TIME-DEPENDENT PHENOMENA IN NANOSCOPIC SUPERCONDUCTORS

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UMCS Lublin

"Condensed Matter Physics Seminar"

Lublin, 20 Oct. 2020

OUTLINE

- **Nanoscopic superconductors**

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$\Longrightarrow$ **quantum dots, nanowires, nanoislands**

proximitized to bulk superconductors

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- **Time-dependent phenomena**

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- **Nanoscopic superconductors**

$\Longrightarrow$ **quantum dots, nanowires, nanoislands**

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- **Time-dependent phenomena**

$\Longrightarrow$ **transient effects**

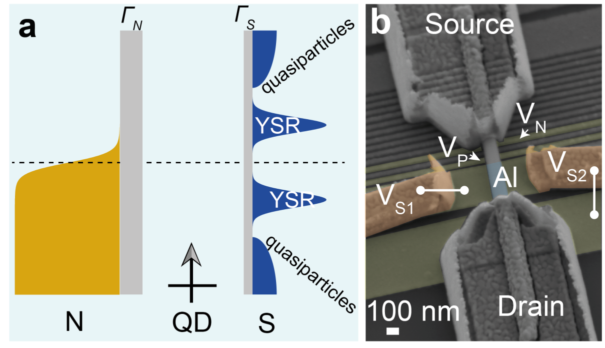
$\Longrightarrow$ **quench dynamics**

a) dynamical quantum phase transition

b) gradual leakage of Majorana modes

HETEROSTRUCTURES WITH SUPERCONDUCTOR(S)

normal metal (N) - quantum dot (QD) - superconductor (S)



J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, Commun. Phys. 3, 125 (2020).

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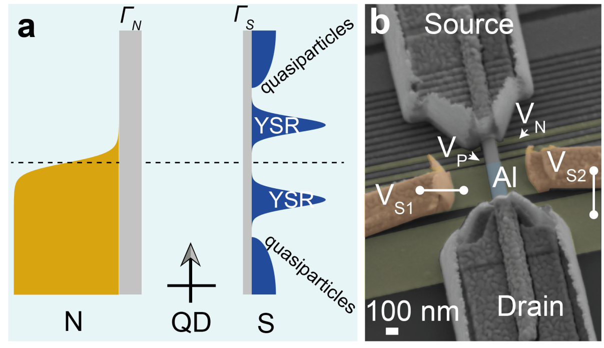

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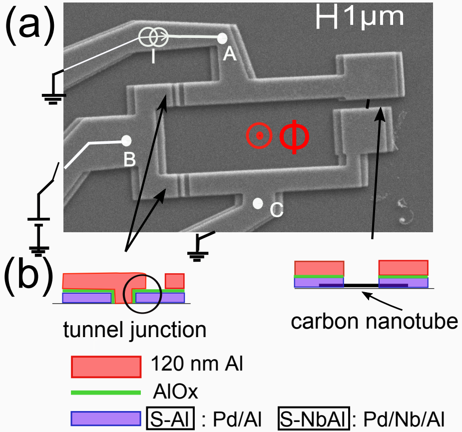
N - metallic gold **QD - InAs wire** **S - Aluminum**

HETEROSTRUCTURES WITH SUPERCONDUCTOR(S)

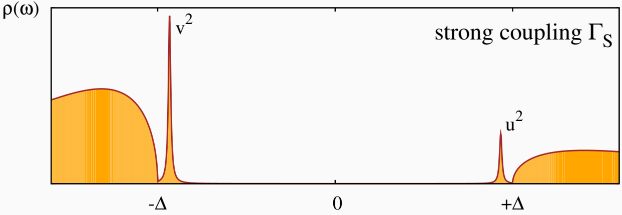
superconductor (S) - quantum dot (QD) - superconductor (S)



R. Delagrange, R. Weil, A. Kasumov, M. Ferrier, H. Bouchiat, R. Deblock, Phys. Rev. B 93, 195437 (2016).

IN-GAP STATES

Spectrum of a single impurity coupled to bulk superconductor:



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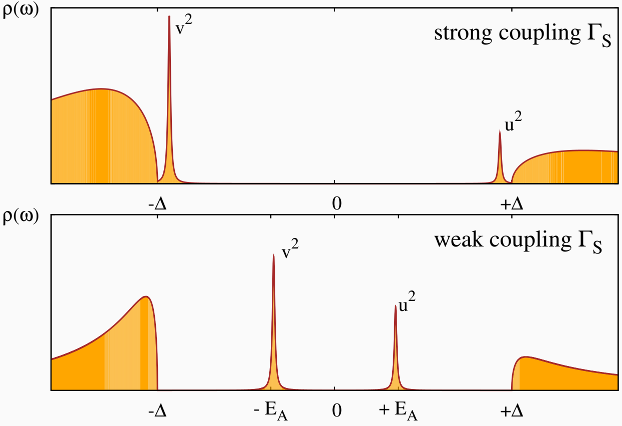

Bound states appearing in the subgap region $-\Delta < \omega < \Delta$.

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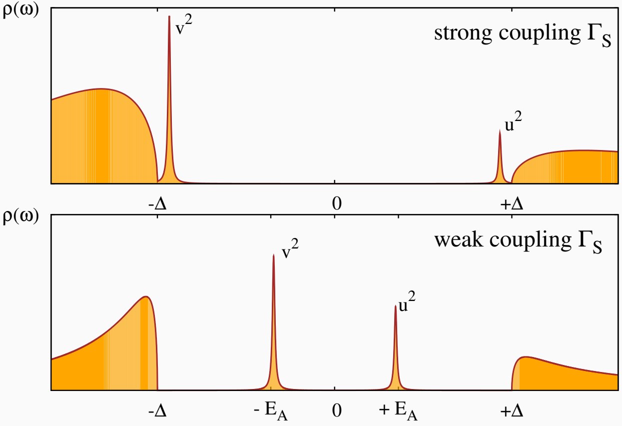


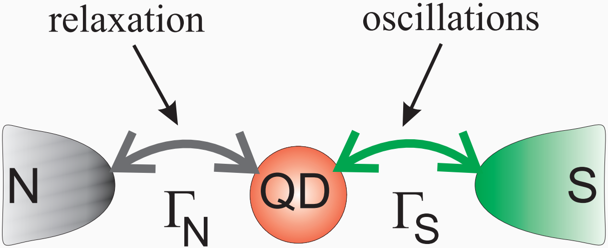
Bound states appearing in the subgap region $-\Delta < \omega < \Delta$.

