Nonequilibrium phenomena on interfaces with topological superconductors

Tadeusz Domański

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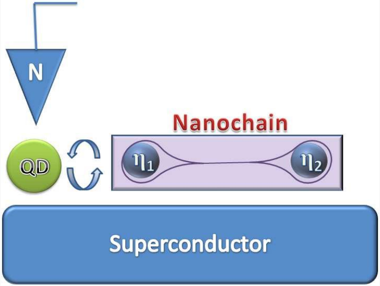
Lublin, Poland

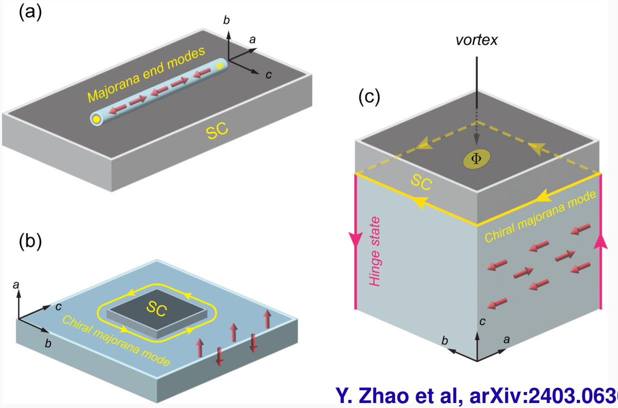


MagTop Focused Expert Meeting

Warsaw, 23/10/2024

MOTIVATION

The boundary modes (localized, chiral or Hinge states) of topological superconductors realized in different dimensions



Y. Zhao et al, arXiv:2403.06304 (2024).

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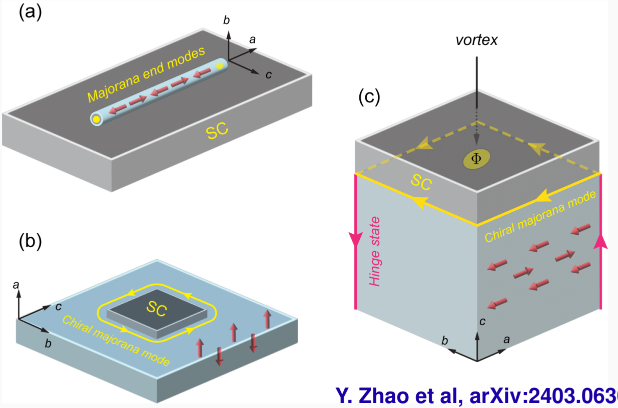

Y. Zhao et al, arXiv:2403.06304 (2024).

can be detected, using the charge tunneling spectroscopies (with attachment of external electrodes) in nonequilibrium conditions.

Topological superconductors are connected to other (topologically trivial) objects:

$\Longrightarrow$ **through some interface**

$\Longrightarrow$ **which affects the edge modes.**

HYBRID STRUCTURES

Topological superconductors are connected to other (topologically trivial) objects:

$\Longrightarrow$ **through some interface**

$\Longrightarrow$ **which affects the edge modes.**

The simplest situation could captured by:

$\Longrightarrow$ **single-level impurity + Majorana mode(s).**

EARLY PROPOSALS

Hybrid structure: quantum dot + topological superconductors

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PHYSICAL REVIEW B **84**, 140501(R) (2011)

Scheme to measure Majorana fermion lifetimes using a quantum dot

Martin Leijnse and Karsten Flensberg
Nano-Science Center & Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark

(Received 30 August 2011; published 3 October 2011)

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PHYSICAL REVIEW B **84**, 201308(R) (2011)

Detecting a Majorana-fermion zero mode using a quantum dot

Dong E. Liu and Harold U. Baranger
Department of Physics, Duke University, Box 90305, Durham, North Carolina 27708-0305, USA
(Received 22 July 2011; revised manuscript received 13 September 2011; published 16 November 2011)

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(Received 22 July 2011; revised manuscript received 13 September 2011; published 16 November 2011)

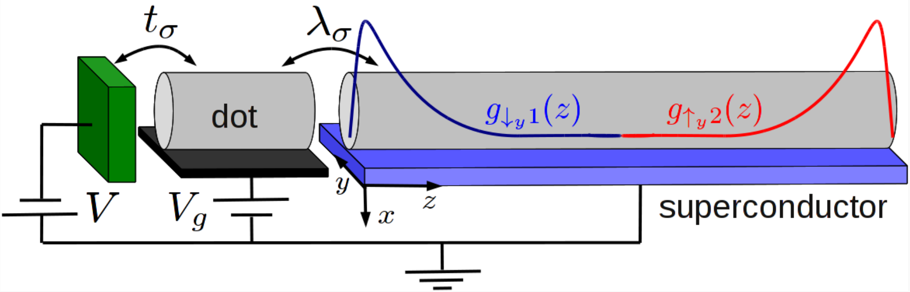
PHYSICAL REVIEW B **89**, 165314 (2014)

Subtle leakage of a Majorana mode into a quantum dot

E. Vernek,[1,2] P. H. Penteado,[2] A. C. Seridonio,[3] and J. C. Egues[2]
[1]*Instituto de Física, Universidade Federal de Uberlândia, Uberlândia, Minas Gerais 38400-902, Brazil*
[2]*Instituto de Física de São Carlos, Universidade de São Paulo, São Carlos, São Paulo 13560-970, Brazil*
[3]*Departamento de Física e Química, Universidade Estadual Paulista, Ilha Solteira, São Paulo 15385-000, Brazil*
(Received 15 August 2013; revised manuscript received 10 April 2014; published 30 April 2014)

MAJORANA MODE LEAKAGE ONTO QD

Hybrid structure: quantum dot + topological superconductor

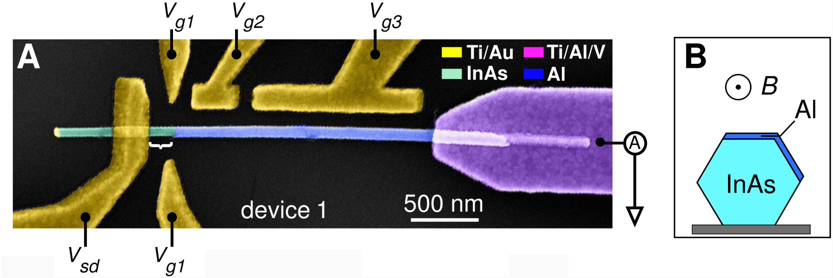


Idea: Majorana mode is partly transferred onto quantum dot where it can be detected by tunneling spectroscopy

