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Residual diamagnetism driven by the superconducting fluctuations

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Outline

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★ **Preliminaries**

/ Cooper pairing & Higgs mechanism /

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/ pre-pairing for BE condensation /

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$\Rightarrow$ *Bogoliubov quasiparticles above* T_c

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- **Results**

  - *Bogoliubov quasiparticles above* T_c
  - *Diamagnetism above* T_c

- **Summary**

Preliminaries

Superconducting state – properties

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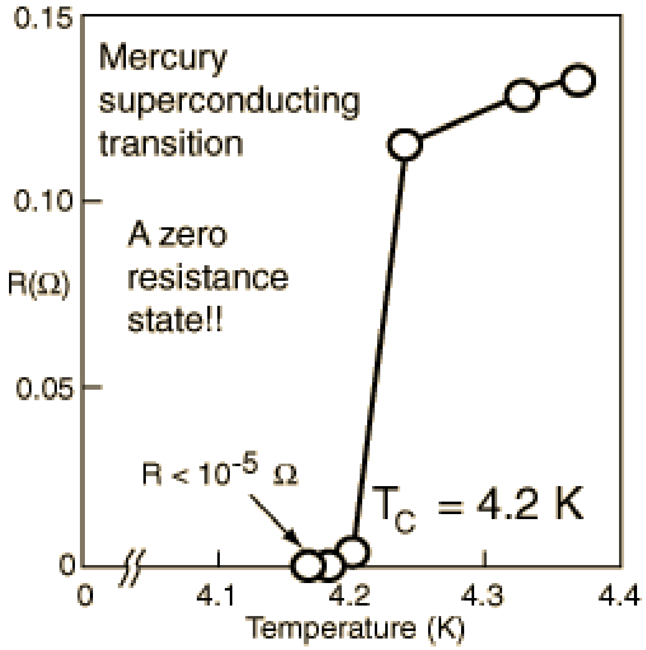
★ **ideal d.c. conductance**



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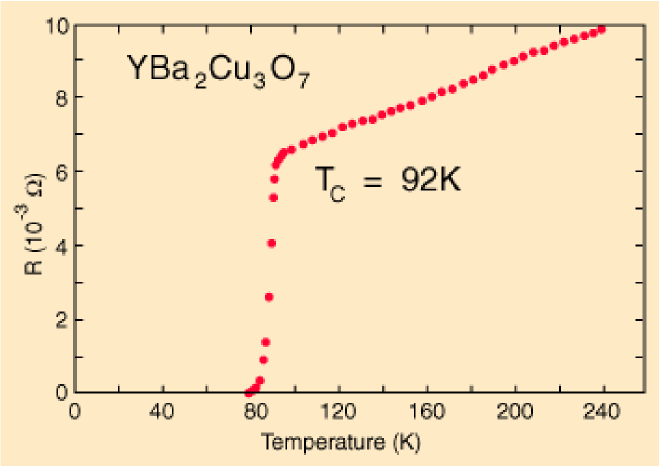

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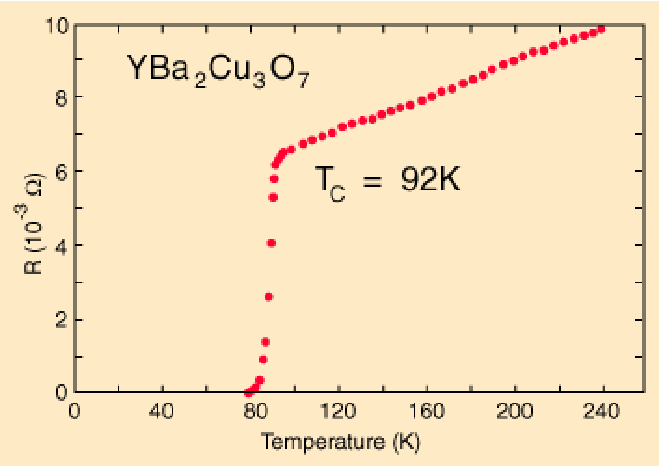

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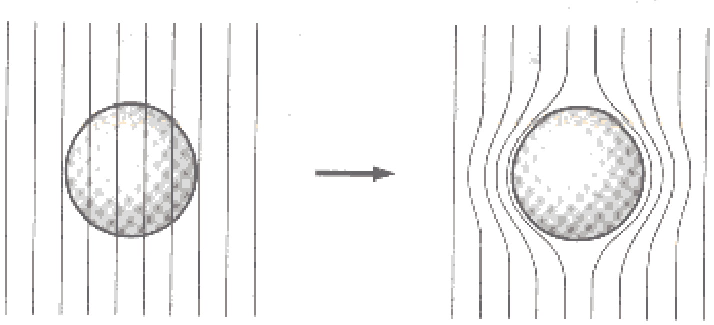
/perfect screening of the external magnetic field/



$T > T_c$ $\qquad$ $T < T_c$

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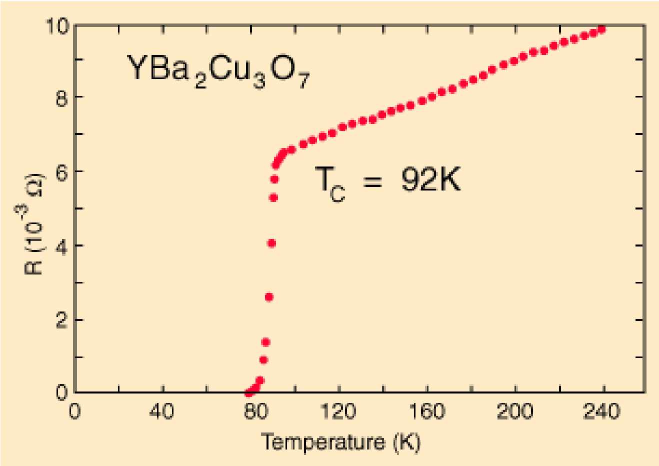

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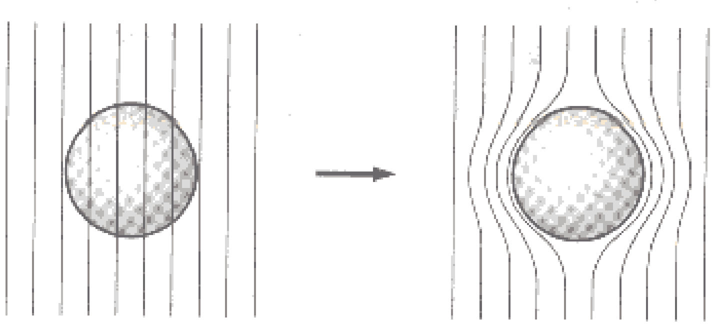

⇒ **Both features originate from the pairing of fermions.**

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Appearance of fermion pairs usually goes hand in hand with **superconductivity/superfluidity** but it needn't be the rule.

Conventional superconductors

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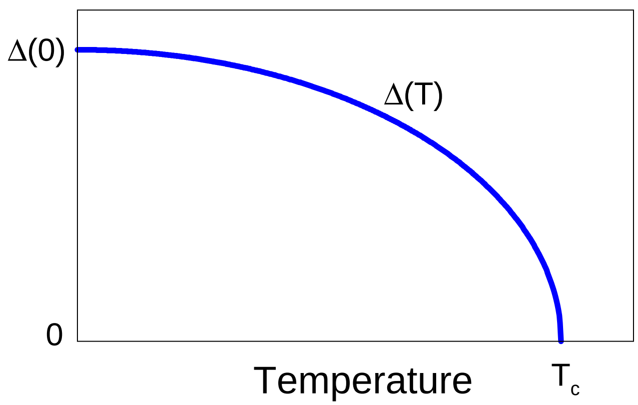
Pair formation coincides with an onset of coherence at T_c



Pairing causes the energy gap $\boldsymbol{\Delta(T)}$ *in a single particle spectrum*

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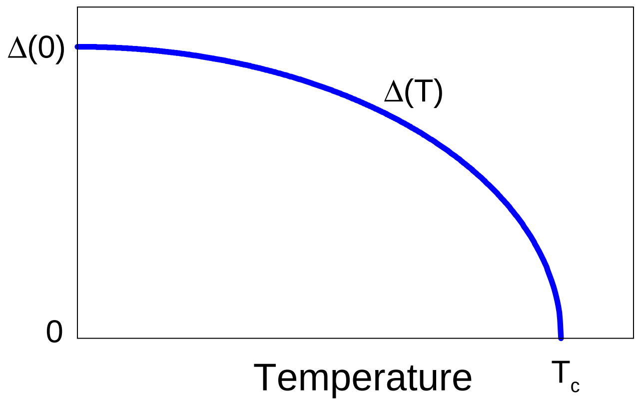
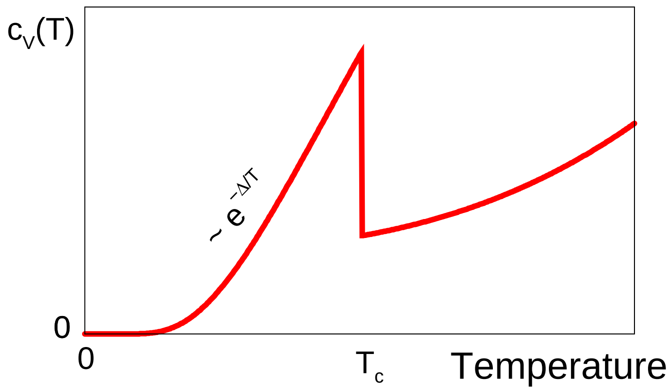


Pairing causes the energy gap $\Delta(T)$ in a single particle spectrum

2-nd order phase transition



Below T_c there appears the order parameter (ODLRO)

$\chi \propto \Delta(T)$

Conventional superconductors – meaning of T_c

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Bose-Einstein condensate of these Cooper pairs is described by a common wave-function

$$\boxed{\chi(\vec{r}, t)}$$

Formal issues – generalities

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The order parameter

$$\chi(\vec{r},t) \equiv \int d\vec{\rho}\ \ \langle \hat{c}_{\downarrow}(\vec{r}+\frac{\vec{\rho}}{2})\ \ \hat{c}_{\uparrow}(\vec{r}-\frac{\vec{\rho}}{2})\rangle$$

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$\nabla\theta \neq 0 \quad \longrightarrow$ **phase slippage induces supercurrents**

Anderson-Higgs mechanism – outline

The low energy excitations of the BE condensed pairs are characterized by the collective (Goldstone) phasal mode.

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$\Rightarrow$ **Can any part of this mechanism survive above T_c ?**

Motivation

Phase transitions – classification

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2. **disordering the phase** .... *(HTSC compounds)*

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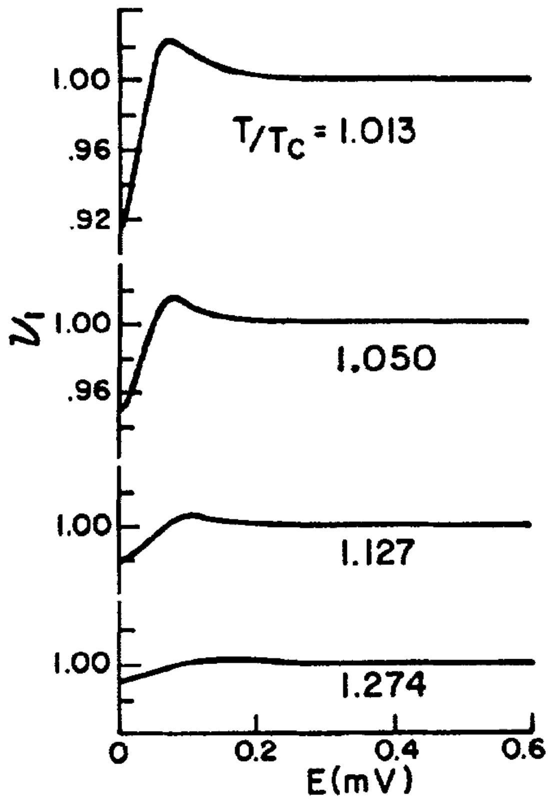
Tunneling conductance revealed a pseudogap surviving above T_c*.*



