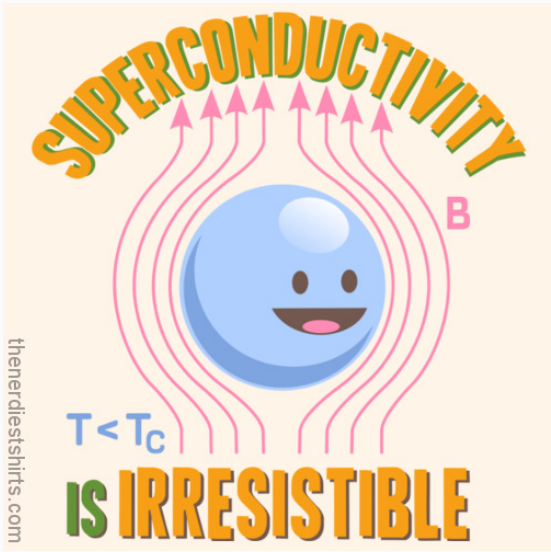
MAJORANA QUASIPARTICLES IN NANOSCOPIC HYBRID STRUCTURES

T. Domański

M. Curie-Skłodowska University (Lublin)



MagTop, Warsaw (26 Feb. 2019)

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Superconductivity in systems of dimensionality reduced to:

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  in-gap bound states ……(Shiba/Andreev qps)

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- **$1 \leq$ dim ≤ 2**
  topological phases .....(Majorana qps)

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Finite-size (nanoscopic) objects, like:

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- ⇒ **magnetic atoms ..... (for instance Fe, Co)**
- ⇒ **correlated quantum dots ..... (Anderson-type impurities)**
- ⇒ **molecules ..... (multi-level or vibrating)**
- ⇒ **magnetic islands ..... (Shiba glasses and/or lattices)**
- ⇒ **nanowires ..... (carbon nanotubes, Fe-chains)**
- ⇒ **etc.**

existing inside or on surfaces of superconducting materials

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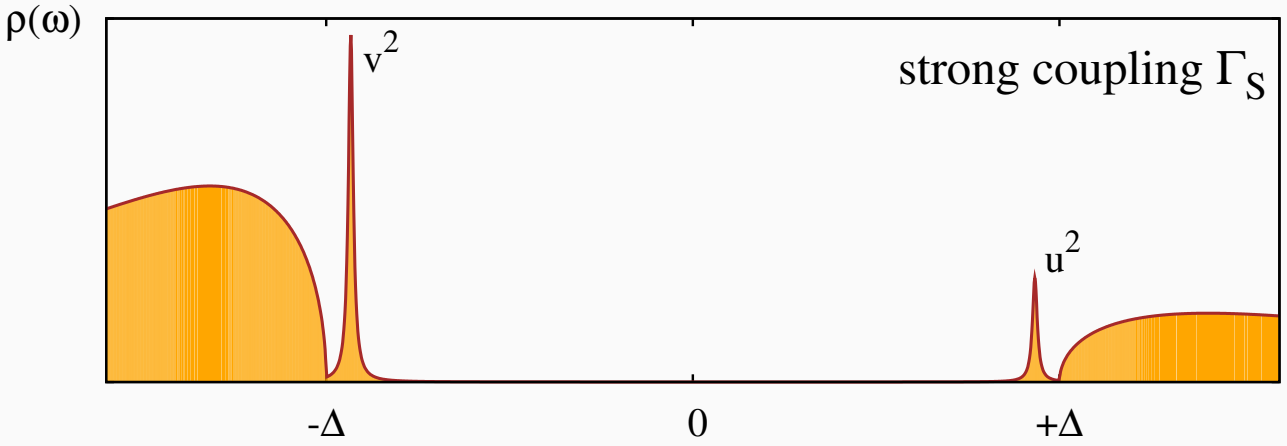
existing inside or on surfaces of superconducting materials

can acquire the electron pairing via *proximity effect*.

IN-GAP STATES

Spectrum of an uncorrelated impurity coupled to superconductor:



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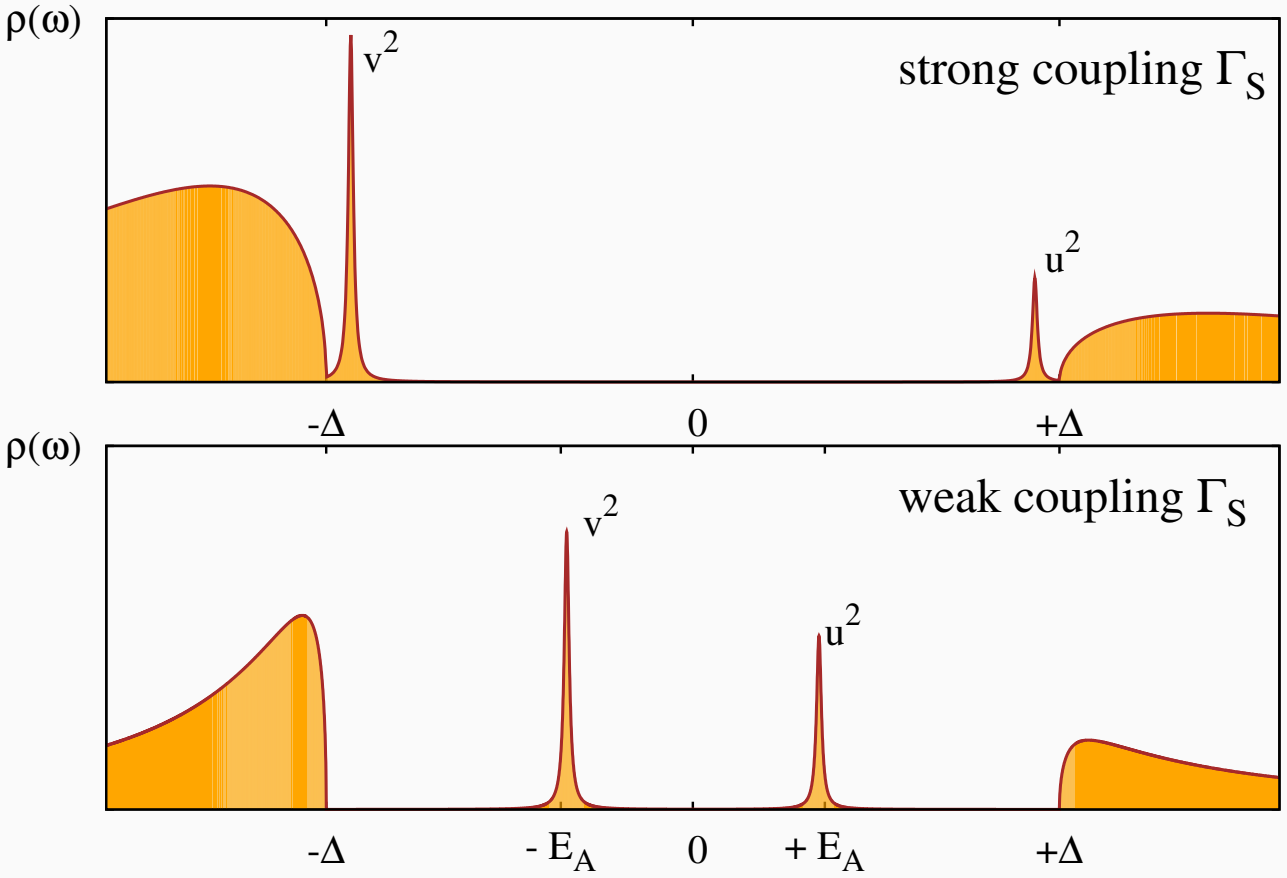

Bound states appearing in the subgap region $E \in \langle -\Delta, \Delta \rangle$.

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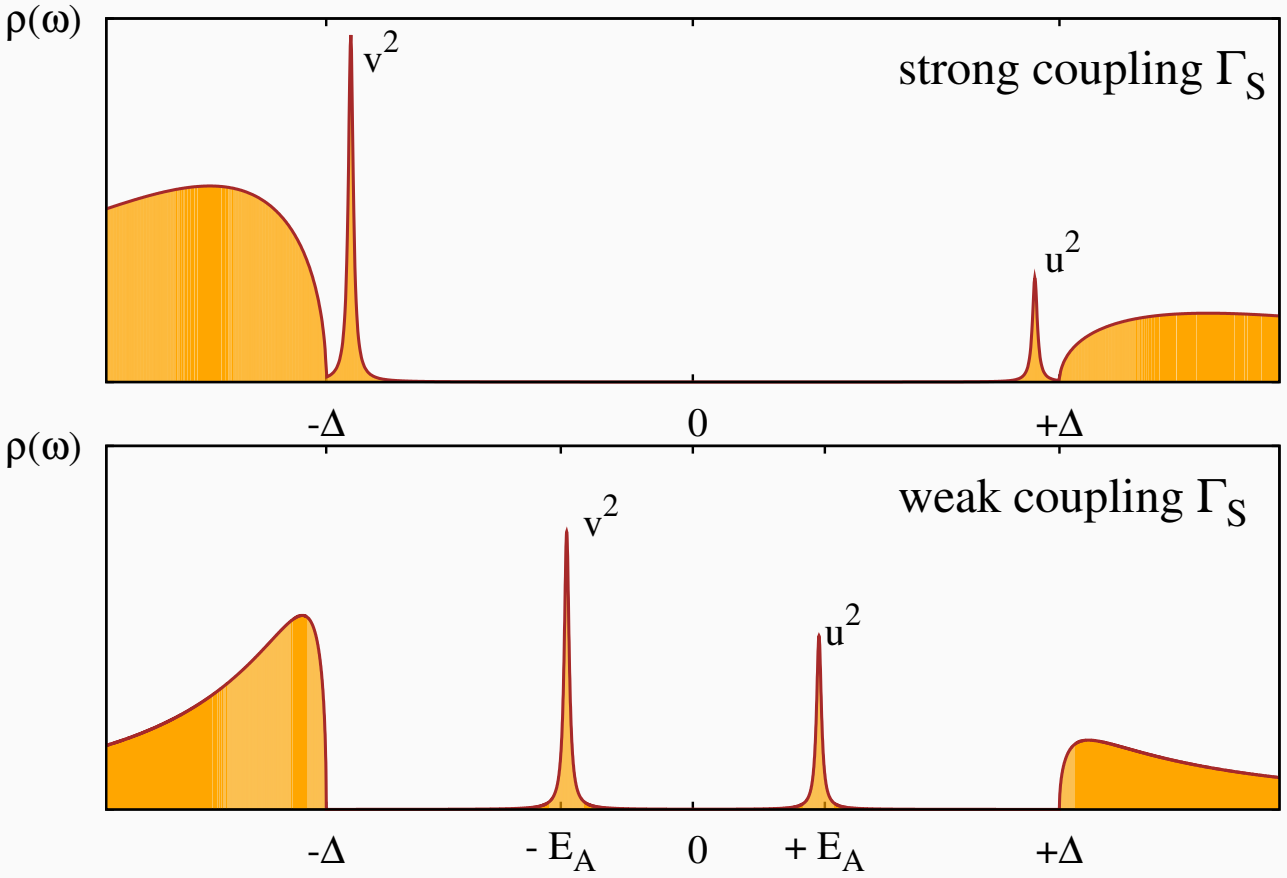

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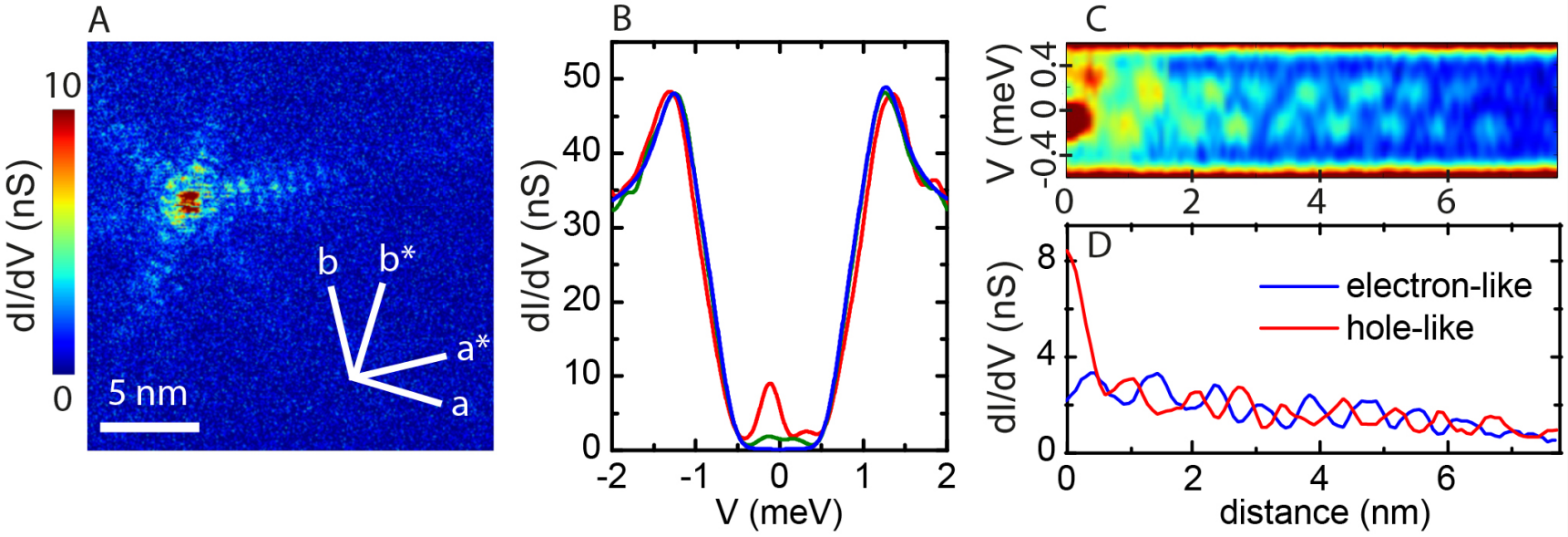
Yu-Shiba-Rusinov (Andreev) quasiparticles

TOPOGRAPHY AND SPATIAL EXTENT

Empirical data obtained from STM measurements for $NbSe_2$



a) bound states extending to 10 nm

b) alternating particle-hole oscillations

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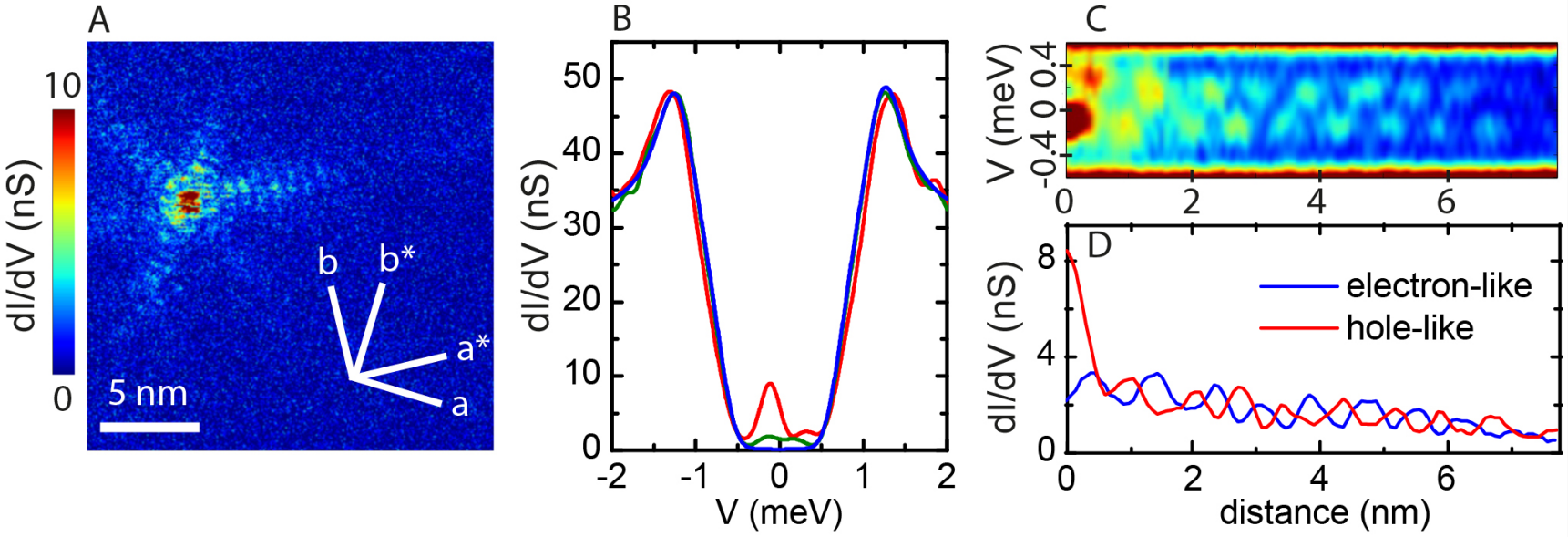

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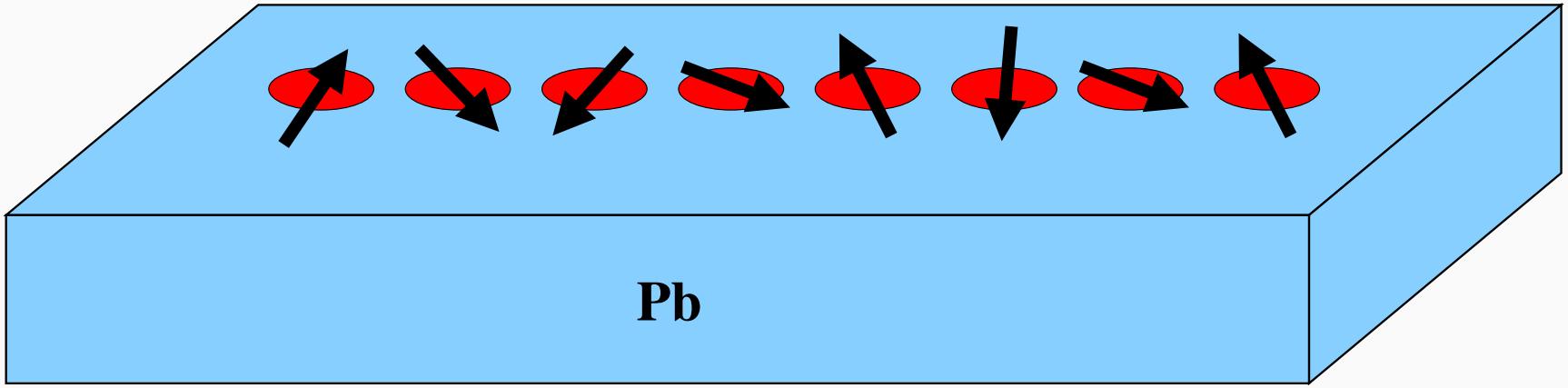
MAGNETIC CHAINS IN SUPERCONDUCTORS

Nanochain of magnetic impurities embedded in superconductor:



T.-P. Choy, J.M. Edge, A.R. Akhmerov, and C.W.J. Beenakker,

Phys. Rev. B 84, 195442 (2011).

