Poznań, 28 June 2017

Spin-resolved Andreev spectroscopy for probing the Majorana quasiparticles

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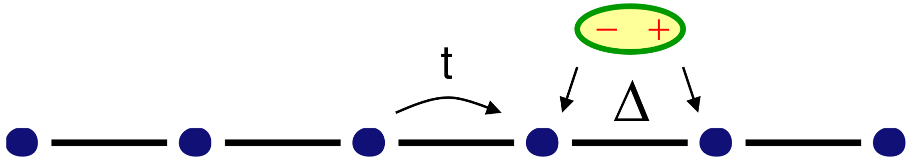
Kitaev chain – a paradigm for Majorana modes



p-wave pairing of spinless 1D fermions

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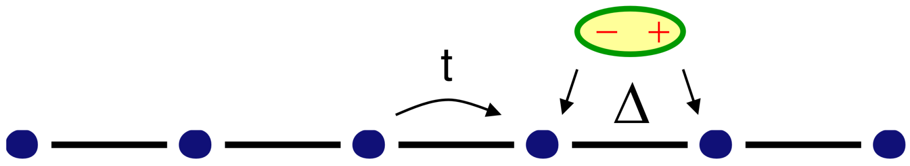

p-wave pairing of spinless 1D fermions

$$\hat{H} = t\sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1} + \mathrm{h.c.}\right) - \mu \sum_i \hat{c}_i^\dagger \hat{c}_i + \Delta \sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1}^\dagger + \mathrm{h.c.}\right)$$

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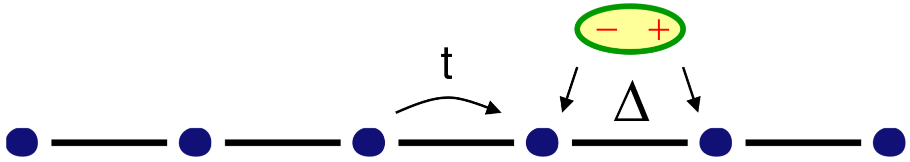

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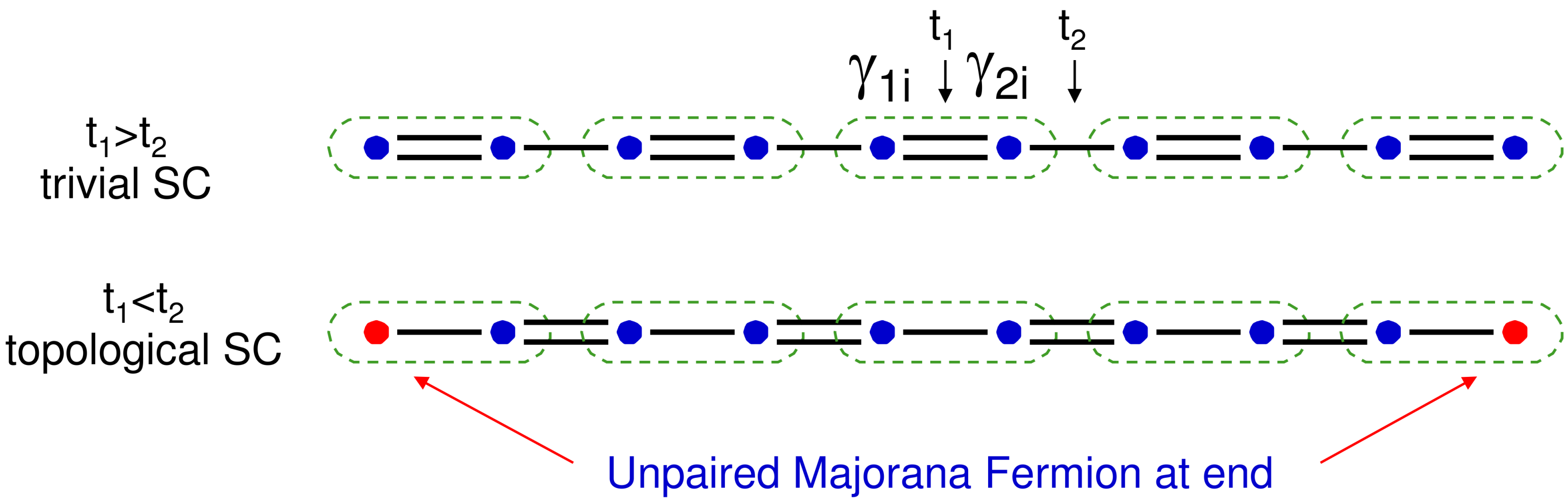
This toy-model can be exactly solved in Majorana basis. For $\Delta = t$ one obtains:



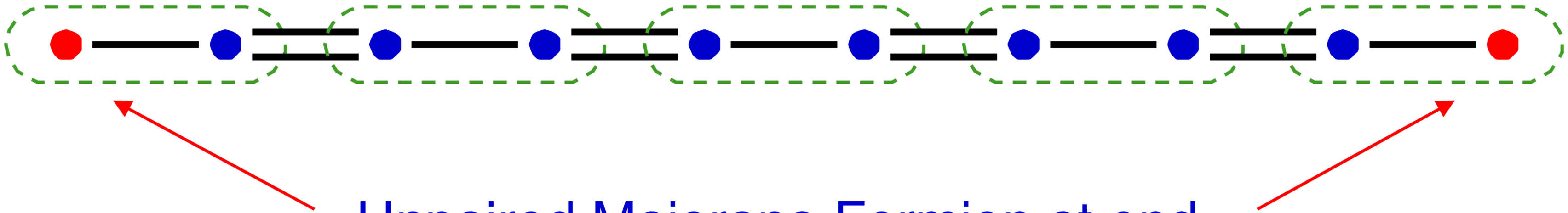
$t_1 \equiv \mu$

$t_2 \equiv 2t$

Kitaev toy model

/ Phys. Usp. 44, 131 (2001) /

In the special case $\Delta = t$ and $|\mu| < 2t$



Unpaired Majorana Fermion at end

operators $\hat{\gamma}_{1,1}$ and $\hat{\gamma}_{2,N}$ are decoupled from all the rest. This implies

zero-energy modes appearing at the chain edges

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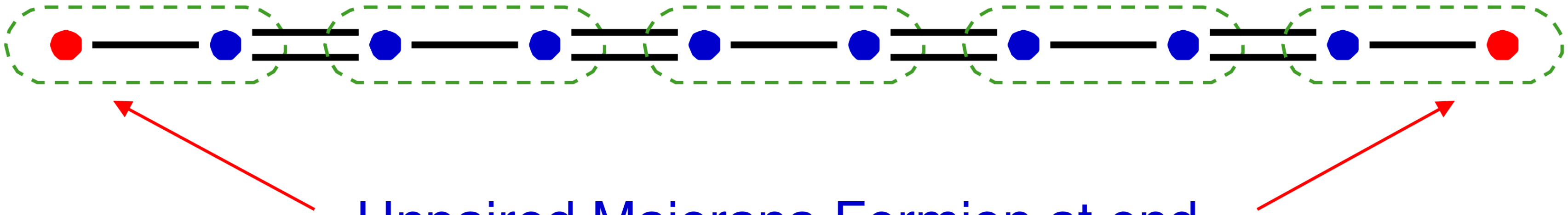

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zero-energy modes appearing at the chain edges

★ **Similar ideas have been considered for 1D Heisenberg chain of 1/2 spins**

$$\bullet\!\!-\!\!-\!\!\bullet \; = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

F.D.M. Haldane, Phys. Rev. Lett. 50, 1153 (1983)

Nobel Prize, 2016

Majoranization – of normal fermions

- Normal fermions (e.g. electrons) obey the anticommutation relations

$$\left\{\hat{c}_i, \hat{c}_j^\dagger\right\} = \delta_{i,j}$$

$$\{\hat{c}_i, \hat{c}_j\} = 0 = \left\{\hat{c}_i^\dagger, \hat{c}_j^\dagger\right\}$$

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- $c_j^{(\dagger)}$ **can be recast in terms of Majorana operators**

$$\hat{c}_j \equiv \left(\hat{\gamma}_{j,1} + i\hat{\gamma}_{j,2}\right)/\sqrt{2}$$
$$\hat{c}_j^\dagger \equiv \left(\hat{\gamma}_{j,1} - i\hat{\gamma}_{j,2}\right)/\sqrt{2}$$

'real' and 'imaginary' parts

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- **Exotic properties**

$$\hat{\gamma}_{i,n}^\dagger = \hat{\gamma}_{i,n}$$
$$\left\{\hat{\gamma}_{i,n}, \hat{\gamma}_{j,m}^\dagger\right\} = \delta_{i,j}\delta_{n,m}$$

creation = annihilation !

fermionic antisymmetry

Majoranization – of normal fermions

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- Exotic properties (cd)

$$\hat{\gamma}_{i,n}\,\hat{\gamma}_{i,n} = 1/2$$
$$\hat{\gamma}_{i,n}^\dagger\,\hat{\gamma}_{i,n} = 1/2$$

no Pauli principle !

half 'occupied' & half 'empty'

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$\Rightarrow$ should be immune to decoherence ... be cautious about that !