M. Leijnse and K. Flensberg, Phys. Rev. B 84, 140501(R) (2011).

EVIDENCE FOR MAJORANA LEAKAGE

Setup: **Epitaxial Al shell (blue) grown on two facets of the hexagonal InAs core (cyan), with a thickness of $\sim$ 10 nm.**



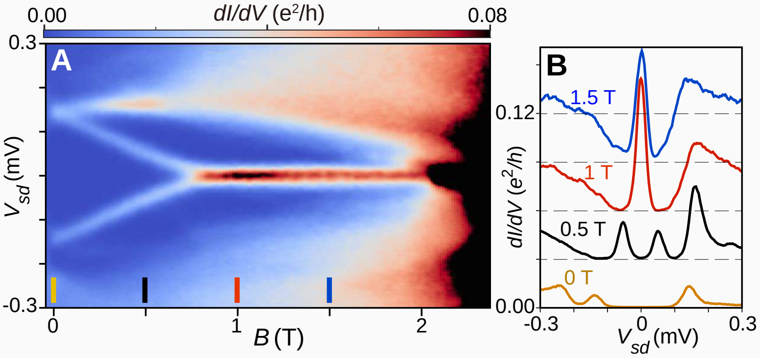
Data: **Transport measurements have been collected, varying the magnetic field oriented parallelly to the nanowire.**

M.T. Deng et al, Science 354, 1557 (2016).

EVIDENCE FOR MAJORANA LEAKAGE

Panel (A): **Tunneling spectrum for resonant dot-wire coupling obtained at $V_{bg} = -8.5$ V, $V_{g1} = 22$ V, and $V_{g2} = V_{g3} = -10$ V.**

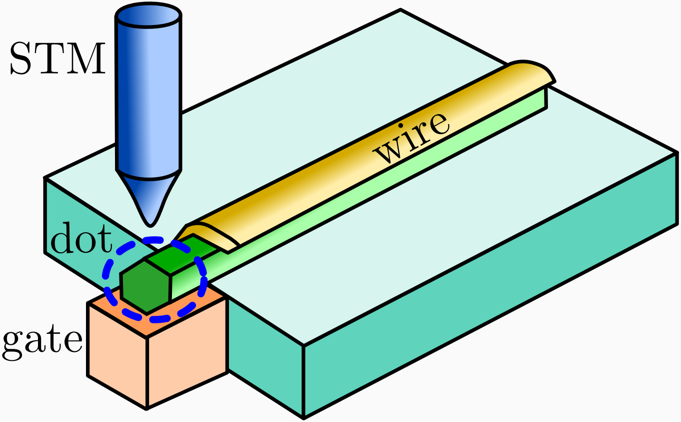
Panel (B): **Differential conductance at various values of the magnetic field.**



M.T. Deng et al, Science 354, 1557 (2016).

GATE-CONTROLLED BOUND STATES

Hybrid structure: **trivial + topological segments of nanowire**

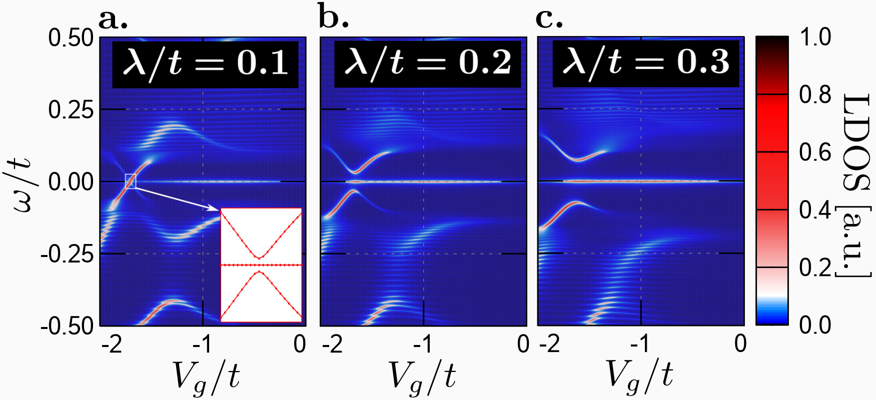


Issue: **bound states of trivial segment attached to topological sc**

A. Ptok, A. Kobiałka, T. Domański, Phys. Rev. B <u>96</u>, 195430 (2017).

GATE-CONTROLLED BOUND STATES

Hybrid structure: trivial + topological segments of nanowire



Variation the trivial (Andreev) & topological (Majorana) states vs the gate potential V_g for several spin-orbit couplings λ.

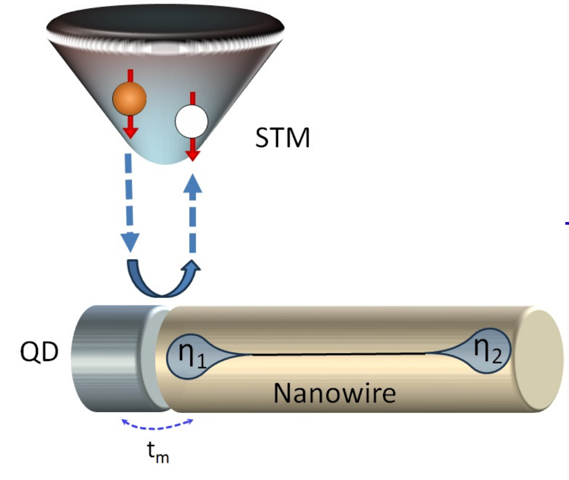
A. Ptok, A. Kobiałka, T. Domański, Phys. Rev. B 96, 195430 (2017).

What about the correlations ?