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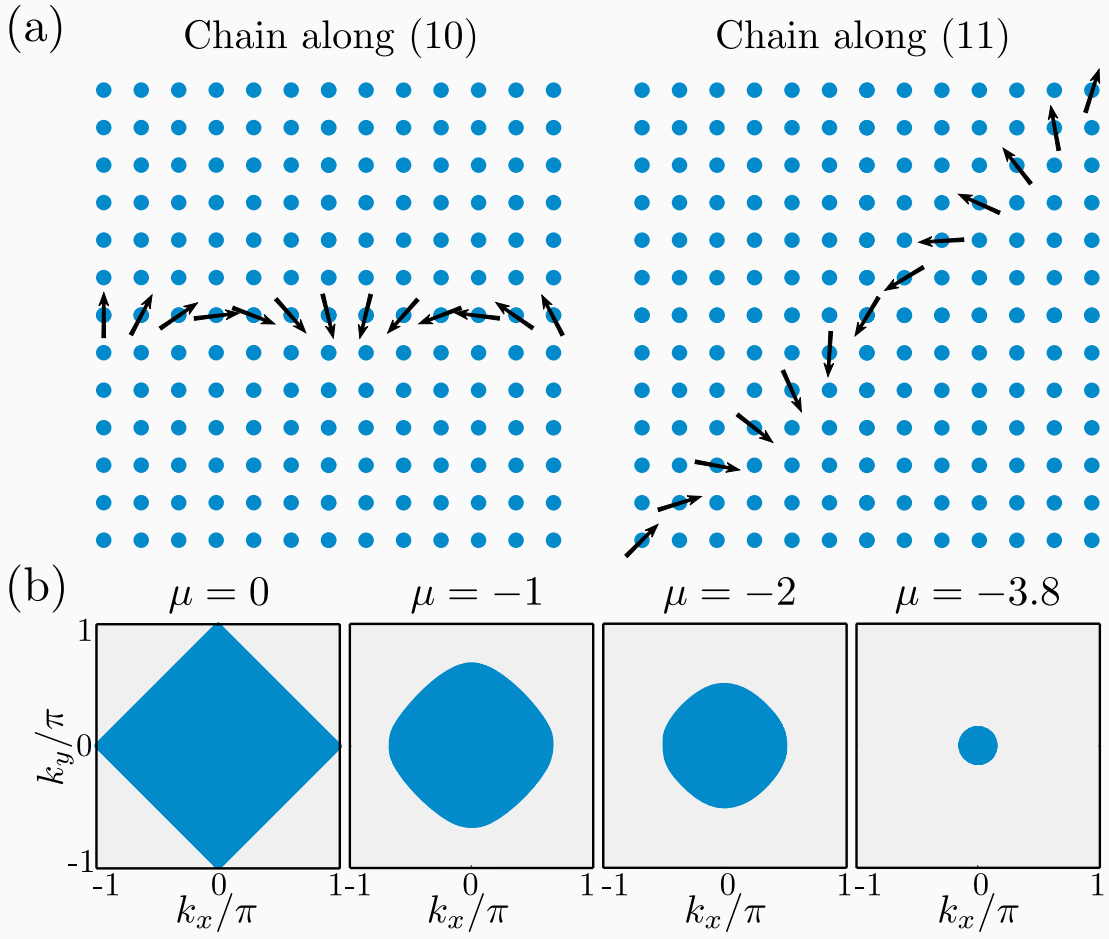

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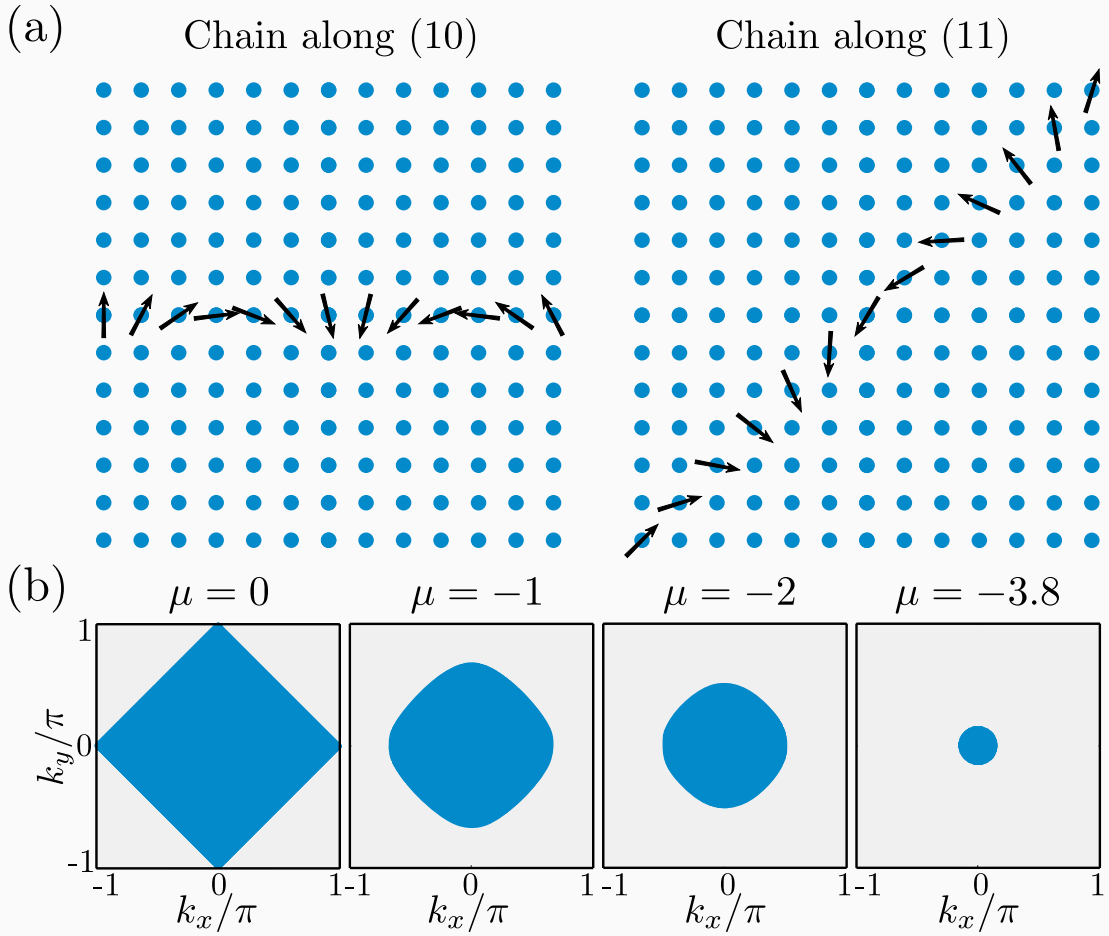
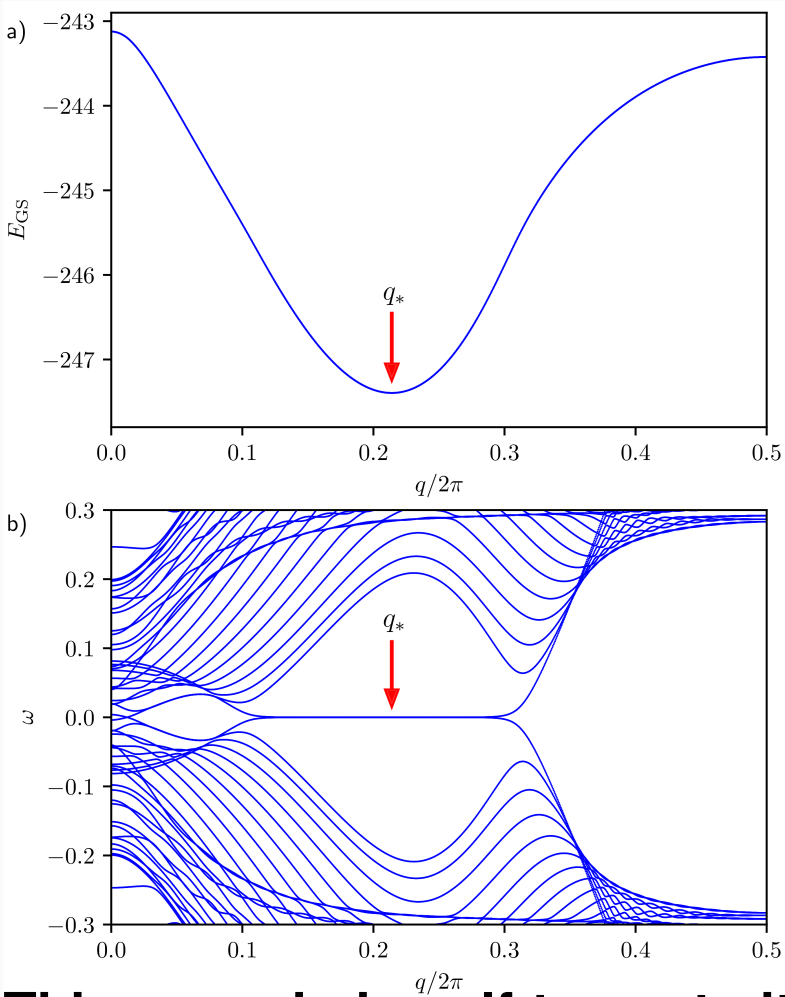


arrange the in-gap bound states into Shiba-band(s).

M.H. Christensen ... J. Paaske, Phys. Rev. B 94, 144509 (2016).

MAGNETIC CHAINS IN SUPERCONDUCTORS



Ground state energy vs the pitch vector q

In-gap Shiba states

This nanochain self-tunes to its *topological phase* (topofilia)

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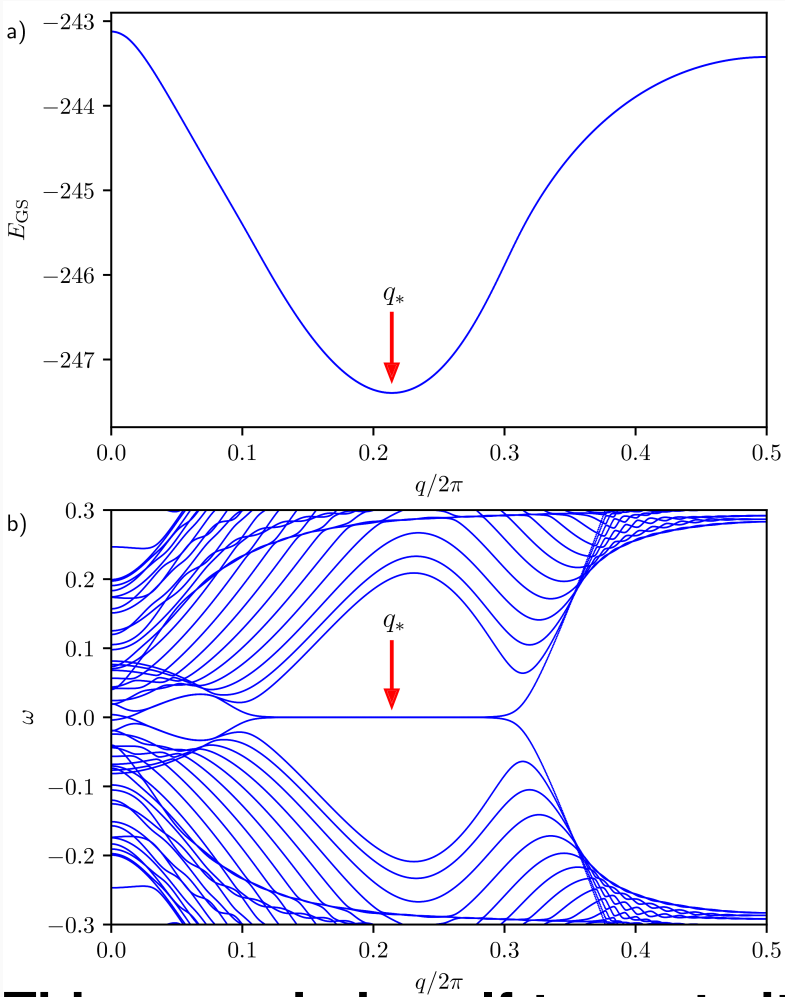

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Details will be given in the next lecture by Maciek Maśka

KITAEV CHAIN: PARADIGM FOR MAJORANA QPS

Itinerant 1D fermions with intersite (*p-wave*) pairing

$$\hat{H} = t \sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1} + \text{h.c.} \right) - \mu \sum_i \hat{c}_i^\dagger \hat{c}_i + \Delta \sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1}^\dagger + \text{h.c.} \right)$$

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This toy-model can be recast in the Majorana basis

$$\begin{aligned} \hat{\gamma}_{j,1} &\equiv \frac{1}{\sqrt{2}} \left(\hat{c}_j + \hat{c}_j^\dagger \right) \\ \hat{\gamma}_{j,2} &\equiv \frac{1}{i\sqrt{2}} \left(\hat{c}_j - \hat{c}_j^\dagger \right) \end{aligned}$$

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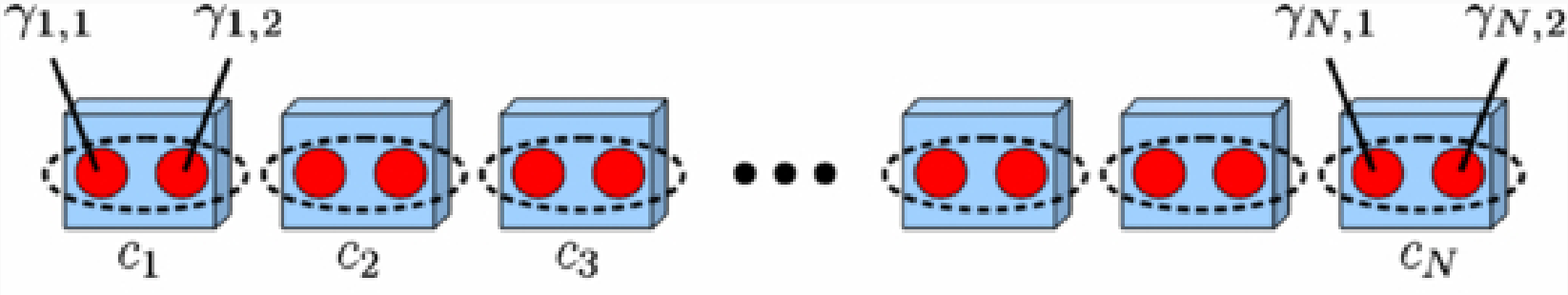
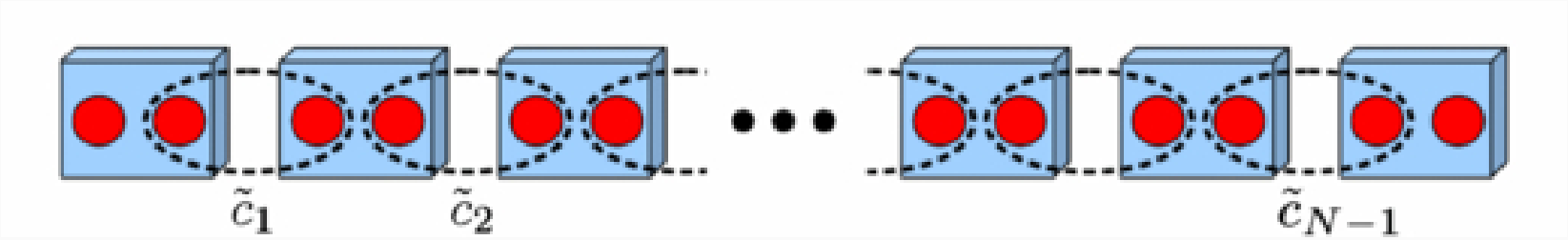


Yu. Kitaev, Phys. Usp. 44, 131 (2001).

KITAEV CHAIN: PARADIGM FOR MAJORANA QPS

In particular, for $\Delta = t$ and when $|\mu|$ is inside the band

two operators $\hat{\gamma}_{1,1}$ and $\hat{\gamma}_{2,N}$ *decouple* from all the rest



inducing the zero-energy modes at the chain edges.

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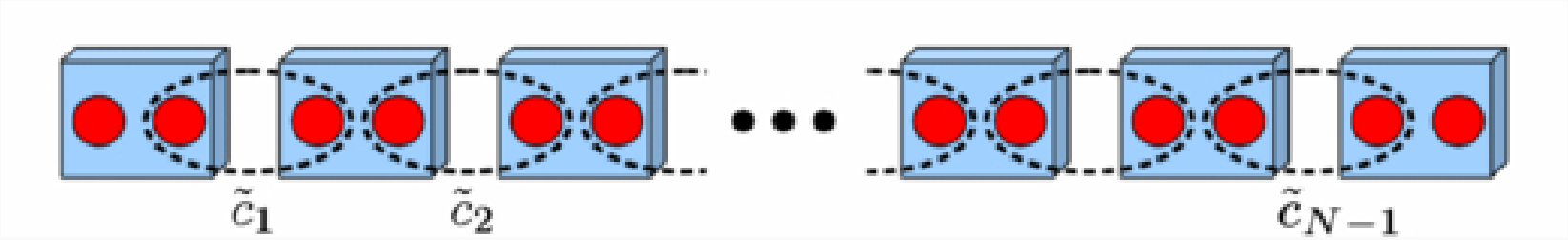


inducing the zero-energy modes at the chain edges.

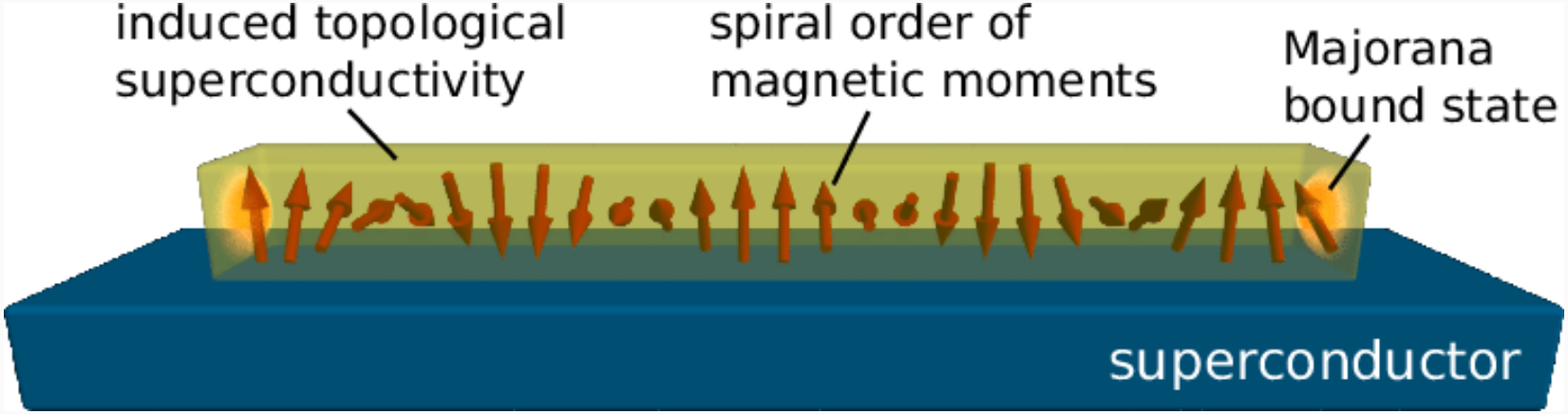
They can be regarded as *fractions* of non-local fermion

$$\hat{c}_{nonlocal} \equiv (\hat{\gamma}_{1,1} + i\hat{\gamma}_{N,2})/\sqrt{2}$$
$$\hat{c}^{\dagger}_{nonlocal} \equiv (\hat{\gamma}_{1,1} - i\hat{\gamma}_{N,2})/\sqrt{2}$$

as manifested by a number of unique phenomena.

MAGNETIC CHAINS IN SUPERCONDUCTORS

Topological superconductivity can be also driven by the spin-orbit Rashba interaction combined with the external magnetic field.