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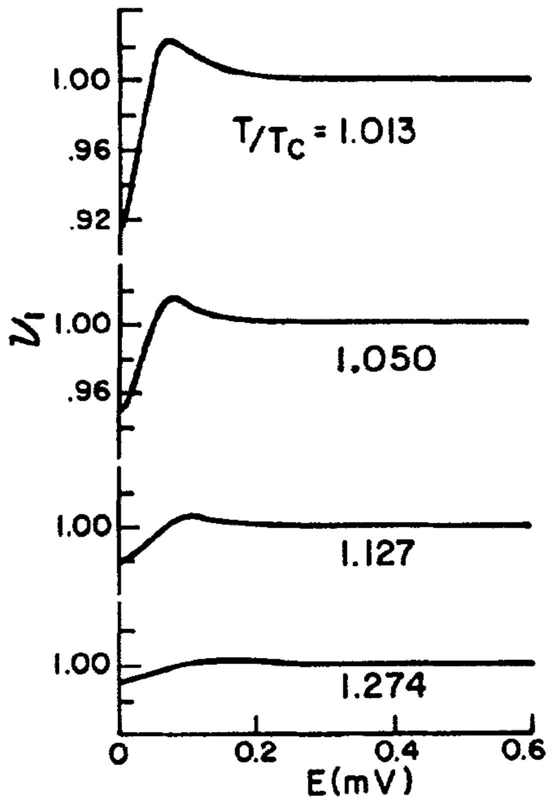

R.W. Cohen and B. Abels, Phys. Rev. **168**, *444 (1968).*

HTSC materials – phase diagram

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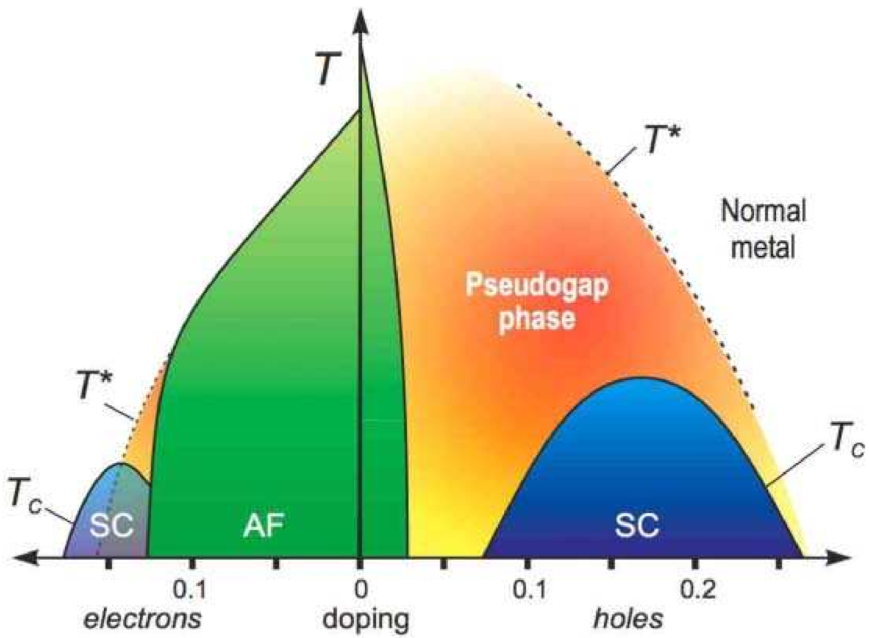

O. Fisher et al, *Rev. Mod. Phys.* **79**, *353 (2007).*

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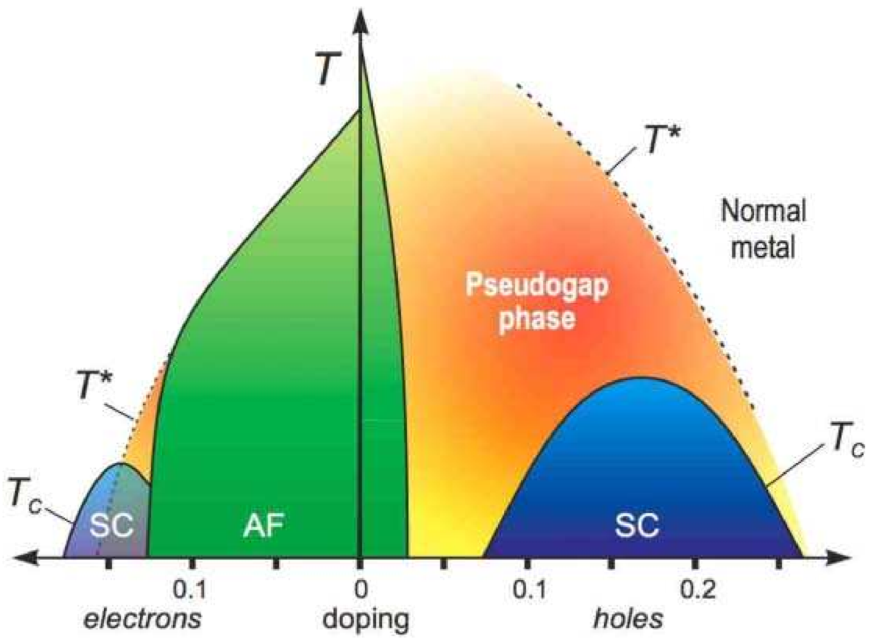

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Unresolved problem:

What causes the pseudogap ?

Incoherent pairs above T_c

experimental fact $\#$ 1

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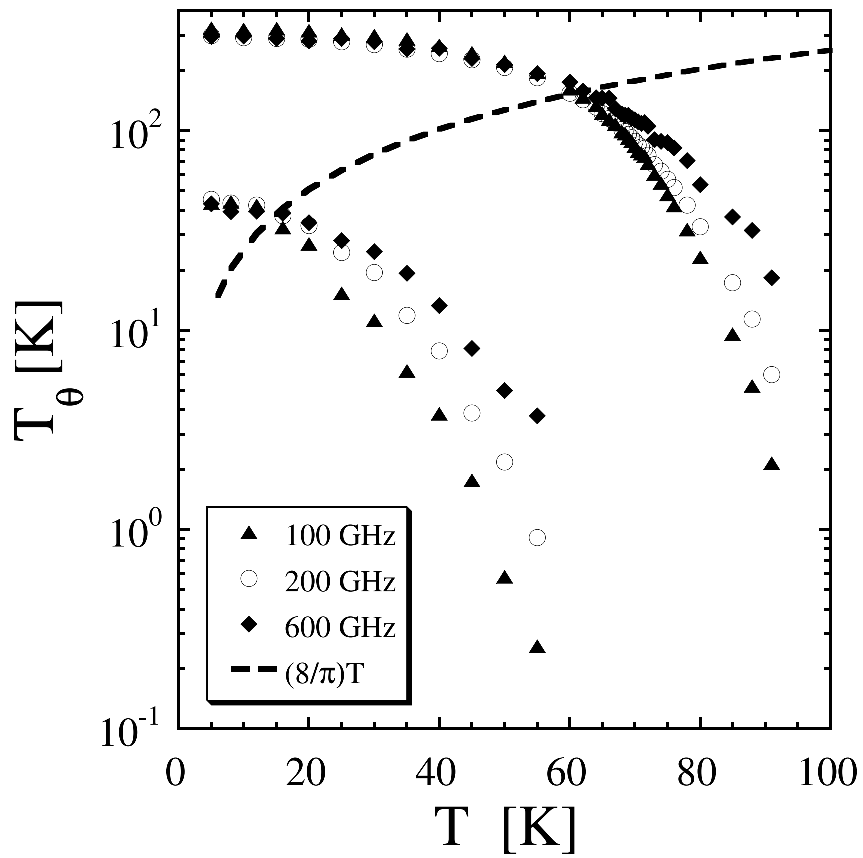

Dynamic phase-stiffness $\quad T_\theta = \omega\ \mathrm{Im}\boldsymbol{\sigma}(\omega, T)/\sigma_Q$

observed at the ultrafast (teraHz) external ac fields.

J. Corson et al, *Nature* **398**, *221 (1999).*

Incoherent pairs above T_c

experimental fact $\#$ 2

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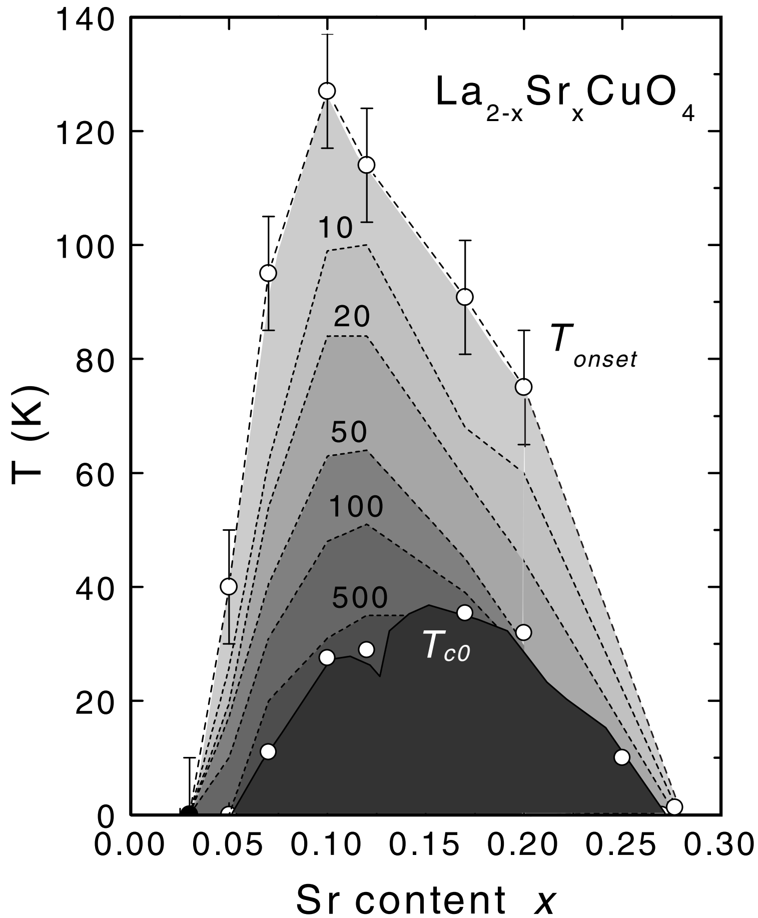
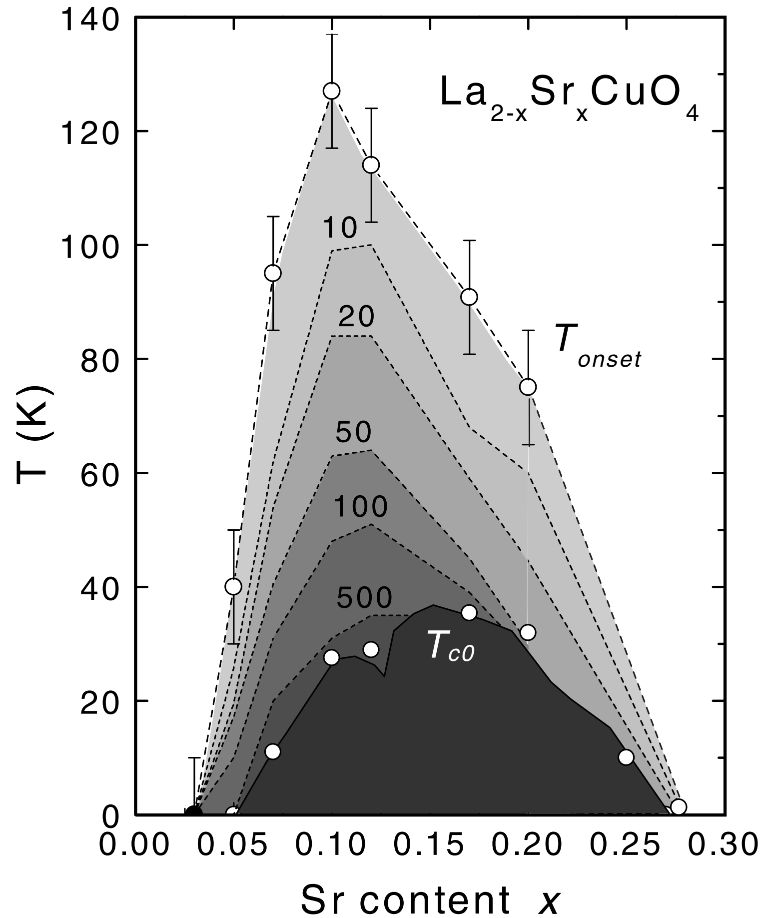


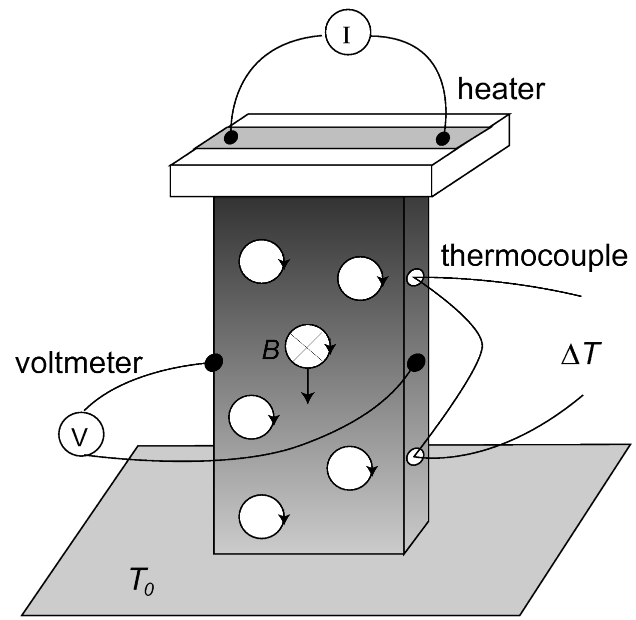
Phase slippage detected in the large Nernst effect.

