Nonlocal dynamical effects transmitted through topological superconductor

Tadeusz Domański

Maria Curie-Skłodowska University
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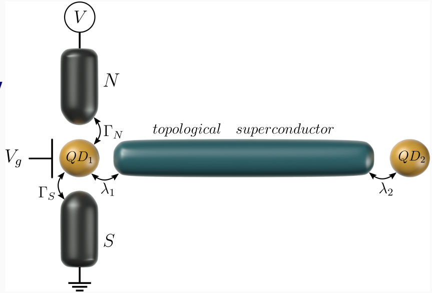


Palma de Mallorca

6 May 2025

OUTLINE

Time-resolved quasiparticles of quantum dot(s) coupled to:

1. **conventional superconductors**

   (Andreev/Yu-Shiba-Rusinov bound states)

2. **topological superconductors**

   (Majorana quasiparticles)

Conventional superconductors

SUPERCONDUCTING PROXIMITY EFFECT

The localized electrons of QD coupled to itinerant electrons of bulk superconductor develop:

$\Rightarrow$ **on-dot pairing**

SUPERCONDUCTING PROXIMITY EFFECT

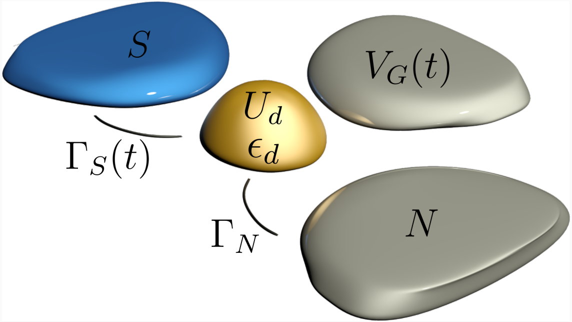
The localized electrons of QD coupled to itinerant electrons of bulk superconductor develop:

$\Rightarrow$ **on-dot pairing**

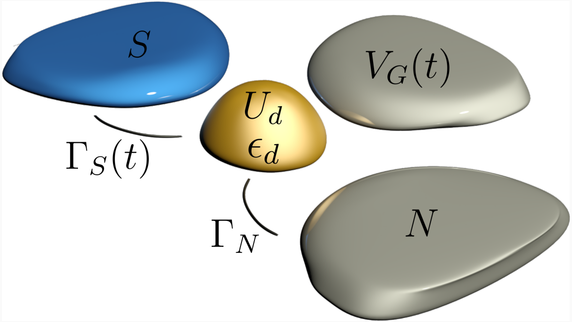
This phenomenon is manifested by emergence of:

$\Rightarrow$ **in-gap bound states**

BUILDUP OF IN-GAP STATES



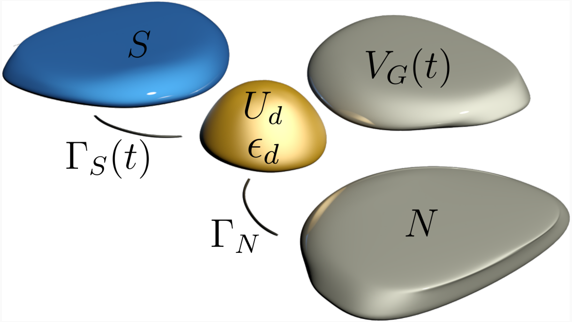
Feasible empirical protocols:



Feasible empirical protocols:

$\Rightarrow$ **variation of the coupling Γ_S to superconductor**

BUILDUP OF IN-GAP STATES



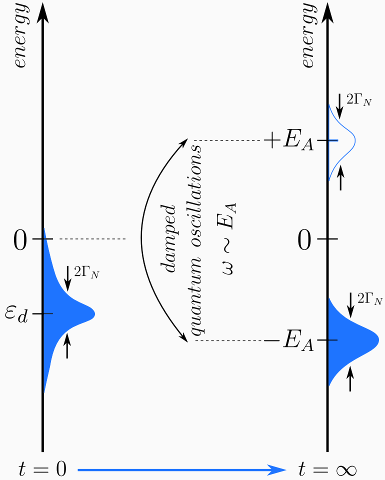
Feasible empirical protocols:

$\Rightarrow$ **variation of the coupling Γ_S to superconductor**

$\Rightarrow$ **abrupt change of energy by the gate potential V_G**

PARTICLE-HOLE INGREDIENTS OF IN-GAP STATES

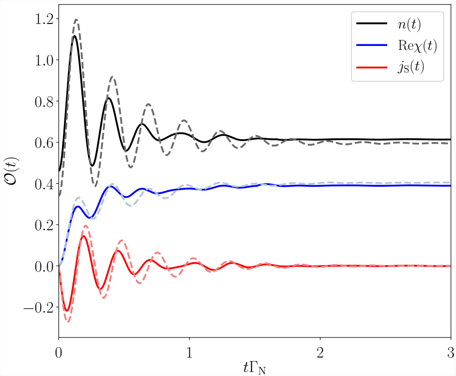
Quench protocol: **abrupt coupling of QD to superconductor $0 \rightarrow \Gamma_S$**