/ due to the Coulomb repulsion /

CORRELATIONS VS LEAKAGE

Hybrid structure: **Anderson impurity + topological superconductor**

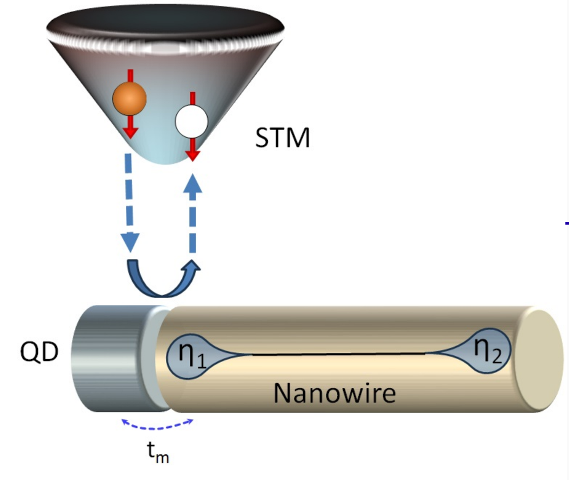


T. Domański et al, (2024).

Question: **Does the Coulomb repulsion affect the Majorana mode(s) leakage ?**

CORRELATIONS VS LEAKAGE

Hybrid structure: Anderson impurity + topological superconductor



T. Domański et al, (2024).

Question: Does the Coulomb repulsion affect the Majorana mode(s) leakage ? Is there any competition ?

LOW ENERGY SCENARIO

For microscopic considerations we used the Anderson-type model

$$\hat{H} = \hat{H}_{QD} + \lambda(\hat{d}_{\downarrow}^{\dagger}\hat{\eta}_1 + \hat{\eta}_1\hat{d}_{\downarrow}) + i\epsilon_m\hat{\eta}_1\hat{\eta}_2$$

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where the correlated quantum dot is described by

$$\hat{H}_{QD} = \sum_{\sigma} \varepsilon_d \hat{d}_{\sigma}^{\dagger}\hat{d}_{\sigma} + U_d \hat{n}_{\uparrow}\hat{n}_{\downarrow}$$

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recasting the Majorana operators in terms of conventional fermions

$$\hat{\eta}_1 = \frac{1}{\sqrt{2}}(\hat{f}^{\dagger} + \hat{f})$$

LOW ENERGY SCENARIO

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recasting the Majorana operators in terms of conventional fermions

$$\hat{\eta}_1 = \frac{1}{\sqrt{2}}(\hat{f}^{\dagger} + \hat{f})$$ and $$\hat{\eta}_2 = \frac{i}{\sqrt{2}}(\hat{f}^{\dagger} - \hat{f})$$

LOW ENERGY SCENARIO

For microscopic considerations we used the Anderson-type model

$$\hat{H} = \hat{H}_{QD} + \lambda(\hat{d}^\dagger_\downarrow \hat{\eta}_1 + \hat{\eta}_1 \hat{d}_\downarrow) + i\epsilon_m \hat{\eta}_1 \hat{\eta}_2$$

where the correlated quantum dot is described by

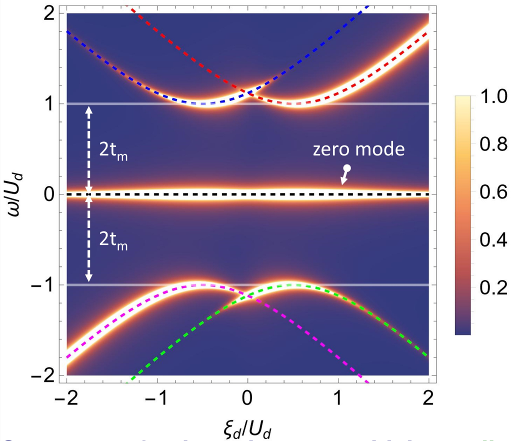
$$\hat{H}_{QD} = \sum_\sigma \varepsilon_d \hat{d}^\dagger_\sigma \hat{d}_\sigma + U_d \hat{n}_\uparrow \hat{n}_\downarrow$$

recasting the Majorana operators in terms of conventional fermions

$$\hat{\eta}_1 = \frac{1}{\sqrt{2}}(\hat{f}^\dagger + \hat{f}) \quad \text{and} \quad \hat{\eta}_2 = \frac{i}{\sqrt{2}}(\hat{f}^\dagger - \hat{f})$$

Quasiparticle states of the quantum dot can be determined analytically.

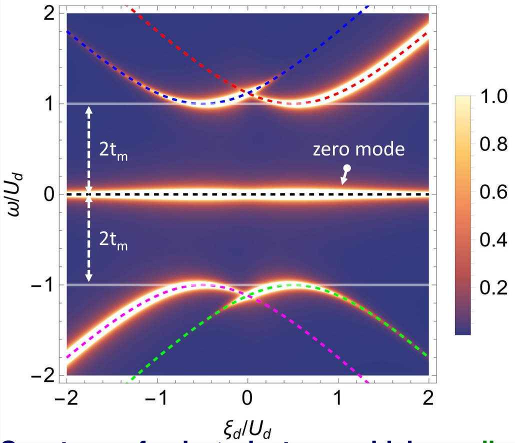
Hybrid structure: Anderson impurity + topological nanowire



T. Domański et al, (2024).

Spectrum of spin-↓ electrons which are directly coupled to the Majorana mode.

Hybrid structure: Anderson impurity + topological nanowire

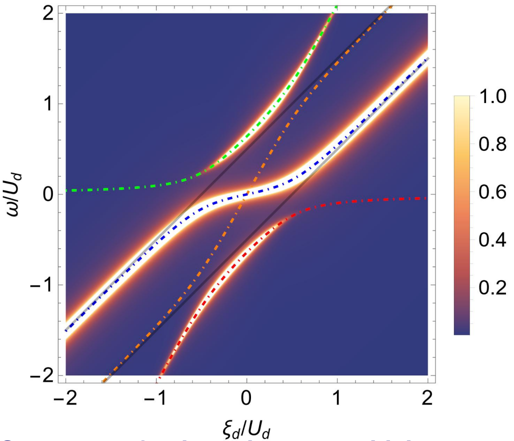


T. Domański et al, (2024).

Spectrum of spin-↓ electrons which are directly coupled to the Majorana mode. Zero-energy mode appears near ϵ_d and $\epsilon_d + U_d$.