Experimental evidence – for Majorana quasiparticles

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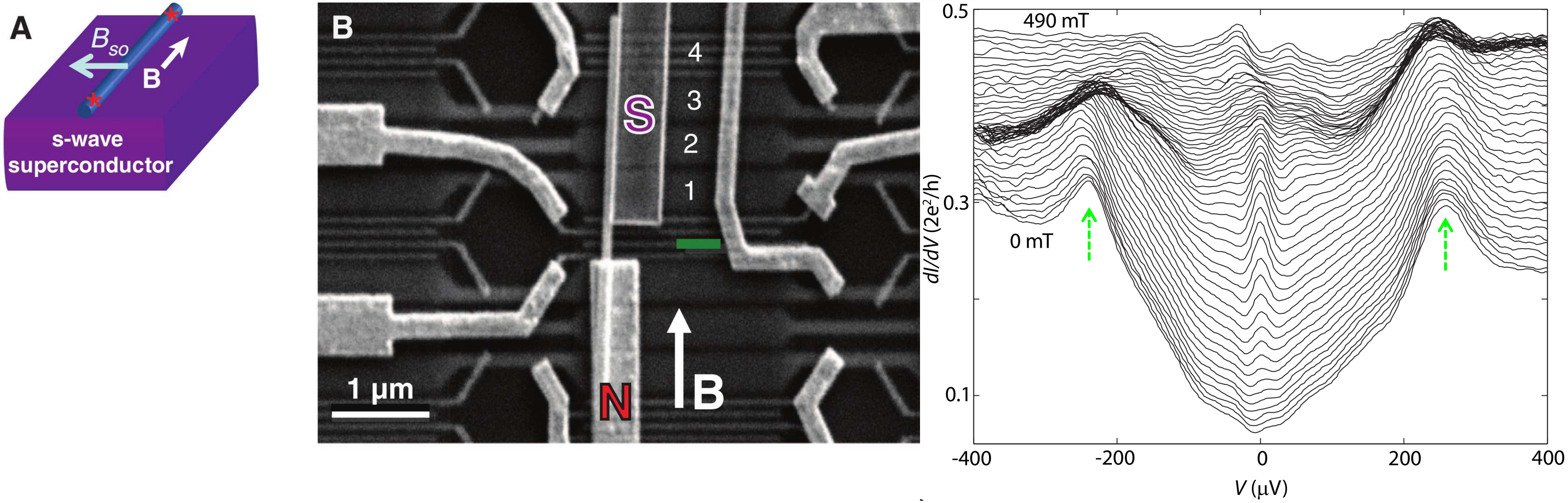
InSb nanowire between a metal (gold) and a superconductor (Nb-Ti-N)



dI/dV **measured at 70 mK for varying magnetic field B indicated:**

$\Rightarrow$ **a zero-bias enhancement due to <u>Majorana state</u>**

V. Mourik, ..., and <u>L.P. Kouwenhoven</u>, Science **336**, 1003 (2012).

/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /

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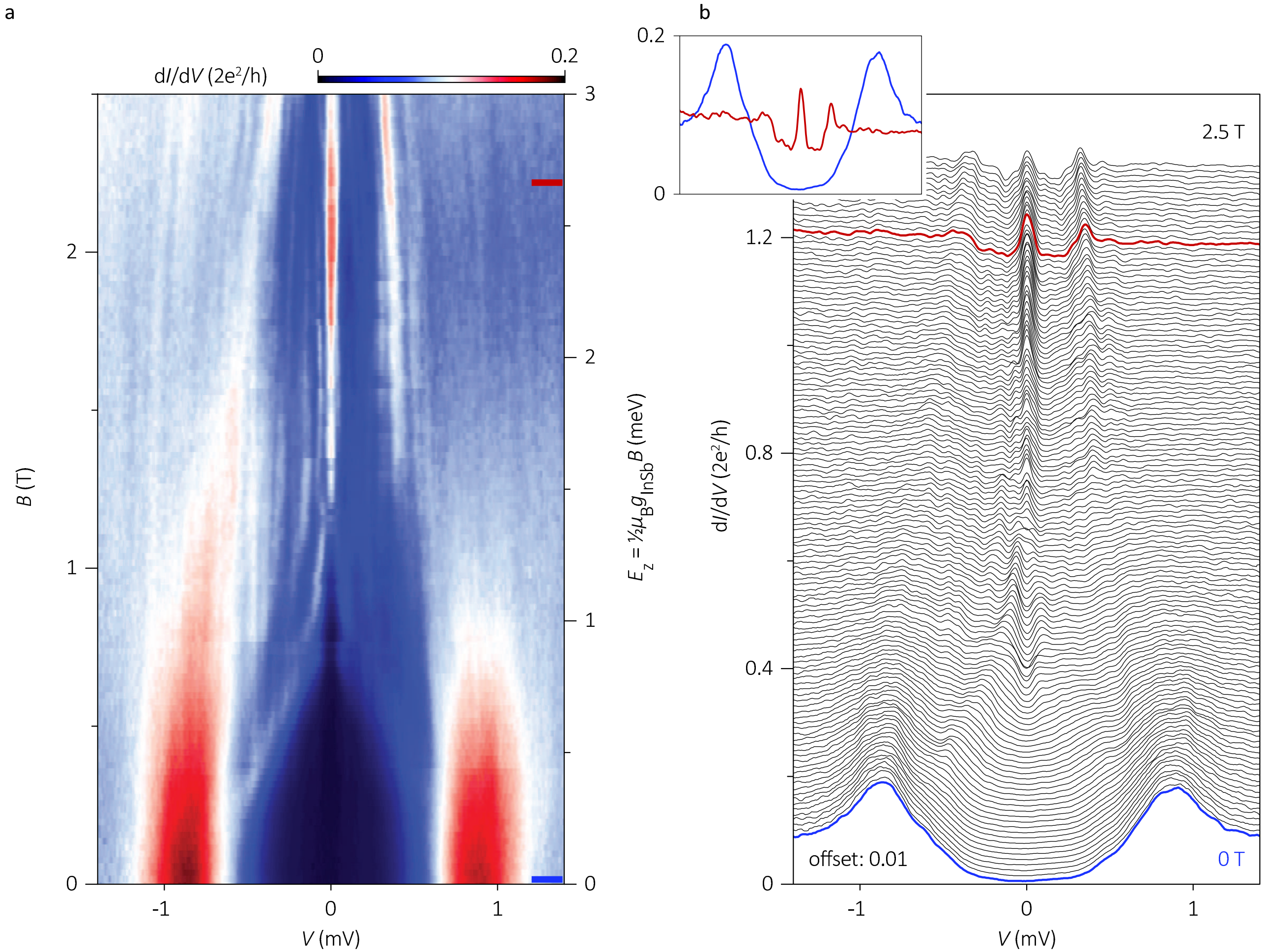
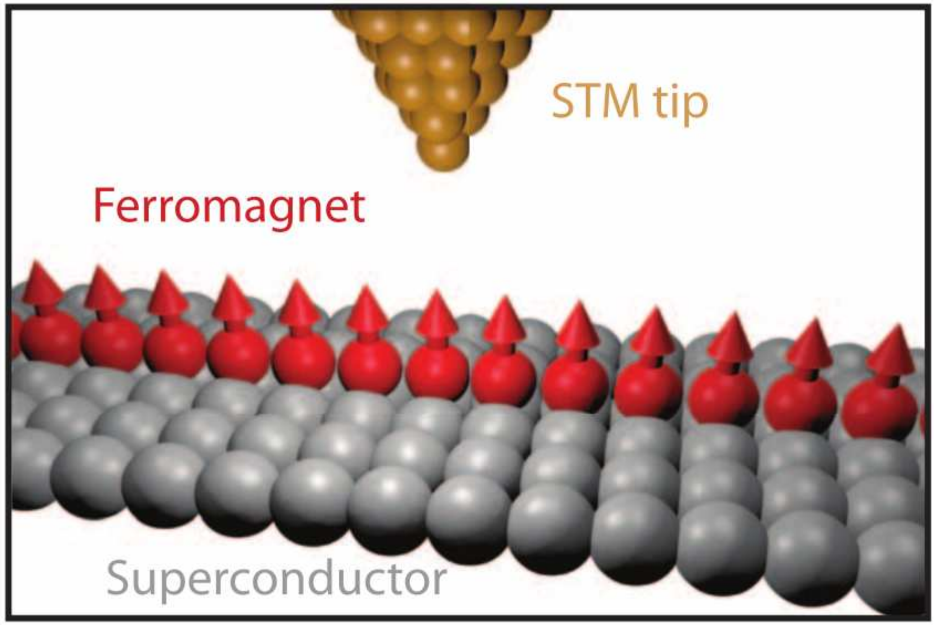


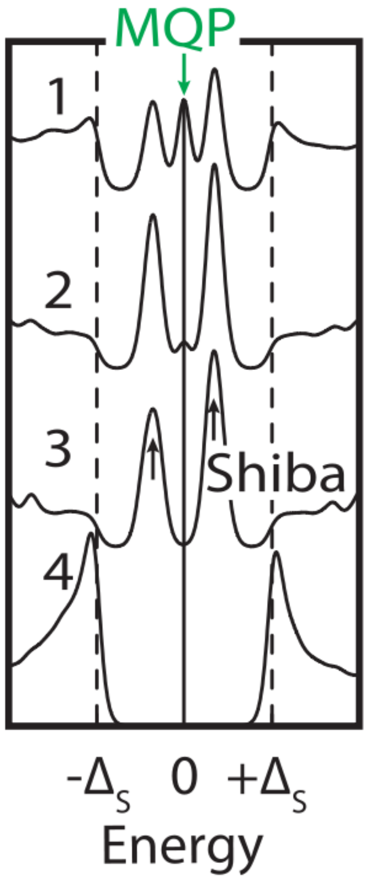
H. Zhang, ..., and L.P. Kouwenhoven, arXiv:1603.04069 (2016).

