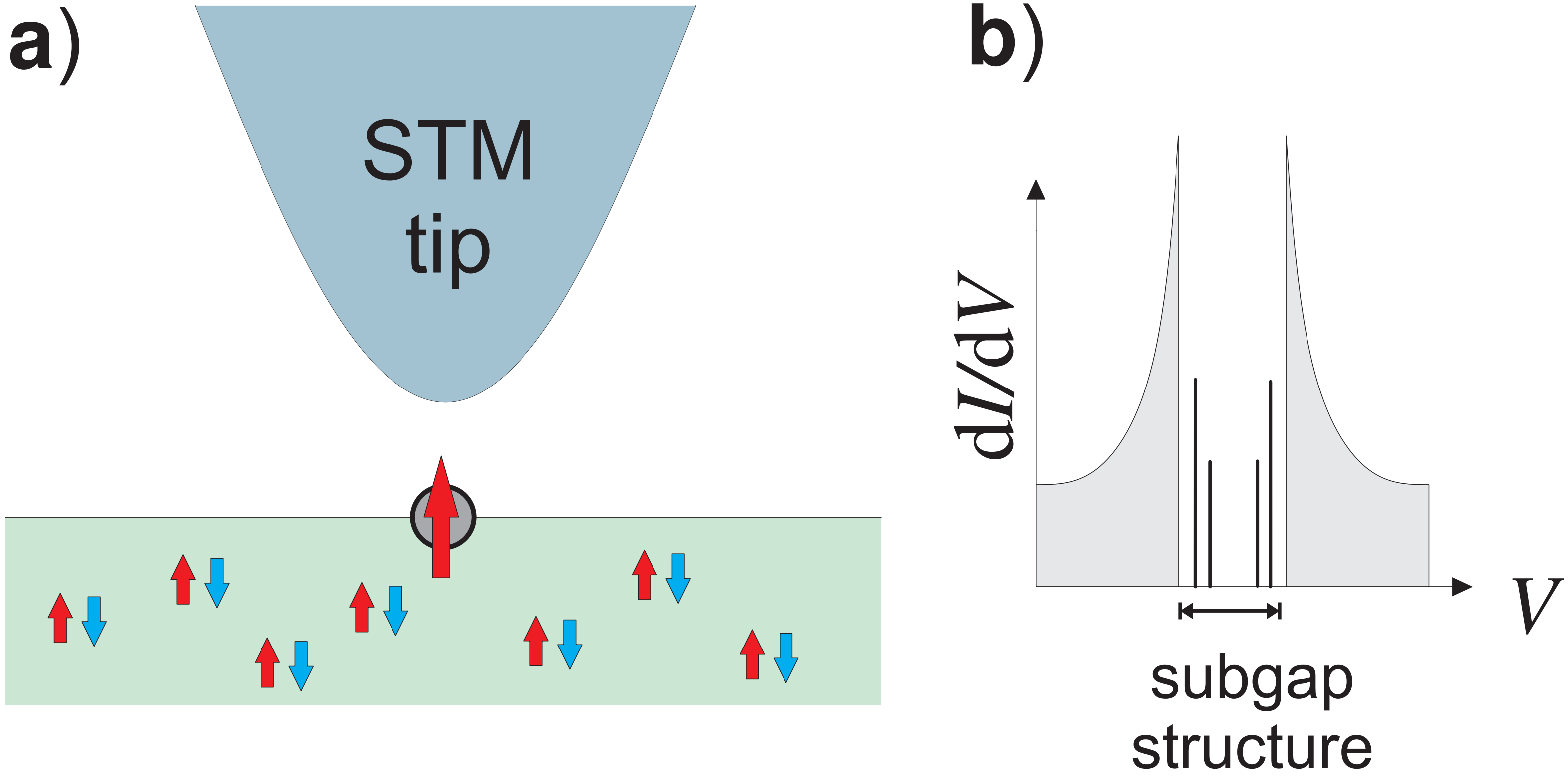


In-gap (Andreev/Shiba) bound states :

⇒ always appear in pairs,

⇒ appear symmetrically at finite energies.

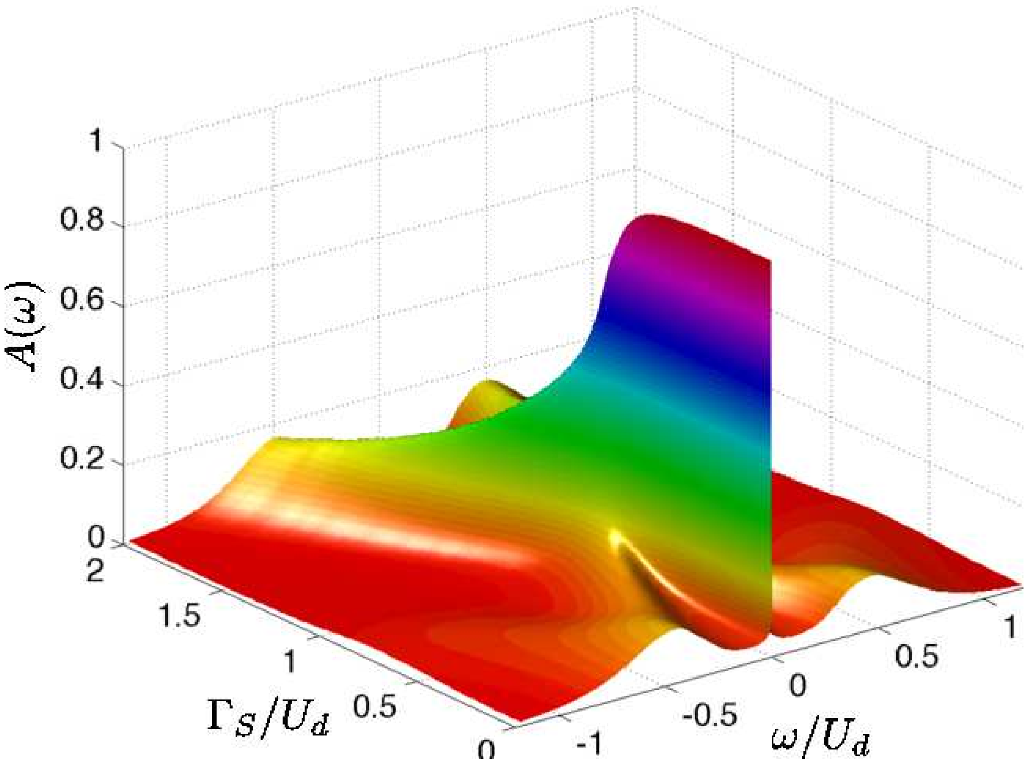
Subgap states of multilevel quantum impurities

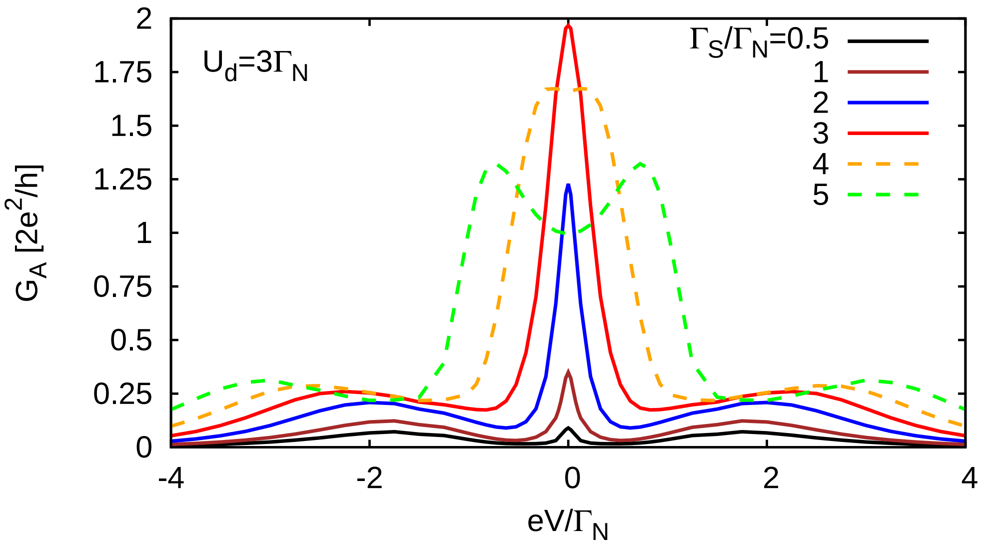


a) STM scheme and b) differential conductance for a multilevel quantum impurity adsorbed on a superconductor surface.