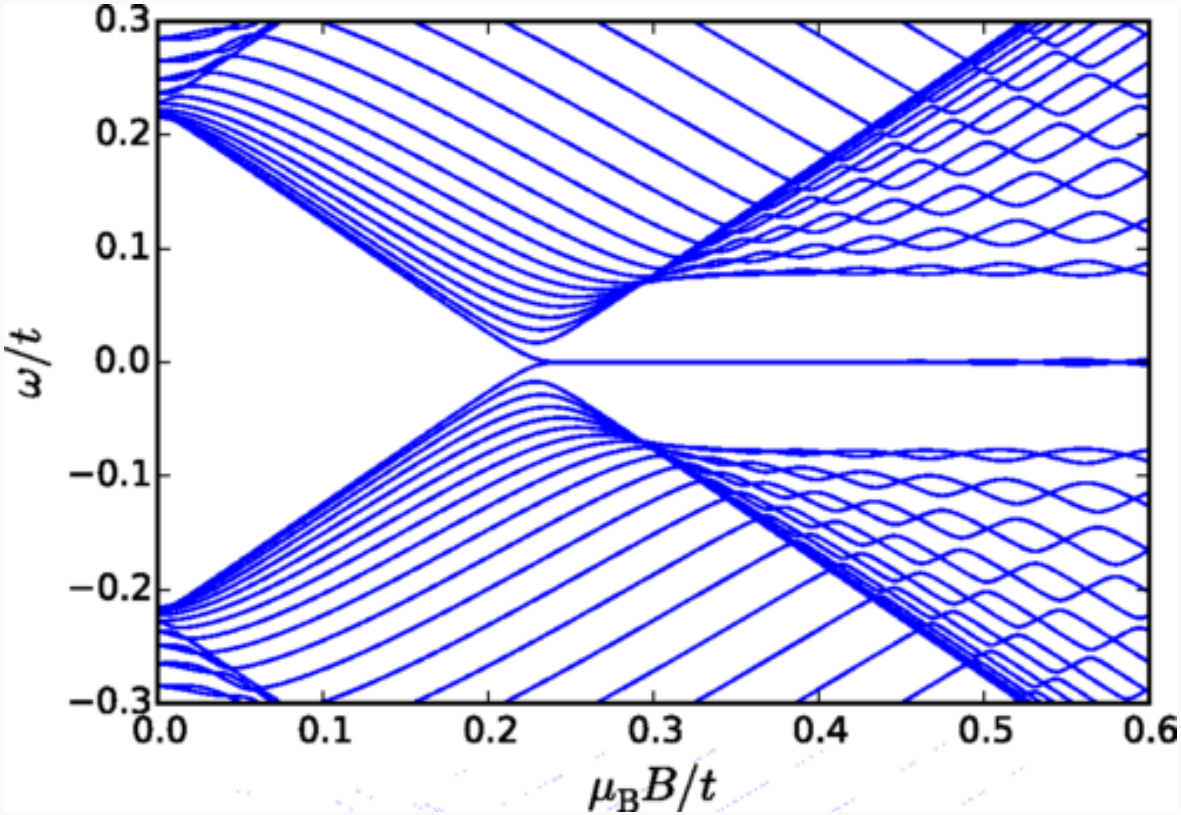


R. Lutchyn, J. Sau, S. Das Sarma, Phys. Rev. Lett. 105, 077001 (2010).

Y. Oreg, G. Refael, F. von Oppen, Phys. Rev. Lett. 105, 177002 (2010).

EVOLUTION FROM TRIVIAL TO TOPOLOGICAL PHASE

A pair of the Shiba (Andreev) states evolve into the Majorana qps

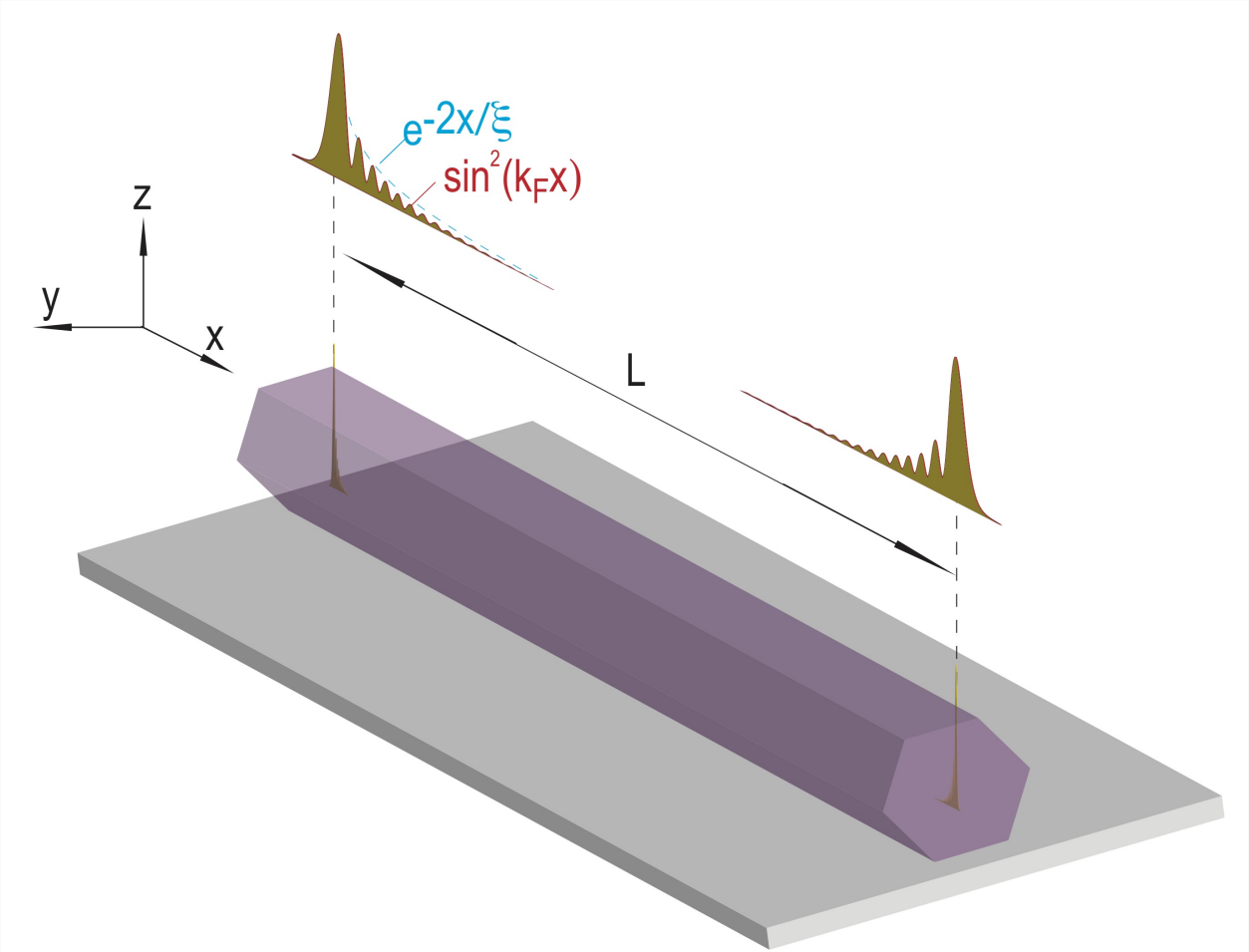


Mutation of the trivial bound states into the nontrivial Majorana modes

M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

SPATIAL PROFILE OF MAJORANA QPS

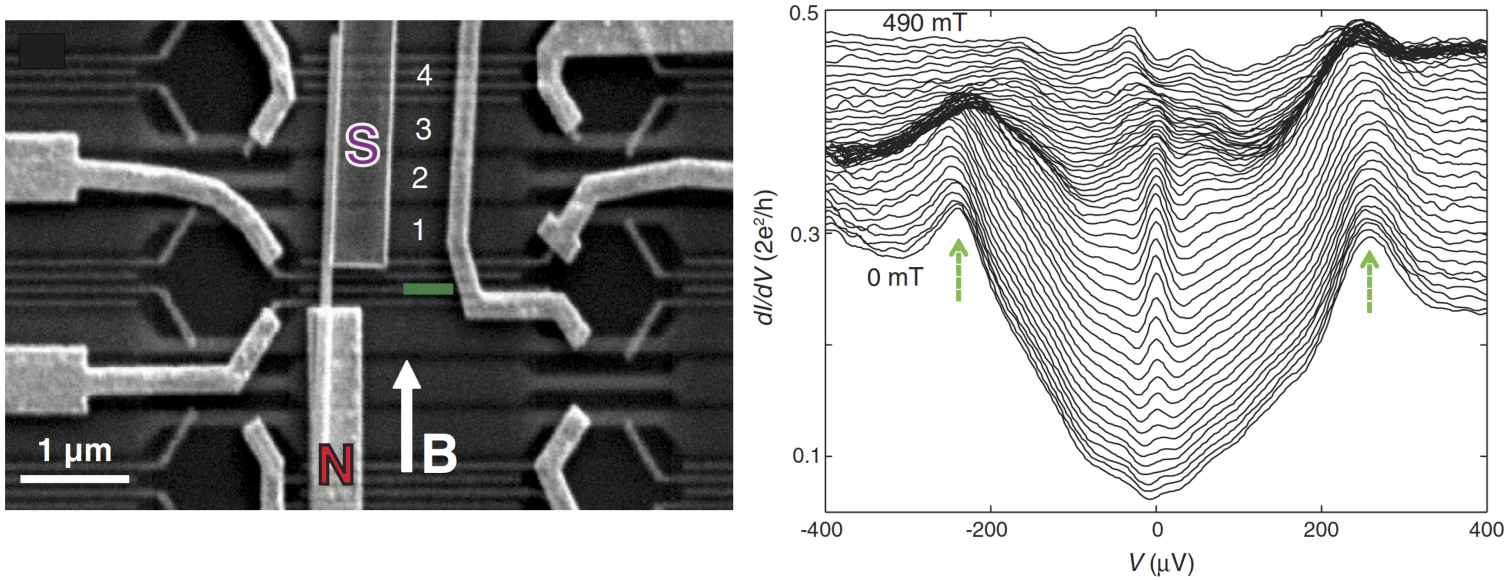
Majorana qps are exponentially localized at the edges



R. Aguado, Riv. Nuovo Cim. 40, 523 (2017).

EXAMPLES OF EMPIRICAL REALIZATION: 1

Differential conductance dI/dV obtained for InSb nanowire at 70 mK upon varying a magnetic field.

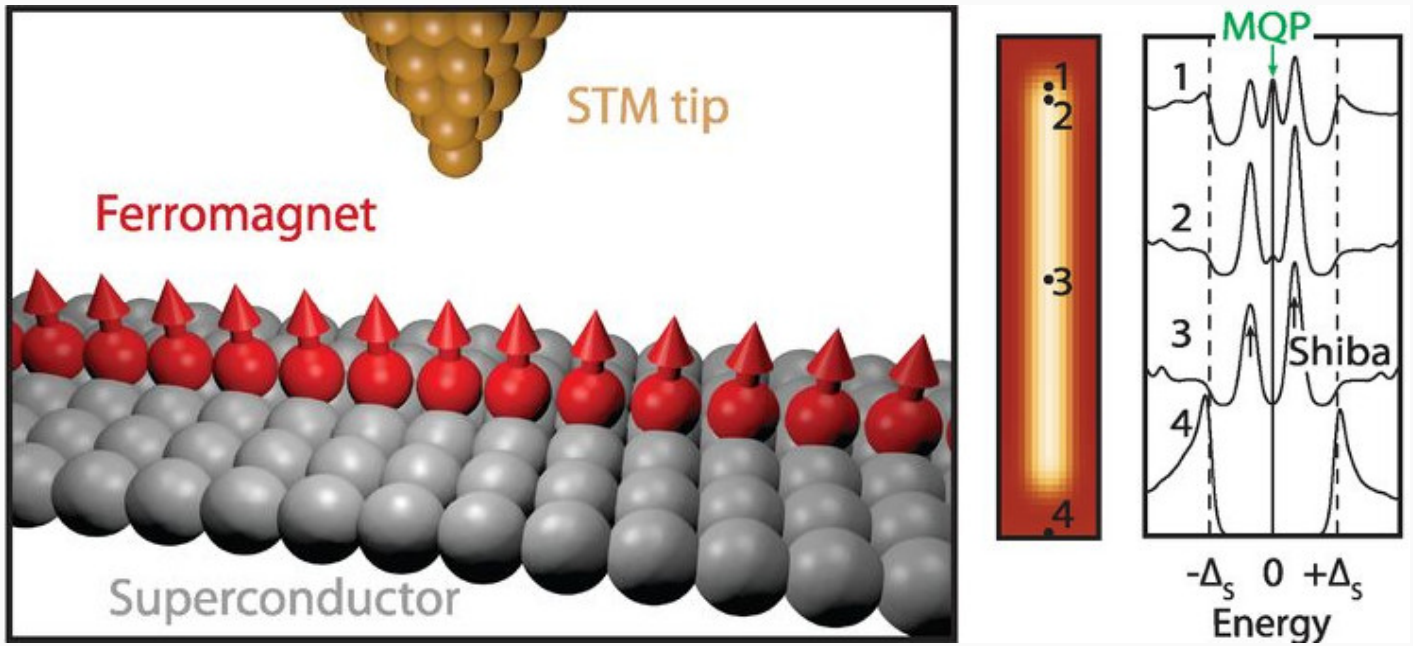


V. Mourik, ..., and L.P. Kouwenhoven, Science **336**, 1003 (2012).

/ Technical Univ. Delft, Netherlands /

EXAMPLES OF EMPIRICAL REALIZATION: 2

STM measurements for the nanochain of Fe atoms self-organized on a surface of superconducting Pb.

