Topological superconductivity and boundary modes of magnetic nanostructures

Tadeusz DOMAŃSKI

M. Curie-Skłodowska Univ. Lublin, Poland





MagTop (Warsaw)

18/Jan/2023

Magnetism vs superconductivity

BULK SUPERCONDUCTORS

Perfect conductor



BULK SUPERCONDUCTORS



Are they friends or foes ?

DESTRUCTIVE INFLUENCE OF MAGNETIC FIELD

Magnetism and electron pairing are rather antagonistic ...

DESTRUCTIVE INFLUENCE OF MAGNETIC FIELD

Magnetism and electron pairing are rather antagonistic ...



Magnetic field penetrates type-II superconductors (vortex-structure)

VORTEX IN SUPERCONDUCTOR





Vortex is a piece of *normal* region inside superconductor.

STATES BOUND TO VORTEX CORE

Volume 9, number 4

where

PHYSICS LETTERS

1 May 1964

BOUND FERMION STATES ON A VORTEX LINE IN A TYPE II SUPERCONDUCTOR

C. CAROLI, P. G. DE GENNES, J. MATRICON Service de Physique des Solides, Faculté des Sciences, Orsay (S & O)

Bound-states of the axisymmetric vortex in s-wave

superconductors take a form:

 $E_n = \omega_0 \left(n + rac{1}{2}
ight) \qquad n = 0, \pm 1, \pm 2, ...$ $\omega_0 pprox rac{\Delta^2}{E_F}$

VORTEX IN P-WAVE SUPERCONDUCTOR

JETP LETTERS

VOLUME 70, NUMBER 9

10 NOV. 1999

Fermion zero modes on vortices in chiral superconductors

G. E. Volovik

Helsinki University of Technology, Low Temperature Laboratory, FIN-02015 HUT, Finland; Landau Institute of Theoretical Physics, Russian Academy of Sciences, 117334 Moscow, Russia

(Submitted 30 September 1999) Pis'ma Zh. Éksp. Teor. Fiz. **70**, No. 9, 601–606 (10 November 1999)

Bound-states of the vortex in p-wave (triplet) superconductors are given by:

$$E_n = \omega_0 n \qquad n = 0, \pm 1, \pm 2, \dots$$

VORTEX IN P-WAVE SUPERCONDUCTOR

JETP LETTERS

VOLUME 70, NUMBER 9

10 NOV. 1999

Fermion zero modes on vortices in chiral superconductors

G. E. Volovik

Helsinki University of Technology, Low Temperature Laboratory, FIN-02015 HUT, Finland; Landau Institute of Theoretical Physics, Russian Academy of Sciences, 117334 Moscow, Russia

(Submitted 30 September 1999) Pis'ma Zh. Éksp. Teor. Fiz. **70**, No. 9, 601–606 (10 November 1999)

Bound-states of the vortex in p-wave (triplet) superconductors are given by:

$$E_n = \omega_0 n \qquad n = 0, \pm 1, \pm 2, \dots$$

implying the bound state at zero-energy !

BOUND STATES IN A VORTEX (EXPERIMENT)

FeTe $_{0.55}$ Se $_{0.45}$ superconductor ($T_c = 14.5$ K, $\Delta = 1.8$ meV, $E_F = 4.4$ meV).



D. Wang et al, Science 362, 333 (2018) /Chinese Academy of Sciences (Beijing)/

Nanoscopic superconductors

SINGLE IMPURITY IN SUPERCONDUCTOR

Typical spectrum of a single impurity in s-wave superconductor:



Bound states appearing in the subgap region $E \in \langle -\Delta, \Delta \rangle$

SINGLE IMPURITY IN SUPERCONDUCTOR

Typical spectrum of a single impurity in s-wave superconductor:



Bound states appearing in the subgap region $E \in \langle -\Delta, \Delta \rangle$ are dubbed Yu-Shiba-Rusinov (or Andreev) quasiparticles.