R. Žitko, O. Bodensiek, and T. Pruschke, Phys. Rev. B **83**, 054512 (2011).

Correlated quantum dot – Kondo meets Cooper





Quantum Phase Transition due the correlations.

Constructive influnce of Γ_S on the Kondo effect.

T. Domański et al, Scientific Reports **6**, 23336 (2016).

Superconductivity in chains

$\Rightarrow$ **Majorana quasiparticles**

Experimental evidence – for Majorana quasiparticles

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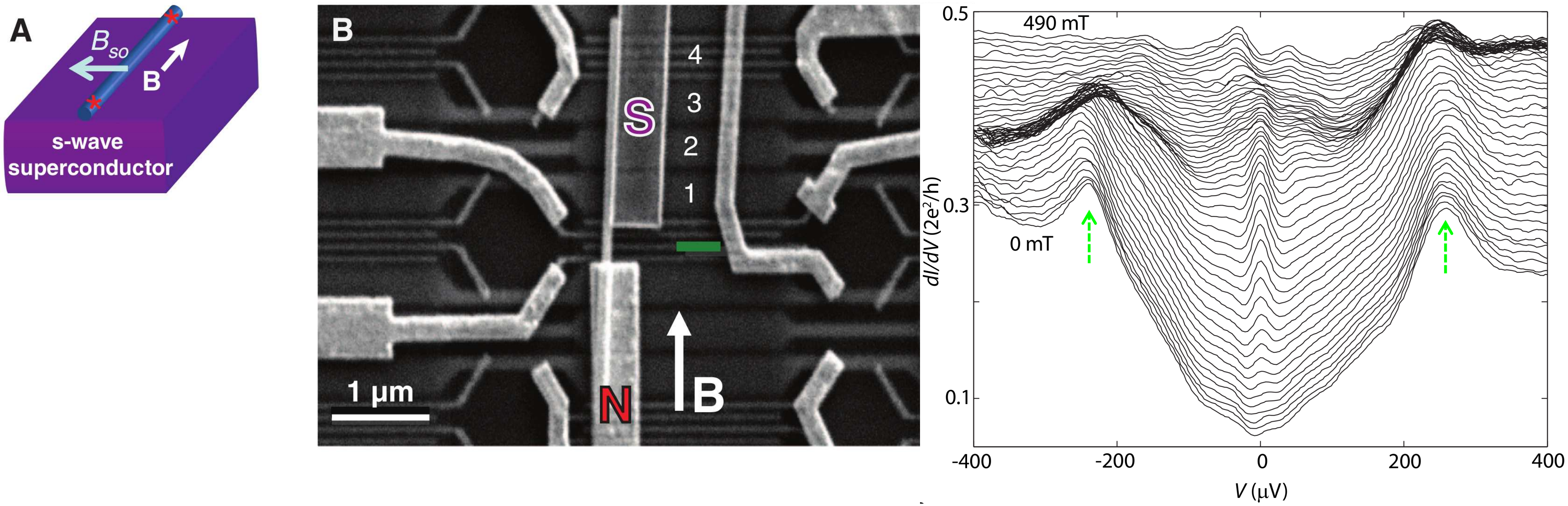
InSb nanowire between a metal (gold) and a superconductor (Nb-Ti-N)



dI/dV **measured at 70 mK for varying magnetic field B indicated:**

$\Rightarrow$ **a zero-bias enhancement due to Majorana state**

V. Mourik, ..., and L.P. Kouwenhoven, Science **336**, 1003 (2012).

/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /

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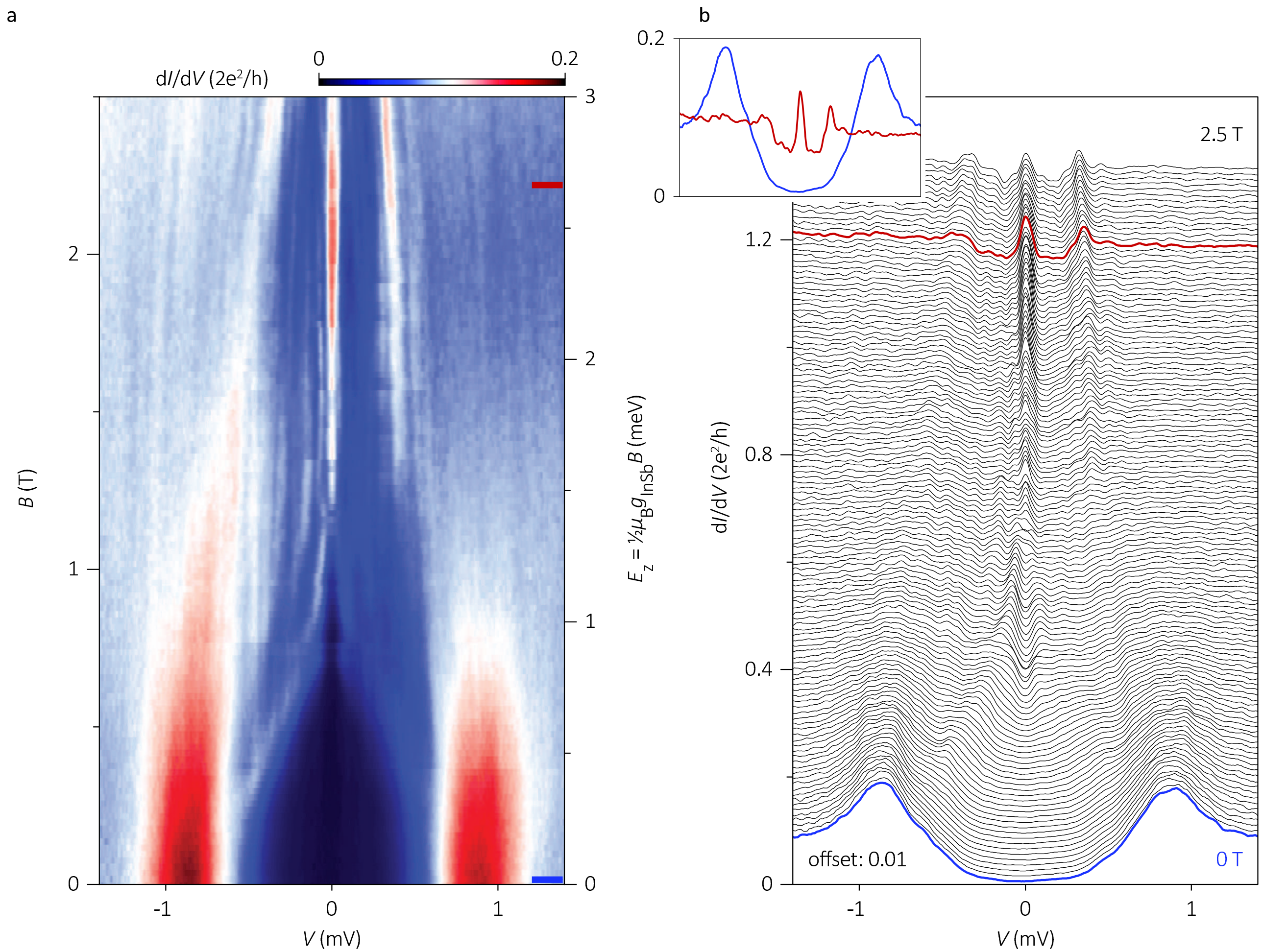
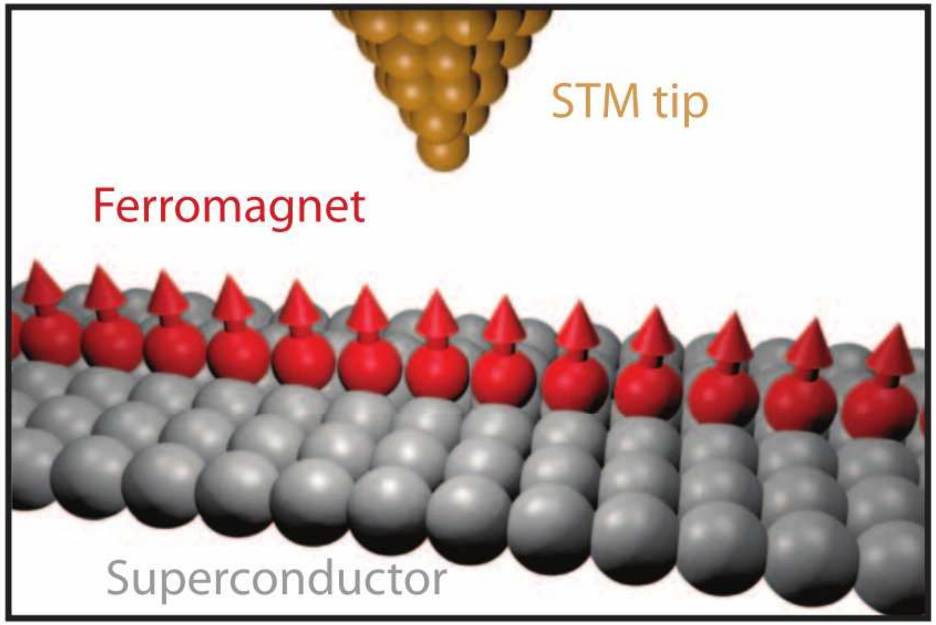