SPIN-SENSITIVE LEAKAGE

Hybrid structure: Anderson impurity + topological nanowire

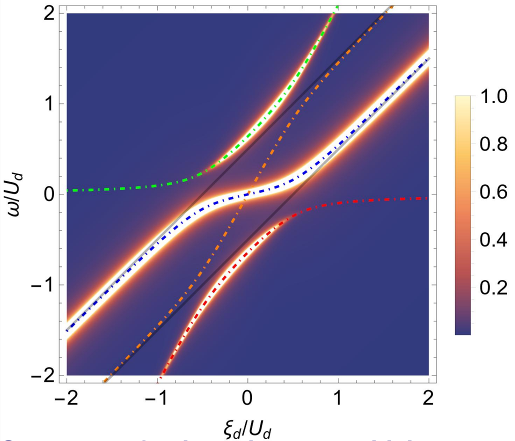


T. Domański et al, (2024).

Spectrum of spin-↑ electrons which are not directly coupled to the Majorana mode.

SPIN-SENSITIVE LEAKAGE

Hybrid structure: Anderson impurity + topological nanowire



T. Domański et al, (2024).

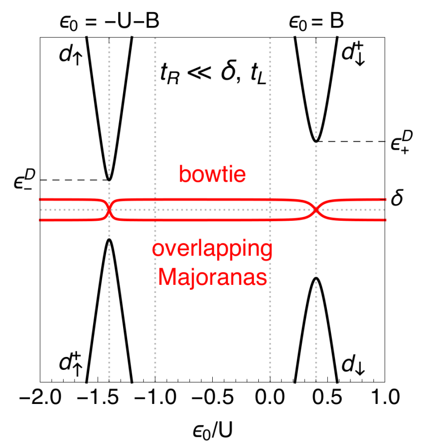
Spectrum of spin-↑ electrons which are not directly coupled to the Majorana mode. Majorana features are missing.

Short topological nanowire

/ overlapping Majorana modes /

OVERLAPPING MAJORANA MODES, $\epsilon_M \neq 0$

Hybrid structure: quantum impurity + short topological nanowire

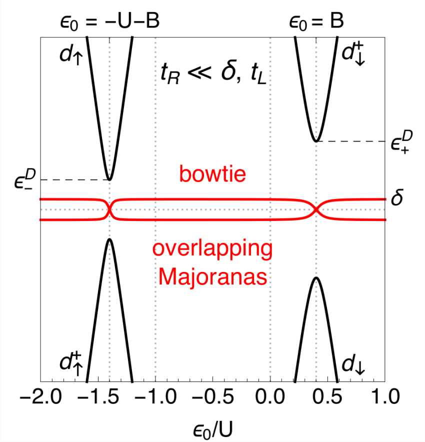


E. Prada et al, PRB 96, 085418 (2017).

Quasiparticle spectrum of the quantum dot obtained for $\epsilon_M \neq 0$.

OVERLAPPING MAJORANA MODES, $\epsilon_M \neq 0$

Hybrid structure: quantum impurity + short topological nanowire



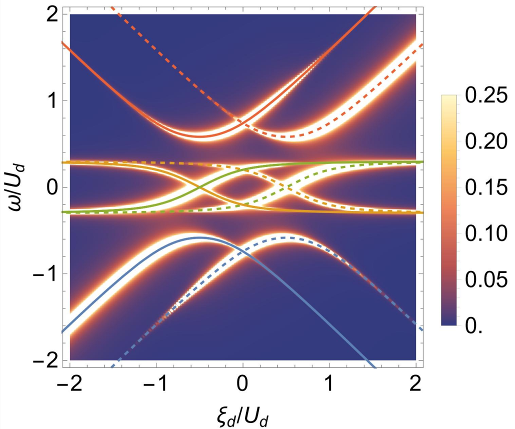
E. Prada et al, PRB 96, 085418 (2017).

Quasiparticle spectrum of the quantum dot obtained for $\epsilon_M \neq 0$.

Notice: bowtie features near the crossing points.

OVERLAPPING MAJORANA MODES, $\epsilon_M \neq 0$

Hybrid structure: Anderson impurity + short topological nanowire

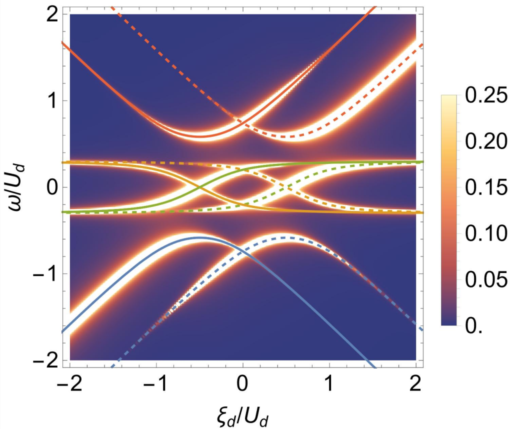


T. Domański et al, (2024).

Quasiparticle spectrum of spin-↓ electrons obtained for $\epsilon_M \neq 0$.

OVERLAPPING MAJORANA MODES, $\epsilon_M \neq 0$

Hybrid structure: Anderson impurity + short topological nanowire



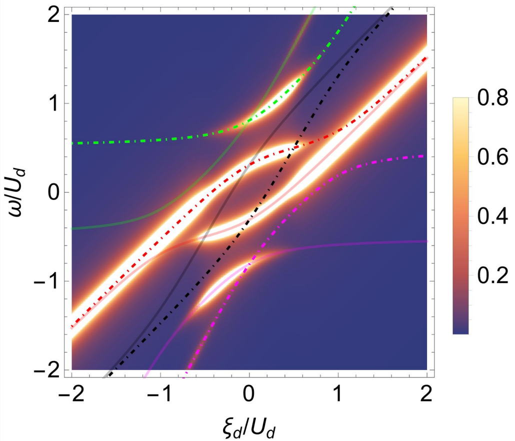
T. Domański et al, (2024).

Quasiparticle spectrum of spin-↓ electrons obtained for $\epsilon_M \neq 0$.

Appearance of two bowtie features inside the topological gap.