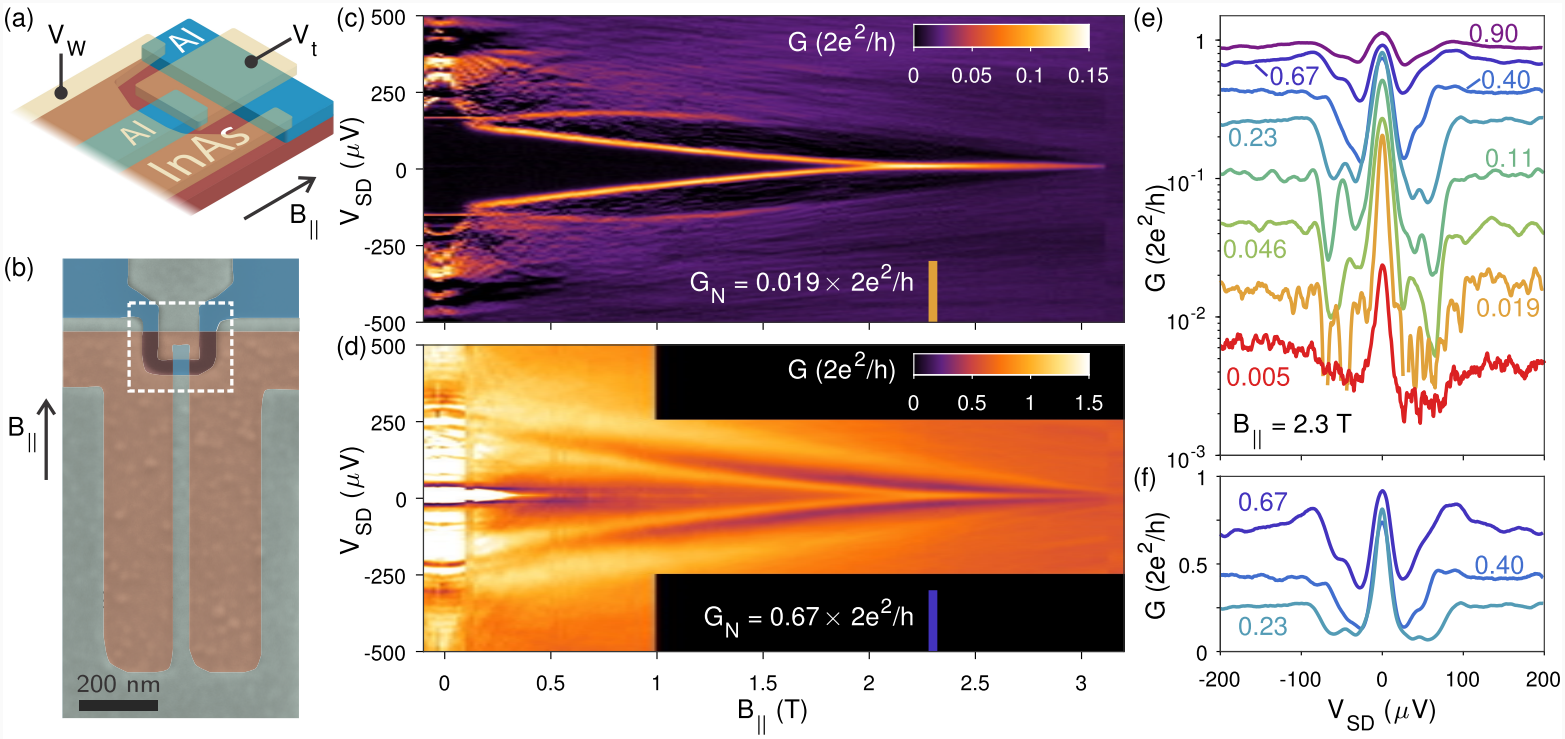


S. Nadj-Perge, ..., and A. Yazdani, Science **346**, 602 (2014).

/ Princeton University, USA /

EXAMPLES OF EMPIRICAL REALIZATION: 3

Results for the litographically obtained Al nanowire

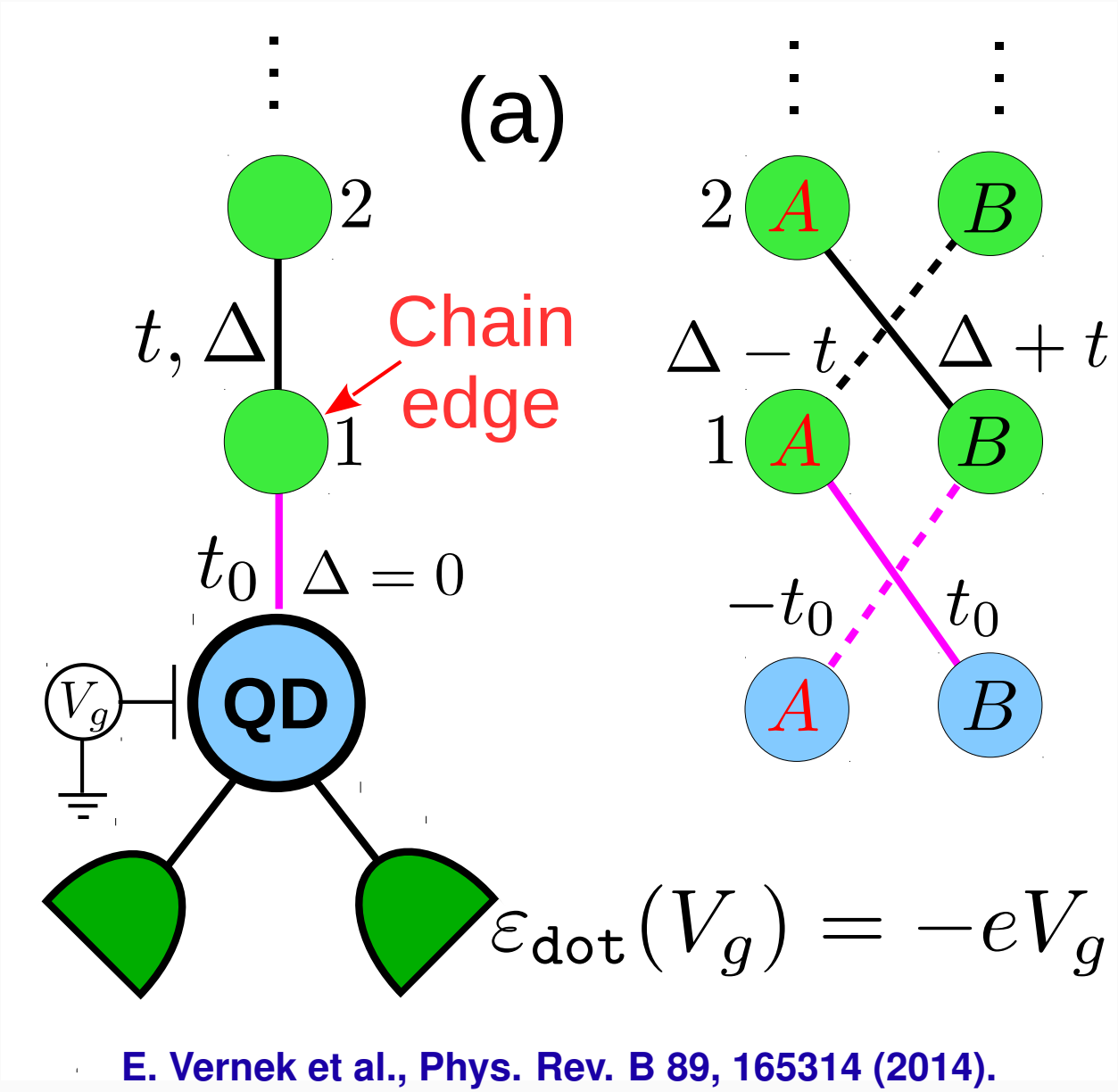


F. Nichele, ..., and Ch. Marcus, Phys. Rev. Lett. **119**, 136803 (2017).

/ Niels Bohr Institute, Copenhagen, Denmark /

Andreev vs Majorana states

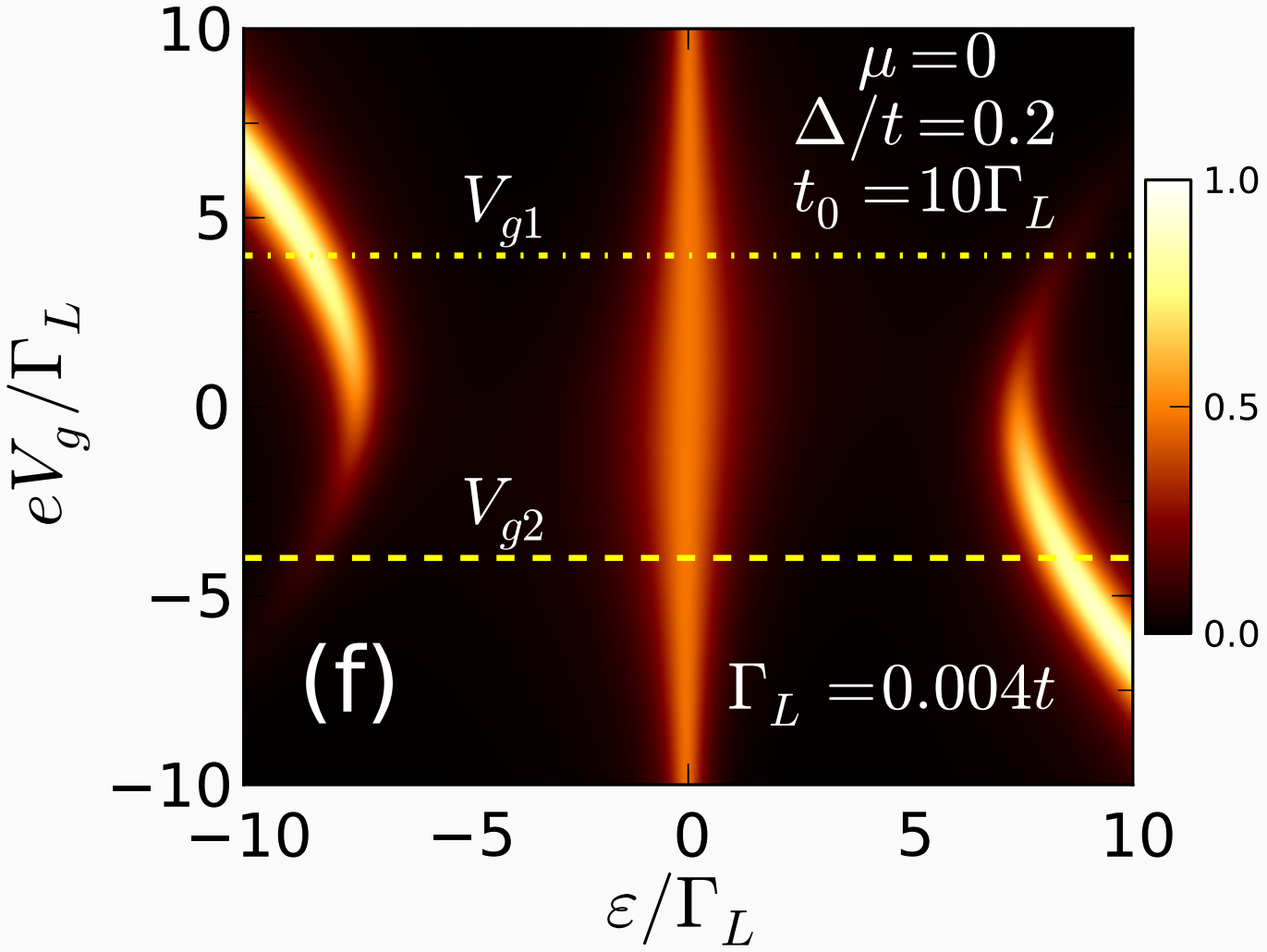
KITAEV CHAIN + NORMAL SITE



E. Vernek et al., Phys. Rev. B 89, 165314 (2014).

KITAEV CHAIN + NORMAL SITE

Subtle leakage of a Majorana mode into a quantum dot



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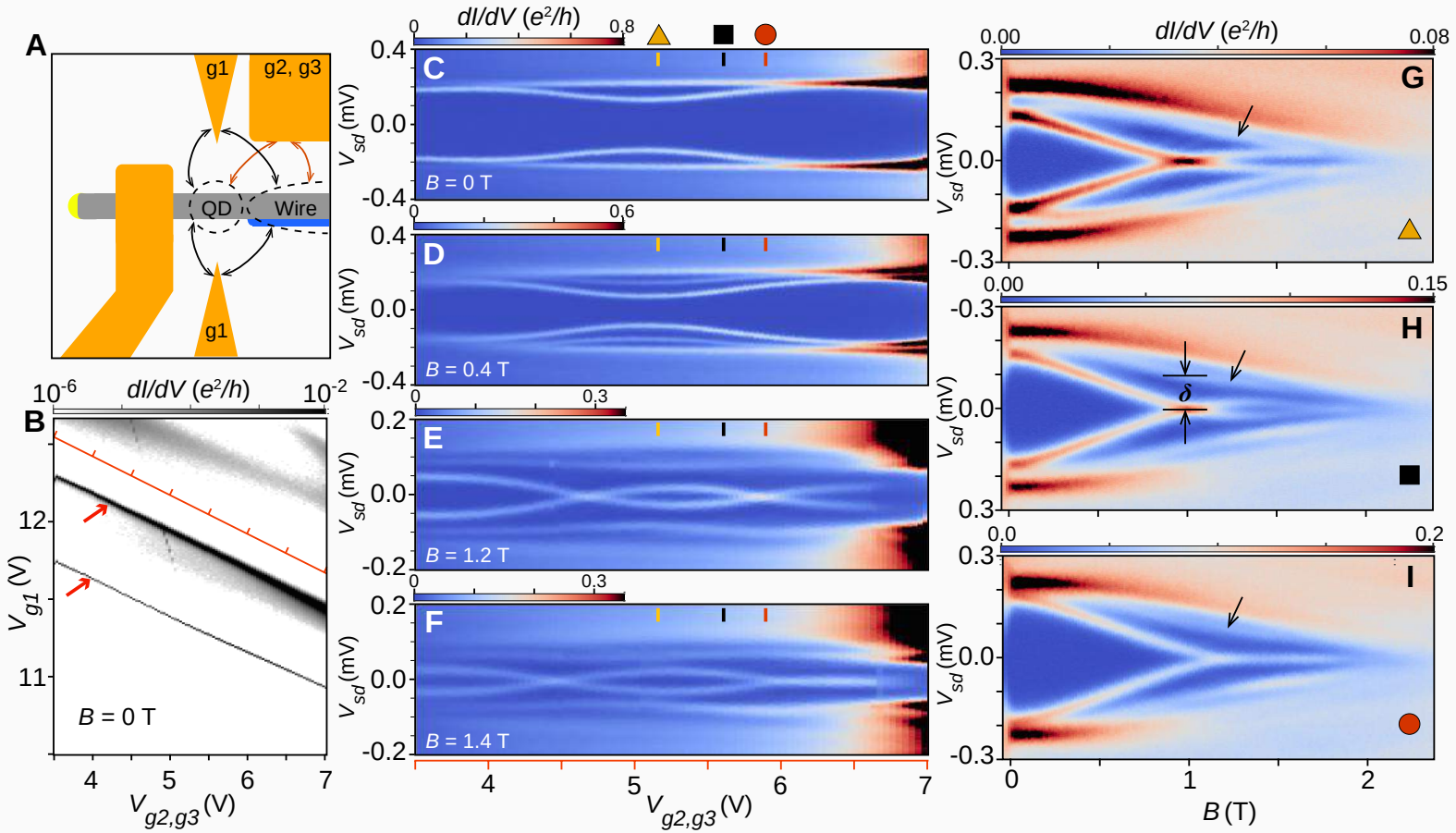
LEAKAGE OF MAJORANAS ON QUANTUM DOT

‘Coalescence’ of the Andreev into Majorana qps

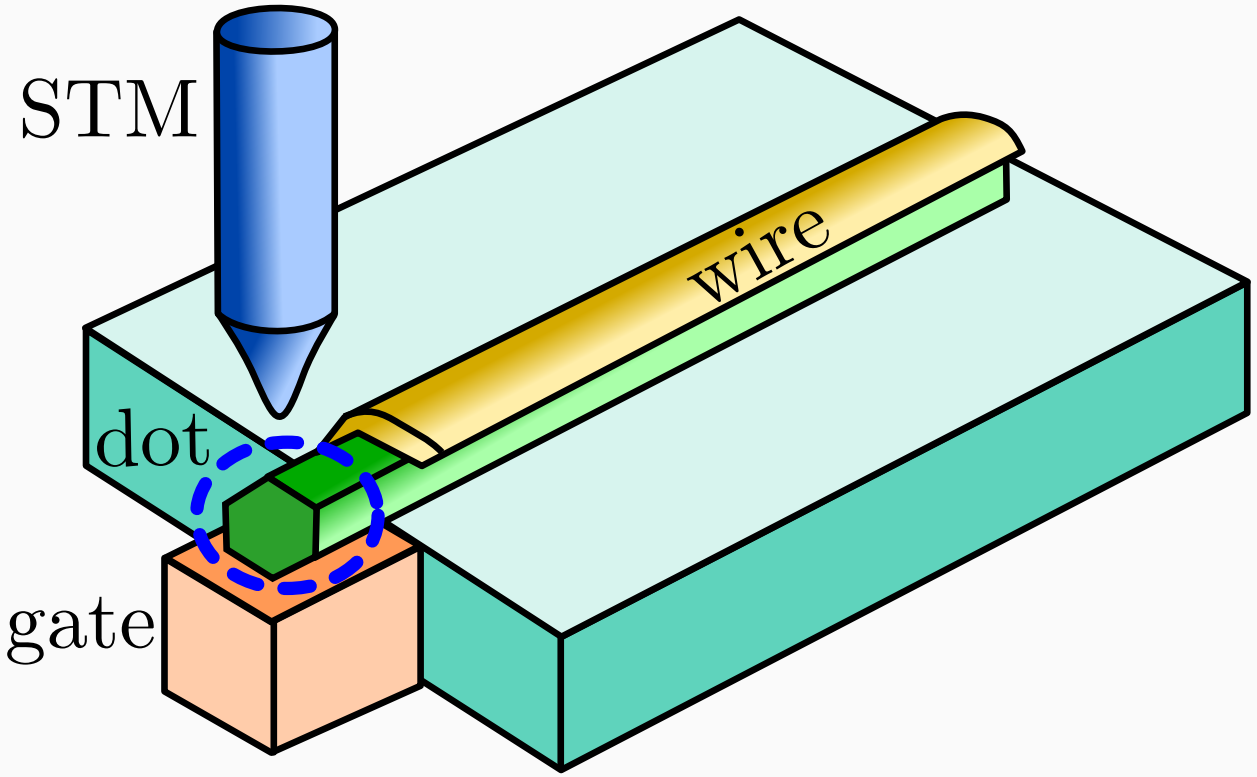


M.T. Deng, ..., and Ch. Marcus, Science 354, 1557 (2016).

/ Niels Bohr Institute, Copenhagen, Denmark /

TRIVIAL VS MAJORANA BOUND STATES

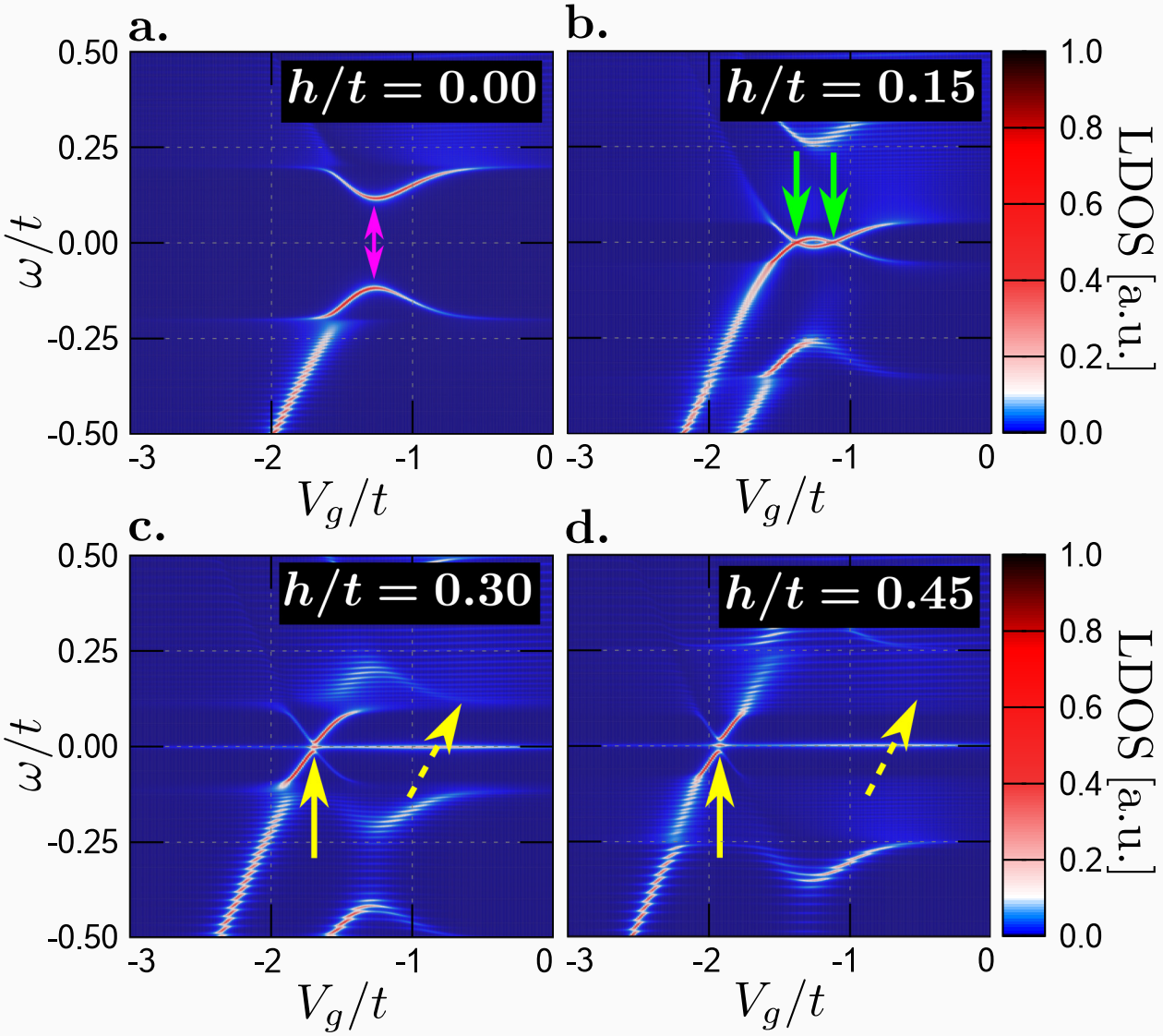
Schematics of a quantum dot – nanowire hybrid structure.



A. Ptok, A. Kobiałka & T. Domański, Phys. Rev. 96, 195403 (2017).

DISTINGUISHING ANDREEV FROM MAJORANA QPS

QD spectrum vs gate potential V_g for several magnetic fields h.



A. Ptok, A. Kobiałka & T. Domański, Phys. Rev. 96, 195403 (2017).

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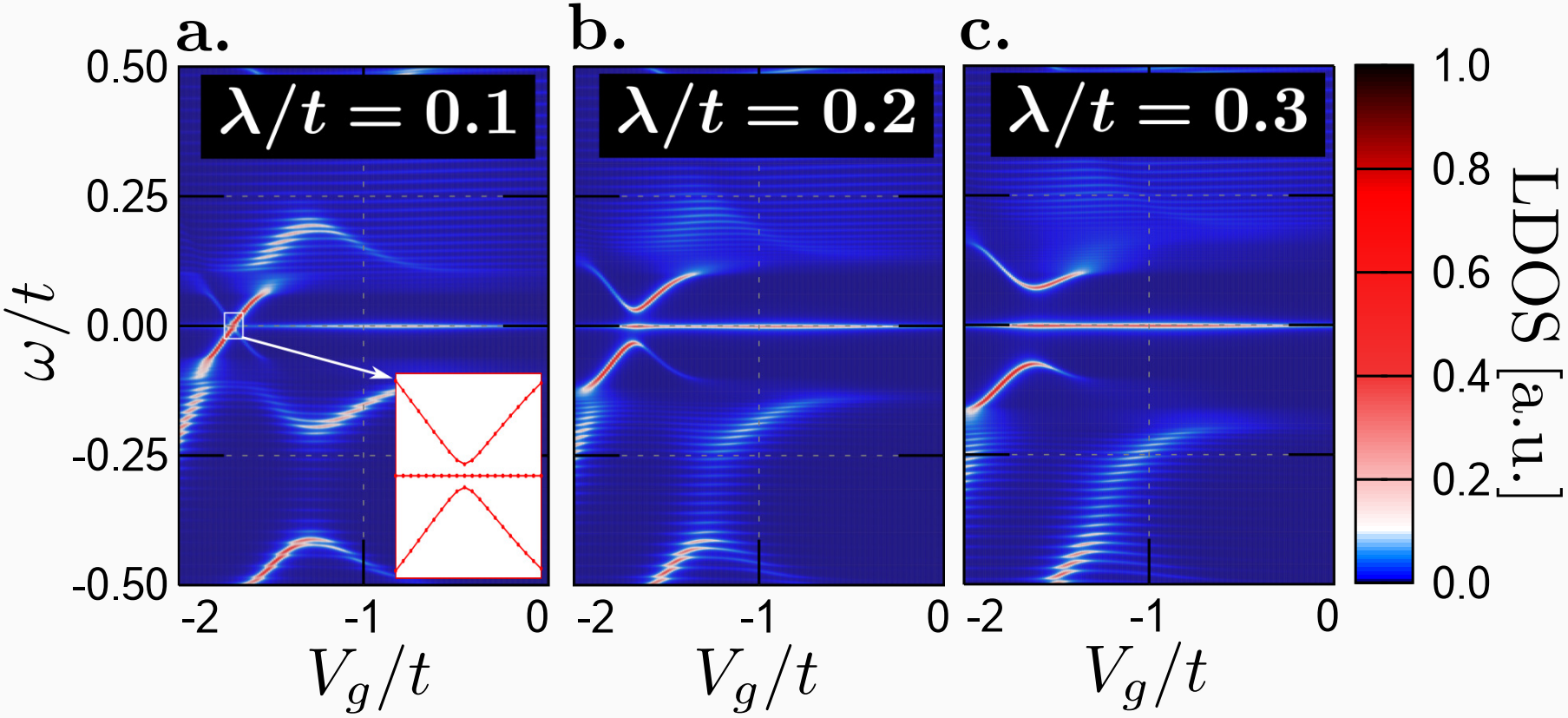
QD spectrum vs gate potential V_g for various spin-orbit couplings λ.



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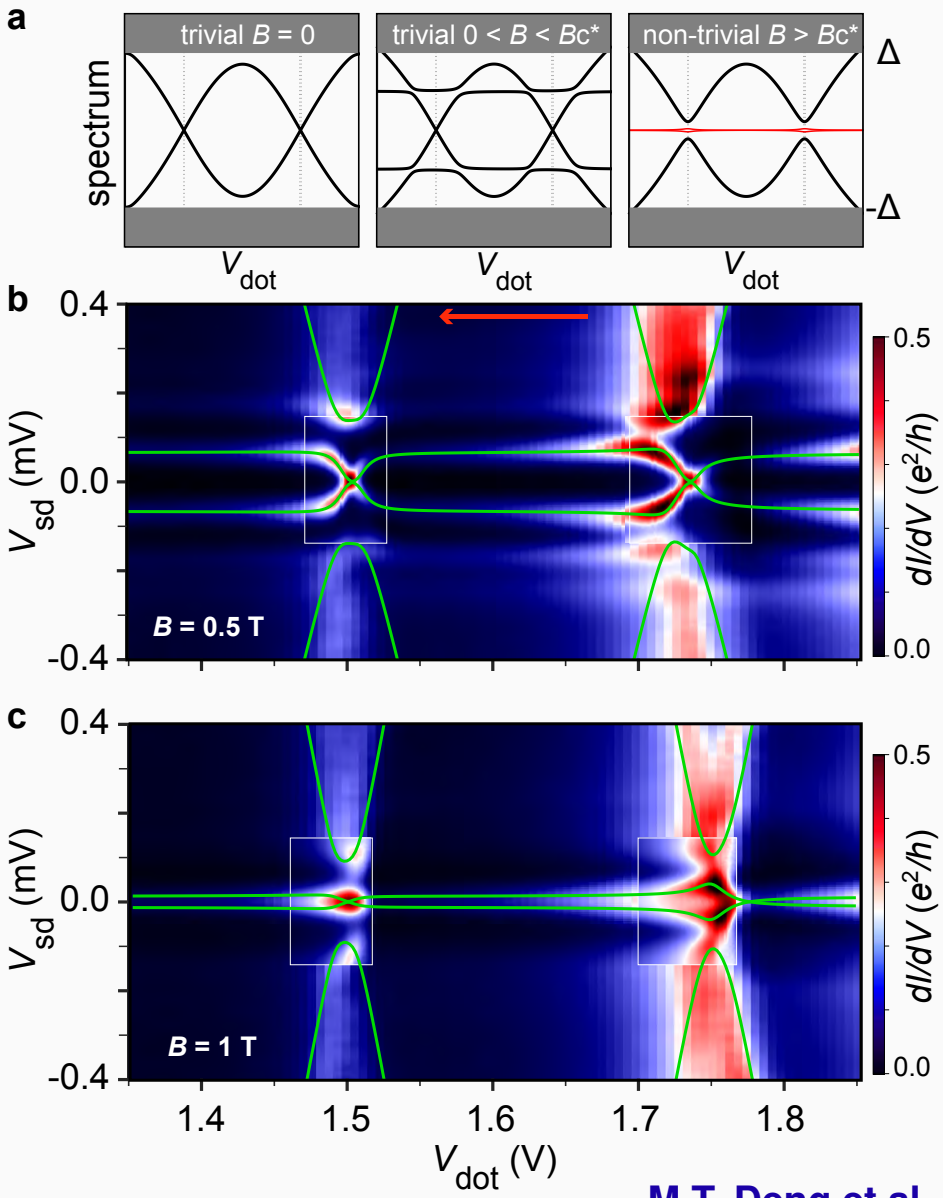

M.T. Deng et al., Phys. Rev. B 98, 085125 (2018).

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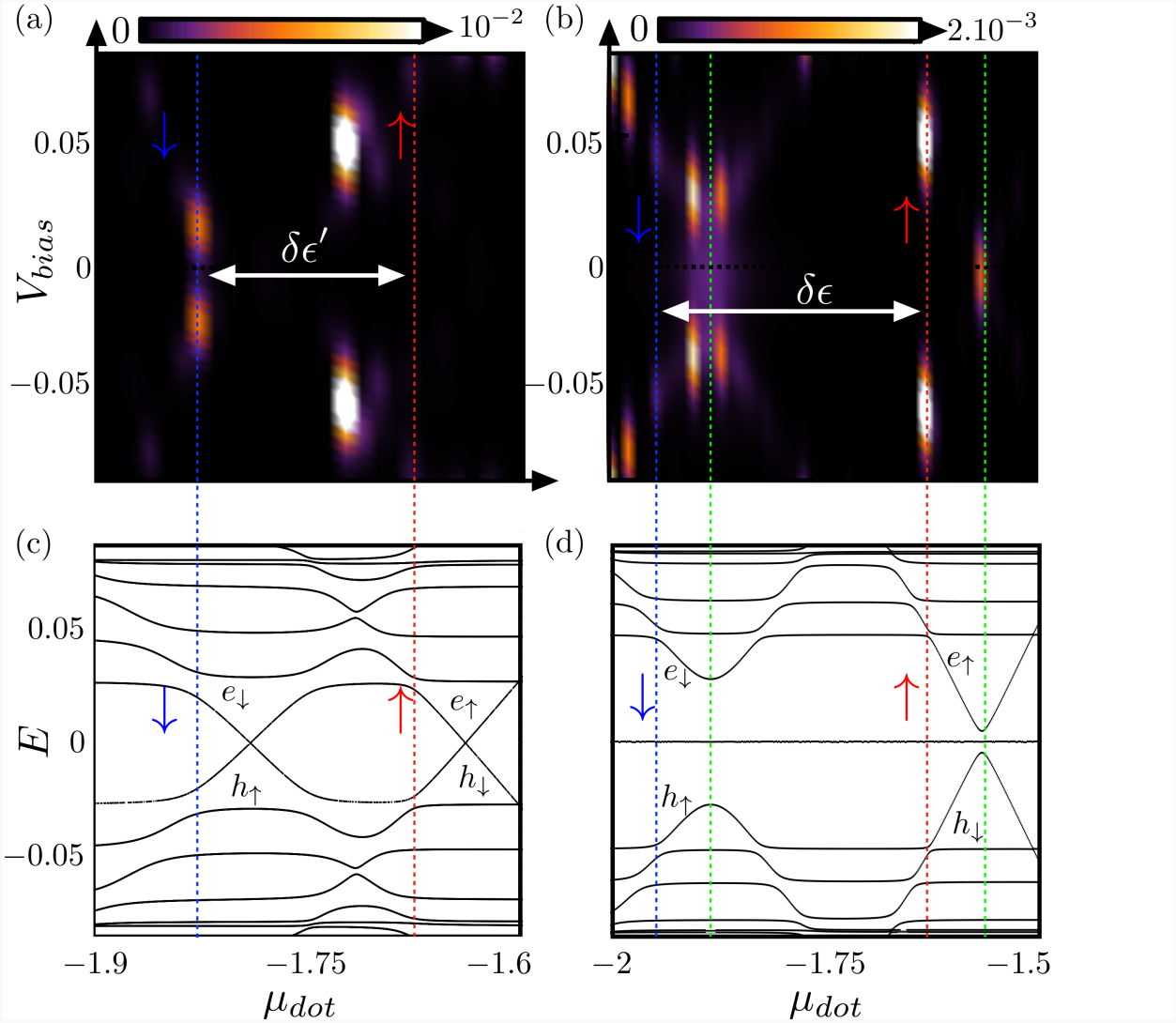