Yu-Shiba-Rusinov (Andreev) bound states

Transient dynamics

TRANSIENT EFFECTS OF IN-GAP STATES

Consider a sudden coupling of QD to external leads



R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

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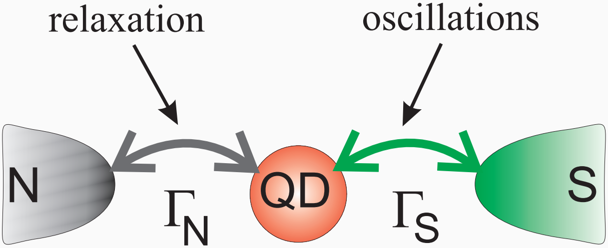

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- **for $t < 0$ QD is isolated** $\Gamma_S = 0 = \Gamma_N$

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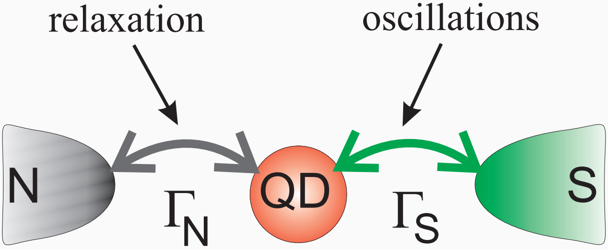

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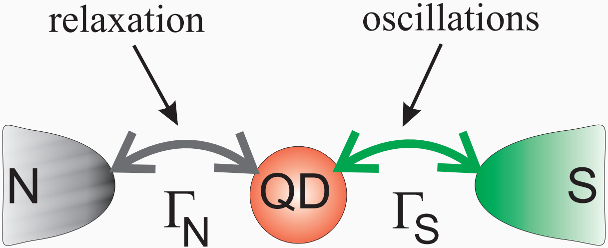
- **for $t < 0$ QD is isolated** $\Gamma_S = 0 = \Gamma_N$
- **for $t \geq 0$ QD is hybridized** $\Gamma_S \neq 0 \neq \Gamma_N$

TRANSIENT EFFECTS OF IN-GAP STATES

Physical questions:



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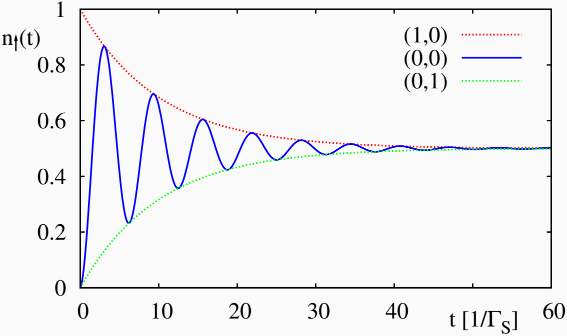
- **how much time is needed to create in-gap states?**
- **can such characteristic time-scale be measured ?**

RELAXATION VS QUANTUM OSCILLATIONS

Time-dependent charge for various initial QD fillings



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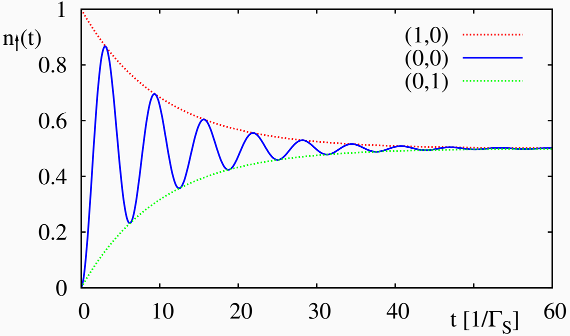

- **importance of the initial QD configuration**

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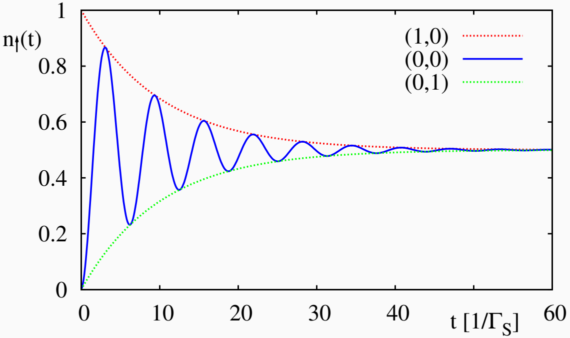

- **importance of the initial QD configuration**
- **oscillations for empty/doubly occupied QD**

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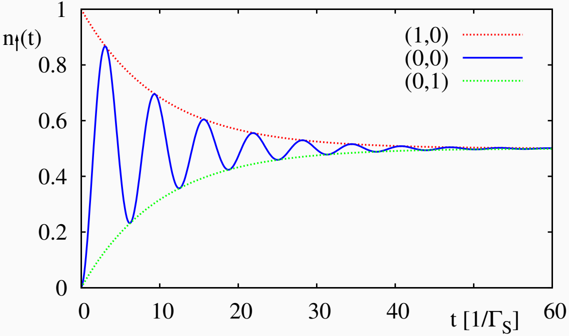
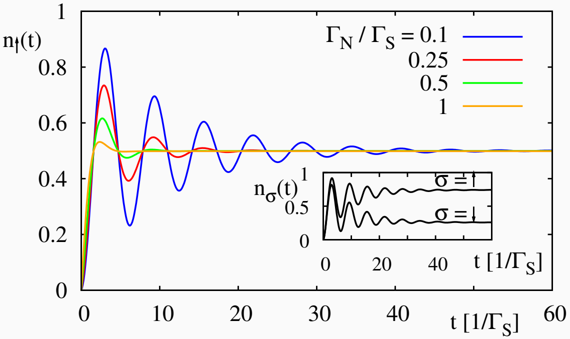


- **importance of the initial QD configuration**
- **oscillations for empty/doubly occupied QD**

These quantum oscillations are reminiscent of the Rabi (two-level) system.

RELAXATION VS QUANTUM OSCILLATIONS

Role of the coupling Γ_N to metallic lead



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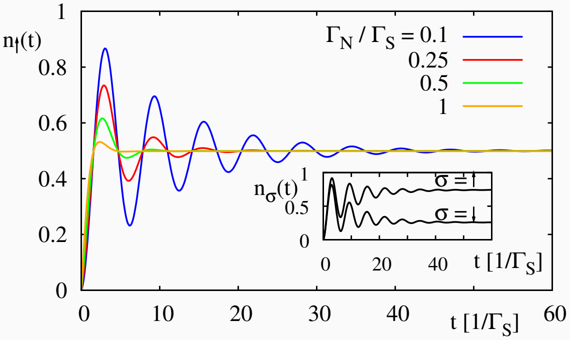

