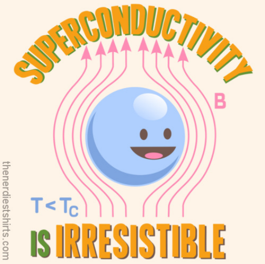
ROLE OF ANDREEV SCATTERING IN BULK SUPERCONDUCTORS & NANOSTRUCTURES

Tadeusz DOMAŃSKI

M. Curie-Skłodowska University, Lublin



IFJ PAN Kraków, 28 May 2019

OUTLINE

- **Quasiparticles in superconductors**

$\Rightarrow$ **particle vs hole dilemma**

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$\Longrightarrow$ **particle vs hole dilemma**

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$\Longrightarrow$ **Andreev bound states**

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$\Longrightarrow$ **Majorana qasiparticles**

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- **Exotics of topological superconductors**

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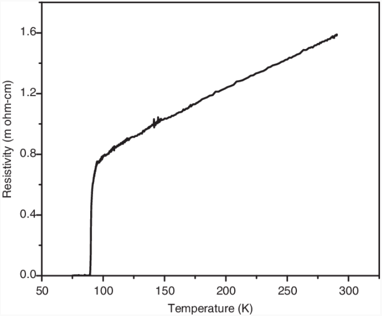
J. Bardeen, A.F. Andreev & E. Majorana

Bulk superconductors

SUPERCONDUCTOR

Perfect conductor



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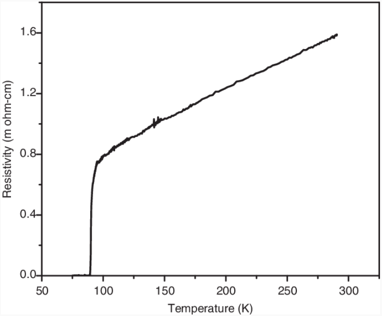

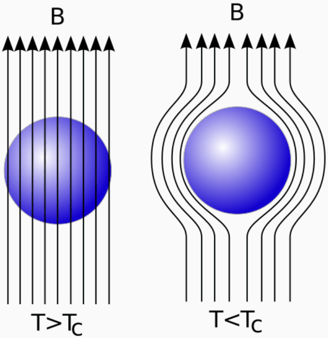

Perfect diamagnet

HALLMARKS OF ELECTRON PAIRING

BCS ground state :

$$|\mathbf{BCS}\rangle = \prod_{k} \left(u_k + v_k \; \hat{c}^{\dagger}_{k\uparrow} \; \hat{c}^{\dagger}_{-k\downarrow} \right) \; |\mathbf{vacuum}\rangle$$

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Effective (Bogoliubov) quasiparticles

$$\begin{aligned} \hat{\gamma}_{k\uparrow} &= u_k \hat{c}_{k\uparrow} + v_k \hat{c}^\dagger_{-k\downarrow} \\ \hat{\gamma}^\dagger_{-k\downarrow} &= -v_k \hat{c}_{k\uparrow} + u_k \hat{c}^\dagger_{-k\downarrow} \end{aligned}$$

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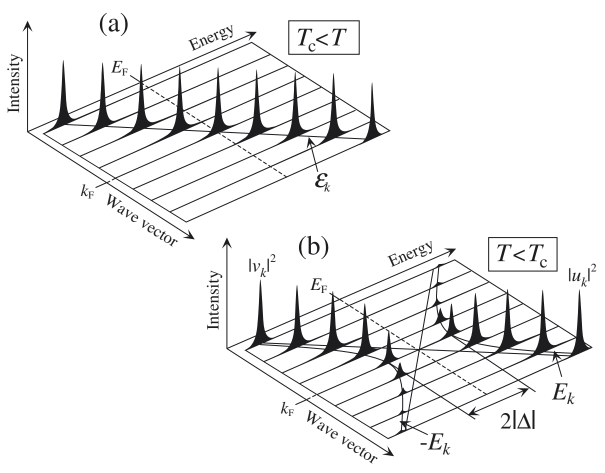
formally due to

$$\begin{aligned} \hat{\gamma}_{k\uparrow} &= u_k \hat{c}_{k\uparrow} + \tilde{v}_k\ \hat{b}_{q=0} \hat{c}^\dagger_{-k\downarrow} \\ \hat{\gamma}^\dagger_{-k\downarrow} &= -\tilde{v}_k\ \hat{b}^\dagger_{q=0}\ \hat{c}_{k\uparrow} + u_k \hat{c}^\dagger_{-k\downarrow} \end{aligned}$$

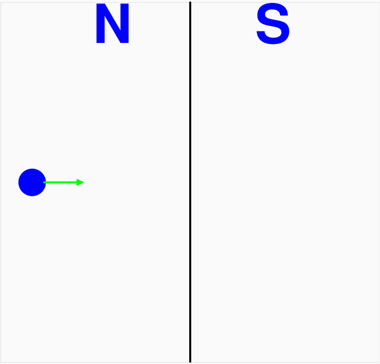
BOGOLIUBOV QUASIPARTICLES

Quasiparticle spectrum of conventional superconductors consists of the Bogoliubov (p/h) branches gaped around E_F



PARTICLE VS HOLE

Let us consider the interface of metal N and superconductor S



where incident electron ...

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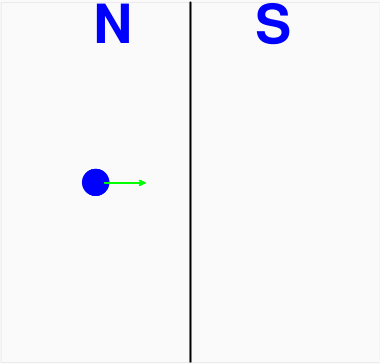

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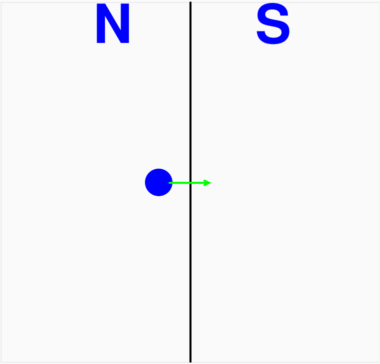

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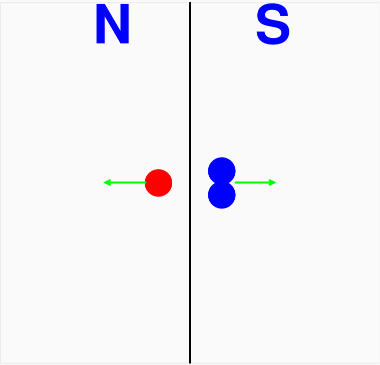

where incident electron is converted into: Cooper pair + hole.

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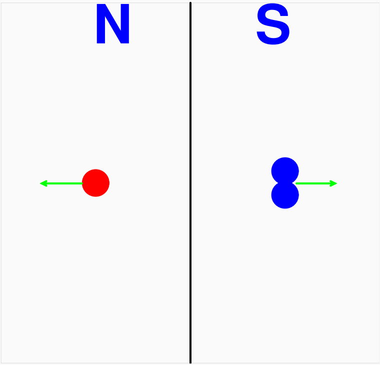

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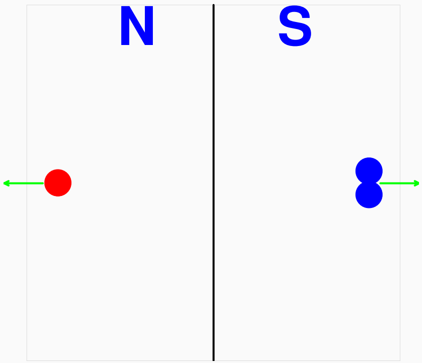

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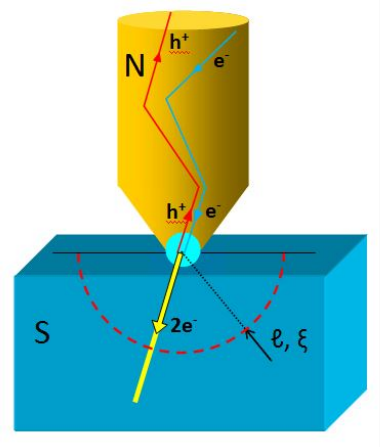
PARTICLE VS HOLE

In superconductors the particle and hole degrees of freedom are mixed via the electron pairing (efficient near the Fermi energy).



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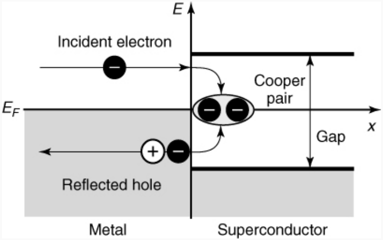

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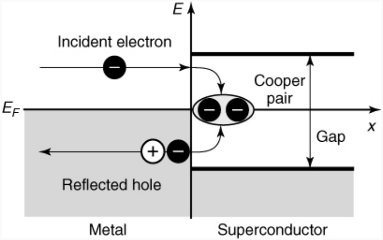
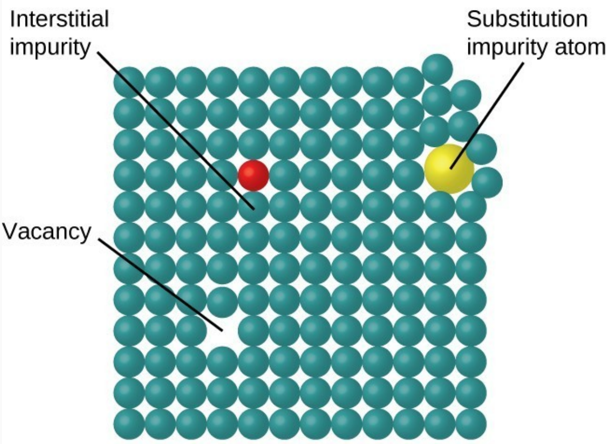


$\Longrightarrow$ **upon injecting an electron to superconductor**

$\Longrightarrow$ **a hole is reflected back (Andreev scattering).**

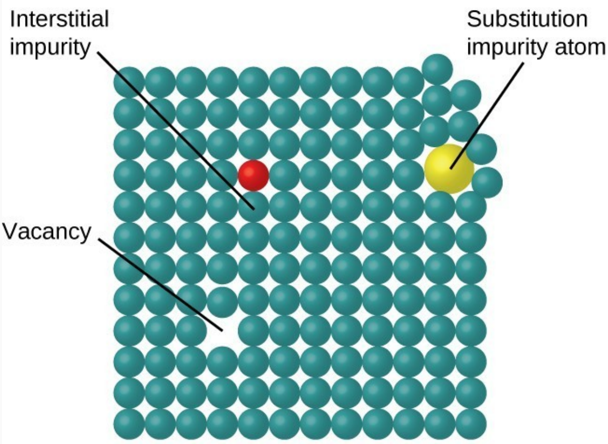
Superconductivity in nanosystems

Various kinds of impurities in solids



IMPURITIES IN SOLIDS

Various kinds of impurities in solids



Are they foes or friends to a superconducting host ?

Impurities/defects:

SPECIFIC EXAMPLES

Impurities/defects:

$\Longrightarrow$ **magnetic atoms** .... **(for instance Fe, Co)**

$\Longrightarrow$ **correlated quantum dots** .... **(Anderson-type)**

$\Longrightarrow$ **molecules** ....**(multi-level or vibrating)**

$\Longrightarrow$ **magnetic islands** .... **(Shiba glasses and/or lattices)**

$\Longrightarrow$ **nanowires** .... **(carbon nanotubes, Fe-chains)**

etc.

existing inside or on surfaces of superconductors.

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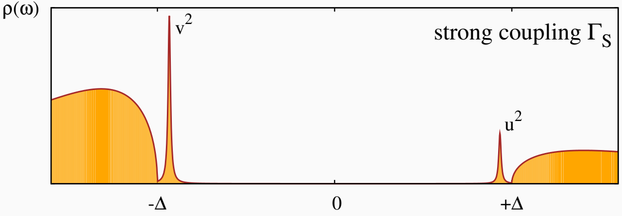
How can we observe the induced electron pairing ?

IN-GAP STATES

Spectrum of a single impurity hybridized with superconductor:



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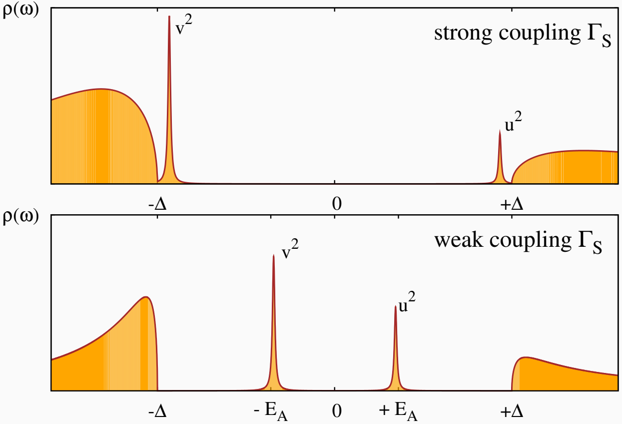

Bound states appearing in the subgap region $E \in \langle -\Delta, \Delta \rangle$

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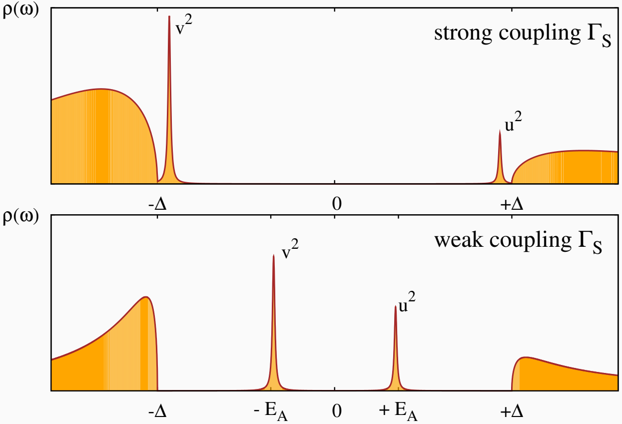
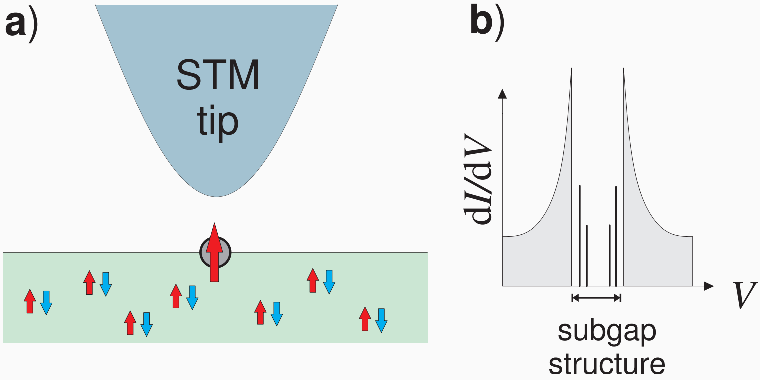


Bound states appearing in the subgap region $E \in \langle -\Delta, \Delta \rangle$

are dubbed Yu-Shiba-Rusinov (or Andreev) quasiparticles.