Magnetic chains



MAGNETIC OBJECTS IN SUPERCONDUCTORS

Magnetic chains



or magnetic islands



MAGNETIC OBJECTS IN SUPERCONDUCTORS

Magnetic chains



or magnetic islands



arrange their in-gap bound states in a form of the Shiba-bands.

MAGNETIC OBJECTS IN SUPERCONDUCTORS

Magnetic chains







arrange their in-gap bound states in a form of the Shiba-bands.

Specific magnetic textures of chains and islands can guarantee topologically non-trivial phases, hosting the Majorana modes !

A few examples ...

1. Rashba nanowires

TOPOLOGICAL SUPERCONDUCTING NANOWIRE

Spin-orbit (Rashba) interaction in presence of magnetic field applied to semiconducting nanowires proximitized to s-wave superconductor induces the triplet pairing of electrons, hosting Majorana modes.



FIG. 1. A nanowire (blue) placed on a superconducting substrate (green). The SOI vector $\boldsymbol{\alpha}$ is taken to lie along the *z* axis and determines the spin quantization axis. A magnetic field *B* is applied along the *x* axis such that it is oriented perpendicularly to the SOI vector. By proximity, the substrate induces a superconducting pairing of strength Δ_{sc} in the nanowire. In the topological phase [see Eq. (41)], MBSs (red) emerge at the wire ends.

TRANSITION TO TOPOLOGICAL PHASE

Effective quasiparticle states of the Rashba nanowire



TRANSITION TO TOPOLOGICAL PHASE

Effective quasiparticle states of the Rashba nanowire



Closing / reopening of a gap \iff band-invertion of topological insulators

M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

SPATIAL PROFILE OF MAJORANA QPS

Majorana qps are localized near the edges



R. Aguado, Riv. Nuovo Cim. 40, 523 (2017).

PROXIMITY TO D-WAVE SUPERCONDUCTOR

Rashba nanowire deposited along *a*-axis of high T_c superconductor



T. Domański, S. Vosoughinia, A. Kobiałka, Acta Phys. Polon. A 142, 679 (2022).

PROXIMITY TO D-WAVE SUPERCONDUCTOR

Transition to topological phase:



T. Domański, S. Vosoughinia, A. Kobiałka, Acta Phys. Polon. A 142, 679 (2022).

2. Selforganised magnetic chains

Magnetic atoms (like Fe) on a surface of s-wave superconductor (for example Pb) arrange themselves into such spiral order, where topological superconducting phase is selfsustained



Itinerant electrons in the chain of magnetic impurities placed on a surface of isotropic superconductor can be described by the Hamiltonian:

$$\begin{split} H &= - t \sum_{i,\sigma} \left(\hat{c}_{i,\sigma}^{\dagger} \hat{c}_{i+1,\sigma} + \text{H.c.} \right) - \mu \sum_{i,\sigma} \hat{c}_{i,\sigma}^{\dagger} \hat{c}_{i,\sigma} \\ &+ J \sum_{i} \vec{S}_{i} \cdot \hat{\vec{S}}_{i} + \sum_{i} \left(\Delta \hat{c}_{i\uparrow}^{\dagger} \hat{c}_{i\downarrow}^{\dagger} + \text{H.c.} \right) \end{split}$$

Here \vec{s}_i are the classical magnetic moments and $\hat{\vec{s}}_i = \frac{1}{2} \sum_{\alpha,\beta} \hat{c}^{\dagger}_{i,\alpha} \vec{\sigma}_{\alpha\beta} \hat{c}_{i,\beta}$ denote the spins of mobile electrons

 \Rightarrow J is the coupling between magnetic atoms and itinerant electrons

 $\Rightarrow \Delta$ is the proximity induced on-site pairing
























Structure factor:
$$A(q) = \frac{1}{L} \sum_{jk} e^{iq(j-k)} \langle \vec{S}_j \cdot \vec{S}_k \rangle$$



Structure factor:
$$A(q) = \frac{1}{L} \sum_{jk} e^{iq(j-k)} \langle \vec{S}_j \cdot \vec{S}_k \rangle$$