- relaxation time is proportional to $1/\Gamma_N$

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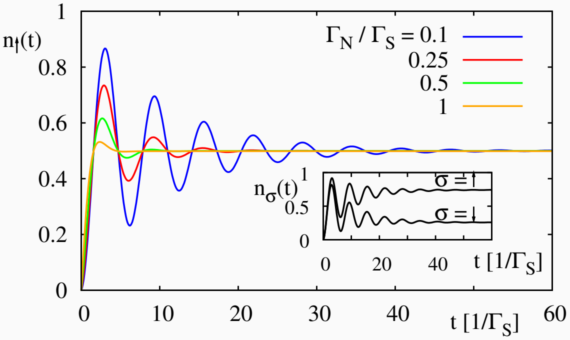
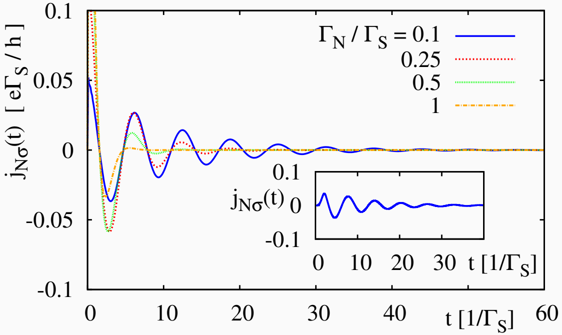


- **relaxation time is proportional to $1/\Gamma_N$**

Thermalization is driven by a continuum electrons from the metallic lead.

RELAXATION VS QUANTUM OSCILLATIONS

Empirically measurable transient current

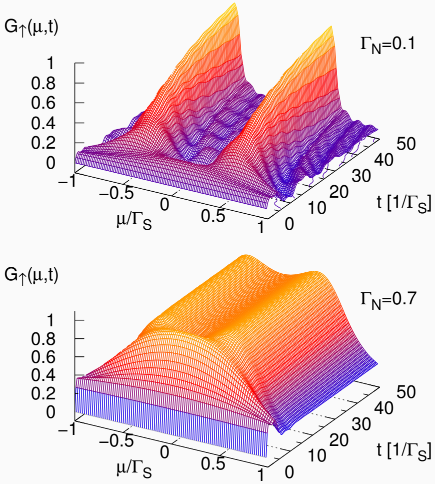


- **relaxation time is proportional to $1/\Gamma_N$**
- **oscillations depend on energies of in-gap states**

R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

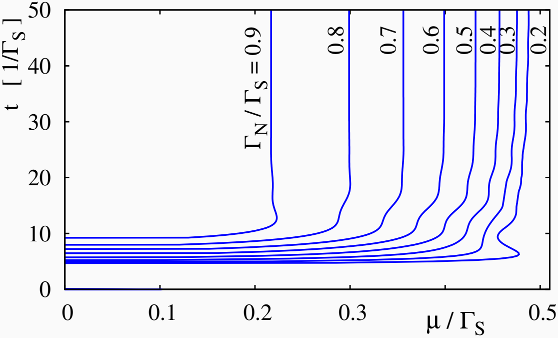
EXPERIMENTALLY ACCESSIBLE QUANTITIES

QD level $\varepsilon_d = 0$



Subgap tunneling conductance $G_\sigma = \frac{\partial I_\sigma}{\partial t}$ vs time (t) and voltage (μ)

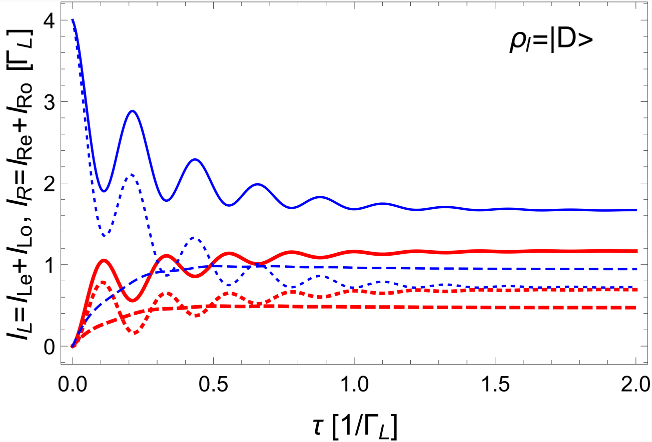
Signatures of the in-gap bound states vs (t, μ)



- **period of oscillations $\tau_2 = \frac{2}{\Gamma_S}$ ....(about picoseconds)**
- **relaxation time $\tau_1 = \frac{2}{\Gamma_N}$ ....(it can be arbitrary)**

R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

STATISTICS OF TUNNELING EVENTS



Transient currents from *'Waiting Time Distribution'* approach

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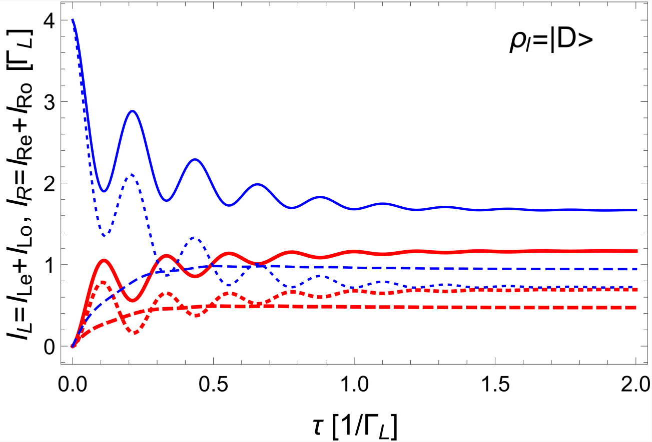

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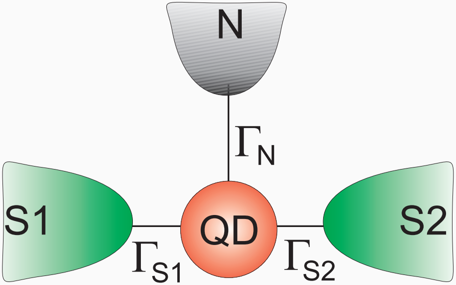
JOSEPHSON/ANDREEV CIRCUITS



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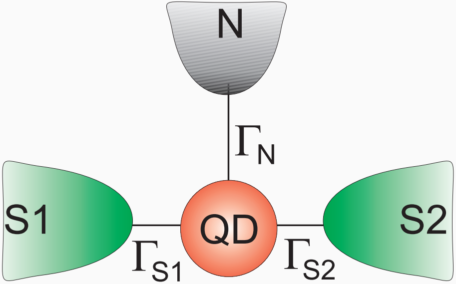

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Issues to be addressed:

- **phase-controlled emergence of in-gap states,**

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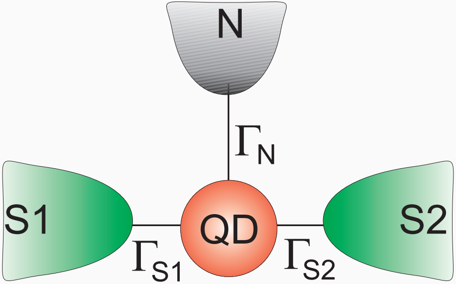

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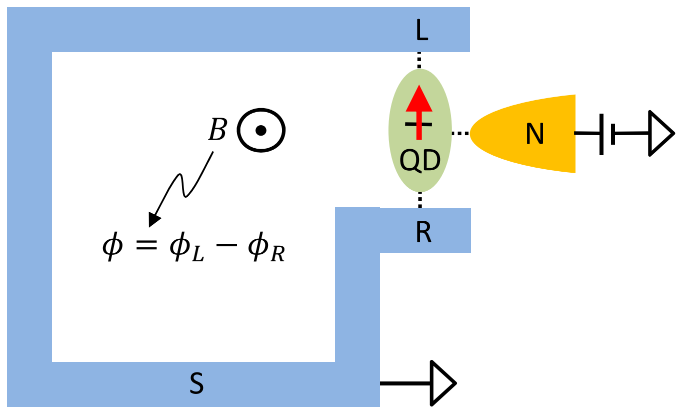
Issues to be addressed:

- **phase-controlled emergence of in-gap states,**
- **dynamical effects observable by $j_S(t)$ and/or $j_N(t)$.**

SCHEME FOR EMPIRICAL REALIZATION

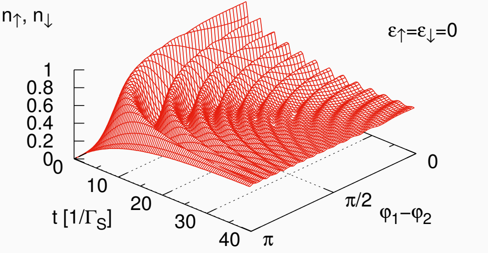
Chart for a practical realization of the Josephson & Andreev circuits

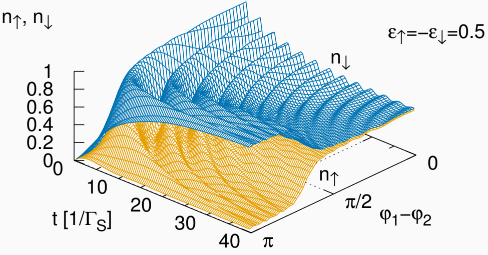


G. Kiršanskas, M. Goldstein, K. Flensberg, L.I. Glazman & J. Paaske,
Phys. Rev. B 92, 235422 (2015)

PHASE-CONTROLLED TRANSIENTS

Time dependent charge $n_\sigma(t)$ of QD





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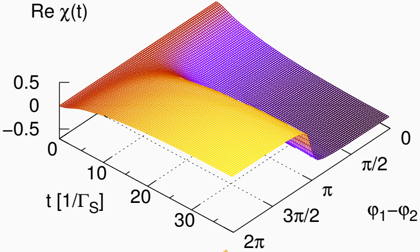
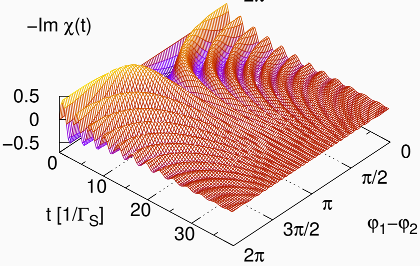
PHASE-CONTROLLED TRANSIENTS

Time dependent order parameter $\langle \hat{d}_{\downarrow} \hat{d}_{\uparrow} \rangle$ vs phase difference





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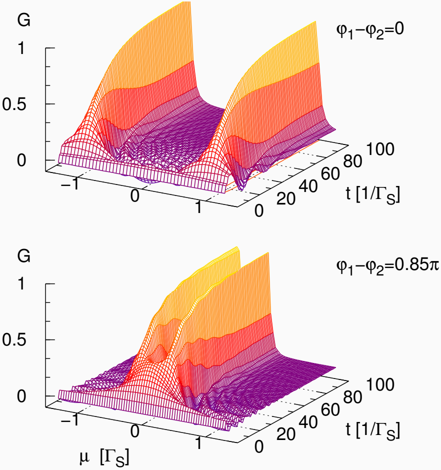
PHASE-CONTROLLED TRANSIENTS

The measurable time-dependent Andreev conductance



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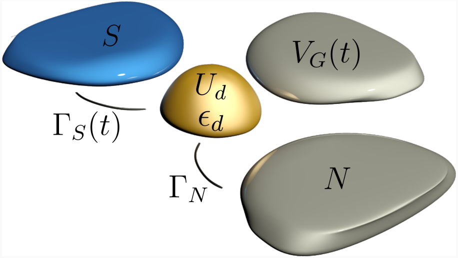
Quench dynamics

QUANTUM QUENCH PROTOCOL



K. Wrześniewski, B. Baran, R. Taranko, T. Domański & I. Weymann, arXiv:2007.10747 (2020).

Two scenarios of quantum quenches:

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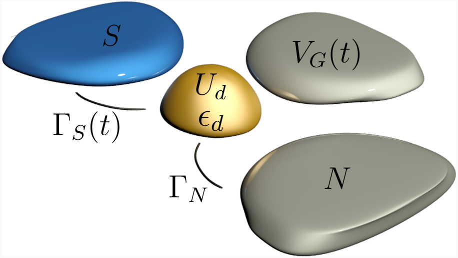

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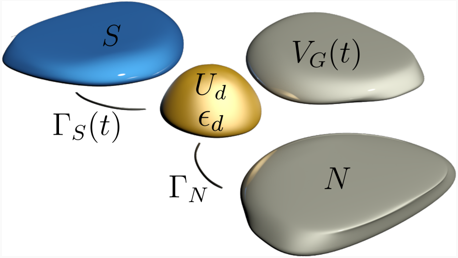

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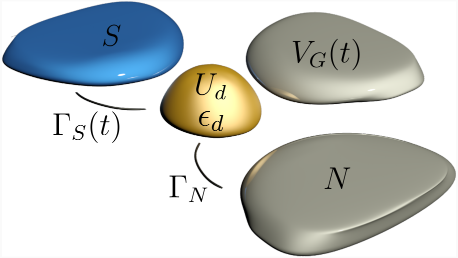
Two scenarios of quantum quenches:

$\Longrightarrow$ **sudden coupling to superconductor $0 \rightarrow \Gamma_S$,**