K. Wrześniewski, B. Baran, R. Taranko, T. Domański & I. Weymann, PRB 103, 155420 (2021).

RABBI-TYPE OSCILLATIONS

Time-dependent observables driven by the quantum quench $0 \to \Gamma_S$



$n(t)$ **electron number**

$\chi(t) = \left\langle \hat{d}_{\downarrow} \hat{d}_{\uparrow} \right\rangle$ **on-dot pairing**

$j_S(t)$ **charge current**

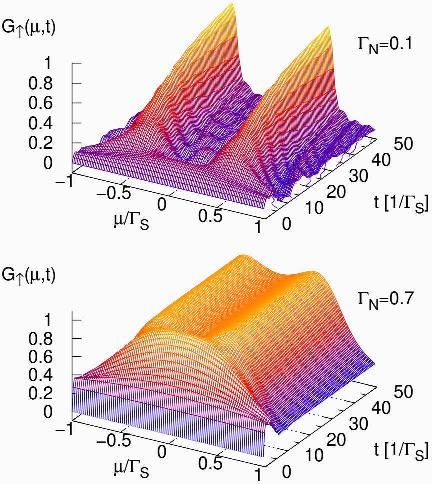
solid lines - time dependent NRG

dashed lines - Hartree-Fock-Bogolubov

K. Wrześniewski, B. Baran, R. Taranko, T. Domański & I. Weymann, PRB 103, 155420 (2021).

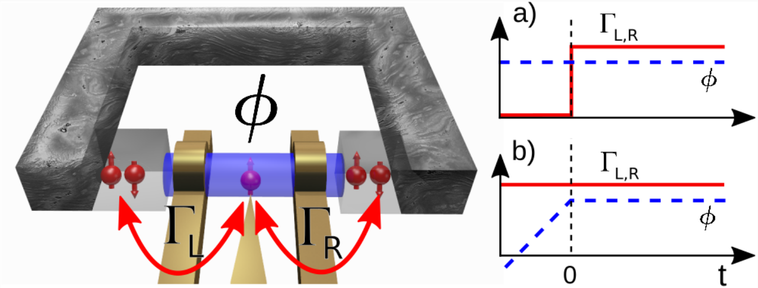
TIME-DEPENDENT TUNNELING CONDUCTANCE

QD level $\varepsilon = 0$



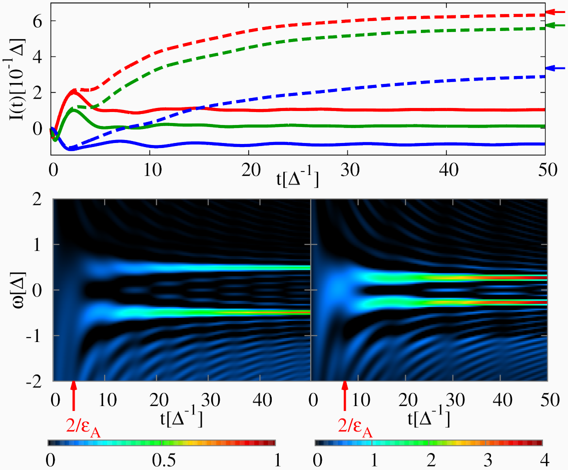
Subgap tunneling conductance $G_\sigma = \frac{\partial I_\sigma}{\partial t}$ vs time (t) and voltage (μ)

Quantum quench imposed on QD in Josephson junction geometry



R. Seoane Souto, A. Martín-Rodero, A. Levy Yeyati, Phys. Rev. Lett. 117, 267701 (2016).

Transient current and quasiparticle spectrum obtained for different ratios of Γ/Δ (from top to bottom: 10, 5 and 1).



R. Seoane Souto, A. Martín-Rodero, A. Levy Yeyati, Phys. Rev. Lett. <u>117</u>, 267701 (2016).

IN-GAP STATES OF CLASSICAL IMPURITY

communications
physics

ARTICLE

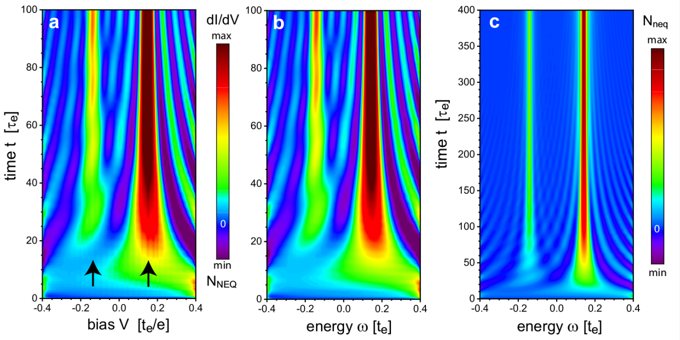
Check for updates

https://doi.org/10.1038/s42005-022-01050-7

OPEN

Emergence and manipulation of non-equilibrium Yu-Shiba-Rusinov states

Jasmin Bedow [1], Eric Mascot [1,2] & Dirk K. Morr [1]✉



Electron pairing vs Coulomb repulsion

Electron pairing vs Coulomb repulsion

[singlet-doublet quantum phase transition]