OVERLAPPING MAJORANA MODES, $\epsilon_M \neq 0$

Hybrid structure: Anderson impurity + short topological nanowire

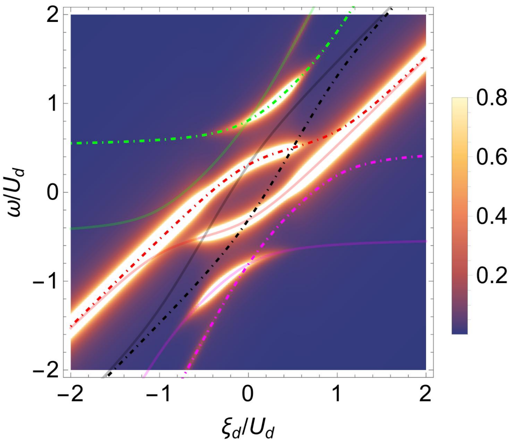


T. Domański et al, (2024).

Quasiparticle spectrum of spin-↑ electrons obtained for $\epsilon_M \neq 0$.

OVERLAPPING MAJORANA MODES, $\epsilon_M \neq 0$

Hybrid structure: Anderson impurity + short topological nanowire



T. Domański et al, (2024).

Quasiparticle spectrum of spin-↑ electrons obtained for $\epsilon_M \neq 0$.

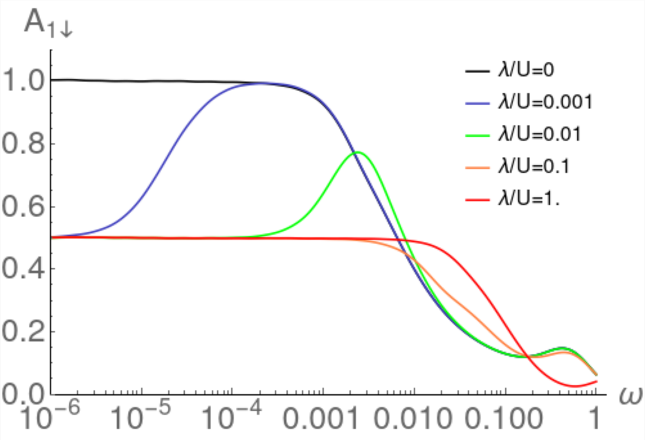
Majorana quasiparticles are completely absent.

Kondo vs Majorana

(means to distinguish them)

MAJORANA-KONDO INTERPLAY

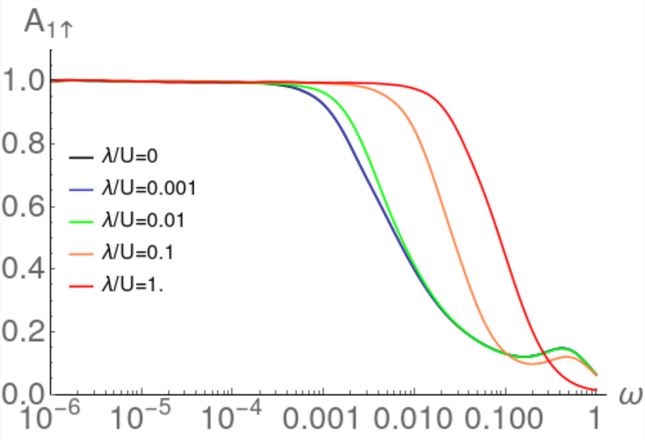
Topological nanowire + quantum dot + metallic electrode



Spin-↓ spectrum: Kondo peak is strongly reshaped by Majorana

NRG results obtained by K.P. Wójcik (2024) in agreement with E. Prada et al, PRB (2014).

Topological nanowire + quantum dot + metallic electrode

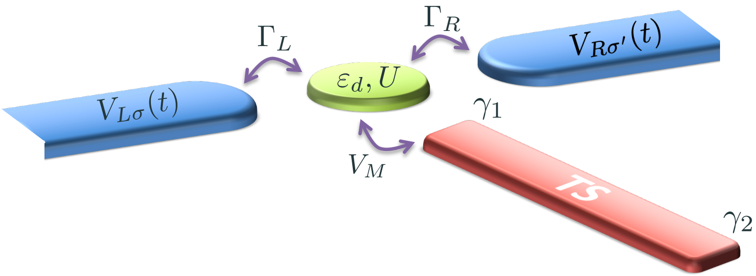


Spin-↑ spectrum: Kondo peak is nearly unaffected by Majorana

NRG results obtained by K.P. Wójcik (2024) in agreement with E. Prada et al, PRB (2014).

MAJORANA SIGNATURES IN AC-CONDUCTANCE

Quantum dot coupled to the topological nanowire under ac-voltage

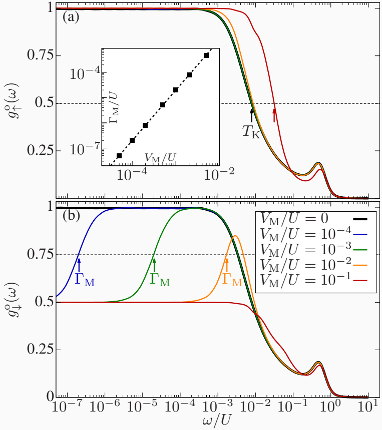


Question:

Can we resolve Majorana and Kondo states in ac-response ?

K.P. Wójcik, T. Domański, I. Weymann, Phys. Rev. B 109, 075432 (2024).

The frequency dependent conductance of *ac*-driven junction



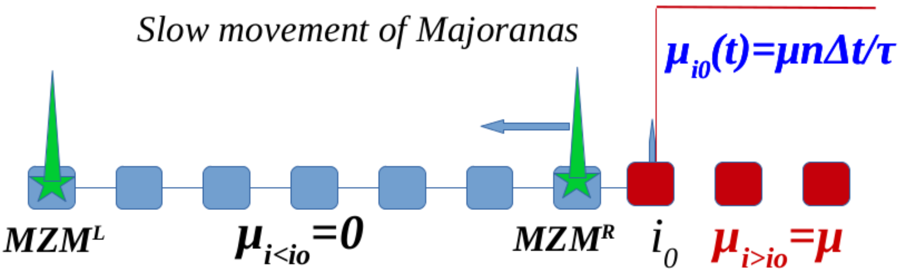
Spin-resolved conductances: **Signatures of the Coulomb peak and the Kondo effect can be clearly distinguished at finite-frequencies.**