$\Longrightarrow$ **abrupt switching of gate potential $0 \rightarrow V_G$.**

DYNAMICS VS CORRELATIONS



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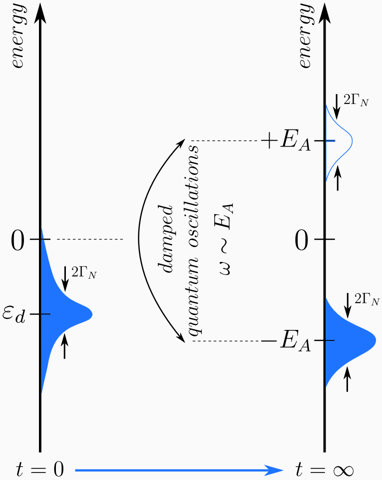
Main issue to be considered:

- **competition between pairing & Coulomb repulsion.**

IN-GAP STATES OF CORRELATED QD

Emergenece of the Andreev states induced by quench $0 \rightarrow \Gamma_S$



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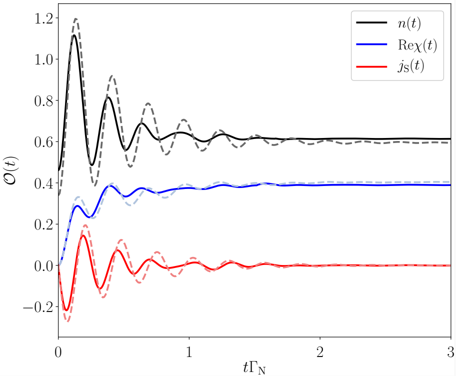
IN-GAP STATES OF CORRELATED QD

Time-dependent observables driven by the quantum quench $0 \to \Gamma_S$



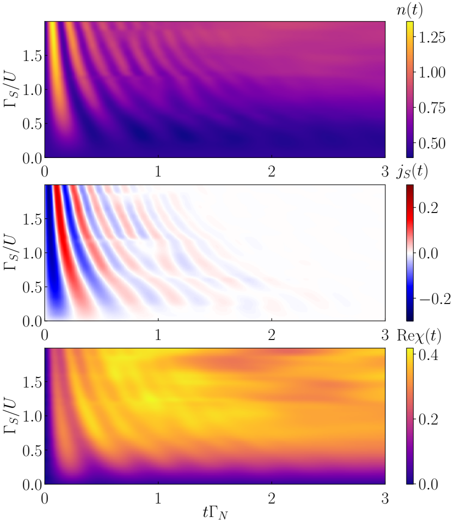
solid lines **- time dependent NRG**

dashed lines - Hartree-Fock-Bogolubov

Results obtained for $\varepsilon_d = 0$, $\Gamma_N/U = 0.1$, quench $\Gamma_S = 0 \longrightarrow \Gamma_S = U$.

IN-GAP STATES OF CORRELATED QD

tNRG results for the quantum quench $0 \rightarrow \Gamma_S$ (as indicated)



QD charge $n(t)$

charge current $j_S(t)$

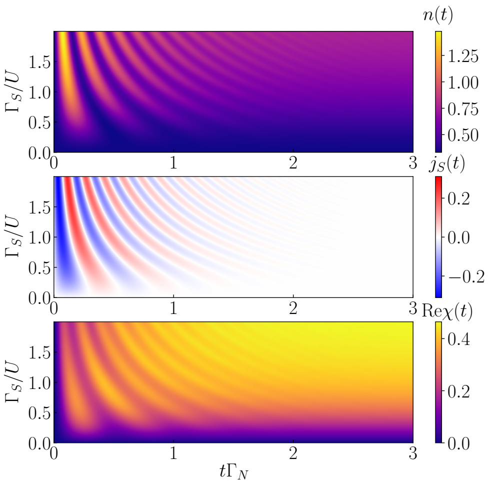
from supercond. to QD

on-dot pairing

$\chi(t) \equiv \langle d_\downarrow d_\uparrow \rangle$

BUILDUP OF IN-GAP STATES

Hartree-Fock-Bogolubov results for the quantum quench $0 \rightarrow \Gamma_S$



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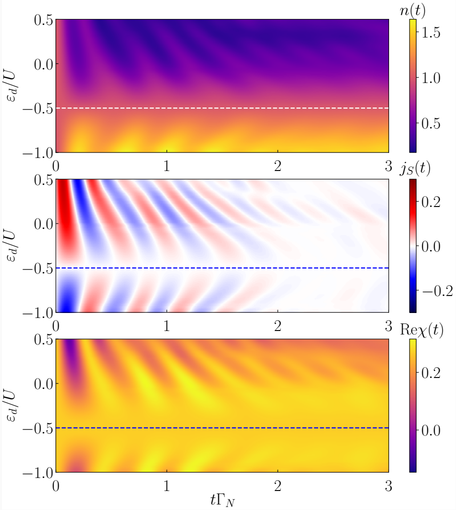
SUDDEN CHANGE OF A GATE POTENTIAL

tNRG results for $\Gamma_S = U/2$, $\Gamma_N = U/10$ imposing the quench lifting QD level from $\varepsilon_d(t<0) = -U/2$ to ε_d (as indicated)



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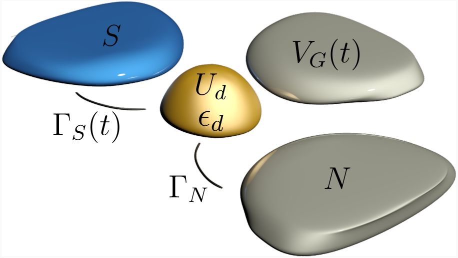
Dynamical phase transition

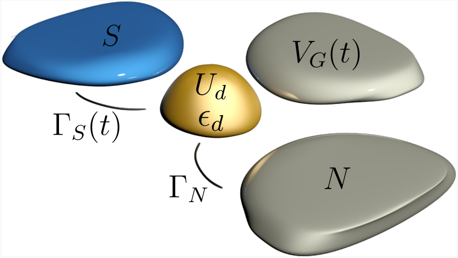
QUENCH-DRIVEN PHASE TRANSITION



K. Wrześniewski et al, (2020) /project in progress/.

Challenging task:

QUENCH-DRIVEN PHASE TRANSITION



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Challenging task:

$\Rightarrow$ **transitions between $|\sigma\rangle$ and $|BCS\rangle$ configurations**

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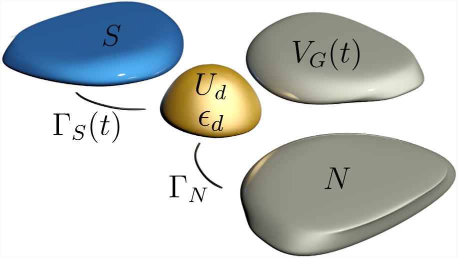

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Challenging task:

⇒ transitions between $|\sigma\rangle$ and $|BCS\rangle$ configurations

⇒ possible signature(s) of critical time(s) ?