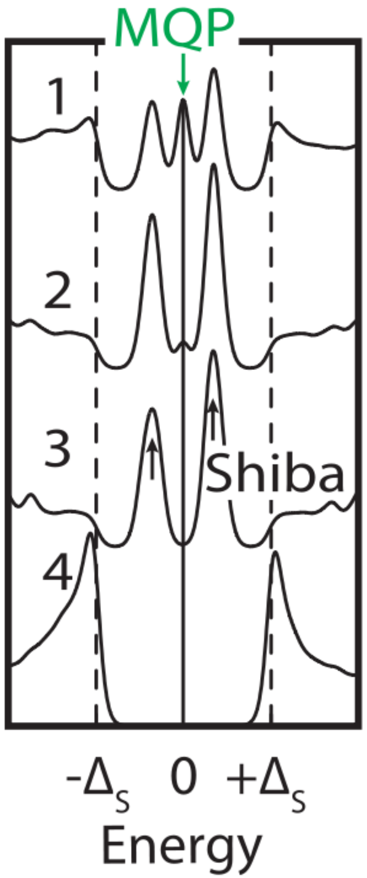
H. Zhang, ..., and L.P. Kouwenhoven, arXiv:1603.04069 (2016).

/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /

Experimental evidence – for Majorana quasiparticles

A chain of iron atoms deposited on a surface of superconducting lead





STM measurements provided evidence for:

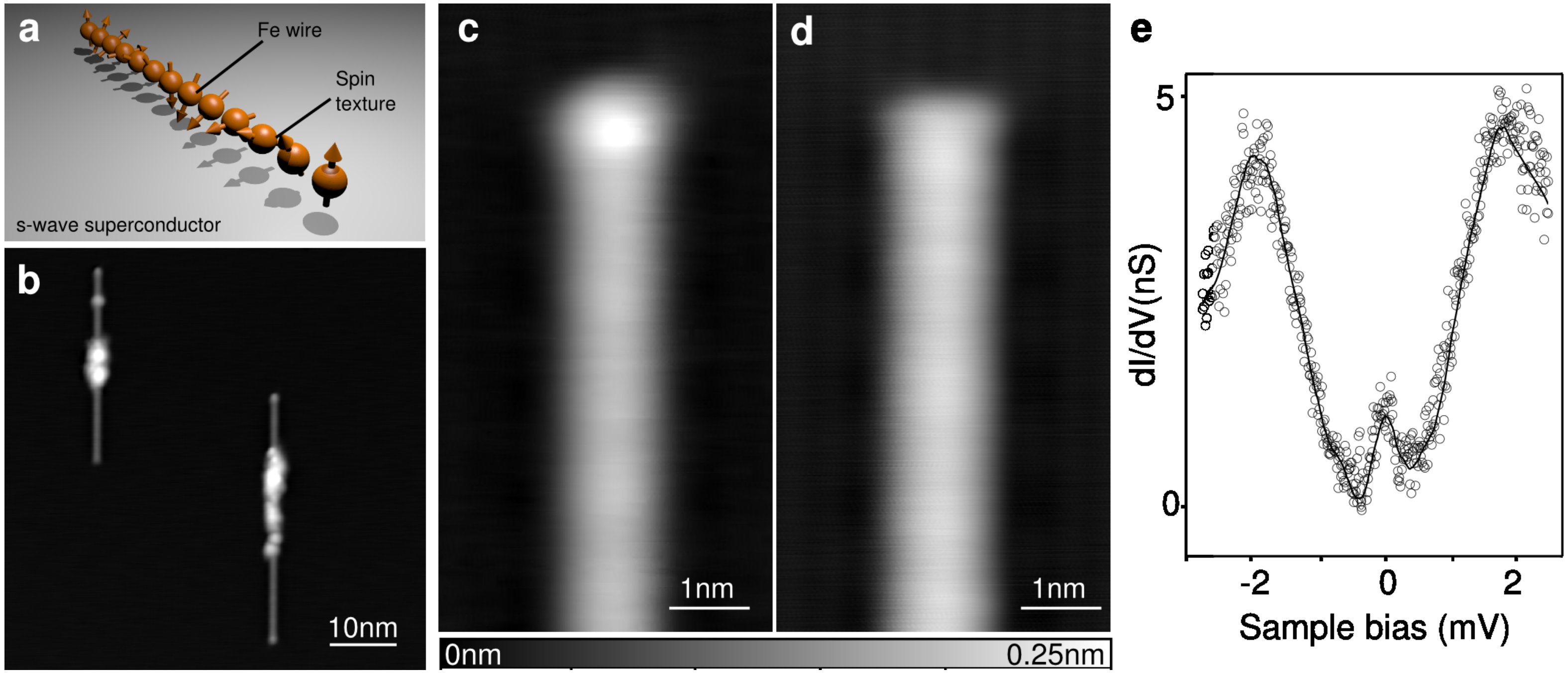
$\Rightarrow$ **Majorana bound states at the edges of a chain.**

S. Nadj-Perge, ..., and A. Yazdani, Science **346**, 602 (2014).

/ Princeton University, Princeton (NJ), USA /

Experimental evidence – for Majorana quasiparticles

Self-assembled Fe chain on superconducting Pb(110) surface



AFM combined with STM provided evidence for:

⇒ Majorana bound states at the edges of a chain.

R. Pawlak, **M. Kisiel**, ..., and E. Meyer, arXiv:1505.06078 (2015).

/ University of Basel, Switzerland /

Question:

$\Rightarrow$ **where does Majorana come from ?**

Basic notions – on Majorana fermions

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- **P. Dirac (1928)**

$$i\dot{\psi} = (\vec{\alpha}\cdot\vec{p} + \beta m)\,\psi$$

/ relativistic description of fermions /

particles ($E > 0$), **anti-particles ($E < 0$)**

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- **E. Majorana (1937)**

noticed that particular choice of $\vec{\alpha}$ and β yields a real wave-function !

Physical implication: **particle = antiparticle**

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$\Rightarrow$ **Basic features:**

chargeless

zero-energy

Majoranization – of normal fermions

- Dirac fermions (e.g. electrons) obey the anticommutation relations

$$\left\{\hat{c}_i, \hat{c}_j^\dagger\right\} = \delta_{i,j}$$

$$\{\hat{c}_i, \hat{c}_j\} = 0 = \left\{\hat{c}_i^\dagger, \hat{c}_j^\dagger\right\}$$

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i,j – any quantum numbers

- $c_j^{(\dagger)}$ can be recast in terms of majorana operators

$$\hat{c}_j \equiv \left(\hat{\gamma}_{j,1} + i\hat{\gamma}_{j,2}\right)/\sqrt{2}$$
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’real’ and ’imaginary’ parts

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'real' and 'imaginary' parts

- $\hat{\gamma}_{i,n}$ obey unconventional algebra

$$\left\{\hat{\gamma}_{i,n}, \hat{\gamma}_{j,m}^\dagger\right\} = \delta_{i,j}\delta_{n,m}$$
$$\hat{\gamma}_{i,n}^\dagger = \hat{\gamma}_{i,n}$$

creation = annihilation !

Majoranization – does it make sense ?