Time - resolved effects

(related with Majorana modes)

RELOCATION OF MAJORANAS

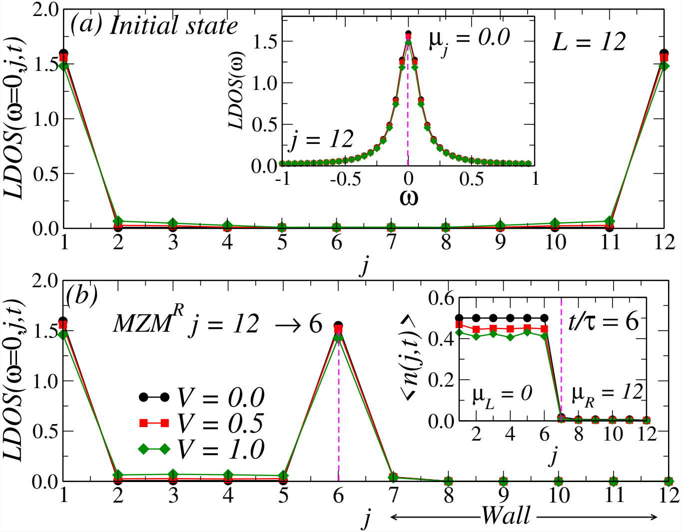
Hybrid structure: **switching on/off topological phase**



Issue: **gate-imposed relocation of the Majorana mode**

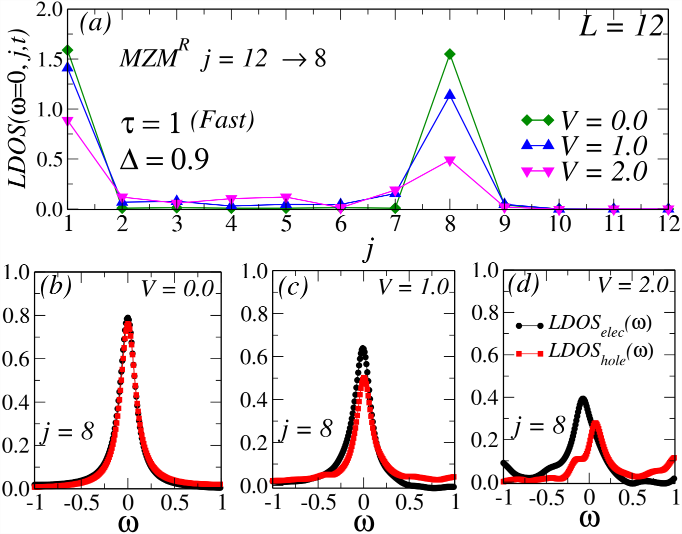
B. Pandey, L. Mohanta and E. Dagotto, Phys. Rev. B 107, L060304 (2023).

Hybrid structure: slow Majorana relocation 12 ⟶ 6



RELOCATION OF MAJORANAS

Hybrid structure: fast Majorana relocation 12 $\longrightarrow$ 6

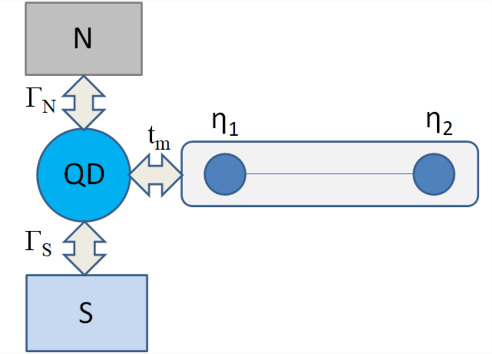


What are typical time-scales ?

/ for transferring Majorana modes /

TIME-RESOLVED LEAKAGE OF MAJORANA MODE

Hybrid structure: **quantum dot attached to topological nanowire**



Question:

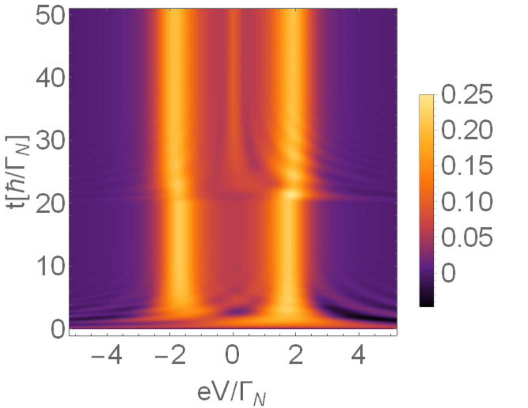
How much time does it take to transfer the Majorana mode on QD ?

J. Barański, M. Barańska, T. Zienkiewicz, R. Taranko, T.Domański, PRB 103, 235416 (2021).

Transient effects:

$\Rightarrow$ **at $t = 0$ QD is coupled to the external N and S electrodes,**

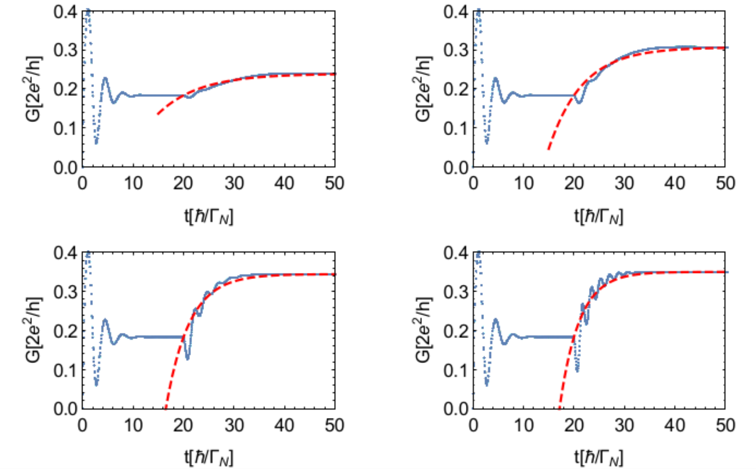
$\Rightarrow$ **at $t = 10$ topological nanowire is attached to N-QD-S setup.**



Gradual development of the trivial (Andreev) and topological (Majorana) states manifested in the differential conductance.