SINGLY OCCUPIED VS BCS-TYPE CONFIGURATIONS

Quantum dot proximitized to superconductor can described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left(\Gamma_S \, \hat{d}^{\dagger}_{\uparrow} \hat{d}^{\dagger}_{\downarrow} + \text{h.c.} \right)$$

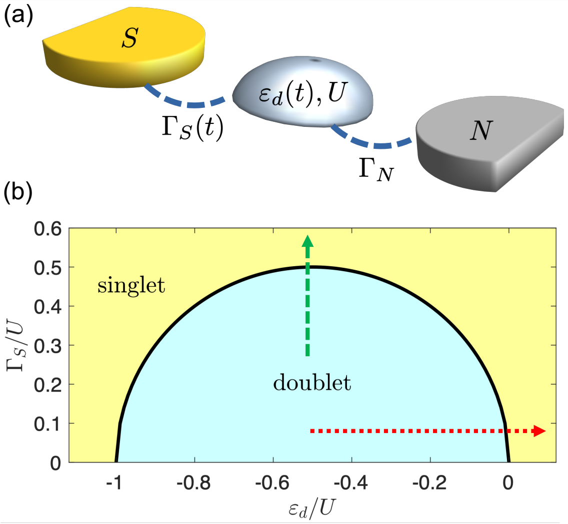
Eigen-states of this problem are represented by:

$|\uparrow\rangle$ **and** $|\downarrow\rangle$ $\Leftarrow$ **doublet states (spin $\frac{1}{2}$)**

$$\left. \begin{array}{l} u\,|0\rangle - v\,|\uparrow\downarrow\rangle \\ v\,|0\rangle + u\,|\uparrow\downarrow\rangle \end{array} \right\} \Leftarrow$$ **singlet states (spin 0)**

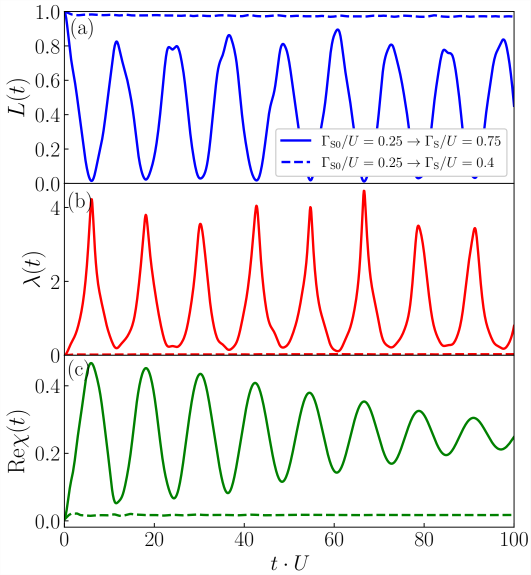
Upon varying the ratio ε_d / U_d or Γ_S / U_d the doublet-singlet transition can be induced between these ground states.

QUENCH ACROSS QPT BOUNDARY



K. Wrześniewski, I. Weymann, N. Sedlmayr & T. Domański, Phys. Rev. B 105, 094514 (2022).

ABRUPT CHANGE OF Γ_S: t-NRG RESULTS



$\varepsilon_d = -U/2$

$\Gamma_N = U/100$

K. Wrześniewski, I. Weymann, N. Sedlmayr & T. Domański, Phys. Rev. B 105, 094514 (2022).

CONCLUSIONS (PART 1)

Quantum impurity contacted with bulk superconductor:

- **induces the Rabi-type oscillations**

  (due to mixing of particle and hole degrees of freedom)

- **leading to emergence of the in-gap states**

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- **induces dynamical quantum phase transition**

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Quantum impurity contacted with bulk superconductor:

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- **leading to emergence of the in-gap states**

Quench imposed across the singlet-doublet boundary:

- **induces dynamical quantum phase transition**

  (manifested by nonanalyticties at critical time-instants)

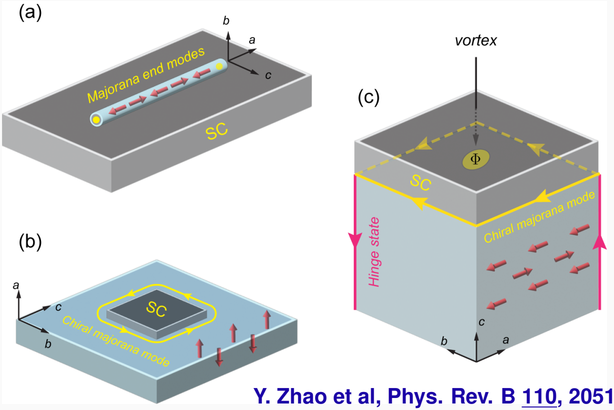
These phenomena are detectable by time-resolved Andreev spectroscopy.