Structure factor:
$$A(q) = \frac{1}{L} \sum_{jk} e^{iq(j-k)} \langle \vec{S}_j \cdot \vec{S}_k \rangle$$



Structure factor:
$$A(q) = \frac{1}{L} \sum_{jk} e^{iq(j-k)} \langle \vec{S}_j \cdot \vec{S}_k \rangle$$



TEMPERATURE EFFECT ON MAJORANA QPS













3. Magnetic ladders

TOPOLOGICAL MAGNETIC LADDER

Spiral magnetic order in a ladder deposited on conventinal superconductor.



M.M. Maśka, N. Sedlmayr, A. Kobiałka, T. Domański, Phys. Rev. B 103, 235419 (2021).

For thermodynamic limit ($N \to \infty$) we have computed the topological invariant \mathbb{Z} of this system, belonging to AllI class.



Regions of the topological superconducting phase coincide either with the antiparallel or parallel spiral arrangements of the magnetic ladder.

UNCONVENTIONAL TOPOLOGICAL TRANSITIONS



UNCONVENTIONAL TOPOLOGICAL TRANSITIONS



Discontinuous transitions to/from topological phase without gap closing!

DISCONTINUOUS TRANSITIONS



Total energy as function of q and Δq obtained for $\Delta = 0.3t$ and several μ .

Red arrows indicate the minimum energy.

Majorana modes in Josephson junctions

Theoretical concept (2017)

Idea: Narrow metallic region with the strong spin-orbit interaction and in presence of magnetic field embedded between external superconductors.



F. Pientka et al., Phys. Rev. X 7,021032 (2017)

Idea: Narrow metallic region with the strong spin-orbit interaction and in presence of magnetic field embedded between external superconductors.



F. Pientka et al., Phys. Rev. X 7,021032 (2017)

Benefit:

Phase-tunable topological superconductivity induced in the metallic stripe.



Diagram of the topological superconducting state with respect to: phase difference ϕ and in-plane magnetic field E_z .

Experimental realization (2019)

Two-dimensional electron gas of InAs epitaxially covered by a thin Al layer



Width: $W_1 = 80 \text{ nm}$

Length:

 $L_1 = 1.6 \ \mu m$

A. Fornieri, ..., <u>Ch. Marcus</u> and F. Nichele, Nature <u>569</u>, 89 (2019). Niels Bohr Institute (Copenhagen, Denmark)

Two-dimensional HgTe quantum well coupled to 15 nm thick Al film



Width: W = 600 nmLength:

 $L = 1.0 \ \mu m$

H. Ren, ..., <u>L.W. Molenkamp</u>, B.I. Halperin & A. Yacoby, Nature <u>569</u>, 93 (2019). Würzburg Univ. (Germany) + Harvard Univ. (USA)

Topography of Majorana modes

TOPOGRAPHY OF MAJORANA MODES

Spatial profile of the zero-energy quasiparticles of a homogeneous metallic strip embedded into the Josephson junction for the phase difference $\phi = \pi$ (which is optimal for topological state).



Complex "Majorana polarization" $u_{\uparrow,n}v_{\uparrow,n} - u_{\downarrow,n}v_{\downarrow,n}$ obtained for eigenvalue $E_n = 0$.

TOPOGRAPHY OF MAJORANA MODES

Spatial profile of the zero-energy quasiparticles of a homogeneous metallic strip embedded into the Josephson junction for the phase difference $\phi = \pi$ (which is optimal for topological state).



Complex "Majorana polarization" $u_{\uparrow,n}v_{\uparrow,n} - u_{\downarrow,n}v_{\downarrow,n}$ obtained for eigenvalue $E_n = 0$. Magnitude of this quantity is measurable by the conductance of SESAR spectroscopy. For details see:

Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

TOPOGRAPHY OF MAJORANA MODES

Selective Equal Spin Andreev Reflection (SESAR) spectroscopy:



Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

Means to localize Majoranas

I. DESHAPED JOSEPHSON JUNCTION

In order to reduce the spatial extent of the Majorana modes one can use zigzag-shape metallic stripe.