TIME-RESOLVED LEAKAGE OF MAJORANA MODE

Time-dependent zero-bias conductance



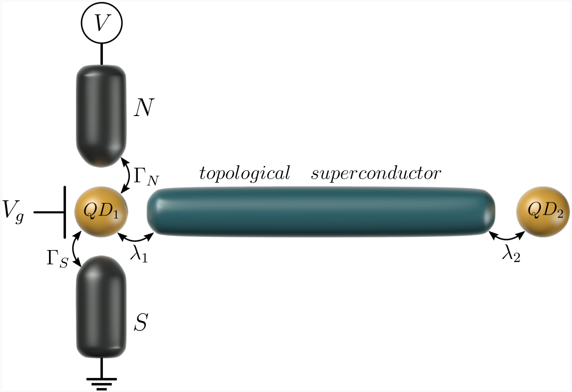
Majorana zero-bias feature establishes in about nanoseconds.

J. Barański, M. Barańska, T. Zienkiewicz, R. Taranko, T.Domański, PRB 103, 235416 (2021).

Are there distant cross-correlations ?

/ transmitted via Majorana modes /

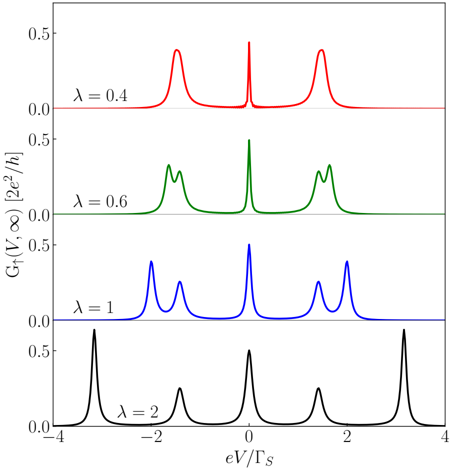
Two quantum dots interconnected via topological superconductor



Question: Is any nonlocal communication transmitted between QD_1 and QD_2 through the Majorana boundary modes ?

R. Taranko, K. Wrześniewski, I. Weymann, T. Domański, Phys. Rev. B 110, 035413 (2024).

STEADY-LIMIT CONDUCTANCE

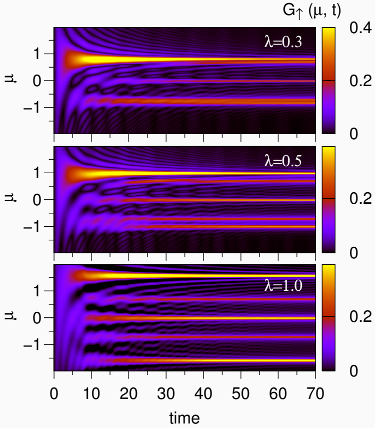


Differential conductance $G(V, t \to \infty)$ versus bias V for several couplings λ between QD$_{1,2}$ and topological superconductor.

R. Taranko, K. Wrześniewski, I. Weymann, T. Domański, Phys. Rev. B 110, 035413 (2024).

TIME-RESOLVED CONDUCTANCE

Time-dependent conductance of the biased N-QD$_1$-S junction

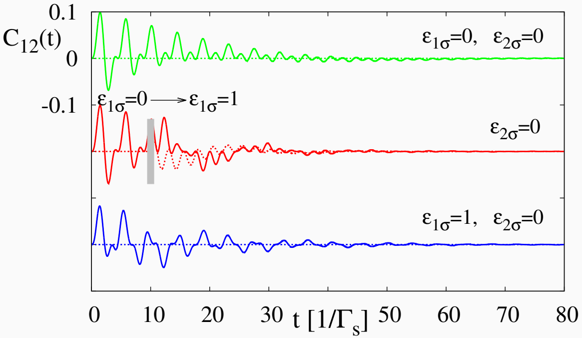


Signatures of the (trivial) molecular bound states and (topological) Majorana mode obtained for $\varepsilon_1 = 0$, $\varepsilon_2 = 2$.

R. Taranko, K. Wrześniewski, I. Weymann, T. Domański, Phys. Rev. B 110, 035413 (2024).

NONLOCAL CROSS-CORRELATIONS

Evolution of the interdot electron pairing $C_{12}(t) = \left\langle \hat{d}_{1\downarrow} \hat{d}_{2\uparrow} \right\rangle$



The nonlocal electron pairing persists only over a short transient time-scale. It could be detected by crossed Andreev refelections.

R. Taranko, K. Wrześniewski, I. Weymann, T. Domański, Phys. Rev. B 110, 035413 (2024).

CONCLUSIONS: PART 1

The uncorrelated/correlated quantum dots coupled via the (non-overlapping) Majorana modes:

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$\Longrightarrow$ **are distantly cross-correlated only briefly after attaching them to topological sc,**

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$\Longrightarrow$ **beyond this transient region they do not show any mutual interdependence**

CONCLUSIONS: PART 1

The uncorrelated/correlated quantum dots coupled via the (non-overlapping) Majorana modes:

$\Longrightarrow$ **are distantly cross-correlated only briefly after attaching them to topological sc,**

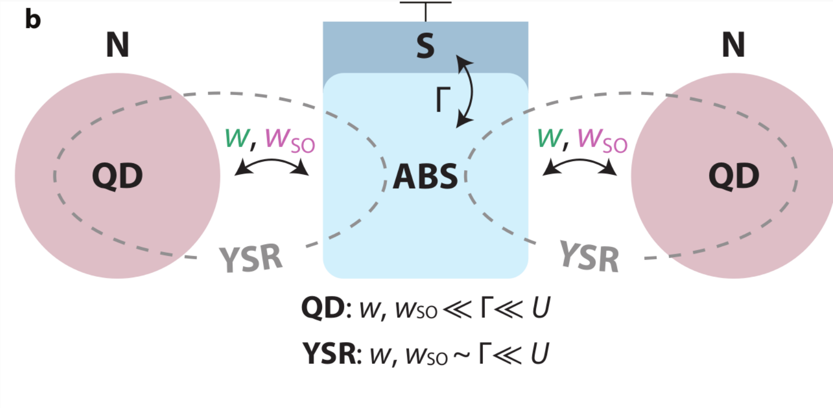
$\Longrightarrow$ **beyond this transient region they do not show any mutual interdependence**