SINGLY OCCUPIED VS BCS-TYPE CONFIGURATIONS

Quantum dot proximitized to superconductor can described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left(\Delta_d \, \hat{d}^{\dagger}_{\uparrow} \hat{d}^{\dagger}_{\downarrow} + \text{h.c.} \right)$$

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Eigen-states of this problem are represented by:

$|\uparrow\rangle$ **and** $|\downarrow\rangle$ $\Leftarrow$ **doublet states (spin $\frac{1}{2}$)**

$$\left. \begin{array}{l} u \, |0\rangle - v \, |\uparrow\downarrow\rangle \\ v \, |0\rangle + u \, |\uparrow\downarrow\rangle \end{array} \right\} \Leftarrow$$

singlet states (spin 0)

SINGLY OCCUPIED VS BCS-TYPE CONFIGURATIONS

Quantum dot proximitized to superconductor can described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left(\Delta_d \, \hat{d}^{\dagger}_{\uparrow} \hat{d}^{\dagger}_{\downarrow} + \text{h.c.} \right)$$

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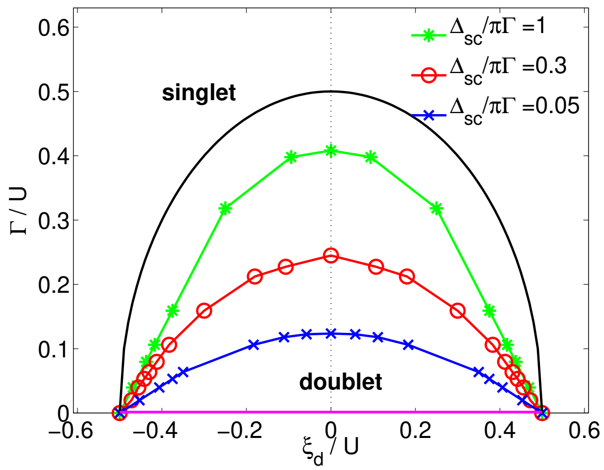
Upon varrying the parameters ε_d, U_d or Γ_S there can be induced quantum phase transition between these doublet/singlet states.

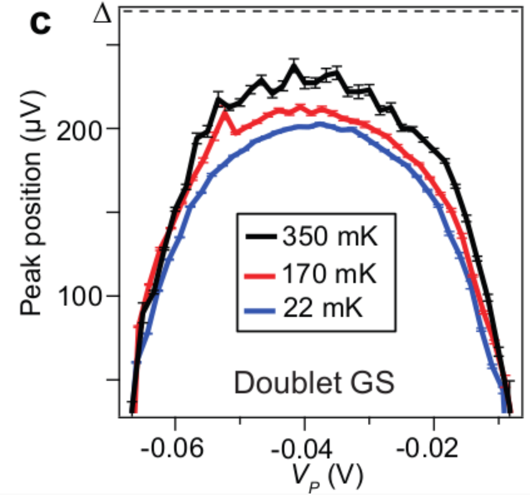
QUANTUM PHASE TRANSITION

Singlet-doublet quantum phase transition: NRG results



J. Bauer, A. Oguri & A.C. Hewson, J. Phys.: Condens. Matter 19, 486211 (2007).

QUANTUM PHASE TRANSITION: EXPERIMENT



J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, Commun. Phys. 3, 125 (2020).

QUENCH DYNAMICS

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Loschmidt amplitude

STATIONARY VS DYNAMICAL PHASE TRANSITION

partition function

$$\mathcal{Z} = \left\langle e^{-\beta \hat{H}} \right\rangle$$

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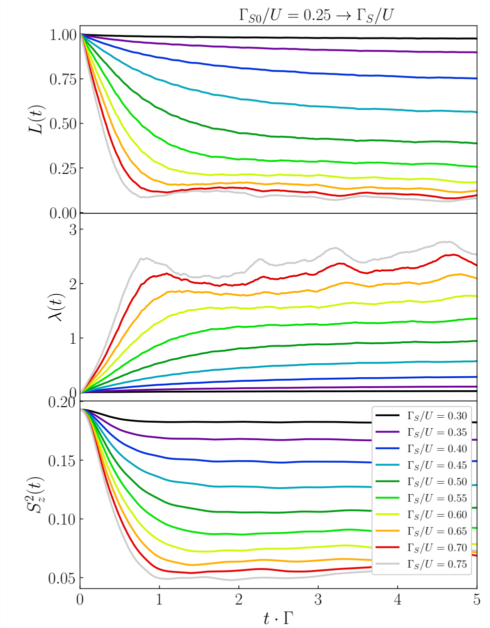
QUENCH FROM $|BCS\rangle$ TO $|\sigma\rangle$



Loschmidt echo

$$L(t) \equiv |\langle\Psi(t)|\Psi(0)\rangle|^2$$

return rate

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squared magnetic moment $\langle S_z^2(t)\rangle$

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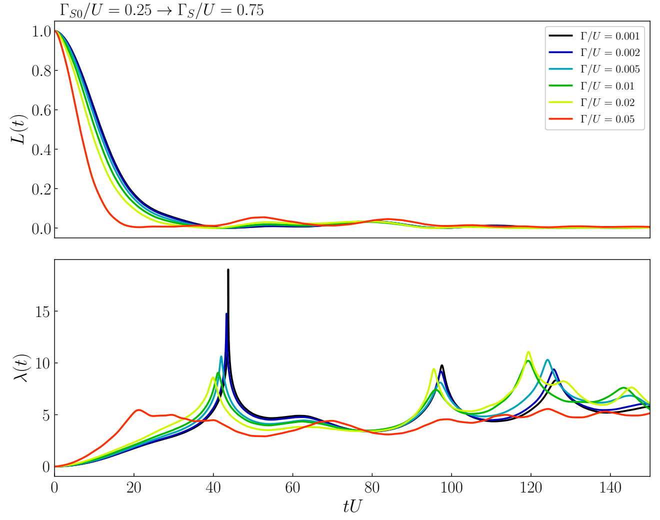
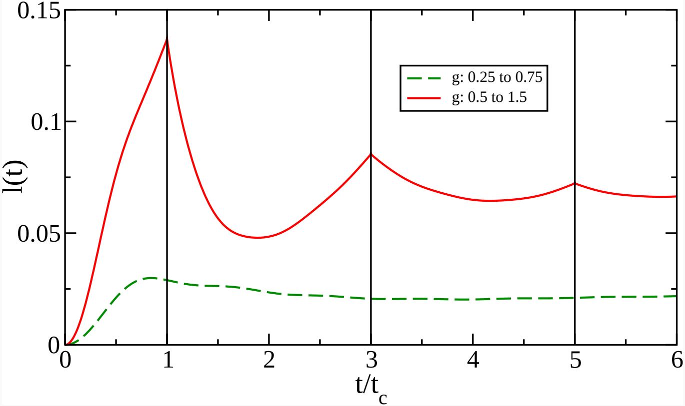