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- Majorana-type quasiparticles

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correspond to <u>neutral</u> objects

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- They resemble Bogoliubov qps of the BCS theory

$$\hat{\beta}_{k\uparrow} \equiv u_k\hat{c}_{k\uparrow} + v_k\hat{c}^\dagger_{-k\downarrow}$$
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- At the Fermi level $u_{k_F} = v_{k_F} = 1/\sqrt{2}$, thus

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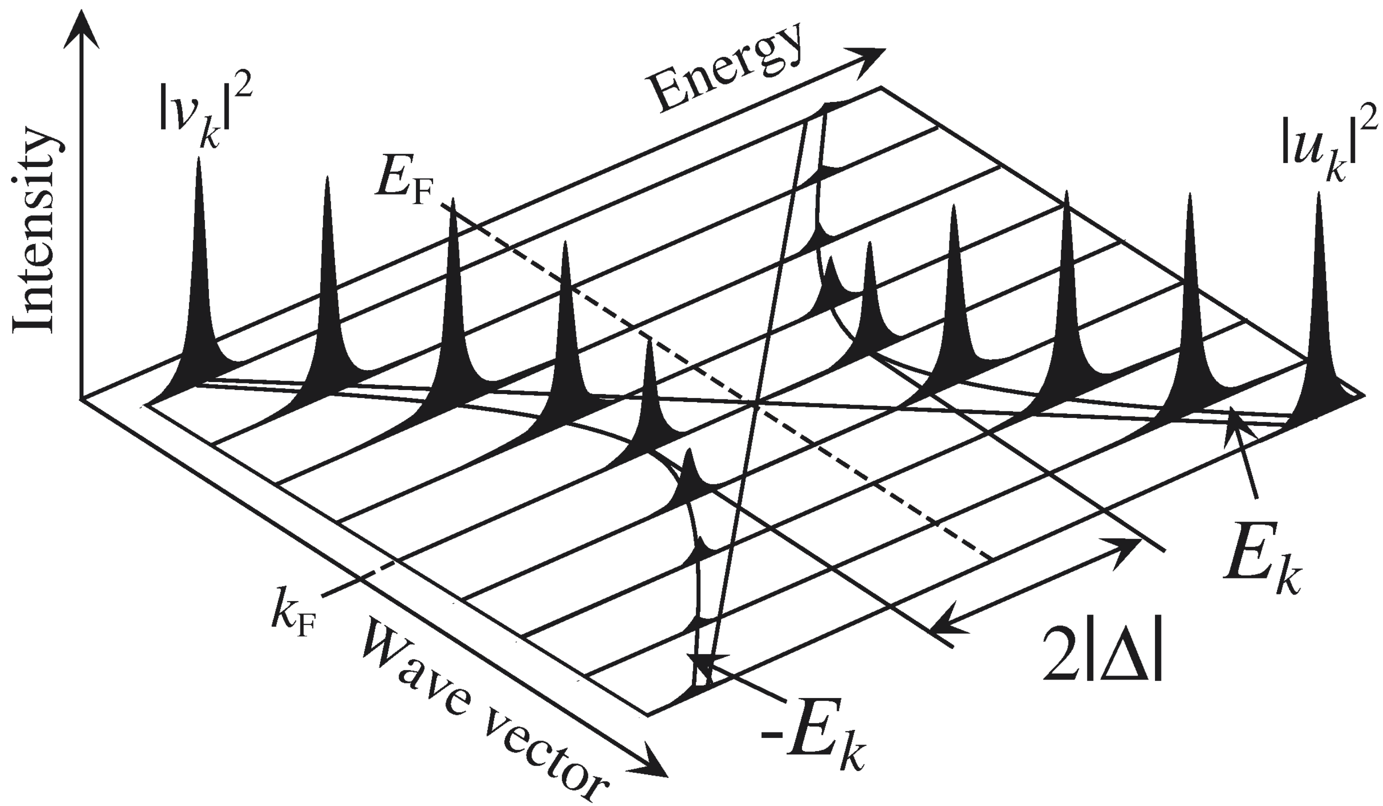
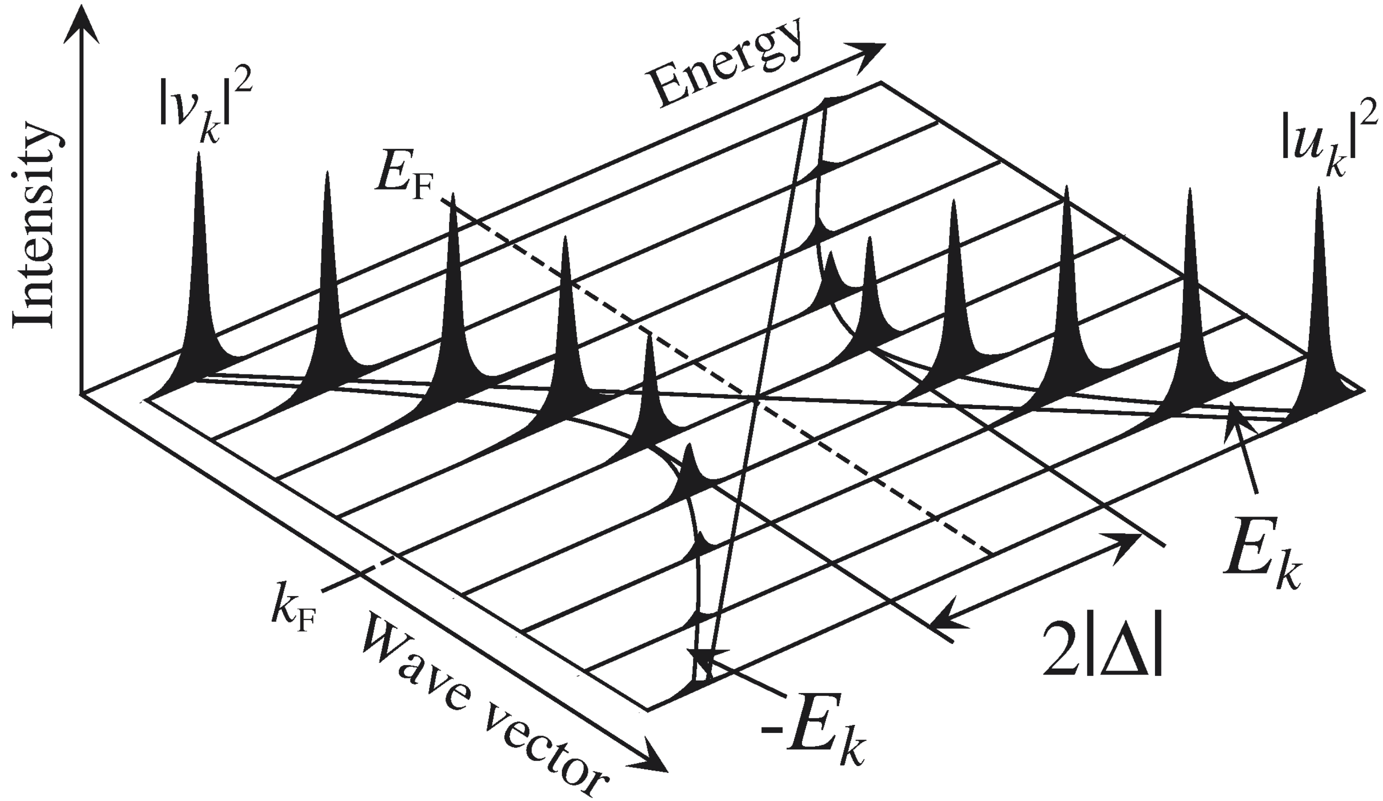
OK, what is their energy ?

Bogoliubov quasiparticles – **in superconductors**



Bogoliubov qps of s-wave superconductors are gapped $\pm\sqrt{\varepsilon_k^2 + \Delta^2}$.

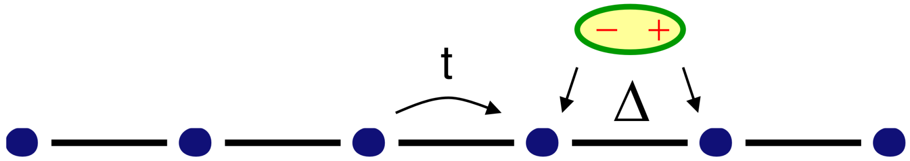
Bogoliubov quasiparticles – in superconductors



Bogoliubov qps of s-wave superconductors are gapped $\pm\sqrt{\varepsilon_k^2 + \Delta^2}$.

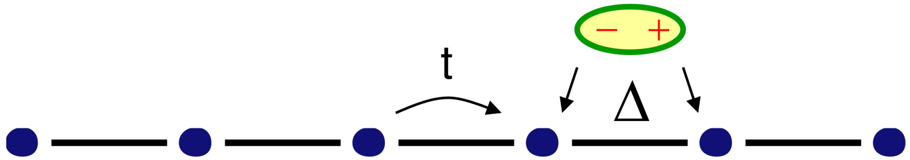
True majoranas have to be the zero energy ($E_k = 0$) quasiparticles !