Part 2. topological superconductors

(Majorana-type quasiparticles)

MAJORANA QUASIPARTICLES

Boundary modes (localized, chiral or Hinge states) existing in topological superconductors of different dimensionality.



Y. Zhao et al, Phys. Rev. B 110, 205111 (2024).

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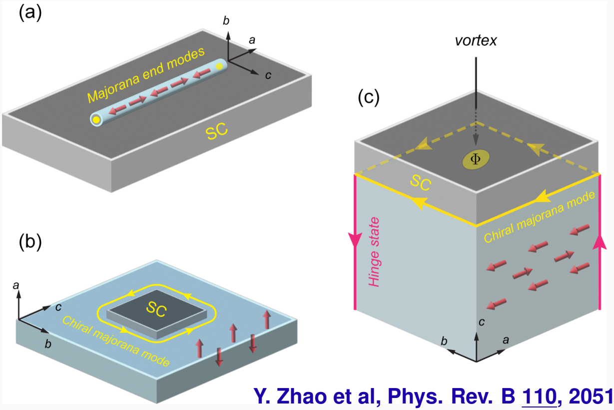
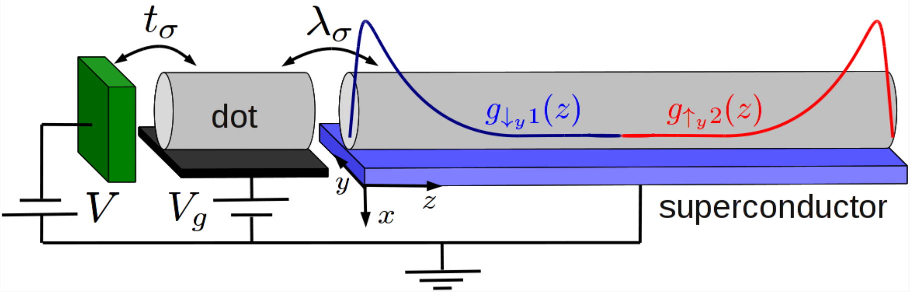


Y. Zhao et al, Phys. Rev. B 110, 205111 (2024).

These modes can be detected by charge tunneling spectroscopy.

MAJORANA MODE LEAKAGE ONTO QD

Hybrid structure: **quantum dot + topological superconductor**



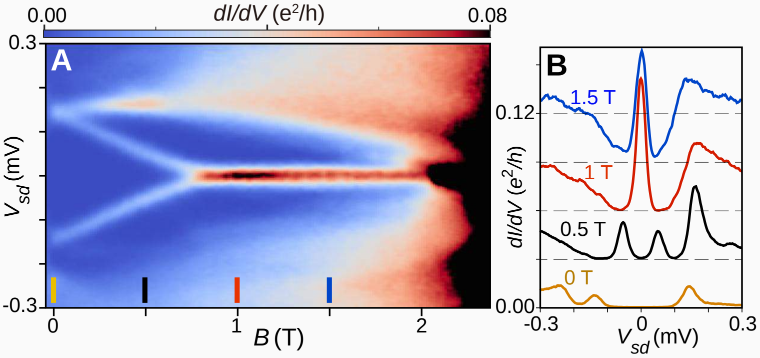
Idea: **Majorana mode is partly transferred onto quantum dot where it can be detected by tunneling spectroscopy**

M. Leijnse and K. Flensberg, Phys. Rev. B 84, 140501(R) (2011).

EVIDENCE FOR MAJORANA LEAKAGE

Panel (A): **Tunneling spectrum for resonant dot-wire coupling obtained at $V_{bg} = -8.5$ V, $V_{g1} = 22$ V, and $V_{g2} = V_{g3} = -10$ V.**

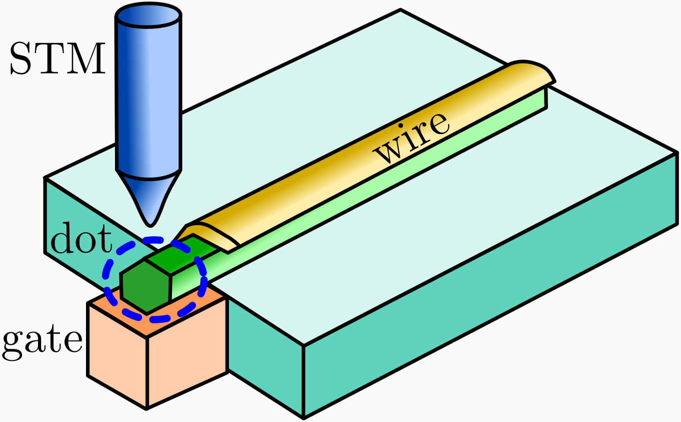
Panel (B): **Differential conductance at various values of the magnetic field.**



M.T. Deng et al, Science 354, 1557 (2016).