D. Chevallier, ... and J. Klinovaja, Phys. Rev. B 97, 04504 (2018).

ANDREEV VS MAJORANA: CONCLUSIONS

- **Low energy features are very distinct:**

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$\Rightarrow$ **avoided-crossing behavior of Andreev/Shiba qps**

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- **Low energy features are very distinct:**

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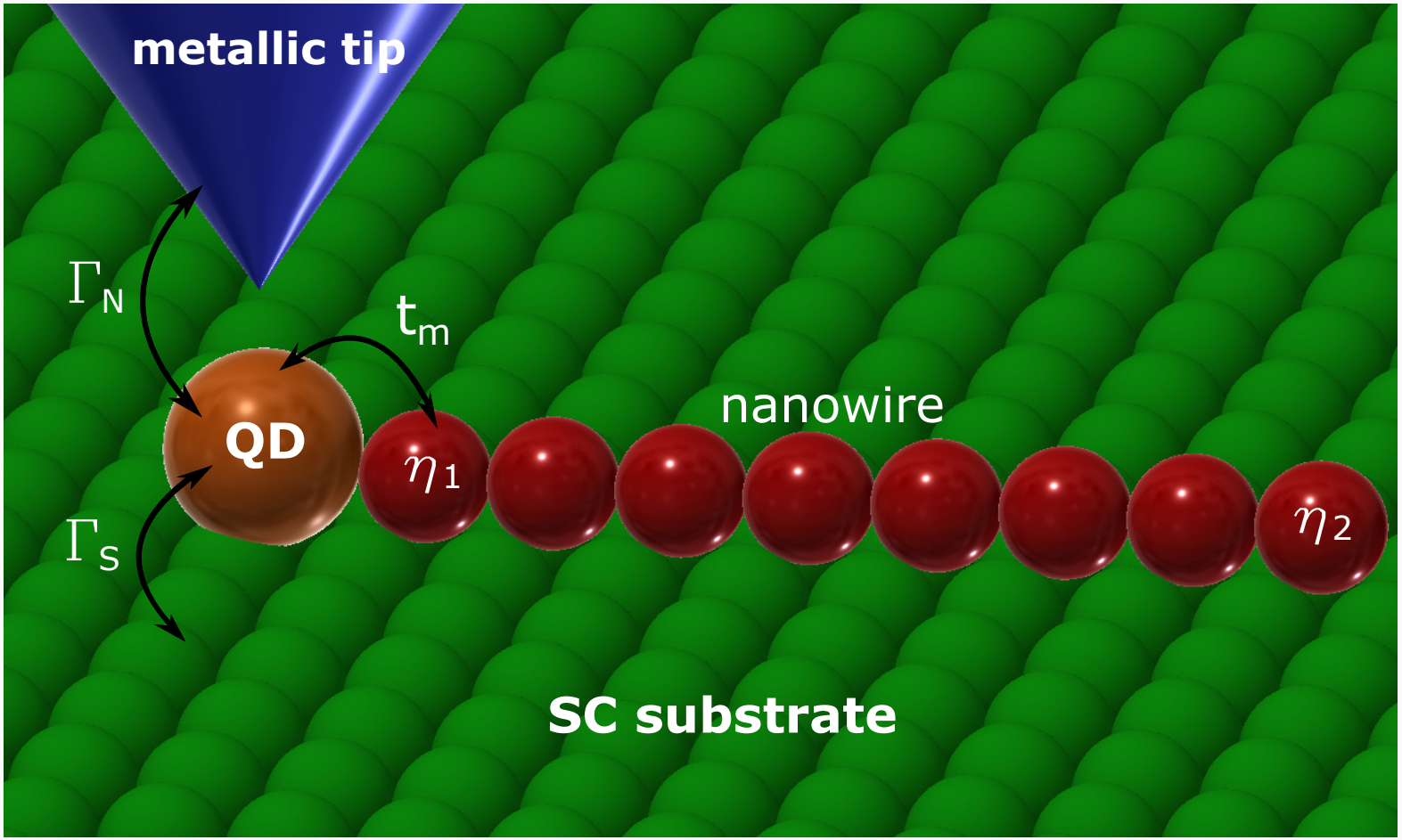
- **Misinterpretation:**

$\Rightarrow$ **coalescence of Andreev into Majorna qps**

Kondo vs Majorana

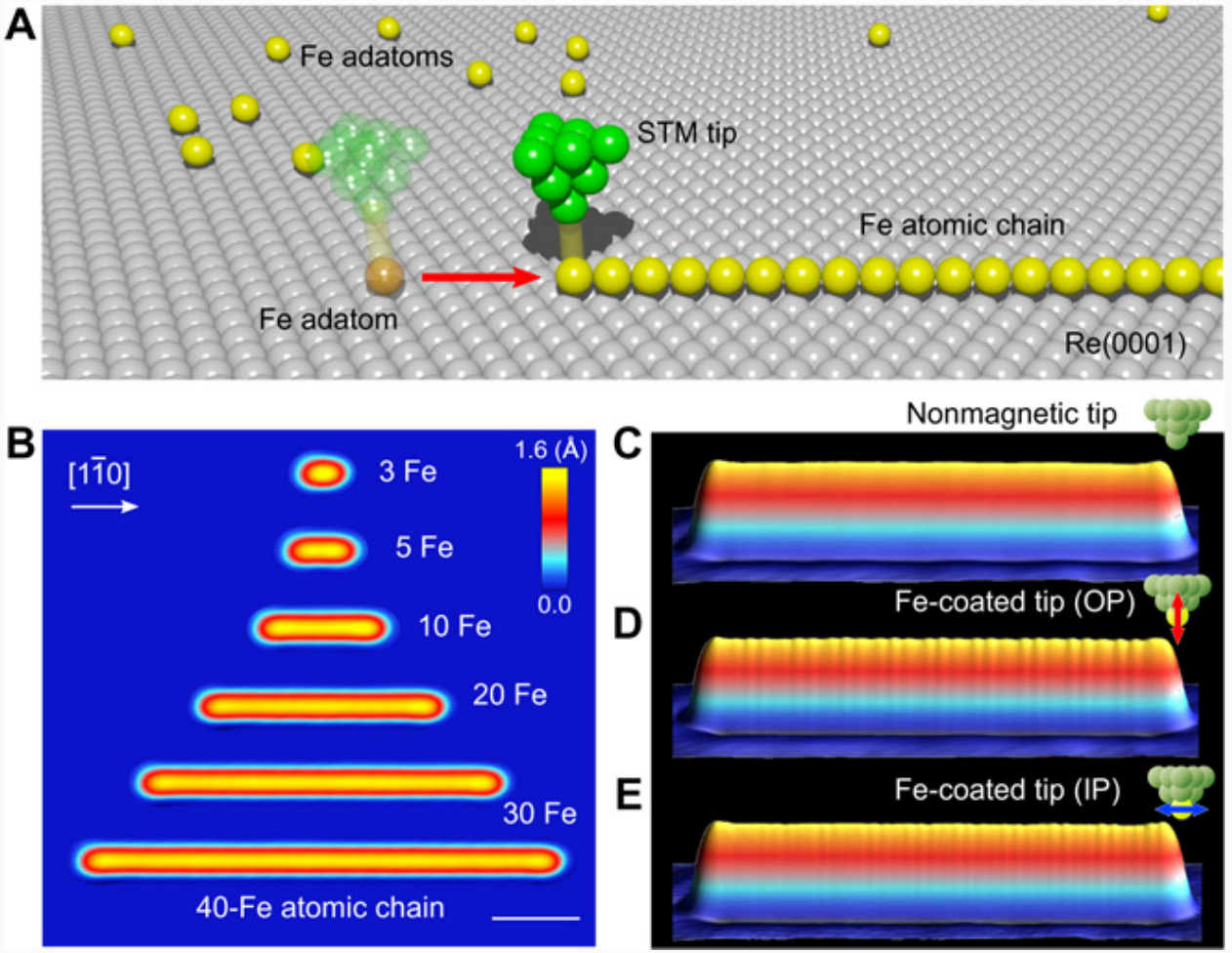
STM-type setup for probing the Kondo – Majorana – pairing effects.



G. Górski, ... and T. Domański, Scientific Reports 8, 15717 (2018).

POSSIBLE EXPERIMENTAL REALISATION

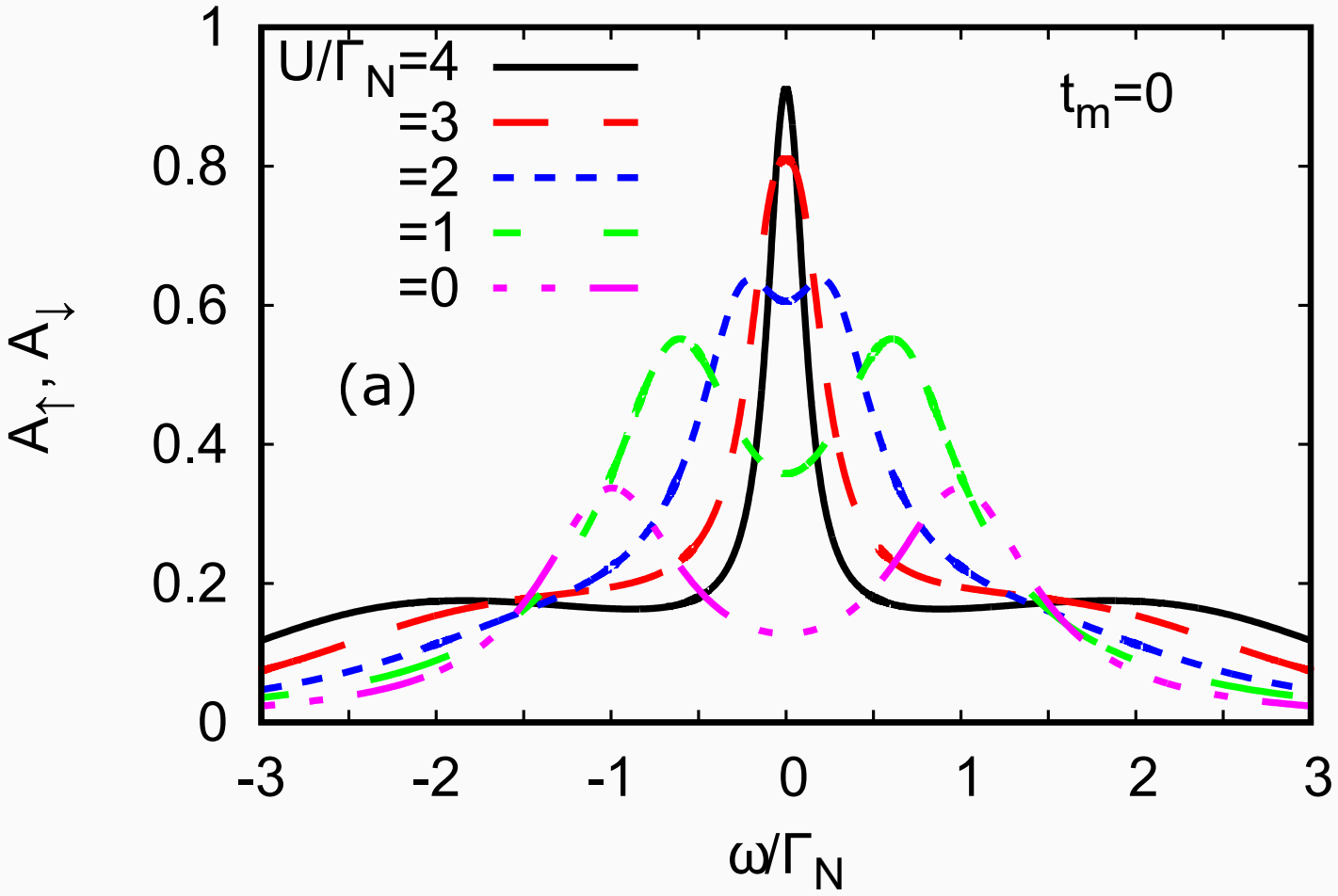
Deposition of individual atoms on superconducting surface



H. Kim, ..., and R. Wiesendanger, Science Adv. 4, eaar5251 (2018).

SUBGAP KONDO PHYSICS

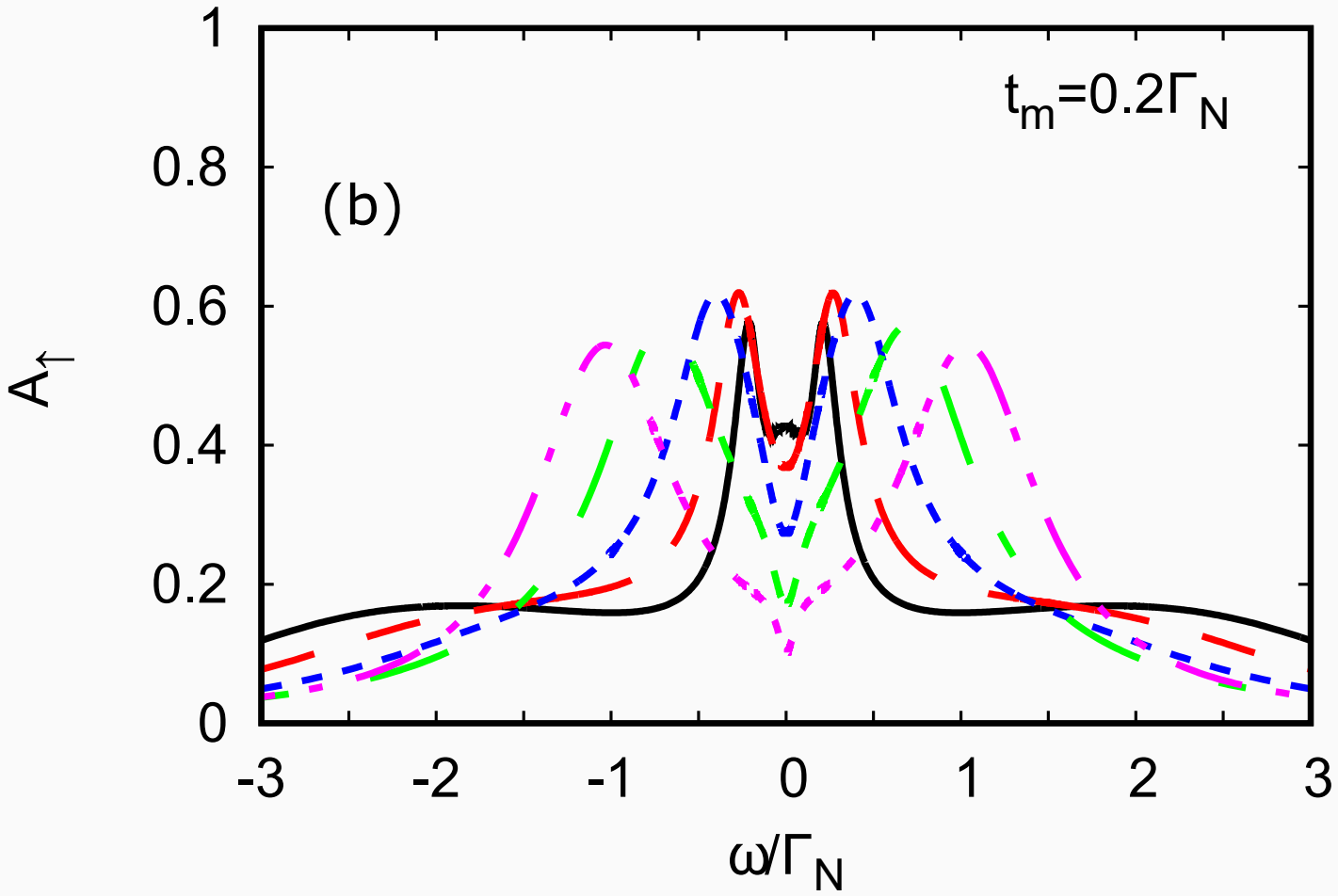
Spectrum of a quantum dot in absence of the Majoranas.



Results obtained for $t_m = 0$

G. Górski, ... and T. Domański, Scientific Reports 8, 15717 (2018).

Spectrum of a quantum dot in its Kondo regime.



Results obtained for ↑ spin, assuming $t_m = 0.2\Gamma_N$

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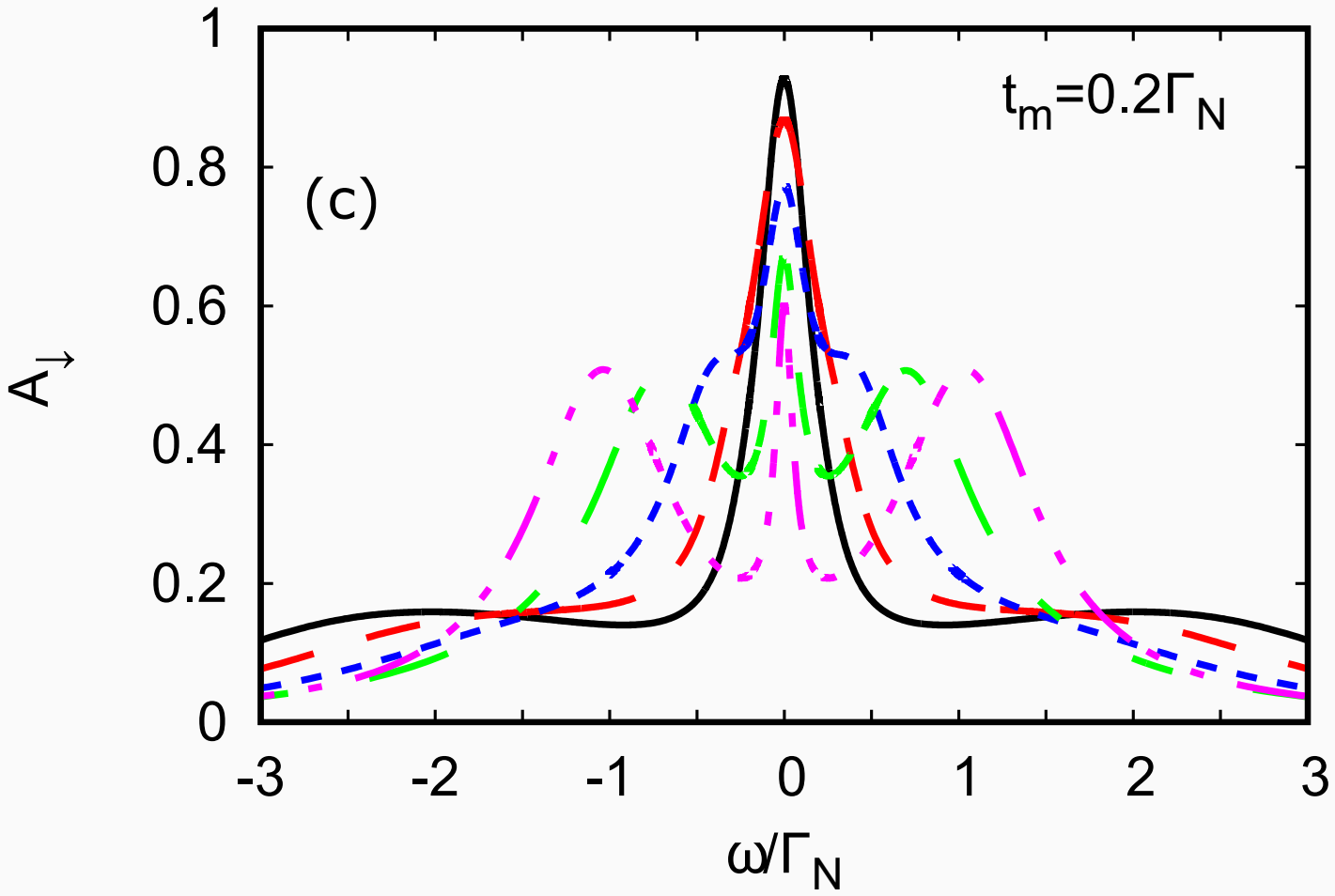
Spectrum of the correlated QD in its Kondo regime.