Y. Wang et al, *Science* **299**, *86 (2003).*

Incoherent pairs above T_c

experimental fact # 2





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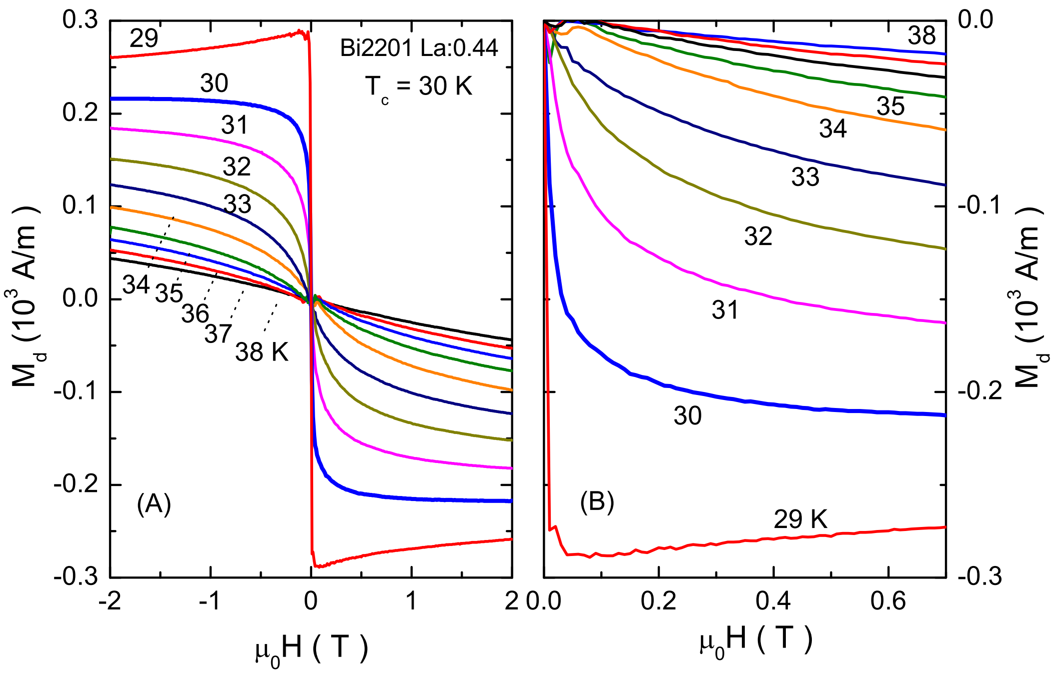

The Van Vleck background $(A + BT)H$ *is subracted*

$$T_c = 30K$$

Enhanced diamagnetic response revealed above T_c by the ultrahigh precission torque magnetometry.

L. Li et al *and N.P. Ong, Phys. Rev. B* **81**, *054510 (2010).*

Incoherent pairs above T_c

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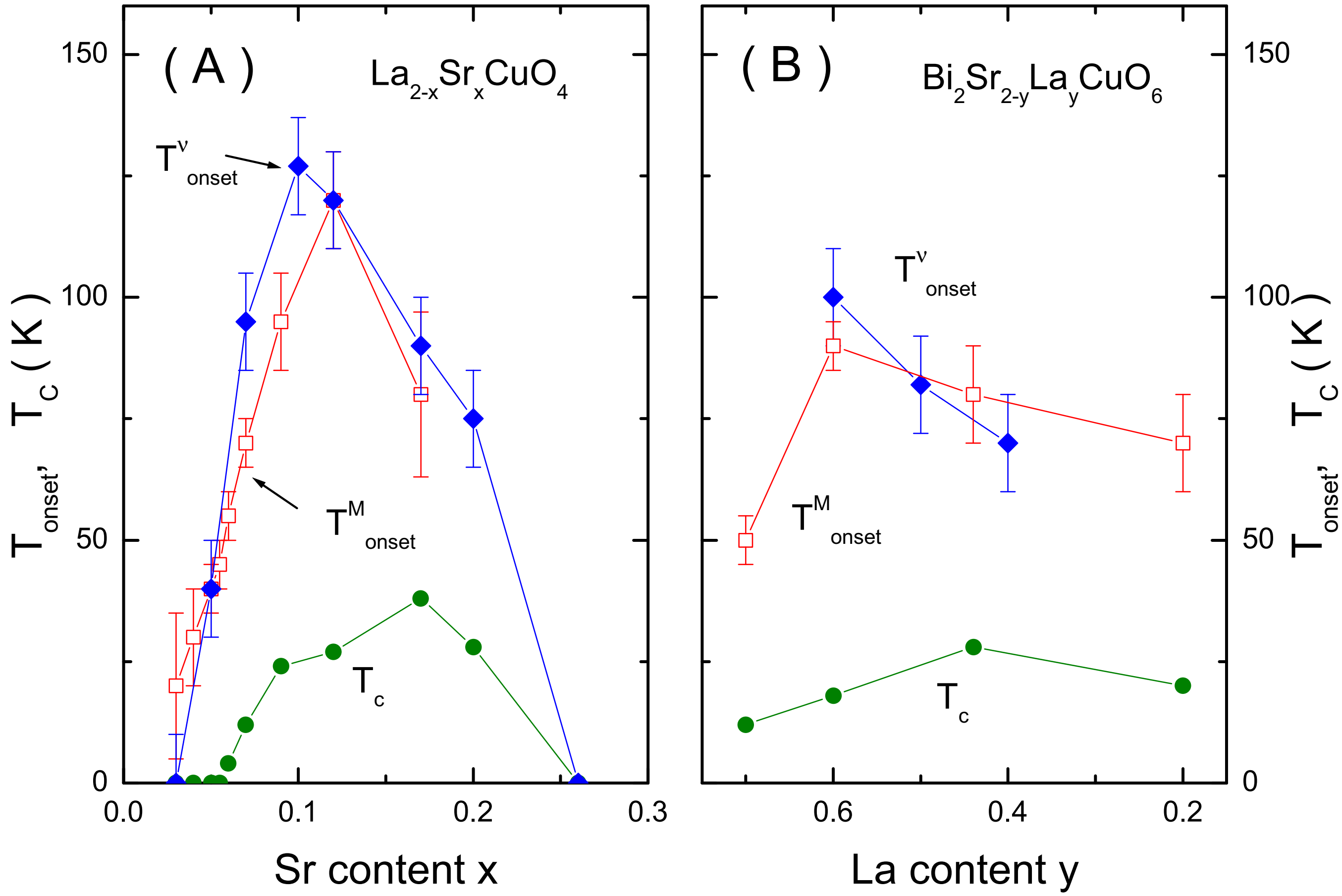

T^{ν} – onset of the Nernst effect

T^{M} – onset of the diamagnetism

L. Li et al *and N.P. Ong, Phys. Rev. B* **81**, *054510 (2010).*

Incoherent pairs above T_c

... continued

Incoherent pairs above T_c

... continued

$\Rightarrow$ **Josephson-like features seen above T_c in the tunneling**

N. Bergeal et al, *Nature Phys.* **4**, *608 (2008).*

Incoherent pairs above T_c ... *continued*

⇒ **Josephson-like features seen above T_c in the tunneling**

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⇒ **smooth evolution of the electronic spectrum observed by ARPES near the superconductor–insulator transition**

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⇒ **spectroscopic fingerprints of the Bogoliubov QPs seen by the unique octet patterns which survive up to $1.5T_c$**

J. Lee, ... and J.C. Davis, Science **325**, *1099 (2009).*

Technical remarks

Boson-Fermion model

[in a lattice representation]

$$\begin{aligned}\hat{H} &= \sum_{i,j,\sigma} (t_{ij} - \mu\ \delta_{i,j})\ \hat{c}^\dagger_{i\sigma}\hat{c}_{j\sigma} + \sum_{l} \left(E_l^{(B)} - 2\mu \right) \hat{b}^\dagger_l \hat{b}_l \\ &+ \sum_{i,j} g_{ij} \left[\hat{b}^\dagger_l \hat{c}_{i,\downarrow}\hat{c}_{j,\uparrow} + \text{h.c.} \right]\end{aligned}$$

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describes a two-component system consisting of:

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$\hat{b}^{(\dagger)}_{l}$ **immobile local pairs** ......*(RVB defines them on the bonds)*

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interacting via:

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[in a lattice representation]

$$\hat{H} \;=\; \sum_{i,j,\sigma} (t_{ij} - \mu \; \delta_{i,j}) \, \hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} + \sum_{l} \left(E_l^{(B)} - 2\mu \right) \hat{b}^{\dagger}_{l} \hat{b}_{l}$$
$$+ \sum_{i,j} g_{ij} \left[\hat{b}^{\dagger}_{l} \hat{c}_{i,\downarrow} \hat{c}_{j,\uparrow} \; + \text{h.c.} \right]$$

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In the Lagrangian language we obtain this kind of physics upon applying the Hubbard-Stratonovich transformation !

Boson-Fermion model

[in the momentum space]

$$\begin{aligned}\hat{H} &= \sum_{\mathbf{k}\sigma} (\varepsilon_{\mathbf{k}} - \mu)\, \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} \left(E^{(B)} - 2\mu \right) \hat{b}^{\dagger}_{\mathbf{q}} \hat{b}_{\mathbf{q}} \\ &+ \frac{1}{\sqrt{N}} \sum_{\mathbf{k},\mathbf{q}} g_{\mathbf{k},\mathbf{q}} \left[\hat{b}^{\dagger}_{\mathbf{q}} \hat{c}_{\mathbf{k},\downarrow} \hat{c}_{\mathbf{q}-\mathbf{k},\uparrow} + \text{h.c.} \right]\end{aligned}$$

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\end{aligned}
$$

This BF scenario has been considered by various groups:

J. Ranninger *with coworkers* . . . . Grenoble

R. Micnas, S. Robaszkiewicz . . . . Poznań

T.D. Lee *with coworkers* . . . . New York

V.B. Geshkenbein, L.B. Ioffe, A.I. Larkin . . . .

E. Altman & A. Auerbach . . . . Technion

A. Griffin *with coworkers* . . . . Toronto

K. Levin *with coworkers* . . . . Chicago

and many others.