Popular toy model – Kitaev (2001)



p-wave pairing of spinless 1D fermions

Popular toy model – **Kitaev (2001)**



p-wave pairing of spinless 1D fermions

$$\hat{H} = t\sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1} + \mathrm{h.c.}\right) - \mu\sum_i \hat{c}_i^\dagger \hat{c}_i + \Delta\sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1}^\dagger + \mathrm{h.c.}\right)$$

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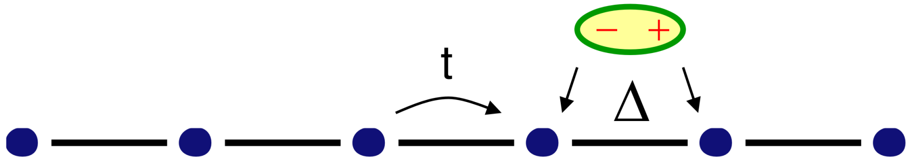
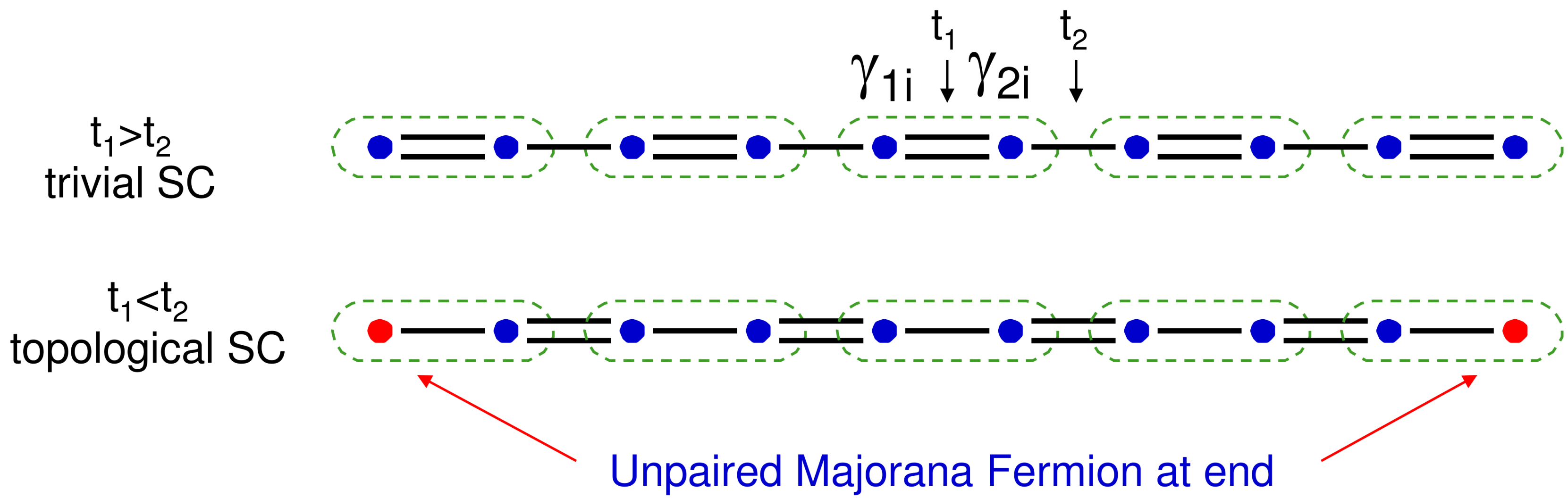


p-wave pairing of spinless 1D fermions

$$\hat{H} = t\sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1} + \text{h.c.}\right) - \mu \sum_i \hat{c}_i^\dagger \hat{c}_i + \Delta \sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1}^\dagger + \text{h.c.}\right)$$

This toy-model can be exactly solved in Majorana basis. For $\Delta = t$ one obtains:



$t_1 \equiv \mu$

$t_2 \equiv 2t$

Popular toy model – Kitaev (2001)

In the special case $\Delta = t$, $|\mu| < 2t$ the nontrivial superconductivity

implies that operators $\hat{\gamma}_{1,1}$ and $\hat{\gamma}_{2,N}$ are decoupled from the all rest.

This implies: **zero-energy modes at the Kitaev chain edges**

Towards more realistic situation

/ Rashba chain + pairing /

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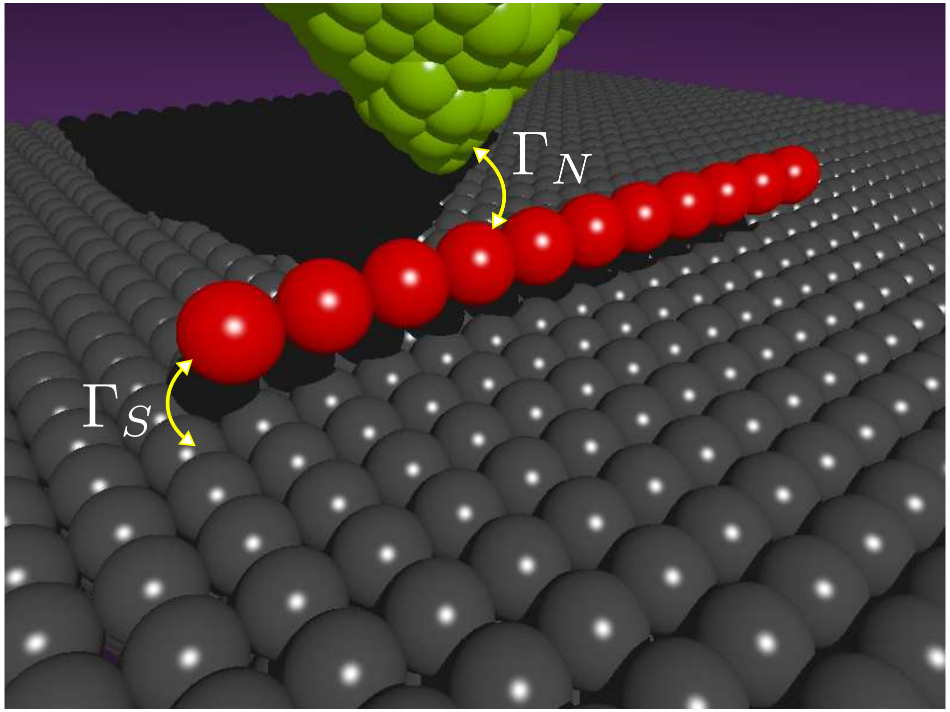
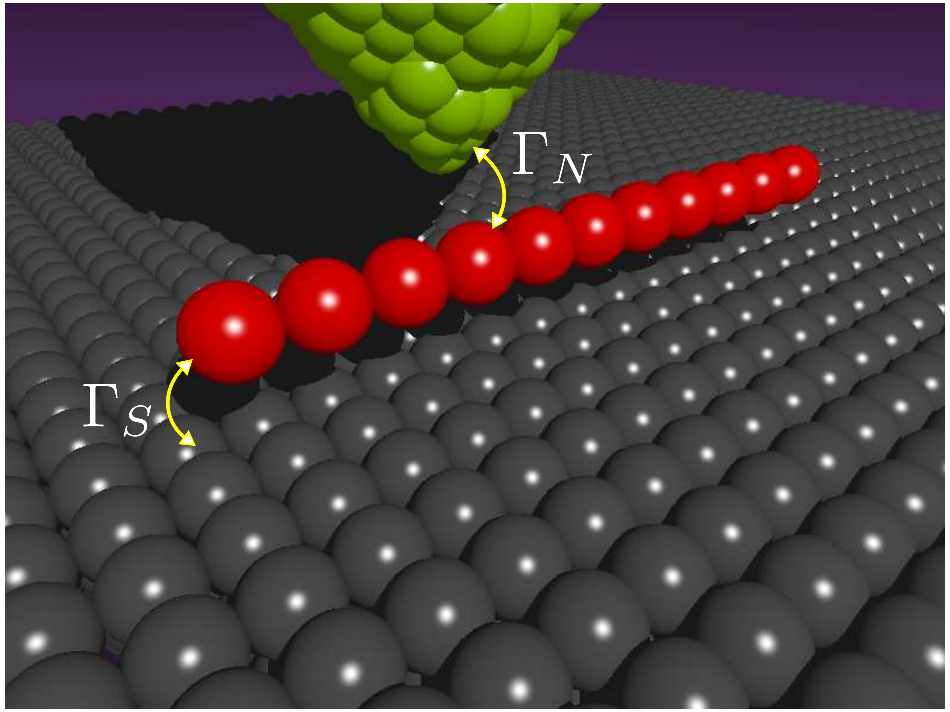