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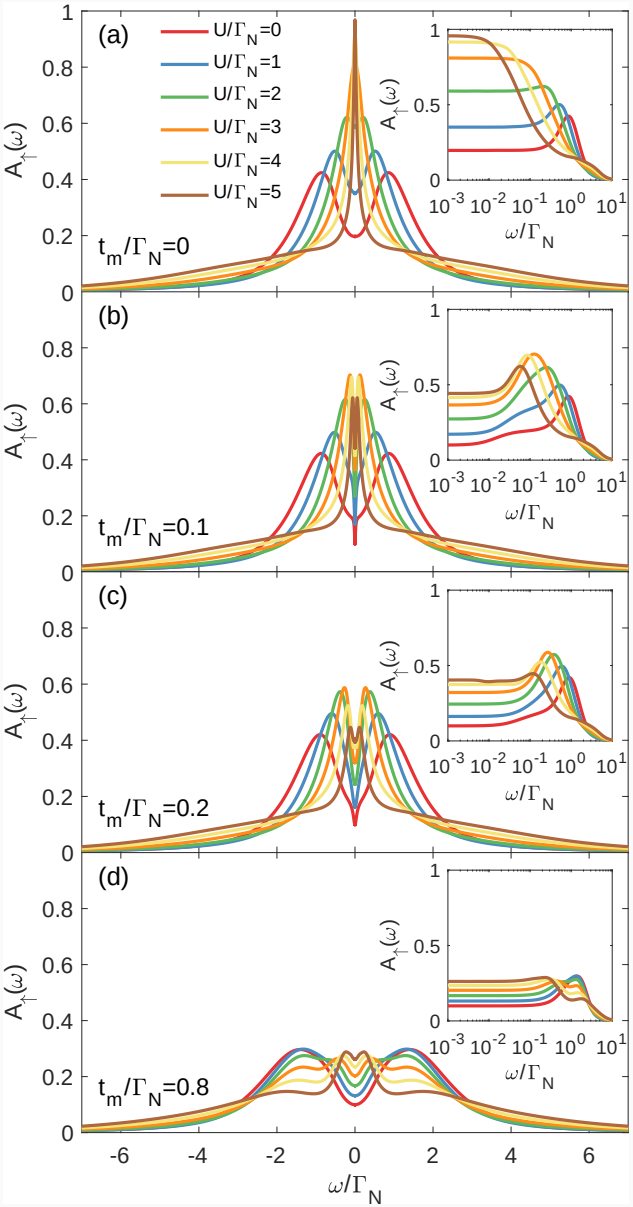
SPIN-RESOLVED NRG DATA



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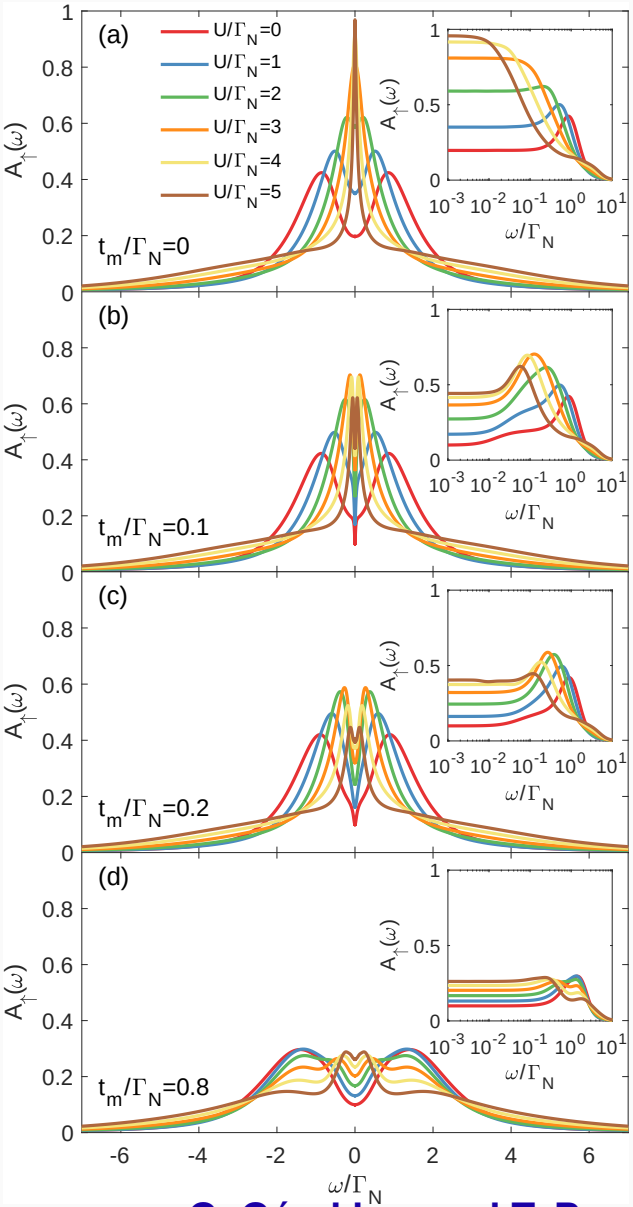

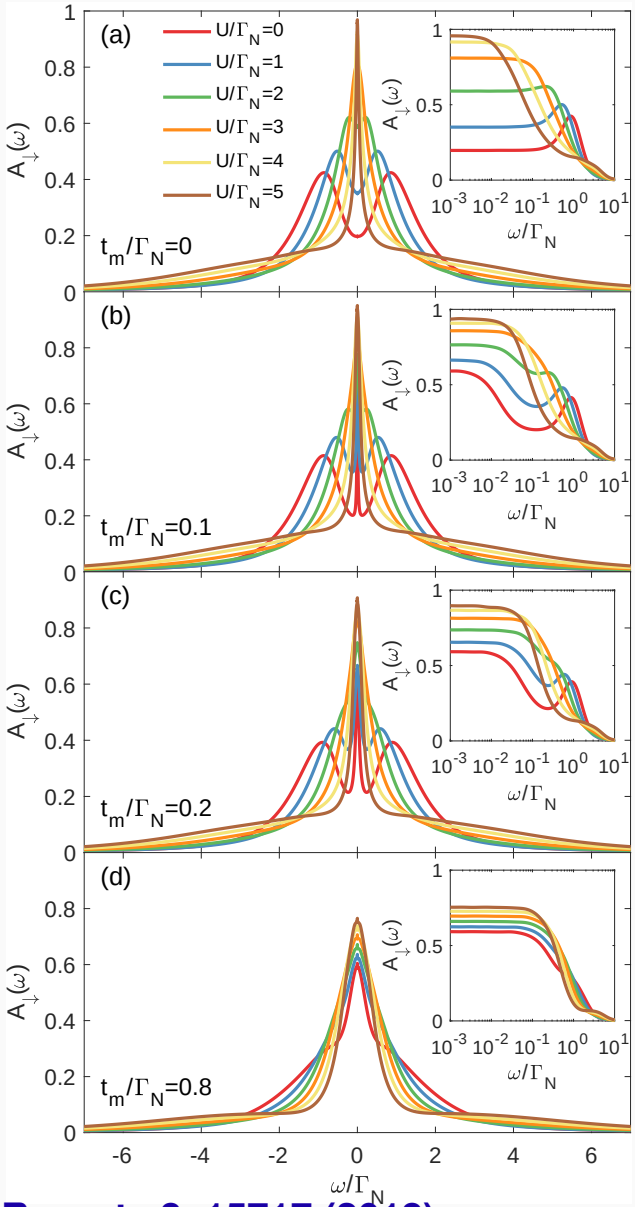

G. Górski, ... and T. Domański, Scientific Reports 8, 15717 (2018).

KONDO VS MAJORANA: CONCLUSIONS

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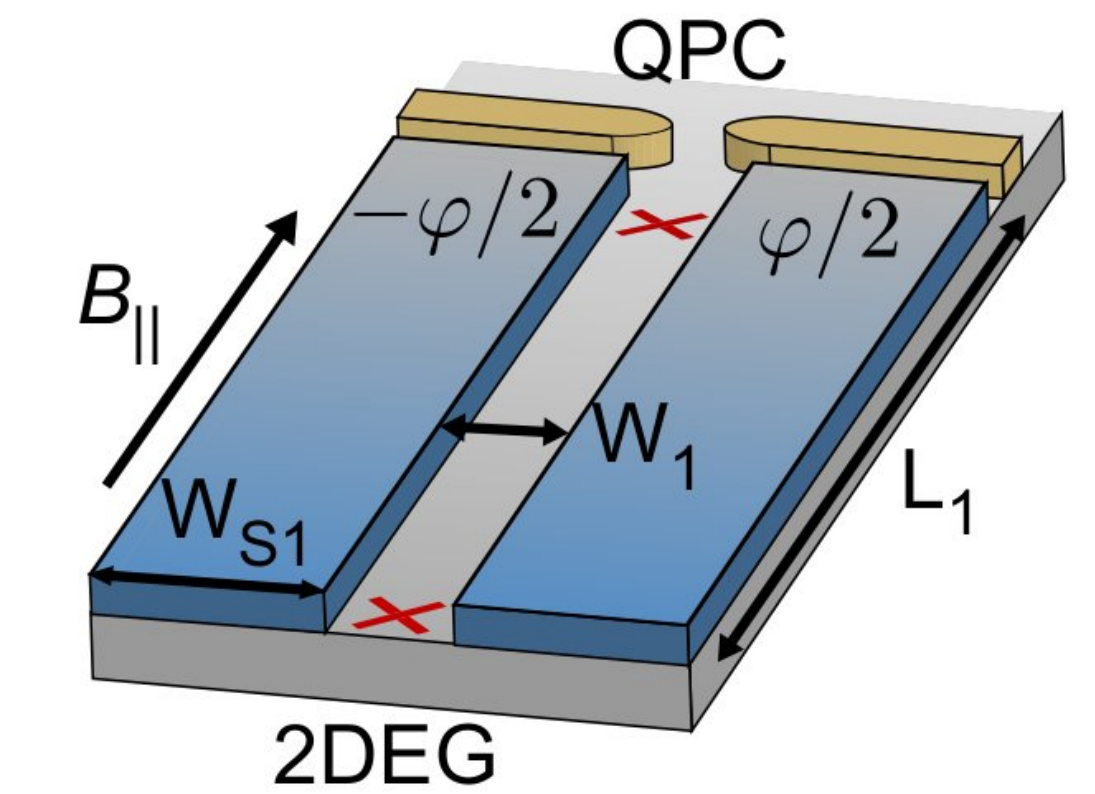
$\Rightarrow$ **selective equal spin Andreev reflections (SESAR)**

Localized Majorana modes in dim=2

PLANAR JOSEPHSON JUNCTIONS

Two-dimensional electron gas of InAs epitaxially covered by a thin Al layer



Width:

W_1 = 80 nm

Length:

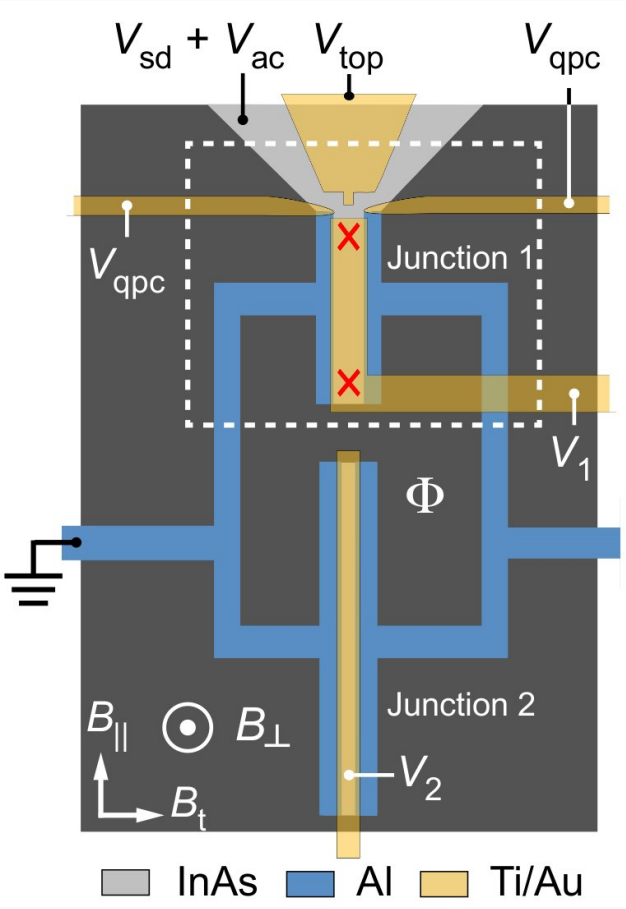
L_1 = 1.6 μm

A. Fornieri, ..., Ch. Marcus and F. Nichele, arXiv:1809.03037 (9 Sept 2018).

/ Niels Bohr Institute (Copenhagen, Denmark) /

PLANAR JOSEPHSON JUNCTIONS

Majorana qps at the ends of 2DEG depend on the phase-difference Φ

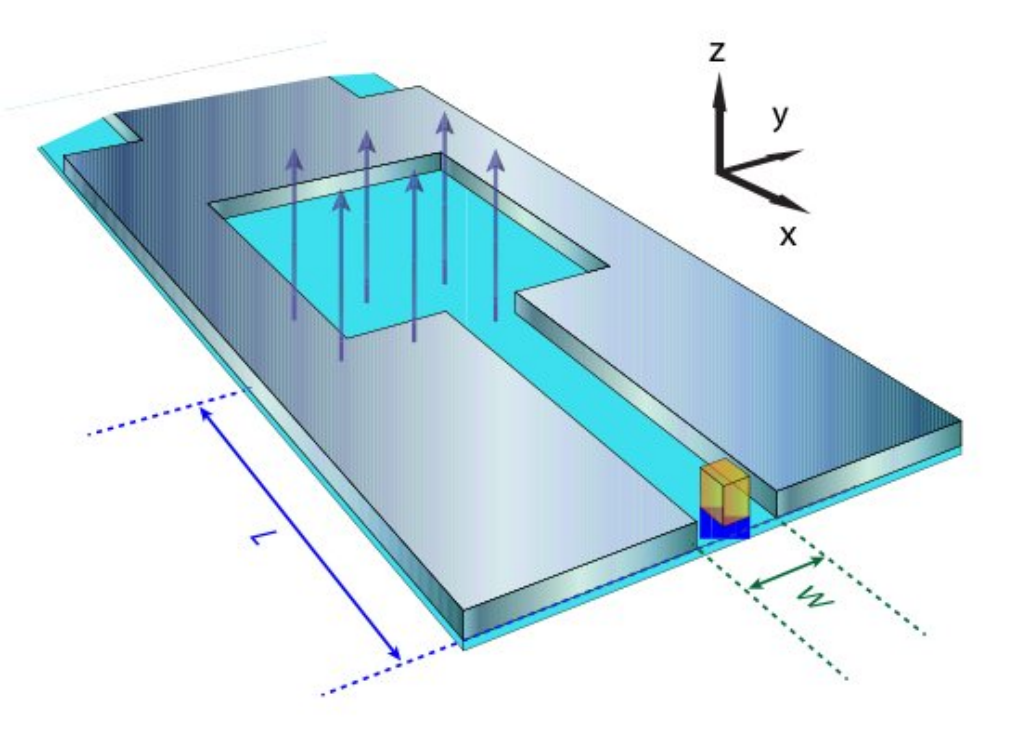


A. Fornieri, ..., Ch. Marcus and F. Nichele, arXiv:1809.03037 (9 Sept 2018).

/ Niels Bohr Institute (Copenhagen, Denmark) /

PLANAR JOSEPHSON JUNCTIONS

Two-dimensional HgTe quantum well coupled to thin Al film

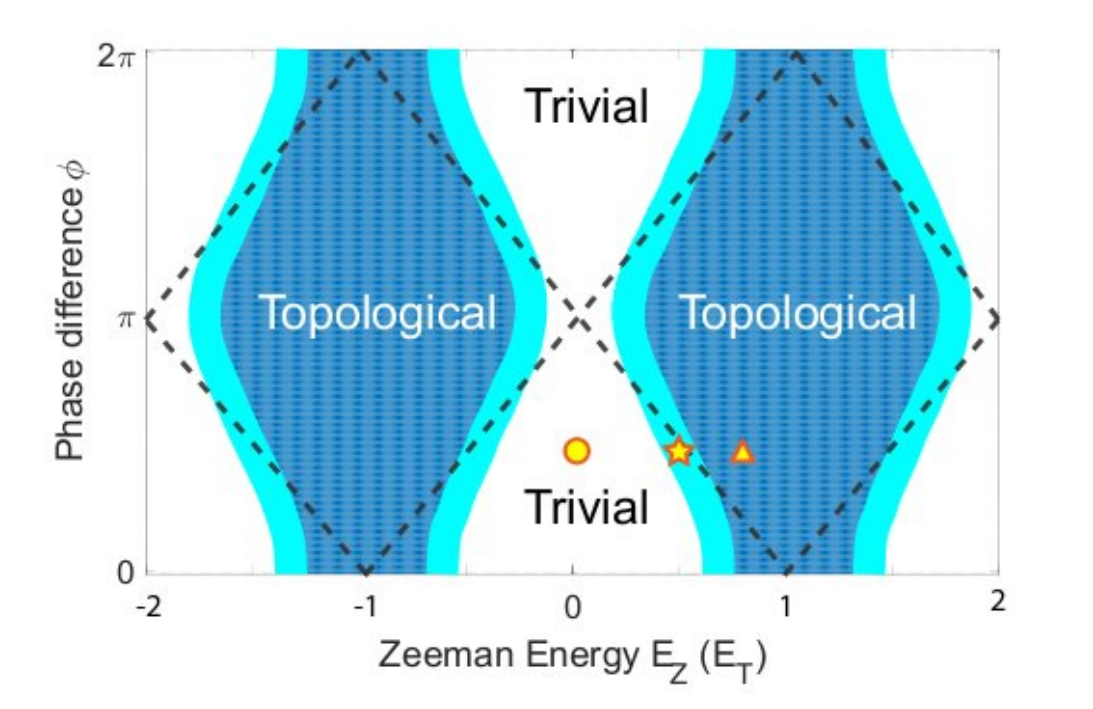


H. Ren, ..., L.W. Molenkamp, B.I. Halperin, A. Yacoby, arXiv:1809.03076 (10 Sept 2018).

/ Würzburg Univ. (Germany) + Harvard Univ. (USA) /

PLANAR JOSEPHSON JUNCTIONS

Tuning between the trivial and topological superconducting state by phase difference ϕ and in-plane magnetic field



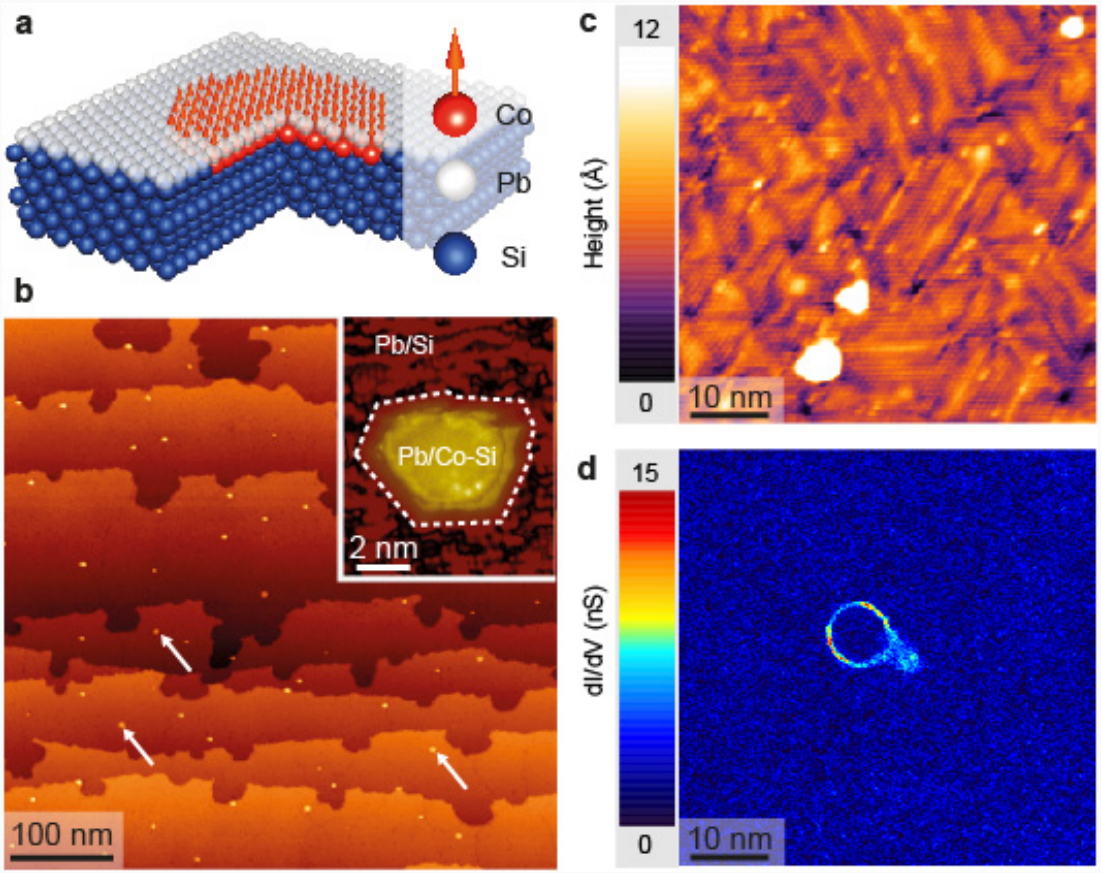
H. Ren, ..., L.W. Molenkamp, B.I. Halperin, A. Yacoby, arXiv:1809.03076 (10 Sept 2018).