/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /

Experimental evidence – for Majorana quasiparticles

A chain of iron atoms deposited on a surface of superconducting lead





STM measurements provided evidence for:

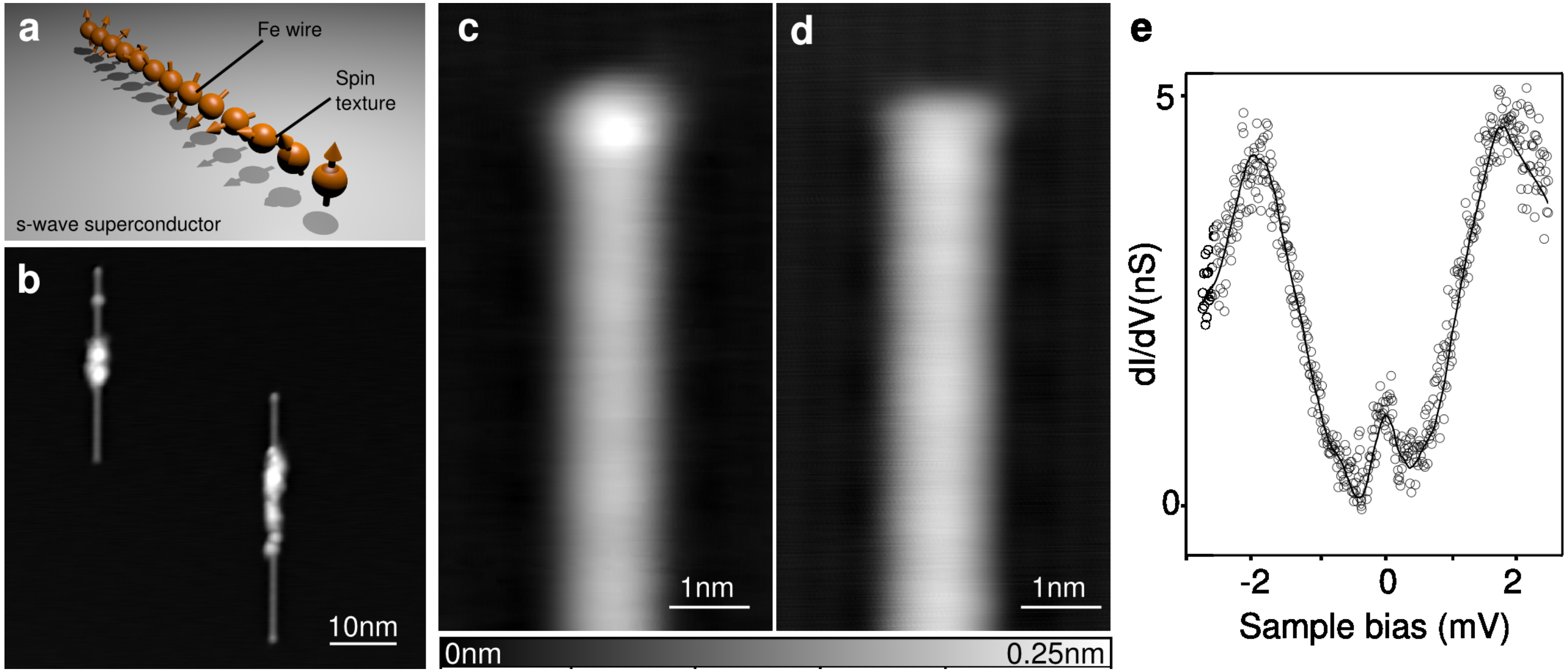
⇒ **Majorana bound states at the edges of a chain.**

S. Nadj-Perge, ..., and A. Yazdani, Science **346**, 602 (2014).

/ Princeton University, Princeton (NJ), USA /

Experimental evidence – for Majorana quasiparticles

Self-assembled Fe chain on superconducting Pb(110) surface



AFM combined with STM provided evidence for:

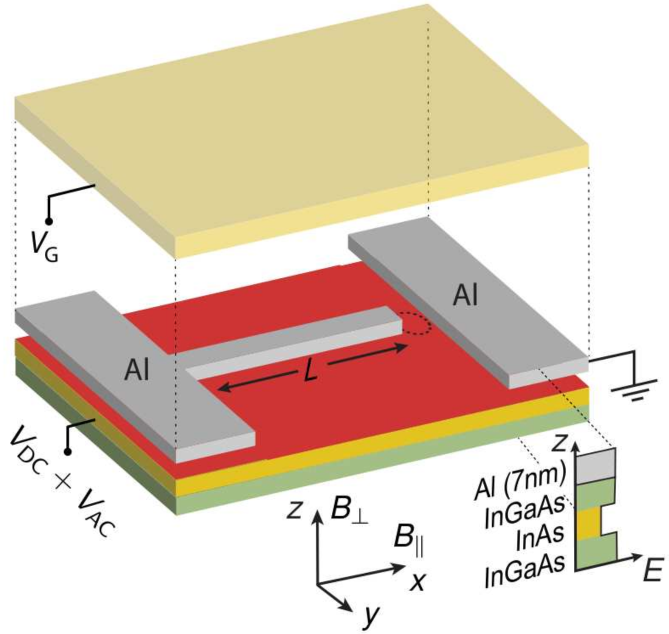
⇒ **Majorana bound states at the edges of a chain.**

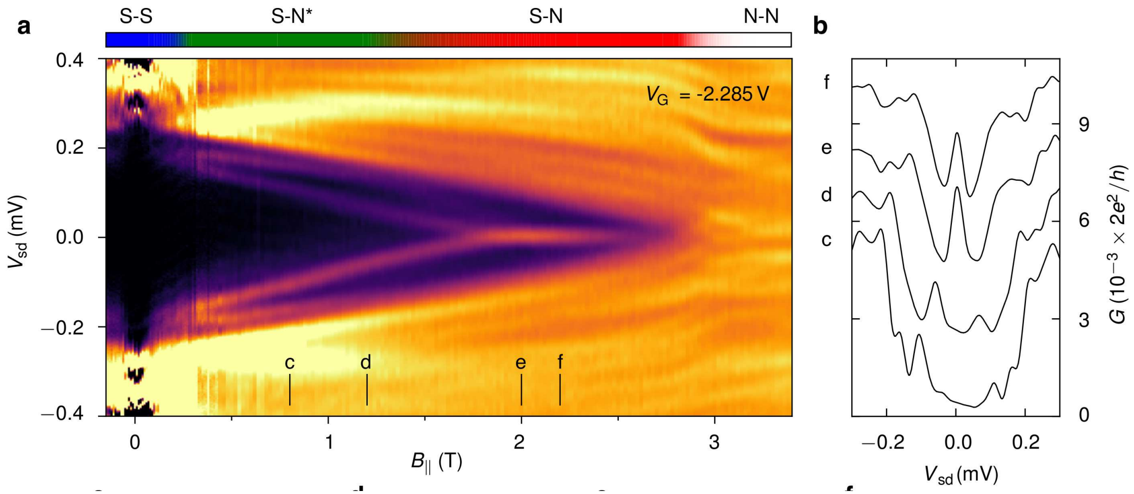
R. Pawlak, M. Kisiel *et al*, npj Quantum Information **2**, 16035 (2016).

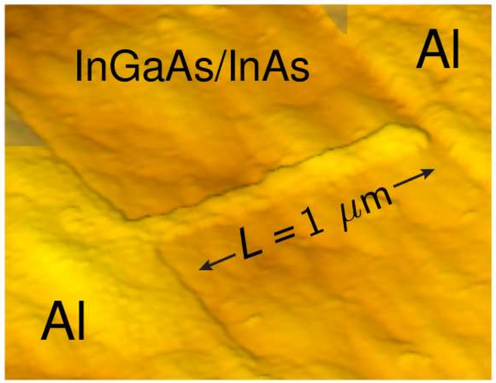
/ University of Basel, Switzerland /

Experimental evidence – for Majorana quasiparticles

Wire-like device constructed lithographically



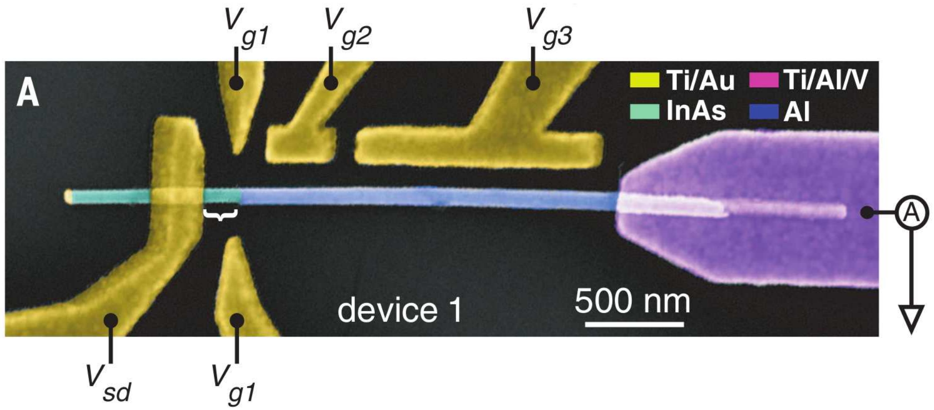




H.J. Suominen *et al*, arXiv:1703.03699 (2017).