Scheme of STM configuration

Towards more realistic situation

/ Rashba chain + pairing /



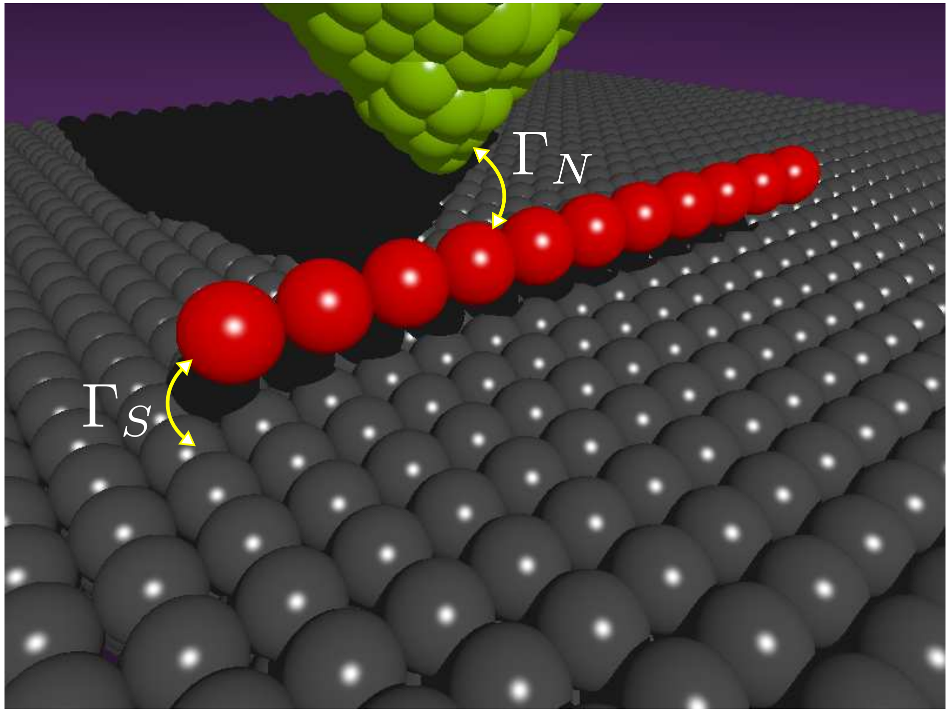
Scheme of STM configuration

$$\hat{H} \;=\; \hat{H}_{tip} + \hat{H}_{chain} + \hat{H}_S + \hat{V}_{hybr}$$

We study this model, focusing on the deep subgap regime $|E| \ll \Delta_{sc}$.

Towards more realistic situation

/ Rashba chain + pairing /



Scheme of STM configuration

where

$$\hat{H}_{chain} = \sum_{i,j,\sigma} (t_{ij} - \delta_{ij}\mu) \hat{d}^{\dagger}_{i,\sigma} \hat{d}_{j,\sigma} + \hat{H}_{Rashba} + \hat{H}_{Zeeman}$$

Majorana states – of the Rashba chain

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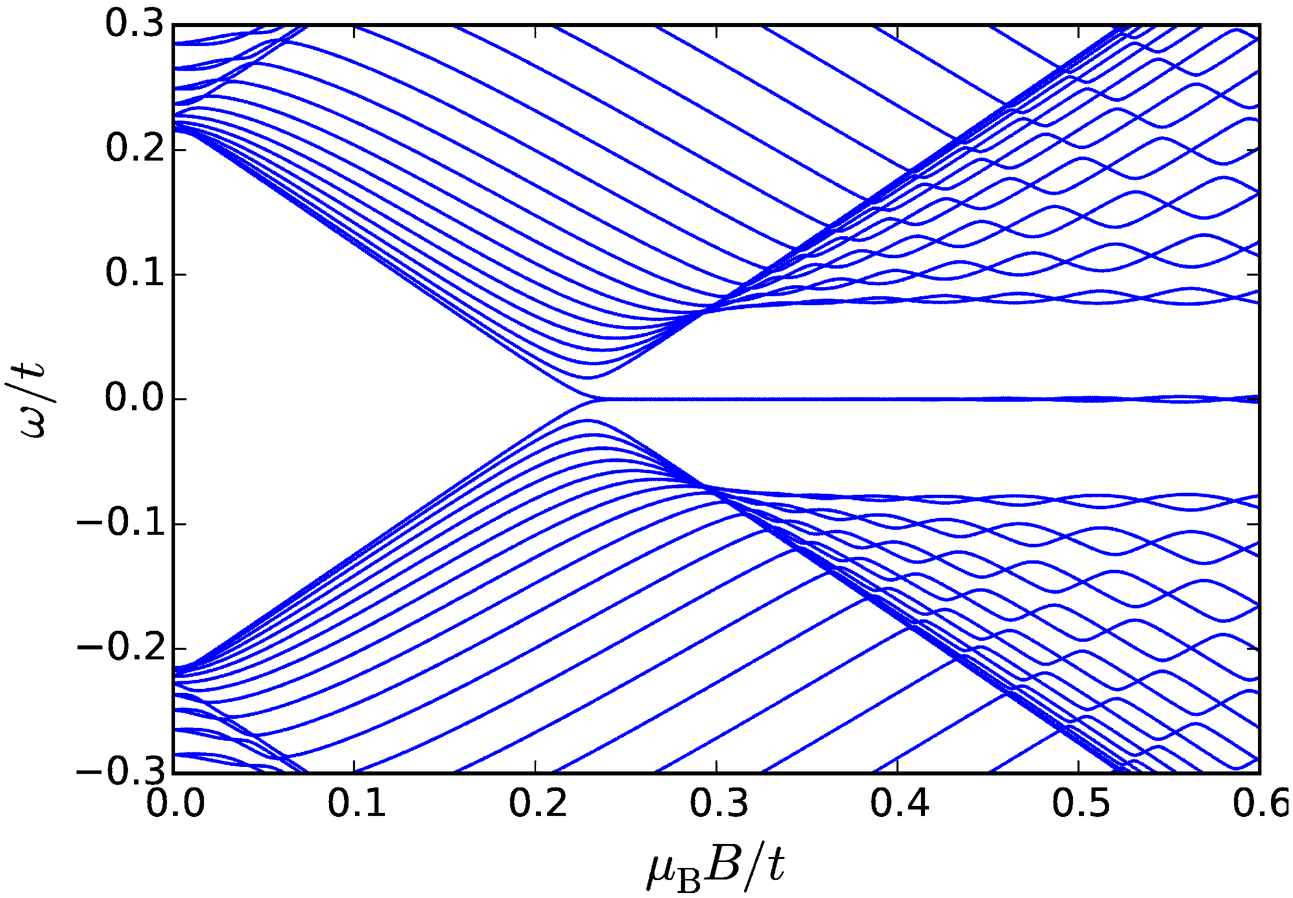
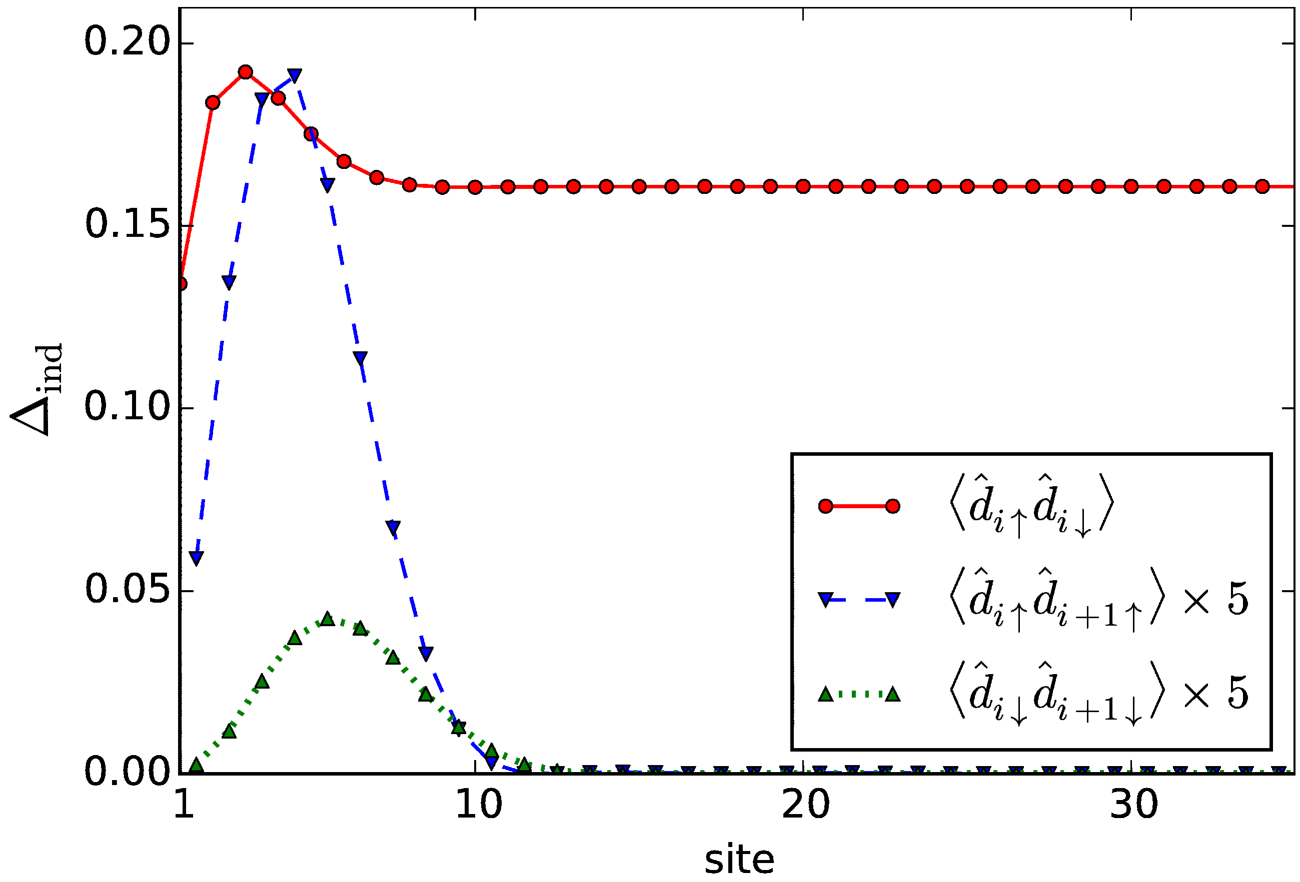
Mutation of Andreev states into zero-energy (Majorana) mode