$\Longrightarrow$ **charge teleportation and/or other nonlocal phenomena would be absent**

Further perspectives

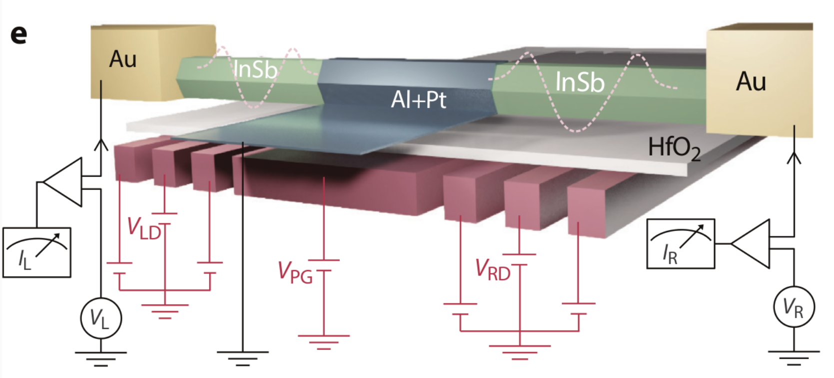
MINIMAL KITAEV CHAIN

Effective triplet pairing has been recently realized using two quantum dots interconnected by superconductor (Poor Man's Majorana states)



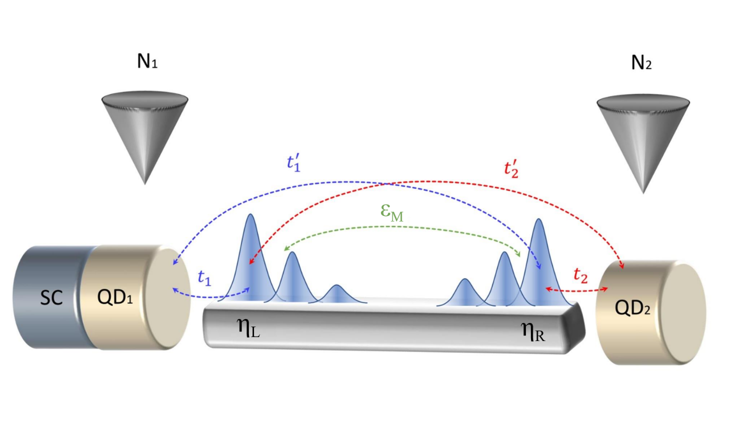
T. Dvir, ... & L.P. Kouwenhoven, Nature 614, 445 (2023).

Two spin-polarized quantum dots in an InSb nanowire strongly coupled by elastic co-tunneling and crossed Andreev reflection



T. Dvir, ... & L.P. Kouwenhoven, Nature 614, 445 (2023).

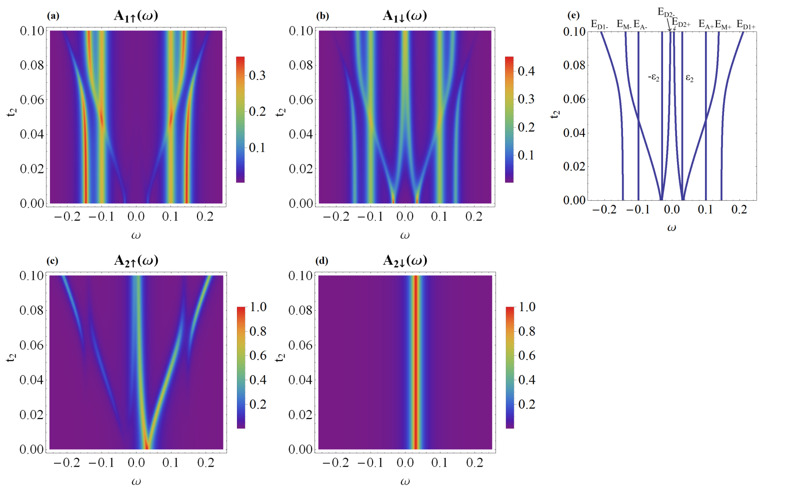
QUASIPARTICLE SPECTRUM OF QUANTUM DOTS



Issue: **Molecular spectrum of the quantum dots connected via the overlapping Majorana modes**

G. Górski, K.P. Wójcik, J. Barański, I. Weymann & T. Domański, Sci. Rep. 14, 13848 (2024).

QUASIPARTICLE SPECTRUM OF QUANTUM DOTS



The same quasiparticle states are present in both quantum dots , however, with very different spectral weights.

G. Górski, K.P. Wójcik, J. Barański, I. Weymann & T. Domański, Sci. Rep. 14, 13848 (2024).

QUANTUM ENTANGLEMENT OF DOUBLE DOTS

Setup: **Quantum dots interconnected via short topological nanowire**

Scientific issue:

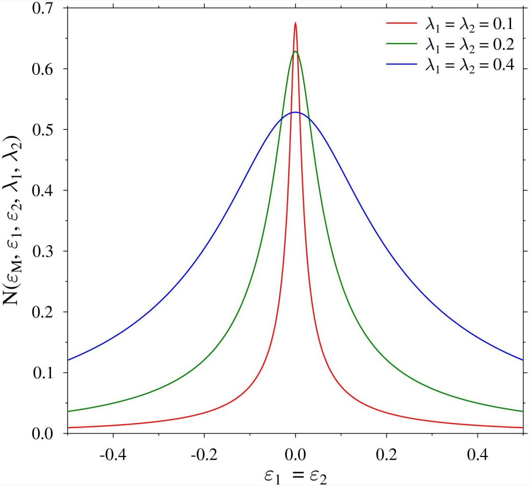
Entanglement of QD's quantified by their fermionic negativity

C. Jasiukiewicz, A. Sinner, I. Weymann, T. Domański & L. Chotorlishvili, (2024)

/to be submitted/.

QUANTUM ENTANGLEMENT OF DOUBLE DOTS

Setup: Quantum dots interconnected via short topological nanowire



Logarithmic negativity versus the energy levels QD's obtained for $\varepsilon_M \neq 0$.

FINAL CONCLUSIONS

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http://kft.umcs.lublin.pl/doman/lectures