/ Harvard Univ. (USA) + Würzburg Univ. (Germany) /

Edge modes of dim=2 systems

TWO-DIMENSIONAL MAGNETIC STRUCTURES

Magnetic island of Co atoms deposited on the superconducting Pb surface



Diameter of island:

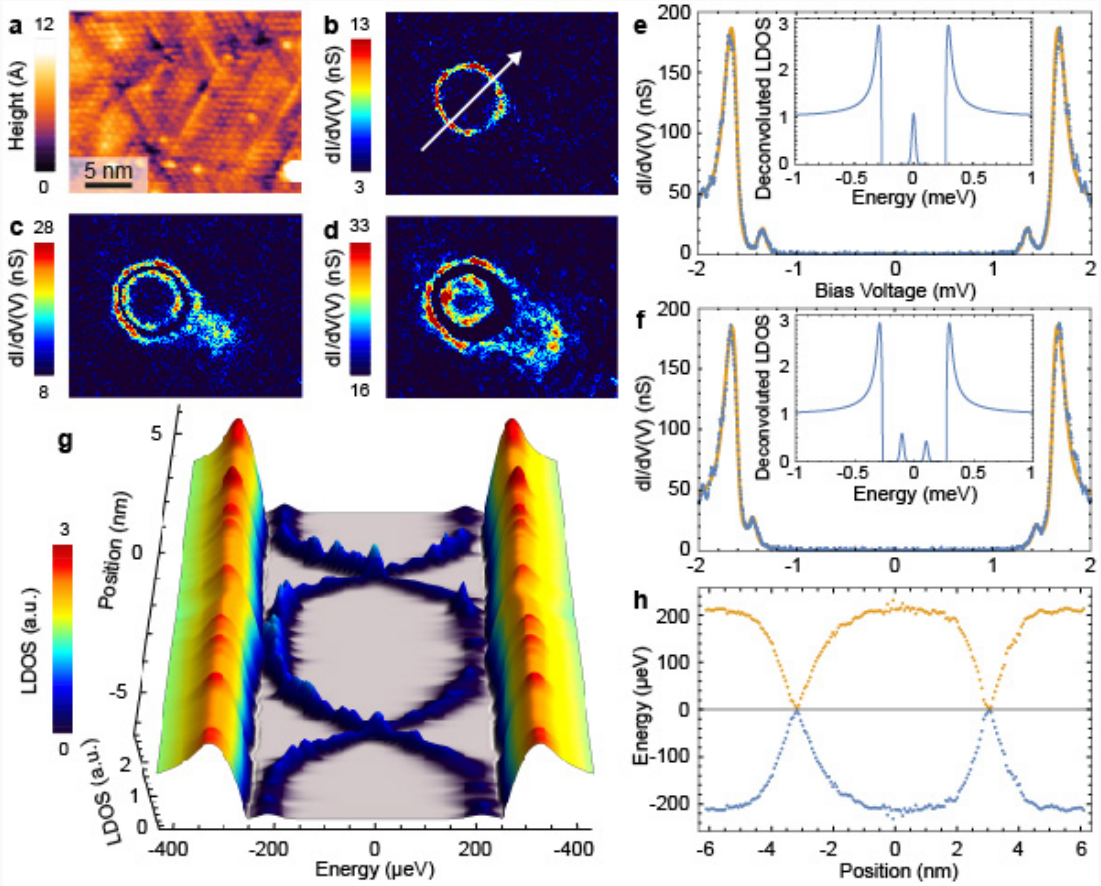
5 – 10 nm

G. Ménard, ..., and P. Simon, Nature Commun. **8**, 2040 (2017).

/ P. & M. Curie University (Paris, France) /

EVIDENCE FOR DELOCALIZED MAJORANA MODES

Majorana modes propagating along magnetic islands

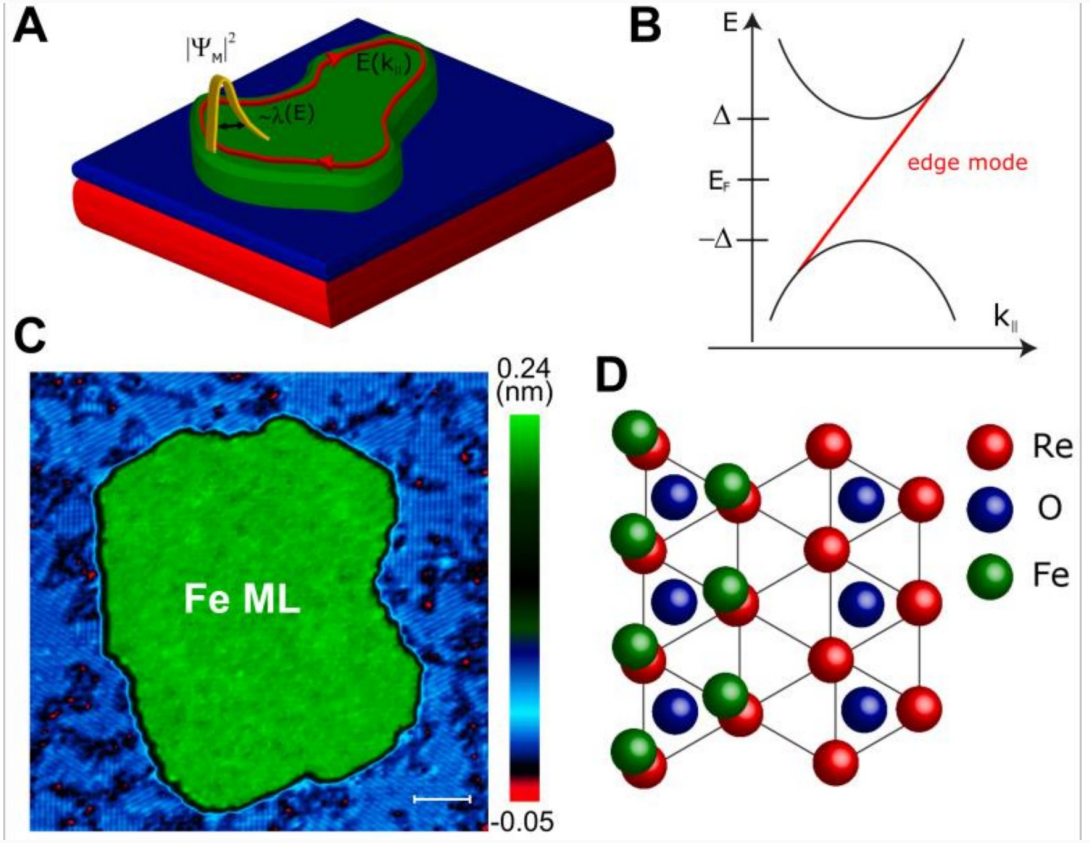


G. Ménard, ..., and P. Simon, Nature Commun. **8**, 2040 (2017).

/ P. & M. Curie University (Paris, France) /

PROPAGATING MAJORANA EDGE MODES

Magnetic island of Fe atoms deposited on the superconducting Re surface



Chern number:

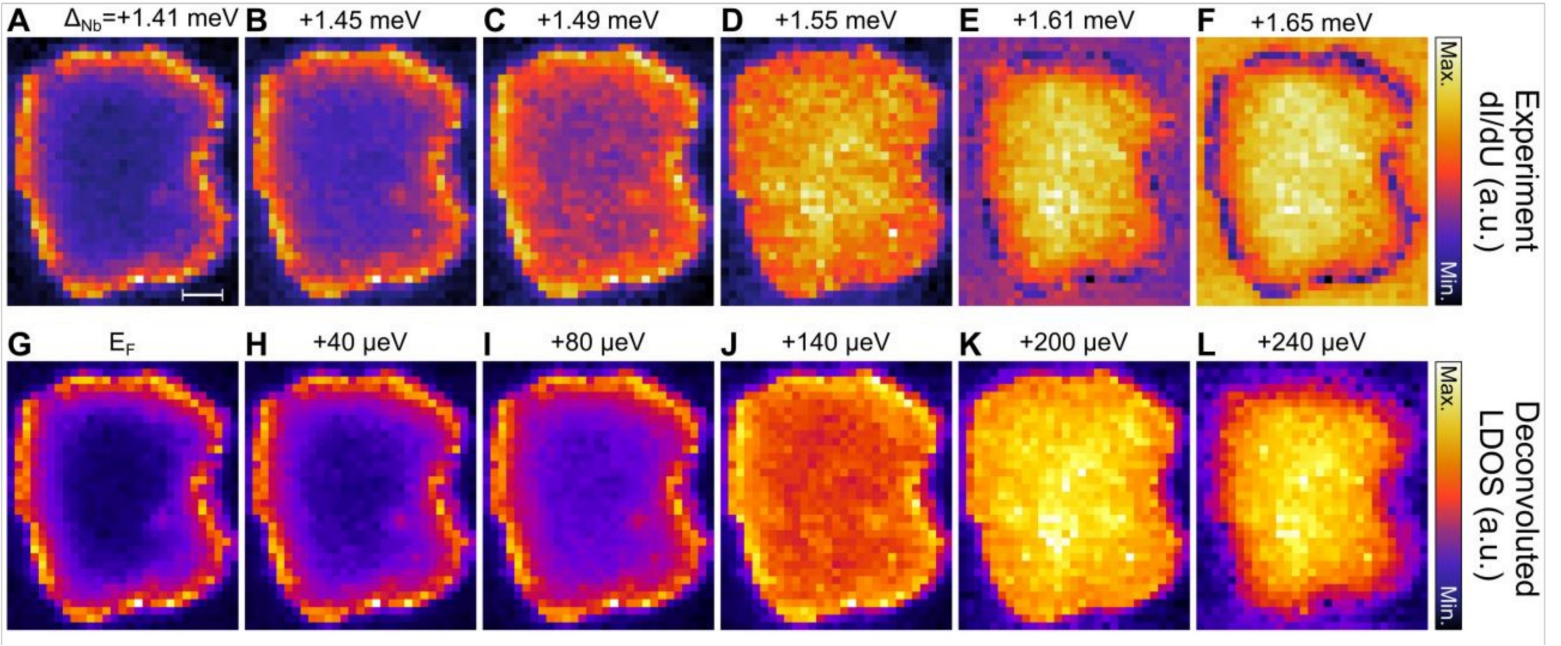
C = 20

A. Palacio-Morales, ..., and R. Wiesendanger, arXiv:1809.04503 (preprint).

/ University of Hamburg (Germany) /

PROPAGATING MAJORANA EDGE MODES

Real space maps of the tunneling conductance (top panel) and deconvoluted DOS (bottom panel) obtained for various energies (as indicated) in the subgap regime ($\Delta = 240 \mu eV$).



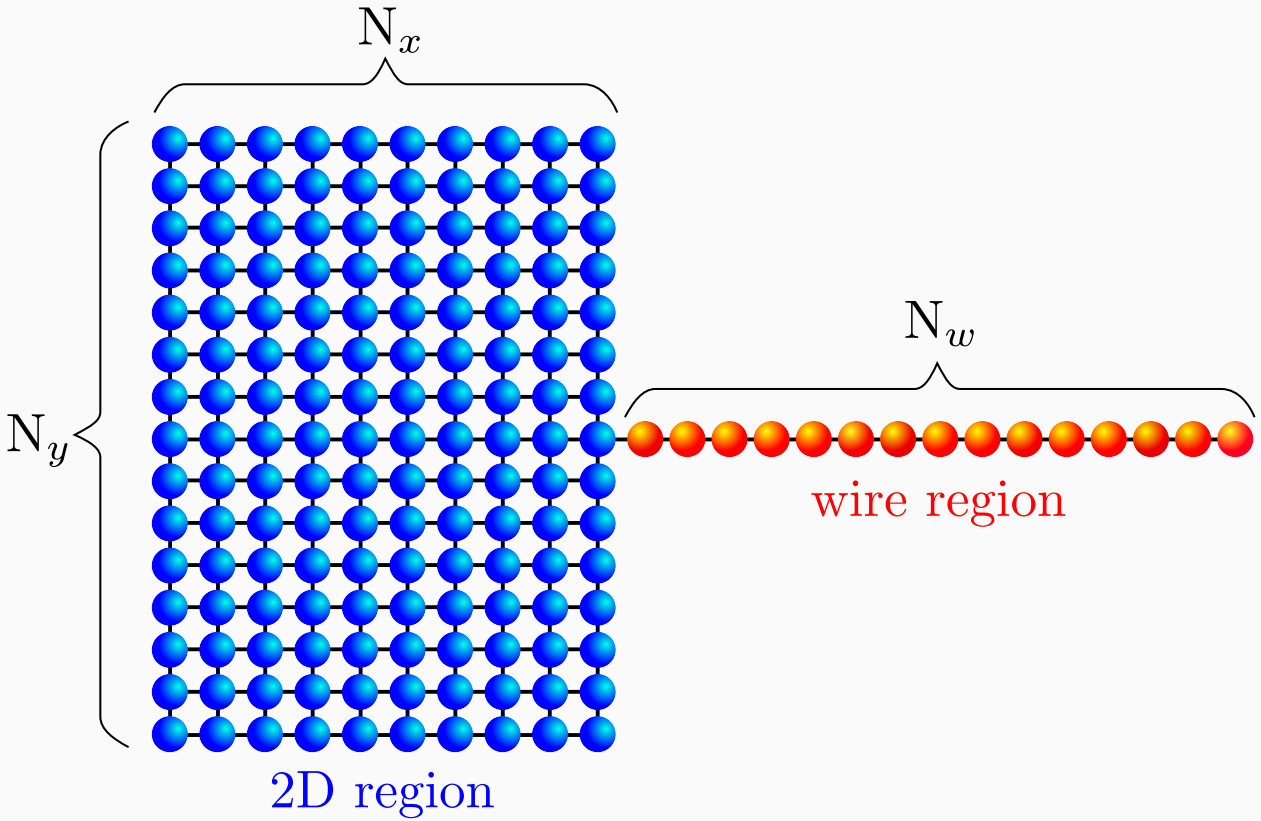
A. Palacio-Morales, ..., and R. Wiesendanger, arXiv:1809.04503 (preprint).

/ University of Hamburg (Germany) /

Mixed – dimensionality structures

CAN MAJORANA QPS BE DECONFINED ?

Our project: **Majorana qps of the 1D–2D hybrid structure**



A. Kobiałka, T. Domański & A. Ptok, arXiv:1808.05281

TOPOLOGICAL INVARIANTS

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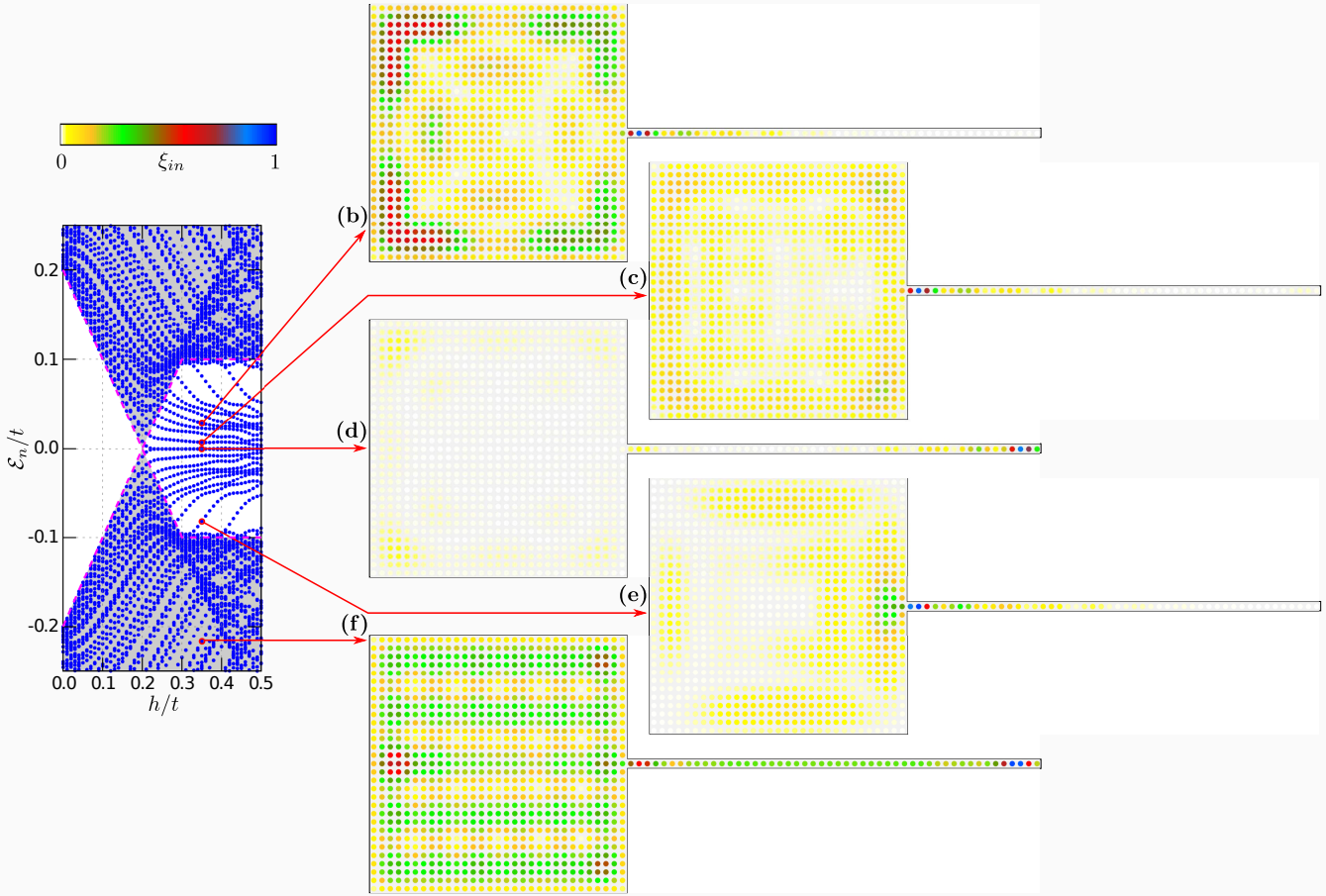
which can be characterized by the Chern number, that is also equivalent to the Thouless–Kohmoto–Nightingale–den Nijs number.

For details, concerning the topological criteria see e.g.

- **A. Kitaev, AIP Conf. Proc. 1134, 22 (2009);**
- **M.Z. Hasan & C.L. Kane, Rev. Mod. Phys. 82, 3045 (2010);**
- **X.-L. Qi & S.-C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).**

TRIVIAL VS MAJORANA MODES

Majorana/Andreev quasiparticles of a wire-plaquette hybrid

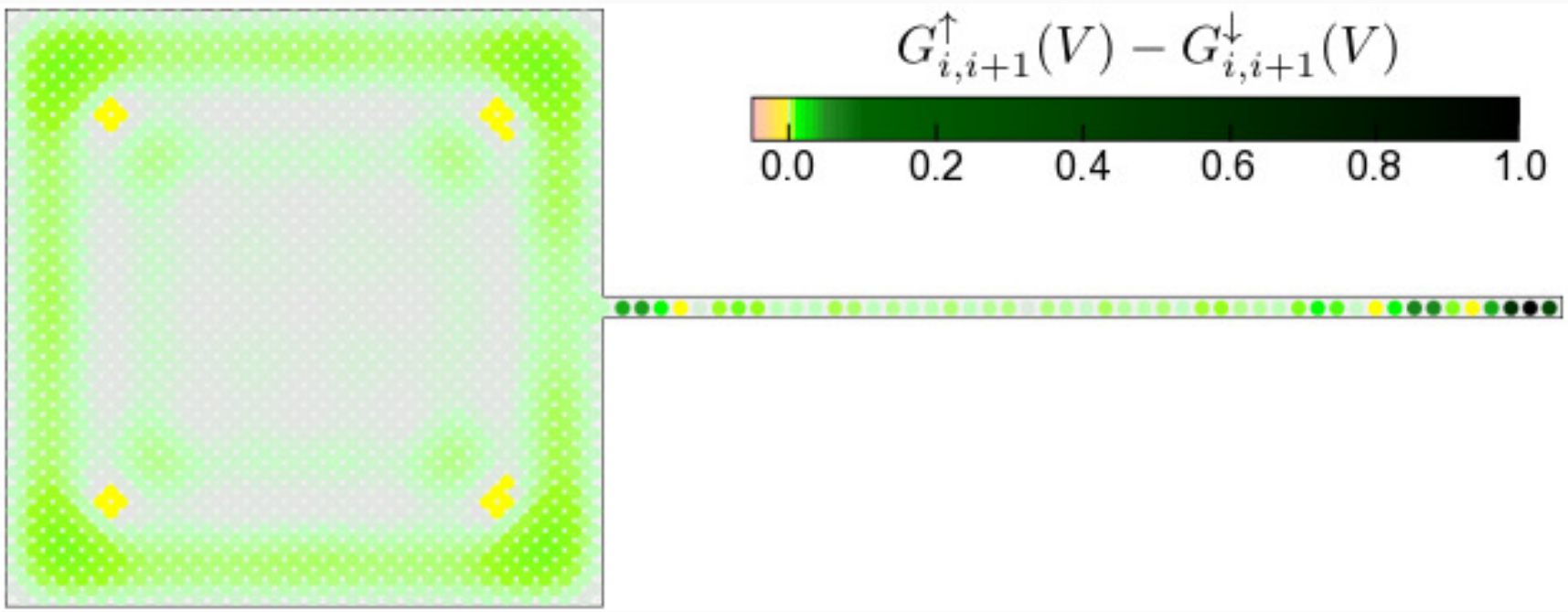


Both regions are assumed to be in topological sc phase.

A. Kobiałka, T. Domański & A. Ptok, arXiv:1808.05281

HOW TO DETECT (DE)LOCALIZED MAJORANA QPS

Maps of the tunneling SESAR conductance at zero-bias.