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T. Domański and J. Ranninger, Phys. Rev. **B 63**, *134505 (2001).*

Continuous unitary transformation – algorithm

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where

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For more details see for instance:

S. Kehrein, Springer Tracts in Modern Physics **217**, (2006);

F. Wegner, J. Phys. A: Math. Gen. **39**, 8221 (2006).

Continuous unitary transformation – algorithm

Similar ideas have been also earlier developed also in the field of **control theory** under the names:

- *”double bracket flow”*

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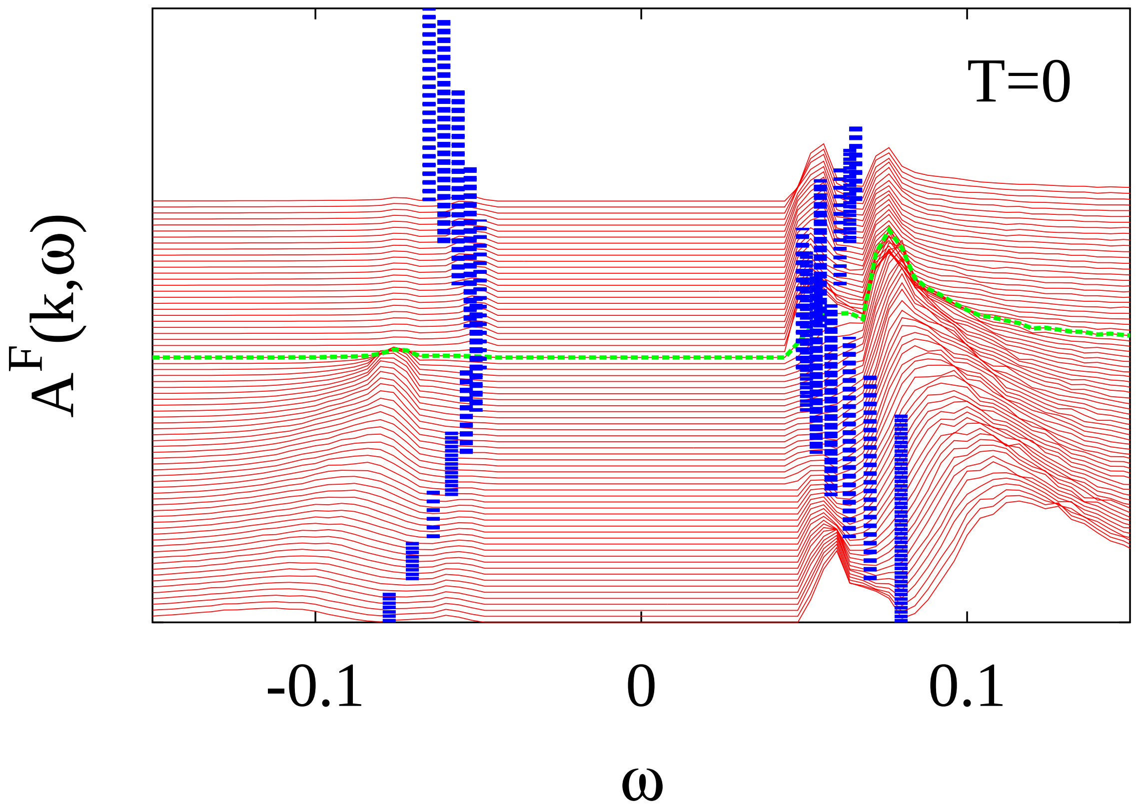
S.R. White, J. Chem. Phys. **117***, 7472 (2002).*

Results :

1. **Bogoliubov quasiparticles above T_c**

Effective spectrum: BF model

$T < T_c$



T. Domański and J. Ranninger, Phys. Rev. Lett. **91**, *255301 (2003).*

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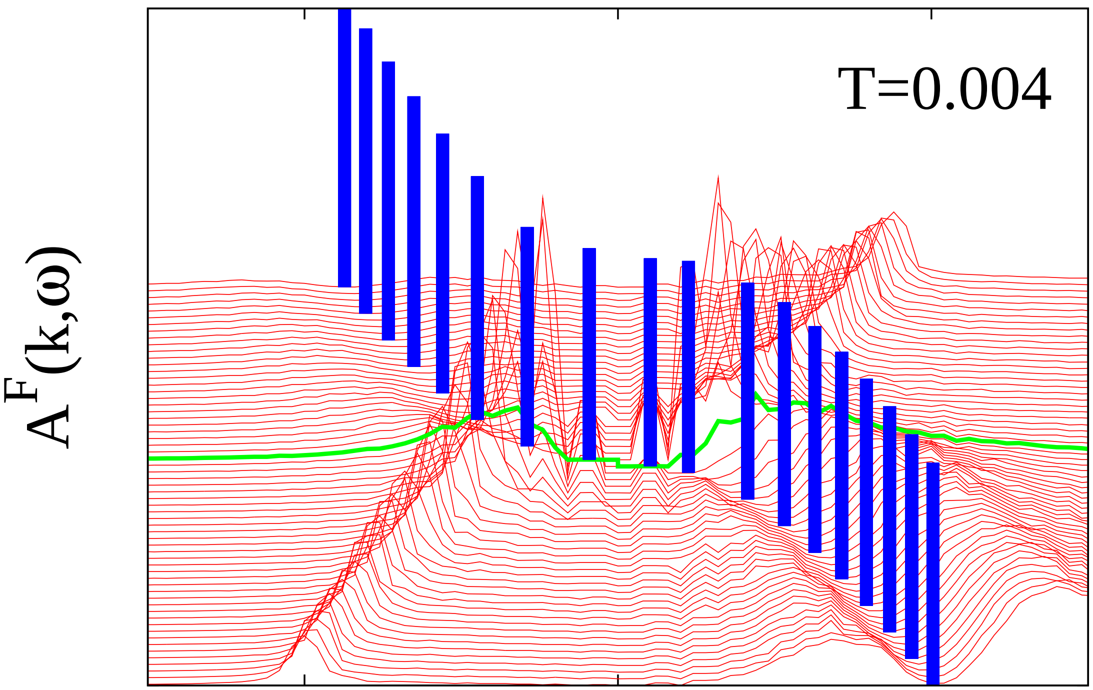

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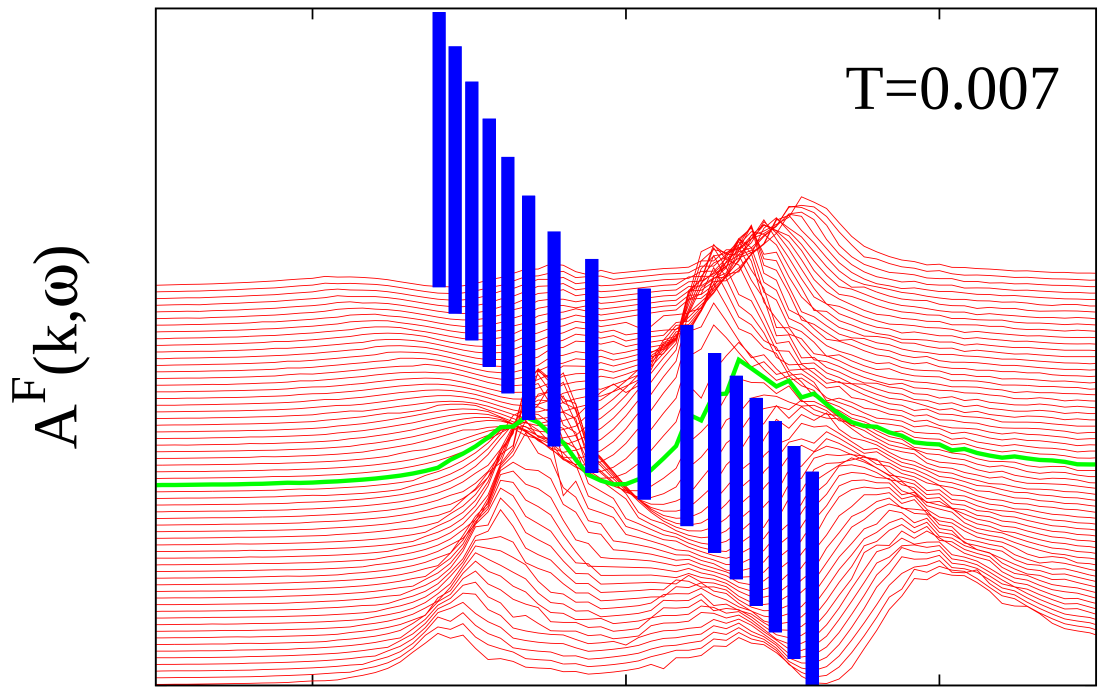

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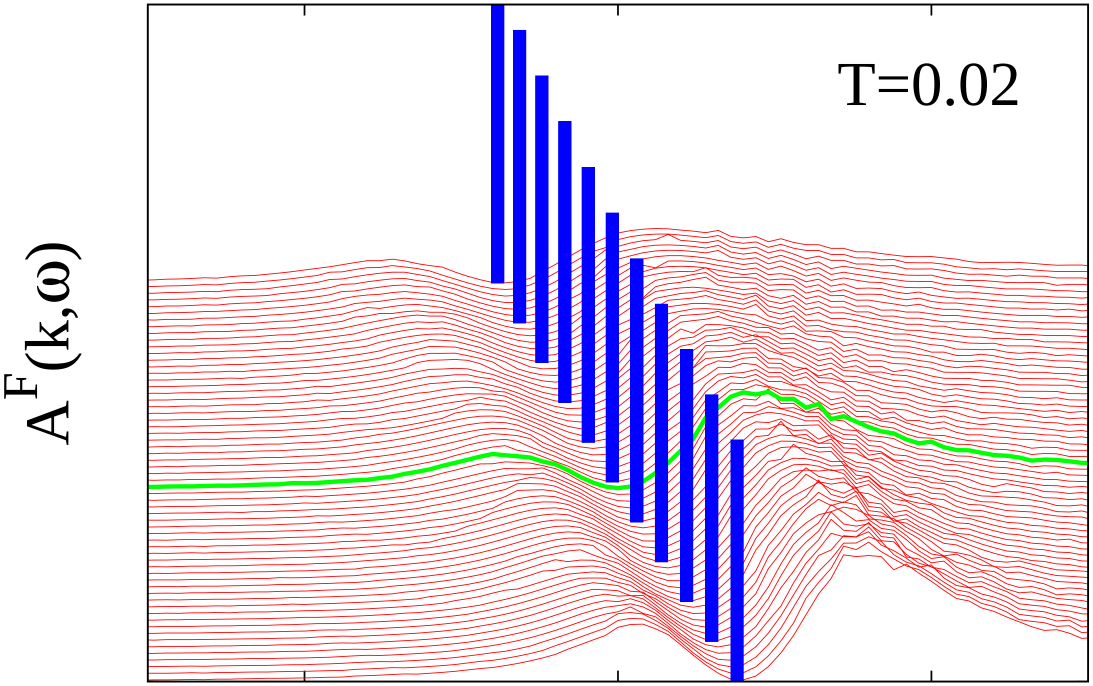
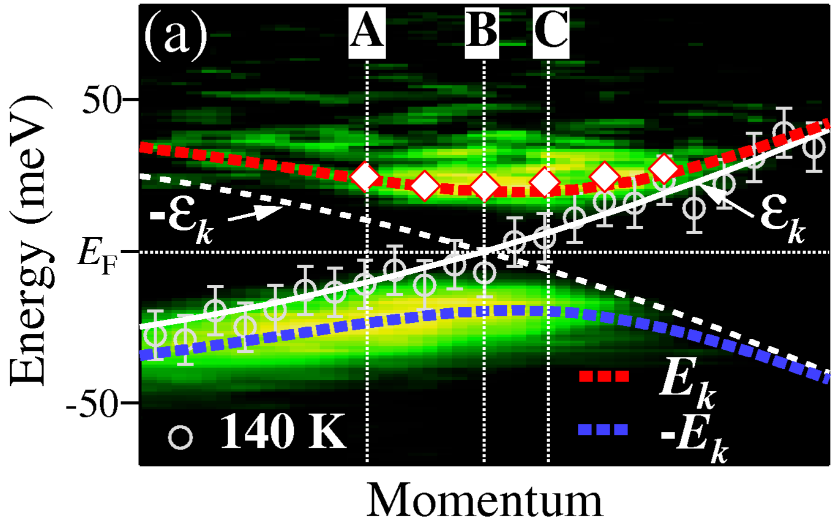
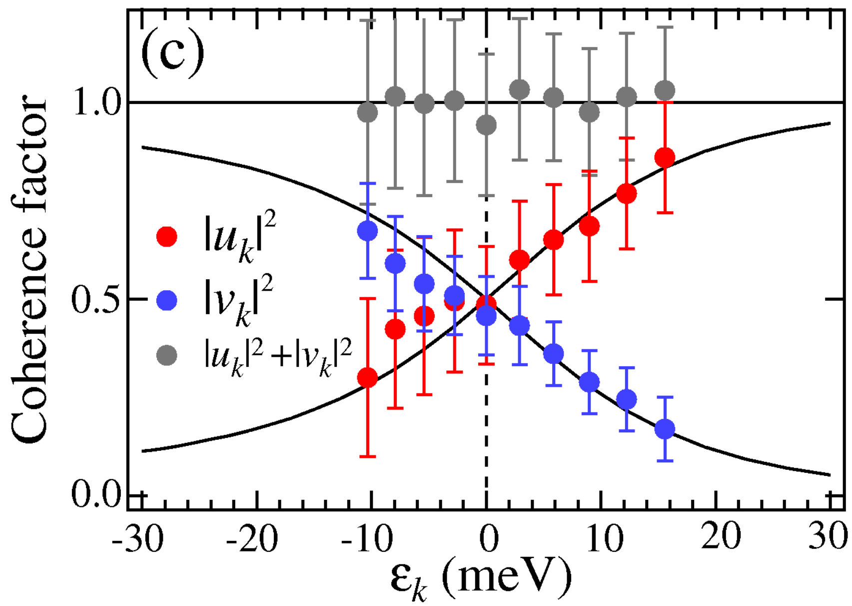