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv:1609.00685 (2016).

Majorana states – **of the Rashba chain**

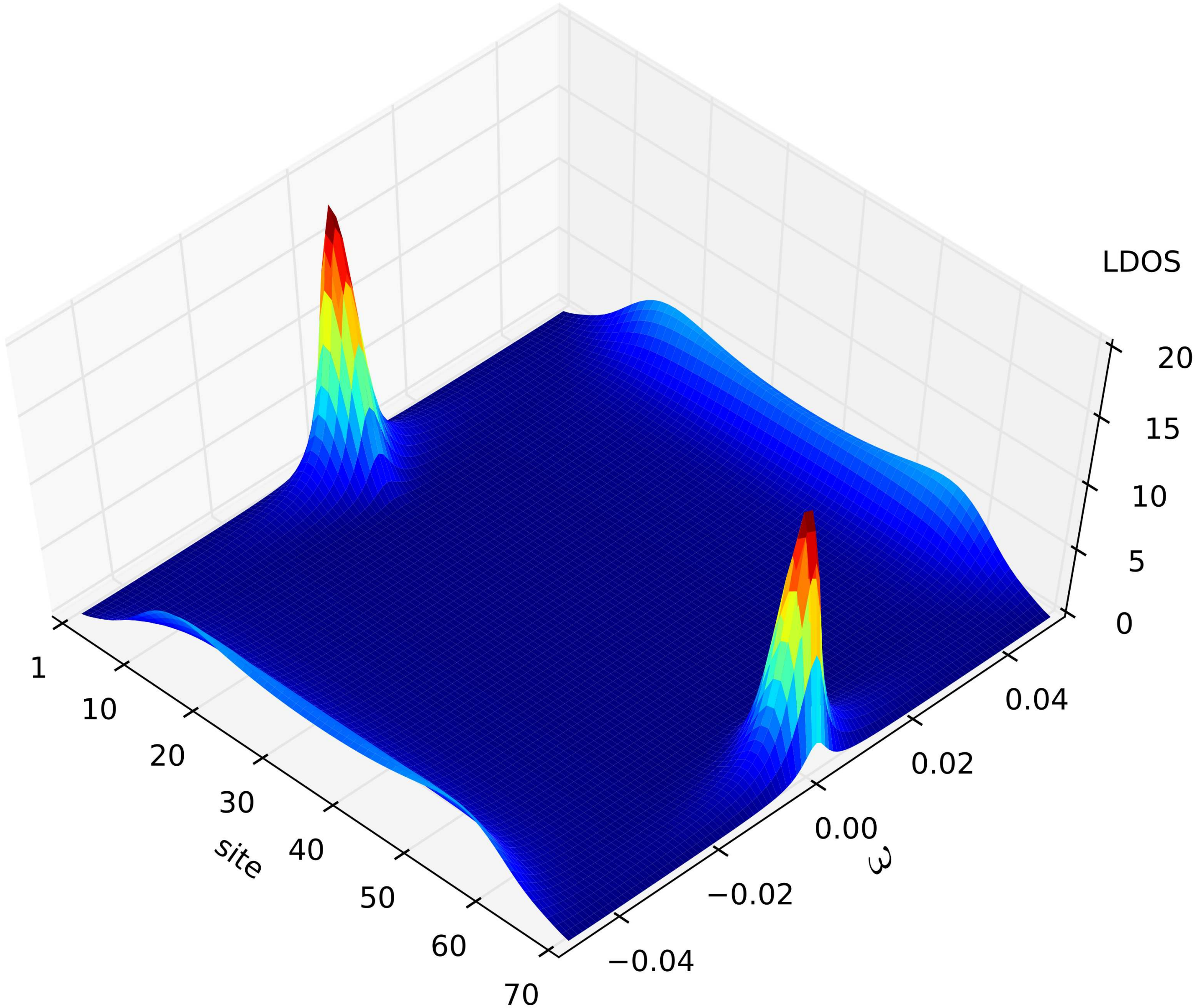
Spatial variation of the trivial and nontrivial pairings



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv:1609.00685 (2016).