/ University of Copenhagen, Denmark /

Proximity effect – experimental realization (27 Dec 2016)

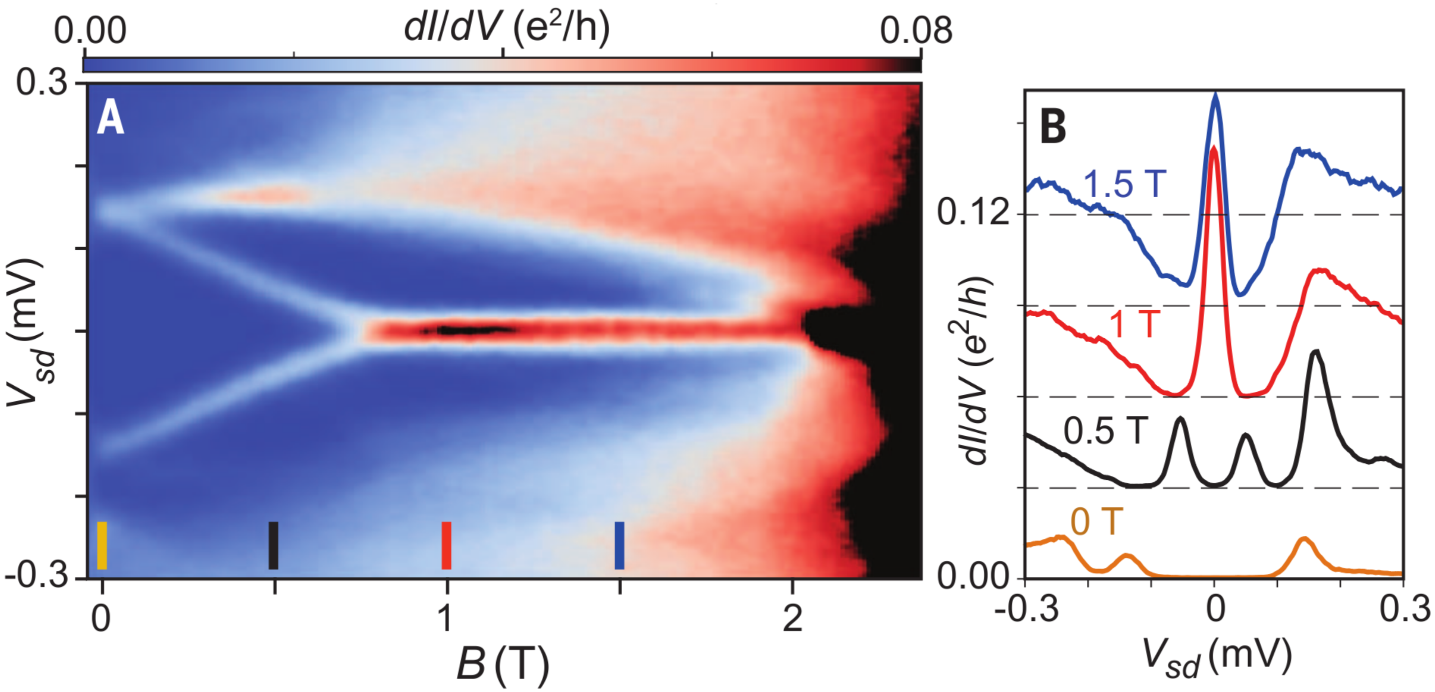


TOPOLOGICAL MATTER

Majorana bound state in a coupled quantum-dot hybrid-nanowire system

M. T. Deng,[1,2] S. Vaitiekėnas,[1,3] E. B. Hansen,[1] J. Danon,[1,4] M. Leijnse,[1,5] K. Flensberg,[1] J. Nygård,[1] P. Krogstrup,[1] C. M. Marcus[1]*

Science **354**, *1557 (2016).*



Microscopic description

/ Rashba chain + pairing /

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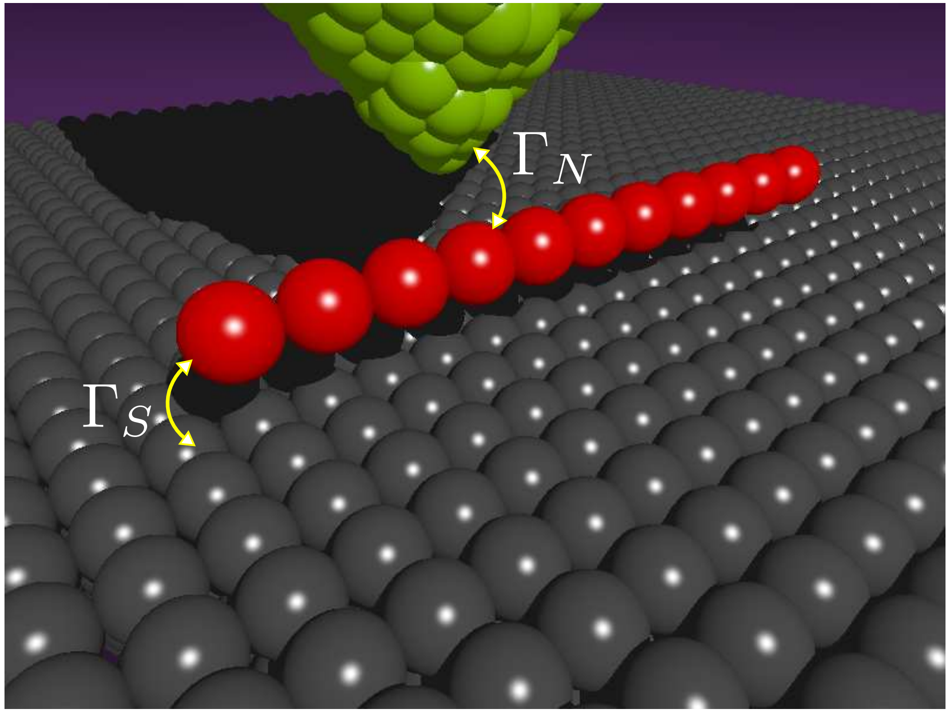
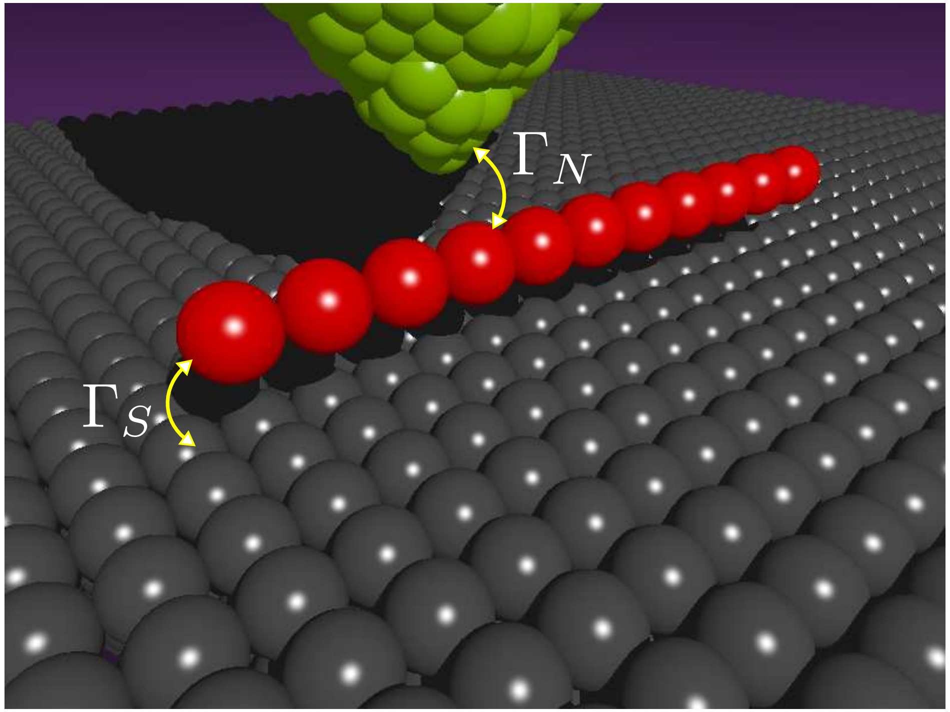