T. Domański and J. Ranninger, Phys. Rev. Lett. **91**, *255301 (2003).*

Experimental data for $T < T_c$





H. Matsui, T. Sato, and T. Takahashi et al, *Phys. Rev. Lett.* **90**, *217002 (2003).*

Evidence for Bogoliubov QPs above T_c

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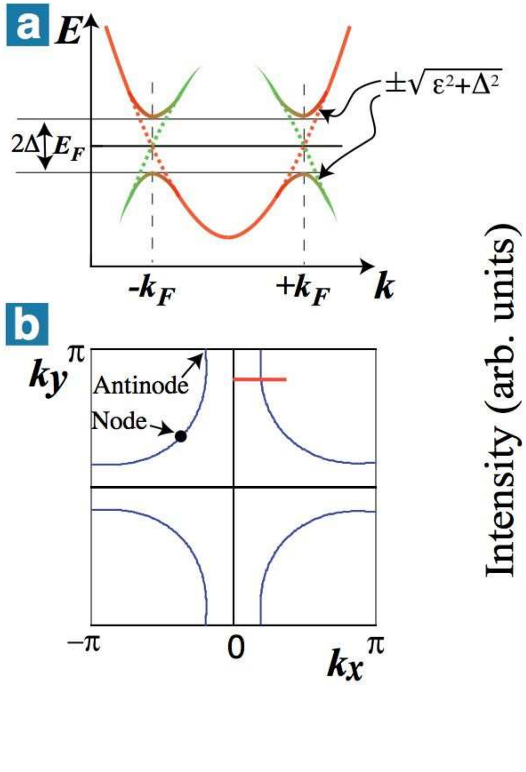

Results for: $Bi_2Sr_2CaCu_2O_8$

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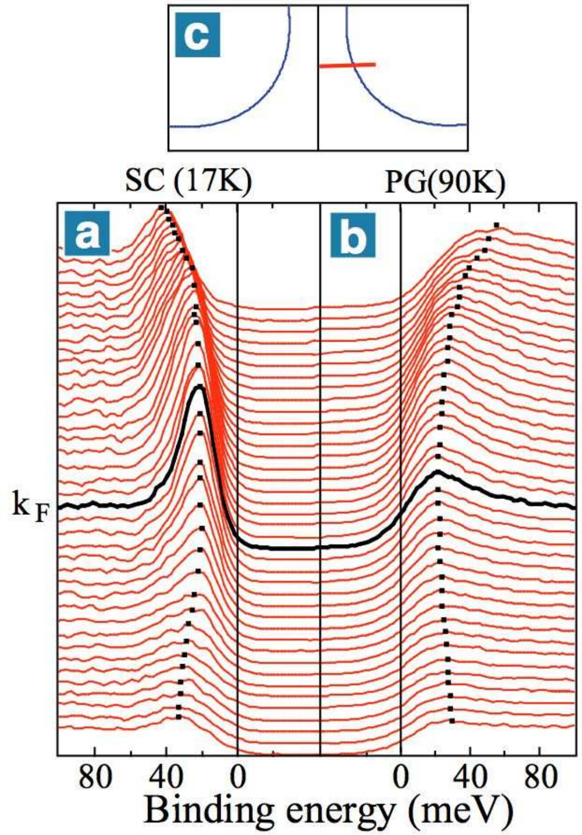

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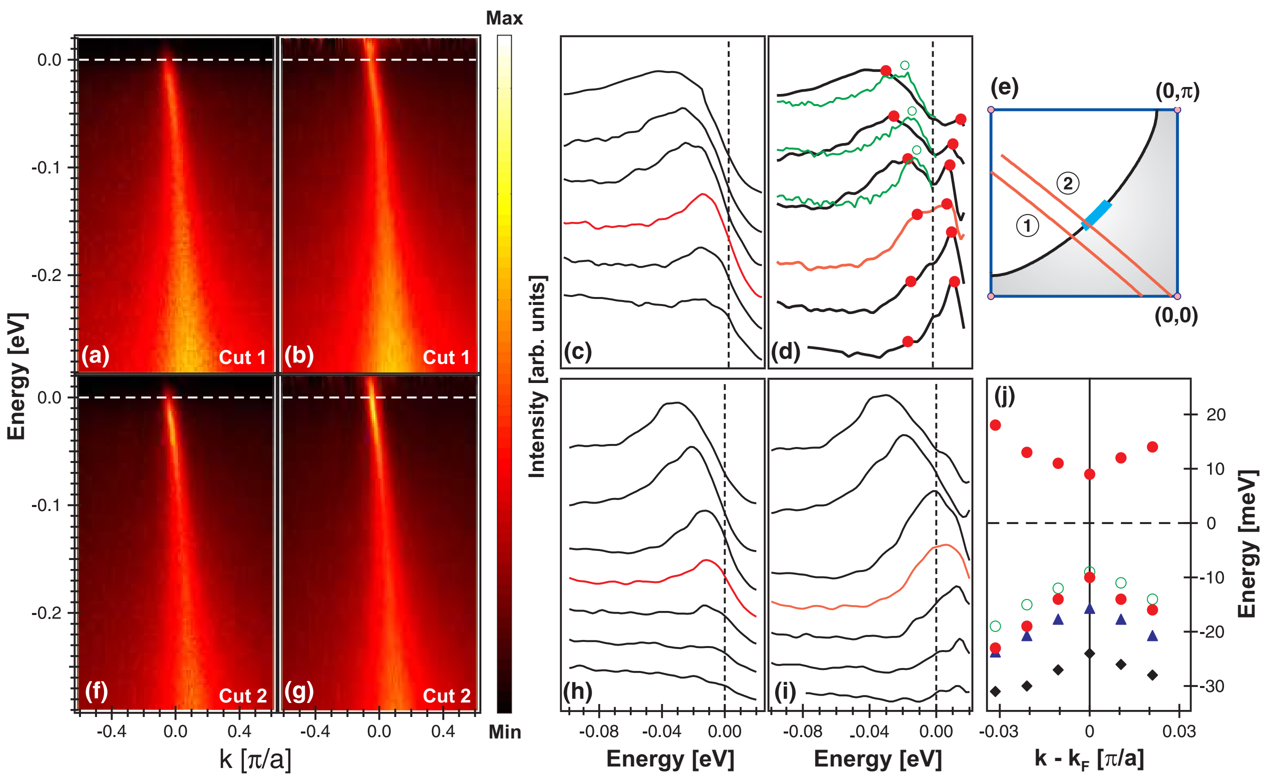
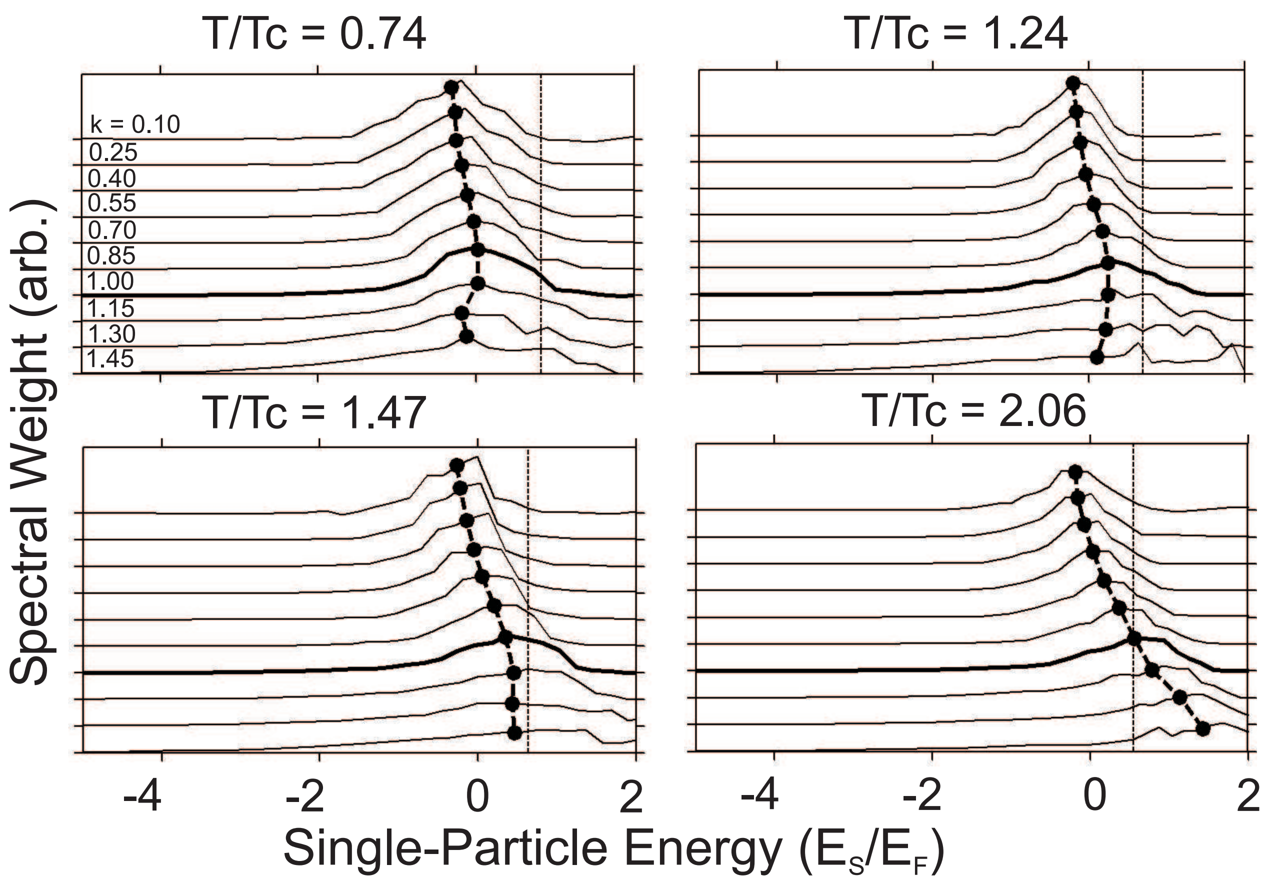


Results for: $La_{1.895}Sr_{0.105}CuO_4$

M. Shi et al, Eur. Phys. Lett. **88**, *27008 (2009).*

Evidence for Bogoliubov QPs above T_c

D. Jin group (Boulder, USA)



Results for: ultracold ^{40}K atoms

J.P. Gaebler et al, Nature Phys. **6***, 569 (2010).*

Results :

2. Diamagnetism above T_c

Correlation functions

For studying the diamagnetic response (in the Kubo formalism) we have to determine the current-current correlation function

$$-\hat{T}_\tau \langle \hat{j}_{\mathbf{q}}(\tau)\, \hat{j}_{-\mathbf{q}}(0) \rangle$$

with statistical averaging defined as

$$\langle \ldots \rangle = \text{Tr}\left\{ e^{-\beta \hat{H}} \ldots \right\} / \text{Tr}\left\{ e^{-\beta \hat{H}} \right\}$$

and $\beta^{-1} = k_B T$.