GATE-CONTROLLED BOUND STATES

Hybrid structure: **trivial + topological segments of nanowire**

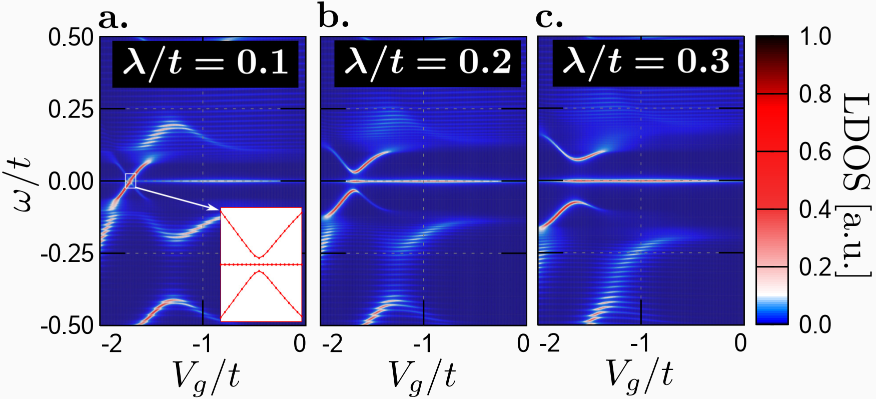


Issue: **bound states of trivial segment attached to topological sc**

A. Ptok, A. Kobiałka, T. Domański, Phys. Rev. B 96, 195430 (2017).

GATE-CONTROLLED BOUND STATES

Hybrid structure: trivial + topological segments of nanowire



Variation the trivial (Andreev) & topological (Majorana) states vs the gate potential V_g for several spin-orbit couplings λ.

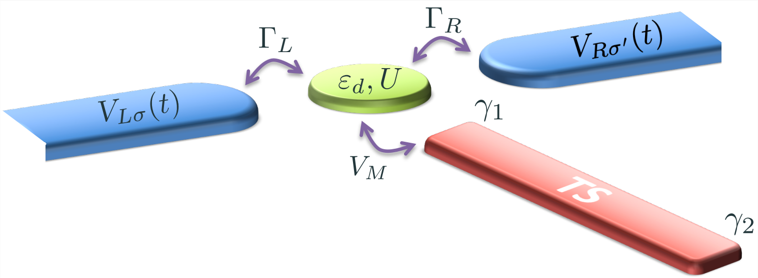
A. Ptok, A. Kobiałka, T. Domański, Phys. Rev. B 96, 195430 (2017).

Kondo vs Majorana

(means to distinguish them)

EXPERIMENTALLY FEASIBLE SETUP

Quantum dot coupled to the topological nanowire under ac-voltage



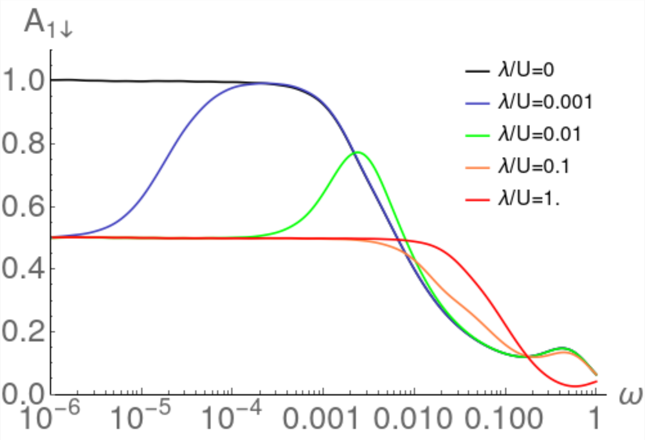
Question:

Can we resolve Majorana and Kondo states in ac-response ?

K.P. Wójcik, T. Domański, I. Weymann, Phys. Rev. B 109, 075432 (2024).

MAJORANA-KONDO INTERPLAY

Topological nanowire + quantum dot + metallic electrode



Spin-↓ spectrum: Kondo peak is strongly reshaped by Majorana

NRG results obtained by K.P. Wójcik (2024) in agreement with E. Prada et al, PRB (2014).