Scheme of STM configuration

Microscopic description

/ Rashba chain + pairing /



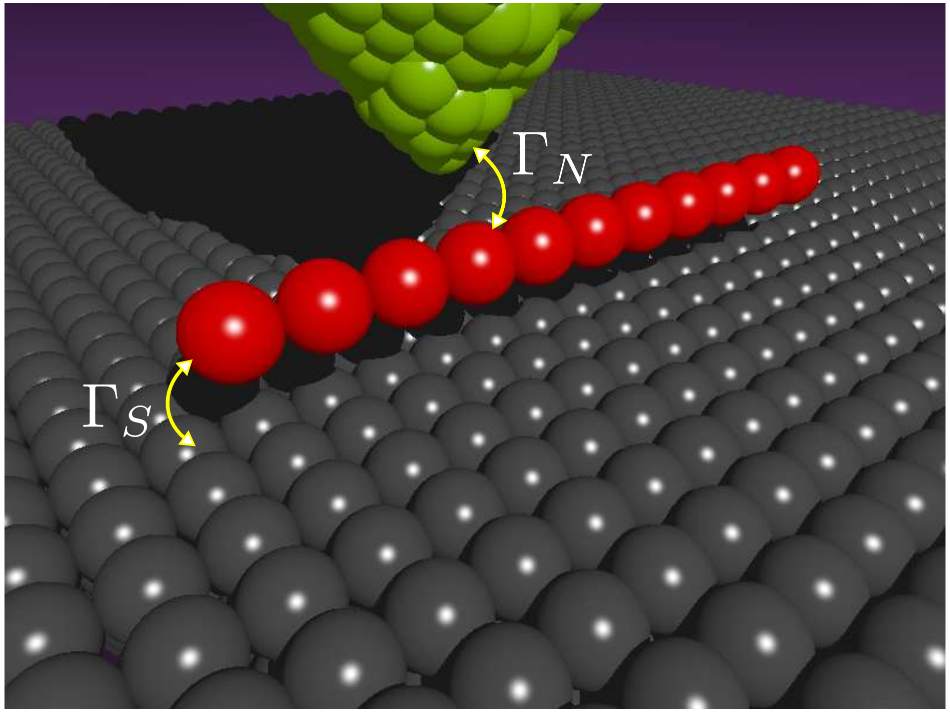
Scheme of STM configuration

$$\hat{H} \;=\; \hat{H}_{chain} + \hat{H}_S + \hat{H}_{tip} + + \hat{V}_{chain-S} + \hat{V}_{chain-tip}$$

We studied this model, focusing on the deep subgap regime $|E| \ll \Delta_{sc}$.

Microscopic description

/ Rashba chain + pairing /



Scheme of STM configuration

where

$$\hat{H}_{chain} = \sum_{i,j,\sigma} (t_{ij} - \delta_{ij}\mu)\hat{d}^{\dagger}_{i,\sigma}\hat{d}_{j,\sigma} + \hat{H}_{Rashba} + \hat{H}_{Zeeman}$$

M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, Phys. Rev. B **95**, *045429 (2017).*

Majorana states – of the Rashba chain

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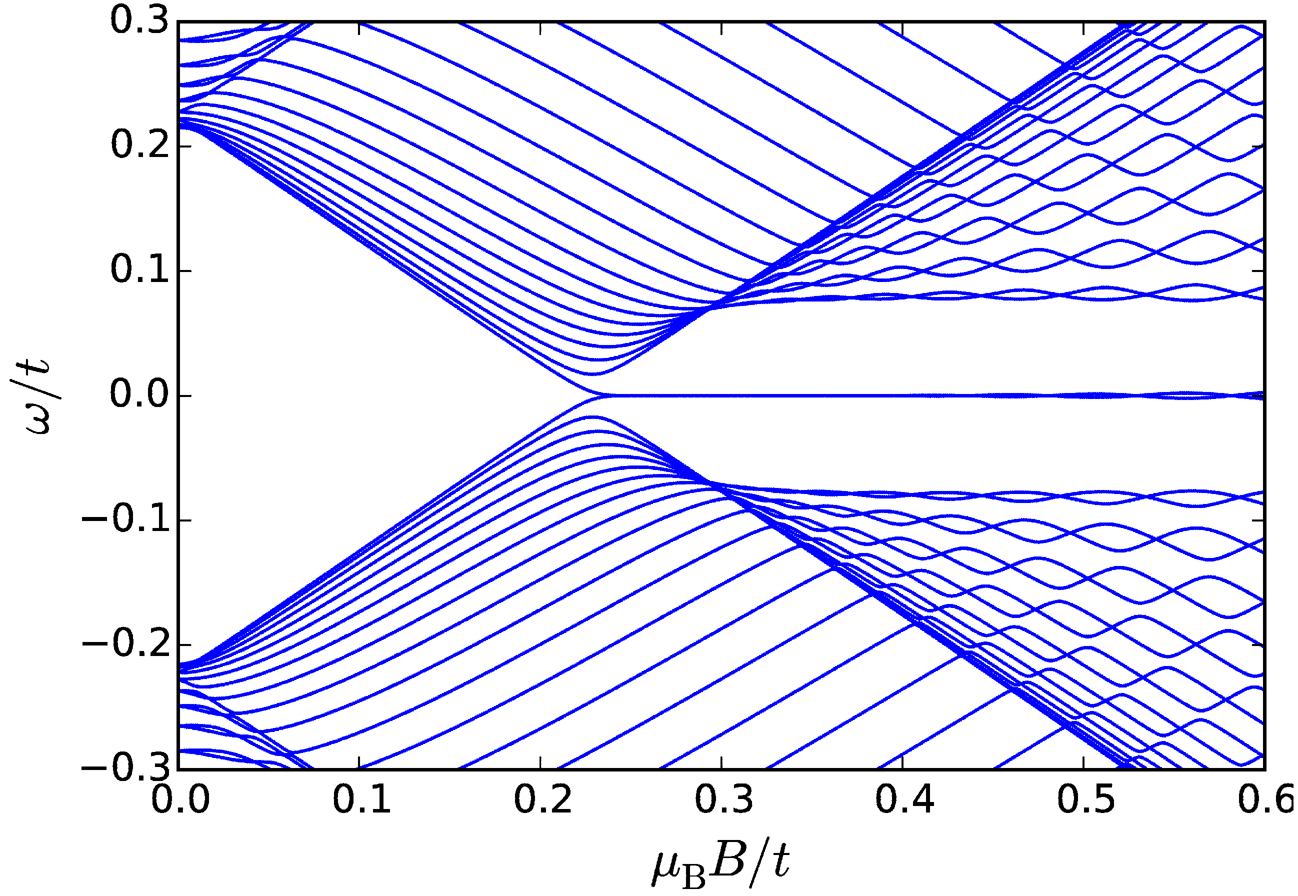
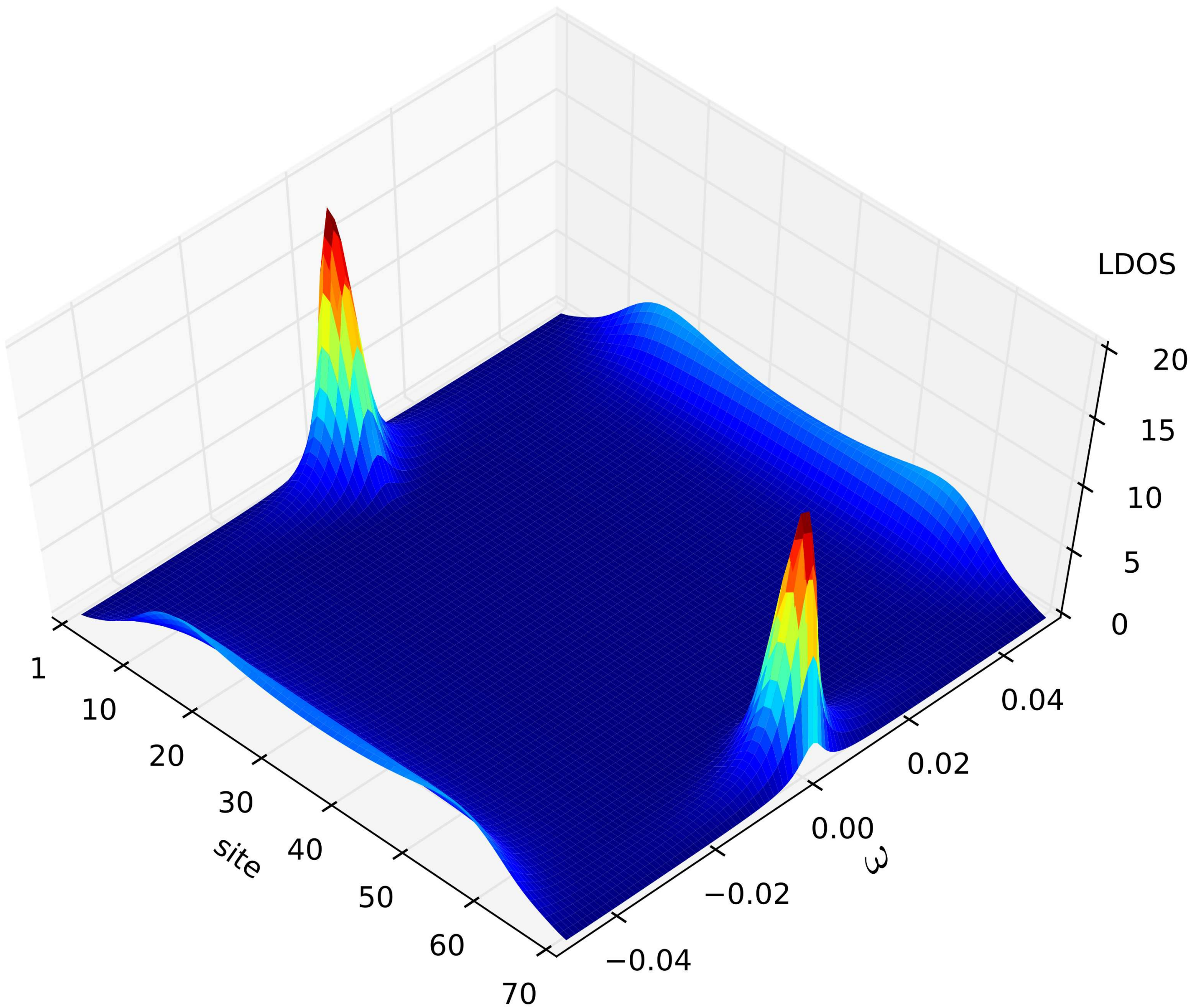
Mutation of Andreev states into zero-energy (Majorana) mode