This can be achieved using the following invariance

$$\begin{aligned} \text{Tr}\left\{ e^{-\beta \hat{H}} \hat{O} \right\} &= \text{Tr}\left\{ e^{\hat{S}(l)} e^{-\beta \hat{H}} \hat{O} e^{-\hat{S}(l)} \right\} \\ &= \text{Tr}\left\{ e^{\hat{S}(l)} e^{-\beta \hat{H}} e^{-\hat{S}(l)} e^{\hat{S}(l)} \hat{O} e^{-\hat{S}(l)} \right\} \\ &= \text{Tr}\left\{ e^{-\beta \hat{H}(l)} \hat{O}(l) \right\} \end{aligned}$$

where

$$\hat{H}(l) = e^{\hat{S}(l)} \hat{H} e^{-\hat{S}(l)} \qquad \hat{O}(l) = e^{\hat{S}(l)} \hat{O} e^{-\hat{S}(l)}$$

Correlation functions – some remarks

- $\star$ The easiest way for calculating $\langle \hat{j}_{\mathbf{q}}(\tau)\, \hat{j}_{-\mathbf{q}} \rangle$ is in a limit $l \longrightarrow \infty$ when $\hat{H}(\infty)$ becomes (block-)diagonal.

- $\star$ The operators $\hat{j}_q$ must however undergo the continuous transformation

$$\boxed{\hat{j} \longrightarrow \dots \longrightarrow \hat{j}(l) \longrightarrow \dots \longrightarrow \hat{j}(\infty)}$$

- $\star$ obeying **the flow equation**:

$$\boxed{\frac{\partial \hat{j}_{\mathbf{q}}(l)}{\partial l} = \left[\hat{\eta}(l), \hat{j}_{\mathbf{q}}(l) \right]}$$

Diamagnetic response above T_c

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Main contributions to the current-current response function:

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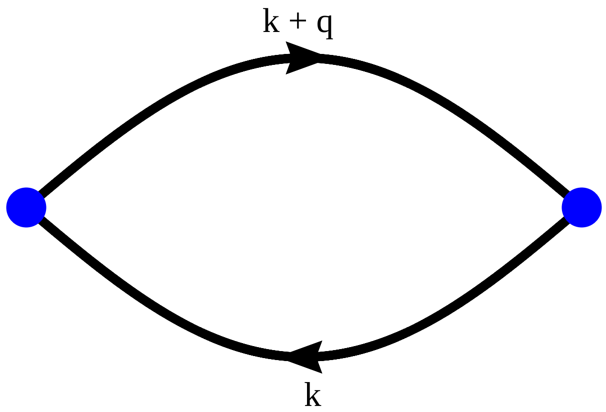

$\Longleftarrow$ **the usual bubble diagram**

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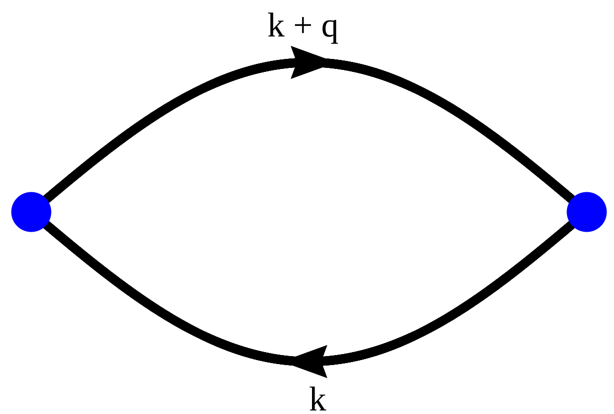

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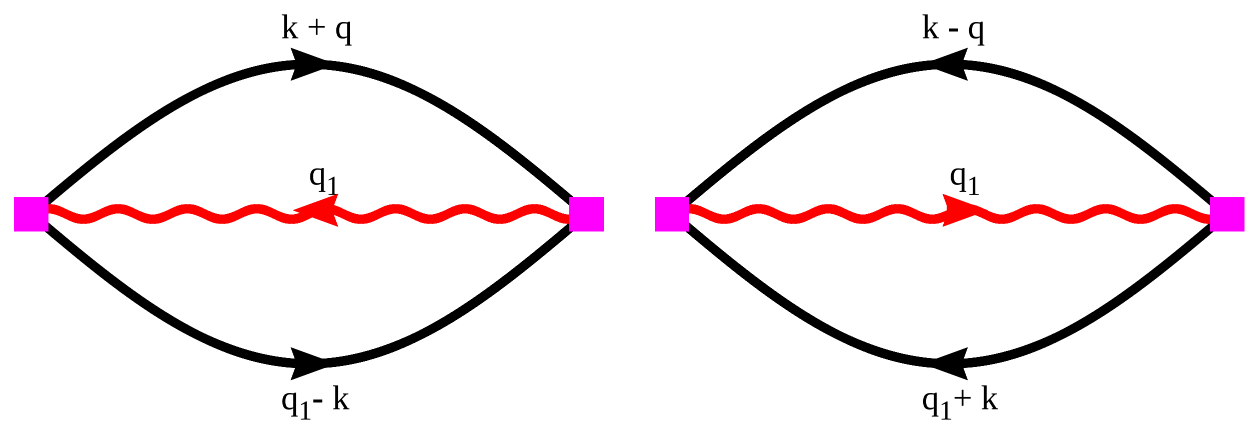


anomalous diagrams

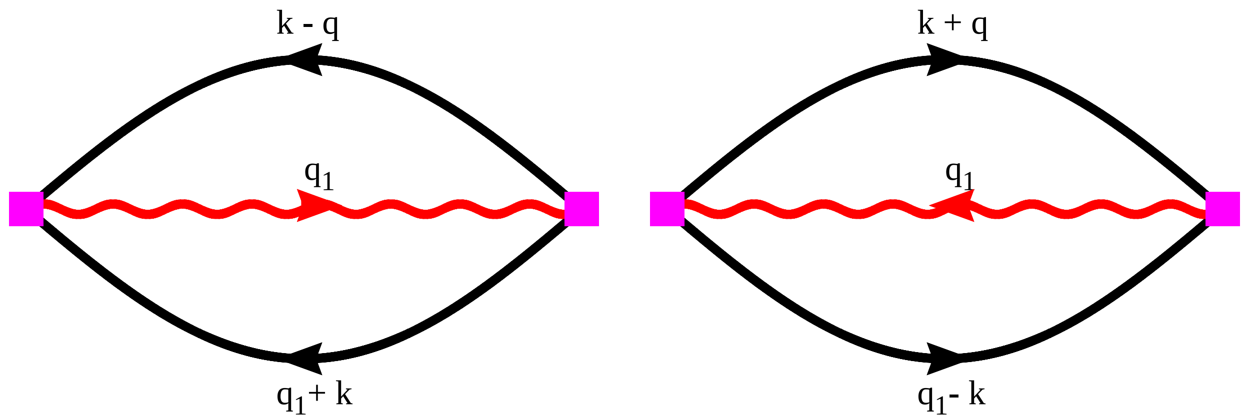
Each vertex has to be determined from the flow equations.

T. Domanski and J. Ranninger, (to be published).

Diamagnetic response above T_c

Diamagnetic response above T_c

The anomalous contributions



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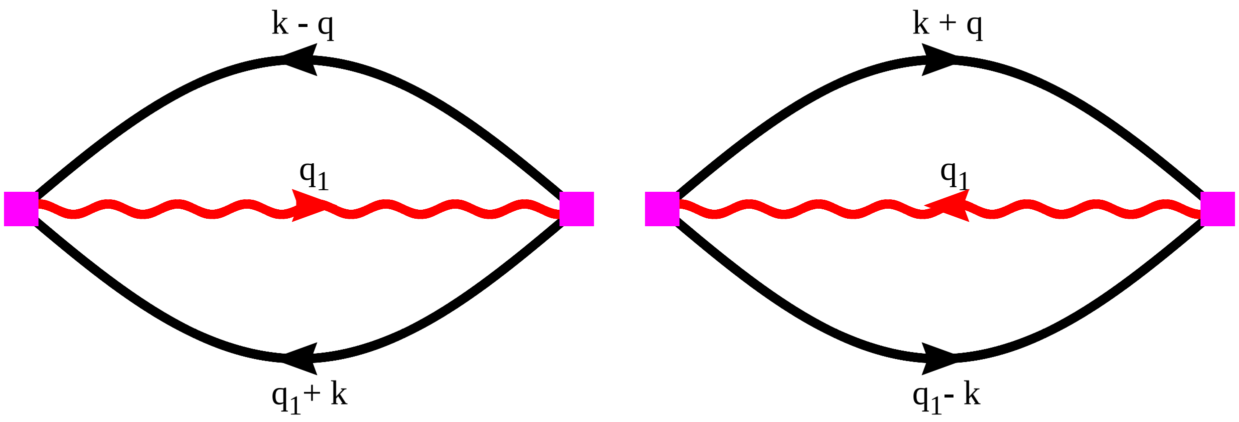
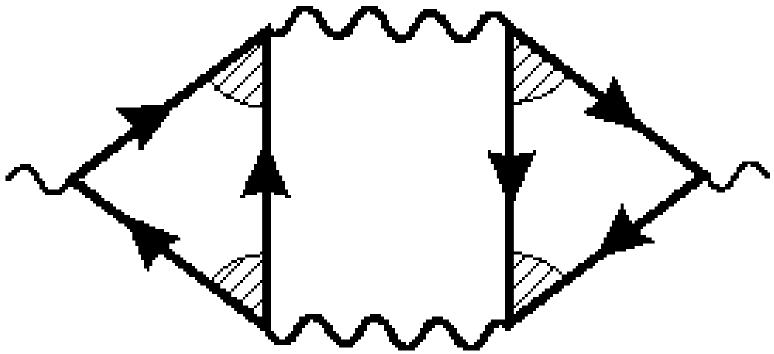


resemble the Aslamasov-Larkin diagram



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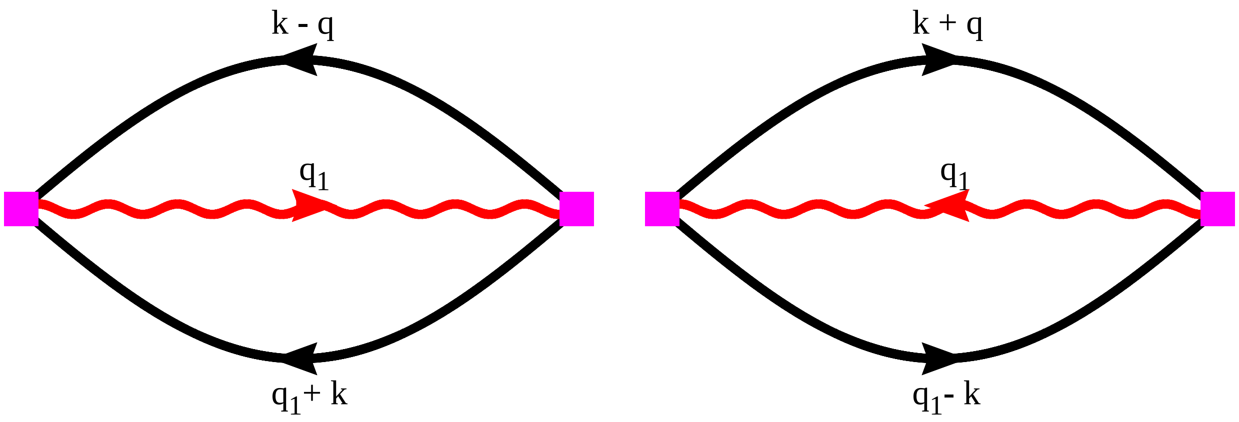

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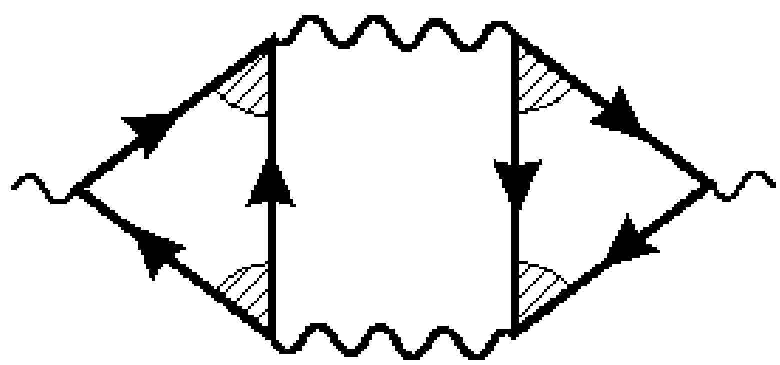

enhancing the conductance/diamagnetism above T_c.