STM as a tool for probing the spectra of proximitized impurities



PROBING IN-GAP STATES

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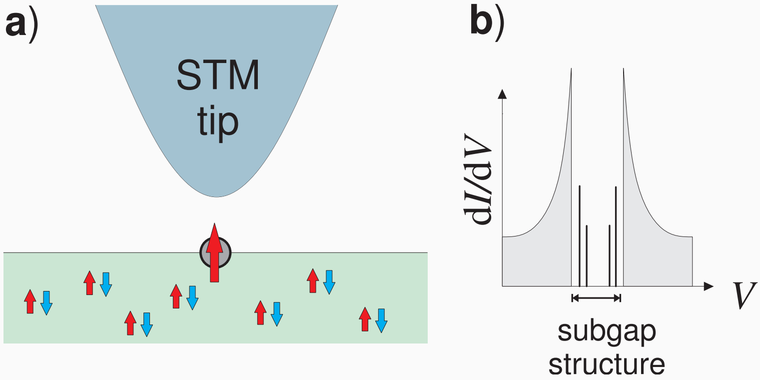
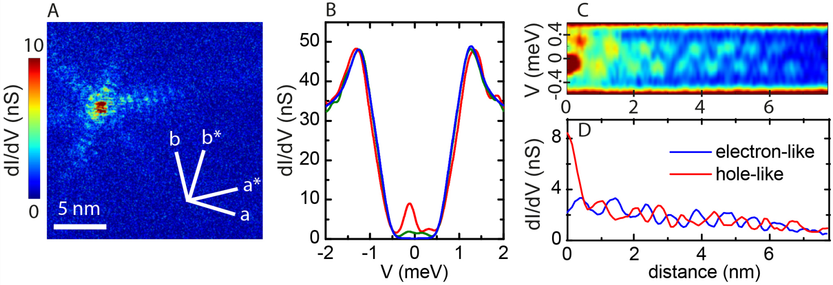


R. Žitko *et al*, Phys. Rev. B 83, 054512 (2011).

TOPOGRAPHY AND SPATIAL EXTENT

Empirical data obtained from STM measurements for $NbSe_2$



a) **bound states extending to 10 nm**

b) **alternating particle-hole oscillations**

G.C. Menard et al., Nature Phys. 11, 1013 (2015).

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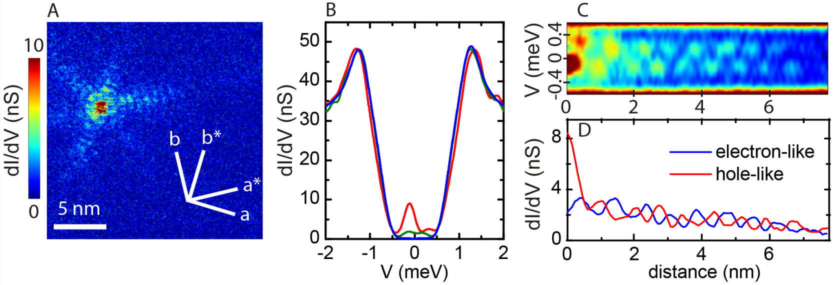

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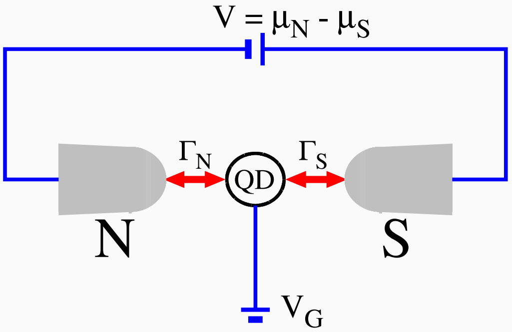
ANDREEV TUNNELING SPECTROSCOPY

For probing the subgap states one can measure the conductance of tunneling current through the quantum dot (QD) coupled between the normal (N) and superconducting (S) electrodes



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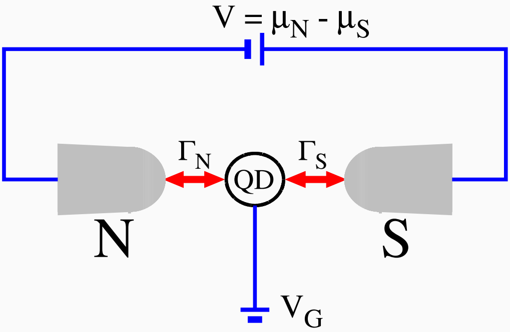

This is a particular realization of the single-electron-transistor.

CORRELATIONS VS PAIRING

The proximitized quantum dot can described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^\dagger_\sigma \, \hat{d}_\sigma \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left(\Delta_d \, \hat{d}^\dagger_\uparrow \hat{d}^\dagger_\downarrow + \text{h.c.} \right)$$

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Eigen-states of this problem are represented by:

$|\uparrow\rangle$ **and** $|\downarrow\rangle$ $\Leftarrow$ **doublet states (spin $\frac{1}{2}$)**

$$\left. \begin{array}{l} u\,|0\rangle - v\,|\uparrow\downarrow\rangle \\ v\,|0\rangle + u\,|\uparrow\downarrow\rangle \end{array} \right\} \Leftarrow$$ **singlet states (spin 0)**

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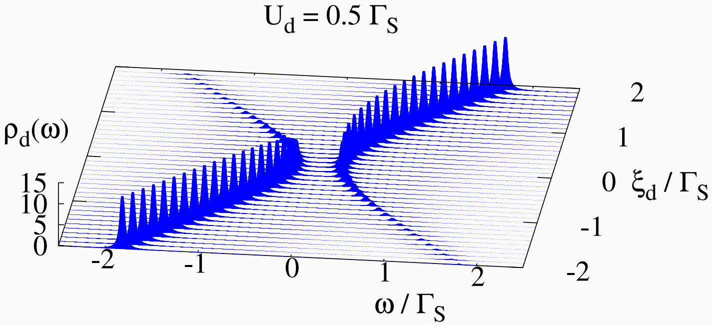
Upon varrying the parameters ε_d, U_d or Γ_S there can be induced quantum phase transition between these doublet/singlet states.

Subgap spectrum of the correlated QD $\xi_d = \varepsilon_d + \frac{1}{2}U_d$



QUANTUM PHASE TRANSITION

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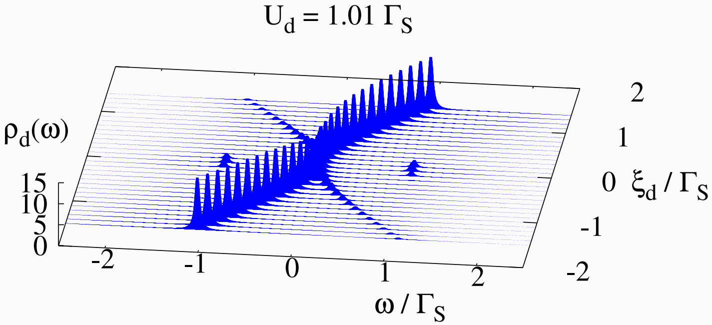
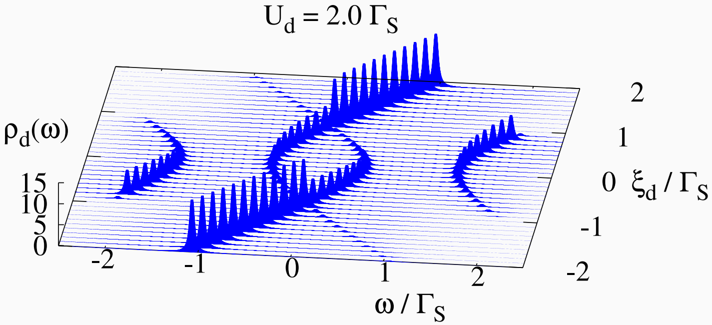
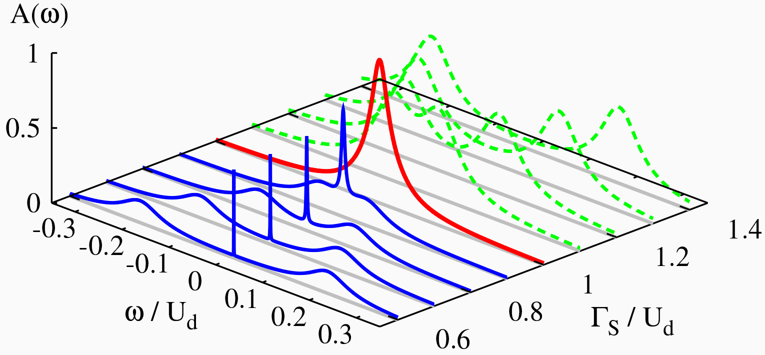


Subgap spectrum of the correlated QD

$\xi_d = \varepsilon_d + \frac{1}{2}U_d$



Kondo effect near the quantum phase transition



SUBGAP KONDO EFFECT

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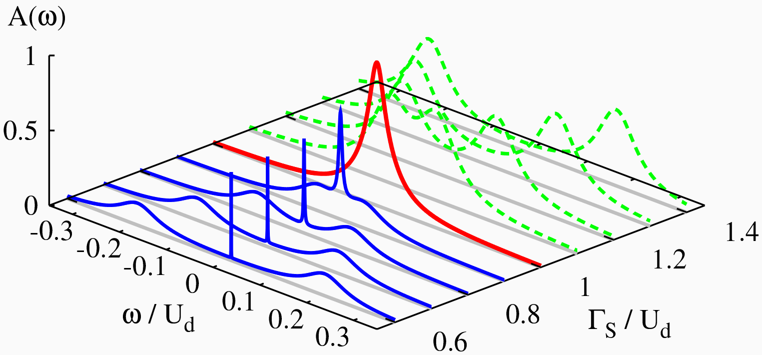
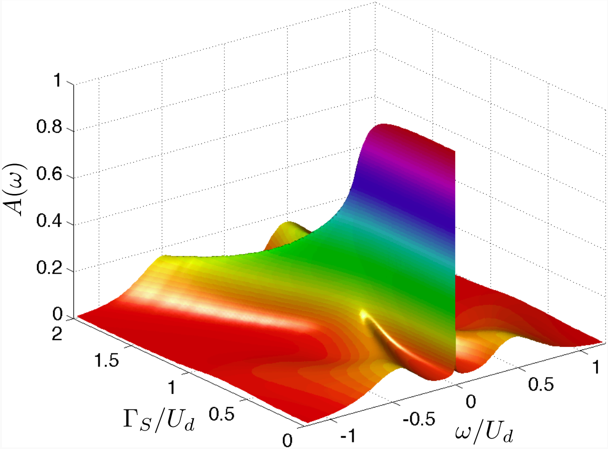


T. Domański *et al*, Scientific Reports 6, 23336 (2016).

SUBGAP KONDO EFFECT

Constructive influence of the induced pairing on the Kondo state



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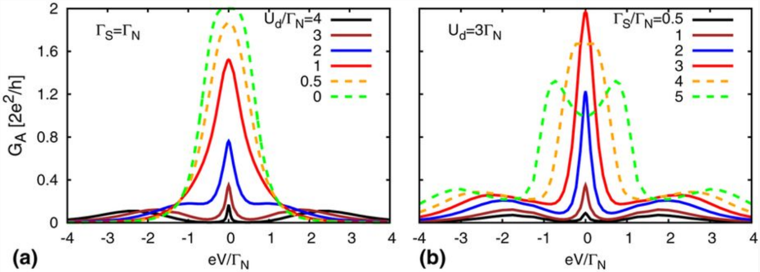
SUBGAP KONDO EFFECT

Physical observability in the Andreev differential conductance



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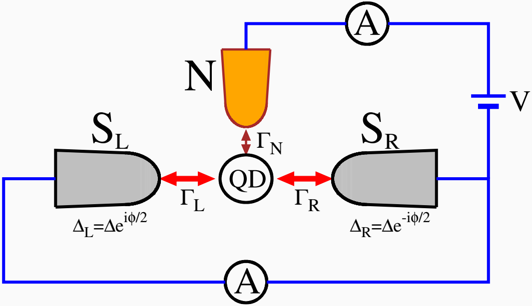
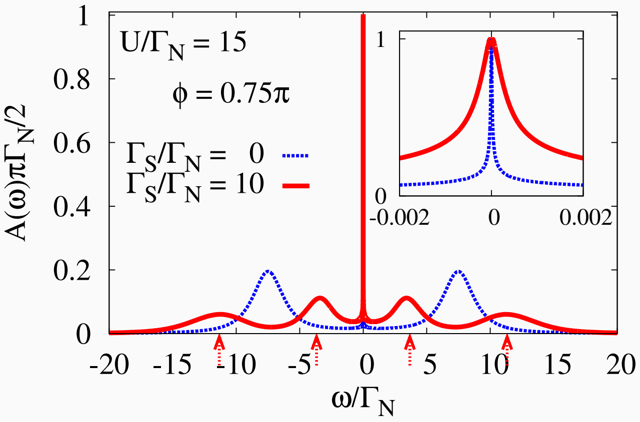
Zero-pi transition

Quantum dot embedded in Josephson & Andreev circuits.



T. Domański ... V. Janiš & T. Novotný, Phys. Rev. B 95, 045104 (2017).

Spectrum of the half-filled quantum dot



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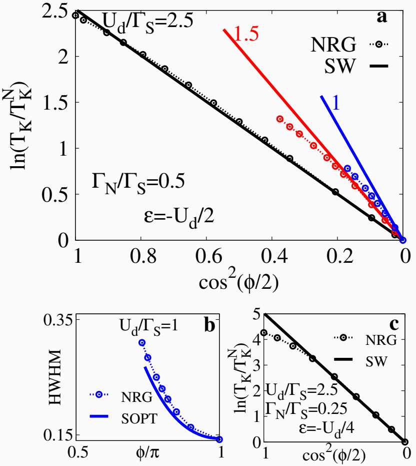
Scaling of the Kondo temperarture T_K



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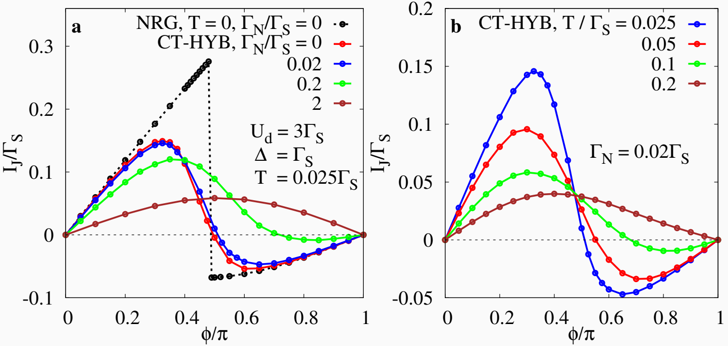
Reversal of Josephson current at 'zero-pi' transition.



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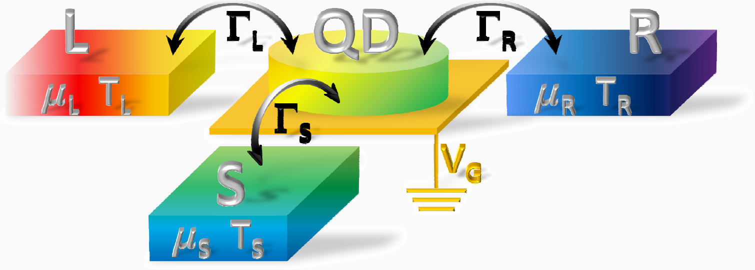
In 3-terminal junctions there can occur:



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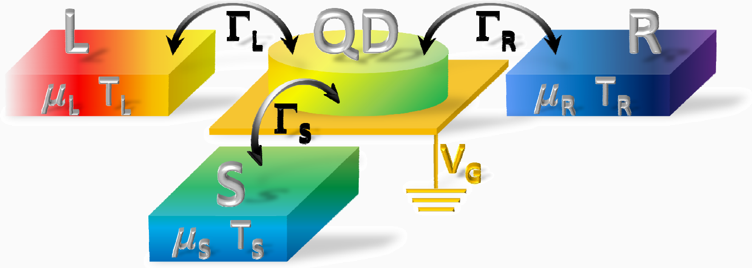
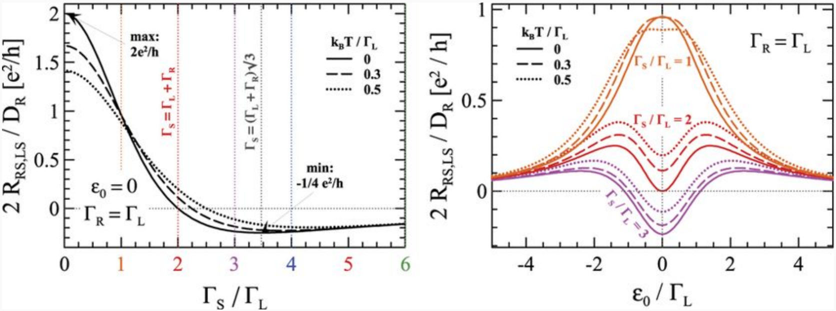


either the <u>direct</u> or <u>crossed</u> Andreev reflections.