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, Phys. Rev. B **95**, *045429 (2017).*

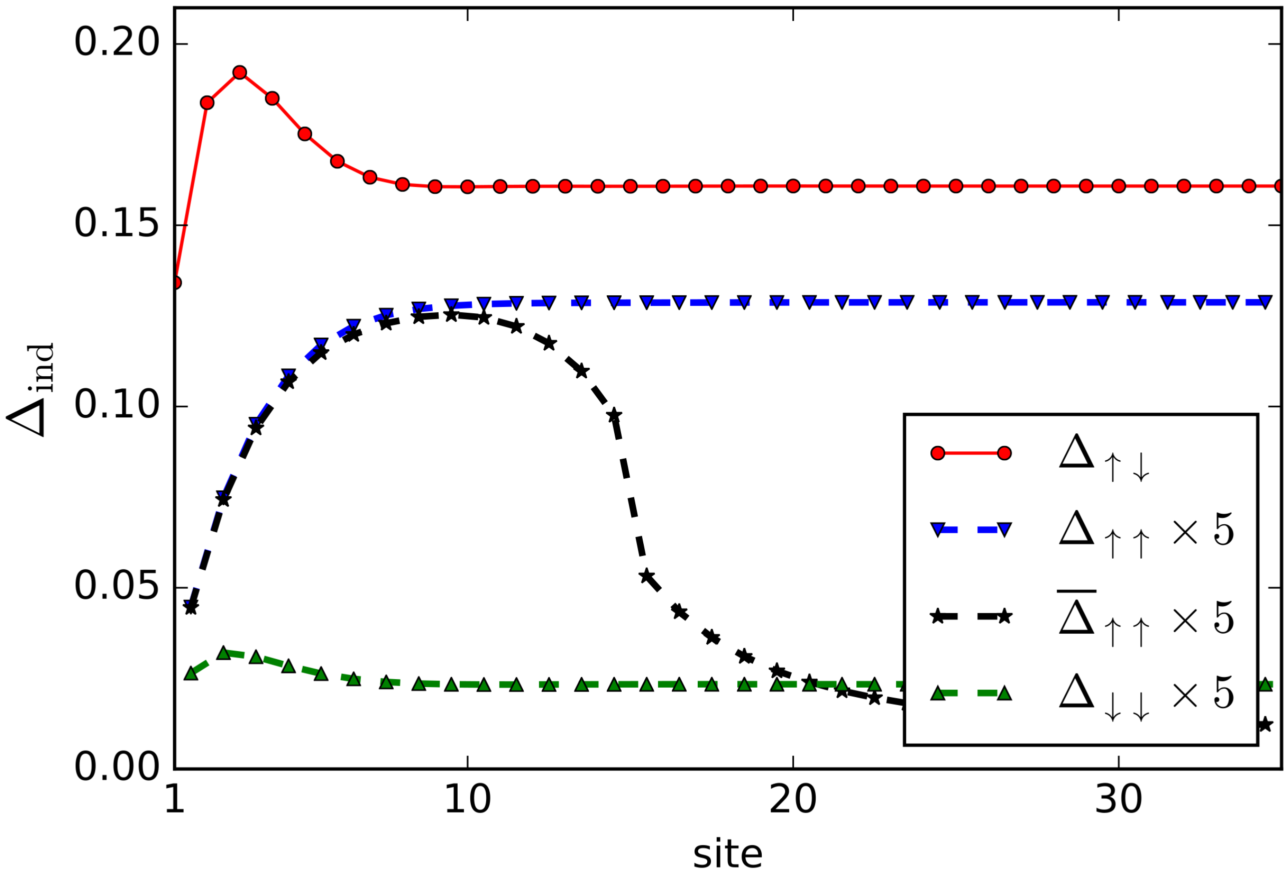
Majorana states – of the Rashba chain

Spectrum with the edge Majorana quasiparticles



Majorana states – of the Rashba chain

Spatial variation of the induced pairings $\Delta_{\sigma,\sigma'} = \left\langle \hat{d}_{i,\sigma} \hat{d}_{i+1,\sigma'} \right\rangle$



M. Maśka, A. Gorczyca-Goraj, J. Tworzydło and T. Domański, Phys. Rev. B **95***, 045429 (2017).*

Spin-polarized Majorana states

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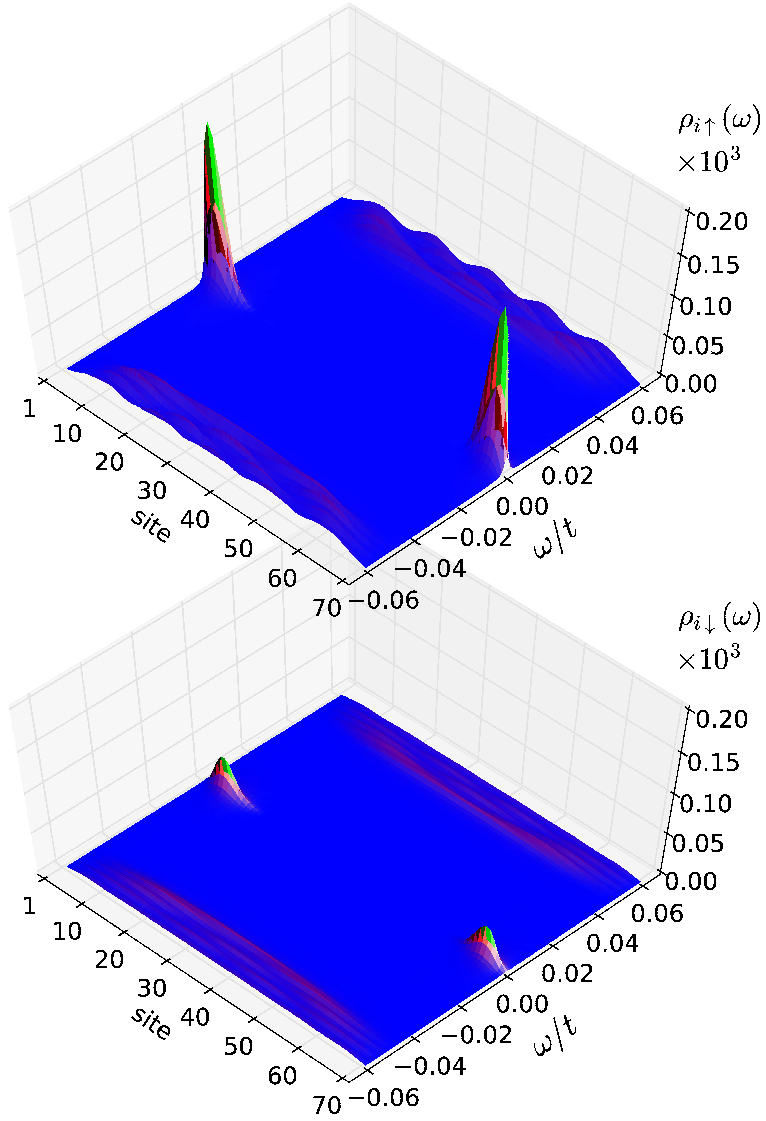
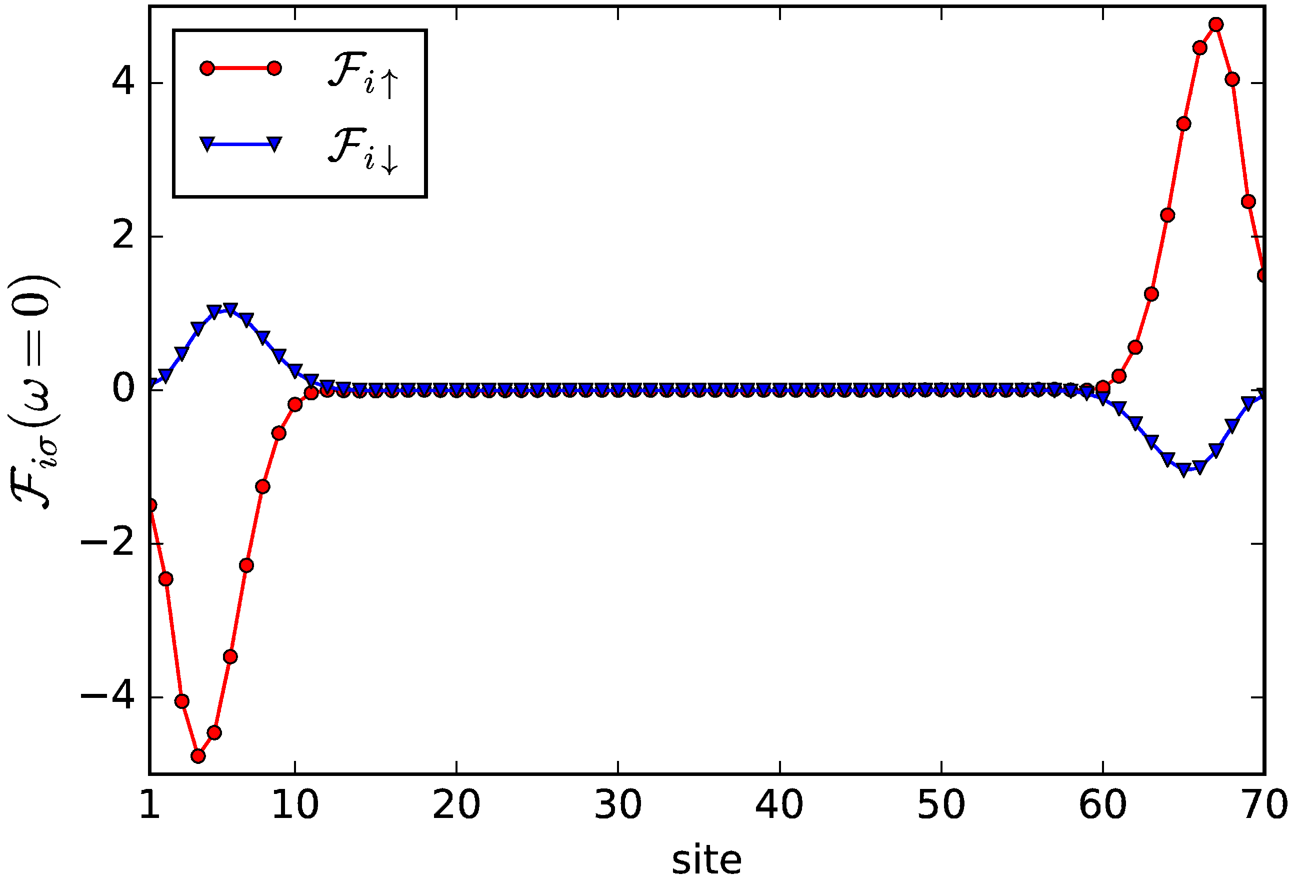
Diagonal spectral functions $\rho_{i\sigma}(\omega) = -\frac{1}{\pi}\mathrm{Im}\left\langle \hat{d}_{i,\sigma}\hat{d}^{\dagger}_{i,\sigma}\right\rangle$