Loschmidt echo $L(t)$ a return rate $\lambda(t)$ induced by the quench of Γ_S

RETURN RATE: ISING MODEL

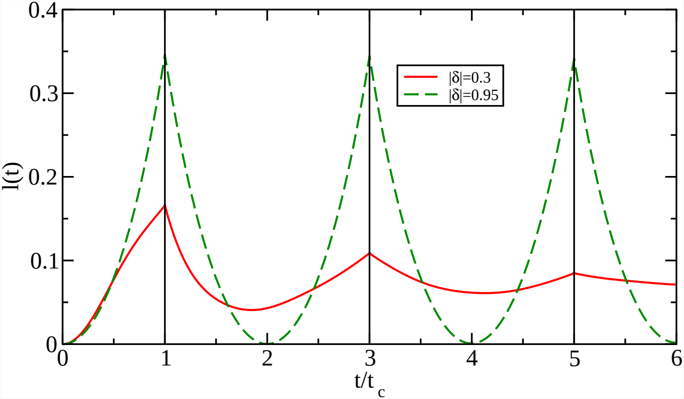


DQPT driven by quench in the Ising model (N. Sedlmayr, 2019)

solid red line - across a phase transition

dashed green line - inside a phase transition

RETURN RATE: SSH MODEL

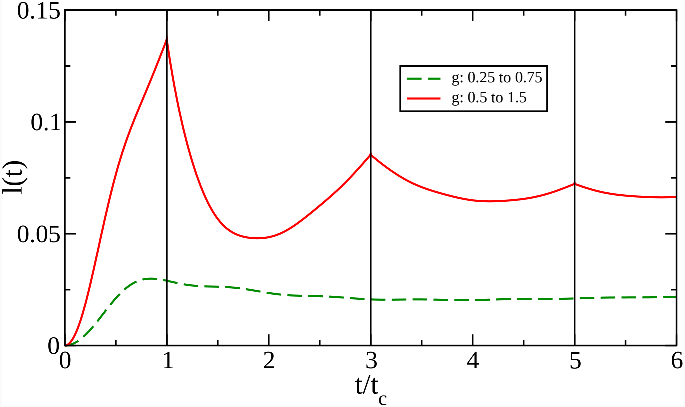


DQPT driven by quench in the SSH model (N. Sedlmayr, 2019)

solid red line $\delta = 0.3$

dashed green line $\delta = 0.95$

RETURN RATE: ISING MODEL



DQPT driven by quench in the Ising model (N. Sedlmayr, 2019)

solid red line - across a phase transition

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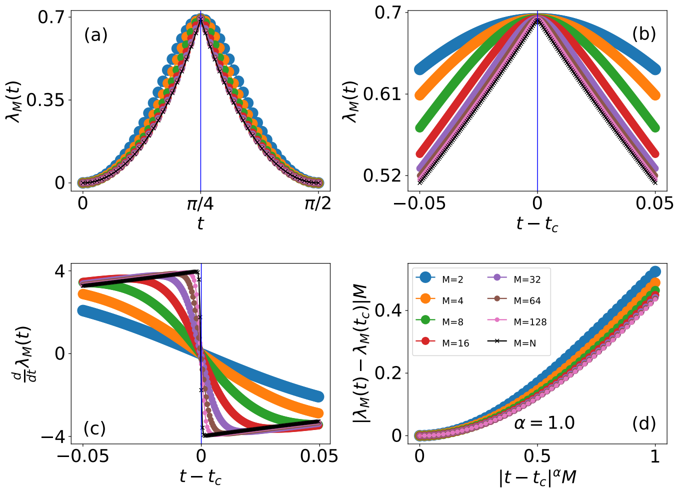
ISING MODEL: FINITE SIZE EFFECTS

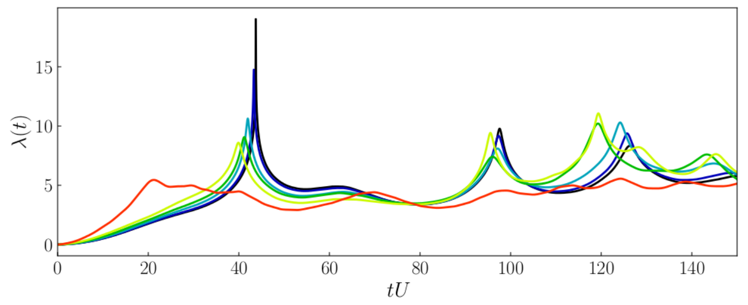


"Local measures of dynamical quantum phase transitions"

J.C. Halimeh, D. Trapin, M. Damme & M. Heyl, arXiv:2010.07307 (2020).

DYNAMICAL QUANTUM PHASE TRANSITION

DQPT driven in N-QD-S by quench across the singlet-doublet transition



Issues to be checked/clarified:

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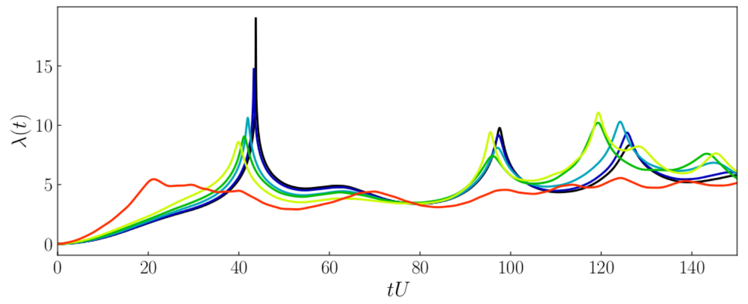

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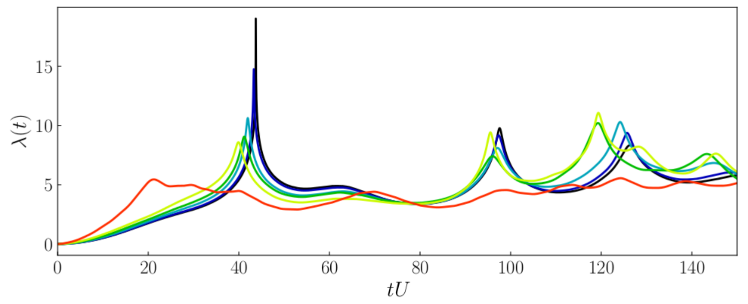