Diamagnetic response above T_c

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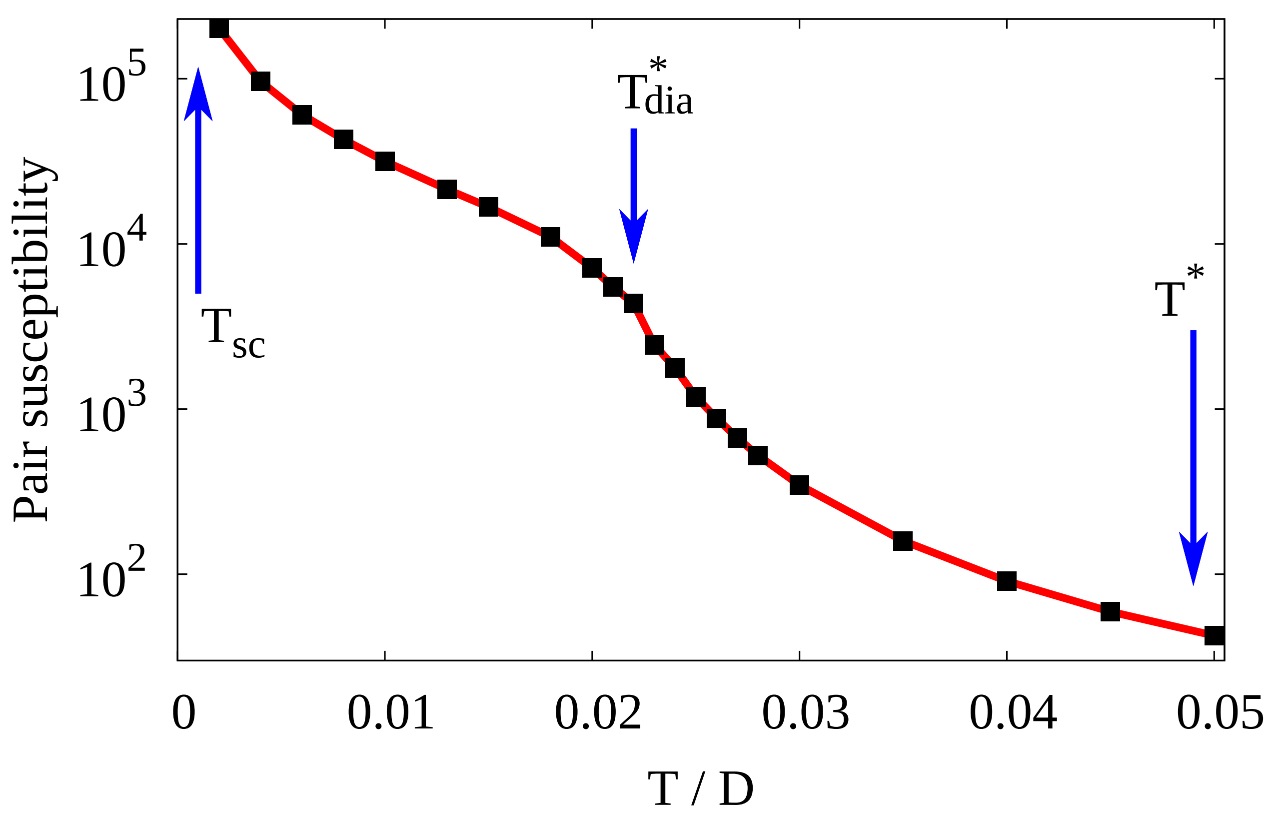

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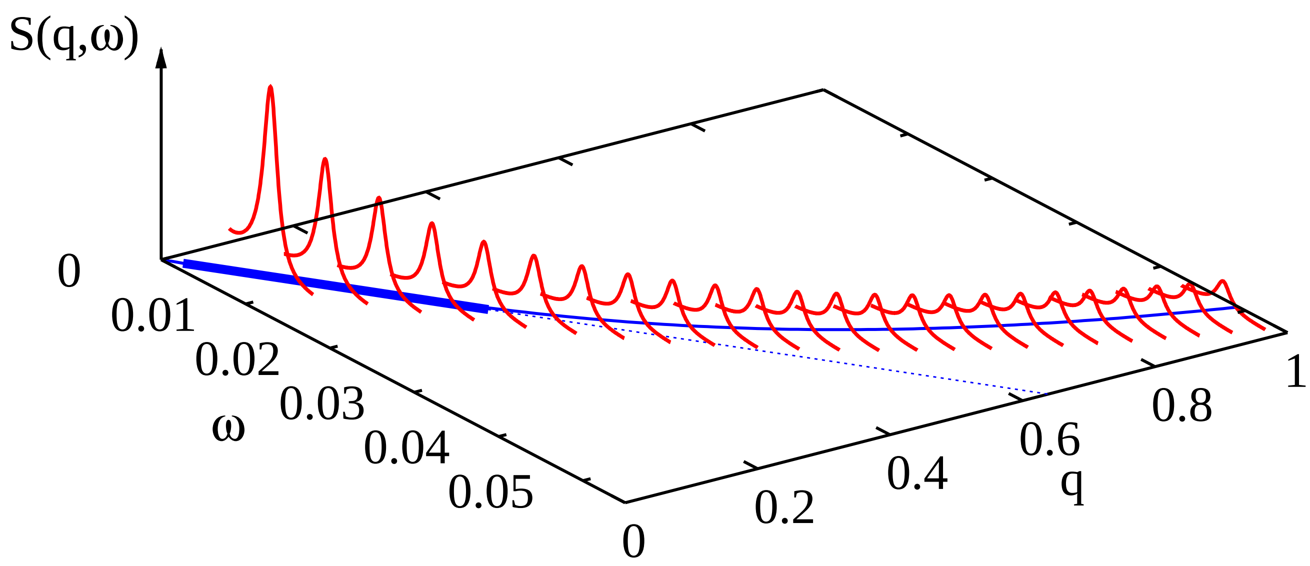

The structure function $\frac{1}{\omega}$ Im $\chi(q,\omega)$ showing a piece of the collective (Goldstone) branch for $q_{c1} < q < q_{c2}$.

Onset of diamagnetism above T_c

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Onset of the diamagnetism coincides with appearance of the collective features in the fermion/boson spectrum.

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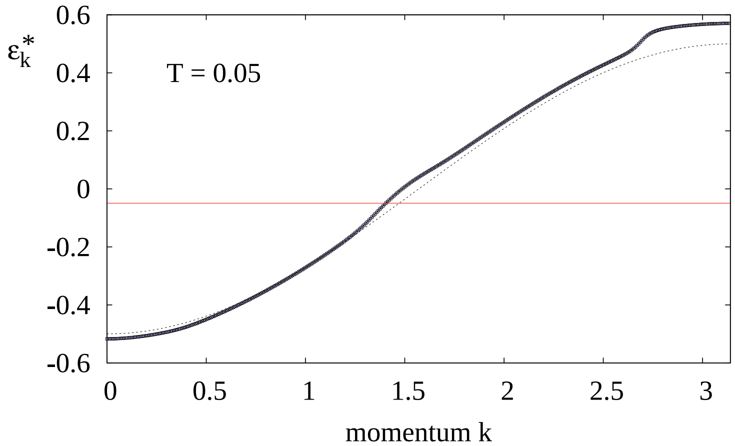
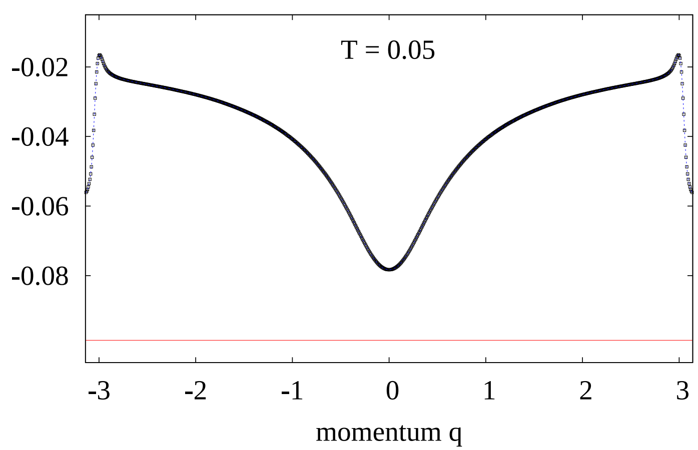


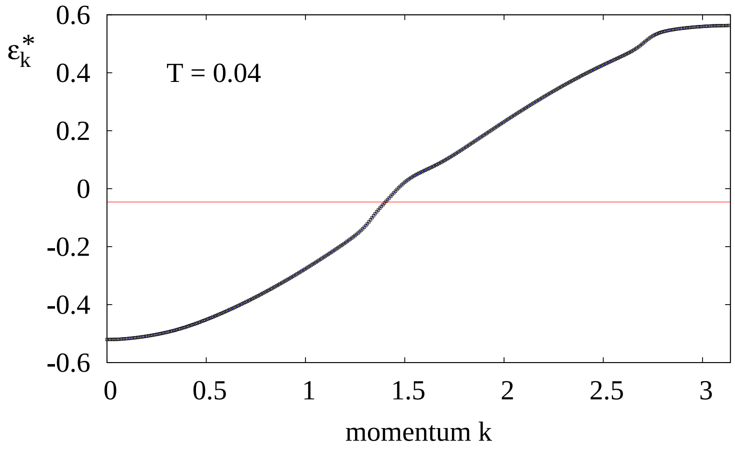
fermion dispersion

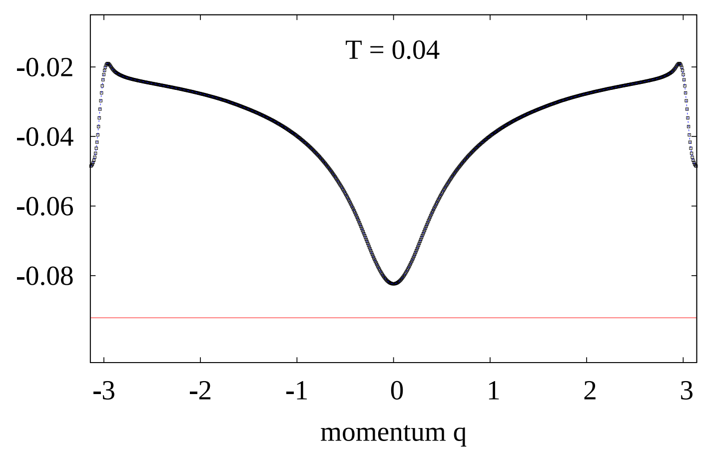


pair dispersion

Onset of diamagnetism above T_c

Onset of the diamagnetism coincides with appearance of the collective features in the fermion/boson spectrum.





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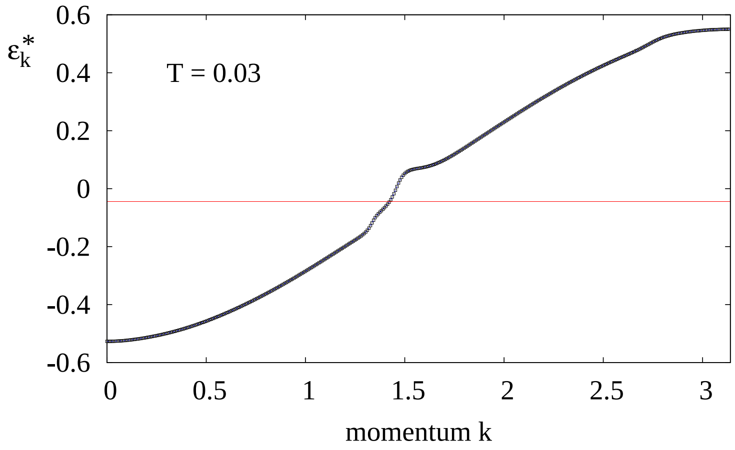