DIRECT VS CROSSED ANDREEV SCATTERING

Physical consequences: selective charge/heat transfer



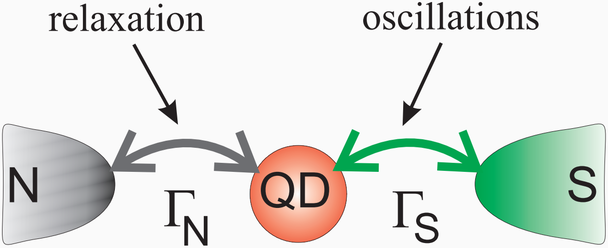
G. Michałek, T. Domański, B.R. Bułka, K.I. Wysokiński, Sci. Rep. 5, 14572 (2015).

G. Michałek, M. Urbaniak, B.R. Bułka, T. Domański, K.I. Wysokiński,

Phys. Rev. B 93, 235440 (2016).

Characteristic temporal scales

Let's consider abrupt coupling of QD to external leads



R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

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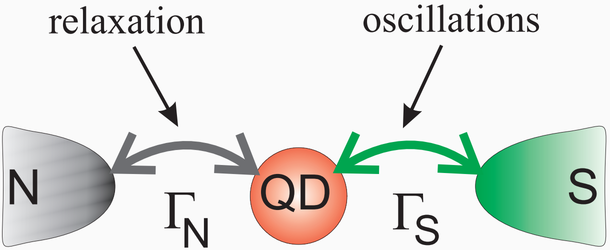

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Important questions:

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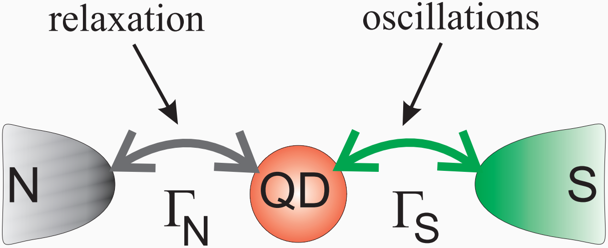

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Important questions:

- **how much time does it take to form the in-gap states?**

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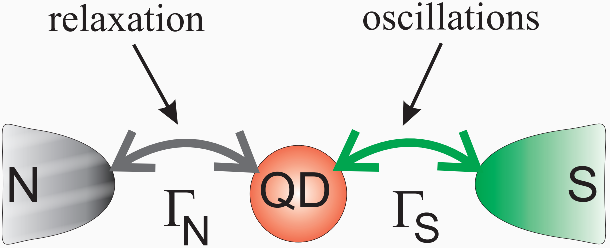

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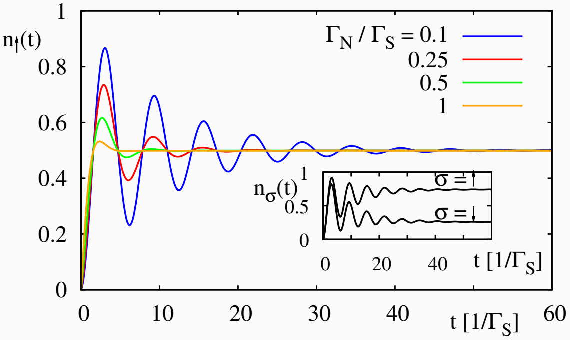
Important questions:

- how much time does it take to form the in-gap states?
- are there some characteristic time-scales?

RELAXATION VS QUANTUM OSCILLATIONS

Time-dependent charge of the quantum dot

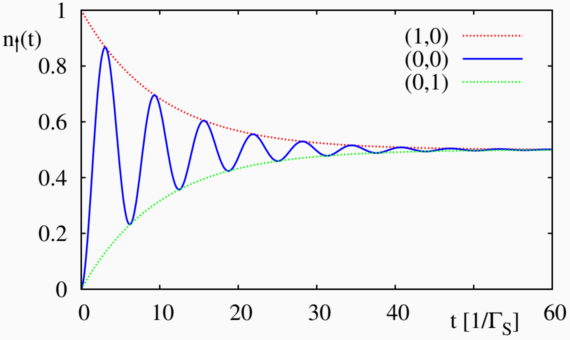


- relaxation time is proportional to $1/\Gamma_N$
- oscillations depend on energies of in-gap states

R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

RELAXATION VS QUANTUM OSCILLATIONS

Time-dependent charge of the quantum dot



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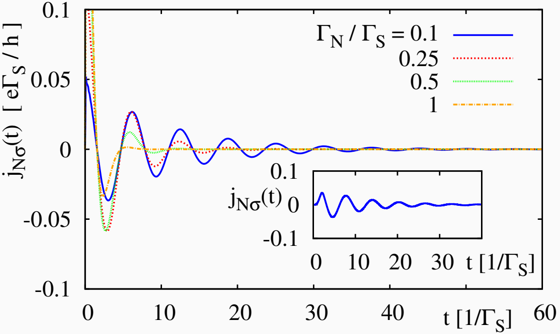
R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

Time-dependent charge current

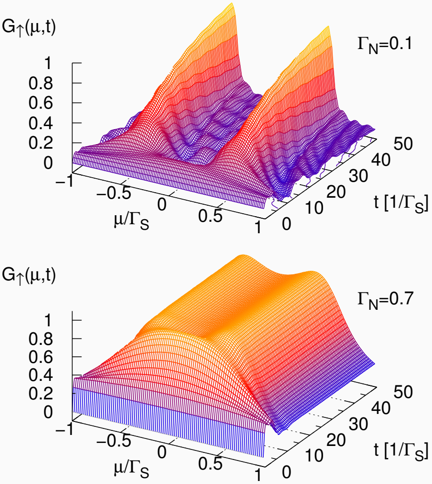


- relaxation time is proportional to $1/\Gamma_N$
- oscillations depend on energies of in-gap states

R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

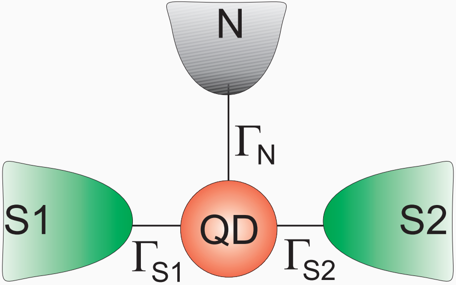
EXPERIMENTALLY ACCESSIBLE QUANTITIES

QD level $\varepsilon = 0$



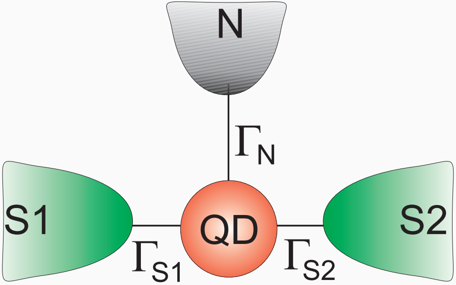
Subgap tunneling conductance $G_\sigma = \frac{\partial I_\sigma}{\partial t}$ vs time (t) and voltage (μ)

PHASE-CONTROLLED TRANSIENT EFFECTS



R. Taranko, T. Kwapiński and T. Domański Phys. Rev. B 99, 165419 (2019).

PHASE-CONTROLLED TRANSIENT EFFECTS

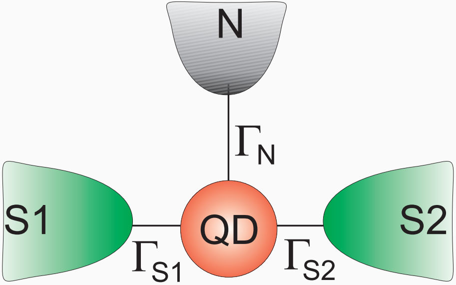


R. Taranko, T. Kwapiński and T. Domański Phys. Rev. B 99, 165419 (2019).

Physical issues:

- **phase-controlled emergence of in-gap states,**

PHASE-CONTROLLED TRANSIENT EFFECTS

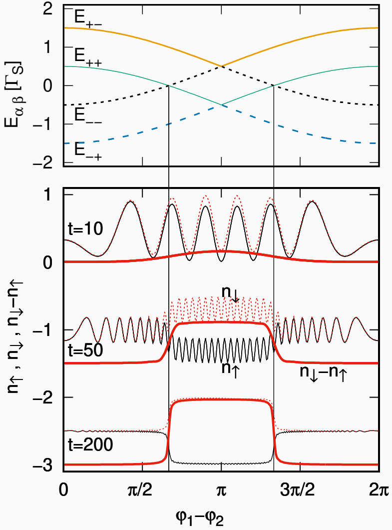


R. Taranko, T. Kwapiński and T. Domański Phys. Rev. B 99, 165419 (2019).

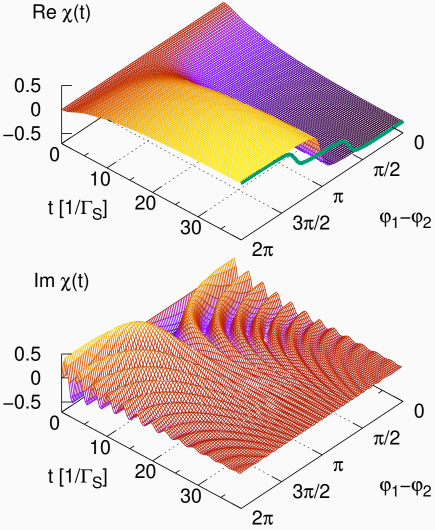
Physical issues:

- **phase-controlled emergence of in-gap states,**
- **dynamics of the $0-\pi$ transition.**

Quasienergies and time-dependent $n_\sigma(t)$ of QD



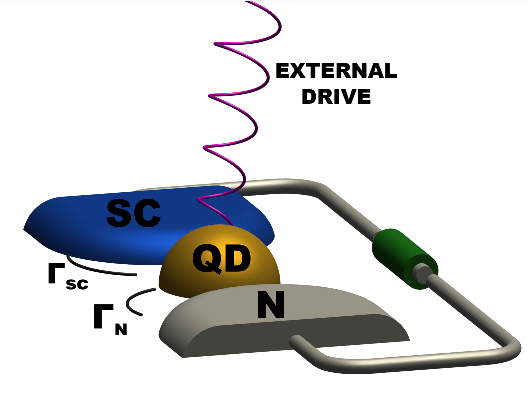
Phase & time dependence of the order parameter



R. Taranko, T. Kwapiński and T. Domański Phys. Rev. B 99, 165419 (2019).

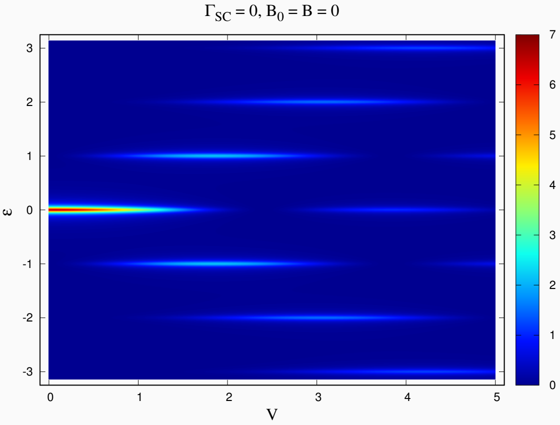
Floquet description of bound states

Quantum impurity with periodically oscillating energy level



$$\varepsilon(t) = \varepsilon_0 + V \times \cos(\omega t)$$

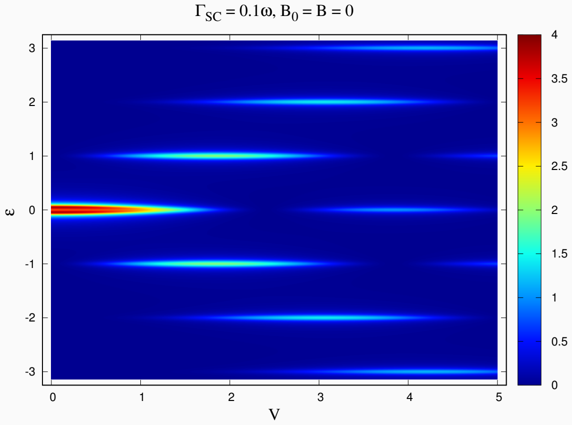
Floquet spectrum averaged over a period $T = 2\pi/\omega$



$\Gamma_S = 0.0$

B. Baran and T. Domański, arXiv:1903.10303 (2019).

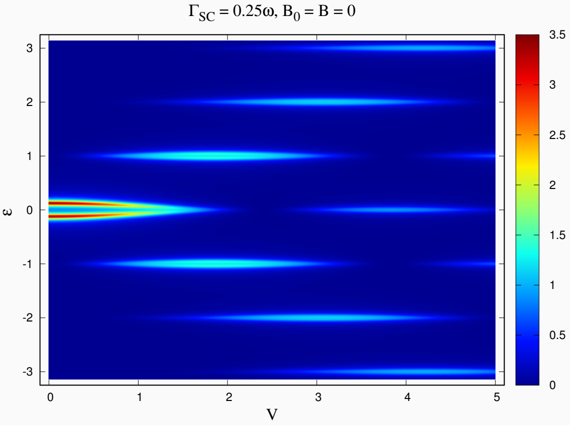
Floquet spectrum averaged over a period $T = 2\pi/\omega$



$\Gamma_S = 0.1\omega$

B. Baran and T. Domański, arXiv:1903.10303 (2019).

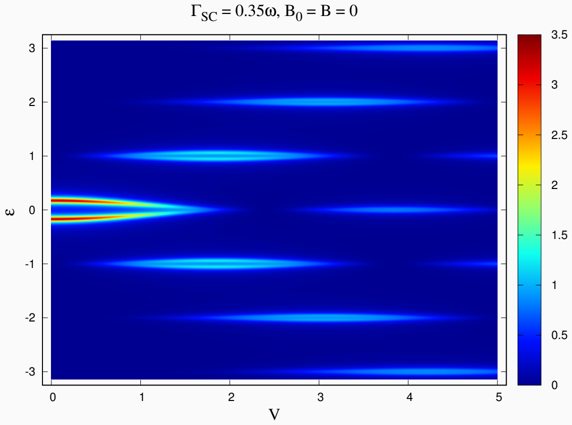
Floquet spectrum averaged over a period $T = 2\pi/\omega$



$\Gamma_S = 0.25\omega$

B. Baran and T. Domański, arXiv:1903.10303 (2019).

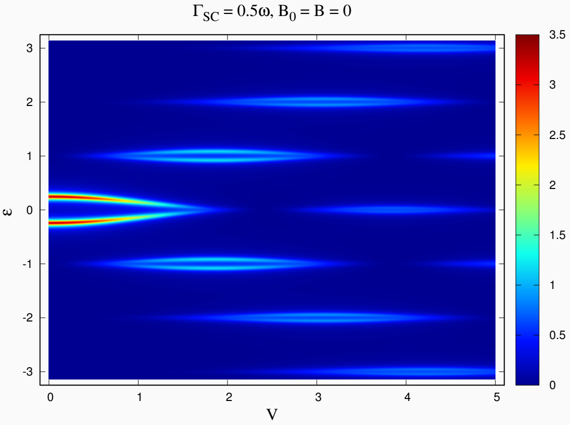
Floquet spectrum averaged over a period $T = 2\pi/\omega$