MAJORANA-KONDO INTERPLAY

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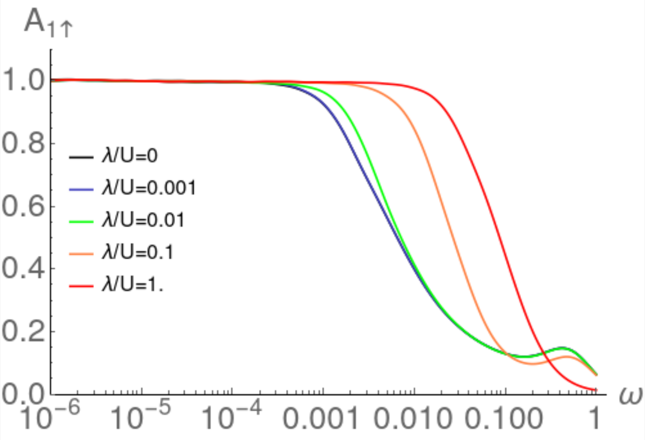
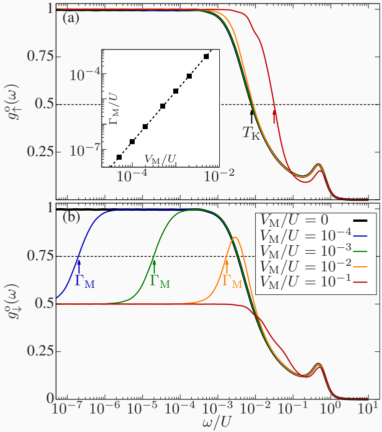


Spin-↑ spectrum: Kondo peak is nearly unaffected by Majorana

NRG results obtained by K.P. Wójcik (2024) in agreement with E. Prada et al, PRB (2014).

MAJORANA SIGNATURES IN AC-CONDUCTANCE

The frequency dependent conductance of *ac*-driven junction



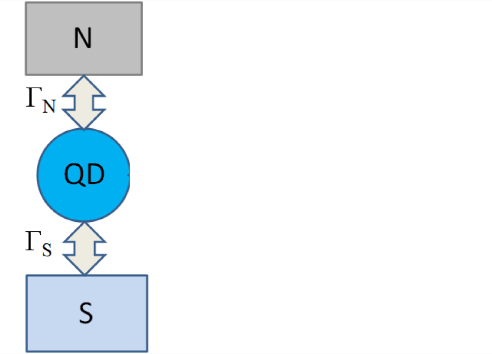
Spin-resolved conductances: **Signatures of the Coulomb peak and the Kondo effect can be clearly distinguished at finite-frequencies.**

Time - resolved effects

(affecting Majorana leakage)

TIME-RESOLVED LEAKAGE OF MAJORANA MODE

Hybrid structure: quantum dot attached to topological nanowire



Question:

How much time does it take to transfer the Majorana mode on QD ?

J. Barański, M. Barańska, T. Zienkiewicz, R. Taranko, T.Domański, PRB 103, 235416 (2021).

TIME-RESOLVED LEAKAGE OF MAJORANA MODE

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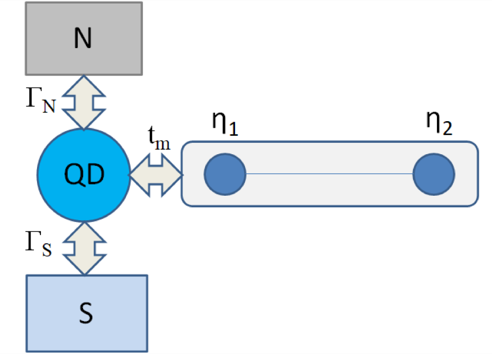


Question:

How much time does it take to transfer the Majorana mode on QD ?

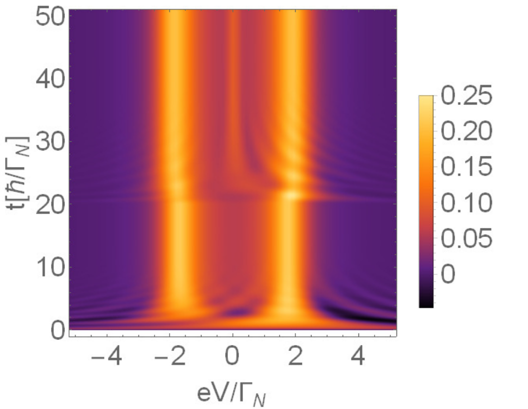
J. Barański, M. Barańska, T. Zienkiewicz, R. Taranko, T.Domański, PRB 103, 235416 (2021).

TIME-RESOLVED LEAKAGE OF MAJORANA MODE

Transient effects:

$\Rightarrow$ at $t = 0$ QD is coupled to the external N and S electrodes,

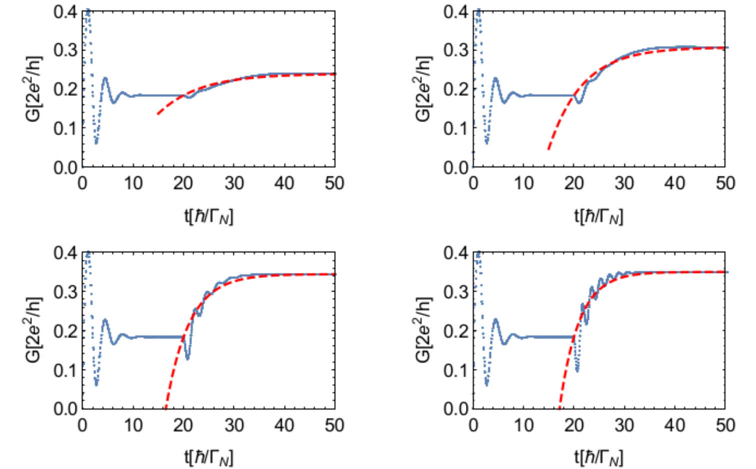
$\Rightarrow$ at $t = 20$ topological nanowire is attached to N-QD-S setup.