T. Laeven, B. Nijholt, M. Wimmer & A.R. Akhmerov, PRL 102, 086802 (2020).
I. DESHAPED JOSEPHSON JUNCTION

$$\begin{array}{c} \overbrace{\blacksquare}\\ & \overbrace{=}\\ & \overbrace{=}\\ & \overbrace{=}\\ & -500 \end{array} \begin{array}{c} E_M = 7 \times 10^{-4} \Delta & (a) \\ & \overbrace{=}\\ & E_{gap} = 9.9 \times 10^{-3} \Delta & \xi_M = 26.7 \ \mu \text{m} \end{array} \\ \overbrace{=}\\ & \overbrace{=}\\ & \overbrace{=}\\ & -500 \end{array} \begin{array}{c} E_M = 8 \times 10^{-5} \Delta & (b) \\ & \overbrace{=}\\ & E_{gap} = 1.1 \times 10^{-1} \Delta & \xi_M = 0.4 \ \mu \text{m} \end{array} \\ \overbrace{=}\\ & \overbrace{=}\\ & \overbrace{=}\\ & \overbrace{=}\\ & -500 \end{array} \begin{array}{c} E_M = 2 \times 10^{-4} \Delta & (c) \\ & \overbrace{=}\\ & E_{gap} = 1.3 \times 10^{-1} \Delta & \xi_M = 0.4 \ \mu \text{m} \end{array} \\ \overbrace{=}\\ & \overbrace{=}\\ & \overbrace{=}\\ & -500 \end{array} \begin{array}{c} E_M = 2 \times 10^{-4} \Delta & (c) \\ & \overbrace{=}\\ & \overbrace{=}\\ & -500 \end{array} \begin{array}{c} E_{gap} = 1.1 \times 10^{-1} \Delta & \xi_M = 0.4 \ \mu \text{m} \\ \hline{=}\\ & \overbrace{=}\\ & 0 \end{array} \\ \overbrace{=}\\ & \overbrace{=}\\ & 0 \end{array} \begin{array}{c} E_{gap} = 1.1 \times 10^{-1} \Delta & \xi_M = 0.4 \ \mu \text{m} \\ \hline{=}\\ & 0 \end{array} \\ \overbrace{=}\\ & 0 \end{array} \begin{array}{c} E_{gap} = 1.1 \times 10^{-1} \Delta & \xi_M = 0.4 \ \mu \text{m} \\ \hline{=}\\ & 0 \end{array} \\ \overbrace{=}\\ & 0 \end{array} \\ \overbrace{=}\\ & 0 \end{array} \begin{array}{c} E_{gap} = 1.1 \times 10^{-1} \Delta & \xi_M = 0.4 \ \mu \text{m} \\ \hline{=}\\ & 0 \end{array} \\ \overbrace{=}\\ & 0 \end{array} \begin{array}{c} E_{gap} = 1.1 \times 10^{-1} \Delta & \xi_M = 0.4 \ \mu \text{m} \\ \hline{=}\\ & 0 \end{array} \\ \overbrace{=}\\ & 0 \end{array} \\ \overbrace{=}\\ & 0 \end{array}$$

T. Laeven, B. Nijholt, M. Wimmer & A.R. Akhmerov, PRL 102, 086802 (2020).

II. LOCAL DEFECT IN JOSEPHSON JUNCTION

Spatial profile of the Majorana modes in presence of the strong electrostatic defect placed in the center.



Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

II. LOCAL DEFECT IN JOSEPHSON JUNCTION

Spatial profile of the Majorana modes in presence of the strong electrostatic defect placed near the edge.



Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

III. RANDOM DISORDER

"Benefits of Weak Disorder in dim=1 Topological Superconductors"



FIG. 1. (a) Phase diagram of the planar Josephson junction Eq. (1) in the clean limit. In the topological phase (Q = -1), the system supports zero-energy MBSs at each end of the junction. (b) The Majorana localization length ξ versus the disorder-induced inverse mean free time τ^{-1} for different points inside the topological phase [see markers in (a)].