Spin-polarized Majorana states

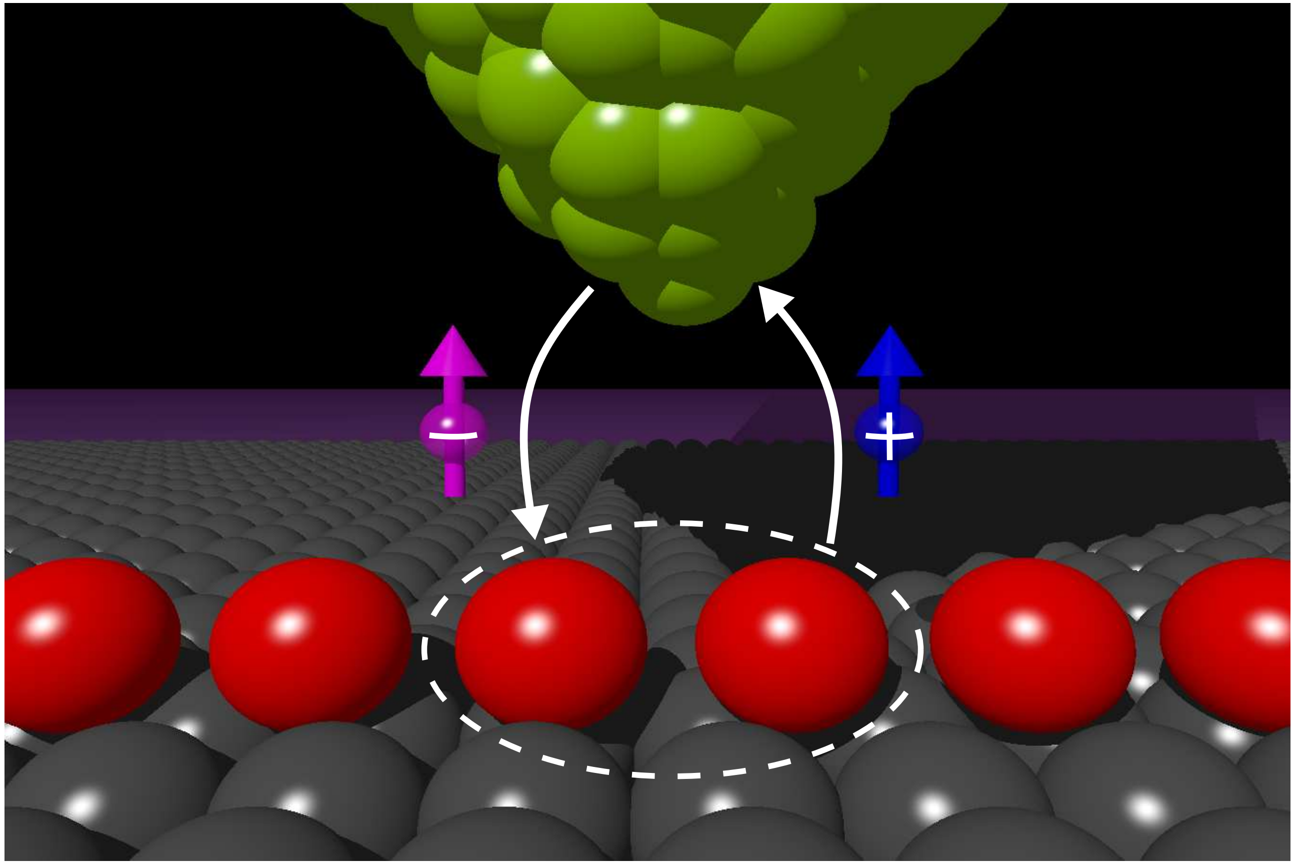
Off-diagonal spectral functions $\mathcal{F}_{i\sigma}(\omega) = -\frac{1}{\pi}\mathrm{Im}\left\langle \hat{d}_{i,\sigma}\hat{d}_{i+1,\sigma}\right\rangle$



M. Maśka and T. Domański, arXiv:1706.01468 (2017).

Spin-polarized Majorana states

Idea of the *Selective-Equal-Spin-Andreev-Reflection* (SESAR)



M. Maśka and T. Domański, arXiv:1706.01468 (2017).

Spin-polarized Majorana states

Probabilities of the Andreeev scattering for various polarizations.