Gradual development of the trivial (Andreev) and topological (Majorana) states manifested in the differential conductance.

TIME-RESOLVED LEAKAGE OF MAJORANA MODE

Time-dependent zero-bias conductance



Majorana zero-bias feature establishes in about nanoseconds.

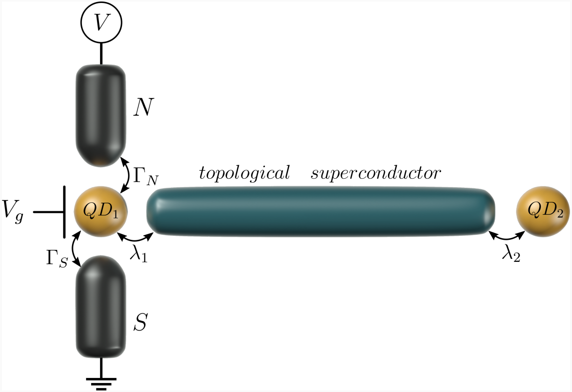
J. Barański, M. Barańska, T. Zienkiewicz, R. Taranko, T.Domański, PRB 103, 235416 (2021).

Nonlocal cross-correlations ?

/ transmitted via Majorana modes /

DYNAMICAL CROSS-CORRELATIONS

Two quantum dots interconnected via topological superconductor

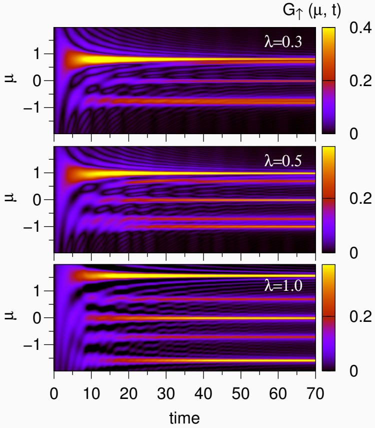


Question: Is nonlocal communication transmitted between QD_1 and QD_2 through the Majorana boundary modes ?

R. Taranko, K. Wrześniewski, I. Weymann, T. Domański, Phys. Rev. B 110, 035413 (2024).

TIME-RESOLVED CONDUCTANCE

Time-dependent conductance of the biased N-QD$_1$-S junction

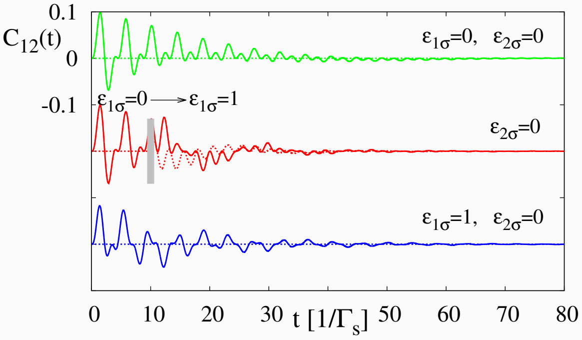


Signatures of the (trivial) molecular bound states and (topological) Majorana mode obtained for $\varepsilon_1 = 0$, $\varepsilon_2 = 2$.

R. Taranko, K. Wrześniewski, I. Weymann, T. Domański, Phys. Rev. B 110, 035413 (2024).

NONLOCAL CROSS-CORRELATIONS

Evolution of the interdot electron pairing $C_{12}(t) = \left\langle \hat{d}_{1\downarrow}\hat{d}_{2\uparrow} \right\rangle$



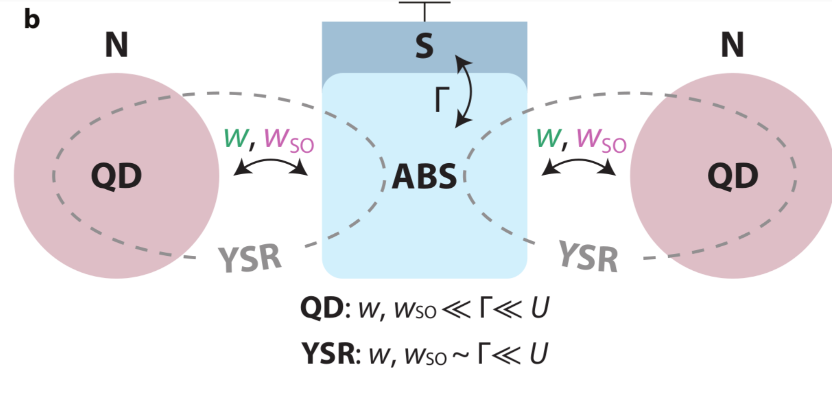
The nonlocal electron pairing persists only over a short transient time-scale. It could be detected by crossed Andreev refelections.