$\Gamma_S = 0.35\omega$

B. Baran and T. Domański, arXiv:1903.10303 (2019).

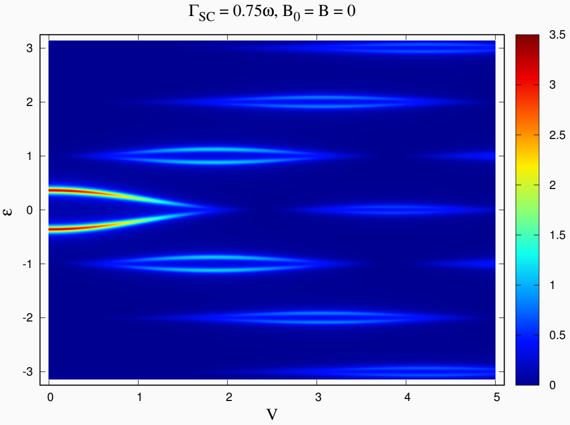
Floquet spectrum averaged over a period $T = 2\pi/\omega$



$\Gamma_S = 0.5\omega$

B. Baran and T. Domański, arXiv:1903.10303 (2019).

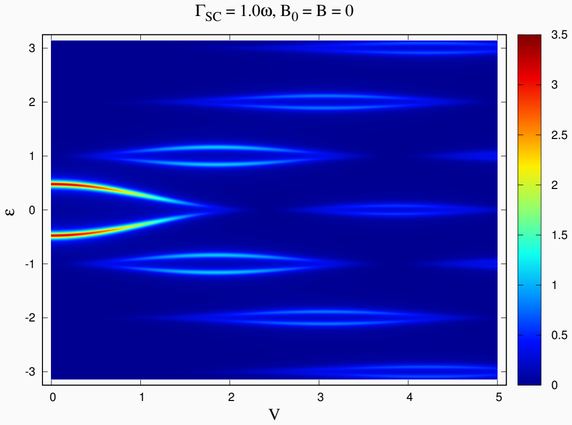
Floquet spectrum averaged over a period $T = 2\pi/\omega$



$\Gamma_S = 0.75\omega$

B. Baran and T. Domański, arXiv:1903.10303 (2019).

Floquet spectrum averaged over a period $T = 2\pi/\omega$



$\Gamma_S = 1.0\omega$

B. Baran and T. Domański, arXiv:1903.10303 (2019).

PERIODICALLY DRIVEN IMPURITY: CONCLUSIONS

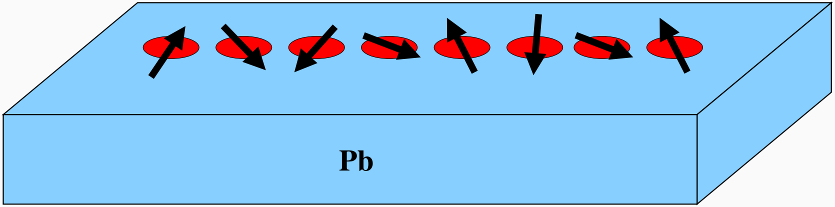
Bound states of an "oscillating" quantum dot:

- **are characterized by a series of side-peaks,**
- **of spectral weights dependent on amplitude**
- **and internal splittings dependent on Γ_S.**

Topological superconductors

MAGNETIC CHAINS IN SUPERCONDUCTORS

Nanochain of magnetic impurities embedded in superconductor:



T.-P. Choy, J.M. Edge, A.R. Akhmerov, and C.W.J. Beenakker,

Phys. Rev. B 84, 195442 (2011).

MAGNETIC CHAINS IN SUPERCONDUCTORS

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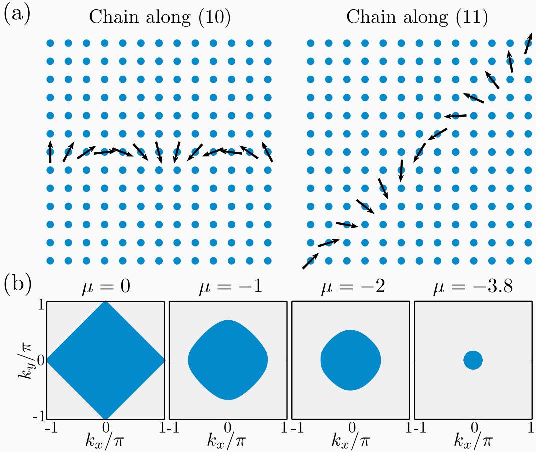

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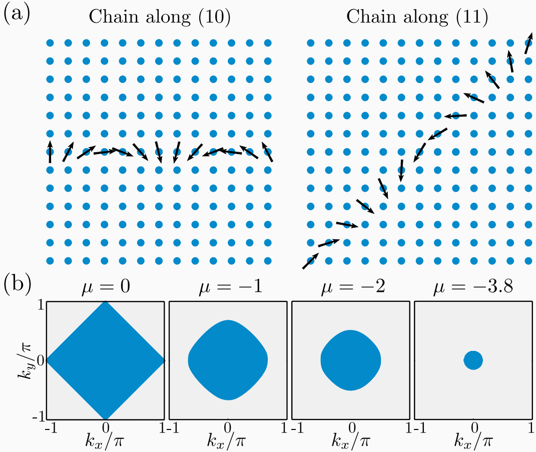

arrange the in-gap bound states into Shiba-band(s).

M.H. Christensen ... J. Paaske, Phys. Rev. B 94, 144509 (2016).

KITAEV CHAIN: PARADIGM FOR MAJORANA QPS

Itinerant 1D fermions with intersite (*p-wave*) pairing

$$\hat{H} = t \sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1} + \text{h.c.} \right) - \mu \sum_i \hat{c}_i^\dagger \hat{c}_i + \Delta \sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1}^\dagger + \text{h.c.} \right)$$

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This toy-model can be recast in the Majorana basis

$$\hat{\gamma}_{j,1} \equiv \frac{1}{\sqrt{2}}\left(\hat{c}_j + \hat{c}_j^\dagger\right)$$

$$\hat{\gamma}_{j,2} \equiv \frac{1}{i\sqrt{2}}\left(\hat{c}_j - \hat{c}_j^\dagger\right)$$

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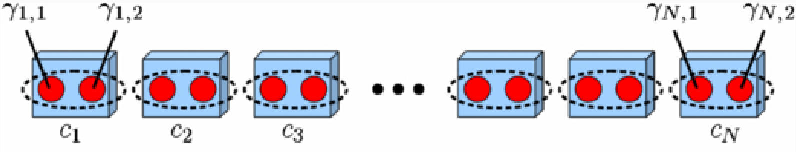
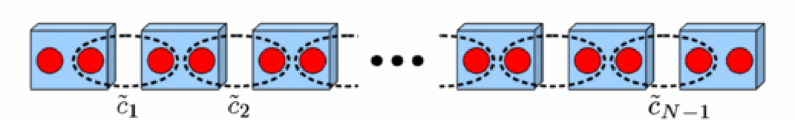


Yu. Kitaev, Phys. Usp. 44, 131 (2001).

KITAEV CHAIN: PARADIGM FOR MAJORANA QPS

In particular, for $\Delta = t$ and when $|\mu|$ is inside the band

two operators $\hat{\gamma}_{1,1}$ and $\hat{\gamma}_{2,N}$ *decouple* from all the rest



inducing the zero-energy modes at the chain edges.

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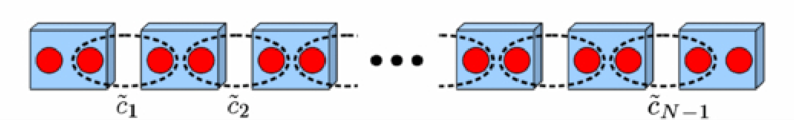


inducing the zero-energy modes at the chain edges.

They can be regarded as *fractions* of non-local fermion

$$\begin{aligned} \hat{c}_{nonlocal} &\equiv \left(\hat{\gamma}_{1,1} + i\hat{\gamma}_{N,2}\right)/\sqrt{2} \\ \hat{c}^{\dagger}_{nonlocal} &\equiv \left(\hat{\gamma}_{1,1} - i\hat{\gamma}_{N,2}\right)/\sqrt{2} \end{aligned}$$

as manifested by a number of unique phenomena.

PROPERTIES OF MAJORANA QPS

- particle = antiparticle $\hat{\gamma}_{i,n}^{\dagger} = \hat{\gamma}_{i,n}$

$\Rightarrow$ **neutral in charge**

$\Rightarrow$ **of zero energy**

- fractional character $\hat{\gamma}_{i,n}^{\dagger}\,\hat{\gamma}_{i,n} = 1/2$

$\Rightarrow$ **half occupied/empty**

- spatially nonlocal

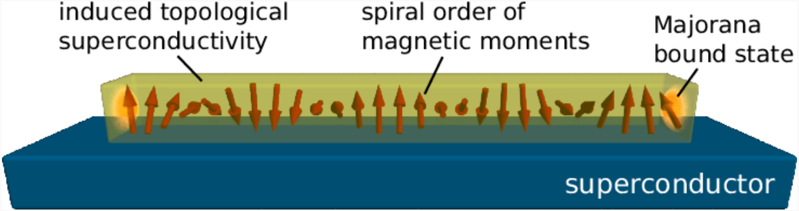
$\Rightarrow$ **exist in pairs near boundaries/defects**

- topologically protected

$\Rightarrow$ **immune to dephasing/decoherence**

FEASIBLE REALIZATION OF KITAEV SCENARIO

Intersite pairing of the same spin electrons can be driven e.g. by the spin-orbit (Rashba) interaction in presence of the external magnetic field, using nanowires proximitized to *s-wave* superconductor.

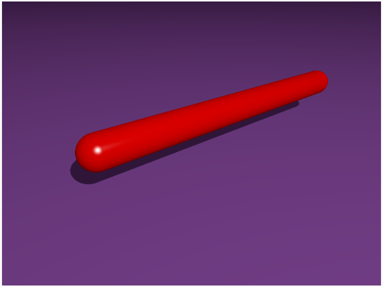


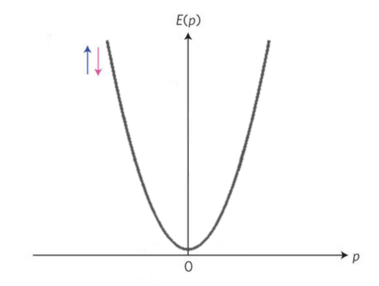
R. Lutchyn, J. Sau, S. Das Sarma, Phys. Rev. Lett. 105, 077001 (2010).

Y. Oreg, G. Refael, F. von Oppen, Phys. Rev. Lett. 105, 177002 (2010).

MAJORANA QPS: UNDERLYING MECHANISM

Nanowire

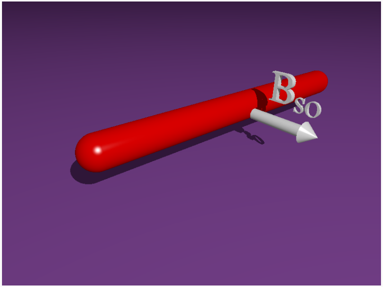


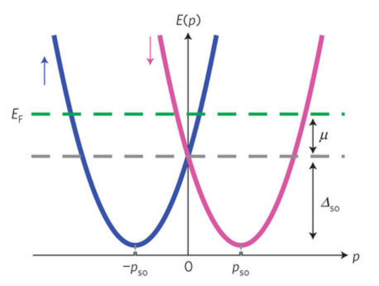


A. Das *et al*, Nature Phys. 8, 887 (2012).

MAJORANA QPS: UNDERLYING MECHANISM

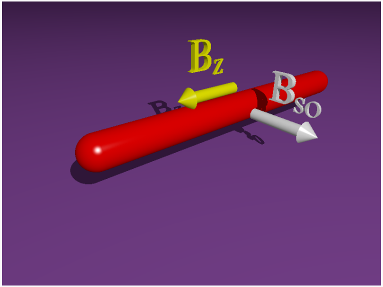
Nanowire + Rashba

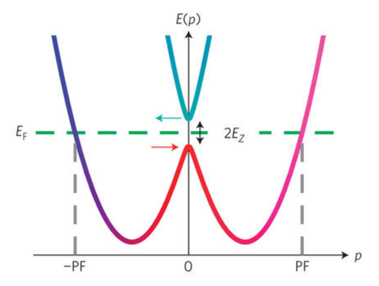




A. Das *et al*, Nature Phys. 8, 887 (2012).

Nanowire + Rashba + magnetic field

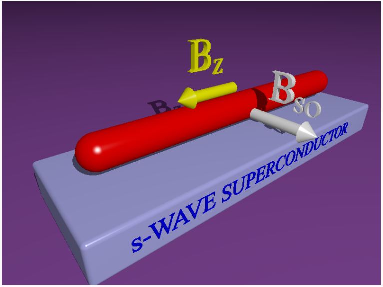


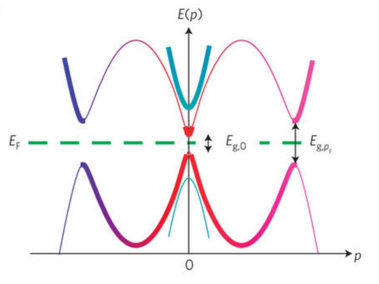


A. Das *et al*, Nature Phys. 8, 887 (2012).

MAJORANA QPS: UNDERLYING MECHANISM

Nanowire + Rashba + magnetic field + superconductor





A. Das *et al*, Nature Phys. 8, 887 (2012).

$B < B_{cr} \longrightarrow$ **trivial superconducting phase**

MAJORANA QPS: UNDERLYING MECHANISM

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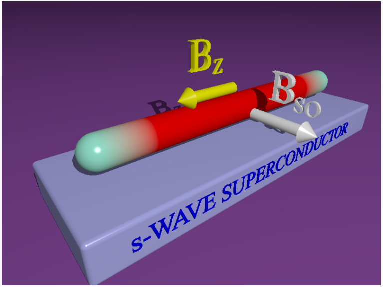
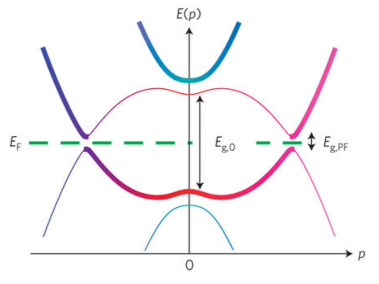
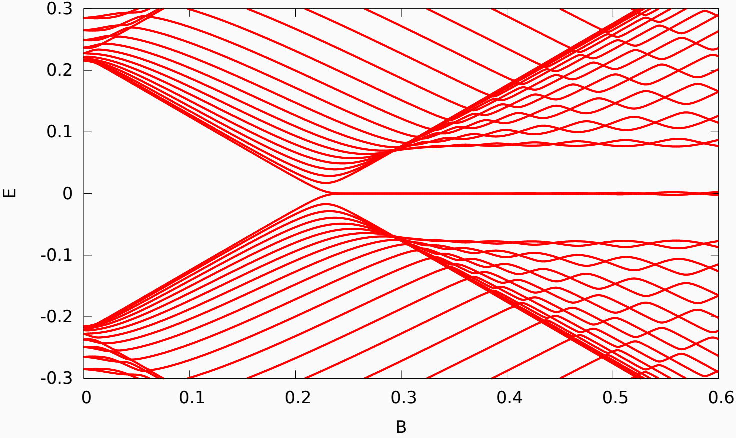




A. Das *et al*, Nature Phys. 8, 887 (2012).

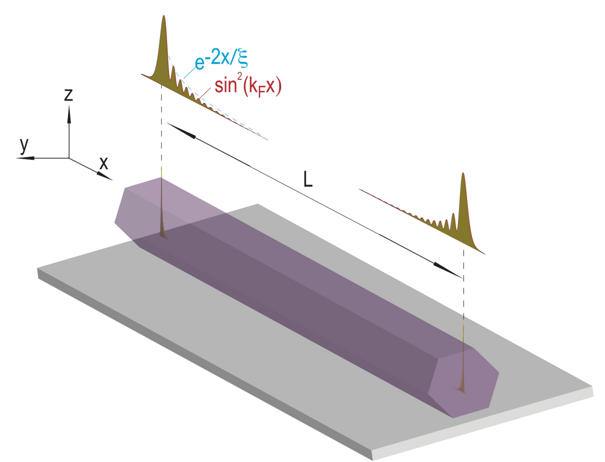
$B > B_{cr}$ $\longrightarrow$ **nontrivial superconducting phase**

Effective quasiparticle states of the Rashba nanowire



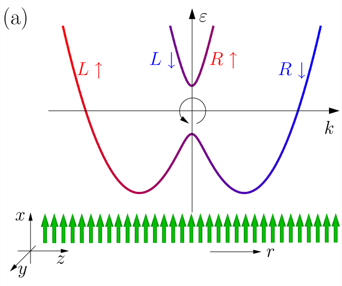
M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

Majorana qps are exponentially localized at the edges



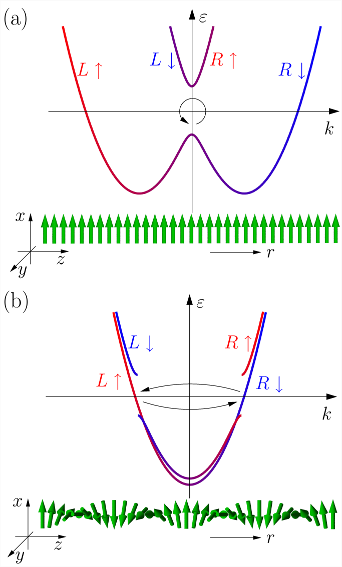
R. Aguado, Riv. Nuovo Cim. 40, 523 (2017).