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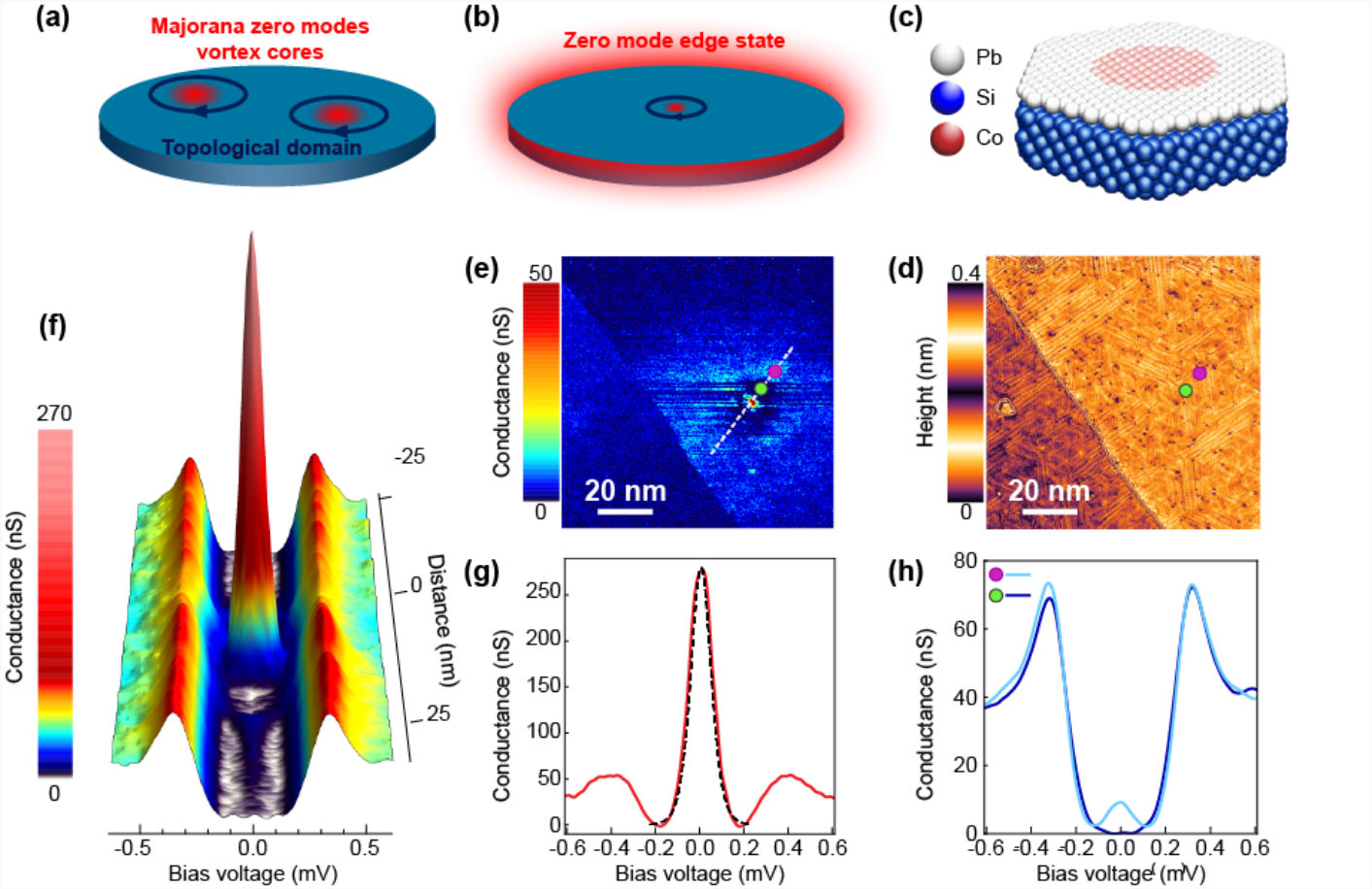
A. Kobiałka, T. Domański & A. Ptok, arXiv:1808.05281

SIMILAR IDEAS: DEFECTS IN MAGNETIC ISLAND

Localized Majorana at point-like defect, coexisting with itinerant Majorana edge mode (observed in Co-Si island on disordered Pb)

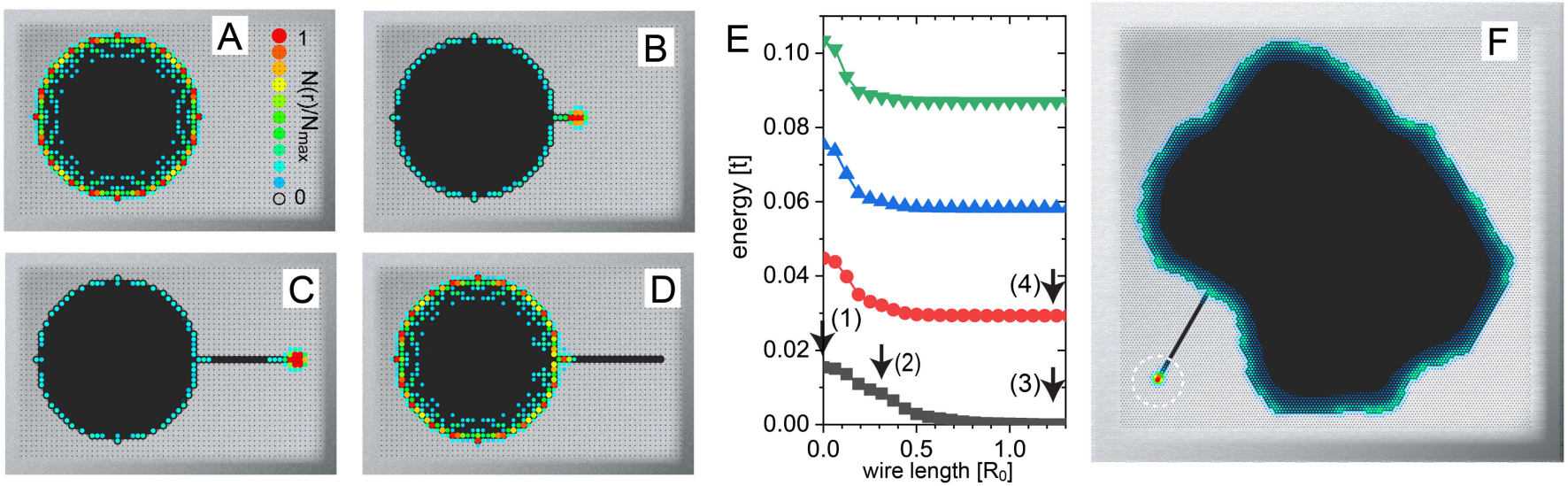


G.C. Ménard, ..., P. Simon and T. Cren, arXiv:1810.09541

Paris (France)

SIMILAR IDEAS: ISLAND + NONOWIRE

Itinerant Majorana mode leaking into side-attached nanowire.

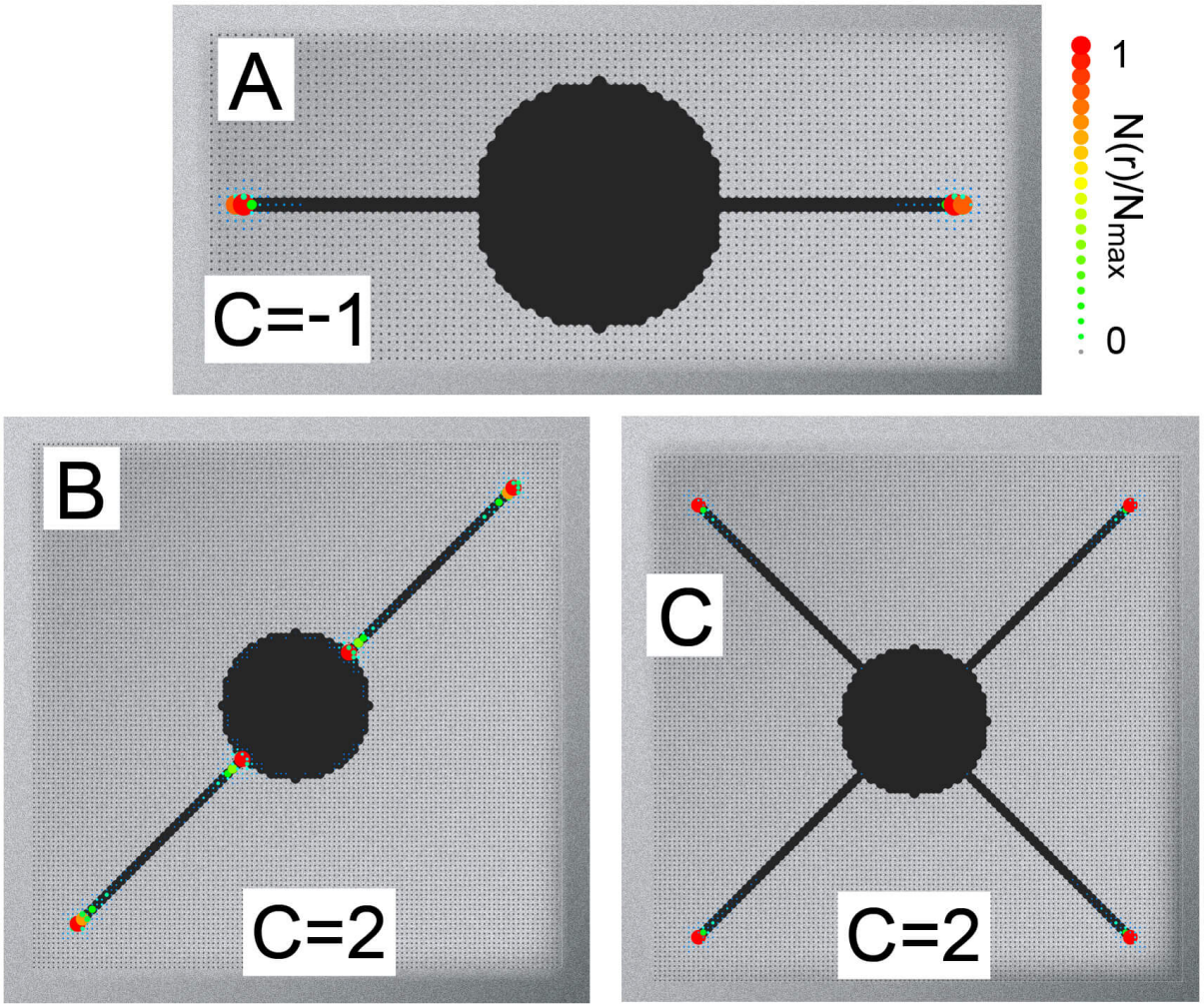


E. Mascot, S. Cocklin, S. Rachel, and D.K. Morr, arXiv:1811.06664

Univ. of Illinois at Chicago (USA)

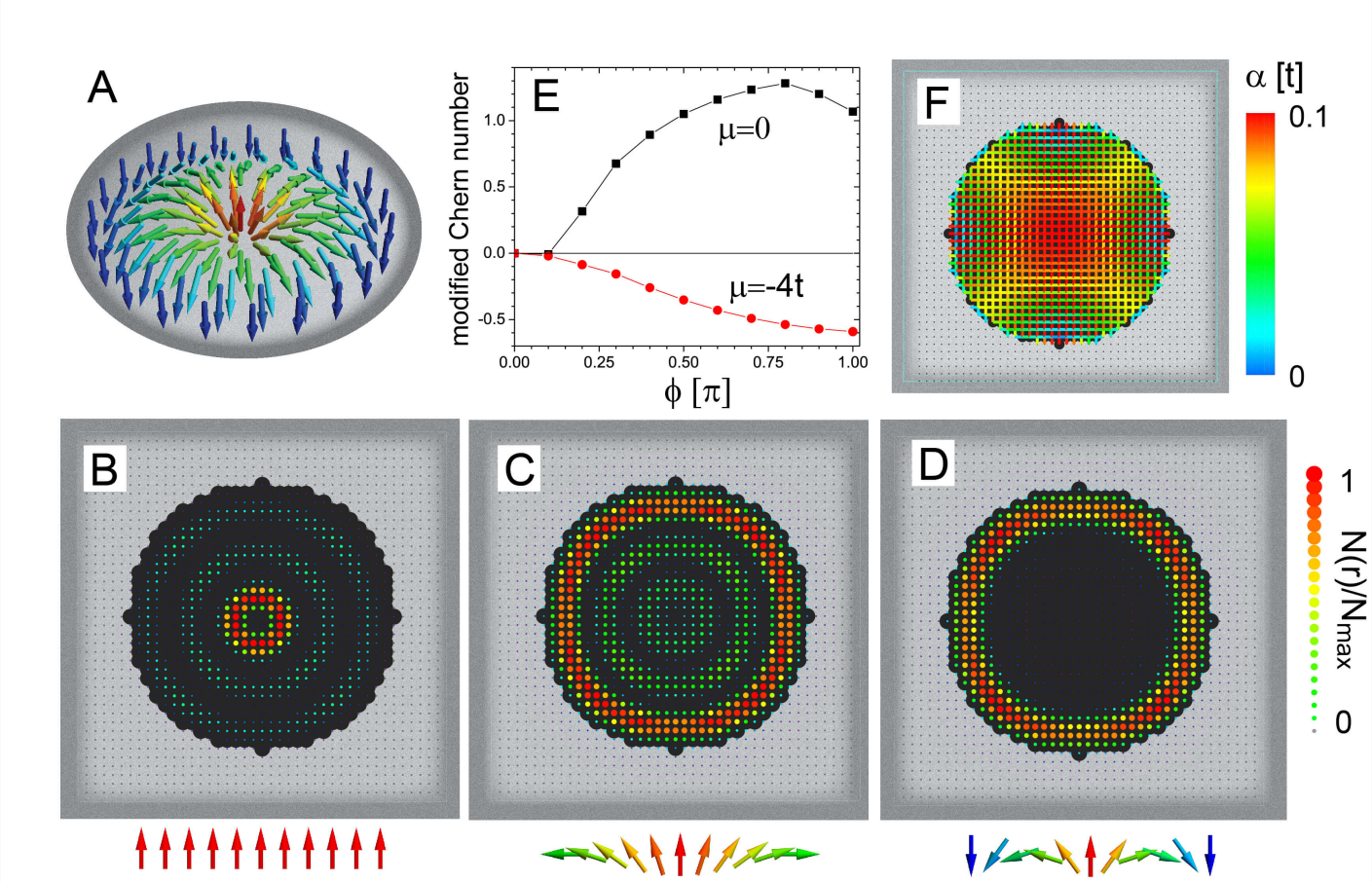
SIMILAR IDEAS: ISLAND + NONOWIRE

Majorana modes leaking to the side-attached nanowires.



PERSPECTIVES: SKYRMIONS IN SUPERCONDUCTORS

Creation of topological phase through skyrmions.



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DIMENSIONAL HYBRIDS: CONCLUSIONS

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Would it help to realise braiding of Majorana qps ?

ACKNOWLEDGEMENTS

- **Majorana quasiparticles**

  ⇒ A. Kobiałka (Lublin), A. Ptok (Kraków),
  M. Maśka & A. Gorczyca-Goraj (Katowice),
  J. Tworzydło (Warszawa), N. Sedlmayr (Rzeszów).

- **Shiba qps in topological phases**

  ⇒ Sz. Głodzik (Lublin)

- **Subgap Kondo effect**

  ⇒ I. Weymann (Poznań), G. Górski (Rzeszów),
  T. Novotný, M. Žonda & V. Janiš (Prague).

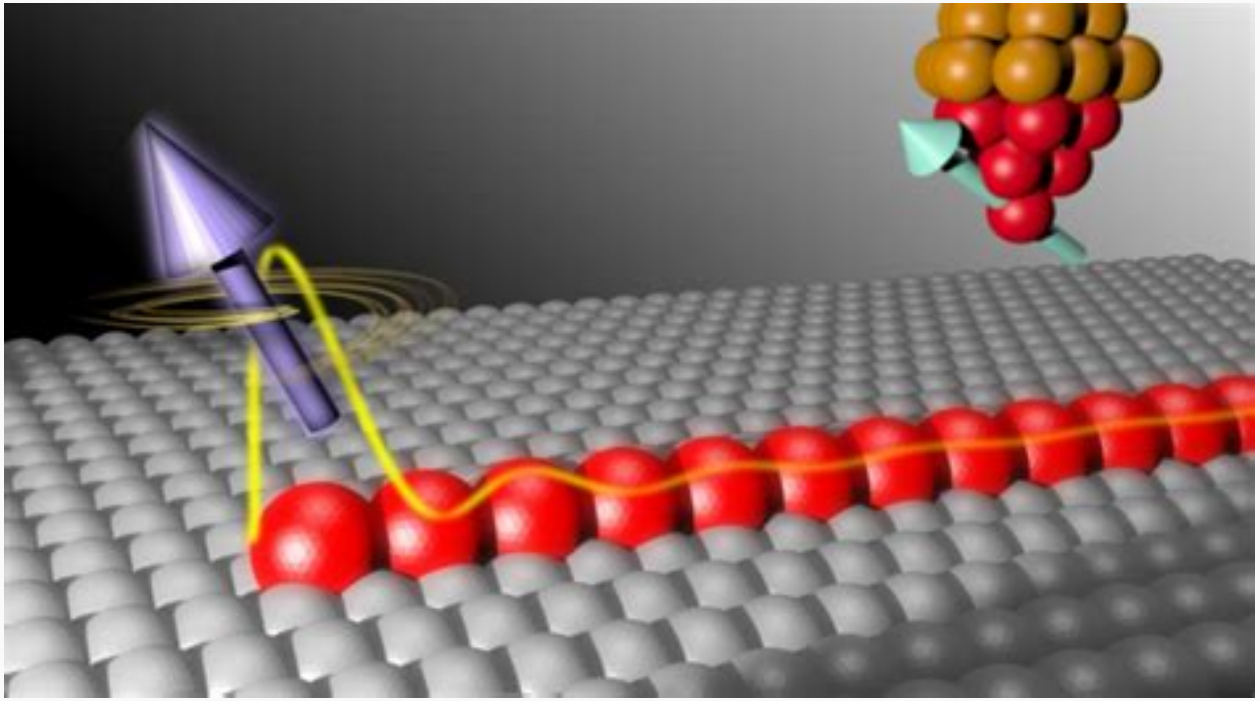
- **Nonlocal Andreev processes**

  ⇒ K.I. Wysokiński (Lublin), G. Michałek & B.R. Bułka (Poznań)

- **Dynamics of in-gap states**

  ⇒ R. Taranko, B. Baran & T. Kwapiński (Lublin)

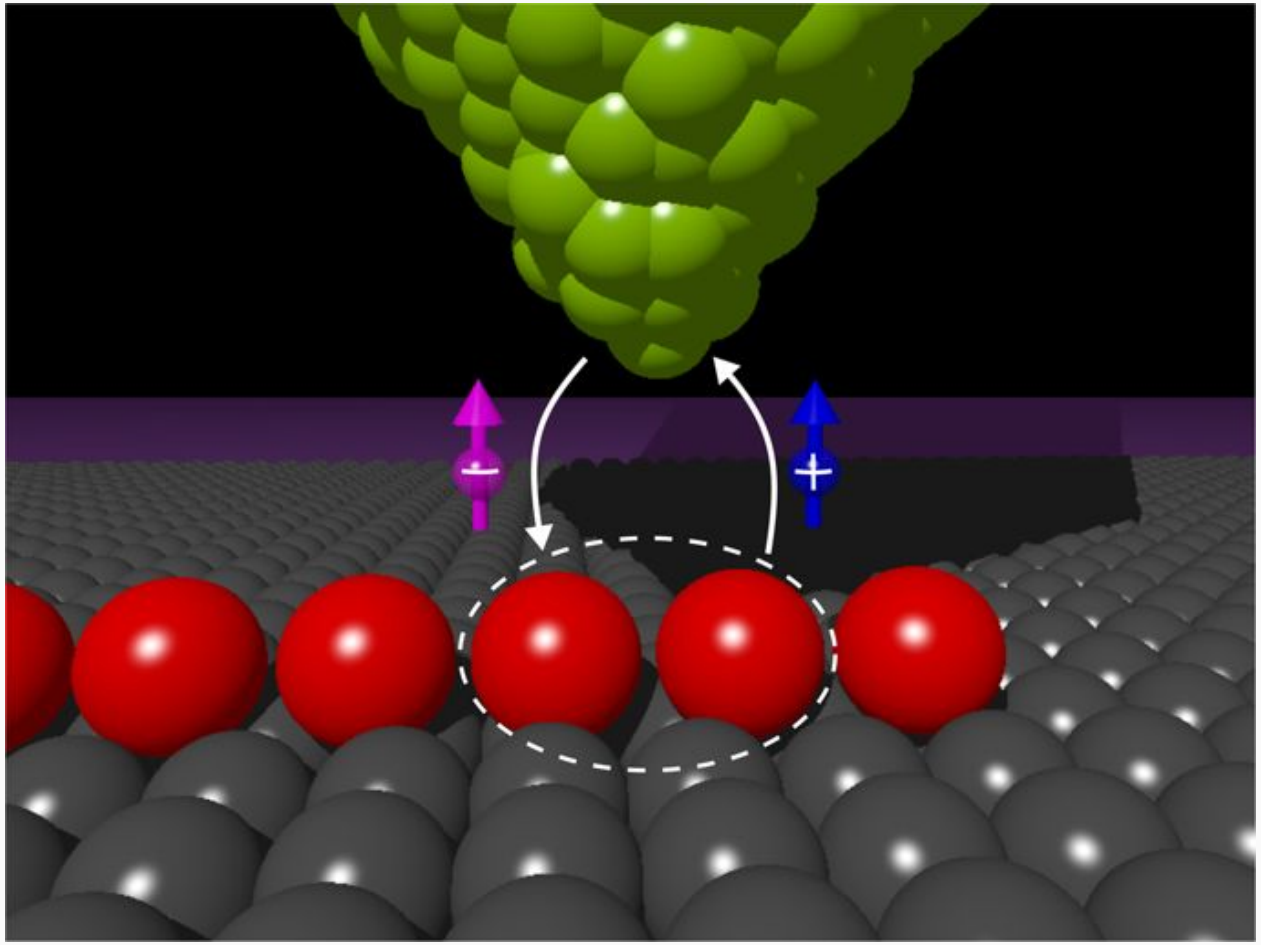
STM-type measurements for probing the Majorana qps



S. Jeon, ... and A. Yazdani, Science **358**, 772 (2017).

/ Princeton University, USA /

Microscopic idea of the SESAR mechanism



M. Maśka and T. Domański, Scientific Reports **7**, 16193 (2017).