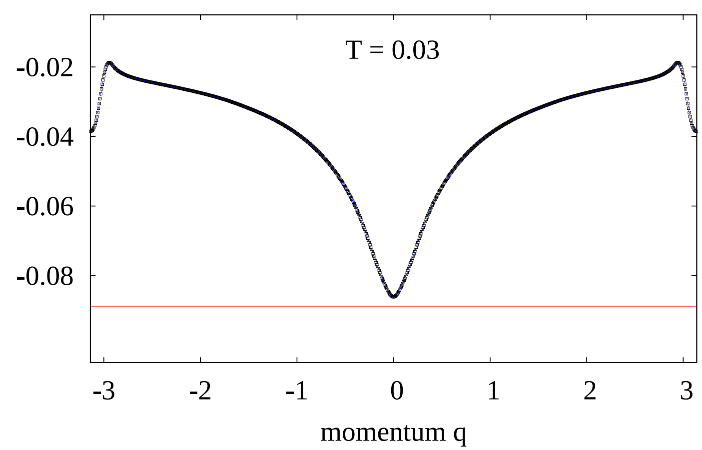

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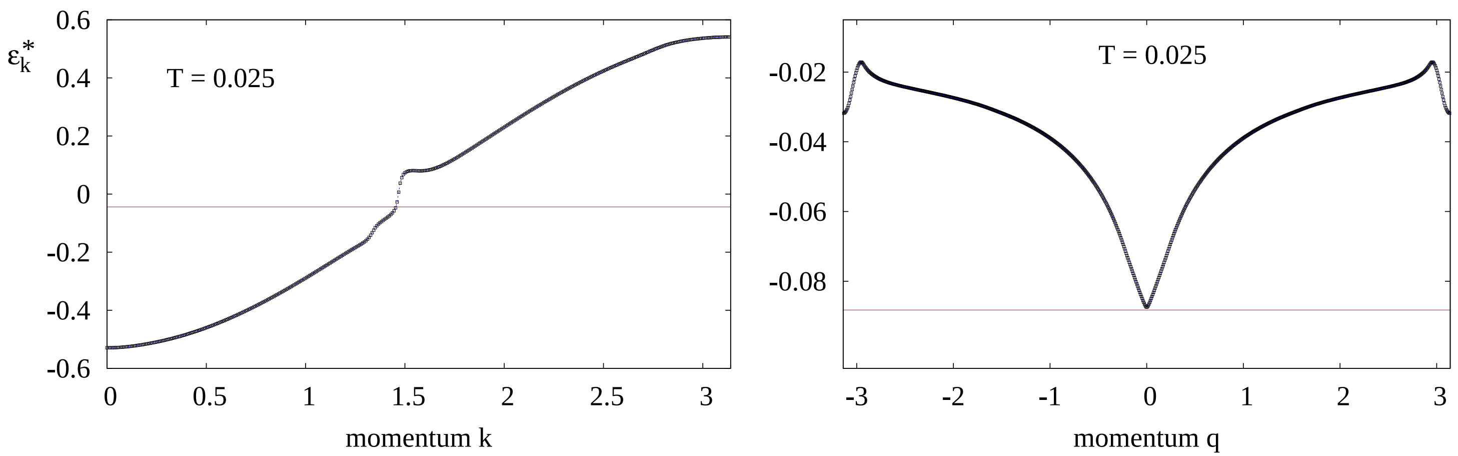

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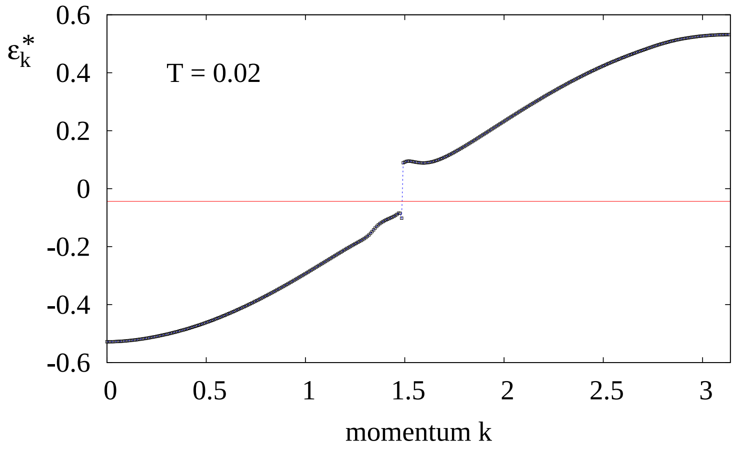

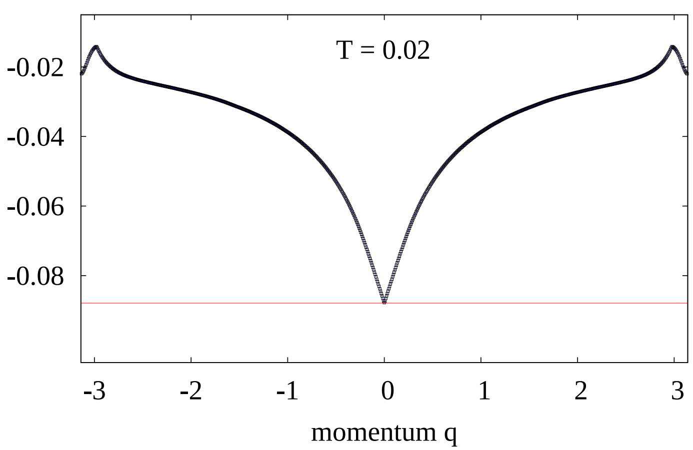

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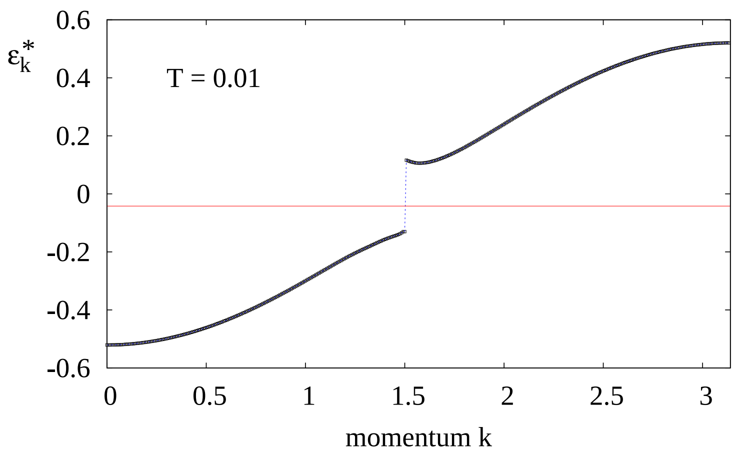

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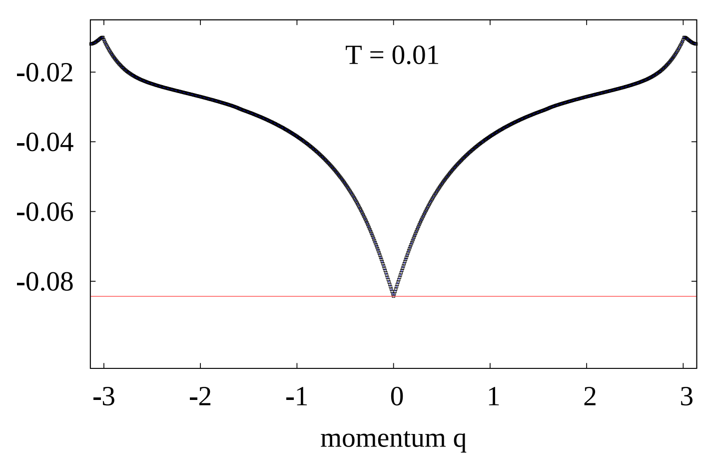

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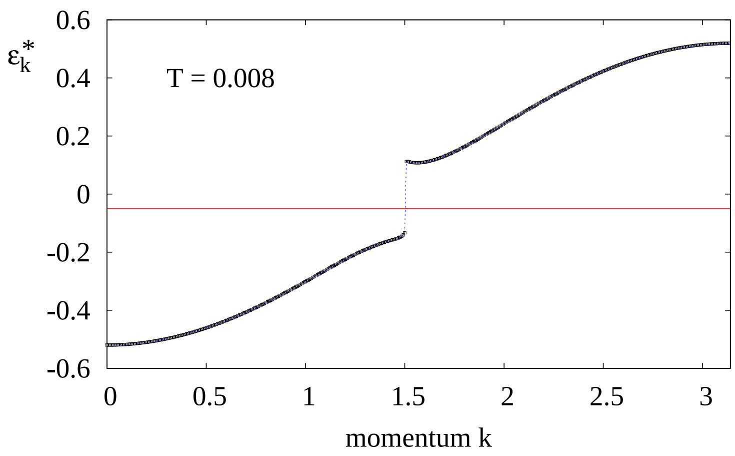

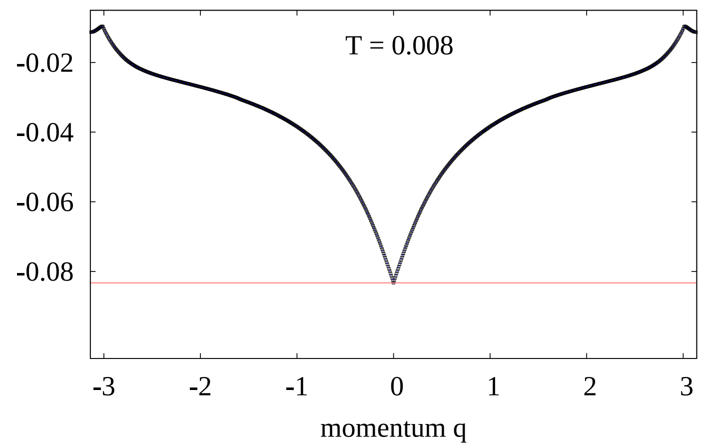

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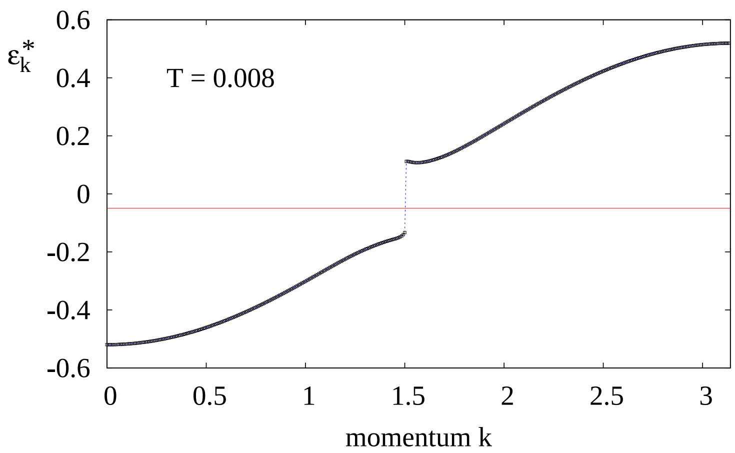

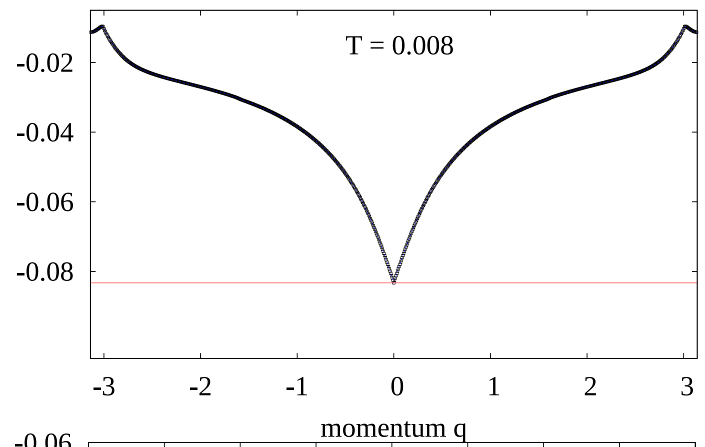

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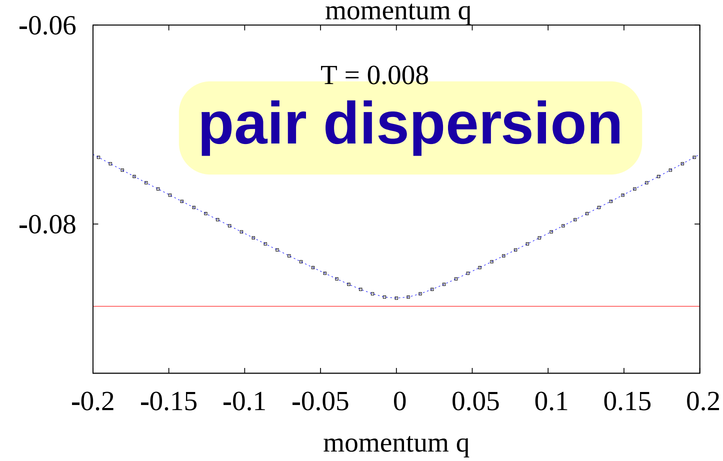

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