DIFFERENT SCENARIO FOR MAJORANA QPS IN DIM=1



B. Braunecker *et. al.* Phys. Rev. B **82**, 045127 (2010)

DIFFERENT SCENARIO FOR MAJORANA QPS IN DIM=1



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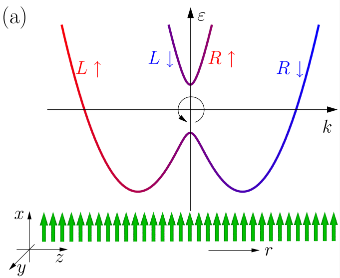
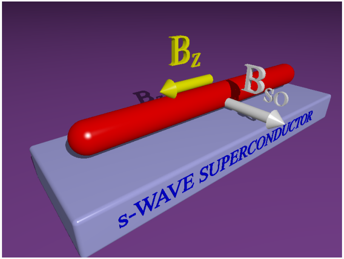
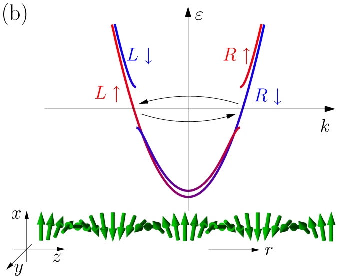
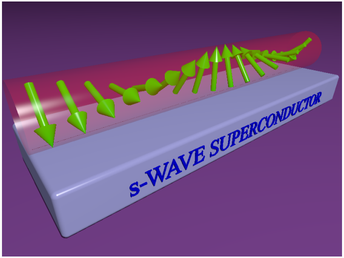








SPIRAL MAGNETIC MOMENTS + PAIRING

$$H = -t\sum_{i\sigma}\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+1,\sigma} - \mu\sum_{i\sigma}\hat{c}^\dagger_{i,\sigma}\hat{c}_{i,\sigma} + J\sum_{i} S_i\cdot\hat{s}_i + \Delta\sum_{i}\hat{c}_{i\uparrow}\hat{c}_{i\downarrow} + \text{H.c.},$$

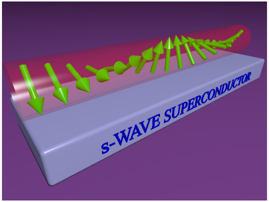
electron spin: $\hat{s}_i = \frac{1}{2}\sum_{\alpha,\beta}\hat{c}^\dagger_{i,\alpha}\sigma_{\alpha\beta}\hat{c}_{i,\beta}$

magnetic moment: $S_i = S\,(\sin\theta_i\cos\phi_i,\ \sin\theta_i\sin\phi_i,\ \cos\theta_i)$

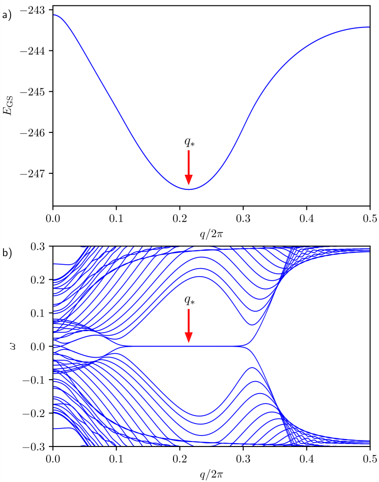
$S \to \infty,\ J \to 0,\ JS$ – finite

This scenario has been studied by:

- M. M. Vazifeh and M. Franz, PRL **111**, 206802 (2013)
- I. Reis *et al.*, PRB **90**, 085124 (2014)
- W. Hu *et al.*, PRB **92**, 115133 (2015)
- T.-P. Choy *et al.*, PRB **84**, 195442 (2011)
- M. H. Christensen *et al.*, PRB **94**, 144509 (2016)
- . . . many other



MAGNETIC CHAINS IN SUPERCONDUCTORS



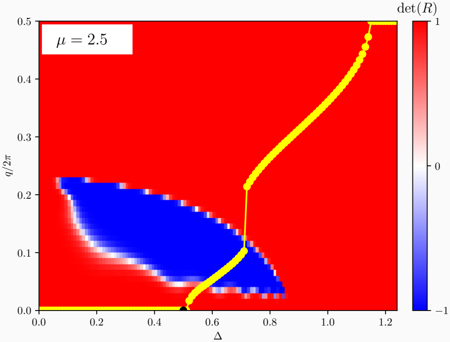
Ground state energy vs the pitch vector q

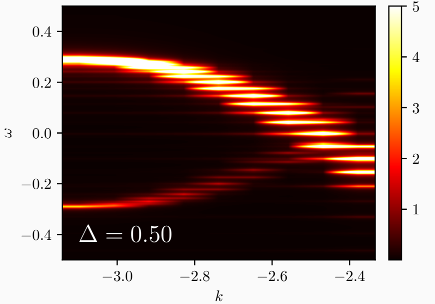
In-gap Shiba states

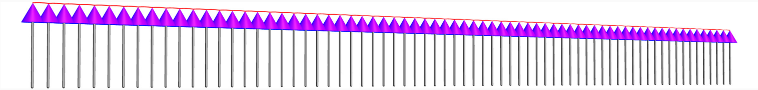
This nanochain self-tunes to its *topological phase* (topofilia)

A. Gorczyca-Goraj, T. Domański & M.M. Maśka, arXiv:1902.1902.06750.

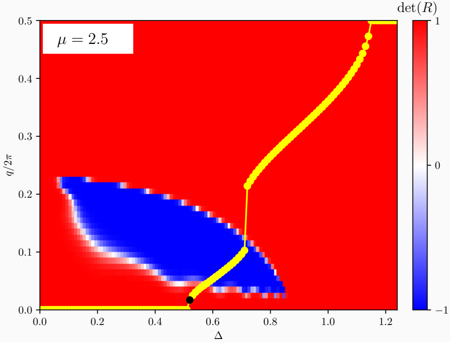
q–DEPENDENCE & SELFORGANISATION

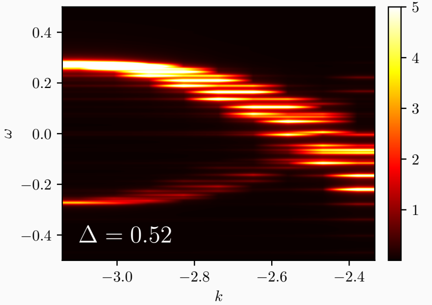


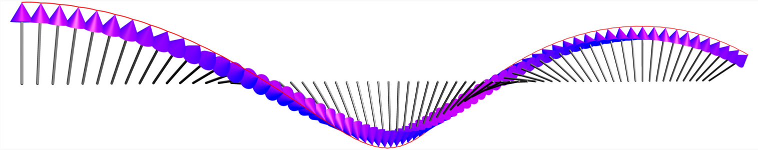




q–DEPENDENCE & SELFORGANISATION







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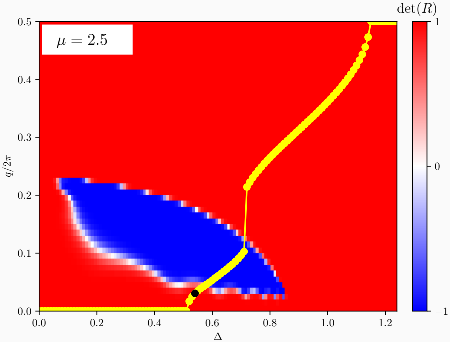

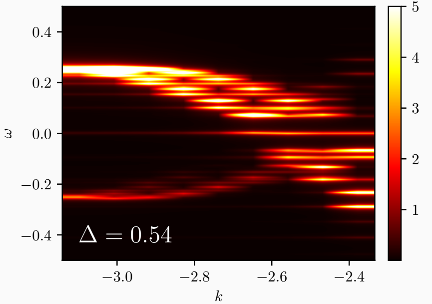

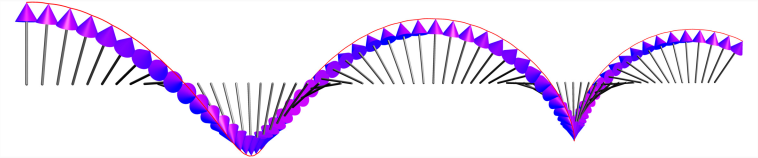

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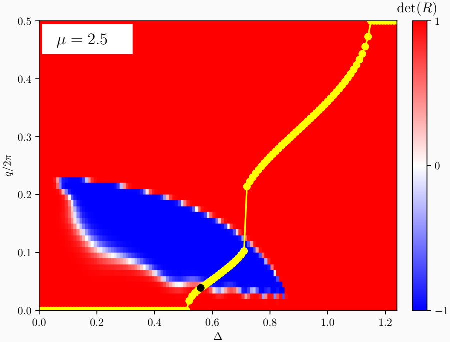

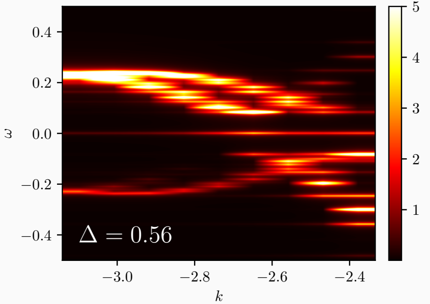

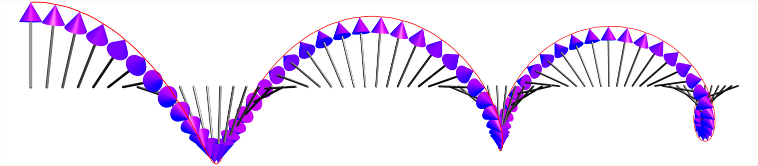

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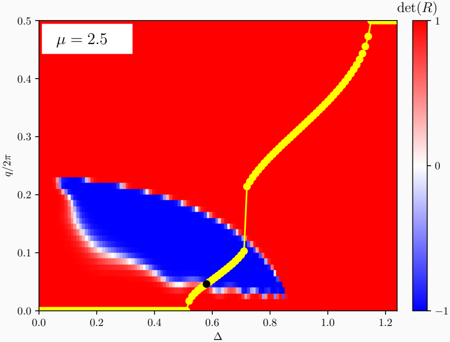

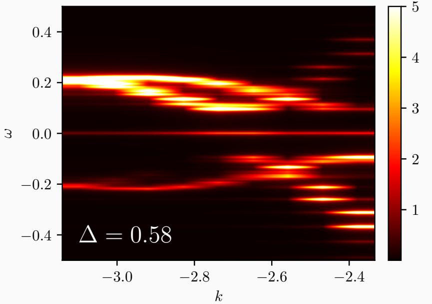

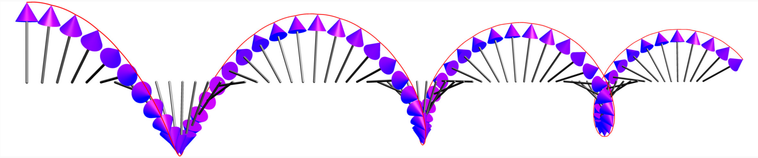

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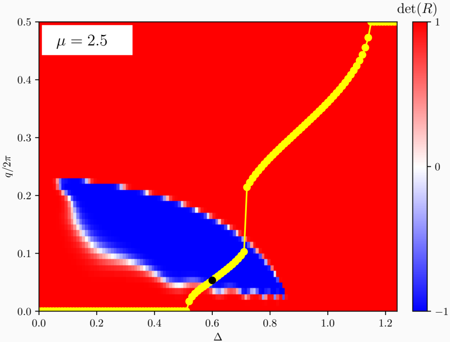

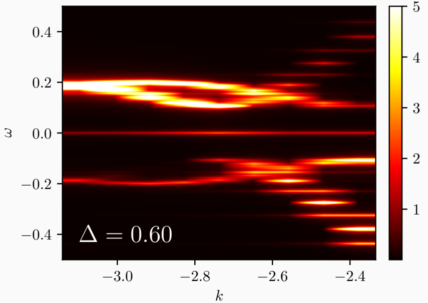

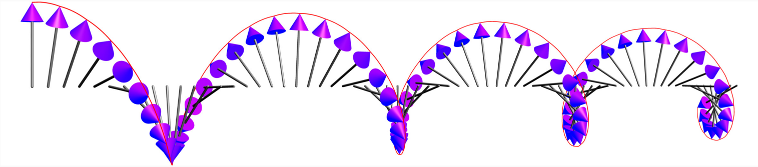

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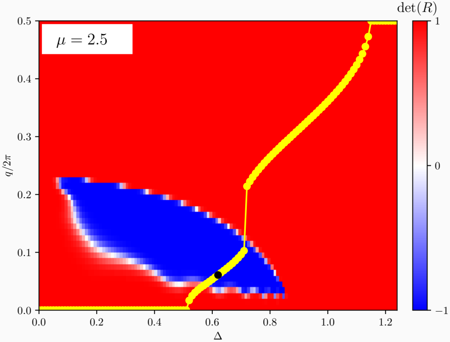

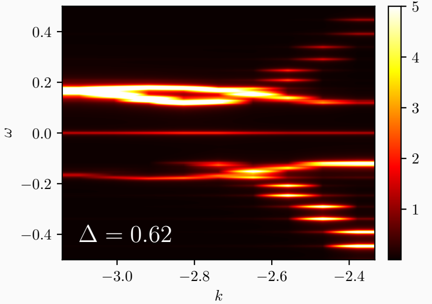

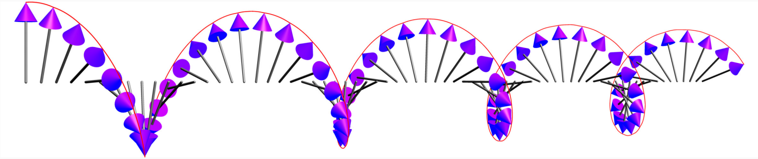

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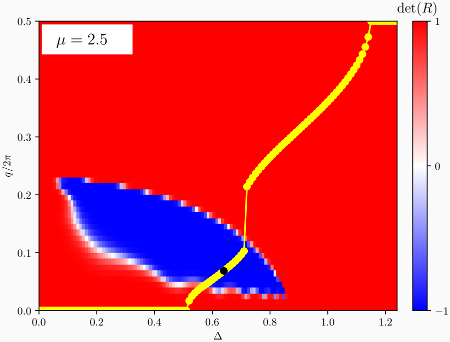