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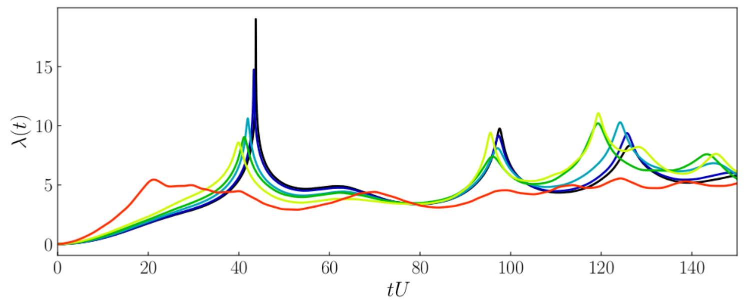

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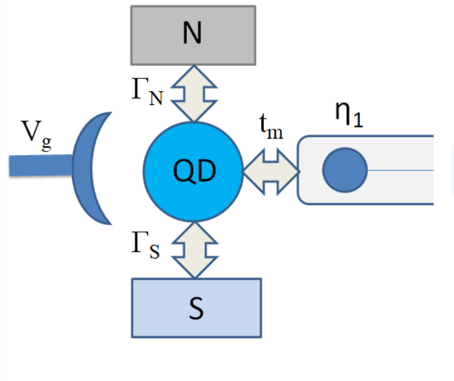
- **how can we observe it ? .... (detectable features)**

Dynamics of Majorana modes

DYNAMICS OF TOPOLOGICAL SUPERCONDUCTORS



J. Barański et al (2020)

/work in progress/.

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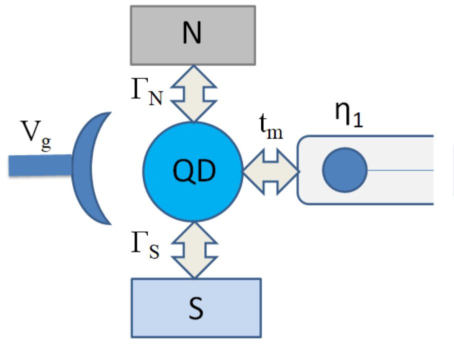

J. Barański et al (2020)

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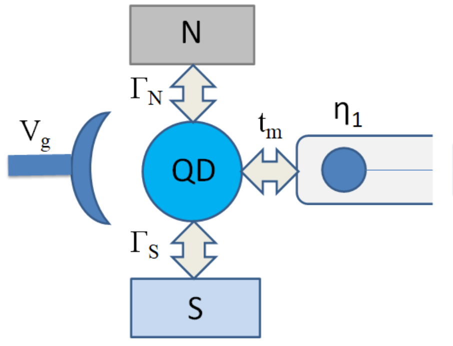

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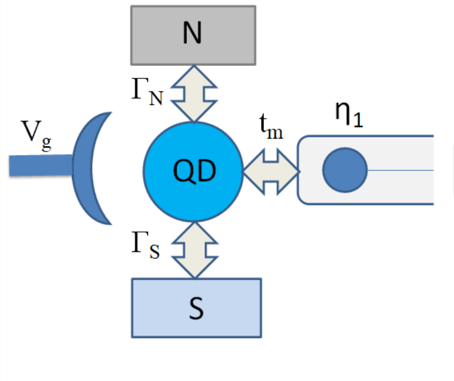

J. Barański et al (2020)

/work in progress/.

Issues to be addressed:

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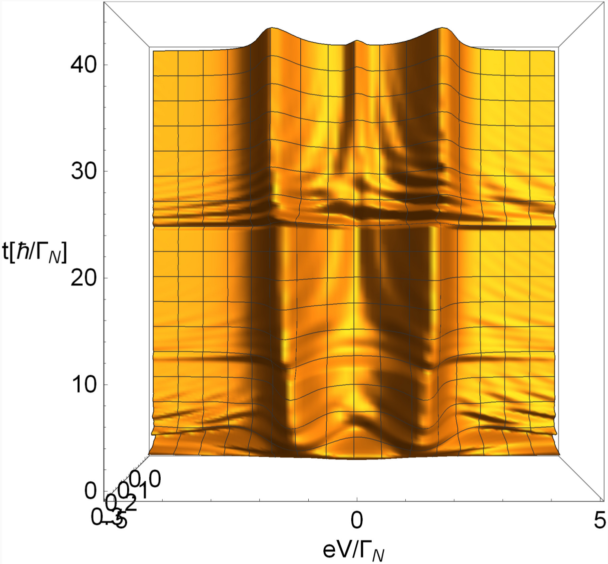
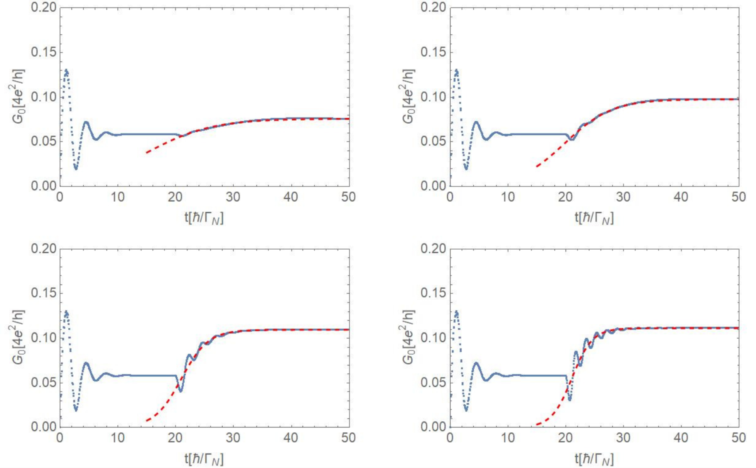
- time-resoveld conductance.

TIME-RESOLVED MAJORANA LEAKAGE



The differential Andreev conductance vs bias voltage V and time

TIME-RESOLVED ZERO BIAS CONDUCTANCE



The zero-bias differential conductivity obtained for $\Gamma_S = 3\Gamma_N$ and $\epsilon_d = \Gamma_N$, assuming: $t_m = 0.25$ (upper left), 0.5 (upper right), 1 (lower left), 1.5 (lower right) Γ_N. QD is abruptly connected to Majorana mode at time $t = 20\hbar/\Gamma_N$.

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- time-dependent Andreev quasiparticles etc.

⇒ R. Taranko (Lublin), B. Baran (Lublin),

- correlations & dynamical quantum phase transition

⇒ K. Wrześniewski (Poznań), I. Weymann (Poznań),

N. Sedlmayr (Lublin),

- time-dependent leakage of Majorana

⇒ J. Barański (Dęblin), M. Barańska (Dęblin),

- quenches in topological nanowires

⇒ A. (Kobiałka), G. Wlazłowski (Warsaw).