R. Taranko, K. Wrześniewski, I. Weymann, T. Domański, Phys. Rev. B 110, 035413 (2024).

Further developments

MINIMAL KITAEV CHAIN

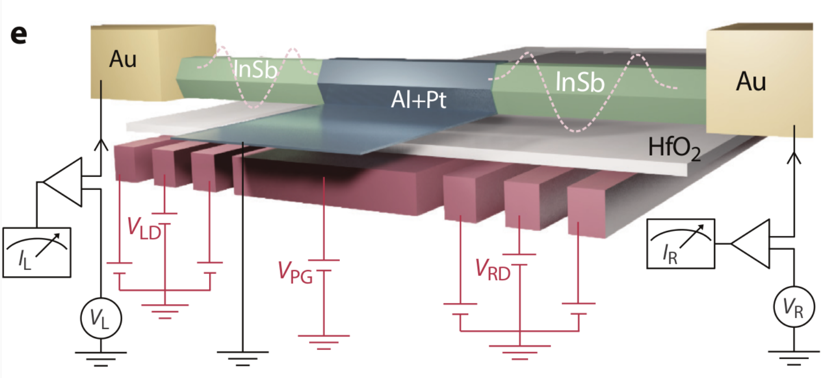
Effective triplet pairing can be realized using two quantum dots interconnected by superconductor (Poor Man's Majorana states)



T. Dvir, ... & L.P. Kouwenhoven, Nature 614, 445 (2023).

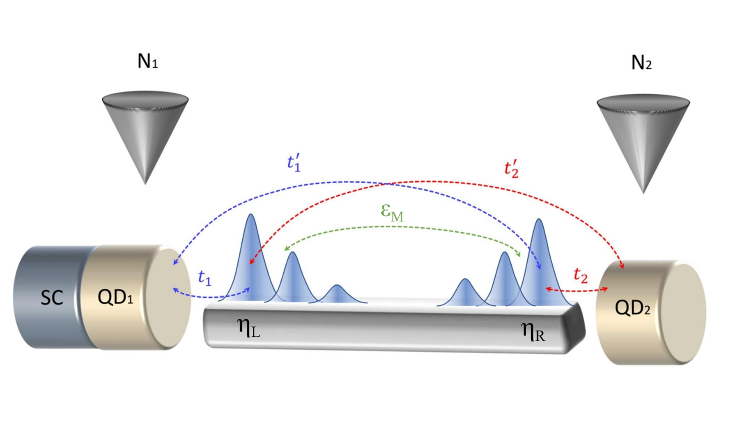
MINIMAL KITAEV CHAIN

Two spin-polarized quantum dots in an InSb nanowire strongly coupled by elastic co-tunneling and crossed Andreev reflection



T. Dvir, ... & L.P. Kouwenhoven, Nature 614, 445 (2023).

QUASIPARTICLE SPECTRUM OF QUANTUM DOTS



Issue: **Molecular quasiparticle spectrum of the quantum dots connected via the overlapping Majorana modes**

G. Górski, K.P. Wójcik, J. Barański, I. Weymann & T. Domański, Sci. Rep. 14, 13848 (2024).

QUANTUM ENTANGLEMENT OF DOUBLE DOTS

Setup: **Quantum dots interconnected via short topological nanowire**

Issue: **Quantum dots become entangled via Majorana qps**

C. Jasiukiewicz, A. Sinner, I. Weymann, T. Domański, & L. Chotorlishvili, PRB 111, 075415 (2025).

Similar results: $\Longrightarrow$ V.K. Vimal and J. Cayao, Phys. Rev. B 110, 224510 (2024).

QUANTUM ENTANGLEMENT OF DOUBLE DOTS

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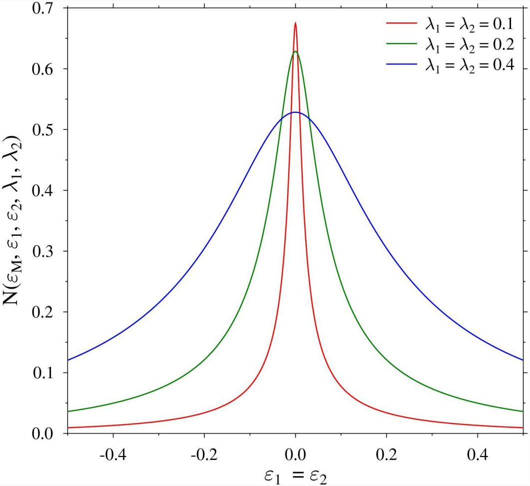

Logarithmic negativity versus the energy levels of QD's for $\varepsilon_M \neq 0$.

CONCLUSIONS (PART 2)

Quantum dots attached to topological superconductors:

$\Longrightarrow$ **enable leakage of the Majorana modes**

(preserving topological protection),

$\Longrightarrow$ **reveal nonlocal cross-correlations**

(under non-equilibrium conditions),

$\Longrightarrow$ **acquire quantum entanglement**

(via short topological superconductors).