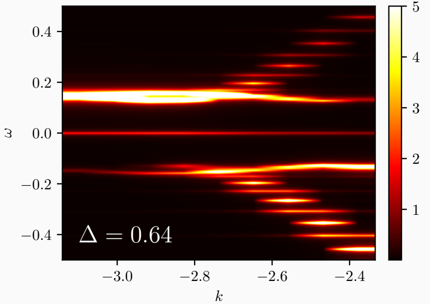

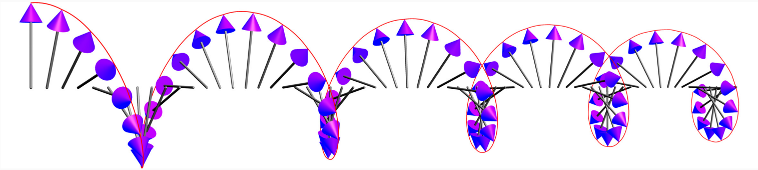

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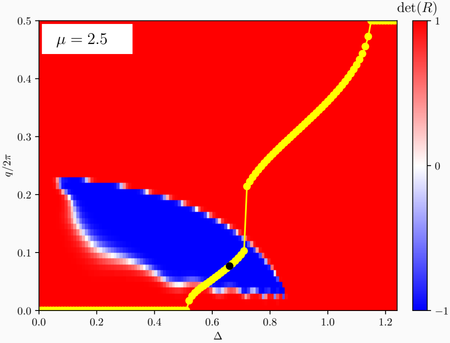

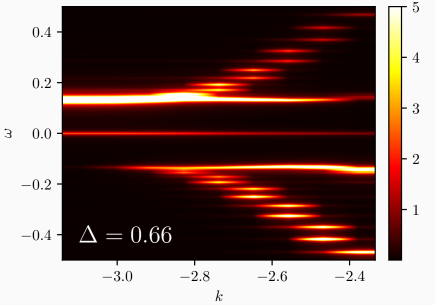

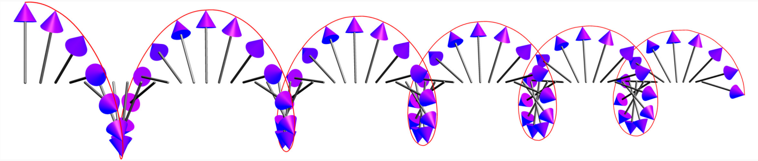

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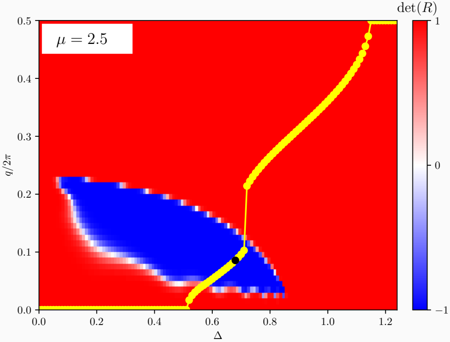

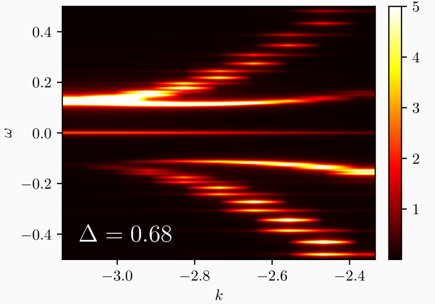

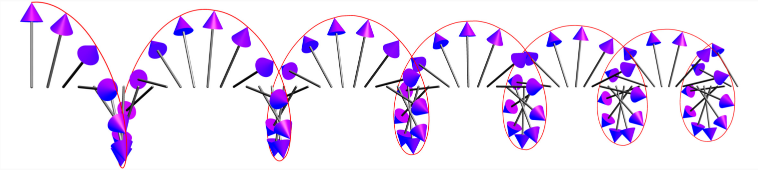

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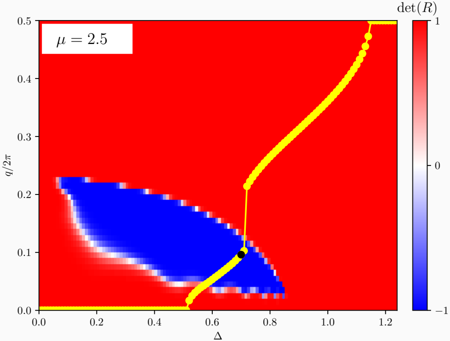

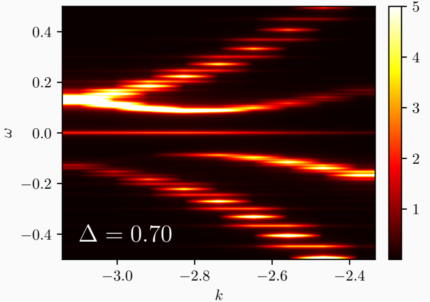

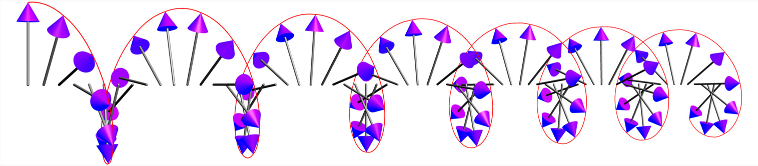

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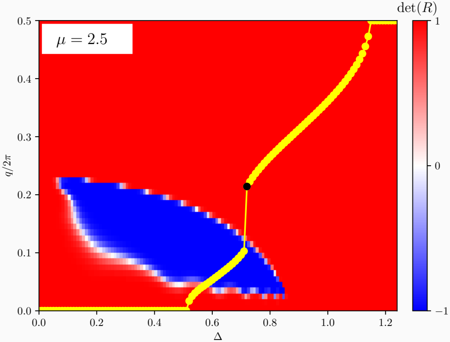
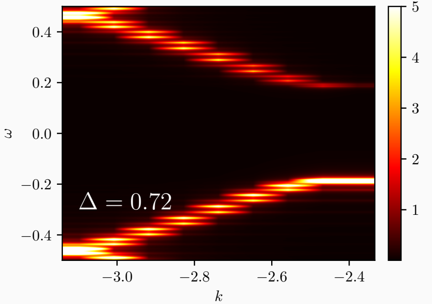
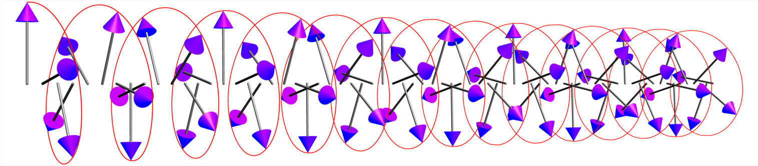


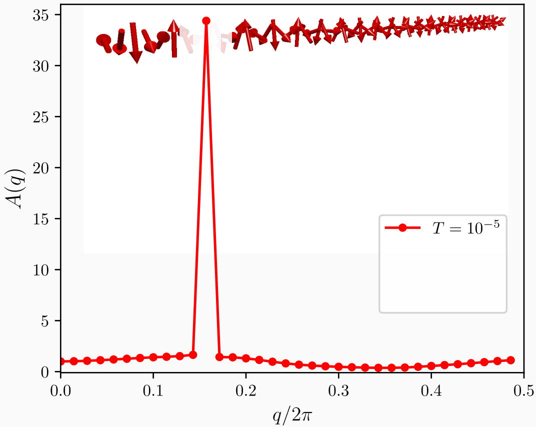




Monte Carlo results for $T \neq 0$

Structure factor:

$$A(q) = \frac{1}{L} \sum_{jk} e^{iq(j-k)} \langle S_j \cdot S_k \rangle$$



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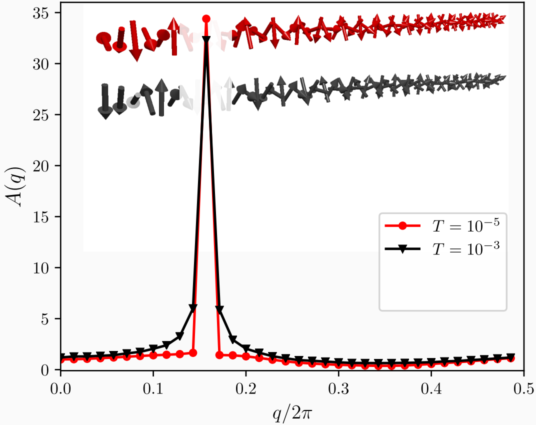

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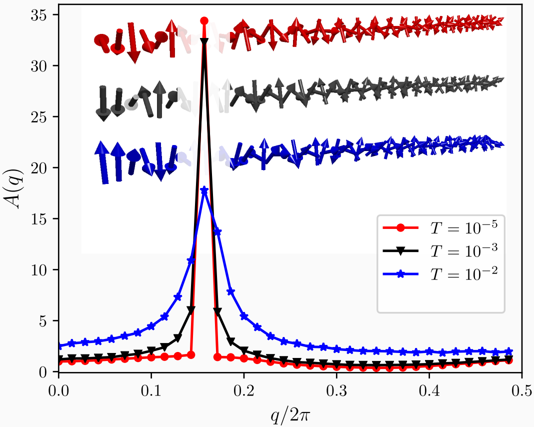

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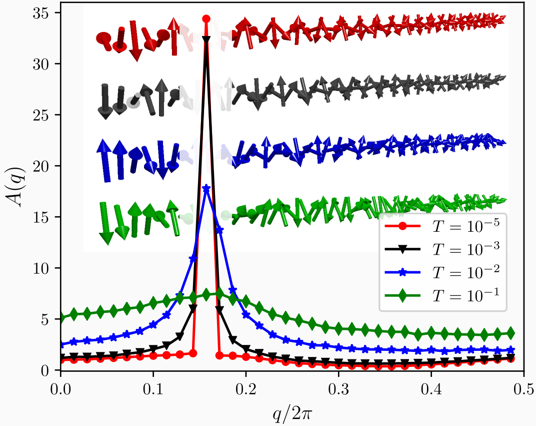
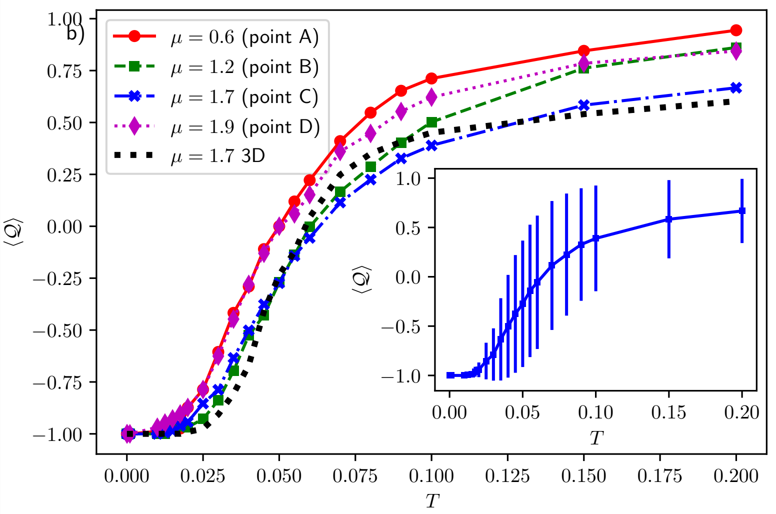
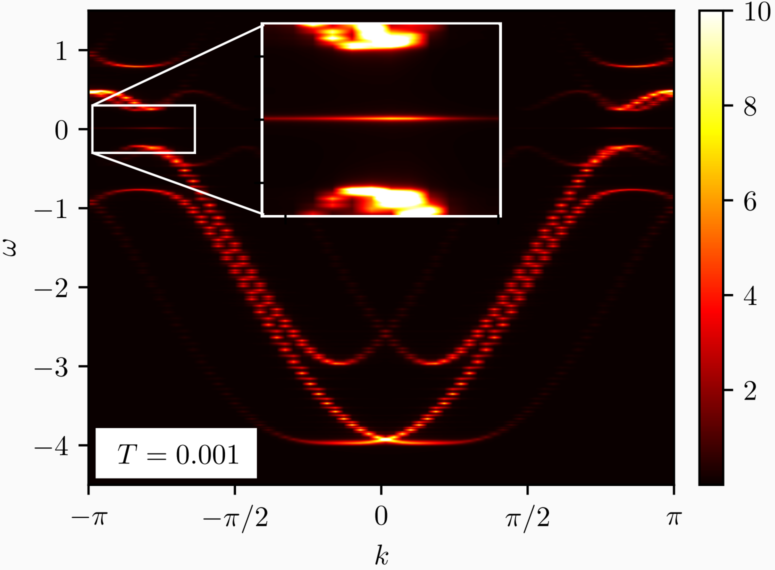


INFLUENCE OF TEMPERATURE ON TOPOLOGY



INFLUENCE OF TEMPERATURE ON MAJORANA QPS



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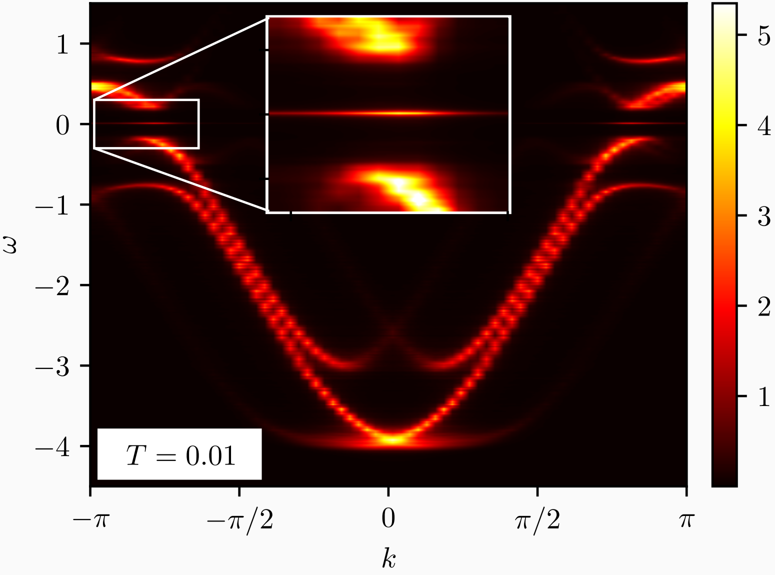

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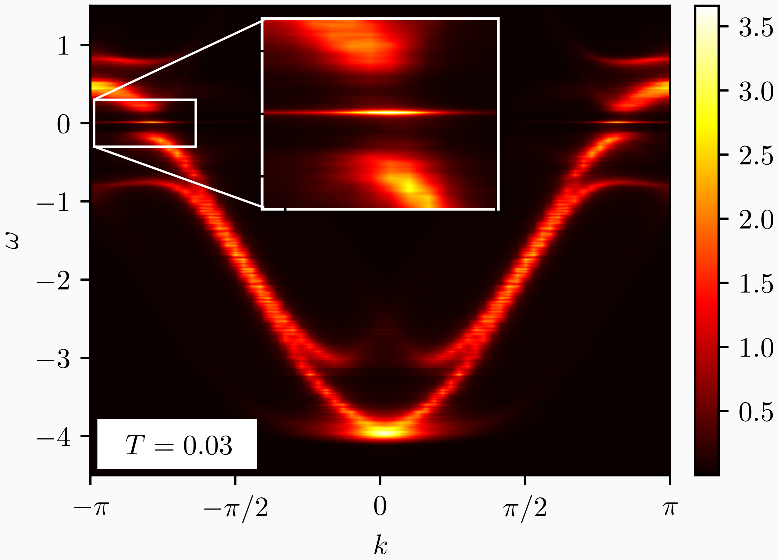