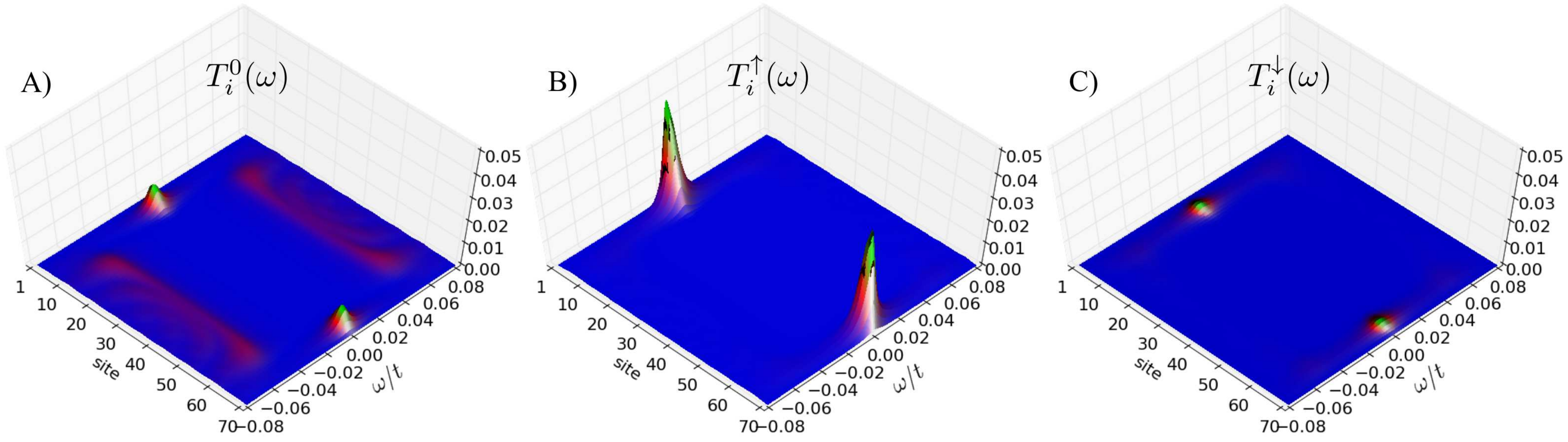
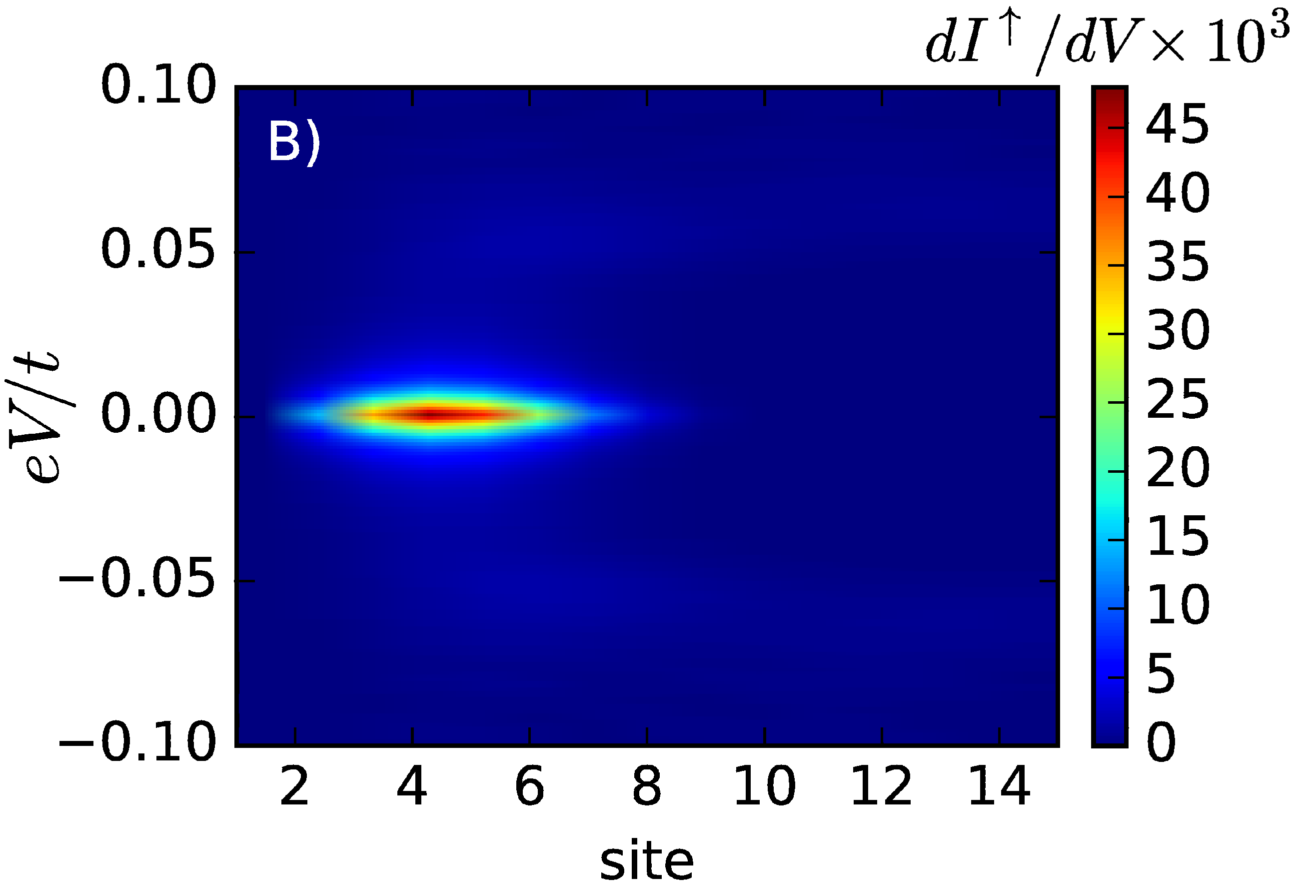


FIG. 4. The spatially resolved transmittances $T_i^\gamma(\omega)$ obtained at low energies ($|\omega| \ll \Delta$) for the nonmagnetic $\gamma = 0$ (panel A) and the spin-polarized Andreev reflections $\gamma =\uparrow$ (panel B) and $\gamma =\downarrow$ (panel C).

M. Maśka and T. Domański, arXiv:1706.01468 (2017).

Spin-polarized Majorana states

Differential conductance of the SESAR spectroscopy for $\sigma = \uparrow$.



M. Maśka and T. Domański, arXiv:1706.01468 (2017).

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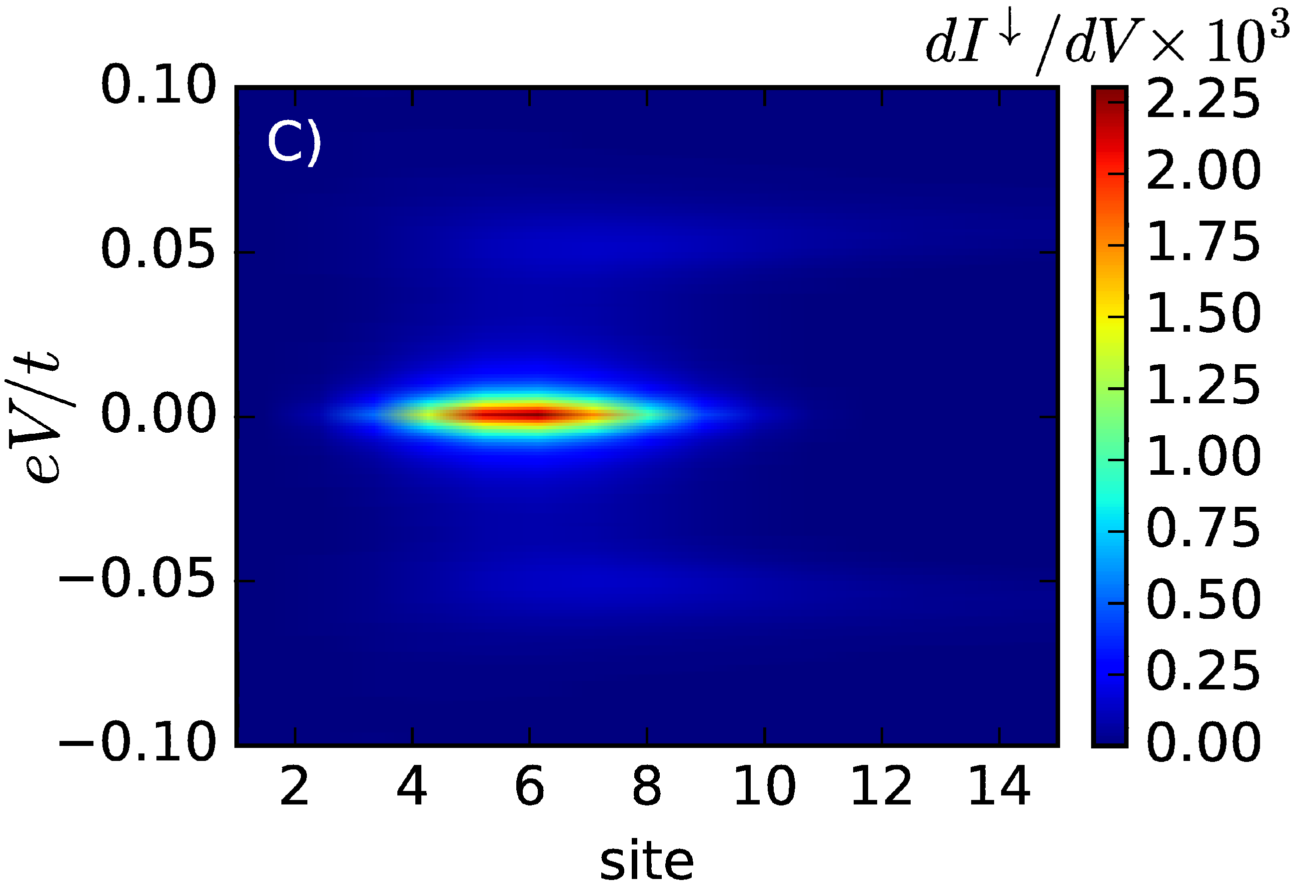
Differential conductance of the SESAR spectroscopy for $\sigma = \downarrow$.



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http://kft.umcs.lublin.pl/doman/lectures

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- topologically protected

Summary: – general remarks about Majorana qps

- creation = annihilation $\hat{\gamma}_{i,n}^{\dagger} = \hat{\gamma}_{i,n}$

$\Rightarrow$ chargeless, spinless, zero-energy objects

- spatially nonlocal

$\Rightarrow$ they always exist in pairs, even though (infinitely) far apart

- fractionality $\hat{\gamma}_{i,n}^{\dagger} \, \hat{\gamma}_{i,n} = 1/2$

$\Rightarrow$ half-empty $\&$ half-filled entities

- topologically protected

$\Rightarrow$ immune to decoherence ... only in the topological sc phase !