A. Haim & A. Stern, Phys. Rev. Lett. 122, 126801 (2019).

III. RANDOM DISORDER

The left-hand-side part of the metallic stripe is randomly disordered



III. RANDOM DISORDER

The left-hand-side part of the metallic stripe is randomly disordered



moderate disorder

Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

Selforganized magnetic stripes

PLANAR MAGNETIC JOSEPHSON JUNCTION

A small metallic piece with the classical magnetic moments placed between two external superconductors, differing in phase Φ .



M. Dziurawiec, M. Strzałka, M.M. Maśka – work in progress (Technical University in Wrocław)

PLANAR MAGNETIC JOSEPHSON JUNCTION

Preliminary results: numerical calculations reveal that magnetic moments arrange themselves in such textures, which sustain the topological superconducting state of the metallic stripe.

PLANAR MAGNETIC JOSEPHSON JUNCTION

Preliminary results: numerical calculations reveal that magnetic moments arrange themselves in such textures, which sustain the topological superconducting state of the metallic stripe.



Spatial profile of Majorana modes obtained for phase difference $\Phi = \pi$.

M. Dziurawiec, M. Strzałka, M.M. Maśka – work in progress (Technical University in Wrocław)

Higher-dimensional topological textures

Higher-dimensional topological textures

(platform for chiral Majorana modes)

TWO-DIMENSIONAL MAGNETIC STRUCTURES

Magnetic island of Co atoms deposited on the superconducting Pb surface



Diameter of island: 5 - 10 nm

G. Ménard, ..., and <u>P. Simon</u>, Nature Commun. 8, 2040 (2017). Pierre & Marie Curie University (Paris, France)

PROPAGATING MAJORANA EDGE MODES

Magnetic island of Fe atoms deposited on the superconducting Re surface



A. Palacio-Morales, ... & <u>R. Wiesendanger</u>, Science Adv. <u>5</u>, eaav6600 (2019). University of Hamburg (Germany)

VAN DER WAALS HETEROSTRUCTURES

Ferromagnetic island CrBr₃ deposited on superconducting NbSe₂



S. Kezilebieke ... Sz. Głodzik ... P. Lilieroth, Nature 424, 588 (2020).

Scenario for topological superconductivity induced in 2D magnetic thin film hosting a skyrmion deposited on conventional s-wave superconductor



Scenario for topological superconductivity induced in 2D magnetic thin film hosting a skyrmion deposited on conventional s-wave superconductor



M. Garnier, A. Mesaros, P. Simon, Comm. Phys. 2, 126 (2019).

 \Rightarrow allows for their constructive relationship

- \Rightarrow allows for their constructive relationship
- \Rightarrow inducing the topological states of matter

 \Rightarrow allows for their constructive relationship

- \Rightarrow inducing the topological states of matter
- \Rightarrow hosting the Majorana boundary modes

ACKNOWLEDGEMENTS

 \Rightarrow Maciek Maśka & coworkers

(Technical University, Wrocław)

\Rightarrow Nick SedImayr

(M. Curie-Skłodowska University, Lublin)

 \Rightarrow Aksel Kobiałka

(University of Basel, Switzerland)

 \Rightarrow Szczepan Głodzik

(University of Ljubljana, Slovenia)









 $t_{35}/t = 1.0$ LDOS 20 15 10 5 0 1 0.04 10 20 0.02 30 ^Sit_e40 0.gqt 50 -0.02 60 -0.04 70

 $t_{35}/t = 0.8$ LDOS 20 15 10 5 0 1 0.04 10 20 0.02 30 ^Sit_e40 0.gqt 50 -0.02 60 -0.04 70

 $t_{35}/t = 0.6$



 $t_{35}/t = 0.4$



 $t_{35}/t = 0.2$



 $t_{35}/t = 0.1$



 $t_{35}/t = 0.0$