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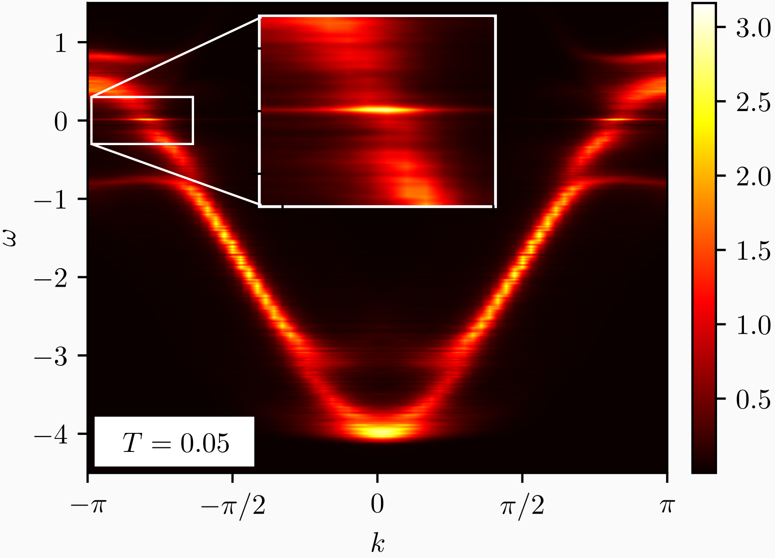

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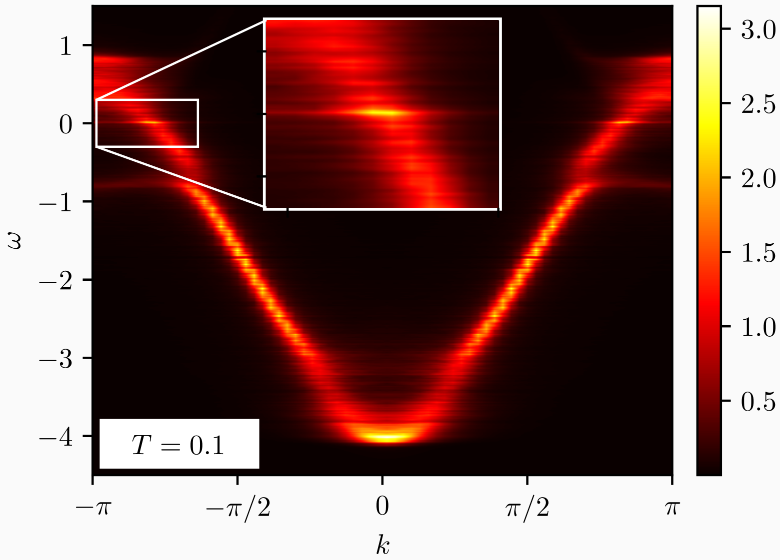

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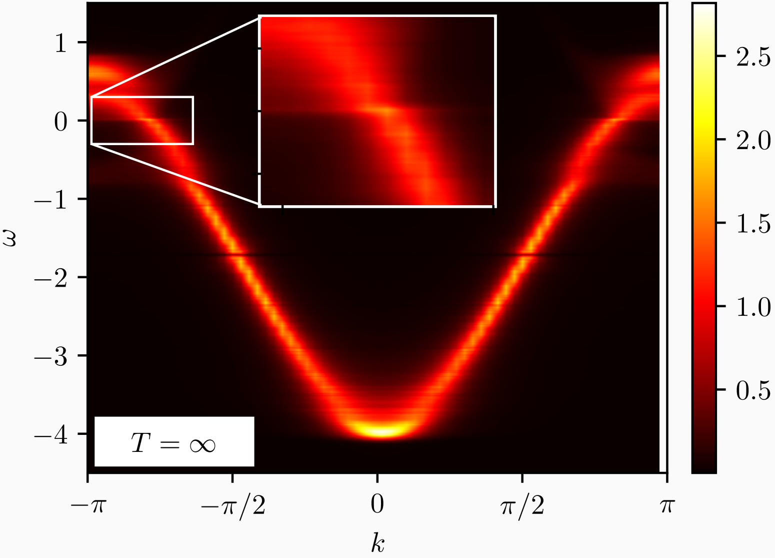

THERMAL EFFECTS: CONCLUSIONS

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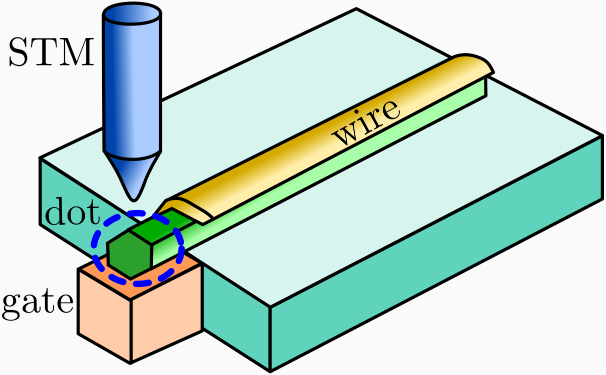
In realistic systems such critical temperature:

$\Longrightarrow$ $T_c \approx$ **5 K**

Trivial vs Majorana qps

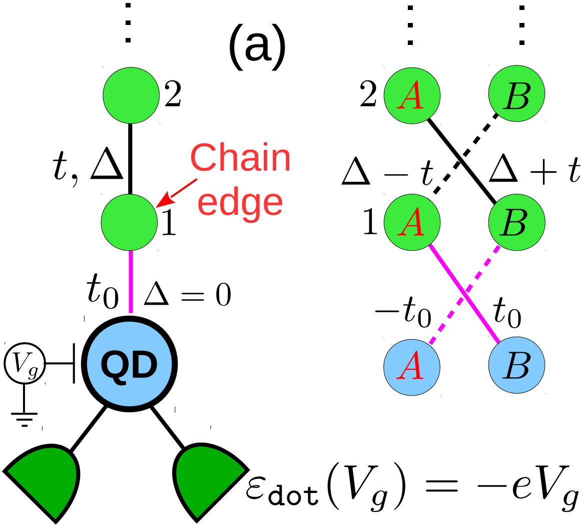
TRIVIAL VS MAJORANA BOUND STATES

Schematics of a quantum dot – nanowire hybrid structure.



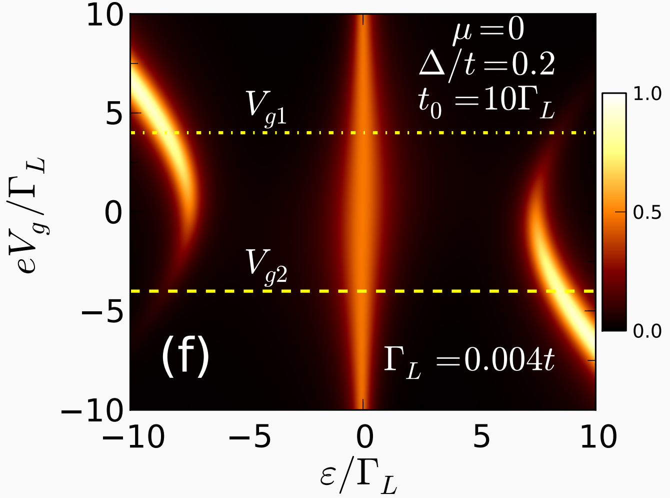
A. Ptok, A. Kobiałka & T. Domański, Phys. Rev. 96, 195403 (2017).

KITAEV CHAIN + NORMAL SITE



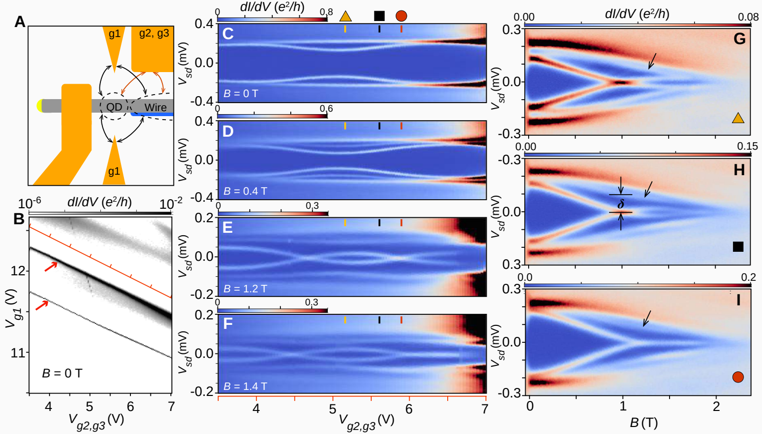
E. Vernek et al., Phys. Rev. B 89, 165314 (2014).

Subtle leakage of a Majorana mode into a quantum dot



E. Vernek et al., Phys. Rev. B 89, 165314 (2014).

‘Coalescence’ of the Andreev into Majorana qps

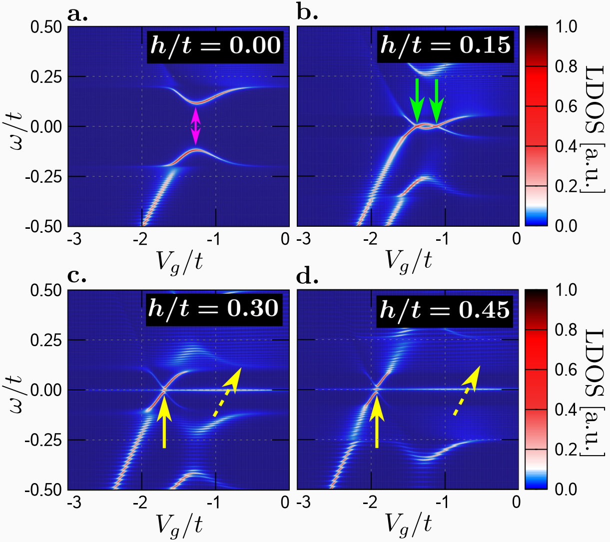


M.T. Deng, ..., and Ch. Marcus, Science 354, 1557 (2016).

/ Niels Bohr Institute, Copenhagen, Denmark /

DISTINGUISHING ANDREEV FROM MAJORANA QPS

QD spectrum vs gate potential V_g for several magnetic fields h.



A. Ptok, A. Kobiałka & T. Domański, Phys. Rev. 96, 195403 (2017).

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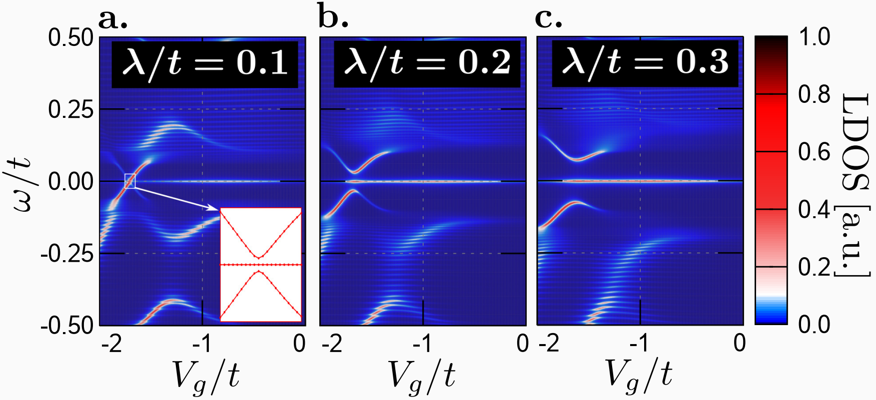
QD spectrum vs gate potential V_g for various spin-orbit couplings λ.



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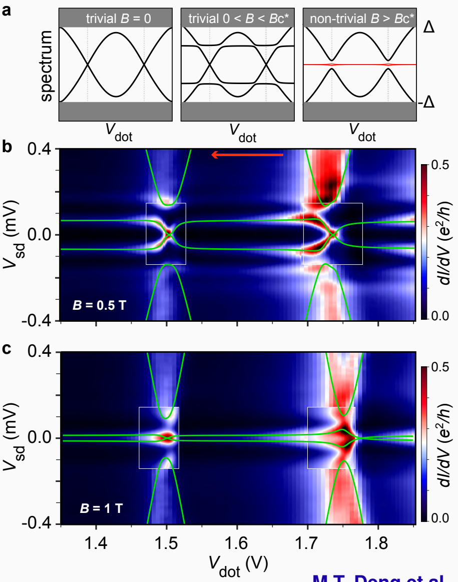

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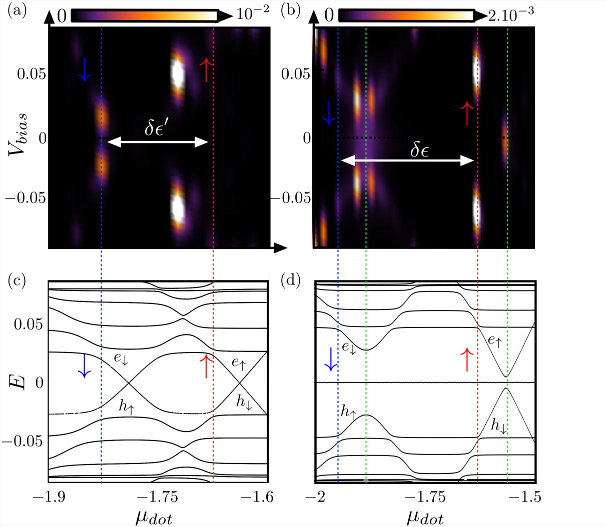

D. Chevallier, ... and J. Klinovaja, Phys. Rev. B 97, 04504 (2018).

ANDREEV VS MAJORANA: CONCLUSIONS

- **Low energy features are very distinct:**

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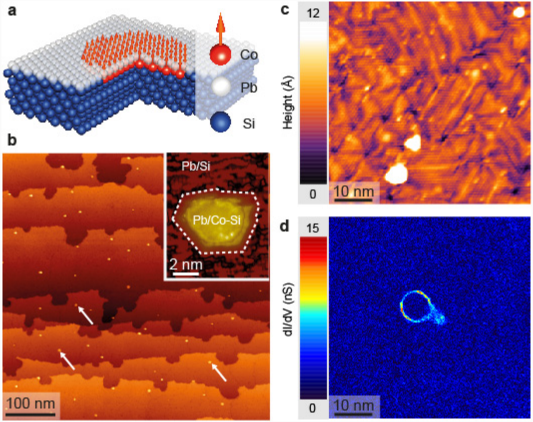
- **Misinterpretation:**

$\Rightarrow$ **coalescence of Andreev into Majorna qps**

Edge modes in dim=2 systems

TWO-DIMENSIONAL MAGNETIC STRUCTURES

Magnetic island of Co atoms deposited on the superconducting Pb surface



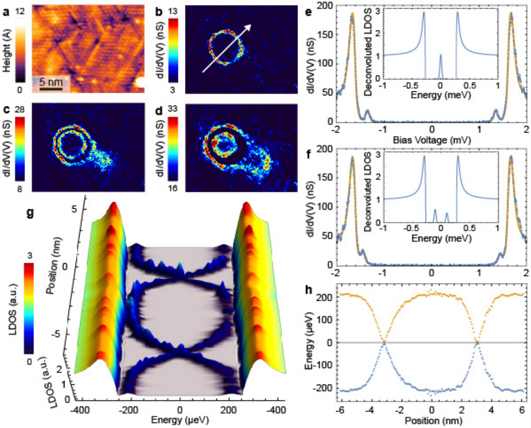
Diameter of island:

5 – 10 nm

G. Ménard, ..., and P. Simon, Nature Commun. **8**, 2040 (2017).

/ P. & M. Curie University (Paris, France) /

Majorana modes propagating along magnetic islands



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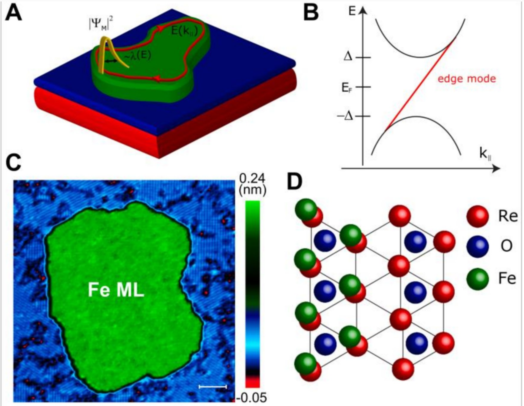
PROPAGATING MAJORANA EDGE MODES

Magnetic island of Fe atoms deposited on the superconducting Re surface



Chern number:

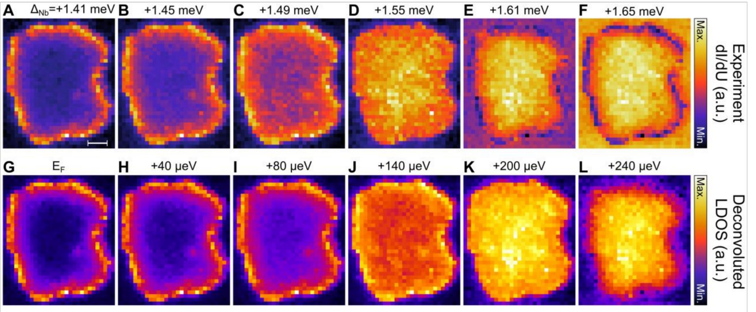
C = 20

A. Palacio-Morales, ..., and R. Wiesendanger, arXiv:1809.04503 (preprint).

/ University of Hamburg (Germany) /

PROPAGATING MAJORANA EDGE MODES

Real space maps of the tunneling conductance (top panel) and deconvoluted DOS (bottom panel) obtained for various energies (as indicated) in the subgap regime ($\Delta = 240\mu eV$).