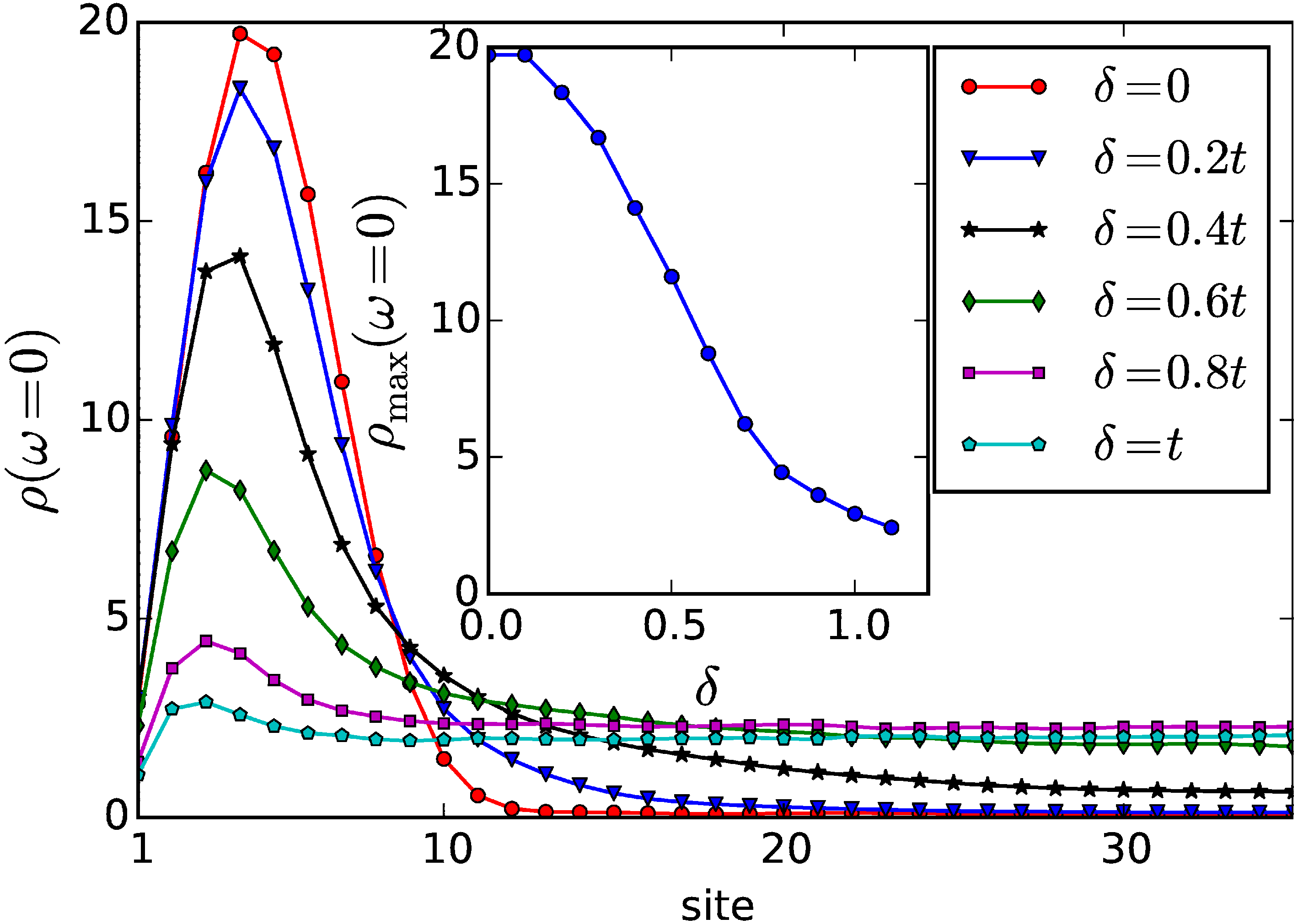
Majorana states – of the Rashba chain

In-gap spectrum with the edge Majorana quasiparticles



Majorana states – of the Rashba chain

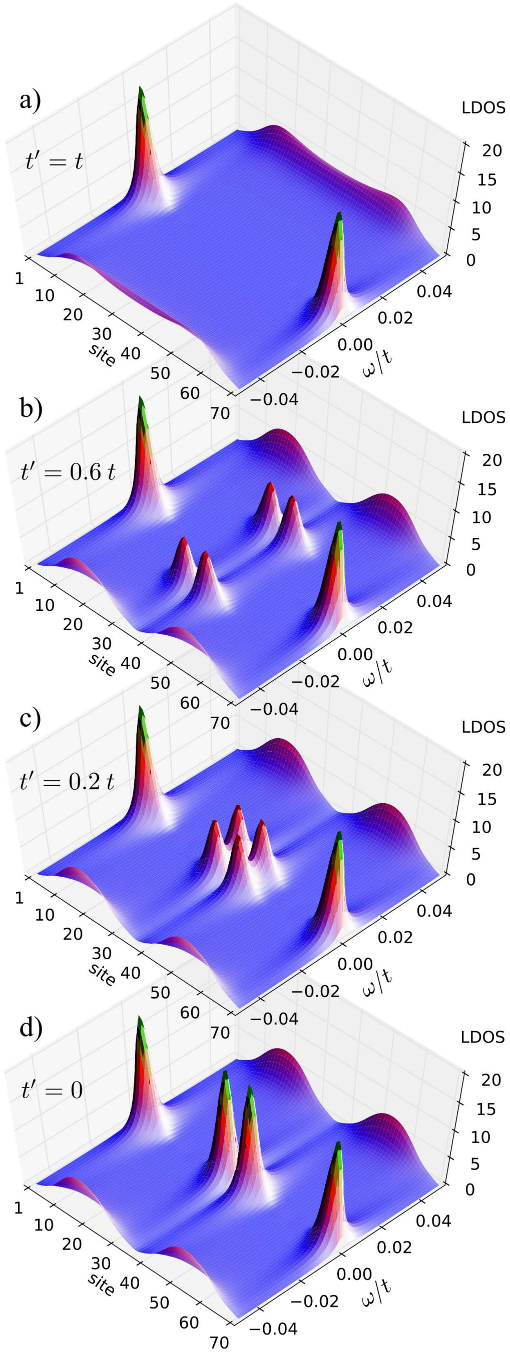
Are Majorana quasiparticles really immune to disorder ?



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, arXiv:1609.00685 (2016).

Majorana states – **of the Rashba chain**

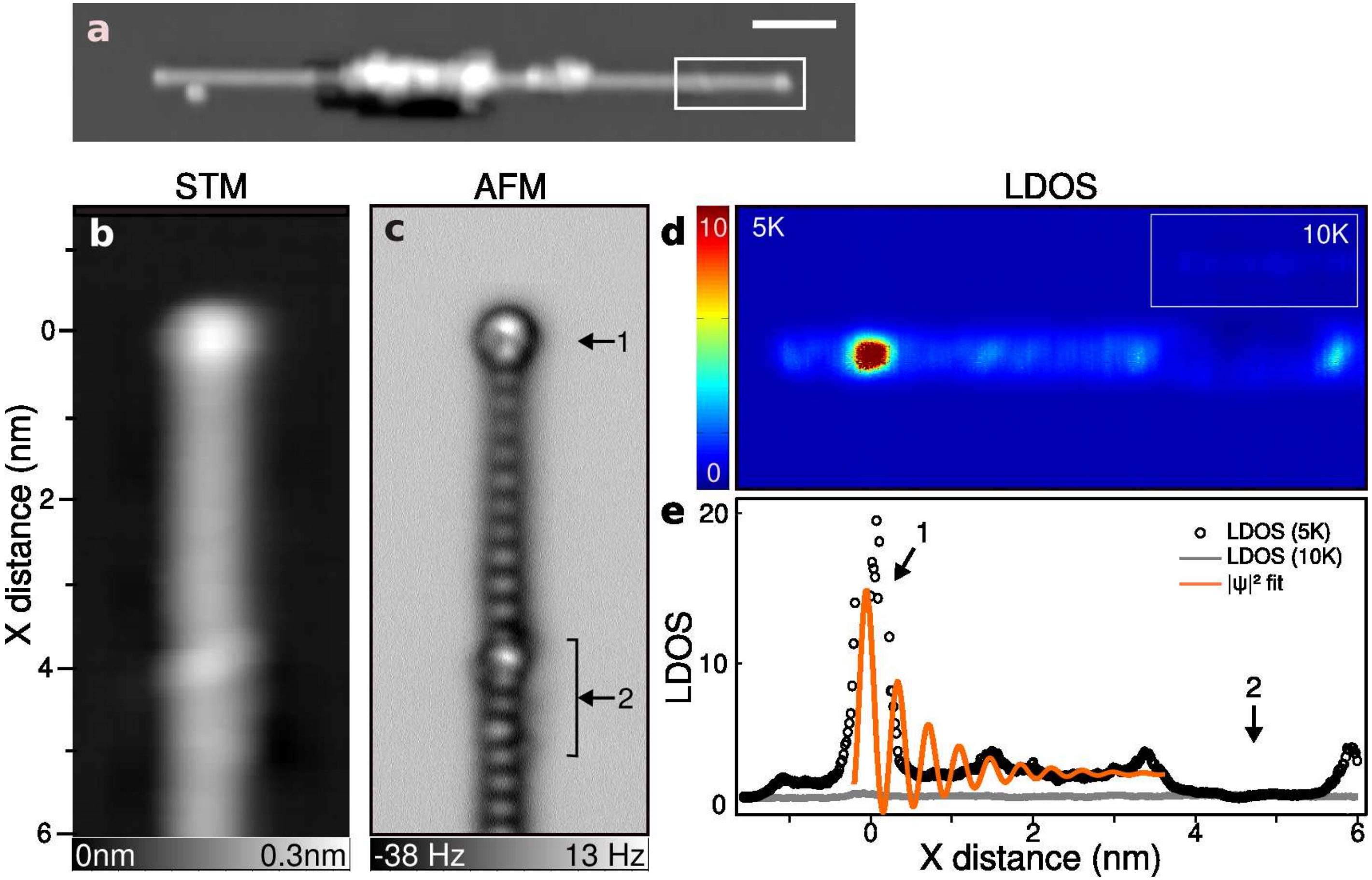
Gradual partitioning of the Rashba chain into separate pieces by reducing hopping t_j at site $j = N/2$



M. Maśka et al, arXiv:1609.00685 (2016).

Majorana states – **of the Rashba chain**

Experimentally observed local deffect



R. Pawlak, **M. Kisiel**, ..., and <u>E. Meyer</u>, arXiv:1505.06078 (2015).

Conclusions

Majorana-type quasiparticles:

$\Longrightarrow$ **emerge from the Andreev/Shiba states**

$\Longrightarrow$ **are not completely immunue to disorder**

$\Longrightarrow$ **represent non-local entities**

$\Longrightarrow$ **’leak’ into normal quantum impurities**

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