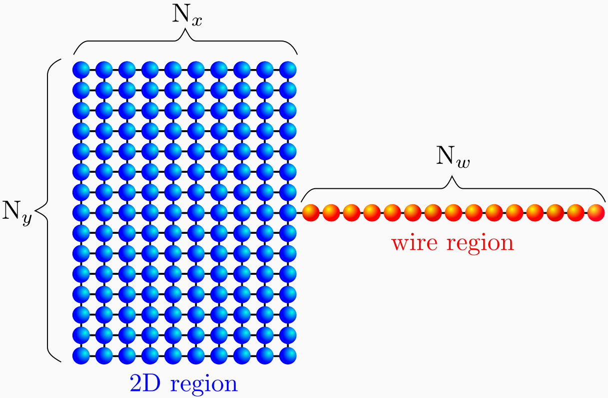
A. Palacio-Morales, ..., and R. Wiesendanger, arXiv:1809.04503 (preprint).

/ University of Hamburg (Germany) /

Mixed – dimensionality structures

CAN MAJORANA QPS BE DECONFINED ?

Our project: **Majorana qps of the 1D–2D hybrid structure**



A. Kobiałka, T. Domański & A. Ptok, arXiv:1808.05281

TOPOLOGICAL INVARIANTS

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dim=1 $\Rightarrow$ **homotopy group Z_2**

featured by the Berry phase ± 1 around the Brillouin zone

TOPOLOGICAL INVARIANTS

Constituents of this hybrid-system belong to different homotopy groups:

dim=1 $\Rightarrow$ **homotopy group** Z_2

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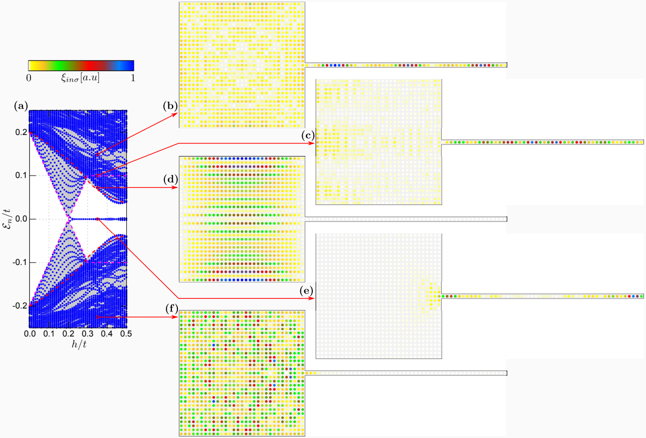
For details, concerning the topological criteria see e.g.

- **A. Kitaev, AIP Conf. Proc. 1134, 22 (2009);**
- **M.Z. Hasan & C.L. Kane, Rev. Mod. Phys. 82, 3045 (2010);**
- **X.-L. Qi & S.-C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).**

TRIVIAL VS MAJORANA MODES

Majorana/Andreev quasiparticles of a wire-plaquette hybrid



plaquette: nontopological ***nanowire: topological***

A. Kobiałka, T. Domański & A. Ptok, arXiv:1808.05281

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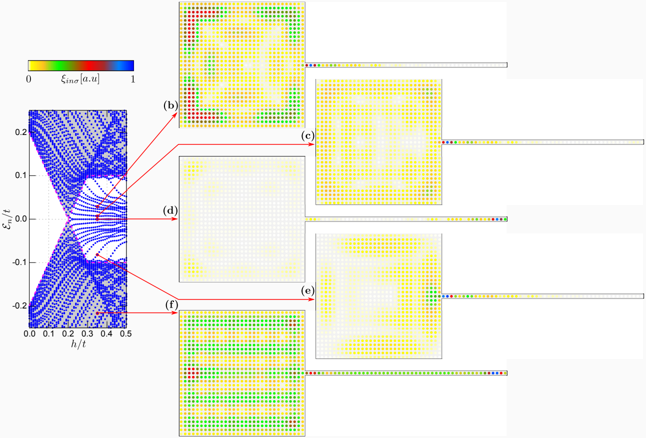
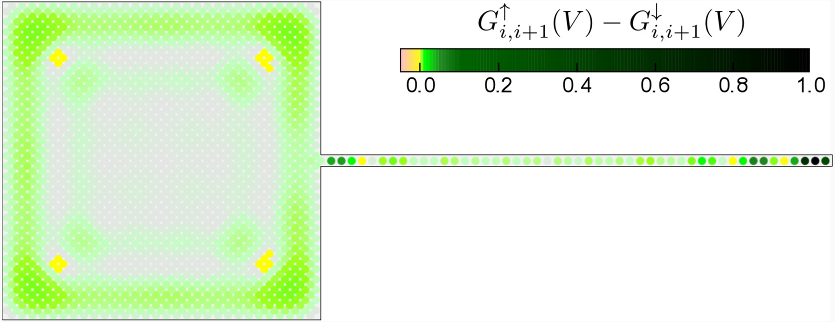


Both regions are assumed to be in topological sc phase.

A. Kobiałka, T. Domański & A. Ptok, arXiv:1808.05281

HOW TO DETECT (DE)LOCALIZED MAJORANA QPS

Maps of the SESAR tunneling conductance at zero-bias.



SESAR = Selective Equal Spin Andreev Reflection

A. Kobiałka, T. Domański & A. Ptok, arXiv:1808.05281

DIMENSIONAL HYBRID: CONCLUSION

Plaquette-nanowire hybrid structures enables:

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Plaquette-nanowire hybrid structures enables:

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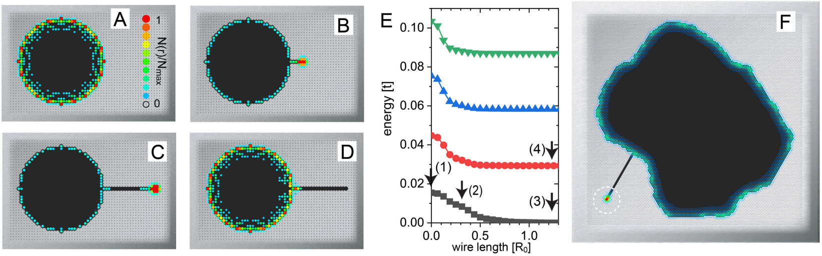
Is this edge mode itinerant ?

Further outlook

ISLAND + NONOWIRE

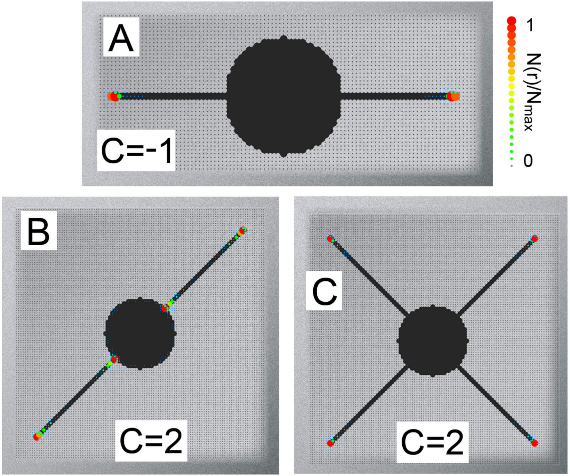
Itinerant Majorana mode leaking into side-attached nanowire.



E. Mascot, S. Cocklin, S. Rachel, and D.K. Morr, arXiv:1811.06664

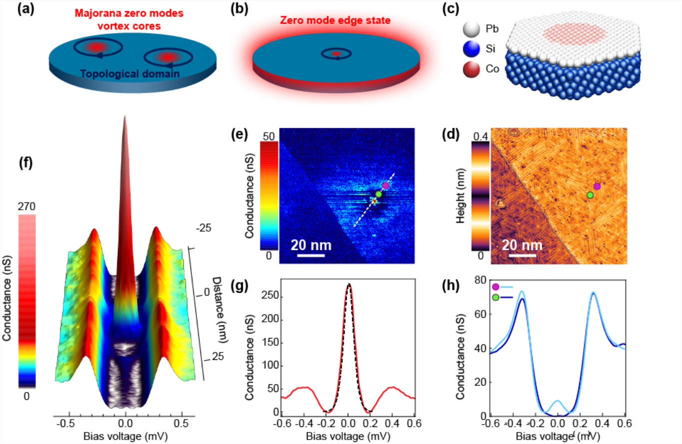
Univ. of Illinois at Chicago (USA)

Majorana modes leaking to the side-attached nanowires.



DEFECTS IN MAGNETIC ISLAND

Localized Majorana at point-like defect, coexisting with itinerant Majorana edge mode (observed in Co-Si island on disordered Pb)



G.C. Ménard, ..., P. Simon and T. Cren, arXiv:1810.09541

Paris (France)

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- **Majorana quasiparticles**

$\Longrightarrow$ **A. Kobiałka (Lublin), A. Ptok (Kraków),**
M. Maśka & A. Gorczyca-Goraj (Katowice)

- **Shiba states/bands in topological phases**

$\Longrightarrow$ **Sz. Głodzik (Lublin)**

- **Subgap Kondo effect**

$\Longrightarrow$ **I. Weymann & K. Wójcik (Poznań), G. Górski (Rzeszów),**
T. Novotný, M. Žonda & V. Janiš (Prague),
M. Barańska & J. Barański (Dęblin).

- **Dynamics of in-gap states**

$\Longrightarrow$ **R. Taranko, B. Baran & T. Kwapiński (Lublin)**

- **Nonlocal Andreev processes**

$\Longrightarrow$ **K.I. Wysokiński (Lublin), G. Michałek & B.R. Bułka (Poznań)**

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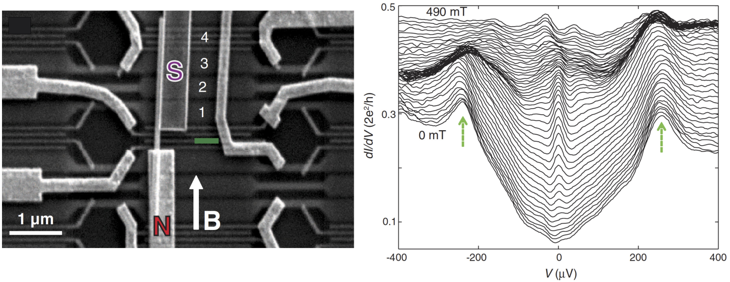
7-10 April 2019, M. Planck Inst. (Dresden, Germany)

EMPIRICAL REALIZATION: EXAMPLE # 1

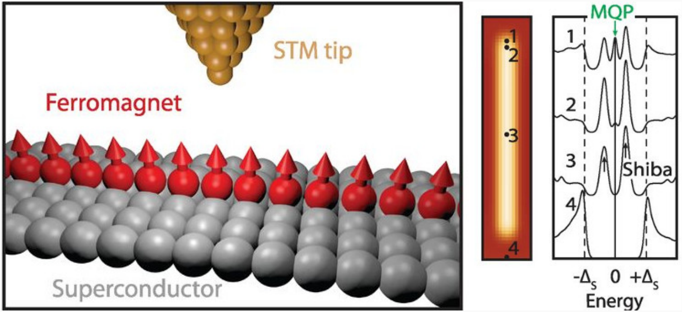
Differential conductance dI/dV obtained for InSb nanowire at 70 mK upon varying a magnetic field.



V. Mourik, ..., and L.P. Kouwenhoven, Science **336**, 1003 (2012).

/ Technical Univ. Delft, Netherlands /

STM measurements for the nanochain of Fe atoms self-organized on a surface of superconducting Pb.

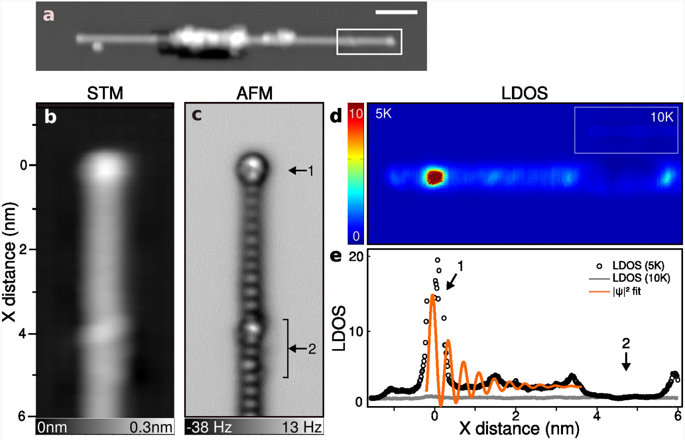


S. Nadj-Perge, ..., and A. Yazdani, Science **346**, 602 (2014).

/ Princeton University, USA /

EMPIRICAL REALIZATION: EXAMPLE # 3

AFM & STM data for Fe chain on Pb(110) surface

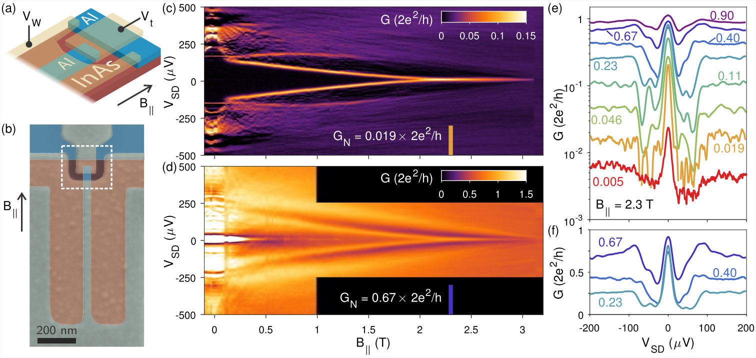


R. Pawlak, M. Kisiel *et al*, npj Quantum Information **2**, 16035 (2016).

/ University of Basel, Switzerland /

EMPIRICAL REALIZATION: EXAMPLE # 4

Results for the litographically fabricated Al nanowire



F. Nichele, ..., and Ch. Marcus, Phys. Rev. Lett. **119**, 136803 (2017).

/ Niels Bohr Institute, Copenhagen, Denmark /